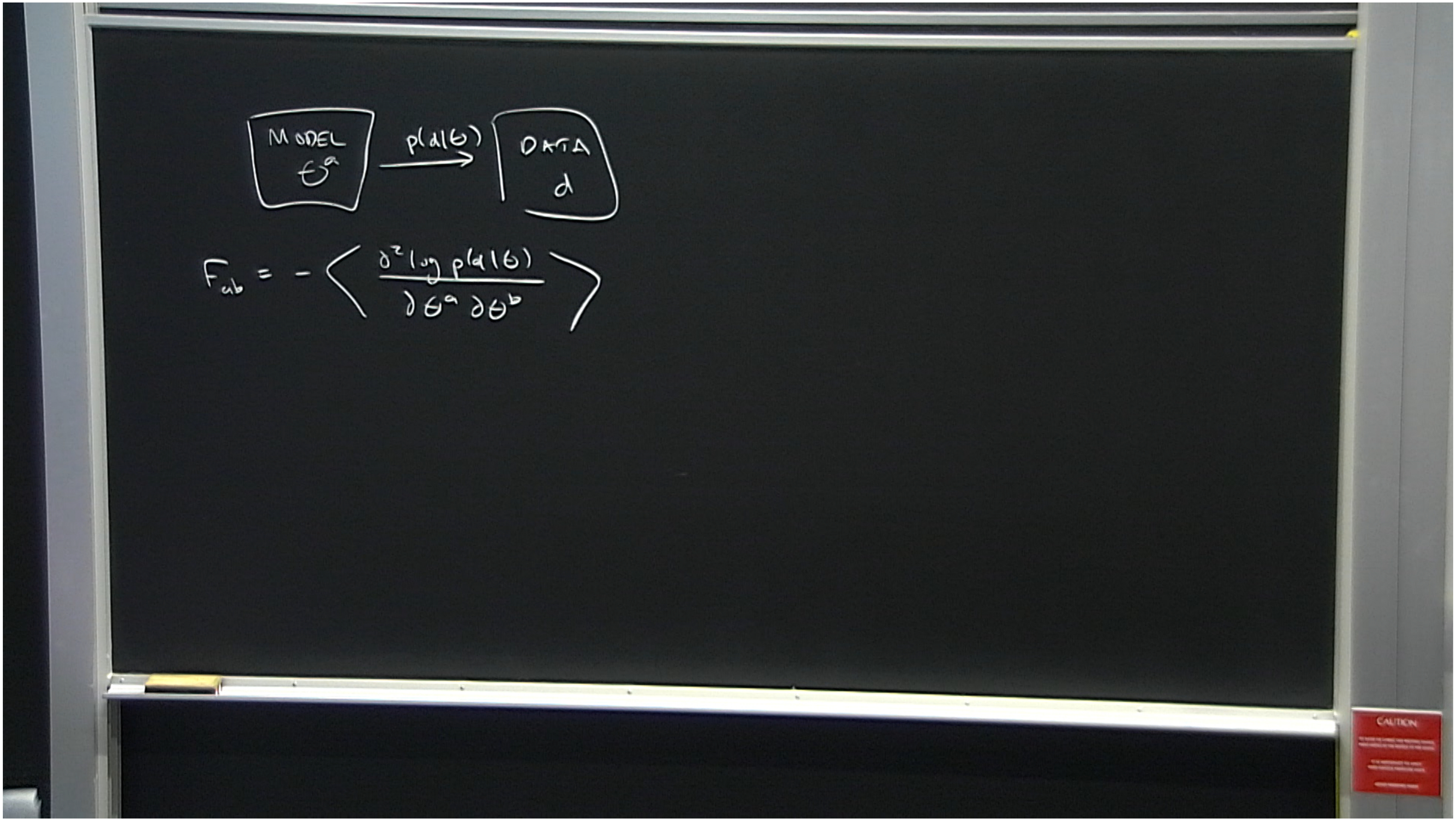
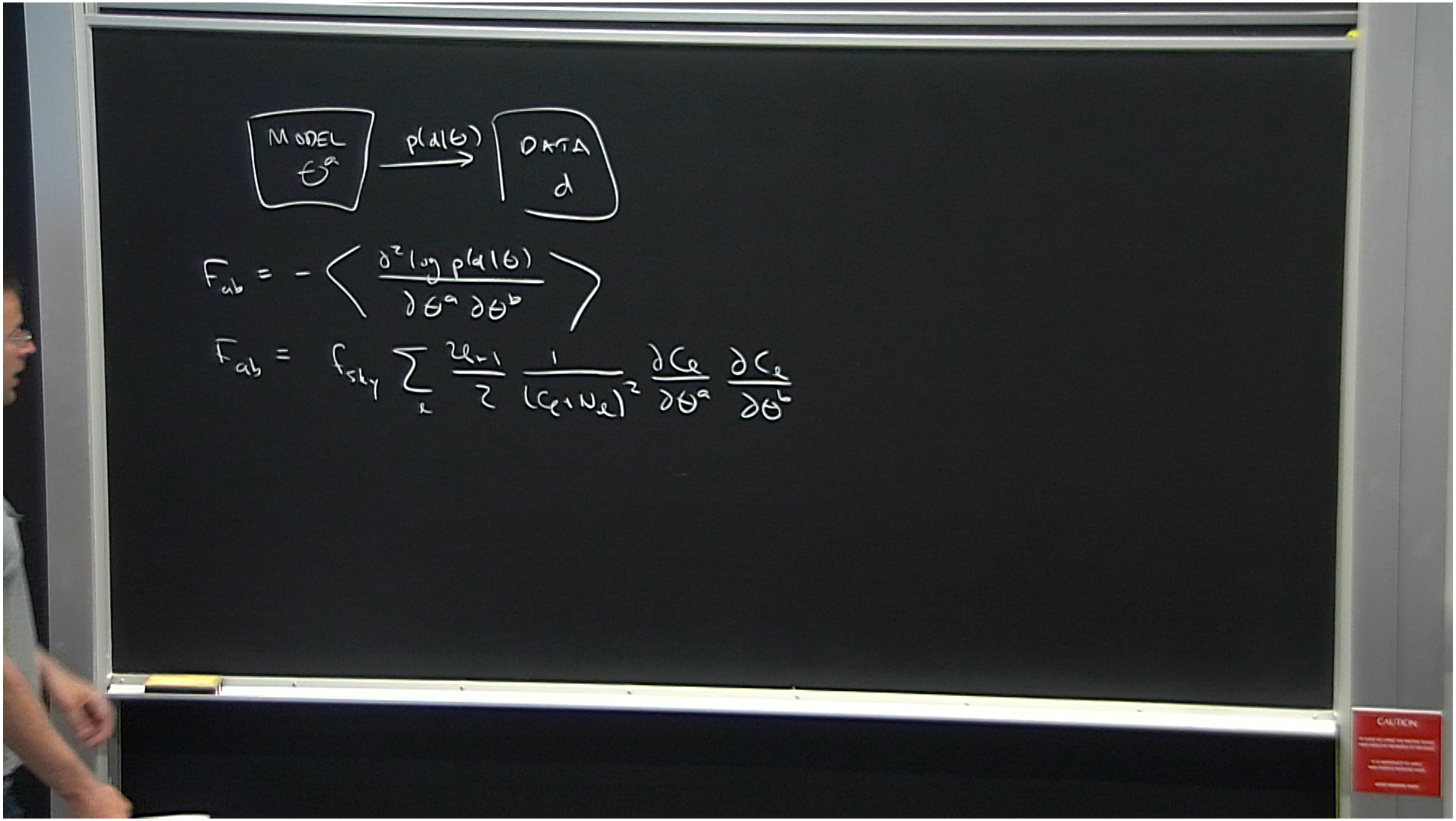
Title: Cosmology Observations 3

