Title: Gravitational Waves Experiments 2

Date: Jul 20, 2018 09:00 AM

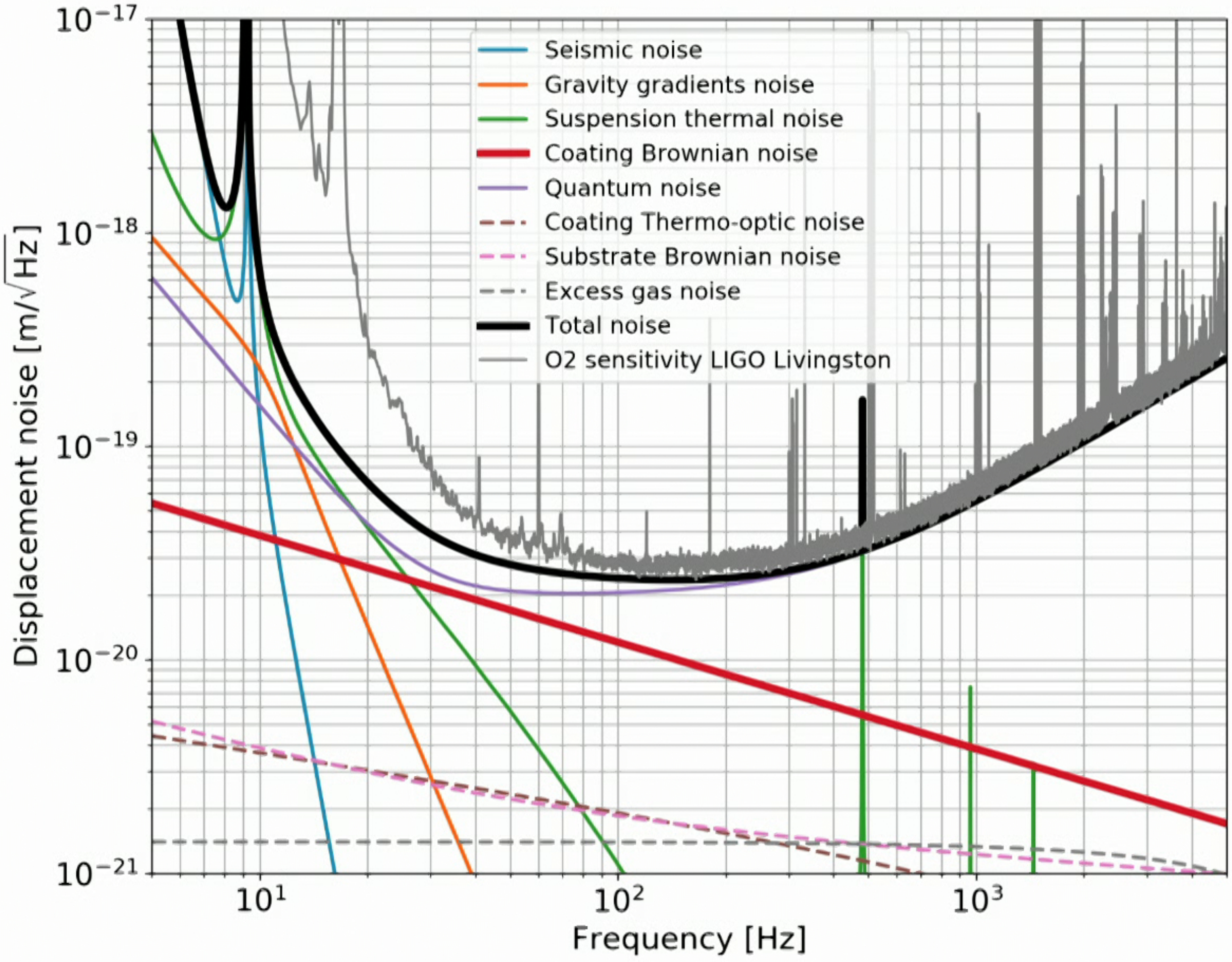
URL: http://pirsa.org/18070009

Abstract:

Outline

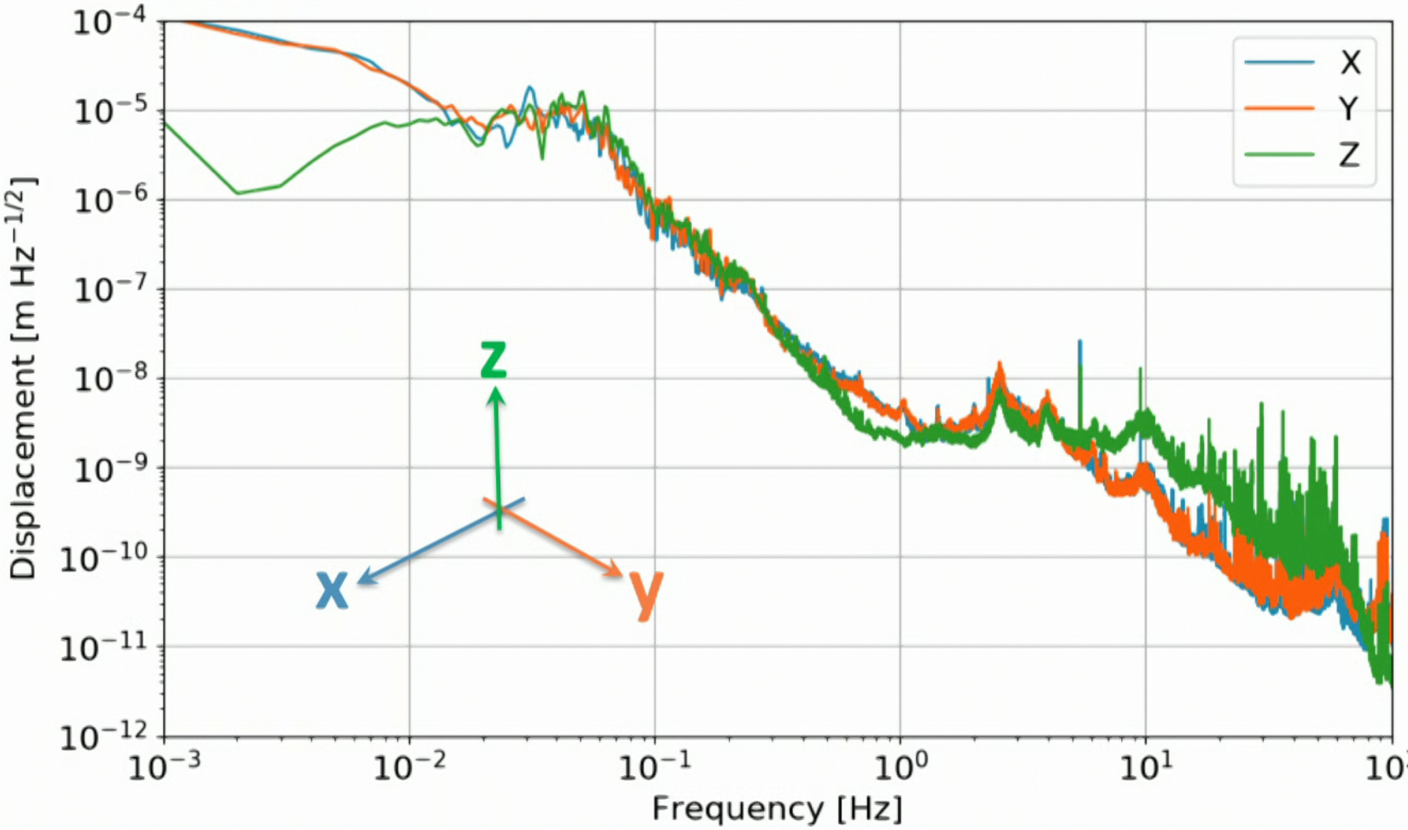
- **Lecture 1**: the basics of interferometric detection of gravitational waves
- **Lecture 2**: fundamental noise sources (seismic, thermal and quantum noises)
- **Lecture 3**: the dirty reality of ”technical noises” (scattered light, control noise, etc…) and prospects for the future

Advanced LIGO design

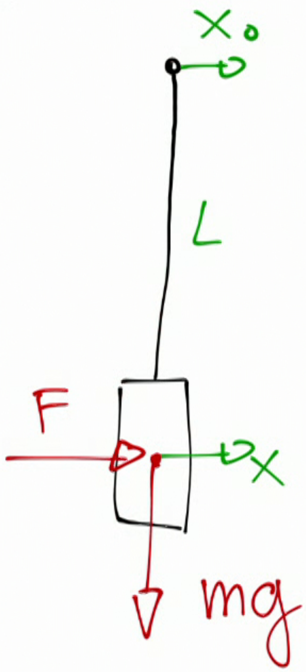


Seismic noise

- Ground motion is way too large!

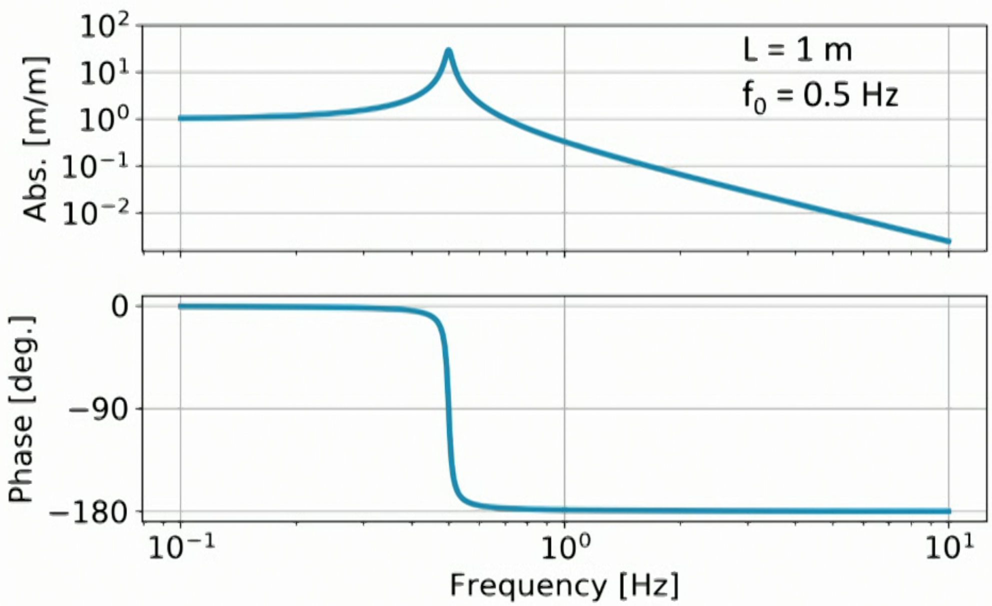


Passive isolation

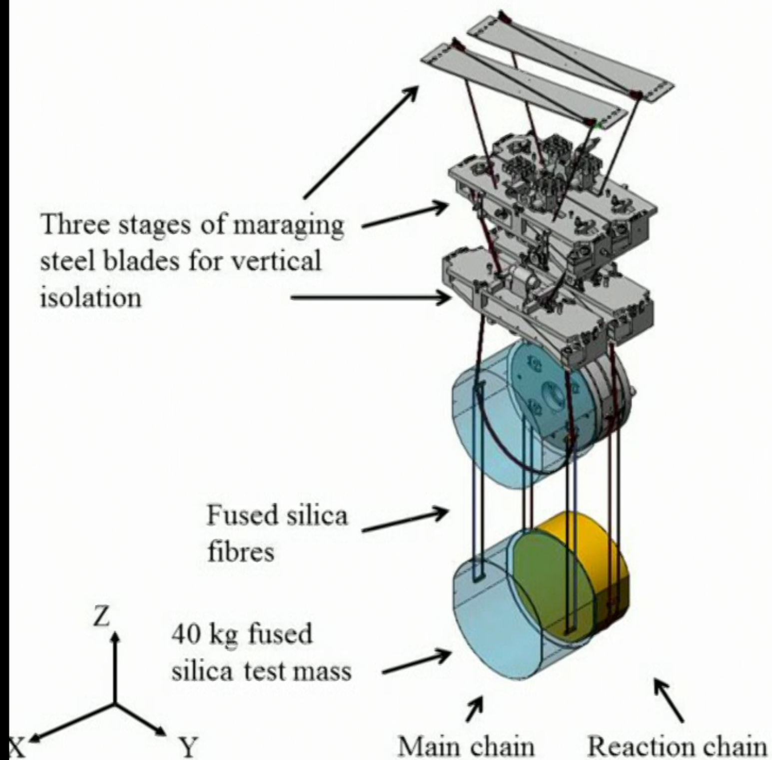


$$m\ddot{x} = -\frac{mg}{L}(x - x_0) - \gamma\dot{x} + F$$

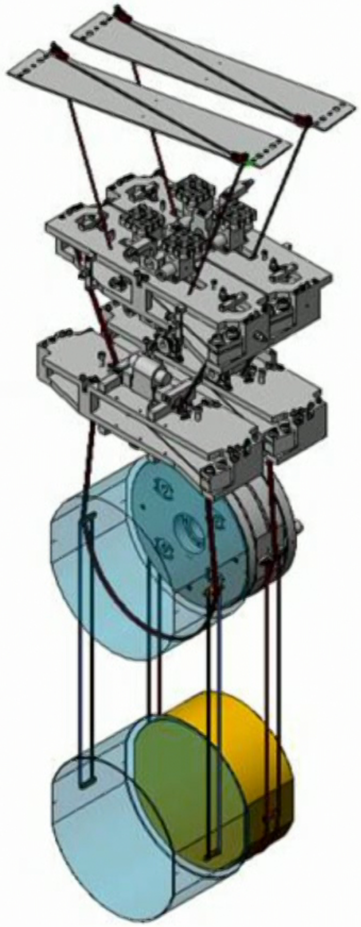
$$\omega_0^2 = \frac{g}{L} \qquad \tilde{x}(\omega) = \frac{\omega_0^2\tilde{x}_0(\omega) + \tilde{F}(\omega)/m}{(\omega_0^2 - \omega^2) + i\omega\frac{\gamma}{m}}$$

