Title: EFT of Gravity 3

Date: Jul 19, 2018 09:00 AM

URL: http://pirsa.org/18070008

Abstract:

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GR as an EFT

TRISEP 7/19/18

Note Title 7/10/2018

Main goal: How to think of gravity as part of our "core theory".

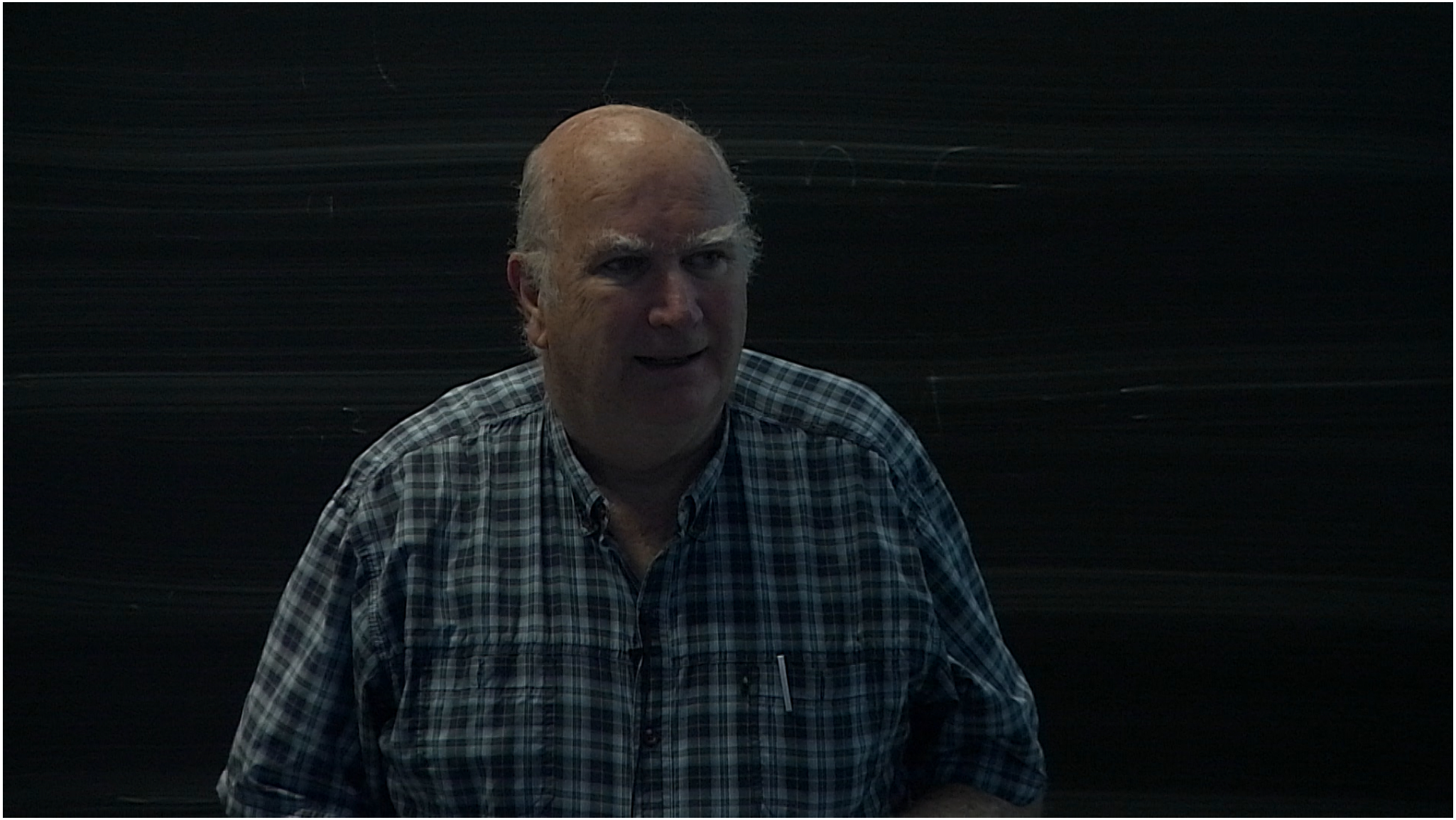
SM + GR

$$Z = \int_{\Lambda} [d\phi\, dA\, d\psi] \exp i \int d^4x \sqrt{-g} \left[-\frac{1}{4} F^2 + \bar{\psi} i \not{D} \psi + 2 \phi^* \partial^\mu \phi - V(\phi) \right.$$
$$\left. - \Lambda + \frac{2}{K^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \cdots \right]$$

QM + GR (at ordinary energies)

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GR as an EFT

TRISEP 7/19/18

Note Title 7/10/2018

Recall:

Main goal: How to think of gravity as part of our "core theory".

SM + GR

$$Z = \int_{\Lambda} [d\phi\, dA\, d\psi] \exp i \int d^4x \sqrt{-g} \left[-\frac{1}{4} F^2 + \bar{\psi} i \not{D} \psi + 2 \phi^* \partial^\mu \phi - V(\phi) \right.$$
$$\left. - \Lambda + \frac{2}{K^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \cdots \right]$$

QM ⊂ GR (at ordinary energies)

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tion? But since I am among equally irrational men I won't be criticized I hope for the fact that there is no possible, practical reason for making these calculations.