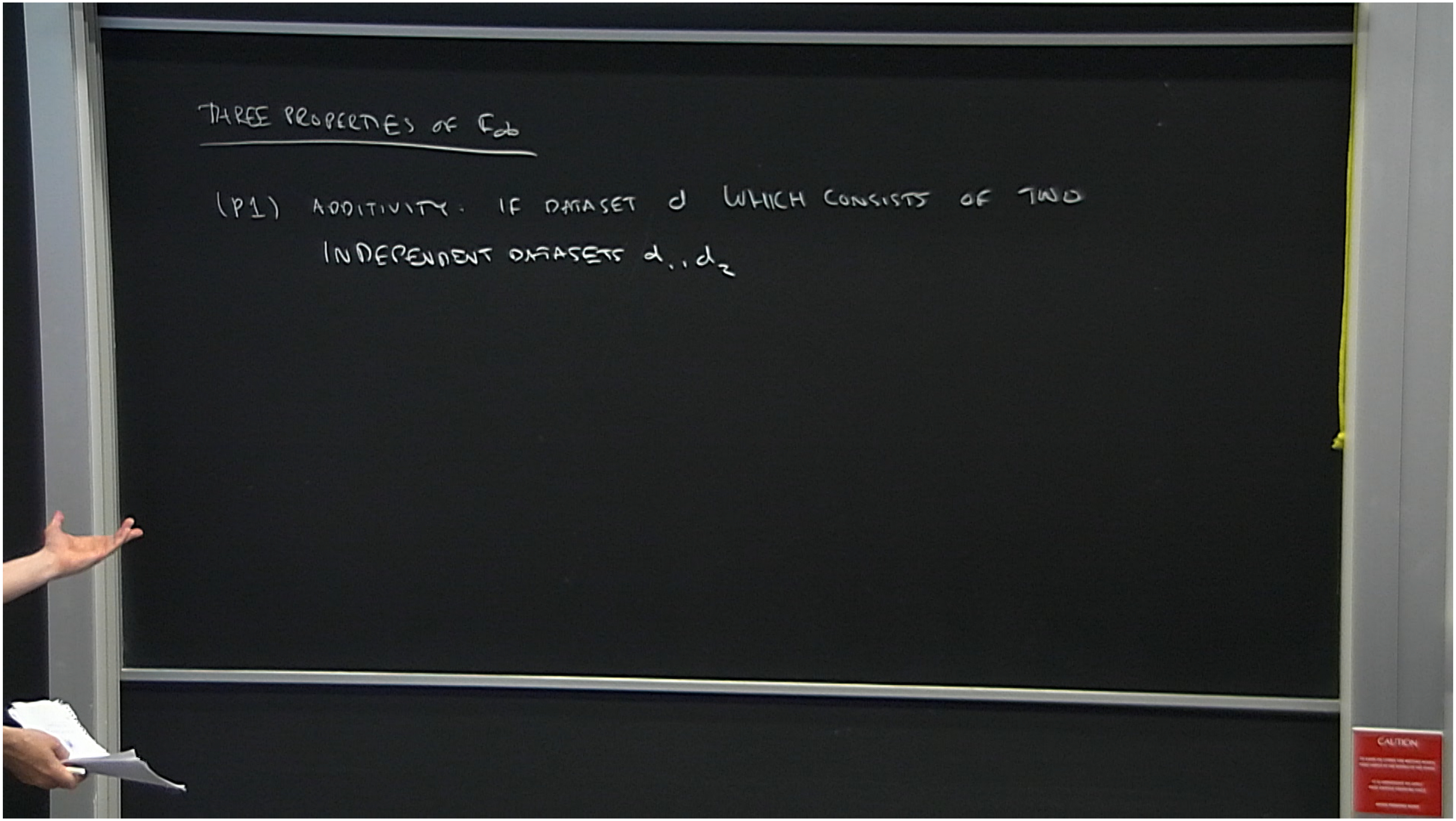
Date: Jul 13, 2018 09:00 AM

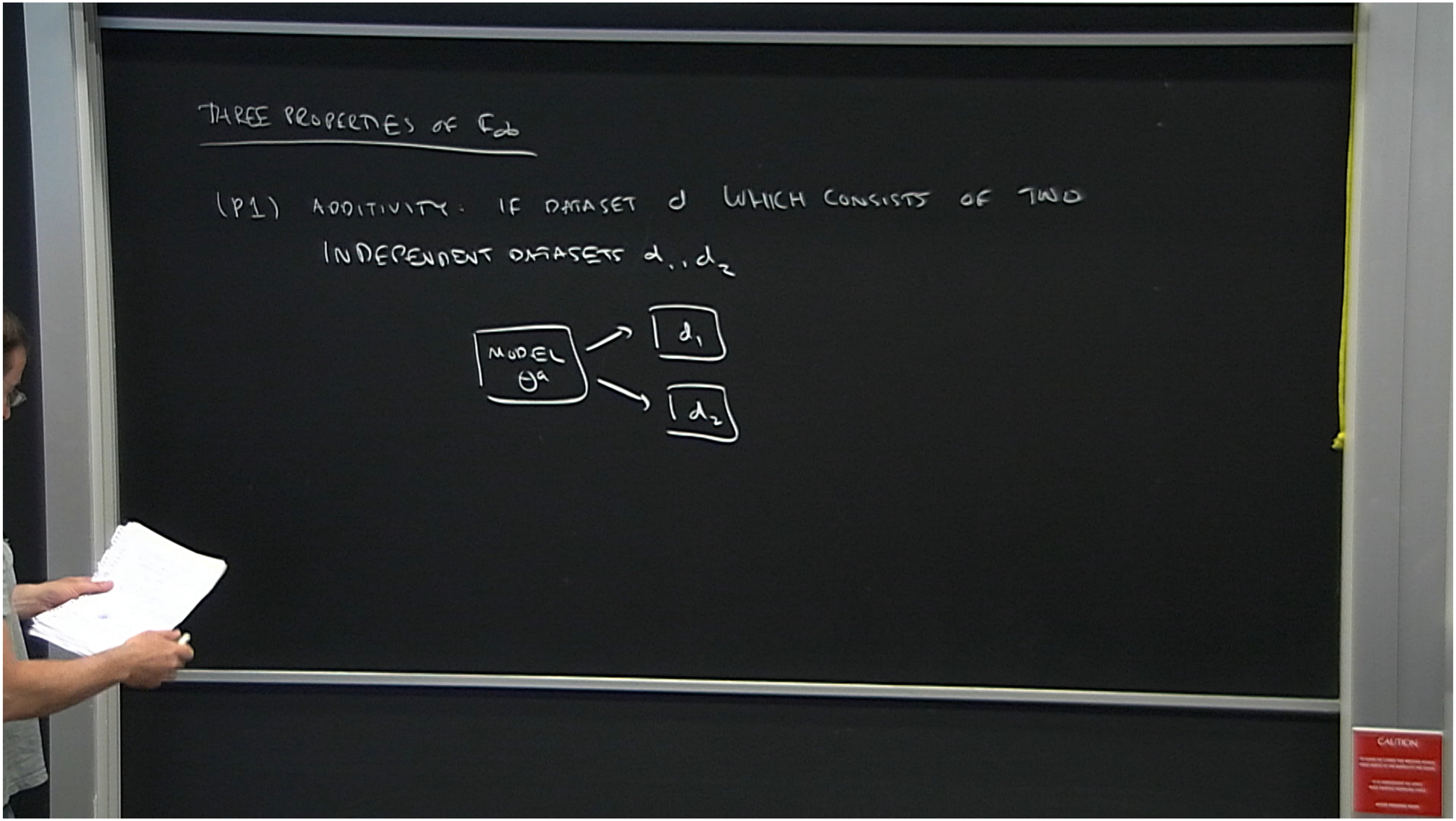
URL: http://pirsa.org/18070003

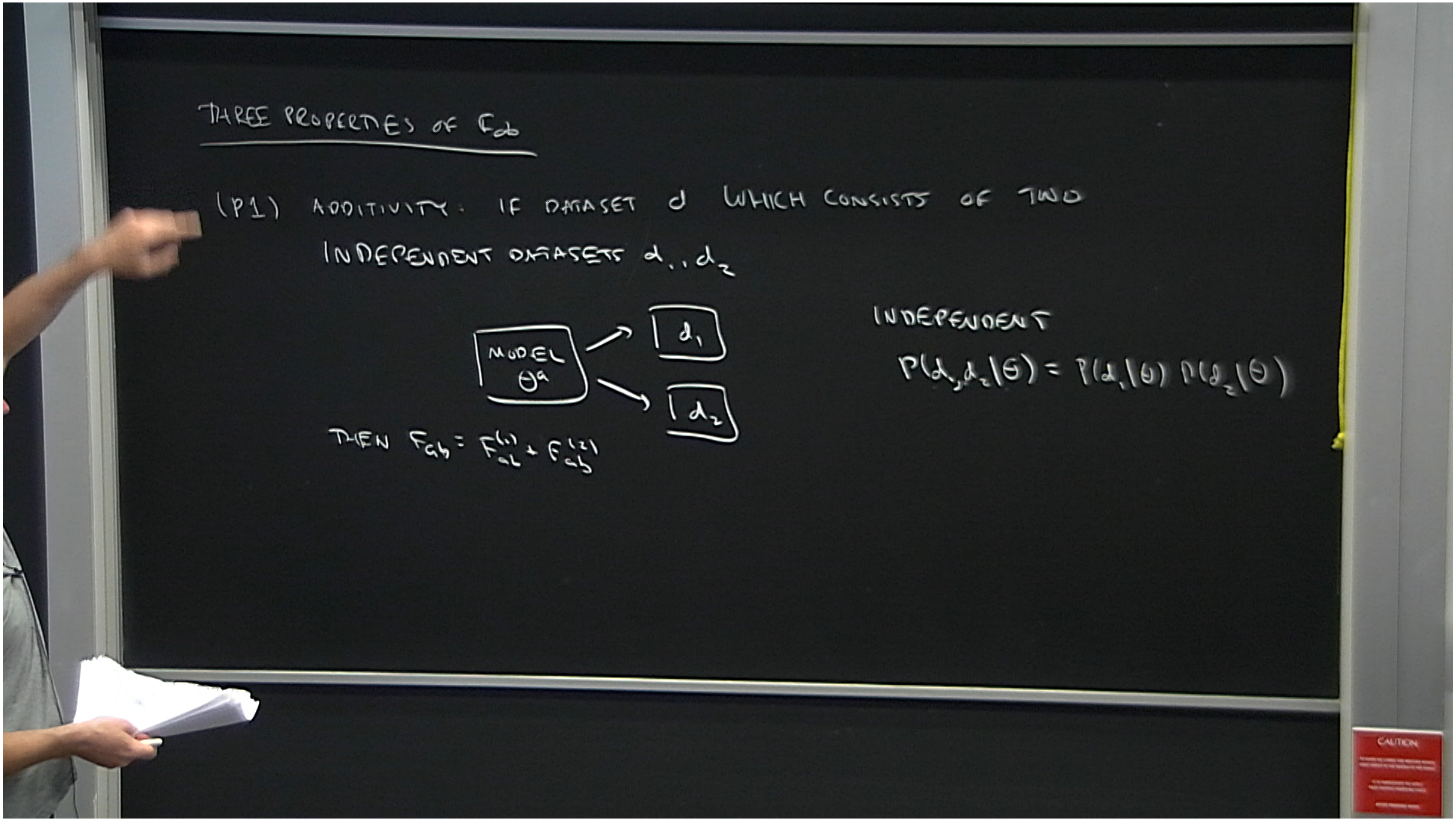
Abstract:


MODEL
θ^a
$p(d|\theta)$
DATA
d
$F_{ab} = -\left\langle \frac{\partial^2 \log p(d|\theta)}{\partial\theta^a \partial\theta^b} \right\rangle$


MODEL
θ^a
$p(d|\theta)$
DATA
d
$F_{ab} = -\left\langle \frac{\partial^2 \log p(d|\theta)}{\partial\theta^a \partial\theta^b} \right\rangle$
$F_{ab} = f_{sky} \sum_\ell \frac{2\ell+1}{2} \frac{1}{(C_\ell + N_\ell)^2} \frac{\partial C_\ell}{\partial\theta^a} \frac{\partial C_\ell}{\partial\theta^b}$

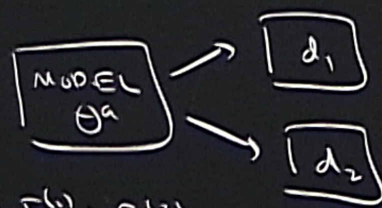

THREE PROPERTIES OF F_{ob}
(P1) ADDITIVITY. IF DATASET d WHICH CONSISTS OF TWO
INDEPENDENT DATASETS d_1, d_2


THREE PROPERTIES OF F_{ab}
(P1) ADDITIVITY: IF DATASET d WHICH CONSISTS OF TWO
INDEPENDENT DATASETS d_1, d_2
MODEL
θ^a
d_1
d_2


THREE PROPERTIES OF F_{ab}
(P1) ADDITIVITY: IF DATASET d WHICH CONSISTS OF TWO
INDEPENDENT DATASETS d_1, d_2
MODEL θ^a
d_1
d_2
THEN $F_{ab} = F^{(1)}_{ab} + F^{(2)}_{ab}$
INDEPENDENT
$P(d_1, d_2 | \theta) = P(d_1|\theta) P(d_2|\theta)$
CAUTION

THREE PROPERTIES OF F_{ab}

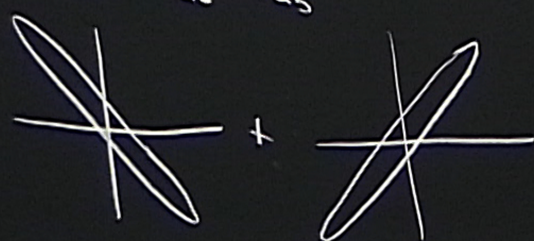
(P1) ADDITIVITY: IF DATASET d WHICH CONSISTS OF TWO INDEPENDENT DATASETS d_1, d_2

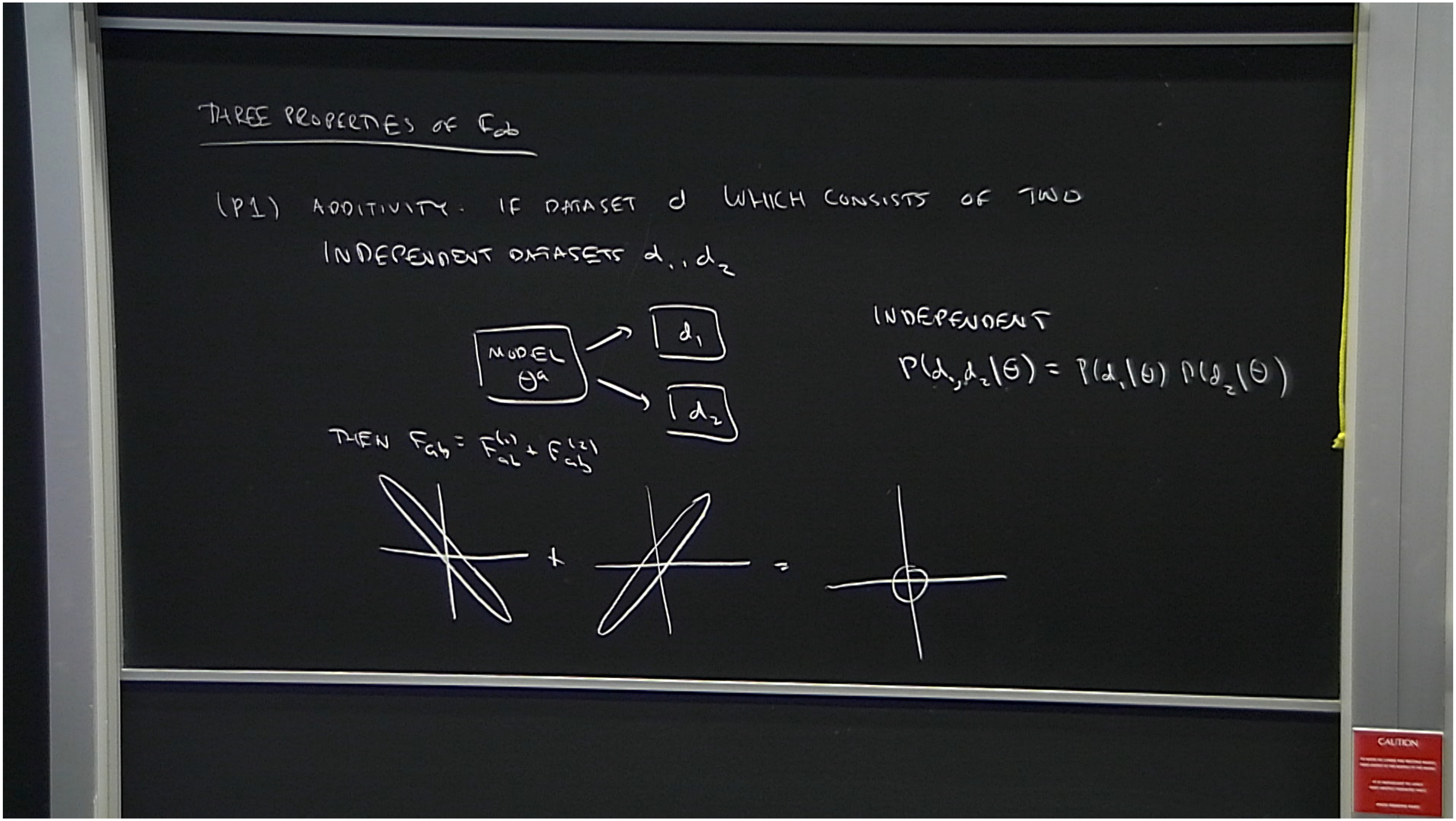


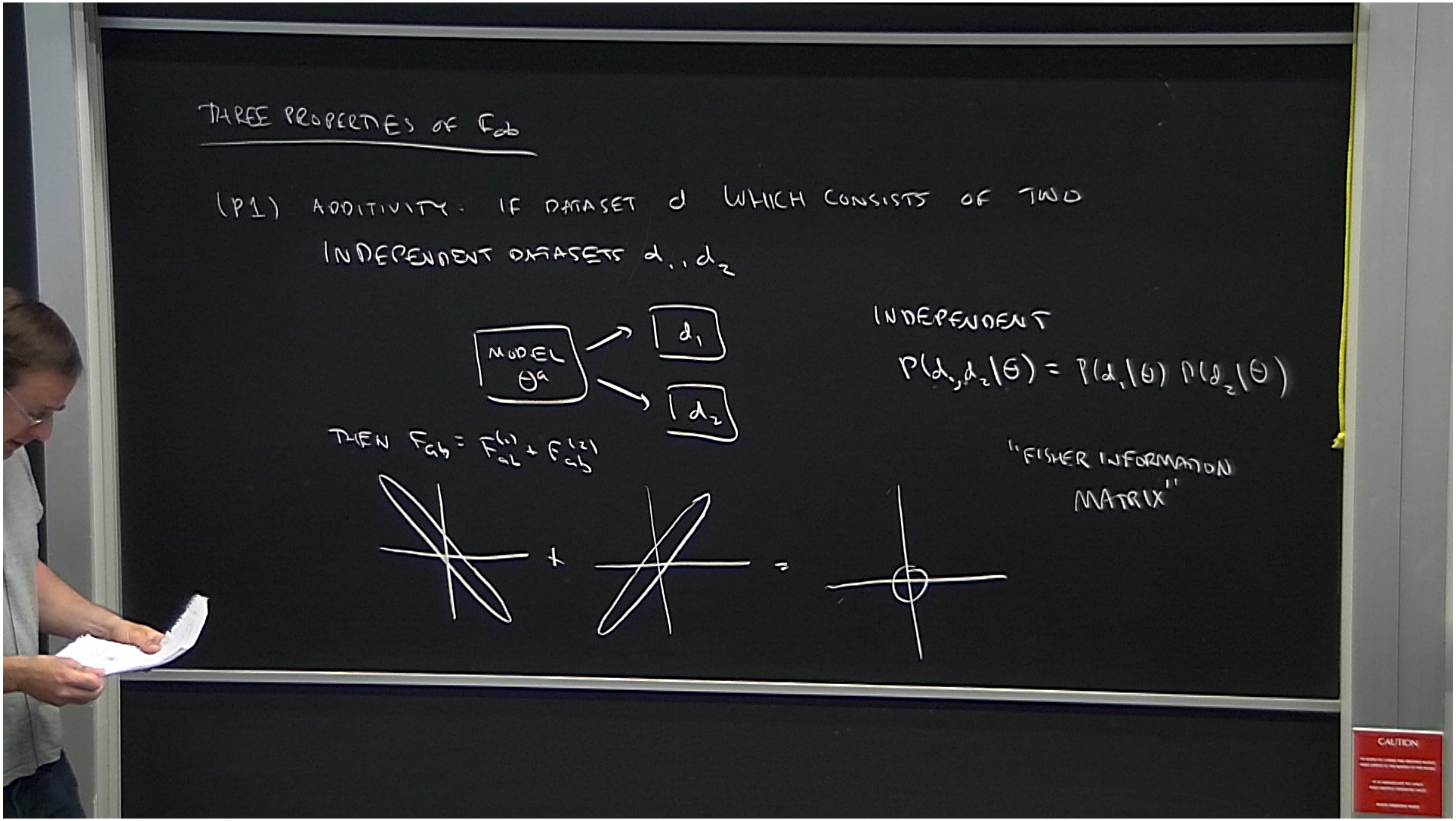
INDEPENDENT

$$P(d_1, d_2|\theta) = P(d_1|\theta)\, P(d_2|\theta)$$

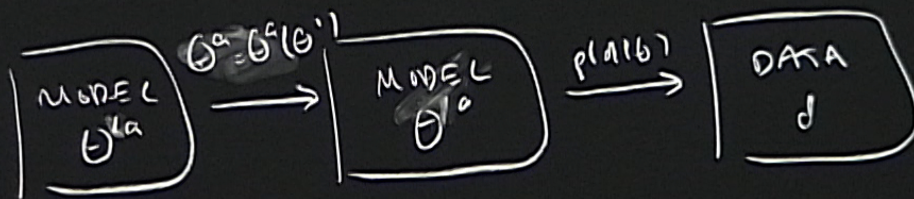
THEN $F_{ab} = F^{(1)}_{ab} + F^{(2)}_{ab}$