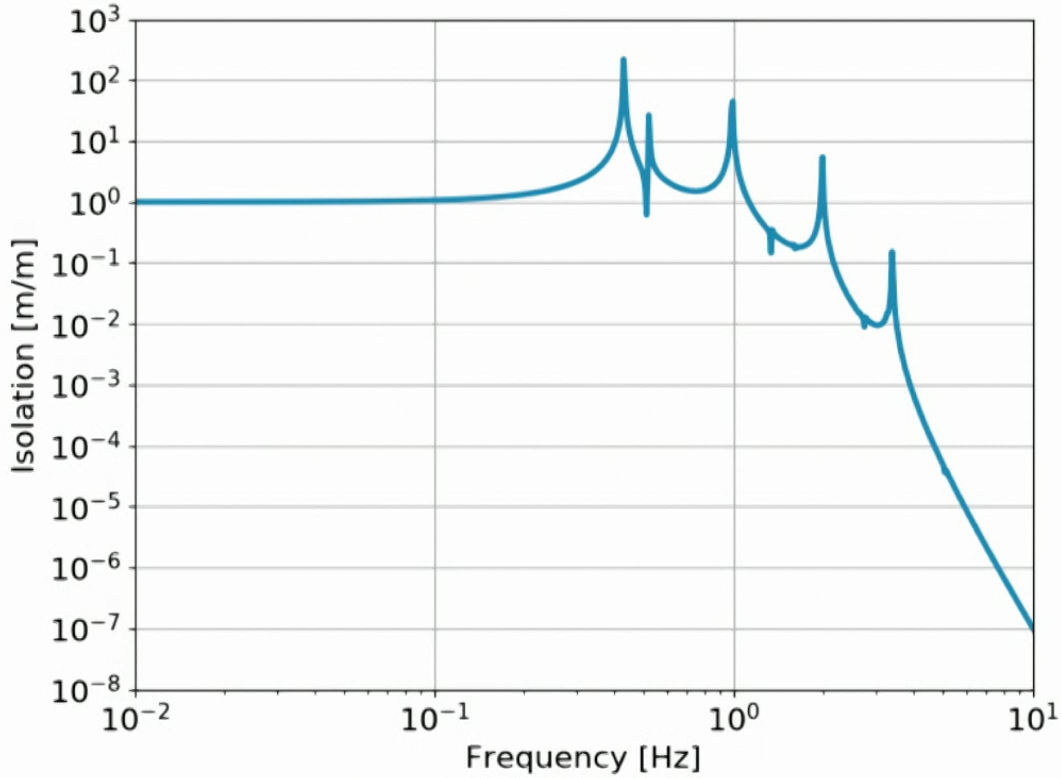


Advanced LIGO: quadruple suspension



Advanced LIGO: quadruple suspension

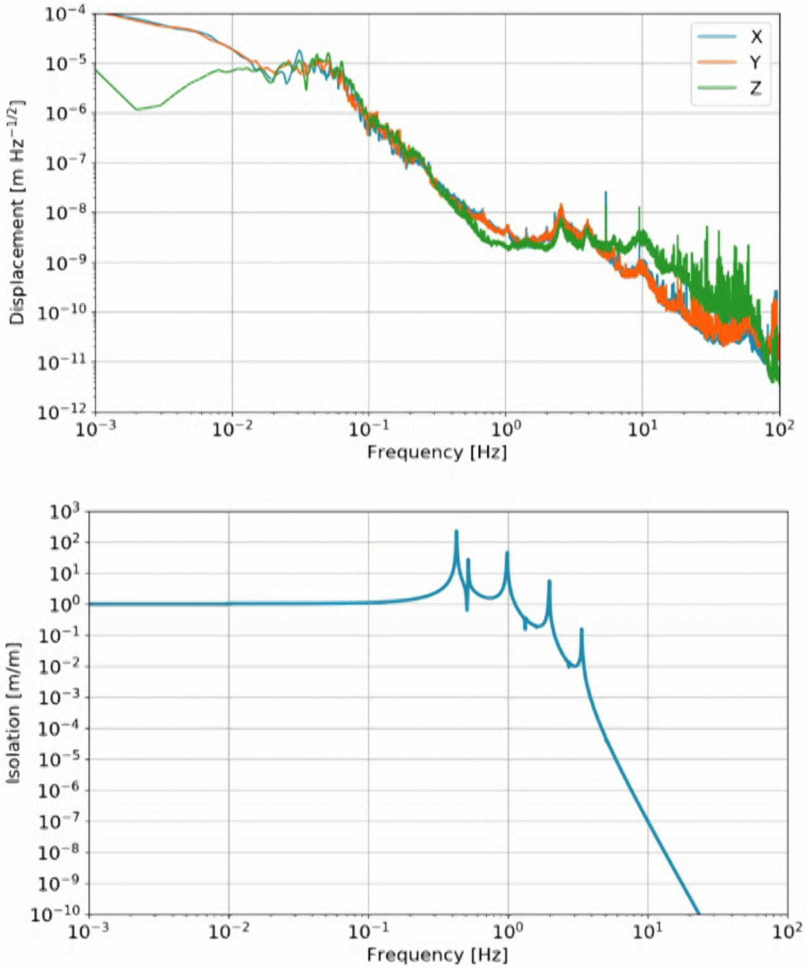




S M Aston *et al* 2012 *Class. Quantum Grav.* **29** 235004

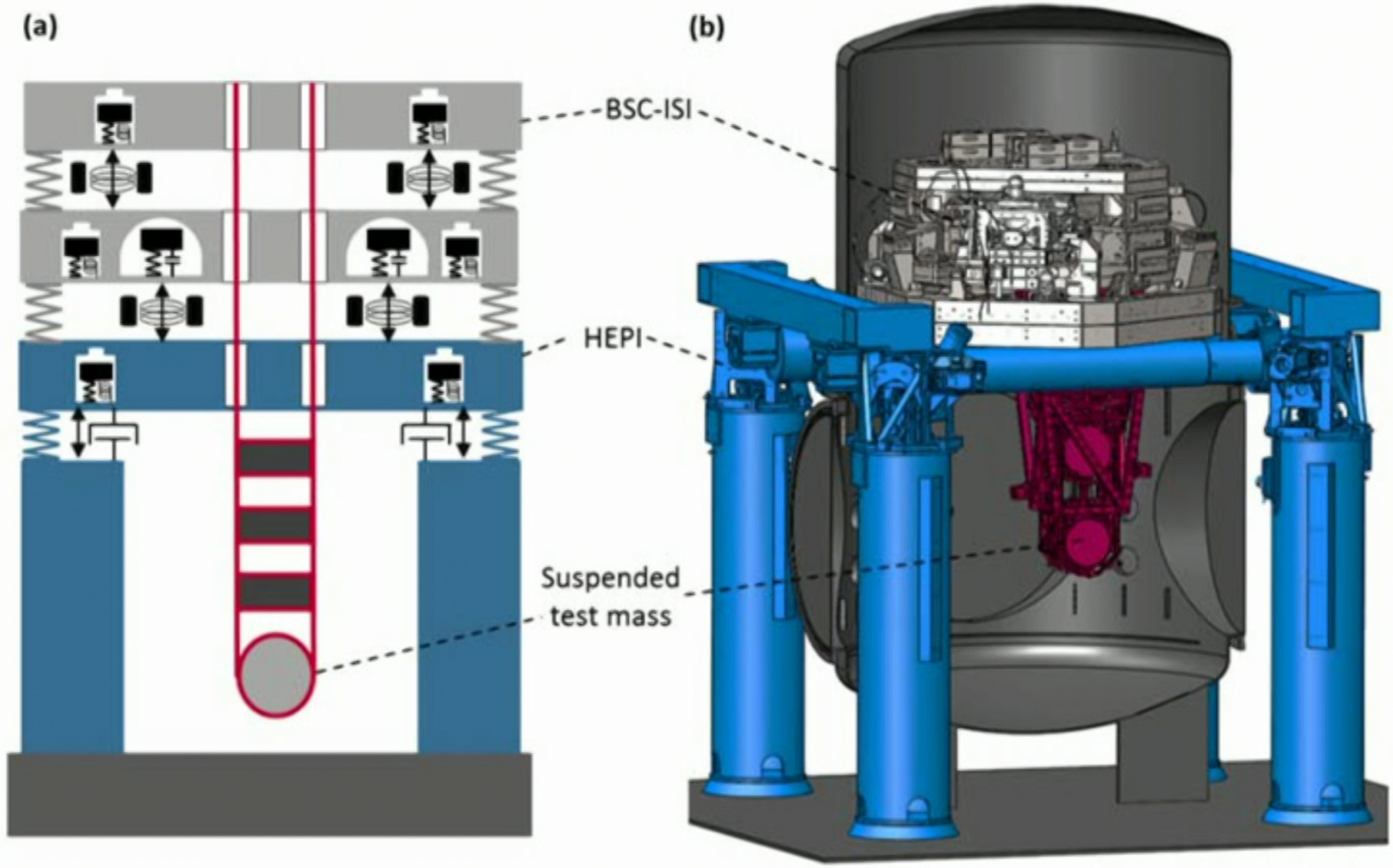
LIGO

But...



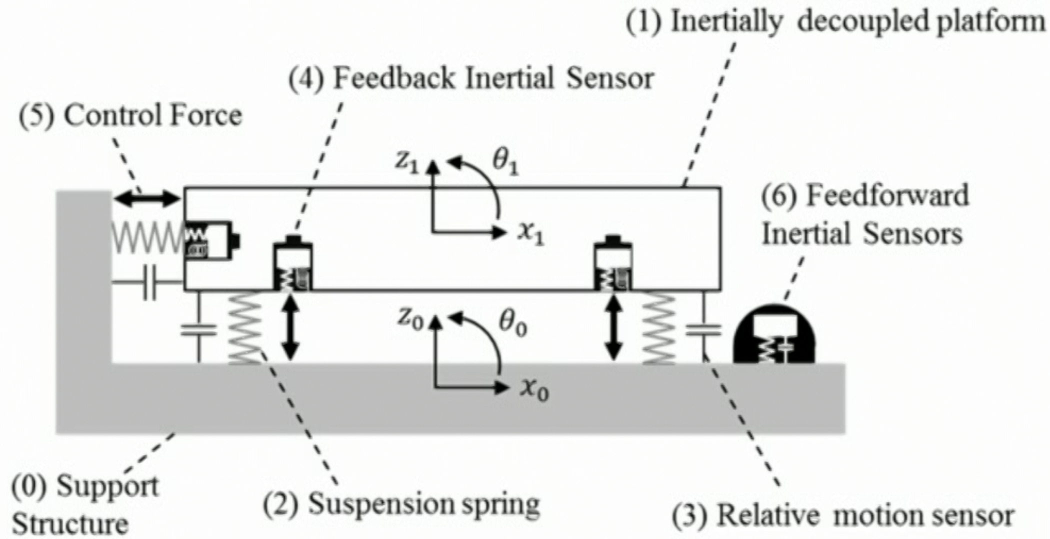
X
Y
Z
Displacement [m Hz$^{-1/2}$]
Frequency [Hz]
Isolation [m/m]
Frequency [Hz]

Active isolation



F Matichard *et al* 2015 *Class. Quantum Grav.* **32** 185003

Active isolation



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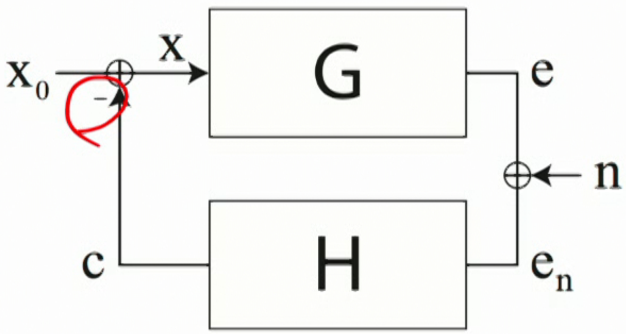
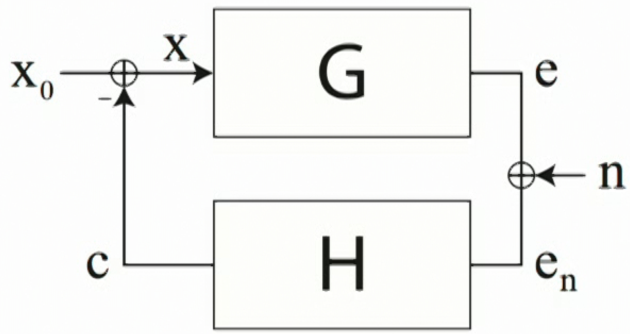
Control systems and sensor noise



$$\hat{e} = G\hat{x}$$

$$= G\left(\hat{x}_0 - H(\hat{e} + \hat{n})\right)$$

Control systems and sensor noise

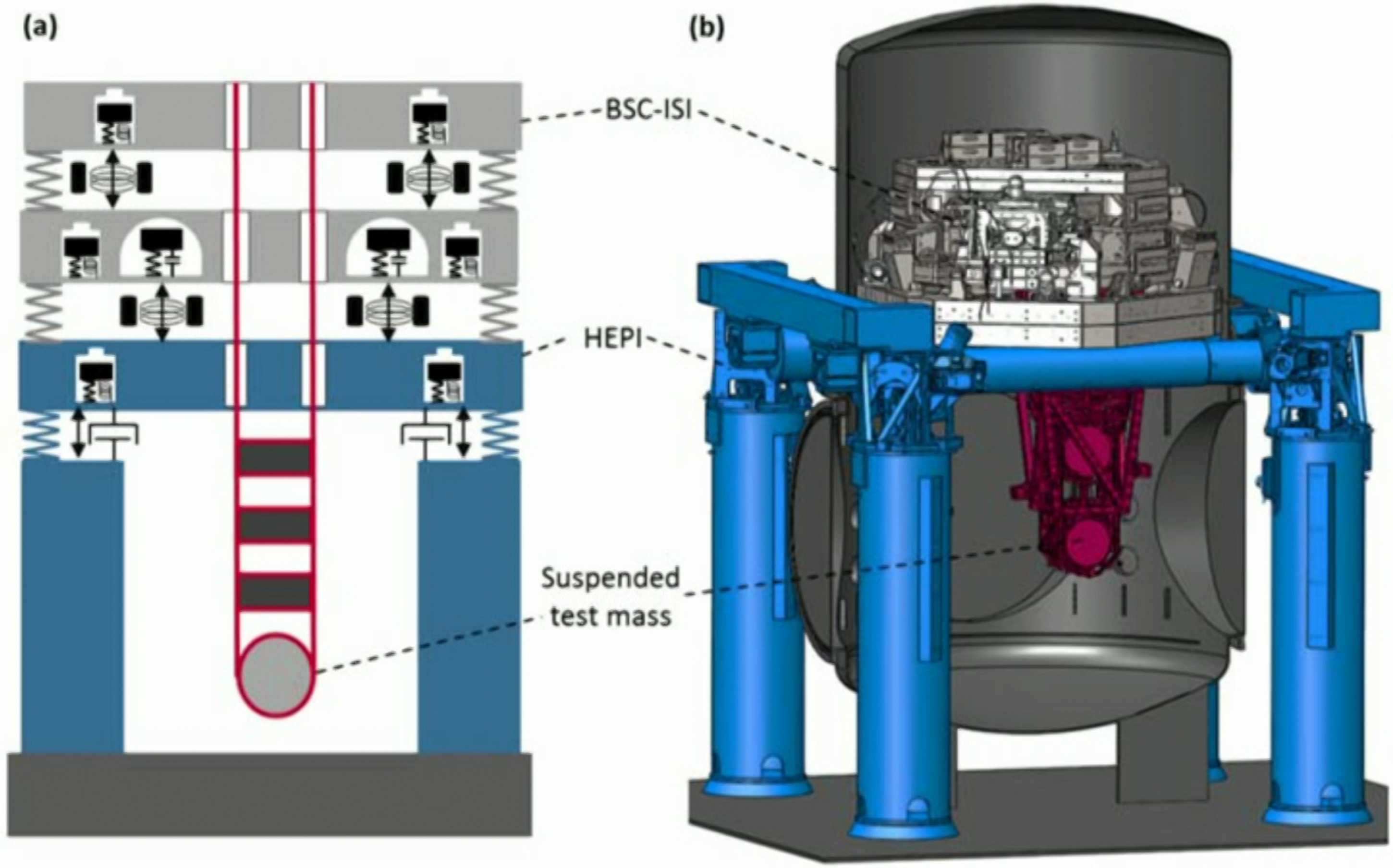


$$\hat{x} = \frac{\hat{x}_0}{1+HG} - \frac{HG}{1+HG}\left(\frac{\hat{n}}{G}\right)$$