Need to overcome outdated claims:

"The existence of gravity clashes with our description of the rest of physics by quantum fields"

"Quantum mechanics and relativity are contradictory to each other and therefore cannot both be correct."

"Attempting to combine general relativity and quantum mechanics leads to a meaningless quantum field theory with unmanageable divergences."

"The application of conventional field quantization to GR fails because it yields a nonrenormalizable theory"

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Need to overcome outdated claims:

"*The existence of gravity clashes with our description of the rest of physics by quantum fields*"

"*Quantum mechanics and relativity are contradictory to each other and therefore cannot both be correct.*"

"*Attempting to combine general relativity and quantum mechanics leads to a meaningless quantum field theory with unmanageable divergences.*"

"*The application of conventional field quantization to GR fails because it yields a nonrenormalizable theory*"

"*Quantum mechanics and general relativity are incompatible*"

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EFT is the key

1) Identify low Energy d.o.f and symmetries

2) Most general $\mathcal{L}$, ordered in energy expansion

3) Quantize

4) Renormalize

5) Match/measure

6) Predictions from low energy propagators

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$$S_g = \int d^4x \sqrt{-g} \left[-\Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \right]$$

$$S_m = S(\phi, g)$$

$\Lambda \sim 10^{-122} M_p^2$

$\kappa^2 = 32\pi G_N$

$c_i < 10^{+65}$

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First scalar

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$\delta S_{div} = \frac{1}{16\pi^2} \frac{1}{180} \left[\frac{1}{\varepsilon} \cdots \right] \left[3 R_{\mu\nu} R^{\mu\nu} - R^2 \right] \quad \leftarrow *$$

$\sim O(\partial^4)$

$$\sim O(E^4) \sim O(\partial^4)$$

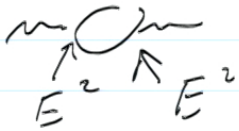
E^2 E^2

First scalar

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

$$\delta S_{div} = \frac{1}{16\pi^2} \frac{1}{18} \left[\frac{1}{\epsilon} \cdots \right] \left[3 R_{\mu\nu} R^{\mu\nu} - R^2 \right] \leftarrow \star$$

$\sim O(\partial^4)$



$\sim O(E^4) \sim O(\partial^4)$

$\lambda \phi^4$  $\sim O(\partial^4)$

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Gravity "simple" 't Hooft Veltman

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}$$

Expansion of $O(\kappa)$

$$\Gamma^{\lambda}_{\mu\nu} = \bar{\Gamma}^{\lambda}_{\mu\nu} + \underline{\Gamma^{\lambda}_{\mu\nu}} + \underline{\underline{\Gamma^{\lambda}_{\mu\nu}}}$$

$$\underline{\Gamma^{\lambda}_{\mu\nu}} = \frac{1}{2}\bar{g}^{\lambda\sigma}\left[\bar{D}_{\mu} h_{\nu\sigma} + \bar{D}_{\nu} h_{\mu\sigma} - \bar{D}_{\sigma} h_{\mu\nu}\right]$$

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$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{-g}R = \sqrt{-g}\left[\frac{2}{\kappa^2}\bar{R} + \frac{1}{2}\bar{D}_\alpha h_{\mu\nu}\bar{D}^\alpha h^{\mu\nu} + \cdots \right.$$

$$\left. h^\lambda{}_\nu \bar{R}^\nu{}_\mu h^{\mu\nu} + \cdots \right]$$

Also $x^\mu \to x'^\mu = x^\mu + \xi^\mu(x)$, ξ^μ small

$$h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} + \bar{D}_\mu \xi_\nu + \bar{D}_\nu \xi_\mu$$

gauge invariance for h

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Gauge fixing

$$S_{gf} = \left(\bar{D}^{\mu} h_{\mu\nu} - \frac{1}{2} \bar{D}_{\nu} h^{\lambda}_{\ \lambda} \right)$$