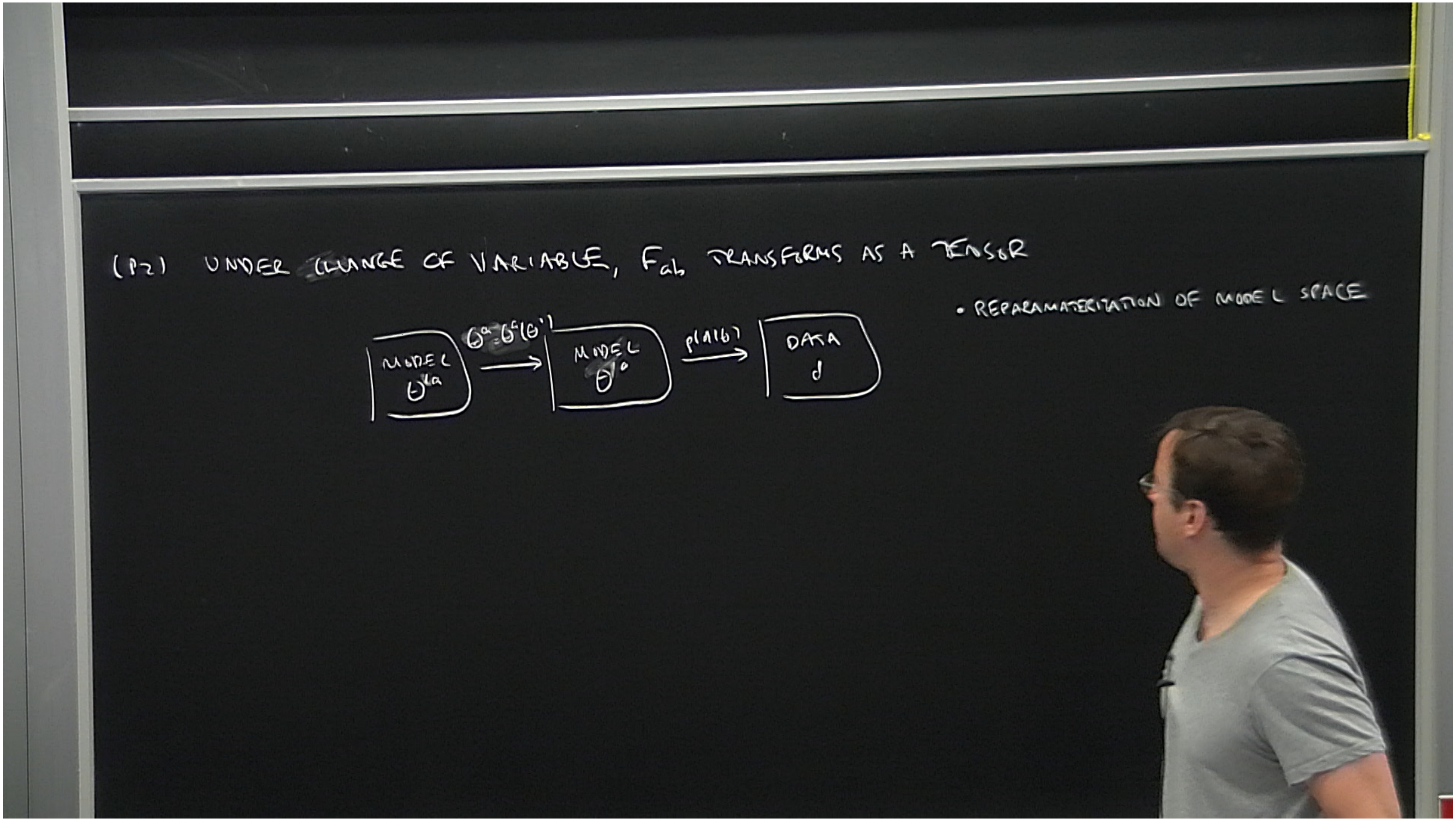

THREE PROPERTIES OF F_ab
(P1) ADDITIVITY. IF DATASET d WHICH CONSISTS OF TWO
INDEPENDENT DATASETS d_1, d_2
MODEL θ^a
d_1
d_2
INDEPENDENT
P(d_1, d_2 | θ) = P(d_1 | θ) P(d_2 | θ)
THEN F_ab = F_ab^(1) + F_ab^(2)

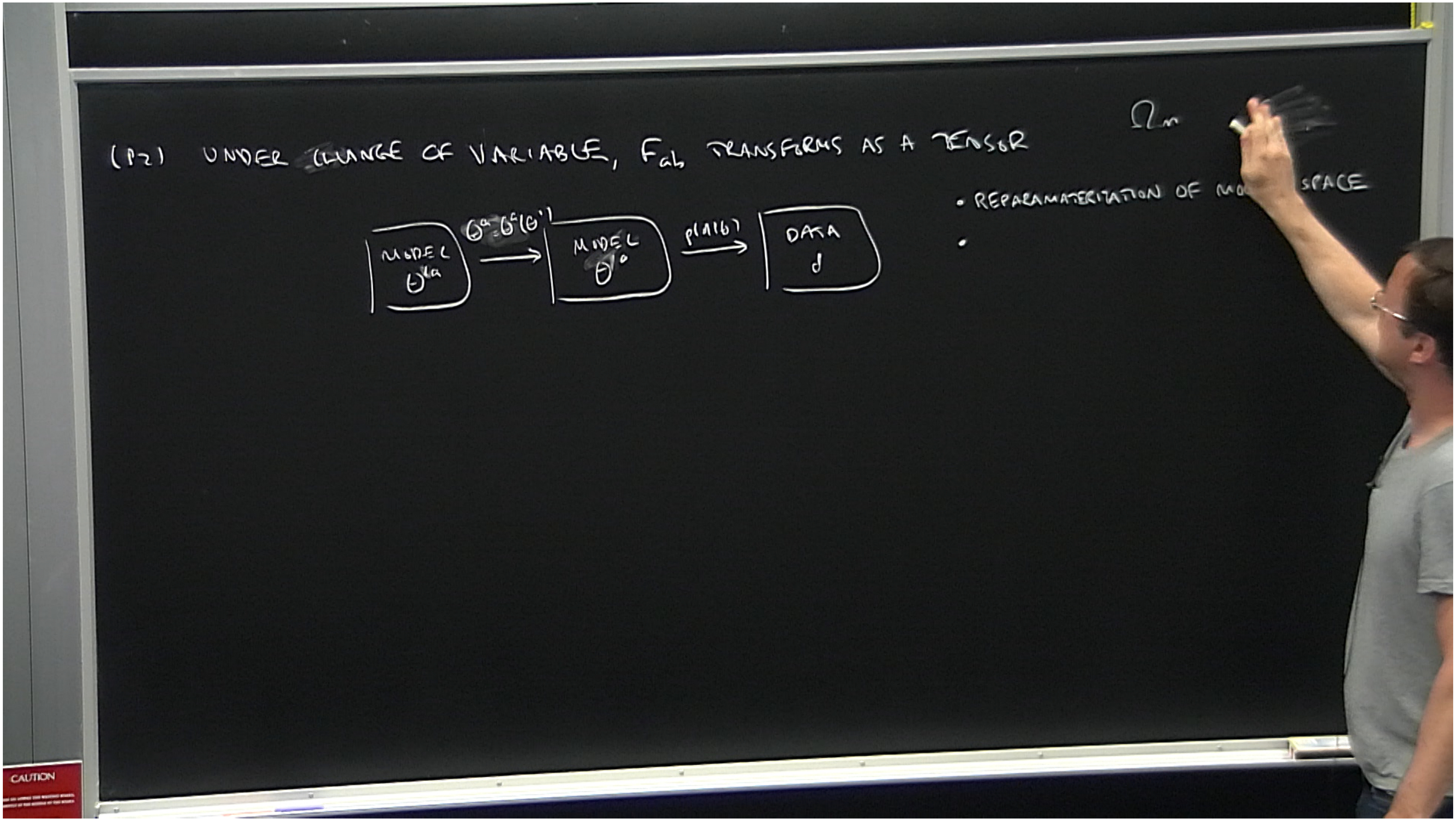

THREE PROPERTIES OF F_{ab}
(P1) ADDITIVITY. IF DATASET d WHICH CONSISTS OF TWO
INDEPENDENT DATASETS d_1, d_2
MODEL θ^a
d_1
d_2
INDEPENDENT
$P(d_1, d_2 | \theta) = P(d_1|\theta)\, P(d_2|\theta)$
THEN $F_{ab} = F^{(1)}_{ab} + F^{(2)}_{ab}$
"FISHER INFORMATION
MATRIX"
CAUTION

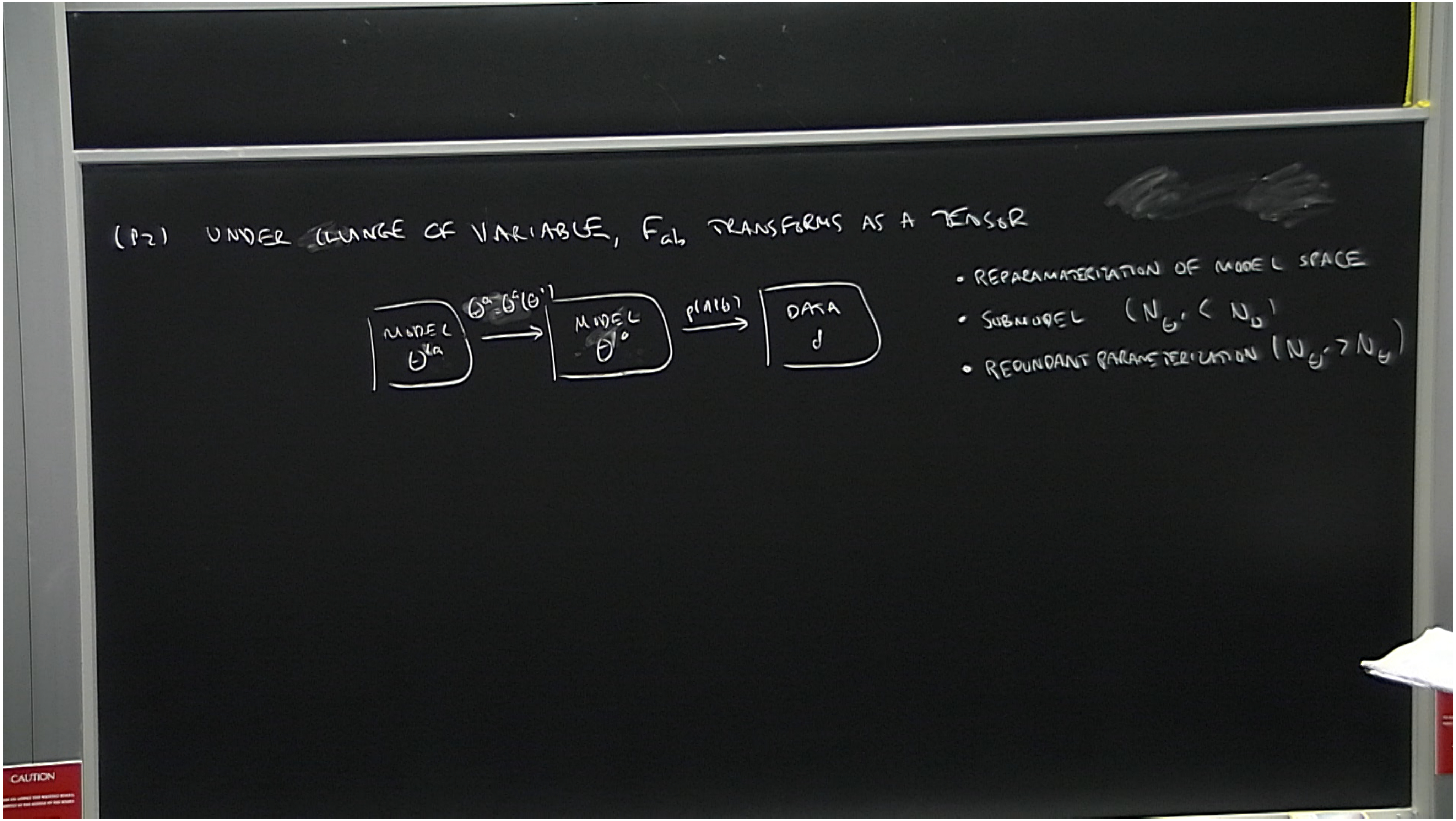
(P2) UNDER CHANGE OF VARIABLE, F_{ab} TRANSFORMS AS A TENSOR