$$\hat{e} = \hat{x}_0\frac{G}{1+HG} - \frac{HG}{1+HG}\hat{n}$$

$$\hat{e}_n = \frac{G\hat{x}_0 + \hat{n}}{1+HG}$$

Active isolation



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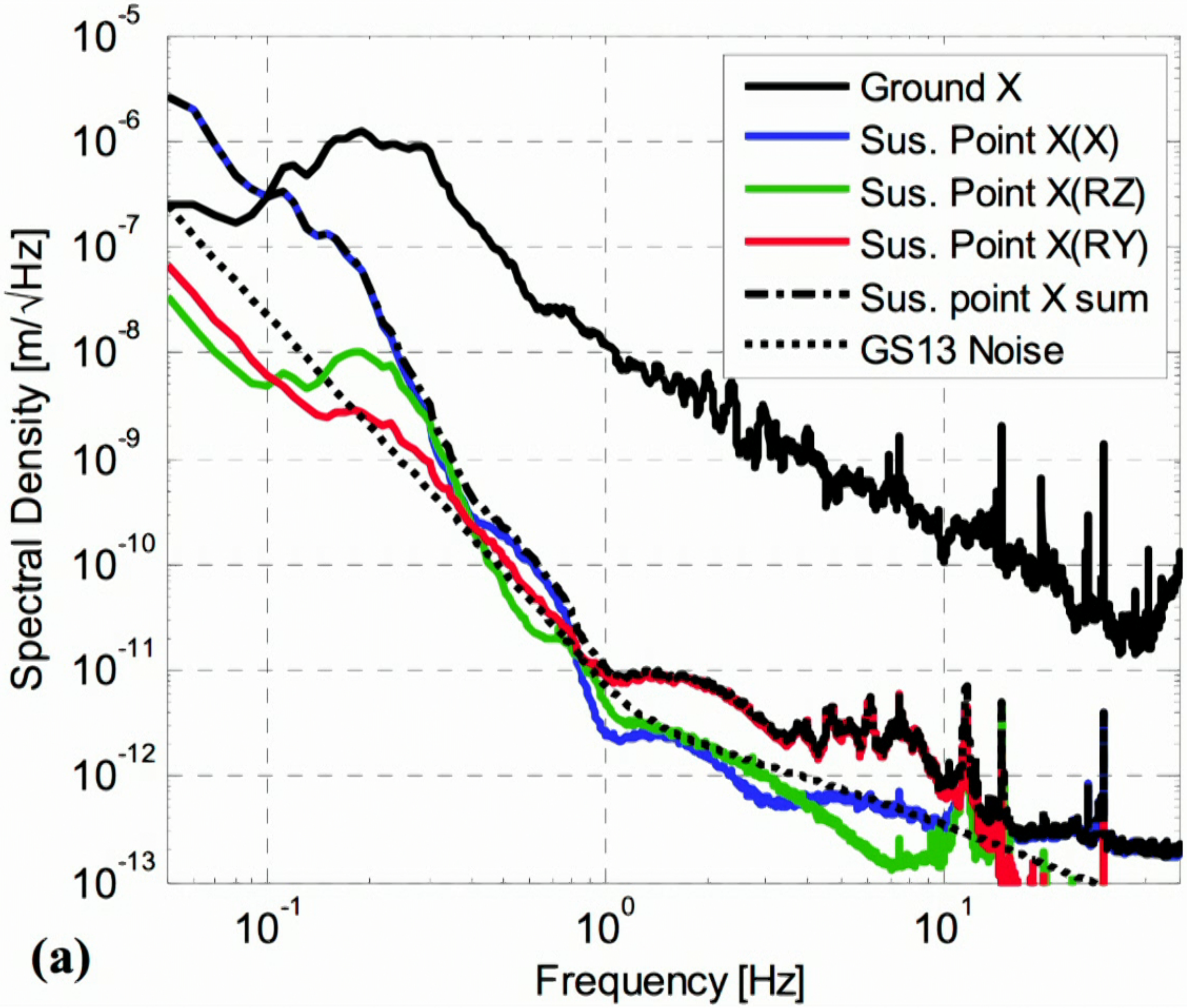
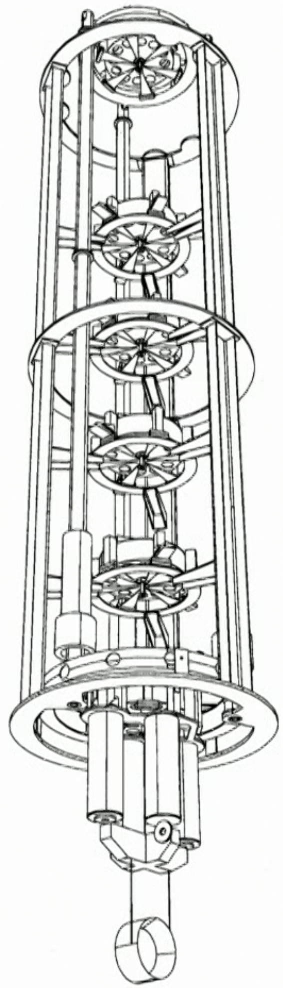
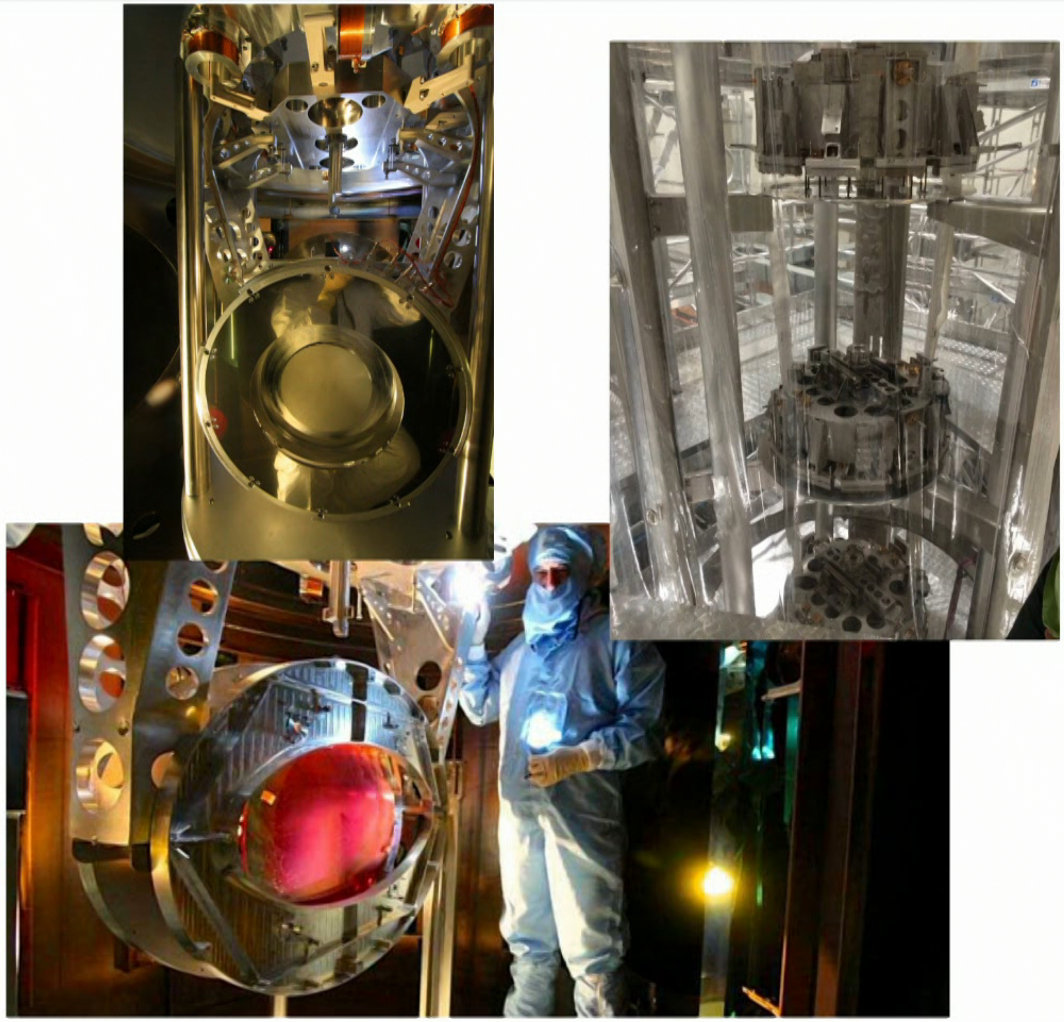
Active isolation

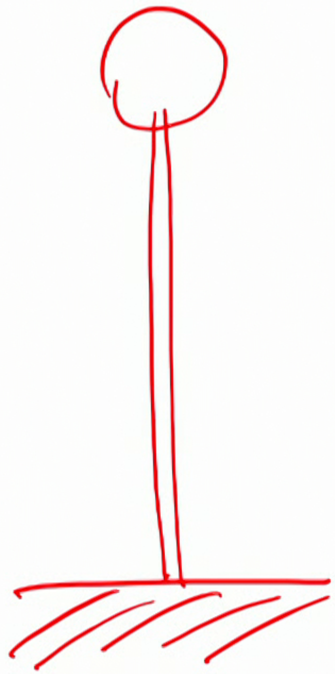


Advanced Virgo: Superattenuator

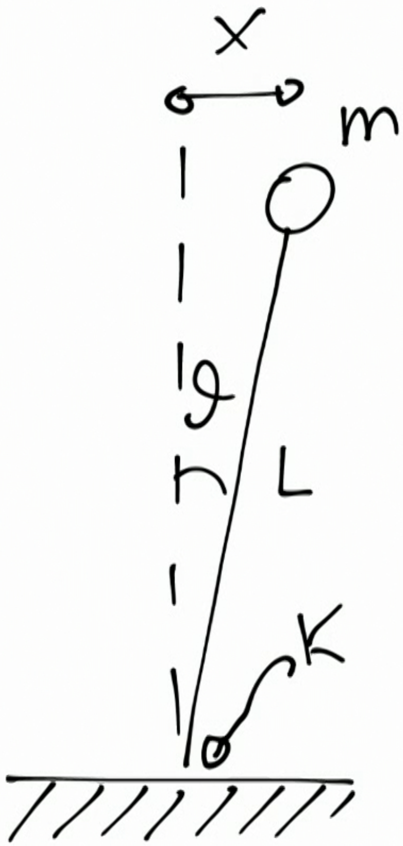




Inverted pendulum

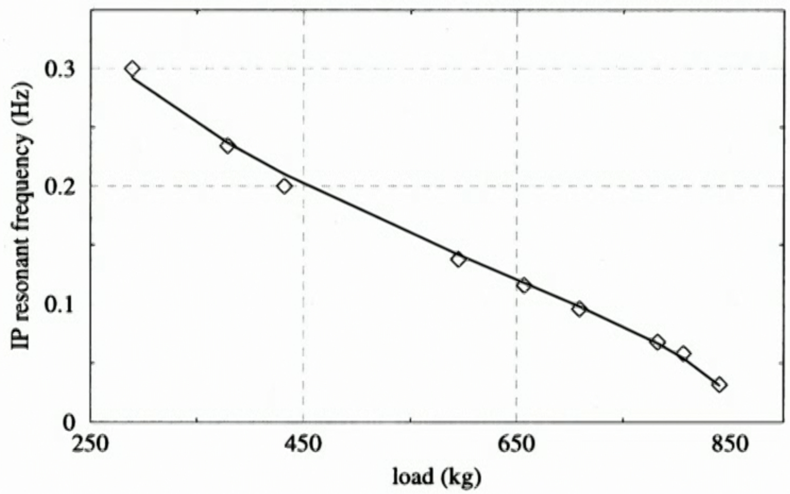


Inverted pendulum



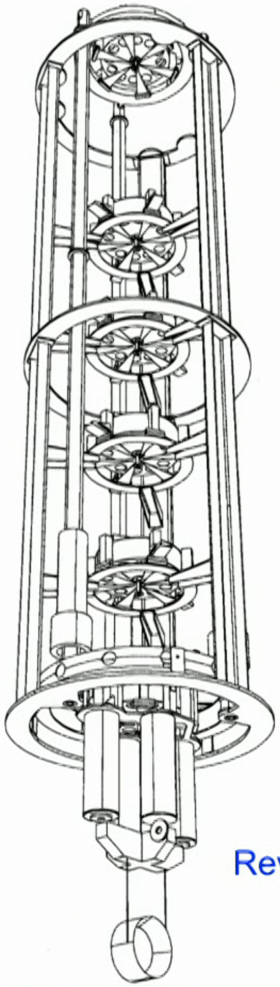
$$m\ddot{x} = \frac{mg}{L}(x - x_0) - k(x - x_0) + \ldots$$

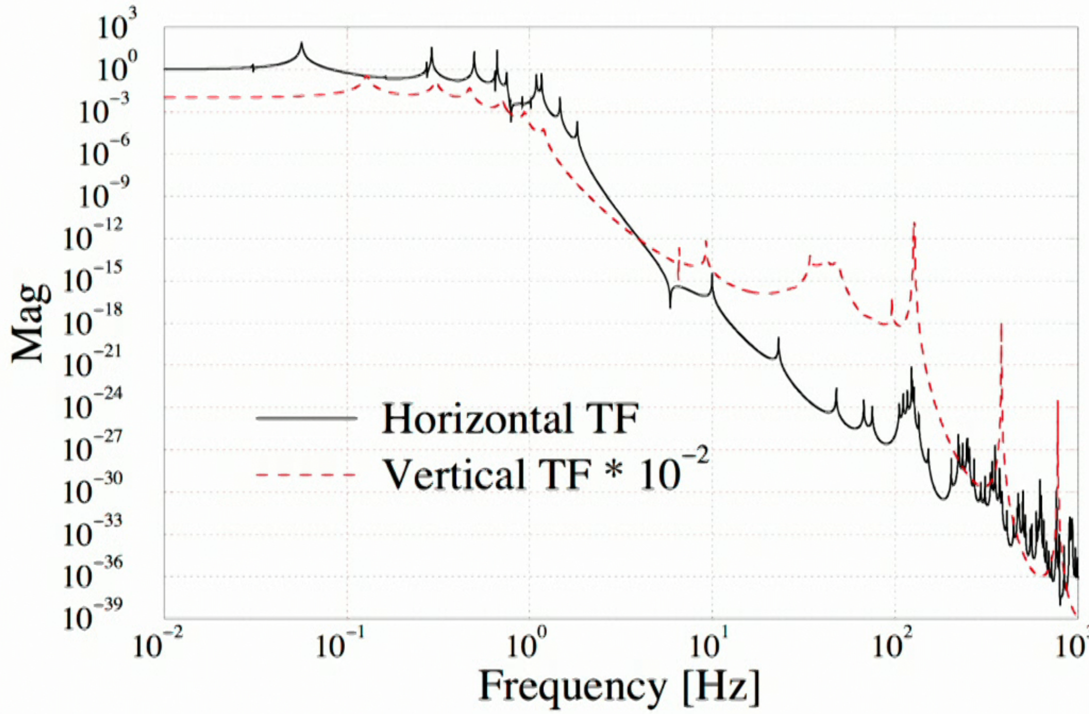
$$\omega_0^2 = \frac{k}{m} - \frac{g}{L} \quad k \approx \frac{mg}{L} = 2000\ \mathrm{N/m}$$