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Recall

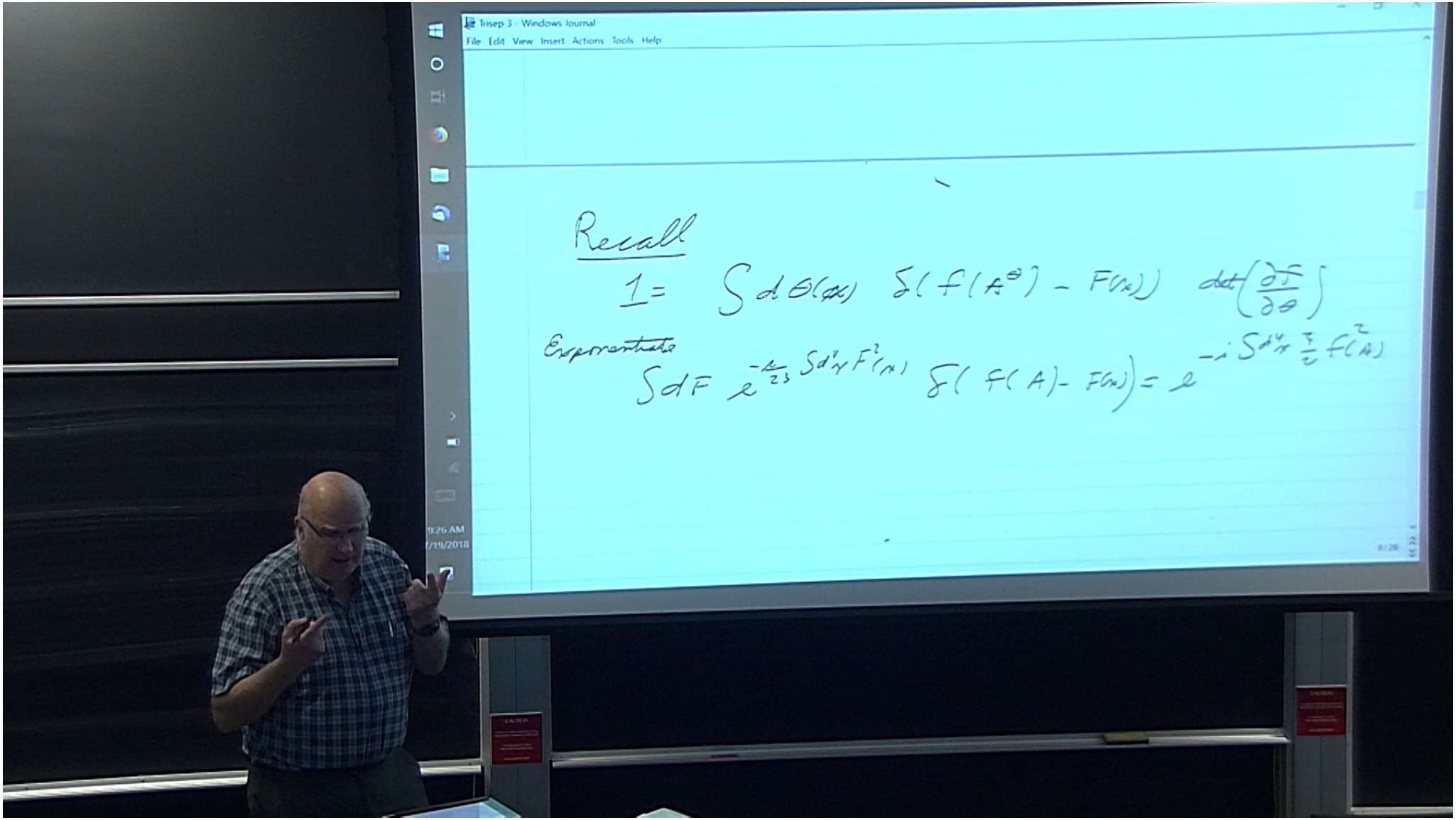
$$1 = \int d\Theta(x)\ \delta(f(A^\Theta) - F(x))\ \det\left(\frac{\partial f}{\partial \theta}\right)$$

Exponentiate

$$\int dF\ e^{-\frac{1}{2\xi}\int d^4x F^2(x)}\ \delta(f(A) - F(x)) = e^{-i\int d^4x \frac{\xi}{2} f^2(A)}$$

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Recall
$1 = \int d\Theta(\phi) \; \delta(f(A^\theta) - F(x)) \; \det\left(\frac{\partial f}{\partial \theta}\right)$
Exponentiate
$\int dF \; e^{-\frac{\alpha}{2\beta}\int d^4x F^2(x)} \; \delta(f(A) - F(x)) = e^{-i\int d^4x \frac{\alpha}{2} f^2(A)}$

Recall

$$1 = \int d\theta(x)\ \delta(f(A^\theta) - F(x))\ \det\left(\frac{\partial f}{\partial \theta}\right)$$

Exponentiate

$$\int dF\ e^{-\frac{1}{2\xi}\int d^4x F^2(x)}\ \delta(f(A) - F(x)) = e^{-i\int d^4x \frac{1}{2} f(A)^2}$$

Evaluate FP det with ghost fields

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Recall:

$$1 = \int [d\theta\, \delta(G(A^\theta))]\, \delta(f(A^\theta) - F(x))\, \det\left(\frac{\partial f}{\partial \theta}\right)$$

$A_\mu \to A_\mu + \partial_\mu \theta(x)$

Exponentiate

$$\int dF\, e^{-\frac{1}{2\xi}\int d^4x F^2(x)}\, \delta(f(A) - F(x)) = e^{-i\int d^4x \frac{1}{2} f(A)^2}$$

Evaluate FP det with ghost fields

$$\det \frac{\partial f}{\partial \theta} = \int dc\, d\bar{c}\, e^{i\int d^4x\, \bar{c}\, \frac{\partial f}{\partial \theta}\, c}$$

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Recall

$$1 = \int [\mathcal{D}\theta(x)]\, \delta(f(A^\theta) - F(x)) \det\left(\frac{\partial f}{\partial \theta}\right)$$

$A_\mu \to A_\mu + \partial_\mu \theta(x)$

Exponentiate

$$\int dF\, e^{-\frac{1}{2\xi}\int d^4x F^2(x)}\, \delta(f(A) - F(x)) = e^{-i\int d^4x \frac{1}{2} f^2(A)}$$