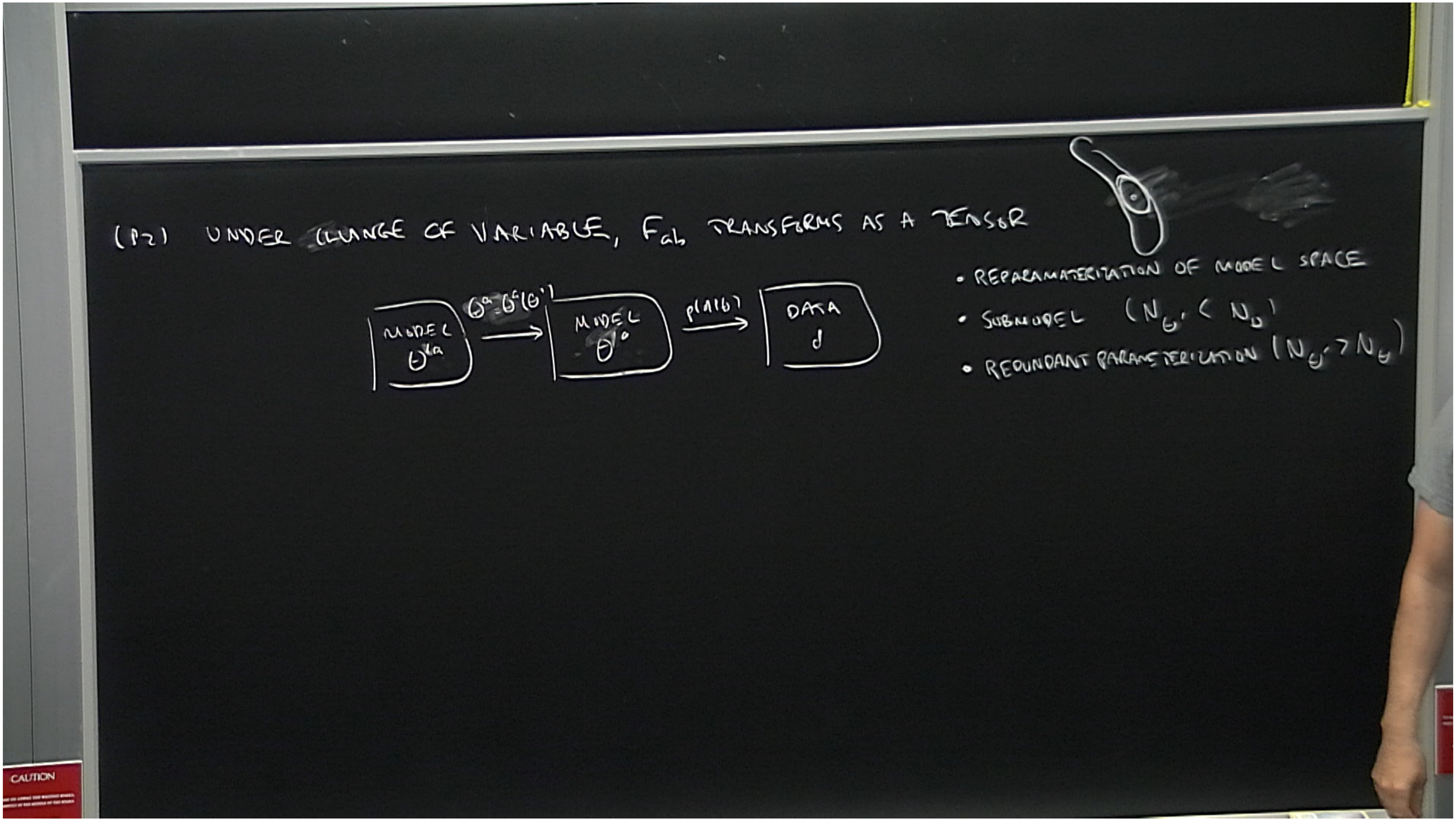


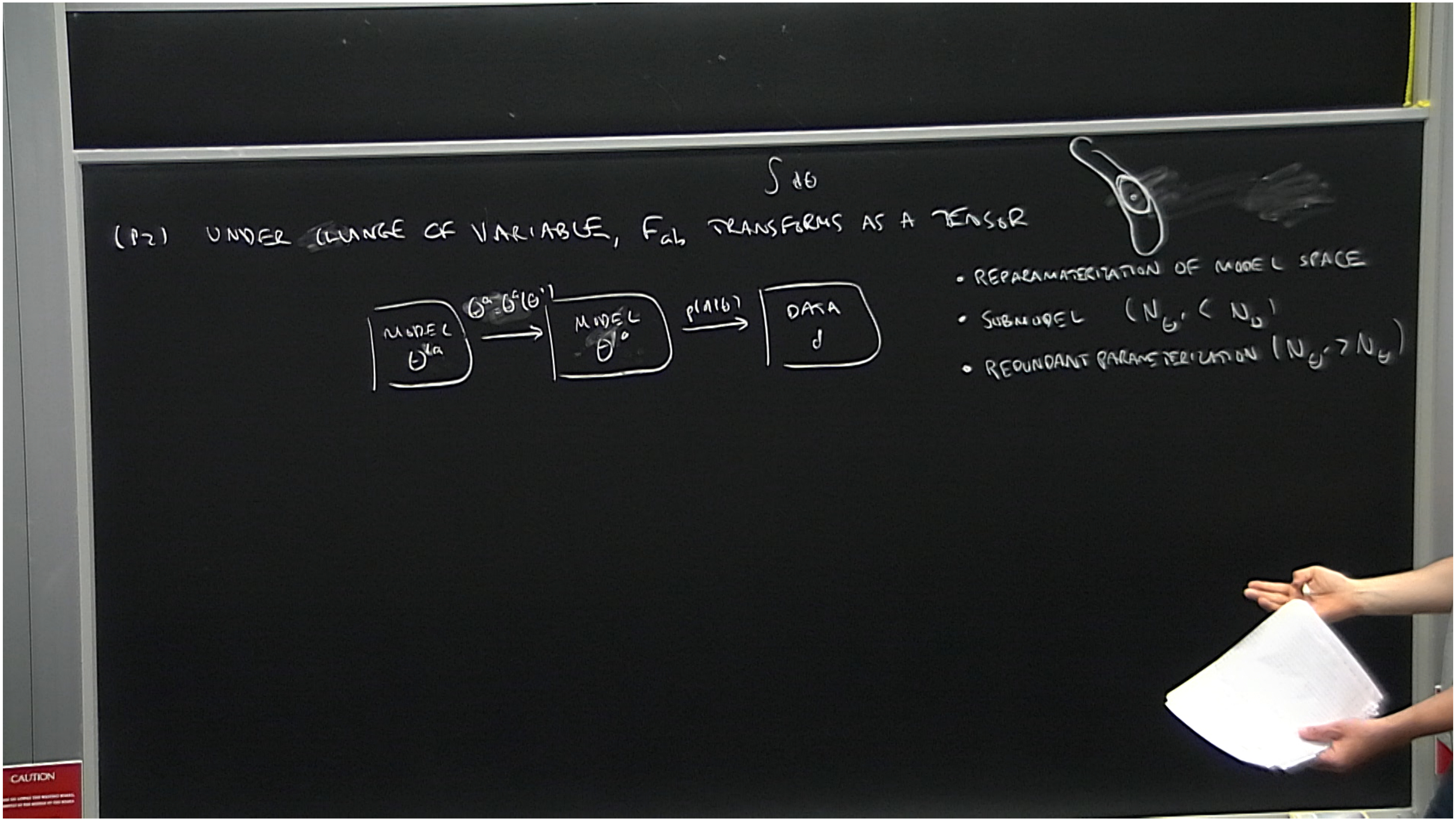
- REPARAMATERIZATION OF

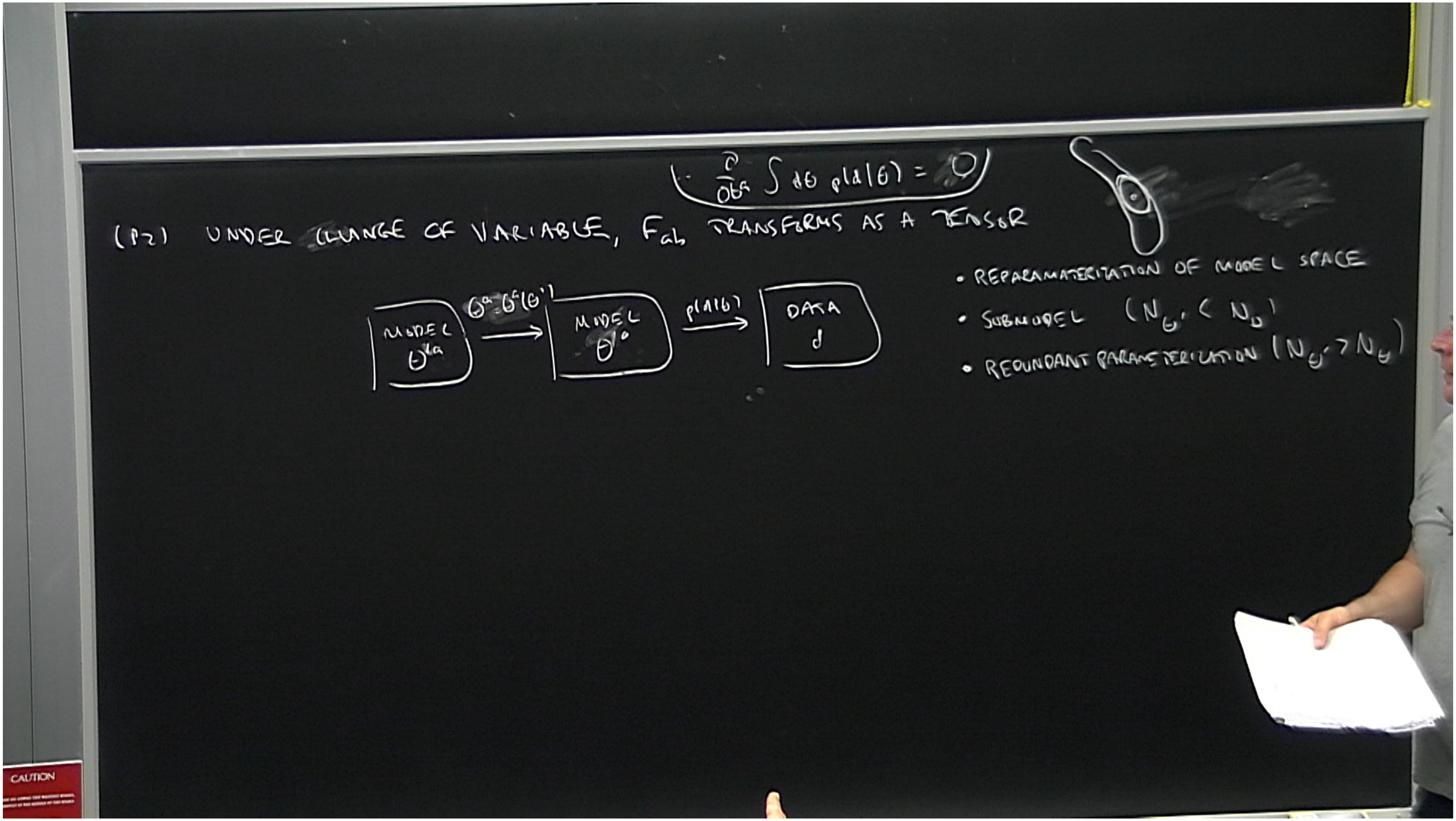

(P2) UNDER CHANGE OF VARIABLE, F_ab TRANSFORMS AS A TENSOR
• REPARAMATERIZATION OF MODEL SPACE
MODEL θ'^a
θ^a = θ^a(θ')
MODEL θ^a
p(d|θ)
DATA d


(P2) UNDER CHANGE OF VARIABLE, F_{ab} TRANSFORMS AS A TENSOR
Ω_m
• REPARAMATERIZATION OF MO SPACE
•
MODEL θ'^a
$\theta^a = \theta^a(\theta')$
MODEL θ^a
$p(d|\theta)$
DATA d
CAUTION

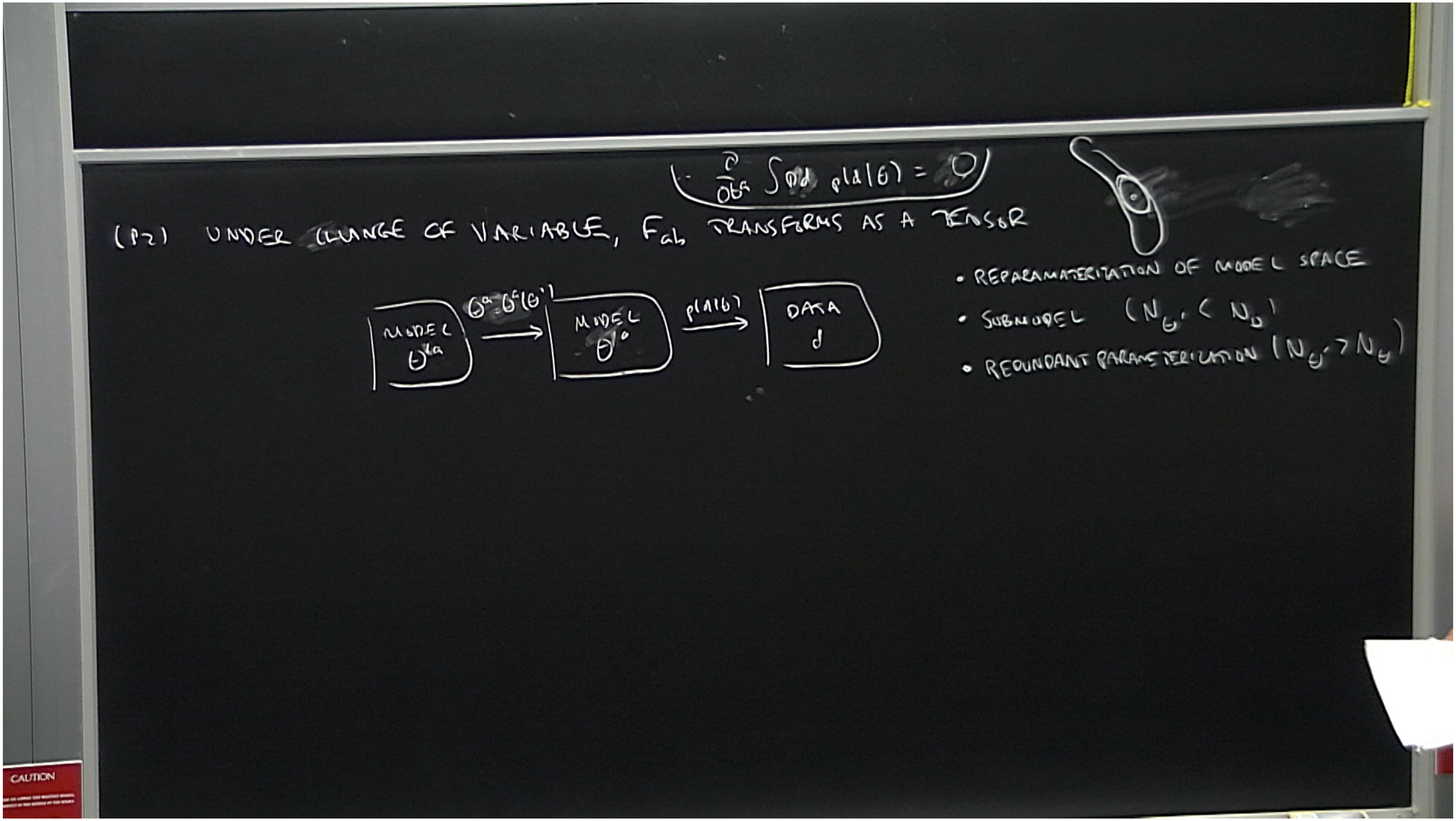

(P2) UNDER CHANGE OF VARIABLE, F_ab TRANSFORMS AS A TENSOR
MODEL θ'^a
θ^a = θ^a(θ')
MODEL θ^a
p(d|θ)
DATA d
• REPARAMATERIZATION OF MODEL SPACE
• SUBMODEL (N_θ' < N_θ)
• REDUNDANT PARAMETERIZATION (N_θ' > N_θ)