Review of Scientific Instruments **70**, 2507 (1999); doi: 10.1063/1.1149783

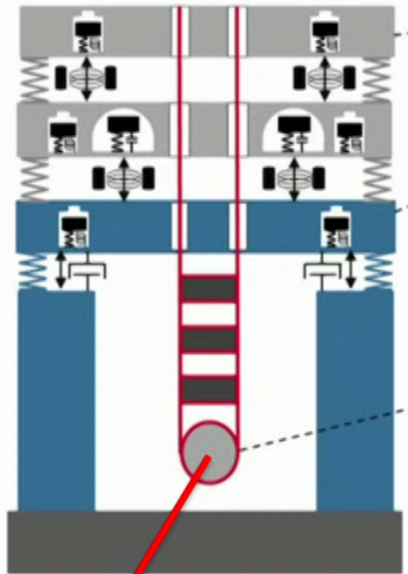
Advanced Virgo: Superattenuator

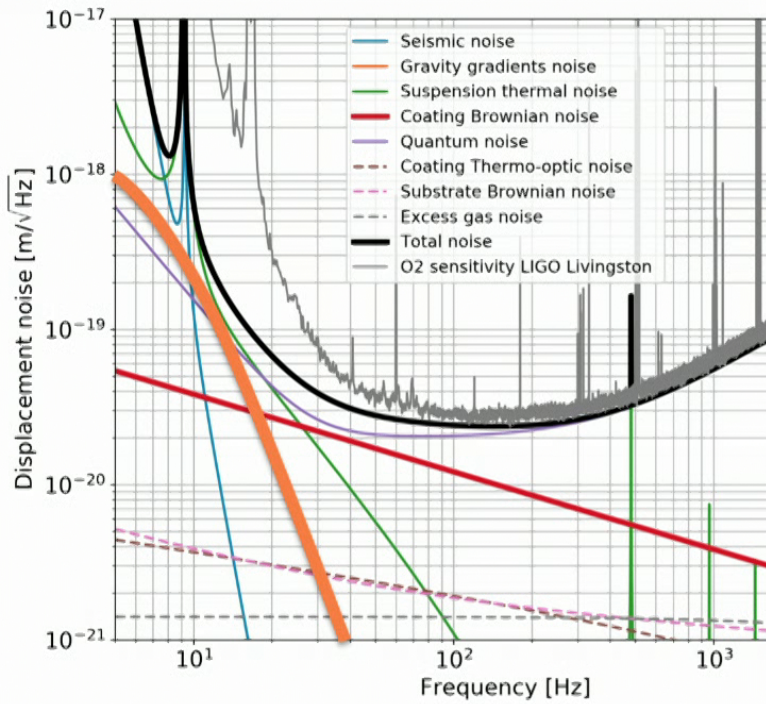


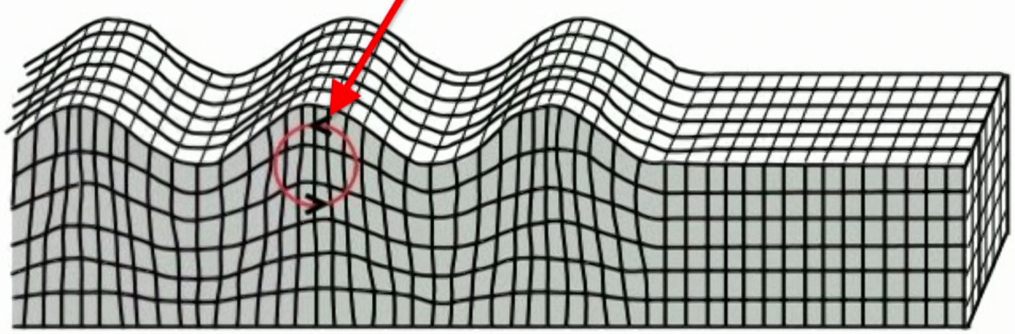


Review of Scientific Instruments **72**, 3643 (2001); doi: 10.1063/1.1392338

Newtonian noise







$$d\mathbf{F} = -G\frac{M\rho dV}{|\mathbf{r}|^2}\frac{\mathbf{r}}{|\mathbf{r}|}$$

THERMAL NOISE

Thermal noise

- All physical objects are in thermal contact with the environment: thermal equilibrium
- Two aspect of the same microscopic physics:
  - Friction: the system loses energy that goes into heat dissipated to the environment
  - Thermal noise: the environment couples back energy into the system

LIGO

Friction

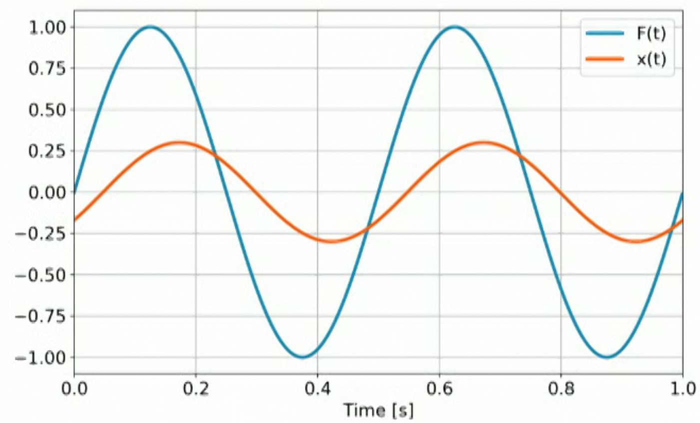
$$\tilde{x}(\omega) = \tilde{T}(\omega)\,\tilde{F}(\omega)$$

$$F(t) = F_0 \cos\omega t$$

$$\tilde{T}(\omega) = T_R(\omega) + i\,T_I(\omega)$$

Friction

$$\tilde{x}(\omega) = T(\omega)\tilde{F}(\omega)$$
$$F(t) = \tilde{F}_0 \cos \omega t$$
$$T(\omega) = T_R(\omega) + iT_I(\omega)$$
$$x(t) = F_0 \left(T_R \cos \omega t + T_I \sin \omega t\right)$$



Work done by the external force:

$$W(t) = \dot{x}(t)F(t) = \omega F_0^2 \left(T_I \cos^2 \omega t - R_R \sin \omega t \cos \omega t\right)$$

$$\langle W \rangle = \frac{1}{T}\int_0^T \dot{x}(t)F(t)\, dt \qquad \langle W \rangle = \frac{\omega T_I(\omega)}{2} F_0^2$$

Thermal motion

- Langevin's equation

$$m\ddot{x} + kx + \gamma\dot{x} = F$$

$$\langle F(t)F(t')\rangle = F_0^2\delta(t-t')$$

$$\langle F\dot{x}\rangle = \frac{2\gamma}{m}\langle E_K\rangle$$

$$m\langle F(t)\dot{x}(t)\rangle = F_0^2$$

$$F_0^2 = 2\gamma\langle E_K\rangle$$

$$\tfrac{1}{2}k_BT$$

$$F_0^2 = \gamma k_B T$$

Fluctuation dissipation theorem

$$S_x(\omega) = \frac{4k_B T}{\omega} \mathfrak{Im}\hat{\chi}(\omega)$$

Fluctuation dissipation theorem

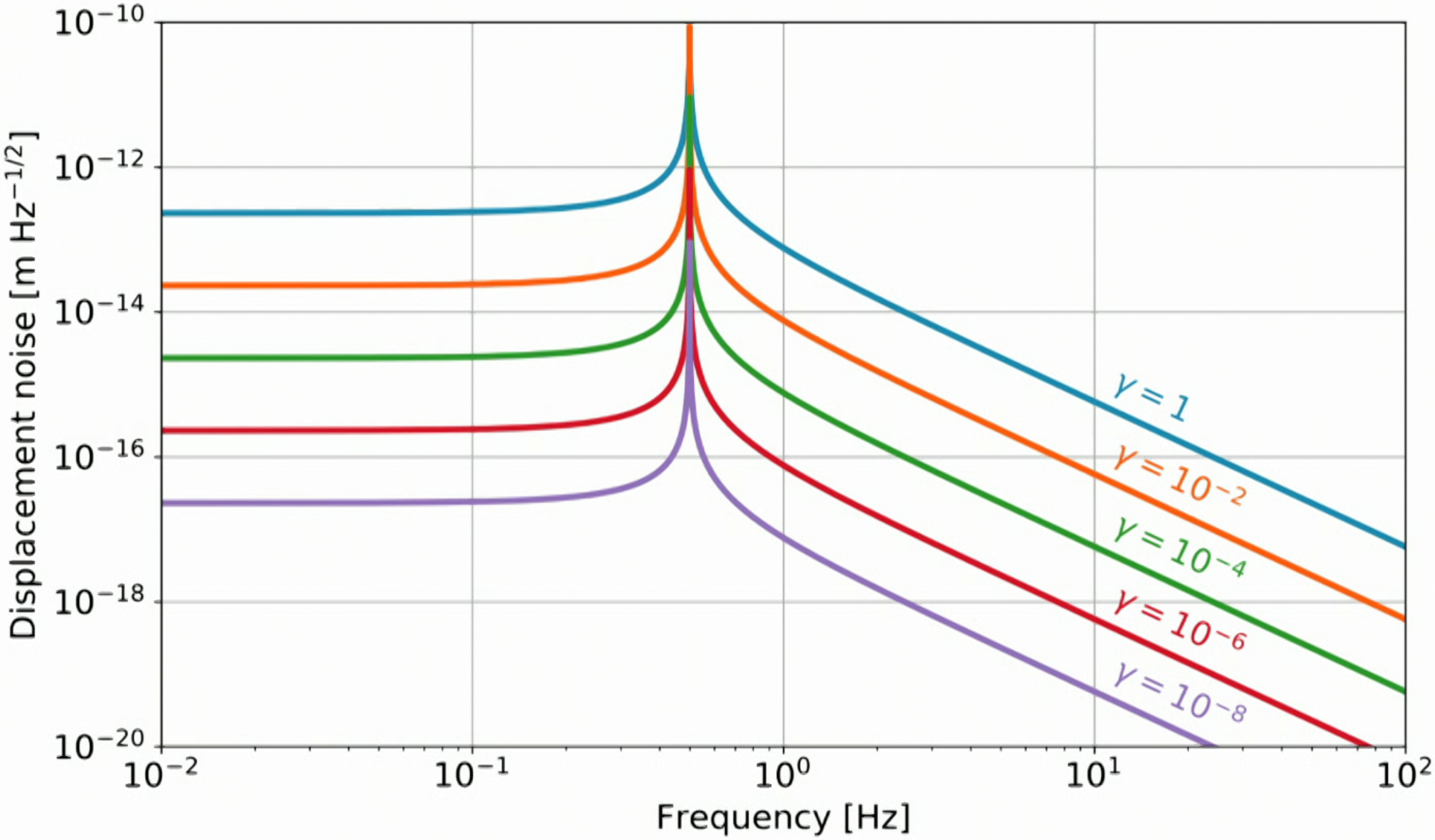
Thermodynamics

$$S_x(\omega) = \frac{4k_B T}{\omega} \mathfrak{Im}\hat{\chi}(\omega)$$

Displacement noise of the suspended mass

Mechanics: response of the system to external disturbance

Thermal noise in a pendulum



Structural damping

Viscous damping: $m\ddot{x} + kx + \gamma\dot{x} = F$ $\qquad \Delta E_{cycle} = \frac{2\gamma}{m}\langle E_K \rangle$