Evaluate FP det with ghost fields

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Gravity

$$C_\nu = D_\mu h_{\mu\nu} - \frac{1}{2} D_\nu h^\mu{}_\mu$$

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Gravity

$$C_\nu = \left(\bar{D}_\mu h^{\mu}{}_{\nu} - \frac{1}{2}\bar{D}_\nu h^{\mu}{}_{\mu}\right)$$

$$\mathcal{L}_{GF} = \frac{1}{2} C_\nu C^\nu$$

$$\delta C_\mu = \bar{D}_\nu \left(\bar{D}_\nu \xi_\mu + \bar{D}_\mu \xi_\nu\right) - \frac{2}{2}\bar{D}_\mu \bar{D}_\nu \xi$$

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Gravity

$$C_\nu = \left(\bar{D}_\mu h^\mu{}_\nu - \frac{1}{2}\bar{D}_\nu h^\lambda{}_\lambda\right)$$

$$\mathcal{L}_{gf} = \frac{1}{2} C_\nu C^\nu$$

$$\delta C_\mu = \bar{D}_\nu\left(\bar{D}^\nu \xi_\mu + \bar{D}_\mu \xi^\nu\right) - \frac{2}{2}\bar{D}_\mu \bar{D}_\nu \xi^\nu$$

$$= \bar{D}^2 \xi_\mu + \bar{R}_{\nu\mu}\xi^\nu$$

$$\frac{\delta C_\mu}{\delta \xi^\nu} = \left[g_{\mu\nu}\bar{D}^2 + \bar{R}_{\mu\nu}\right]$$

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$$= \bar{D}^2 \xi_\mu + \bar{R}_{\nu\mu} \xi^\nu$$

$$\frac{\partial \mathcal{L}_{gh}}{\delta \xi^\nu} = \left[\bar{g}_{\mu\nu} \bar{D}^2 + \bar{R}_{\mu\nu} \right]$$

$$\mathcal{L}_{gh} = \bar{\eta}^\mu \left[\bar{g}_{\mu\nu} \bar{D}^2 + \bar{R}_{\mu\nu} \right] \eta^\nu$$

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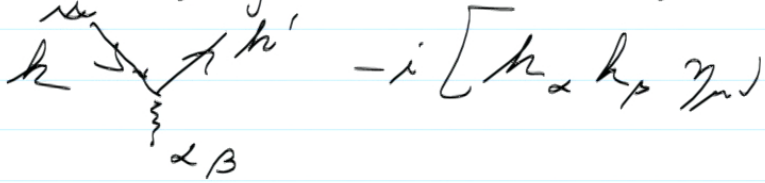
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$$\frac{\delta \mathcal{E}_\mu}{\delta \xi^\nu} = -\left[\bar{g}_{\mu\nu}\bar{D}^2 + \bar{R}_{\mu\nu}\right]$$

$$\mathcal{L}_{gh} = \bar{\eta}^\mu \left[\bar{g}_{\mu\nu}\bar{D}^2 + \bar{R}_{\mu\nu}\right]\eta^\nu$$

New Feynman rule (near flat)



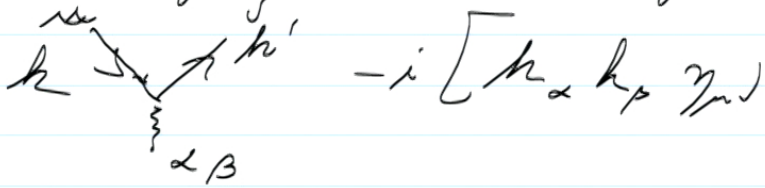
$$-i\left[k_\alpha k_\rho \eta_{\mu\nu}\right.$$

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$\mathcal{L}_{gh} = \eta^{\mu} \left[\bar{\delta}_{\mu\nu} D + K_{\mu\nu} \right] \eta$ ← ghost field

New Feynman rule (near flat)



$-i \left[k_\alpha k_\beta \eta_{\mu\nu} \right]$

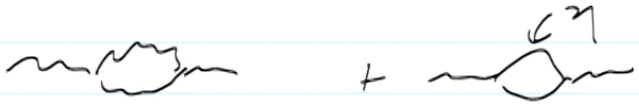
$$Z = \int [dh_{\mu\nu}][d\eta][d\bar{\eta}]\, e^{i \int d^4x \sqrt{-g} \left[\mathcal{L}(h) + \mathcal{L}_{gf}(h) + \mathcal{L}_g(\eta\bar{\eta}) \right]}$$

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$$\frac{i}{\hbar} \Delta \mathcal{L} = \frac{1}{16\pi^2} \frac{1}{\varepsilon} \left[\frac{1}{120} \bar{R}^2 + \frac{7}{120} \bar{R}_{\mu\nu} \bar{R}^{\mu\nu} \right]$$

$O(\lambda^4)$



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First – non-production

$$\left[\frac{2}{\kappa^2}R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu}\right]$$