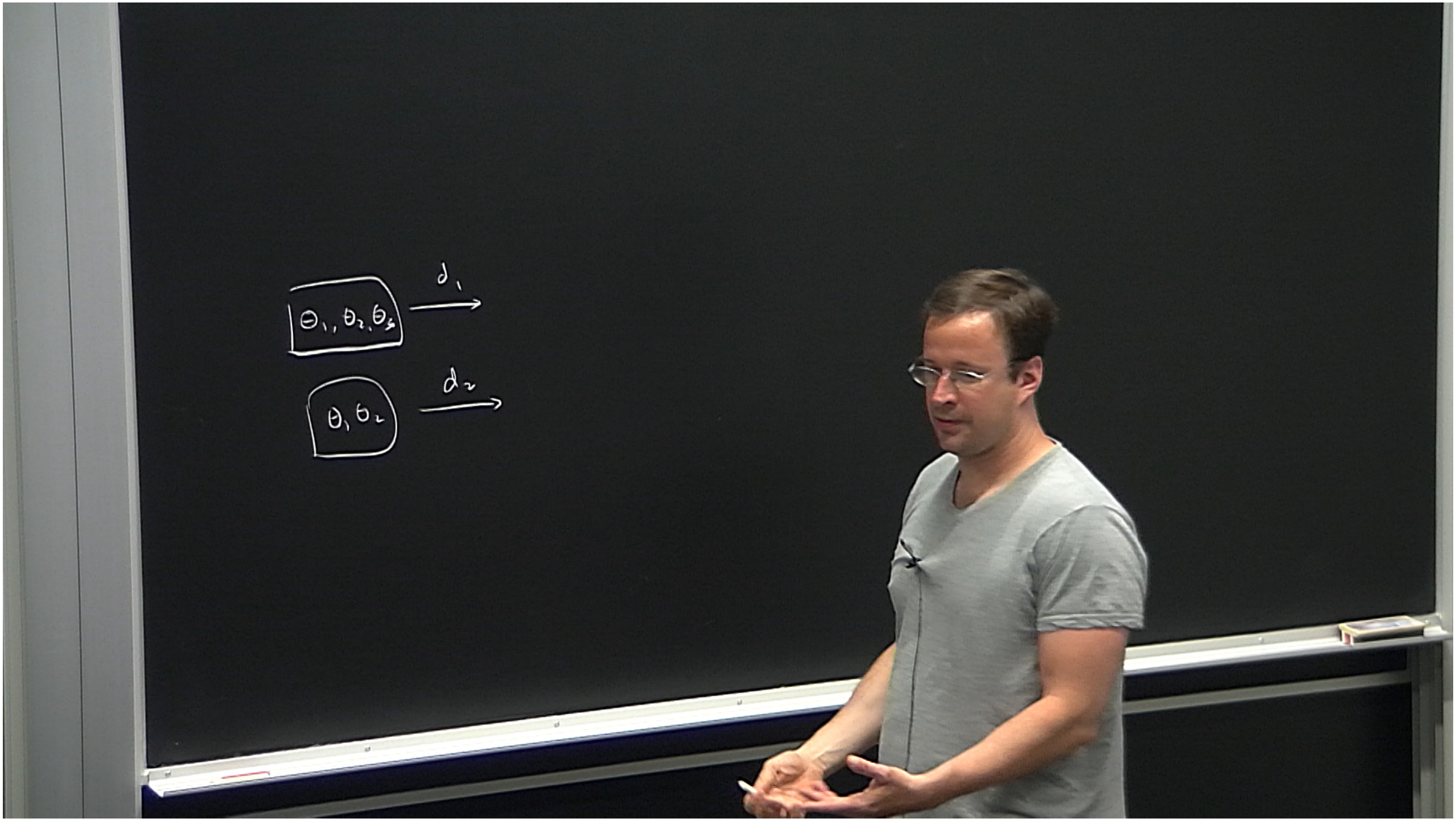

(P2) UNDER CHANGE OF VARIABLE, F_{ab} TRANSFORMS AS A TENSOR
MODEL θ'^a
$\theta^a = \theta^a(\theta')$
MODEL θ^a
$p(d|\theta)$
DATA d
• REPARAMETERIZATION OF MODEL SPACE
• SUBMODEL $(N_{\theta'} < N_\theta)$
• REDUNDANT PARAMETERIZATION $(N_{\theta'} > N_\theta)$
CAUTION

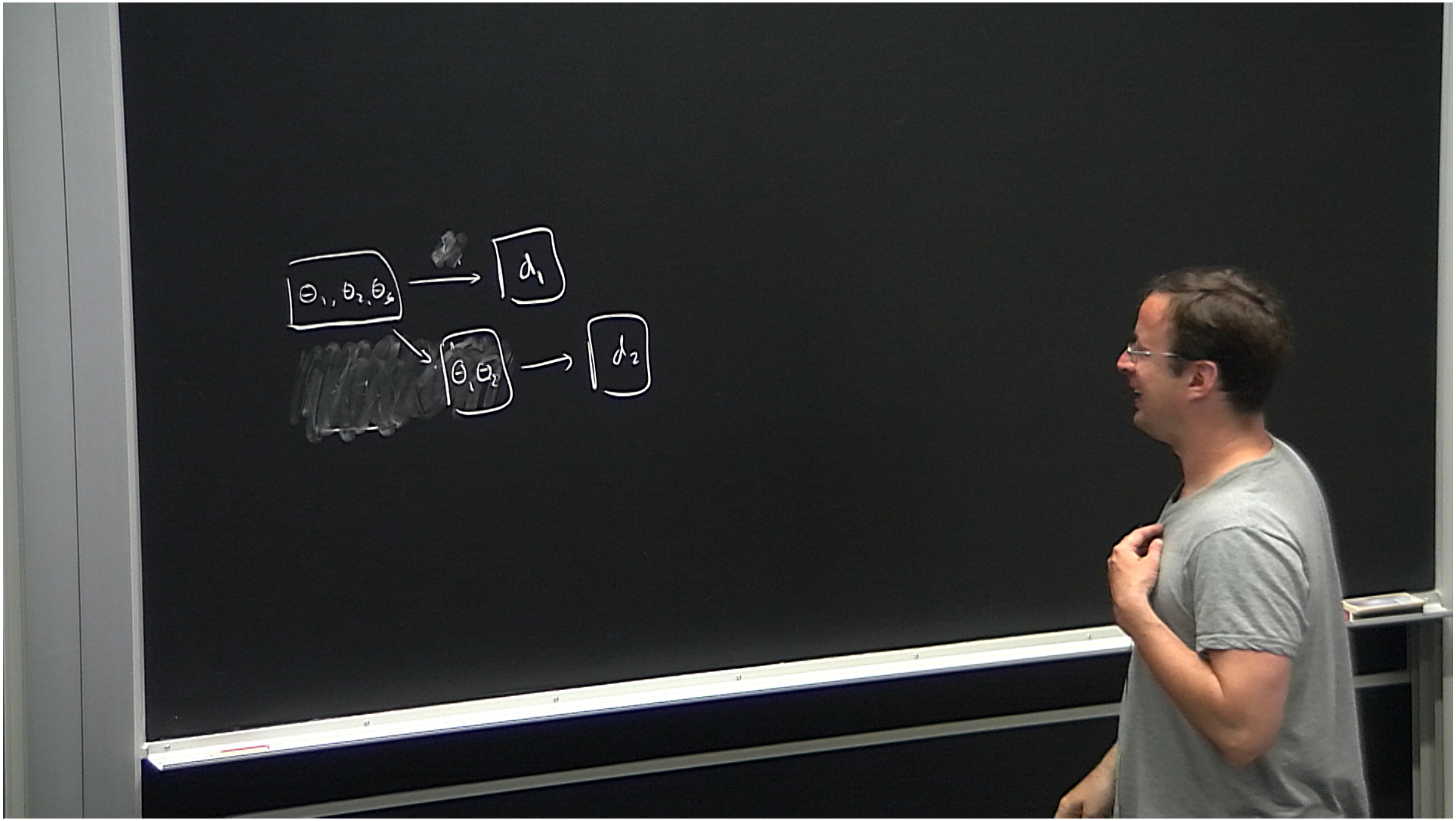

∫ dθ
(P2) UNDER CHANGE OF VARIABLE, F_ab TRANSFORMS AS A TENSOR
MODEL θ'^a
θ^a = θ^a(θ')
MODEL θ^a
p(d|θ)
DATA d
• REPARAMATERIZATION OF MODEL SPACE
• SUBMODEL (N_θ' < N_θ)
• REDUNDANT PARAMETERIZATION (N_θ' > N_θ)
CAUTION