$$k \rightarrow k(1 + i\phi)$$

$$E_E = \frac{1}{2}kx^2 + i\phi\frac{1}{2}kx^2$$

$$-m\omega^2\hat{x} + k(1 + i\phi)\hat{x} = \hat{F}$$

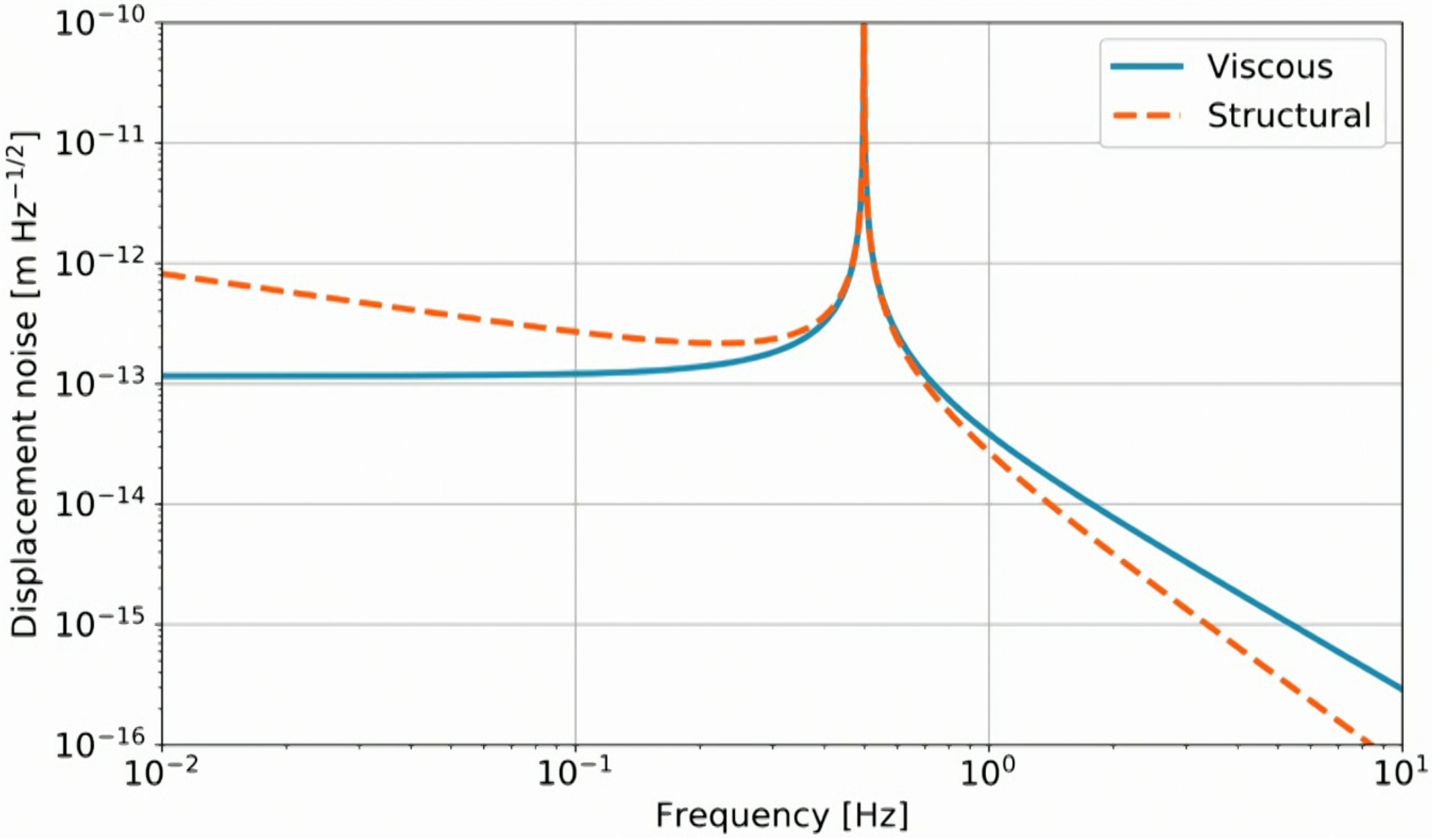
Structural damping

$$T(\omega) = \frac{1}{-m\omega^2 + k(1 + i\phi)}$$

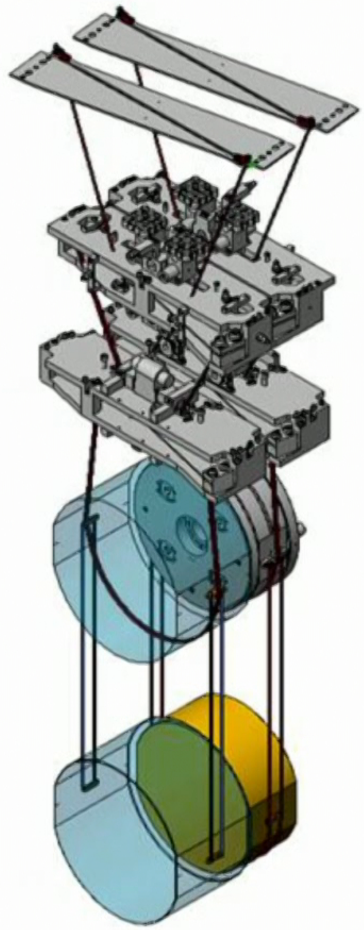
$$= \frac{k - m\omega^2}{(k - m\omega^2)^2 + k^2\phi^2} - i\phi \frac{k}{(k - m\omega^2)^2 + k^2\phi^2}$$

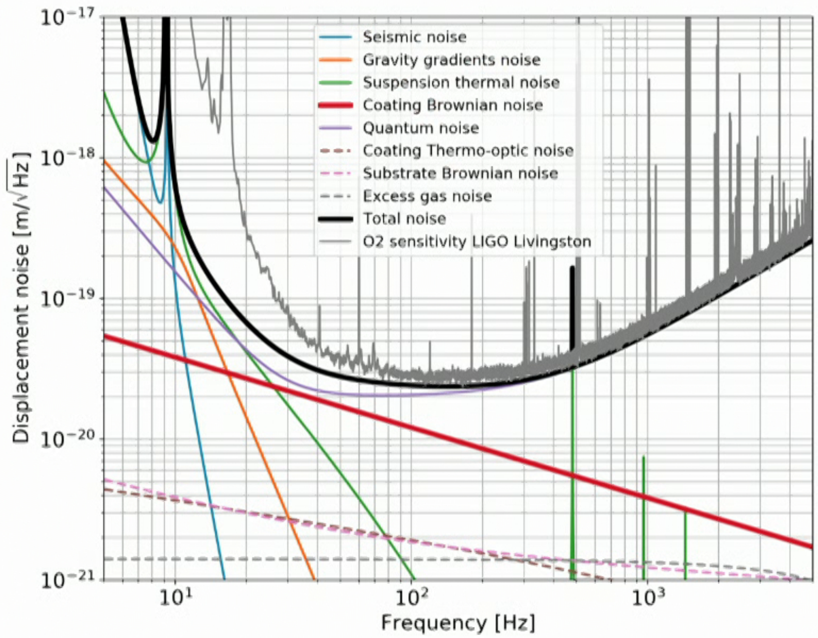
$$S_X(\omega) = \frac{4k_B T}{\omega} \frac{k\phi}{(k - m\omega^2)^2 + k^2\phi^2}$$

Structural vs Viscous

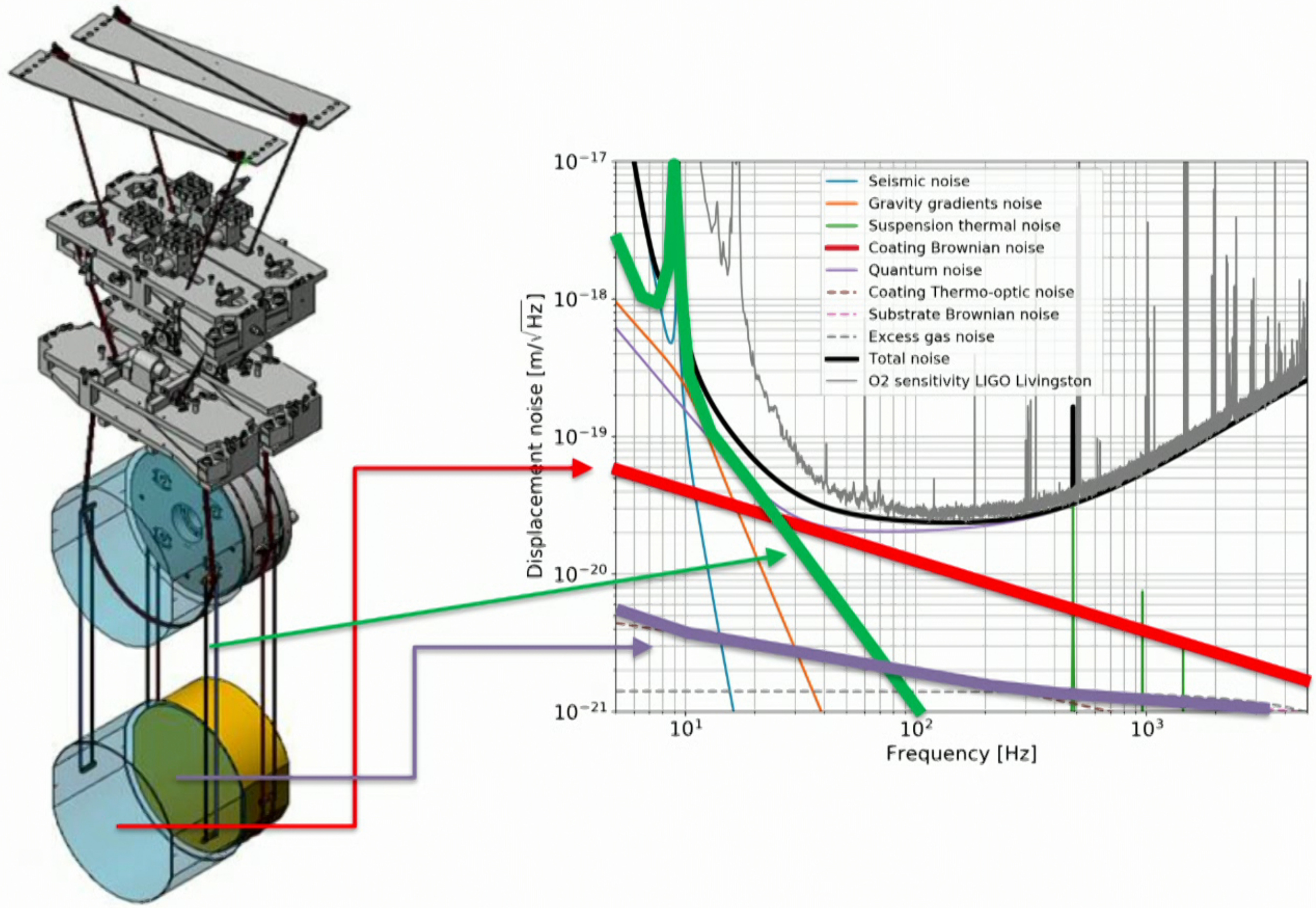


Sources of thermal noise



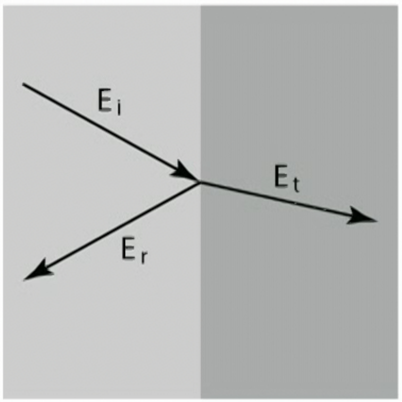


Sources of thermal noise



Coatings

Fresnel reflection



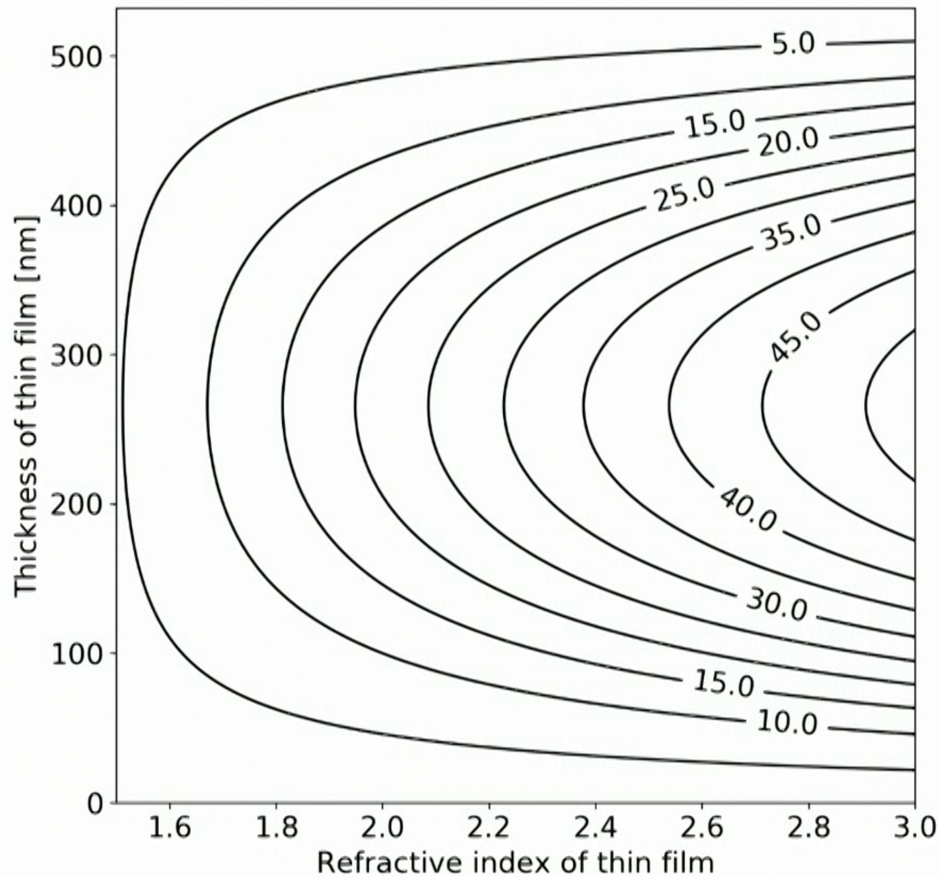
$$r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} \qquad r_s = \frac{n_1 - n_2}{n_1 + n_2}$$

$$t_s = \frac{2 n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2} \qquad t_s = \frac{2 n_1}{n_1 + n_2}$$

$$r_p = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2} \qquad r_p = \frac{n_2 - n_1}{n_2 + n_1}$$

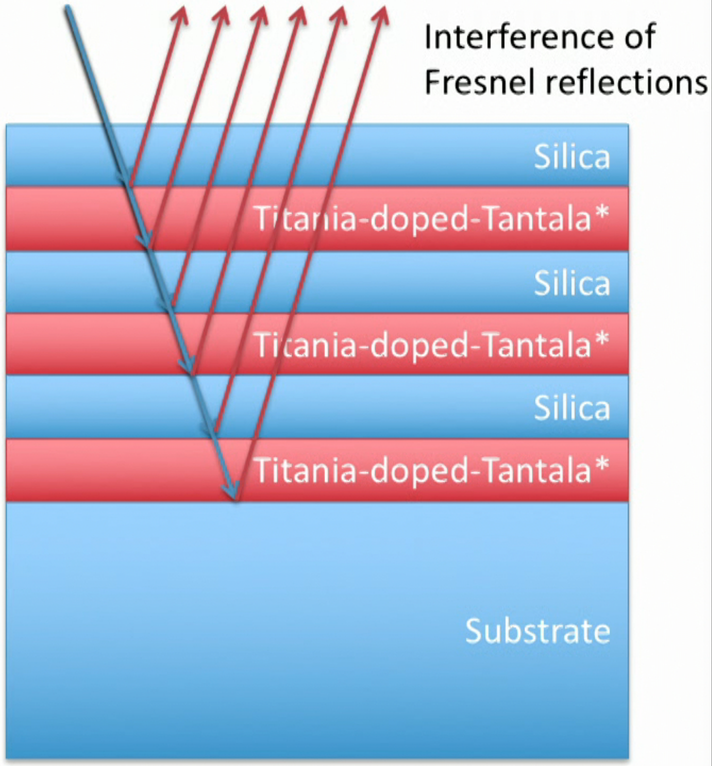
$$t_s = \frac{2 n_1 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2} \qquad t_s = \frac{2 n_1}{n_2 + n_1}$$

Coatings


5.0
15.0
20.0
25.0
35.0
45.0
40.0
30.0
15.0
10.0
500
400
300
200
100
0
Thickness of thin film [nm]
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0
Refractive index of thin film

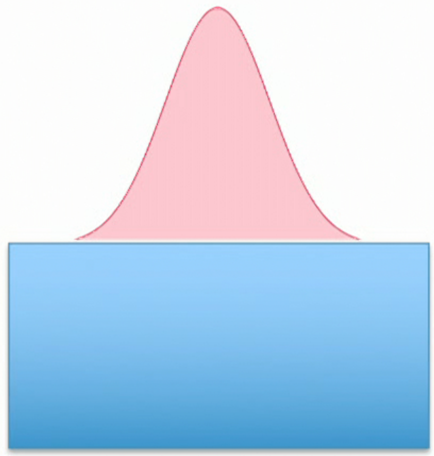
Multi layer coatings

Material	Refractive index	Loss angle
Silica SiO_2	1.45	0.4×10^{-4}
Tantala Ta_2O_5	2.03	3.4×10^{-4}
Titania-doped tantala Ta_2O_5-TiO_2	2.07	**2.3×10^{-4}**