Only graviton $R_{\mu\nu} = 0$ ⟹ no effect of c_1 c_2

Typical · ⟨ ⟩ + ⟨ ⟩ ⟵ $c_1 R^2$ + ·

$$\left(\frac{\kappa}{2}\right)^2 \left[\frac{1}{q^2} + c_1 \frac{1}{q^2} q^4 \frac{1}{q^2}\right] + \cdots$$

const

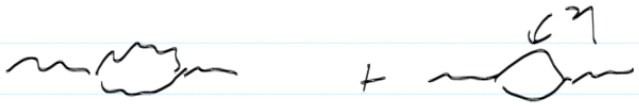
⟶ $V(r)$ = usual + $c_1 \delta^4(x)$

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$$\frac{'t\ H\ V}{\Delta\mathcal{L} = \frac{1}{16\pi^2}\frac{1}{\varepsilon}\left[\frac{1}{120}\bar{R}^2 + \frac{7}{120}\bar{R}_{\mu\nu}\bar{R}^{\mu\nu}\right]}$$

$$(\ O(\partial^4)$$



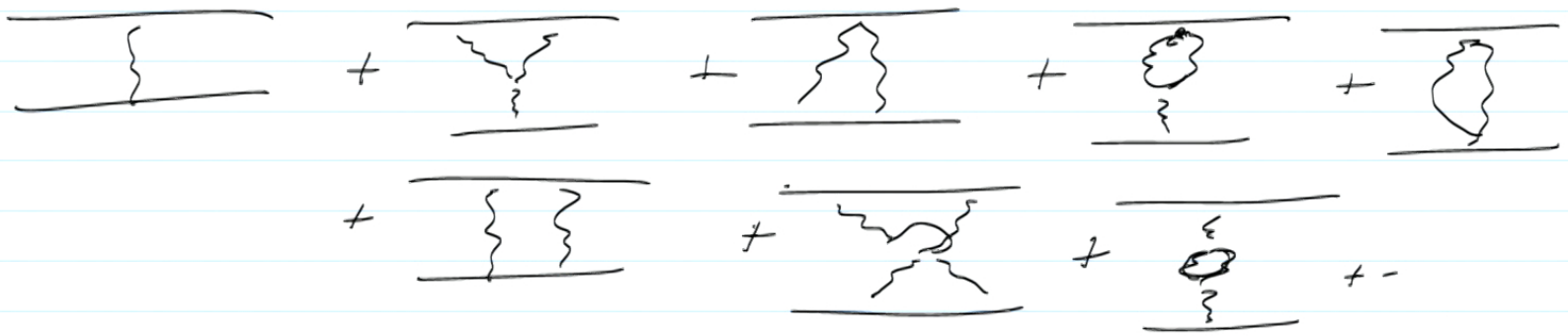
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Newtonian potential



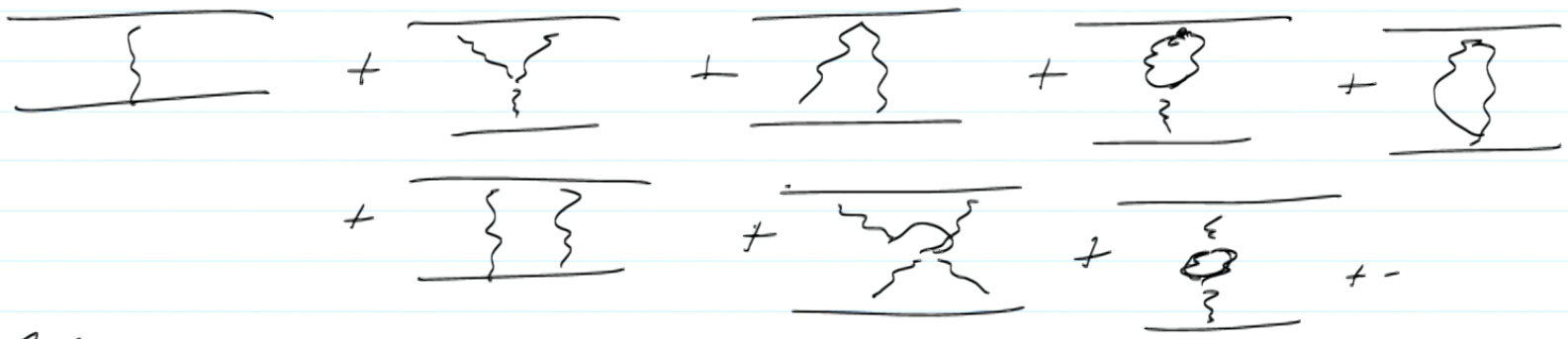
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Newtonian potential



$$\mathcal{M}(q)$$

$$V(r) = \int \mathcal{M}(E) \xleftarrow{\text{W.R}}$$

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$$\mathcal{M} = \frac{GMm}{q^2}\left[1 + aG(m+M)\sqrt{-q^2} + bGq^2\ln q^2 + cGq^2\right]$$

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + c\frac{GMm}{r}\delta(x)$$

classical ↑ quantum ↑ div go here

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$$\mathcal{M} = \frac{GMm}{q^2}\left[1 + aG(m+M)\sqrt{-q^2} + bGq^2\ln q^2 + cGq^2\right]$$

$$V(r) = -\frac{GMm}{r}\left[1 + a\frac{G(M+m)}{rc^2} + b\frac{G\hbar}{r^2c^3}\right] + c\frac{GMm}{r^3}\delta(x)$$

classical ↑ quantum ↑ div go here

Calculate

$$V(r) = -\frac{GmM}{r}\left[1 + 3\frac{G(m+M)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

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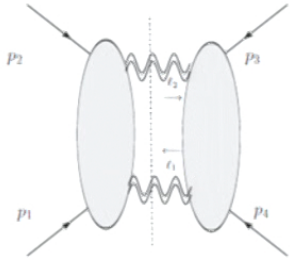
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$$V(r) = -\frac{GmM}{r}\left[1 + 3\frac{G(m+M)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