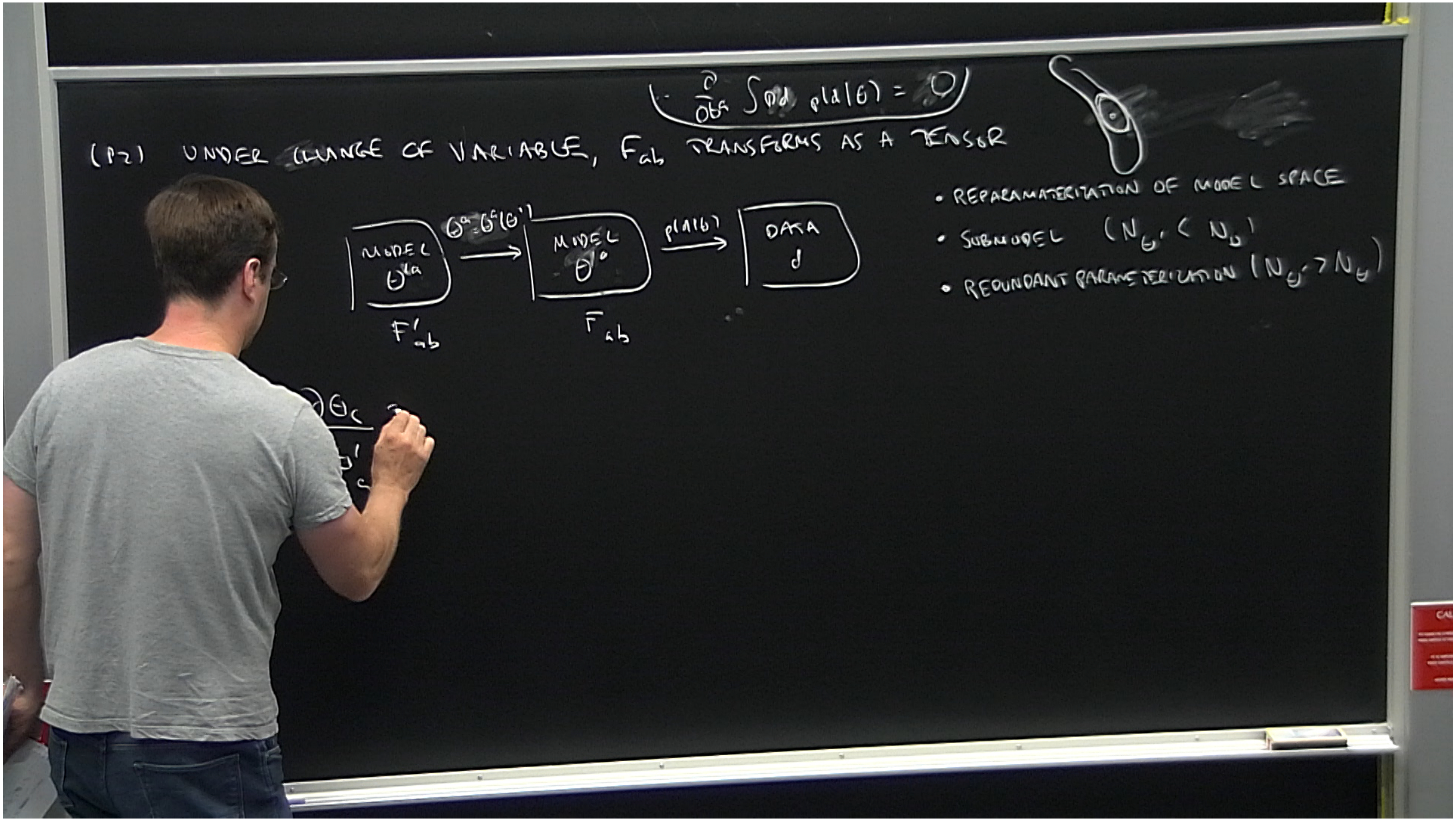


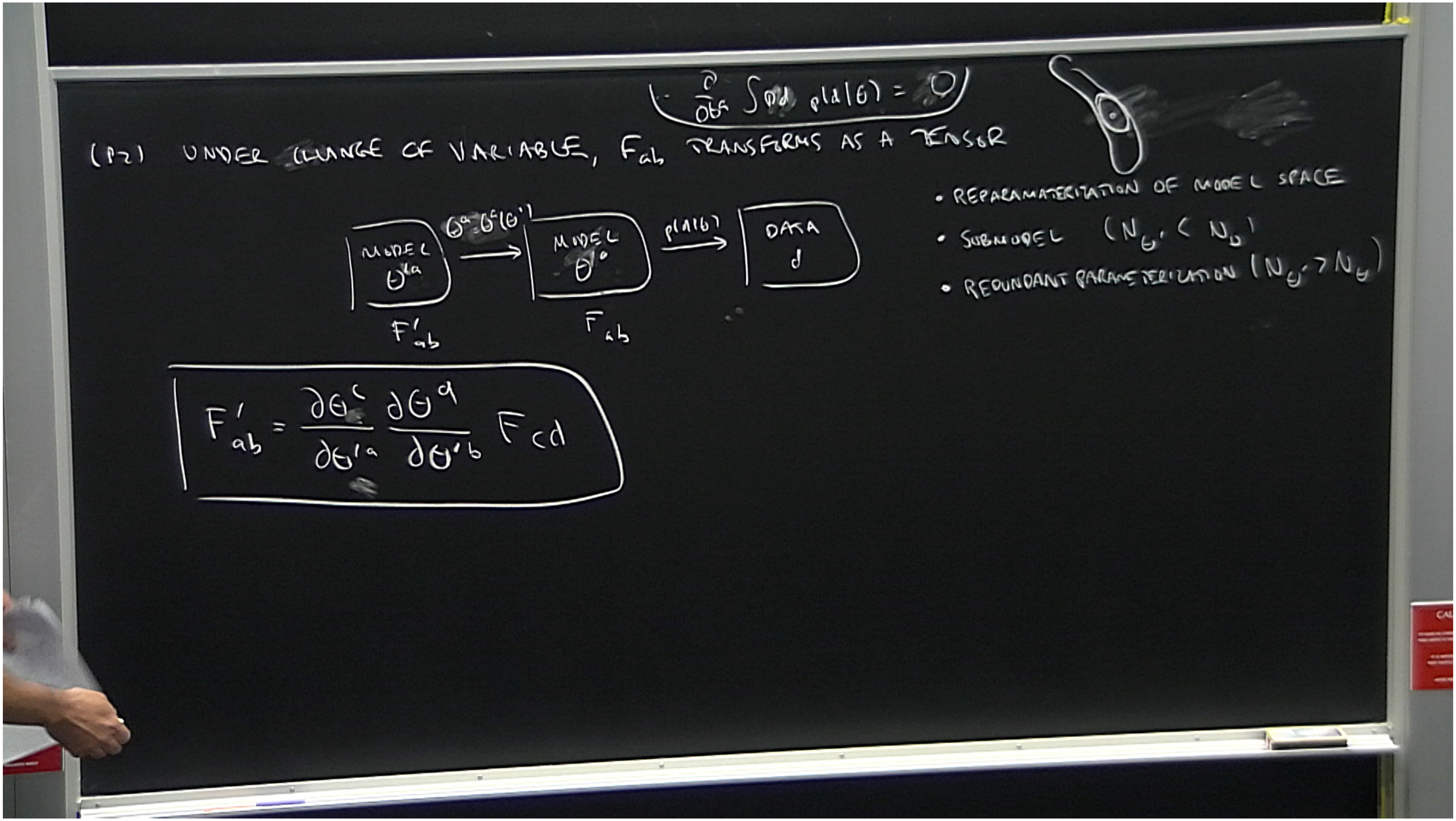
∂/∂θ^a ∫ dθ p(d|θ) = 0
(P2) UNDER CHANGE OF VARIABLE, F_ab TRANSFORMS AS A TENSOR
MODEL θ'^a
θ^a = θ^a(θ')
MODEL θ^a
p(d|θ)
DATA d
• REPARAMATERIZATION OF MODEL SPACE
• SUBMODEL (N_θ' < N_θ)
• REDUNDANT PARAMETERIZATION (N_θ' > N_θ)
CAUTION

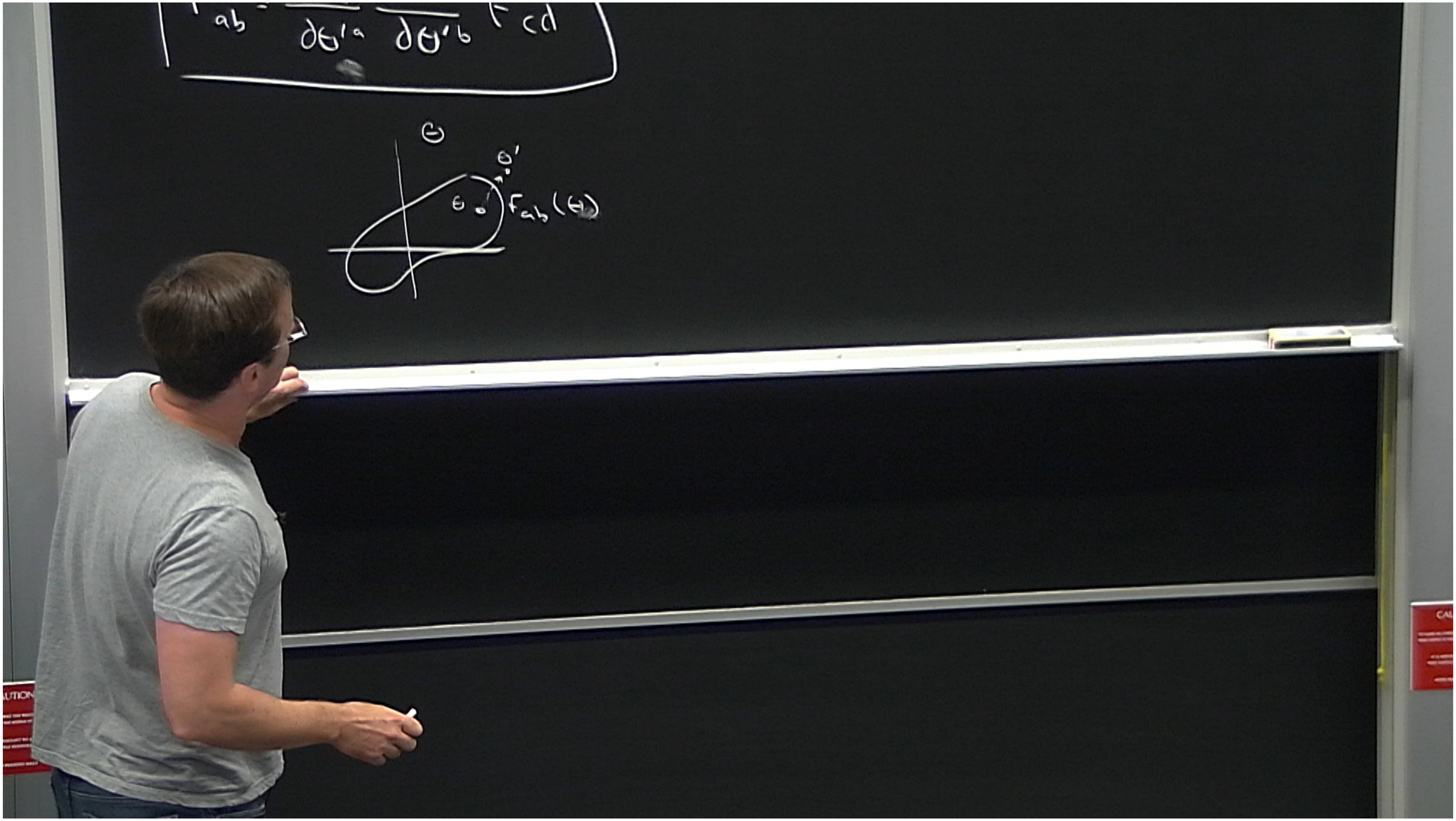

(P2) UNDER CHANGE OF VARIABLE, Fab TRANSFORMS AS A TENSOR
MODEL
MODEL
DATA
d
p(d|θ)
• REPARAMATERIZATION OF MODEL SPACE
• SUBMODEL
• REDUNDANT PARAMETERIZATION


θ1, θ2, θ3
d1
θ1 θ2
d2

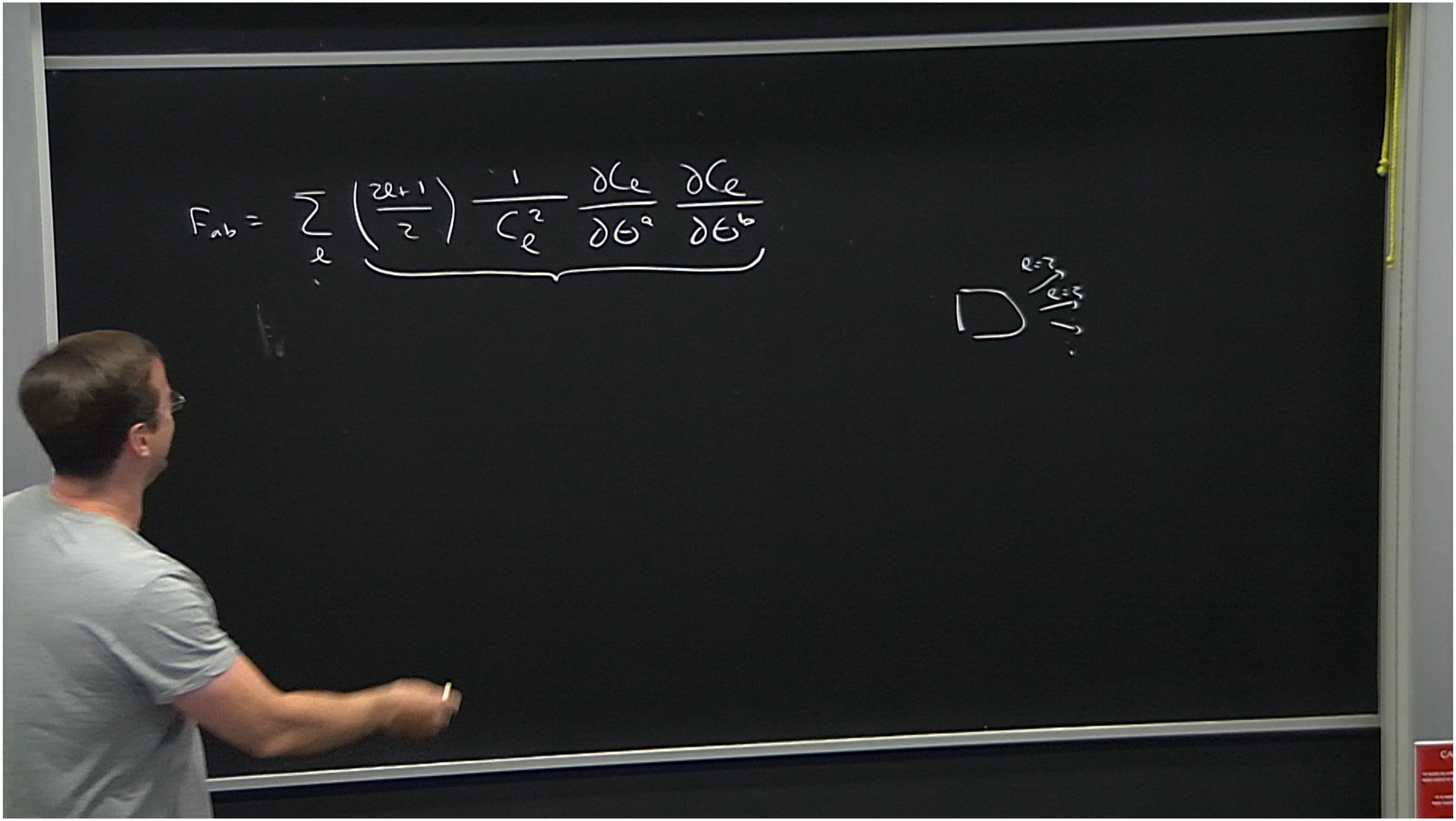

Θ₁, Θ₂, Θ₃
d₁
Θ₁ Θ₂
d₂

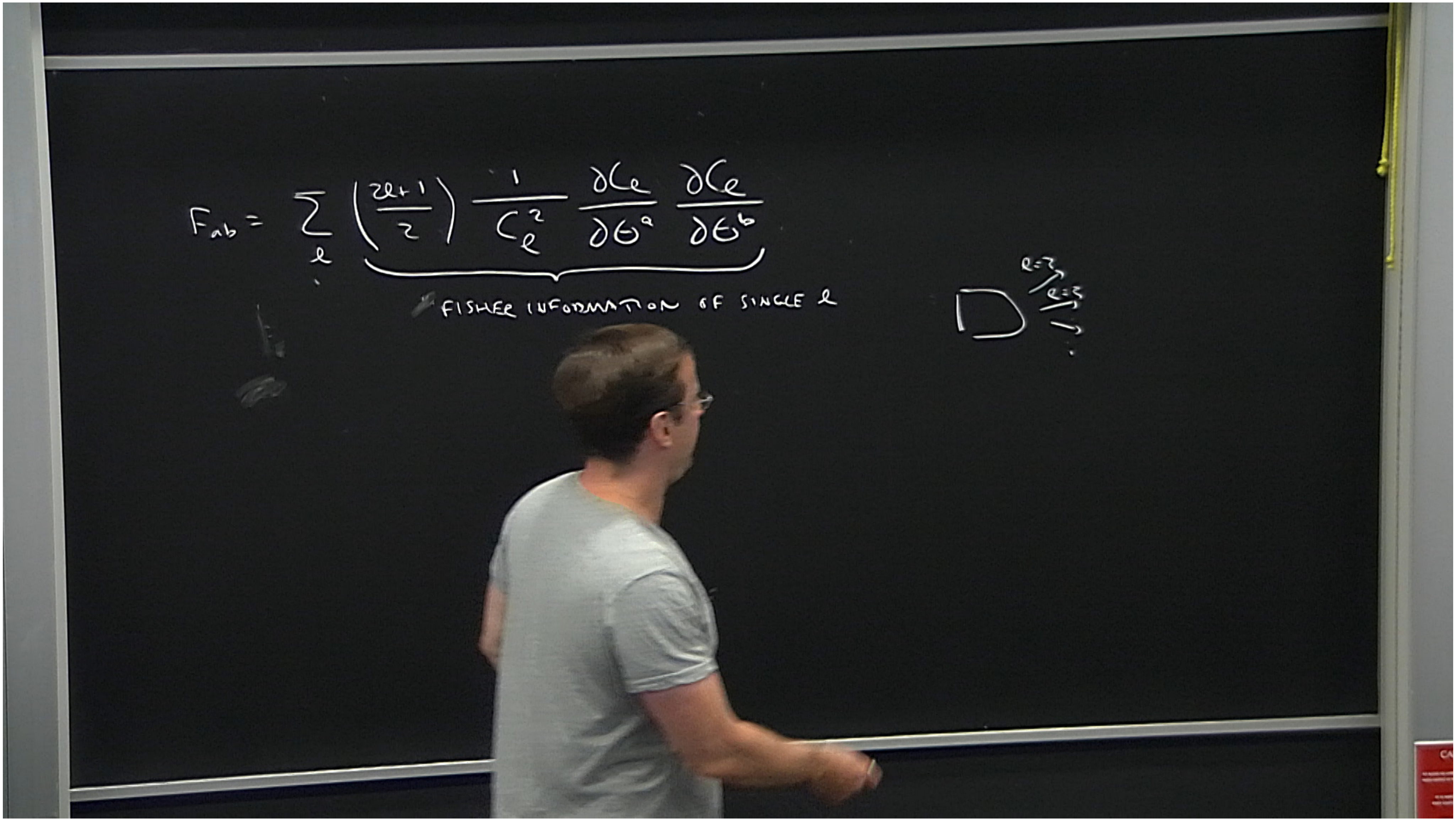

(P2) UNDER CHANGE OF VARIABLE, F_ab TRANSFORMS AS A TENSOR
• REPARAMATERIZATION OF MODEL SPACE
• SUBMODEL
• REDUNDANT PARAMETERIZATION
MODEL
MODEL
DATA
d