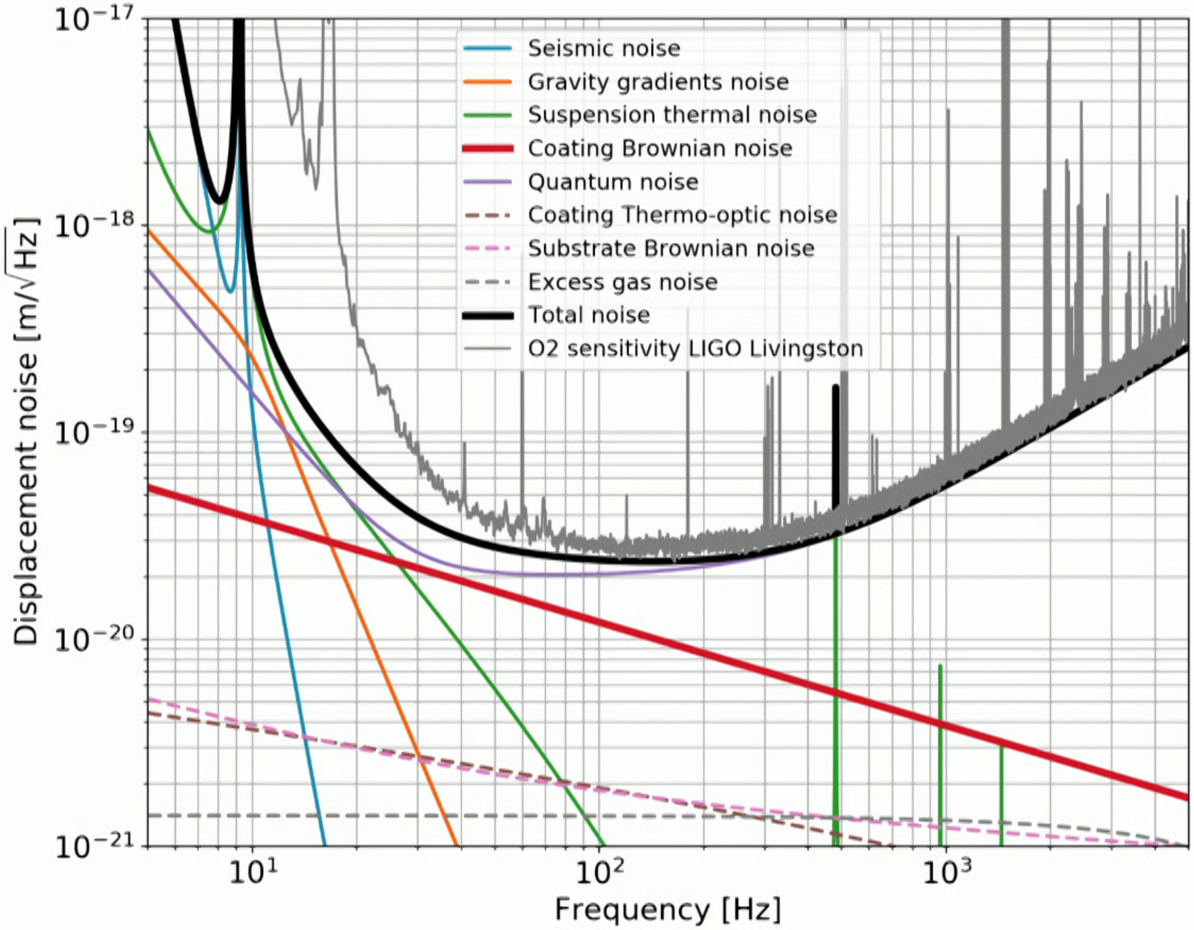


Thermal noise in coatings

- We measure the surface motion of the mirror, monitoring the phase of the reflected beam
  - We average over a large surface, determined by the beam Gaussian distribution (radius ~5-6 cm)
  - The phase changes due to
    - Surface displacement (**Brownian noise**)
    - Change in the material and layer properties:
      thermal expansion, change of refractive index with temperature (**Thermo-optic noise**)



LIGO



Seismic noise
Gravity gradients noise
Suspension thermal noise
Coating Brownian noise
Quantum noise
Coating Thermo-optic noise
Substrate Brownian noise
Excess gas noise
Total noise
O2 sensitivity LIGO Livingston
Displacement noise [m/√Hz]
Frequency [Hz]
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{1}
10^{2}
10^{3}

QUANTUM NOISE

Shot noise again

- Quantum nature of light: photons
- Measuring power = measuring the number of photons
  - Poisson statistics

$$\mathrm{Prob}(n) = e^{-m}\frac{m^n}{n!} \qquad m = \mathcal{E}[n] = \frac{P\Delta t}{\hbar\omega} \qquad S_P^{1/2}(\Omega) = \sqrt{2\hbar\omega\bar{P}}$$

- This is simplified description, useful only in very simple cases
- What is a good quantum description?

$$P = |E + \delta E|^2 = P\left|1 + \frac{\delta E}{E}\right| \simeq P + 2P\frac{\delta E}{|E|} = P + 2\frac{\delta E}{\sqrt{P}}$$

$$S_E^{1/2} = \frac{S_P^{1/2}}{2\sqrt{P}} = \sqrt{\frac{\hbar\omega}{2}}$$

Radiation pressure

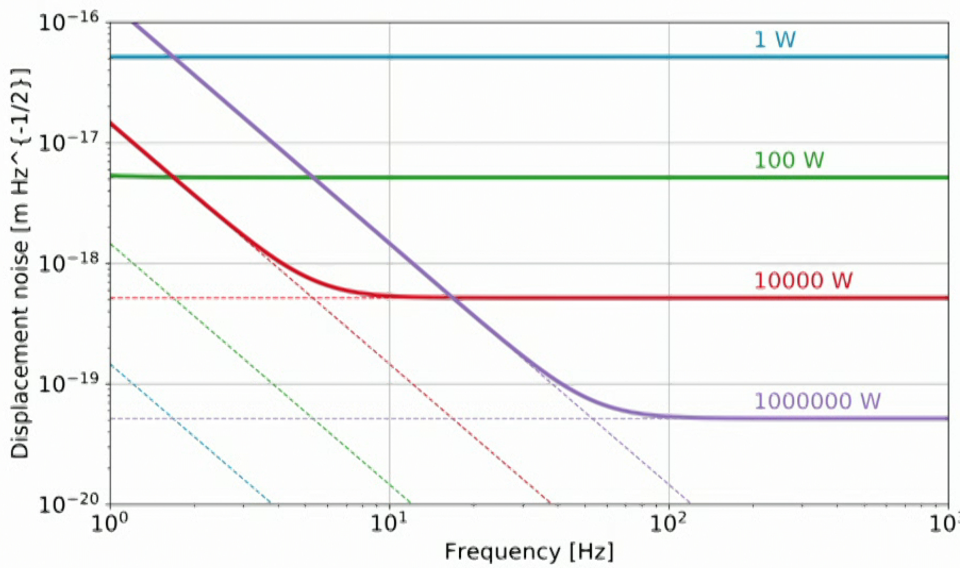
- Photons carry momentum: a laser beam exerts a force on a mirror

$$2p = \frac{2E}{c} = \frac{2\hbar\omega}{c} \simeq 10^{-27}\ \mathrm{N\ m}$$

$$F_{RP} = \frac{2P}{c} = 6.7 \times 10^{-9}\left(\frac{P}{1\mathrm{W}}\right)\ \mathrm{N}$$

$$S_F^{1/2}(\Omega) = \frac{\sqrt{8\hbar\omega P}}{c}$$

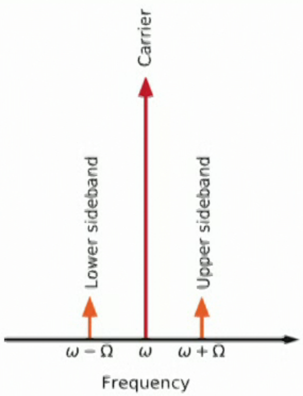
$$S_z^{1/2}(\Omega) = \frac{\sqrt{8\hbar\omega P}}{mc\Omega^2}$$



Quantum description

- Electromagnetic field in the Heisenberg picture

$$E(t)=\sqrt{\frac{2\pi\hbar\,\omega_0}{\mathcal{A}c}}e^{-i\omega_o t}\int_0^{+\infty}[a_+(\Omega)e^{-i\Omega t}+a_-(\Omega)e^{i\Omega t}]\,\frac{d\Omega}{2\pi}+\text{H.c.}$$



$$[a_+,a_{+'}^\dagger]=2\pi\,\delta(\Omega-\Omega')\left(1+\frac{\Omega}{\omega_o}\right),$$

$$[a_-,a_{-'}^\dagger]=2\pi\,\delta(\Omega-\Omega')\left(1-\frac{\Omega}{\omega_o}\right),$$

$$[a_+,a_{+'}]=0=[a_-,a_{-'}],\quad [a_+^\dagger,a_{+'}^\dagger]=0=[a_-^\dagger,a_{-'}^\dagger],$$

$$[a_+,a_{-'}]=0=[a_+,a_{-'}^\dagger],$$

$$a_1=\frac{a_++a_-^\dagger}{\sqrt{2}},\quad a_2=\frac{a_+-a_-^\dagger}{\sqrt{2}i}$$

$$[a_1,a_{2'}^\dagger]=-[a_2,a_{1'}^\dagger]=2\pi i\,\delta(\Omega-\Omega'),$$

$$[a_1,a_{1'}^\dagger]=0=[a_1,a_{1'}],\quad [a_2,a_{2'}^\dagger]=0=[a_2,a_{2'}]$$

PHYSICAL REVIEW D, VOLUME 64, 042006

Quantum description

Quantum: $$E(t) = \sqrt{\frac{2\pi\hbar\,\omega_0}{\mathcal{A}c}}\,e^{-i\omega_o t}\int_0^{+\infty}[a_+(\Omega)e^{-i\Omega t} + a_-(\Omega)e^{i\Omega t}]$$

Classical: $$E(t) = E_0\left(1 + A_+e^{-i\Omega t} + A_-e^{+i\Omega t}\right)$$

Quantum: $$E_j(a_j;t) = \sqrt{\frac{4\pi\hbar\,\omega_0}{\mathcal{A}c}}\int_0^{+\infty}(a_je^{-i\Omega t} + a_j^\dagger e^{i\Omega t})\frac{d\Omega}{2\pi}$$

Classical: $$E(t) = E_0\left(1 + A_c\cos\Omega t + A_s\sin\Omega t\right)$$

LIGO

Quantum Description

- Commutation relations

$$[E_j(t), E_j(t')] = 0$$

$$[E_1(t), E_2(t')] \sim i\,\delta(t - t')$$

- We can treat the two quadrature as classical variable when propagating them
- But when we measure them (simultaneously, using a photodiode) we need to take into account the quantum commutation relations > Heisenberg uncertainty > quantum noise

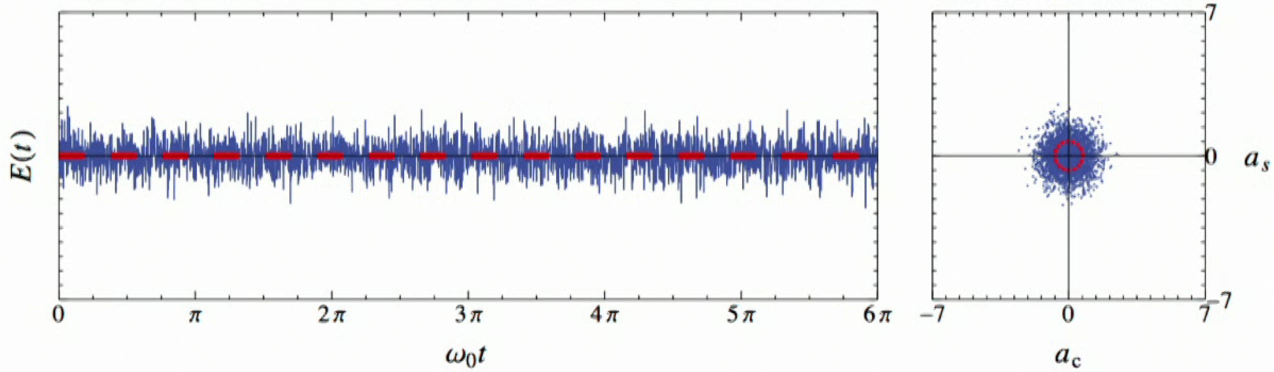
Quantum states

- **Vacuum state**: $|\text{vac}\rangle \equiv \bigotimes_{\omega} |0\rangle_\omega \quad \hat{a}_\omega |0\rangle_\omega = 0$

- Time domain representation of the electric field measurement:

$$\hat{X}_\varepsilon(t) \equiv \frac{1}{\sqrt{\varepsilon}} \int_{t-\varepsilon/2}^{t+\varepsilon/2} d\tau\, \hat{a}_c(\tau) \qquad \hat{Y}_\varepsilon(t) \equiv \frac{1}{\sqrt{\varepsilon}} \int_{t-\varepsilon/2}^{t+\varepsilon/2} d\tau\, \hat{a}_s(\tau)$$

$$\left[\hat{X}_\varepsilon(t),\, \hat{Y}_\varepsilon(t)\right] = i \qquad \begin{aligned}\mathbb{V}_{cc} &= \langle \hat{X}_\varepsilon^2(t)\rangle = 1/2 \\ \mathbb{V}_{ss} &= \langle \hat{Y}_\varepsilon^2(t)\rangle = 1/2\end{aligned} \qquad \mathbb{V}_{\text{vac}} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$