Classical

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$$iM_0^{\rm tree}(p_1,p_2,k_1^+,k_2^+) = \frac{\kappa_{(4)}^2}{16}\,\frac{1}{(k_1\cdot k_2)}\,\frac{m^4\,[k_1\,k_2]^4}{(k_1\cdot p_1)(k_1\cdot p_2)}\,,$$

$$iM_0^{\rm tree}(p_1,p_2,k_1^-,k_2^+) = \frac{\kappa_{(4)}^2}{16}\,\frac{1}{(k_1\cdot k_2)}\,\frac{\langle k_1|p_1|k_2]^2\,\langle k_1|p_2|k_2]^2}{(k_1\cdot p_1)(k_1\cdot p_2)}\,,$$

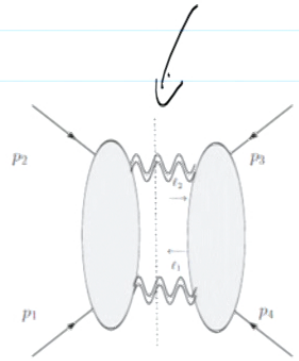
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Unitarity techniques

Passarino Veltman

-> Box, triangle, bubbles



$$iM_0^{\text{tree}}(p_1, p_2, k_1^+, k_2^+) = \frac{\kappa_{(4)}^2}{16} \frac{1}{(k_1 \cdot k_2)} \frac{m^4 \, [k_1 \, k_2]^4}{(k_1 \cdot p_1)(k_1 \cdot p_2)} \,,$$

$$iM_0^{\text{tree}}(p_1, p_2, k_1^-, k_2^+) = \frac{\kappa_{(4)}^2}{16} \frac{1}{(k_1 \cdot k_2)} \frac{\langle k_1 | p_1 | k_2]^2 \, \langle k_1 | p_2 | k_2]^2}{(k_1 \cdot p_1)(k_1 \cdot p_2)} \,,$$

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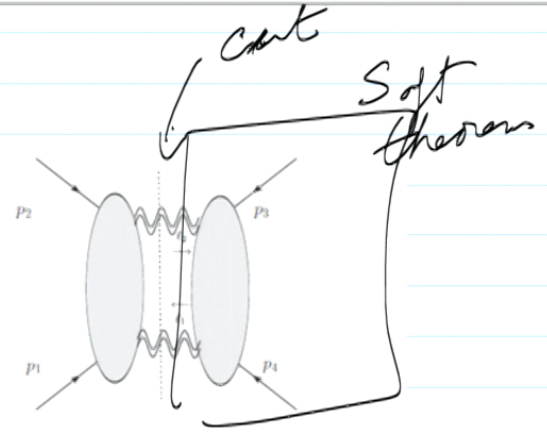
Unitarity techniques

Passarino Veltman

-> Box, triangle, bubbles

(GR) = (YM)^2

Only physical D.O.F.



$$iM_0^{\text{tree}}(p_1,p_2,k_1^+,k_2^+) = \frac{\kappa_{(4)}^2}{16}\frac{1}{(k_1\cdot k_2)}\frac{m^4\,[k_1\,k_2]^4}{(k_1\cdot p_1)(k_1\cdot p_2)},$$

$$iM_0^{\text{tree}}(p_1,p_2,k_1^-,k_2^+) = \frac{\kappa_{(4)}^2}{16}\frac{1}{(k_1\cdot k_2)}\frac{\langle k_1|p_1|k_2]^2\,\langle k_1|p_2|k_2]^2}{(k_1\cdot p_1)(k_1\cdot p_2)},$$

=> same answer

$\frac{4!}{10!}$ is soft theorem - independent of spin

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Massless off of Massive (Bending of light)

$$i\mathcal{M}^{[\eta(p_1)\eta(p_2)]}_{[\phi(p_3)\phi(p_4)]} \simeq \frac{\mathcal{N}^{\eta}}{\hbar}(M\omega)^2\left[\frac{\kappa^2}{t} + \kappa^4\frac{15}{512}\frac{M}{\sqrt{-t}} + \hbar\kappa^4\frac{15}{512\pi^2}\right.$$
$$\times \log\left(\frac{-t}{M^2}\right) - \hbar\kappa^4\frac{bu^{\eta}}{(8\pi)^2}\log\left(\frac{-t}{\mu^2}\right)$$
$$+ \hbar\kappa^4\frac{3}{128\pi^2}\log^2\left(\frac{-t}{\mu^2}\right)$$
$$\left. + \kappa^4\frac{M\omega}{8\pi}\frac{i}{t}\log\left(\frac{-t}{M^2}\right)\right], \qquad (11)$$

$$\theta \simeq \frac{4G_N M}{b} + \frac{15}{4}\frac{G_N^2 M^2\pi}{b^2} + \left(8bu^S + 9 - 48\log\frac{b}{2b_0}\right)\frac{\hbar G_N^2 M}{\pi b^3} + \dots .$$