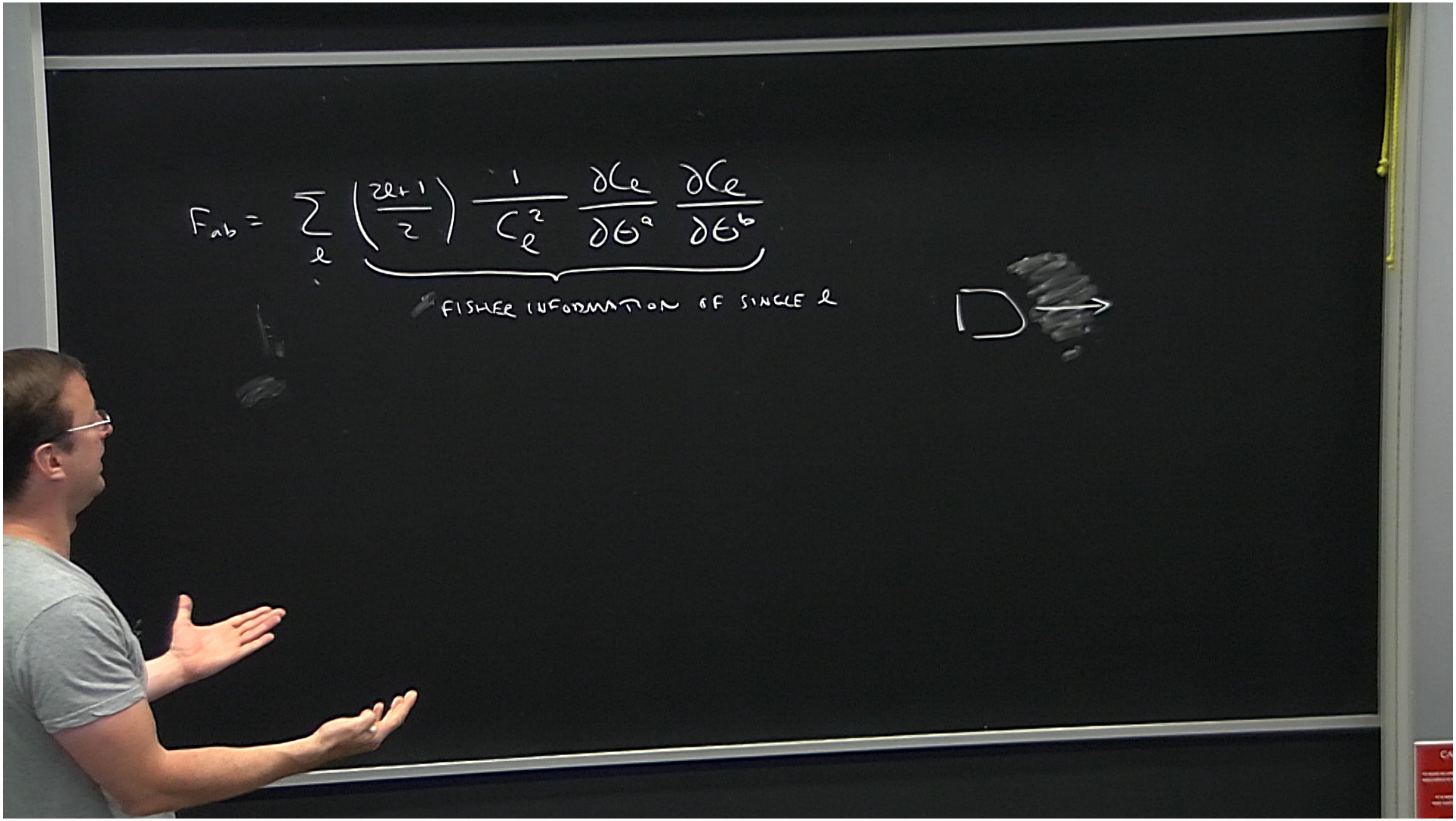

(P2) UNDER CHANGE OF VARIABLE, F_{ab} TRANSFORMS AS A TENSOR
MODEL θ'^a
$\theta^a = \theta^a(\theta')$
MODEL θ^a
$p(d|\theta)$
DATA d
F'_{ab}
F_{ab}
$$F'_{ab} = \frac{\partial \theta^c}{\partial \theta'^a} \frac{\partial \theta^d}{\partial \theta'^b} F_{cd}$$
• REPARAMATERIZATION OF MODEL SPACE
• SUBMODEL $(N_{\theta'} < N_{\theta})$
• REDUNDANT PARAMETERIZATION $(N_{\theta'} > N_{\theta})$

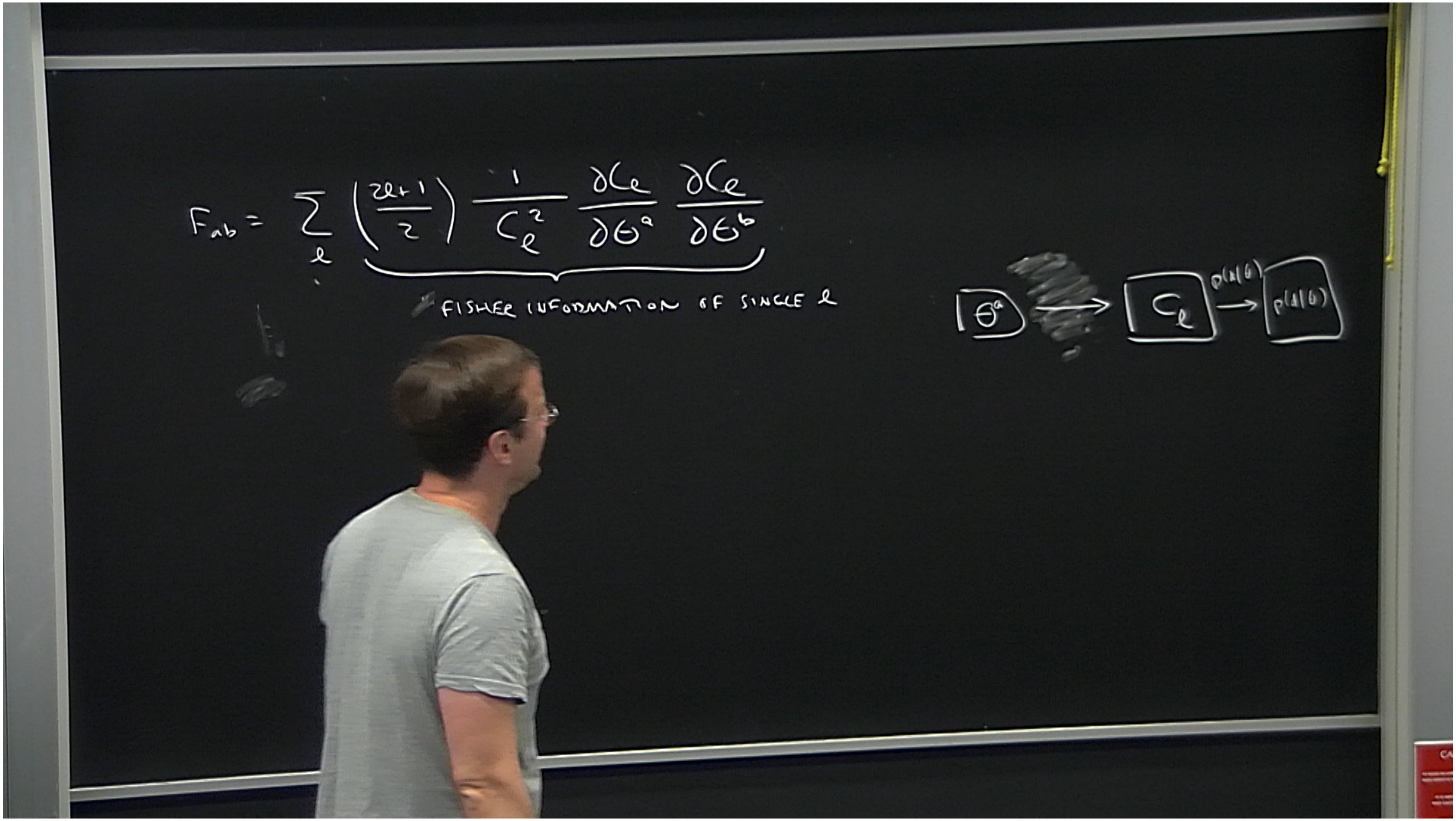

Θ
Θ'
F_ab(Θ)

$$F_{ab} = \sum_{\ell} \underbrace{\left(\frac{2\ell+1}{2}\right) \frac{1}{C_\ell^2} \frac{\partial C_\ell}{\partial \theta^a} \frac{\partial C_\ell}{\partial \theta^b}}$$


$F_{ab} = \sum_\ell \left(\frac{2\ell+1}{2}\right) \frac{1}{C_\ell^2} \frac{\partial C_\ell}{\partial \theta^a} \frac{\partial C_\ell}{\partial \theta^b}$

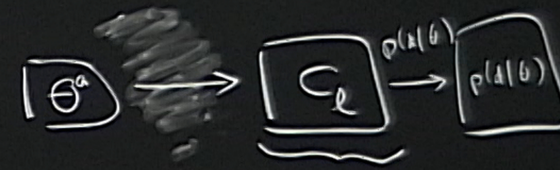

$F_{ab} = \sum_{\ell} \left(\frac{2\ell+1}{2}\right) \frac{1}{C_\ell^2} \frac{\partial C_\ell}{\partial \theta^a} \frac{\partial C_\ell}{\partial \theta^b}$
FISHER INFORMATION OF SINGLE ℓ
ℓ=2
ℓ=3


Fab = Σl ((2l+1)/2) (1/Cl²) (∂Cl/∂θa) (∂Cl/∂θb)
FISHER INFORMATION OF SINGLE l

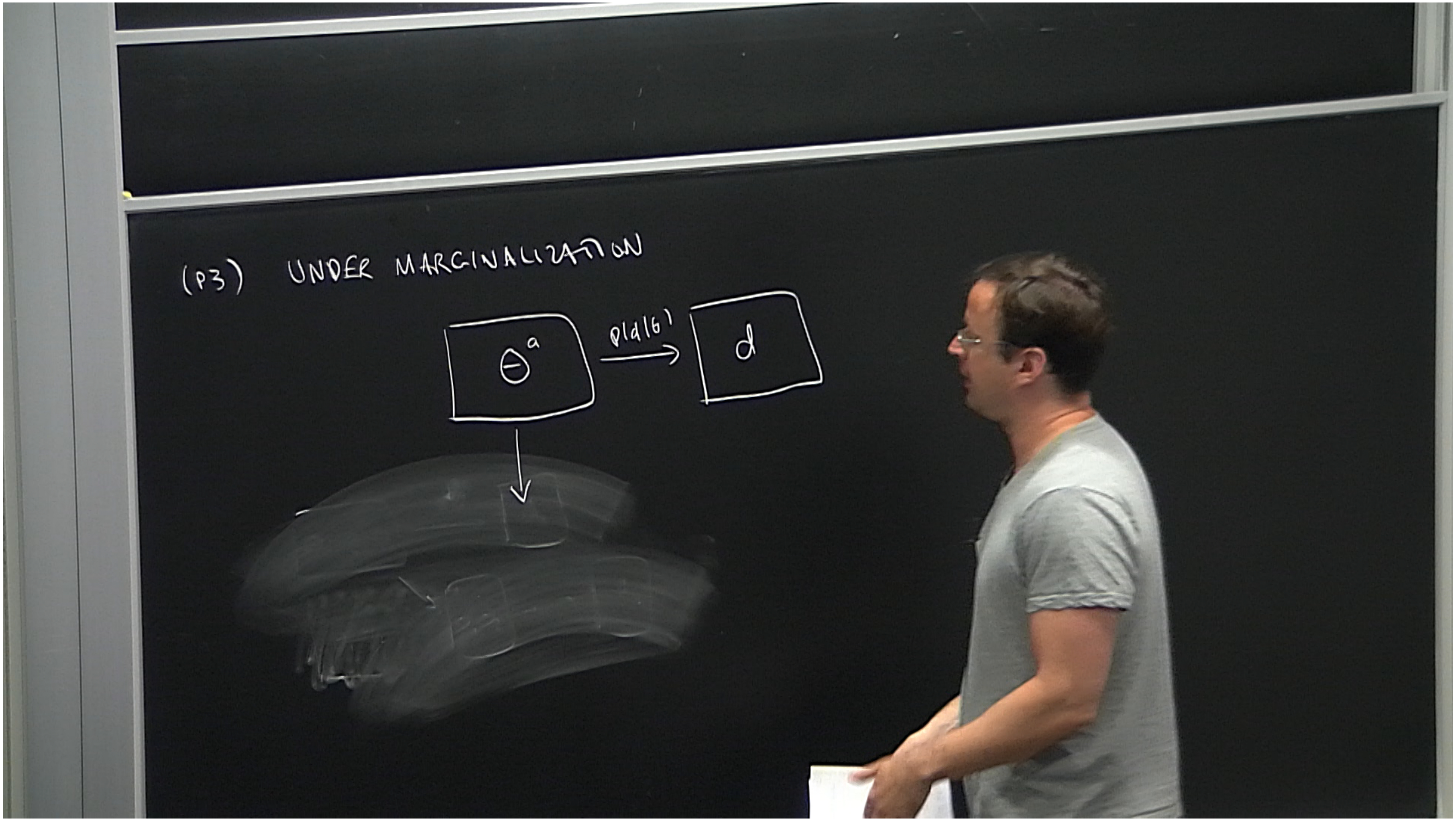

$F_{ab} = \sum_\ell \left(\frac{2\ell+1}{2}\right) \frac{1}{C_\ell^2} \frac{\partial C_\ell}{\partial \theta^a} \frac{\partial C_\ell}{\partial \theta^b}$
FISHER INFORMATION OF SINGLE ℓ
θ^a
C_ℓ
$p(d|\theta)$
$p(d|\theta)$