Living Rev. Relativity, 15, (2012), 5

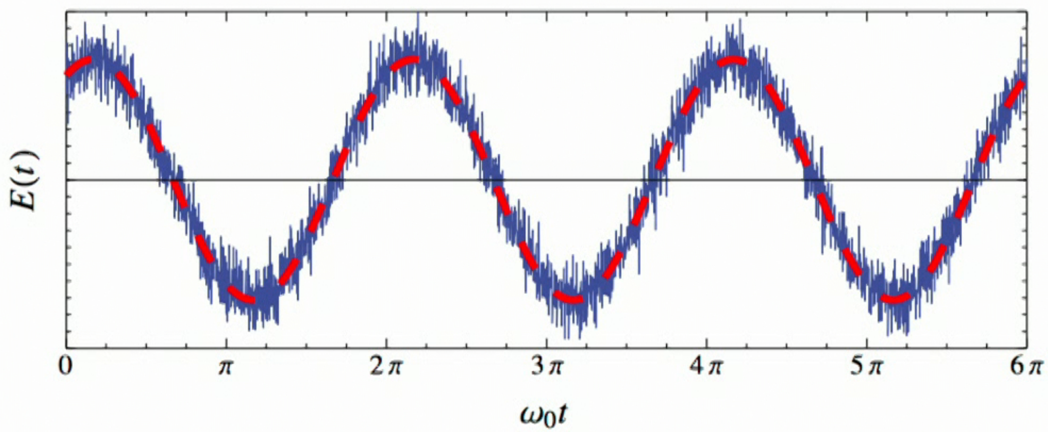
Quantum states

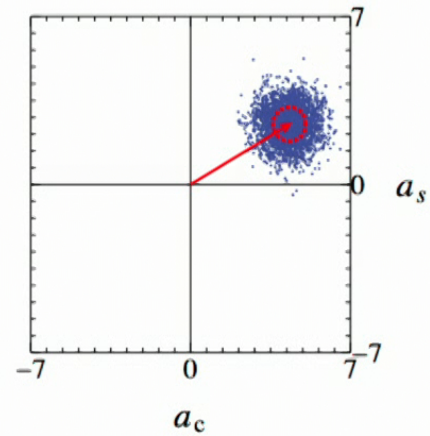
- Coherent states: that's what a laser would produce

$$|\alpha\rangle = \hat{D}[\alpha]|0\rangle \equiv e^{\alpha\hat{a}^\dagger - \alpha^*\hat{a}}|0\rangle$$

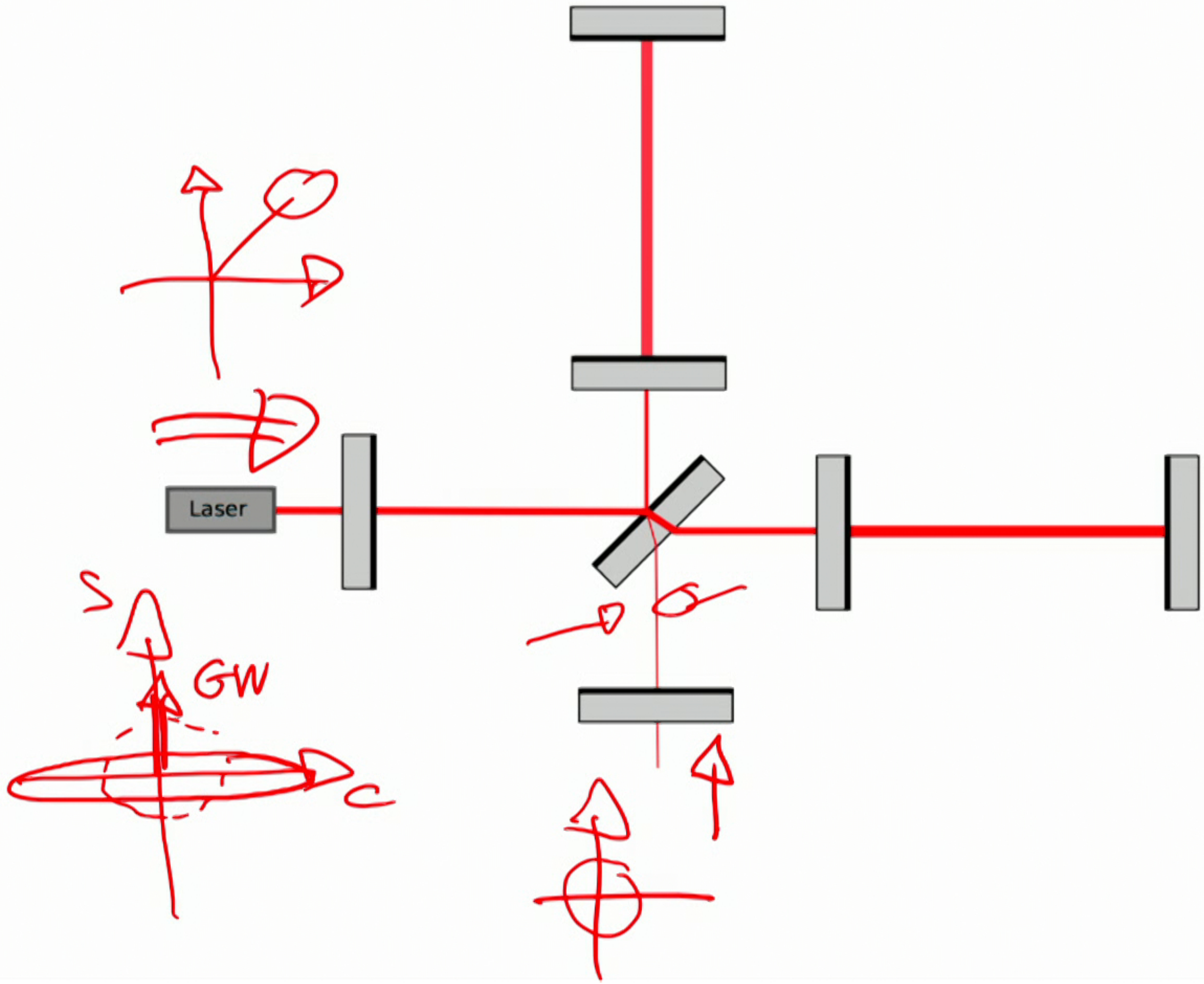
$$\langle\alpha|\hat{X}|\alpha\rangle = \langle 0|\hat{D}^\dagger[\alpha]\hat{X}\hat{D}[\alpha]|0\rangle = \sqrt{2}\mathrm{Re}[\alpha] \qquad \mathrm{Var}[\hat{X}] = \langle\alpha|\hat{X}^2|\alpha\rangle - \Big(\langle\alpha|\hat{X}|\alpha\rangle\Big)^2 = \frac{1}{2}$$

$$\langle\alpha|\hat{Y}|\alpha\rangle = \langle 0|\hat{D}^\dagger[\alpha]\hat{Y}\hat{D}[\alpha]|0\rangle = \sqrt{2}\mathrm{Im}[\alpha] \qquad \mathrm{Var}[\hat{Y}] = \langle\alpha|\hat{Y}^2|\alpha\rangle - \Big(\langle\alpha|\hat{Y}|\alpha\rangle\Big)^2 = \frac{1}{2}$$

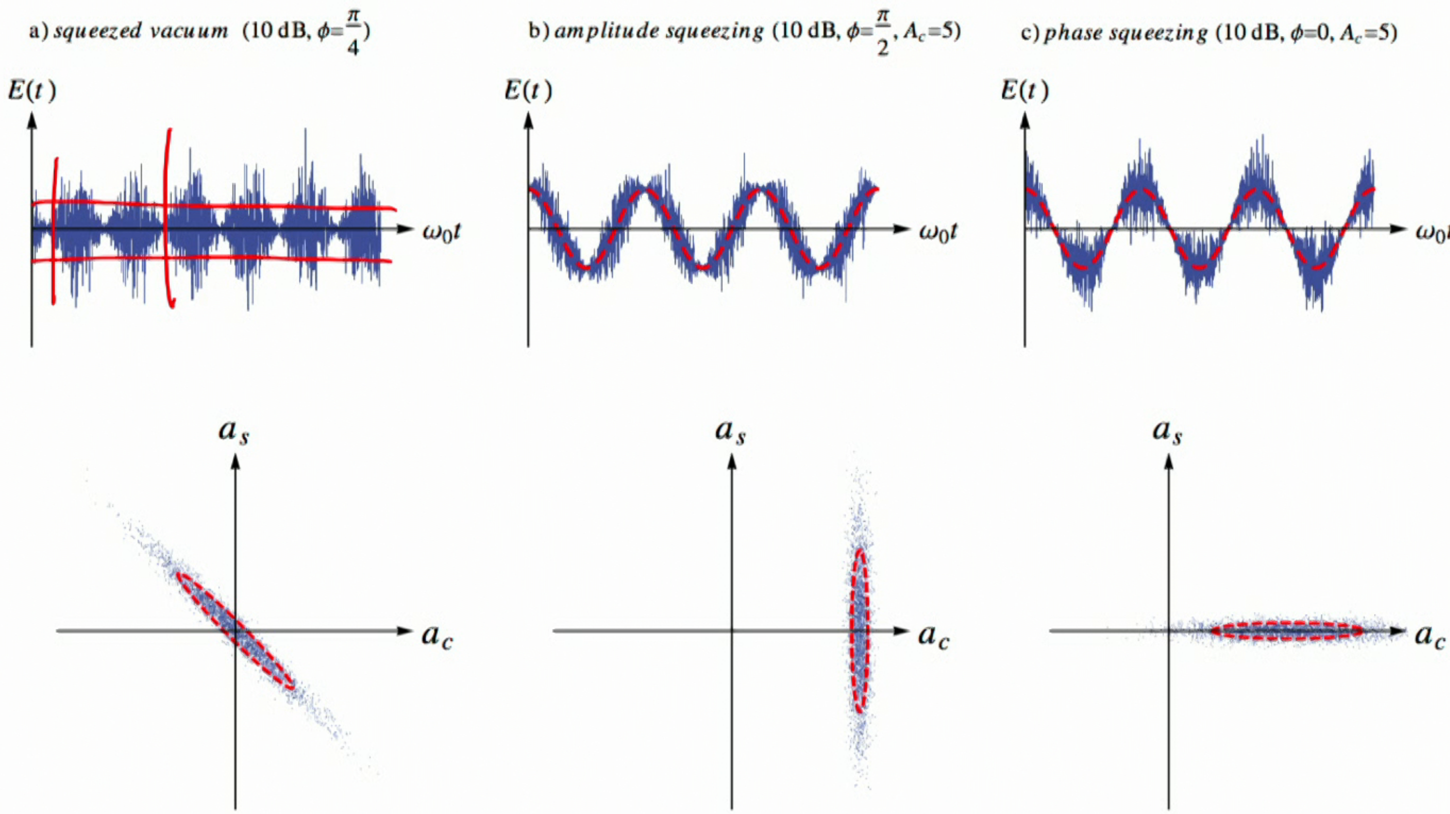




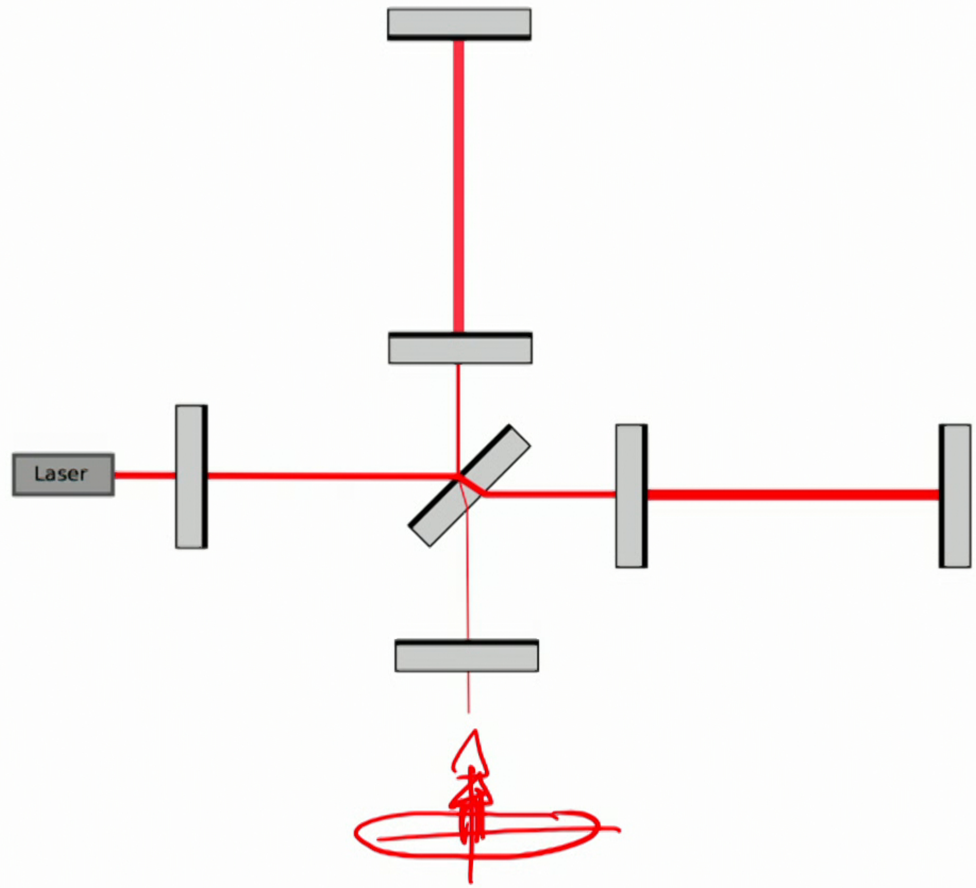
How vacuum enters in the interferometer



Squeezed states

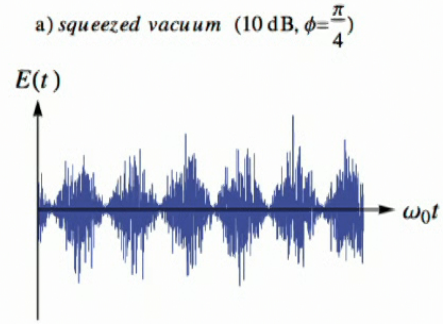


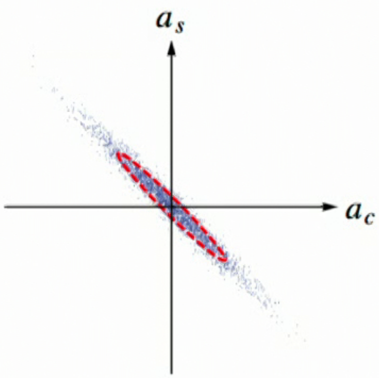
Squeezed vacuum for an interferometer



How to generate squeezed states

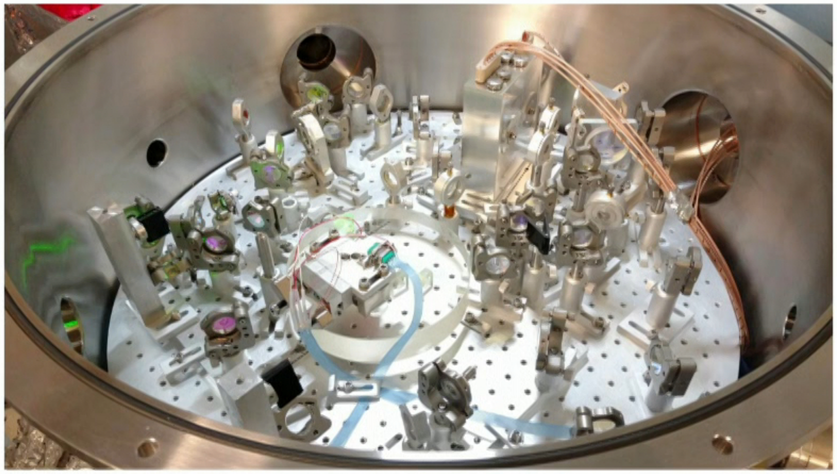
$$a_1 = \frac{a_+ + a_-^\dagger}{\sqrt{2}}, \quad a_2 = \frac{a_+ - a_-^\dagger}{\sqrt{2}i}$$