Massless off of Massive (Bending of light)

$$i\mathcal{M}^{[\eta(p_1)\eta(p_2)]}_{[\phi(p_3)\phi(p_4)]} \simeq \frac{\mathcal{N}^{\eta}}{\hbar}(M\omega)^2\left[\frac{\kappa^2}{t}+\kappa^4\frac{15}{512}\frac{M}{\sqrt{-t}}+\hbar\kappa^4\frac{15}{512\pi^2}\right.$$
$$\times\log\left(\frac{-t}{M^2}\right)-\hbar\kappa^4\frac{bu^{\eta}}{(8\pi)^2}\log\left(\frac{-t}{\mu^2}\right)$$
$$+\hbar\kappa^4\frac{3}{128\pi^2}\log^2\left(\frac{-t}{\mu^2}\right)$$
$$\left.+\kappa^4\frac{M\omega}{8\pi}\frac{i}{t}\log\left(\frac{-t}{M^2}\right)\right], \qquad (11)$$

not universal

(bu) $n^3/4\pi$ scalar

$= -11/120$ γ

$$\theta \simeq \frac{4G_N M}{b}+\frac{15}{4}\frac{G_N^2M^2\pi}{b^2}+\left(8bu^S+9-48\log\frac{b}{2b_0}\right)\frac{\hbar G_N^2 M}{\pi b^3}+\ldots.$$

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$$\theta \simeq \frac{4G_N M}{b} + \frac{15}{4}\frac{G_N M}{b^2}\pi + \left(8bu^S + 9 - 48\log\frac{b}{2b_0}\right)\frac{4G_N M}{\pi b^3} + \dots .$$

Graz

$$\begin{aligned}
\mathcal{A}^{1-loop}(++;--) &= -i\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right)\\
\mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3}\mathcal{A}^{1-loop}(++;--)\\
\mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}}\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\mathcal{A}^{tree}(++;++)\times(s\,t\,u) \qquad (3)\\
&\quad\times\left[\frac{2}{\epsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right.\\
&\quad\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-s)}{ts}\right)\right]
\end{aligned}$$

where

$$\begin{aligned}
f\left(\frac{-t}{s},\frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right)\\
&\quad+\frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u}\\
&\quad+\frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4}. \qquad (4)
\end{aligned}$$

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$$\theta \simeq \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2}{b^2}\pi + \left(8bu^5 + 9 - 48\log\frac{u}{2b_0}\right)\frac{4G^3M^3}{\pi b^3} + \dots .$$

Graviton - Graviton

Dunbar
Norridge

$$\begin{aligned}
\mathcal{A}^{1-loop}(++;--) &= -i\frac{\kappa^4}{30720\pi^2}\left(s^2+t^2+u^2\right)\\
\mathcal{A}^{1-loop}(++;+-) &= -\frac{1}{3}\mathcal{A}^{1-loop}(++;--)\\
\mathcal{A}^{1-loop}(++;++) &= \frac{\kappa^2}{4(4\pi)^{2-\epsilon}}\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\mathcal{A}^{tree}(++;++)\times(s\,t\,u) \qquad (3)\\
&\quad\times\left[\frac{2}{\epsilon}\left(\frac{\ln(-u)}{st}+\frac{\ln(-t)}{su}+\frac{\ln(-s)}{tu}\right)+\frac{1}{s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right.\\
&\quad\left.+2\left(\frac{\ln(-u)\ln(-s)}{su}+\frac{\ln(-t)\ln(-s)}{tu}+\frac{\ln(-t)\ln(-s)}{ts}\right)\right]
\end{aligned}$$

No c_1, c_2

where

$$\begin{aligned}
f\left(\frac{-t}{s},\frac{-u}{s}\right) &= \frac{(t+2u)(2t+u)\left(2t^4+2t^3u-t^2u^2+2tu^3+2u^4\right)}{s^6}\left(\ln^2\frac{t}{u}+\pi^2\right)\\
&\quad+\frac{(t-u)\left(341t^4+1609t^3u+2566t^2u^2+1609tu^3+341u^4\right)}{30s^5}\ln\frac{t}{u}\\
&\quad+\frac{1922t^4+9143t^3u+14622t^2u^2+9143tu^3+1922u^4}{180s^4}. \qquad (4)
\end{aligned}$$

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$\frac{1}{\varepsilon}$ canceled

$$\left(\frac{d\sigma}{d\Omega}\right)_{tree} + \left(\frac{d\sigma}{d\Omega}\right)_{rad.} + \left(\frac{d\sigma}{d\Omega}\right)_{non\,rad.} = \tag{29}$$

$$= \frac{\kappa^4 s^5}{2048\pi^2 t^2 u^2}\left\{1 + \frac{\kappa^2 s}{16\pi^2}\left[\ln\frac{-t}{s}\ln\frac{-u}{s} + \frac{tu}{2s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right.\right.$$

$$\left.\left. - \left(\frac{t}{s}\ln\frac{-t}{s} + \frac{u}{s}\ln\frac{-u}{s}\right)\left(3\ln(2\pi^2) + \gamma + \ln\frac{s}{\Lambda^2} + \frac{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(1)}(\gamma_{ij})}{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(0)}(\gamma_{ij})}\right)\right]\right\}.$$

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$\frac{1}{\varepsilon}$ canceled by soft radiation

$$\left(\frac{d\sigma}{d\Omega}\right)_{tree} + \left(\frac{d\sigma}{d\Omega}\right)_{rad.} + \left(\frac{d\sigma}{d\Omega}\right)_{nonrad.} = \tag{29}$$

$$= \frac{\kappa^4 s^5}{2048\pi^2 t^2 u^2}\left\{1 + \frac{\kappa^2 s}{16\pi^2}\left[\ln\frac{-t}{s}\ln\frac{-u}{s} + \frac{tu}{2s^2}f\left(\frac{-t}{s},\frac{-u}{s}\right)\right.\right.$$

$$\left.\left. - \left(\frac{t}{s}\ln\frac{-t}{s} + \frac{u}{s}\ln\frac{-u}{s}\right)\left(3\ln(2\pi^2) + \gamma + \ln\frac{s}{\Lambda^2} + \frac{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(1)}(\gamma_{ij})}{\sum_{ij}\eta_i\eta_j\mathcal{F}^{(0)}(\gamma_{ij})}\right)\right]\right\}.$$

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Beyond Scattering

QED $\mathcal{L} = \int d^4x \; -\frac{1}{4}F^2 + \int d^4x\, d^4y\, b e^2 F_{\mu\nu}(x) \Delta(x-y) F^{\mu\nu}(y)$

Δ : F.T $\ln g^2$

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Beyond Scattering

QED $\mathcal{L} = \int d^4x \; -\frac{1}{4}F^2 + \int d^4x\, d^4y\, b e^2 F_{\mu\nu}(x) \Delta(x-y) F^{\mu\nu}(y)$

Δ : FT $\ln g^2$

Notation $L(x-y) = \langle x | \ln \Box | y \rangle$

Beyond Scattering

QED $S = \int d^4x \left(-\frac{1}{4}F^2\right) + \int d^4x\, d^4y\, be^2 F_{\mu\nu}(x) \Box(x-y) F^{\mu\nu}(y)$

$\Box$: FT $\ln g^2$

Notation $L(x-y) = \langle x | \ln \Box | y \rangle$

$$S = \int d^4x \left[-\frac{1}{4} F^2 + be^2 F_{\mu\nu} \ln \Box F^{\mu\nu} \right]$$

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— non local L

$$S_4 = \int d^4x \sqrt{-g} \left[c_1 R^2 + \alpha R \ln\Box R + \beta R_{\mu\nu} \ln\Box R^{\mu\nu} \cdots \right]$$

also $R^2 \frac{1}{\Box} R \sim O(\delta^4$

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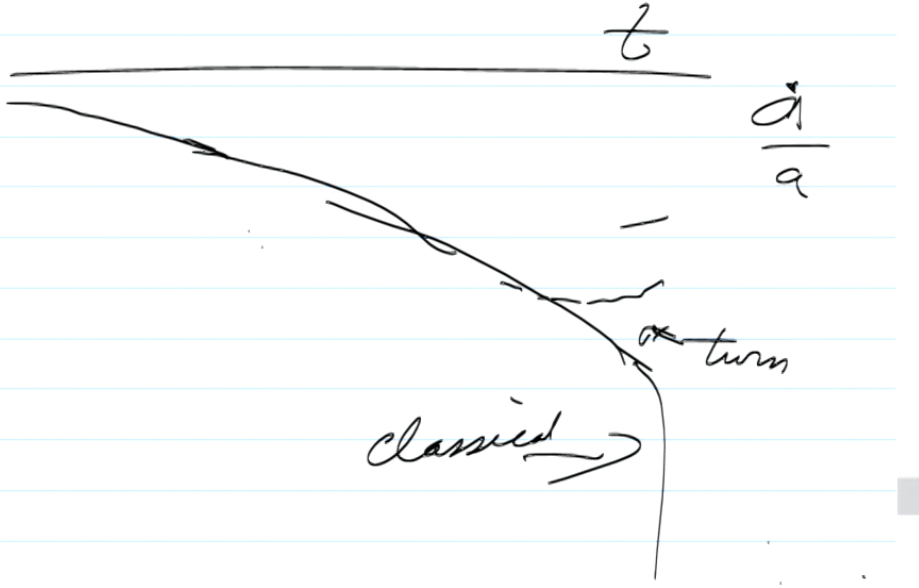
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$$S_4 = \int d^4x \sqrt{-g} \left[c_1 R^2 + \alpha R \ln\Box R + \beta R_{\mu\nu} \ln\Box R^{\mu\nu} \cdots \right]$$

also $R^2 \frac{1}{\Box} R \sim O(\delta^4)$

Example Collapsing FLRW



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Limits of EFT

UV

$$\mathcal{M} = \mathcal{M}_0 \left[1 + k c_i g^2 + k g^2 \ln g^2 \right]$$

Falls apart

$$G g^2 c_i \sim 1$$

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IR limits to calculations

- Grav effects cumulate

$$\frac{1}{1 - \frac{2GM}{r}}$$

small curvature large M

P.T. Sensitivity to very IR effects in non trivial

Cliff B = horizon

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"A lot of portentous drivel has been written about the quantum theory of gravity, so I'd like to begin by making a fundamental observation about it that tends to be obfuscated. There is a perfectly well-defined quantum theory of gravity that agrees accurately with all available experimental data."

Frank Wilczek
Physics Today
2002

$$Z = \int_{\Lambda} [d\phi\, dA\, d\psi] \exp i \int d^4x \sqrt{-g} \left[-\frac{1}{4} F^2 + \bar{\psi} i \not{D} \psi + 2 \phi^* \partial^\mu \phi - V(\phi) \right.$$
$$\left. - \Lambda - \frac{2}{K^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \cdots \right]$$

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