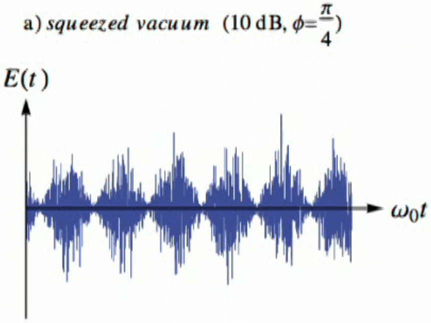


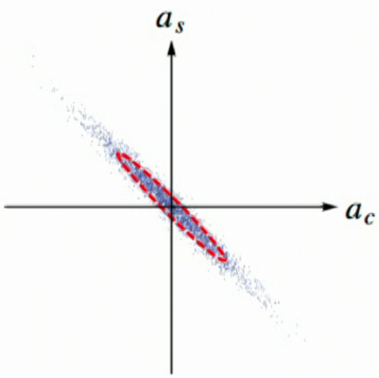


How to generate squeezed states

$$a_1 = \frac{a_+ + a_-^\dagger}{\sqrt{2}}, \quad a_2 = \frac{a_+ - a_-^\dagger}{\sqrt{2}i}$$



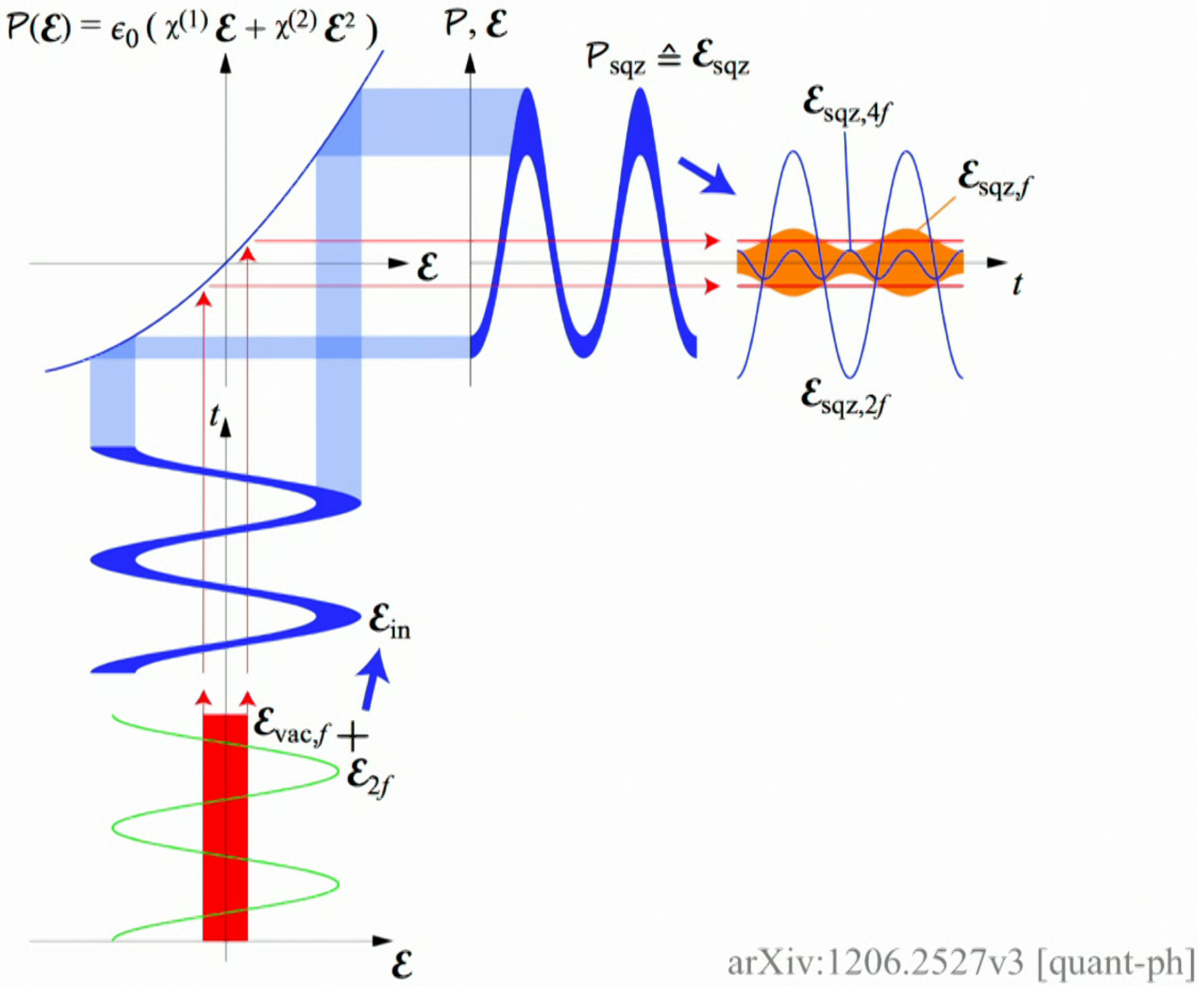




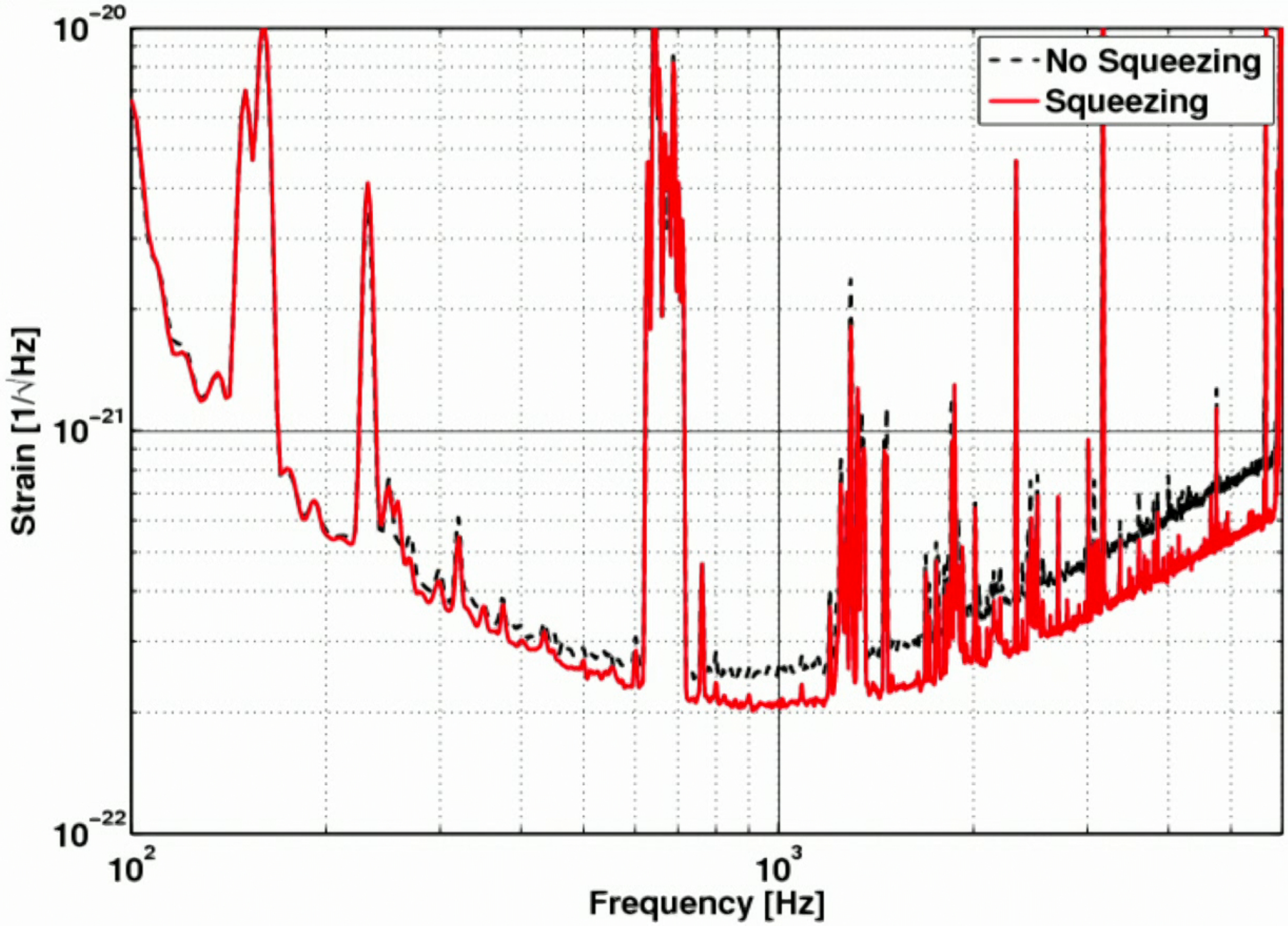
$$\mathcal{P}(\mathcal{E}) = \underbrace{\epsilon_0 \chi^{(1)} \mathcal{E}}_{\mathcal{P}^{(1)}} + \underbrace{\epsilon_0 \chi^{(2)} \mathcal{E}^2}_{\mathcal{P}^{(2)}} + \underbrace{\epsilon_0 \chi^{(3)} \mathcal{E}^3}_{\mathcal{P}^{(3)}} + \cdots$$

arXiv:1206.2527 [quant-ph]

Non linear Optical Parametric Amplifier



Squeezing for real: GEO 600



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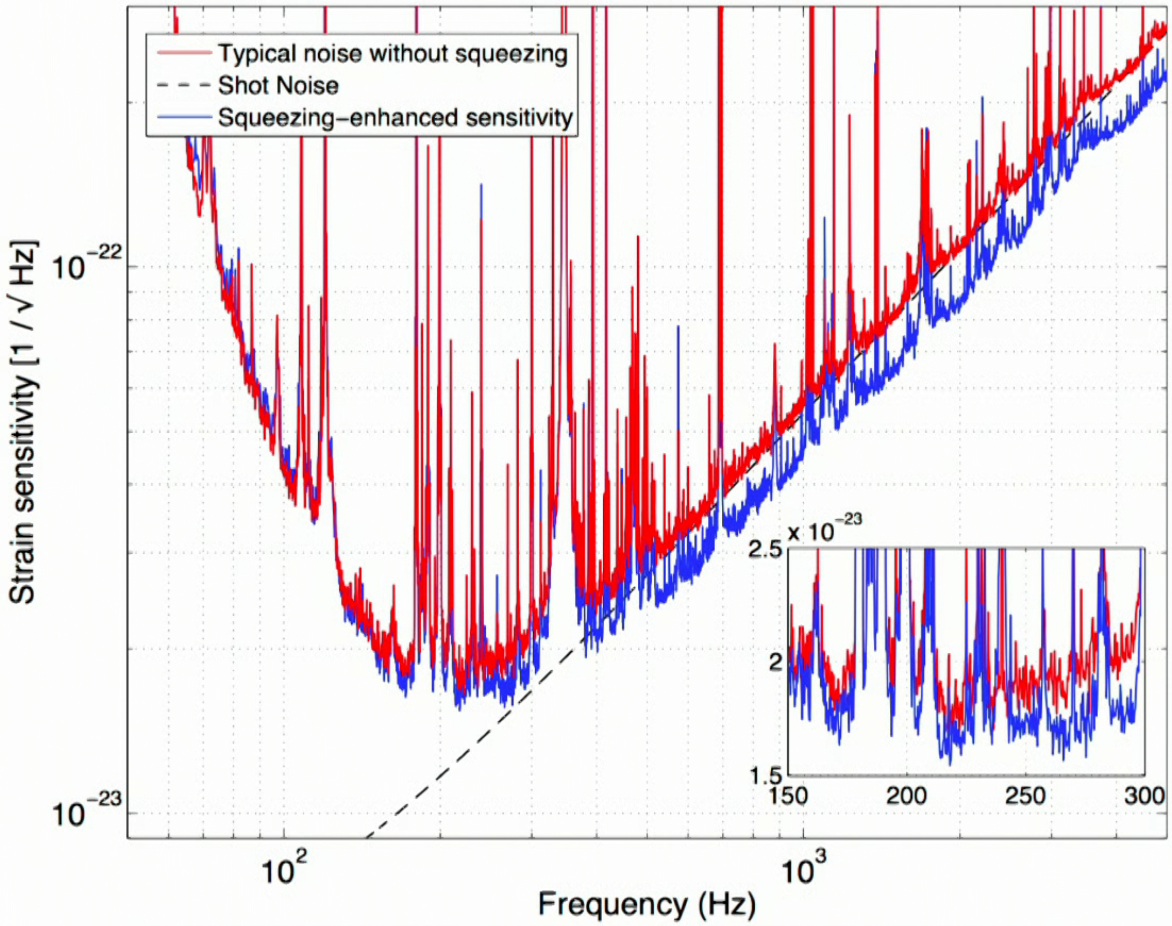
Squeezing for real: eLIGO



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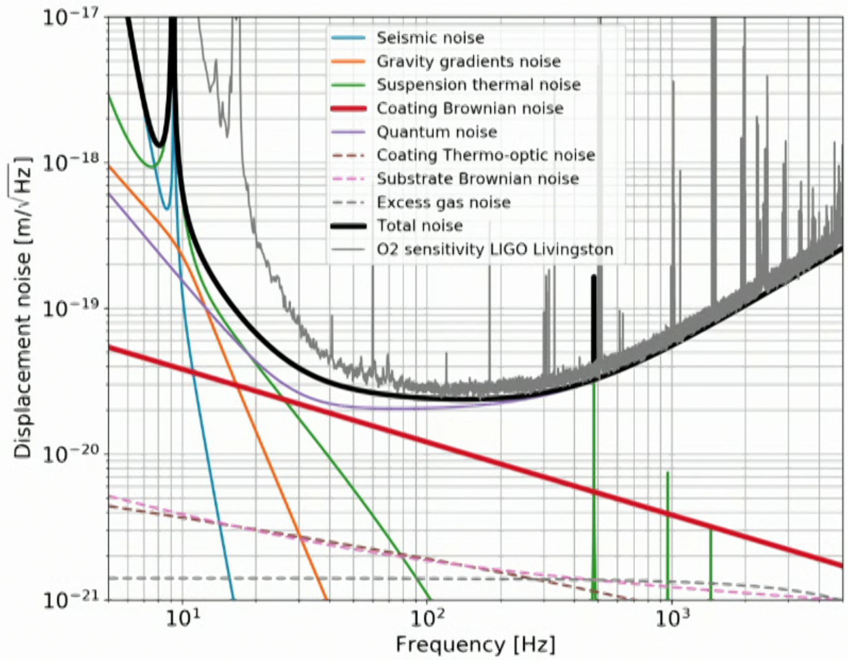
Next time...



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