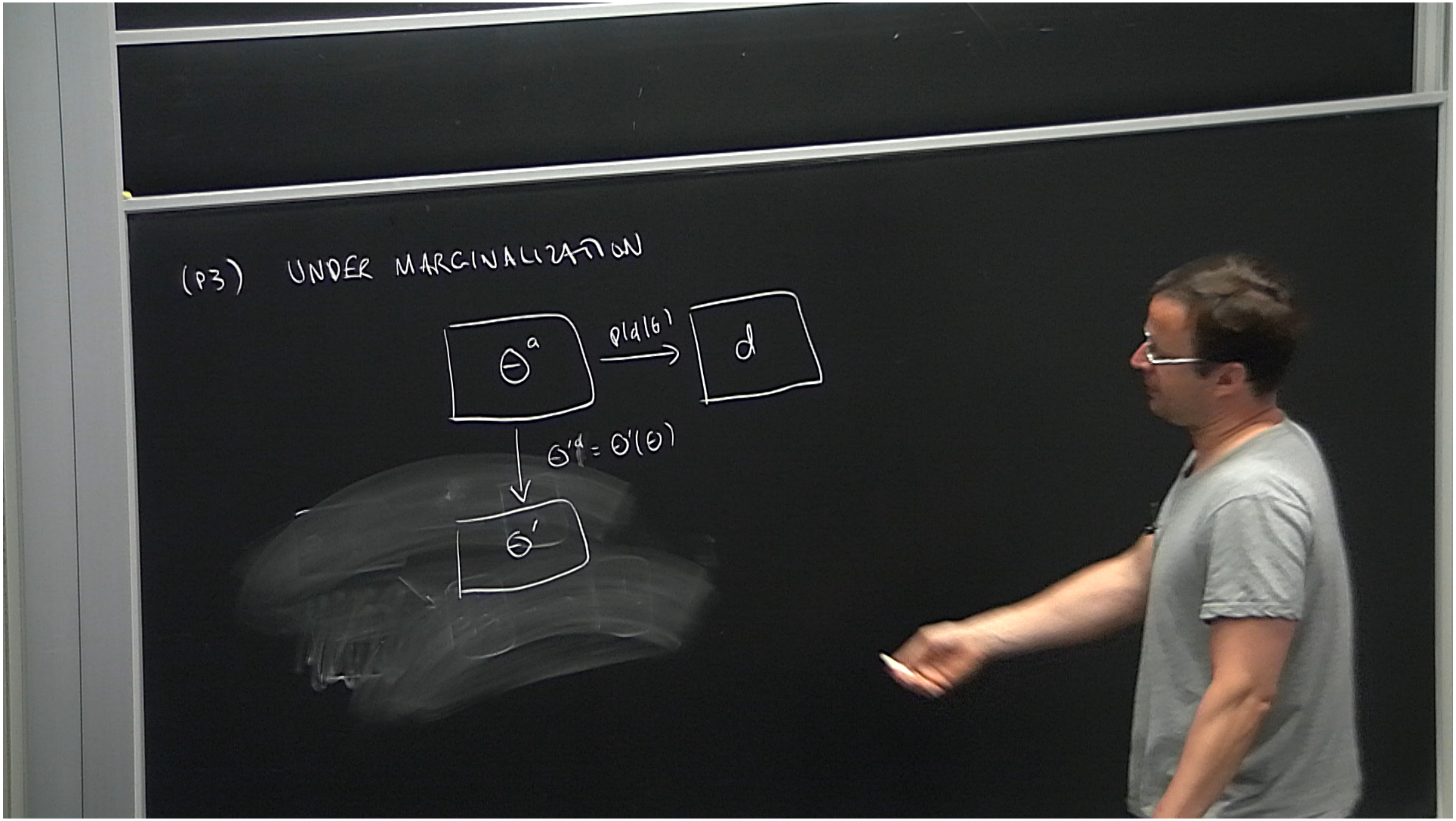
$$F_{ab} = \underbrace{\sum_\ell \left(\frac{2\ell+1}{2}\right) \frac{1}{C_\ell^2} \frac{\partial C_\ell}{\partial \theta^a} \frac{\partial C_\ell}{\partial \theta^b}}_{\text{FISHER INFORMATION OF SINGLE } \ell}$$

$$= \sum_\ell \left(\frac{\partial C_\ell}{\partial \theta^a}\right)\left(\frac{\partial C_\ell}{\partial \theta^b}\right) \underbrace{\frac{(2\ell+1)}{2} \frac{1}{C_\ell^2}}_{\text{1-BY-1 FISHER MATRIX}}$$



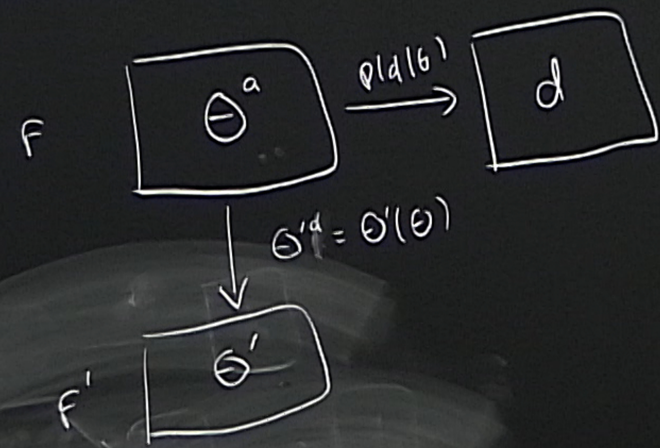
$$F = \frac{2\ell+1}{2} \frac{1}{C_\ell^2}$$


(P3) UNDER MARGINALIZATION
Θ^a
p(d|θ)
d

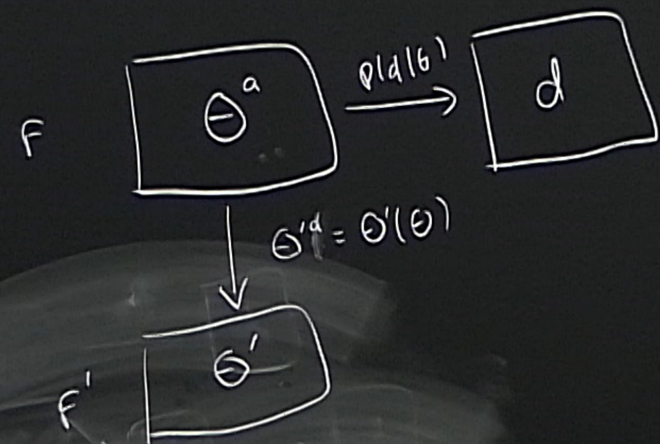

(P3) UNDER MARGINALIZATION
p(d|θ)
θ^a
d
θ'^a = θ'(θ)
θ'

(P3) UNDER MARGINALIZATION, (F^{-1}) TRANSFORMS AS A TENSOR

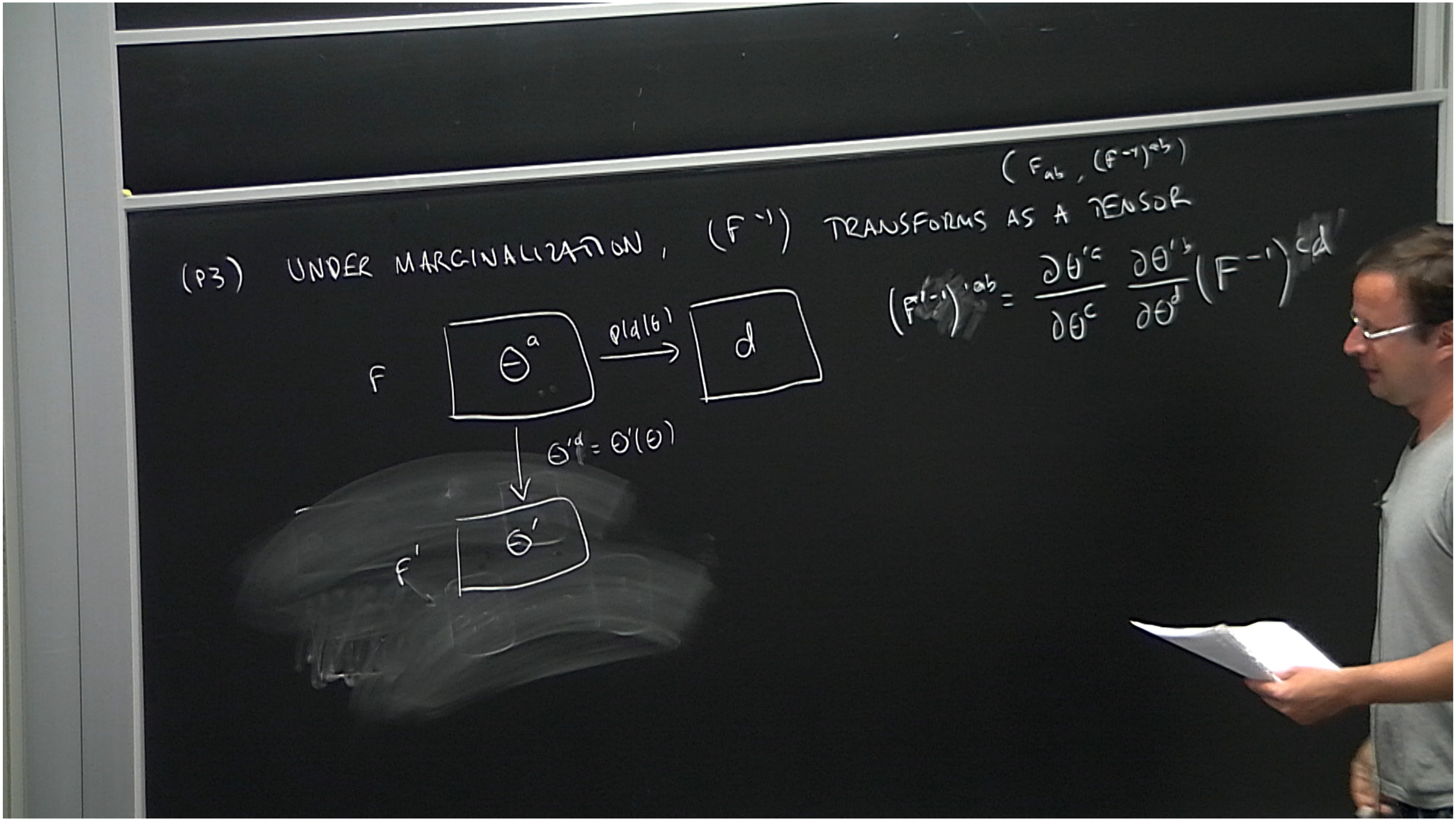
$$(F'^{-1})^{ab} = \frac{\partial\theta'^{a}}{\partial\theta^{c}}\frac{\partial\theta'^{b}}{\partial\theta^{d}}(F^{-1})^{ab}$$



(P3) UNDER MARGINALIZATION, (F^{-1}) TRANSFORMS AS A TENSOR



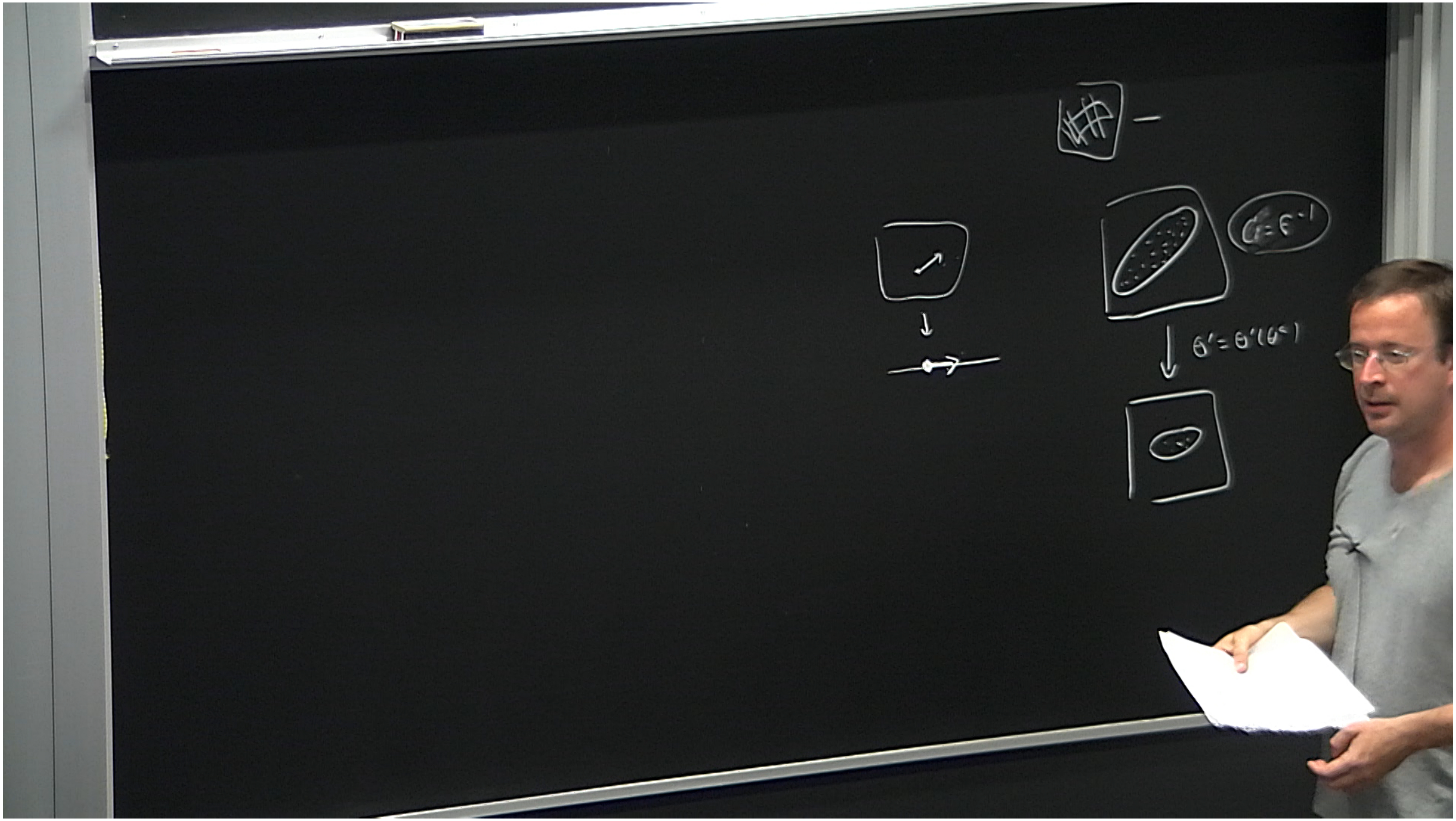
$$(F'^{-1})^{ab} = \frac{\partial\theta'^a}{\partial\theta^c}\frac{\partial\theta'^b}{\partial\theta^d}(F^{-1})^{cd}$$


(F_ab , (F^-1)^ab)
(P3) UNDER MARGINALIZATION, (F^-1) TRANSFORMS AS A TENSOR
(F'^-1)'^ab = ∂θ'^a/∂θ^c ∂θ'^b/∂θ^d (F^-1)^cd
F
θ^a
p(d|θ)
d
θ'^a = θ'(θ)
F'
θ'

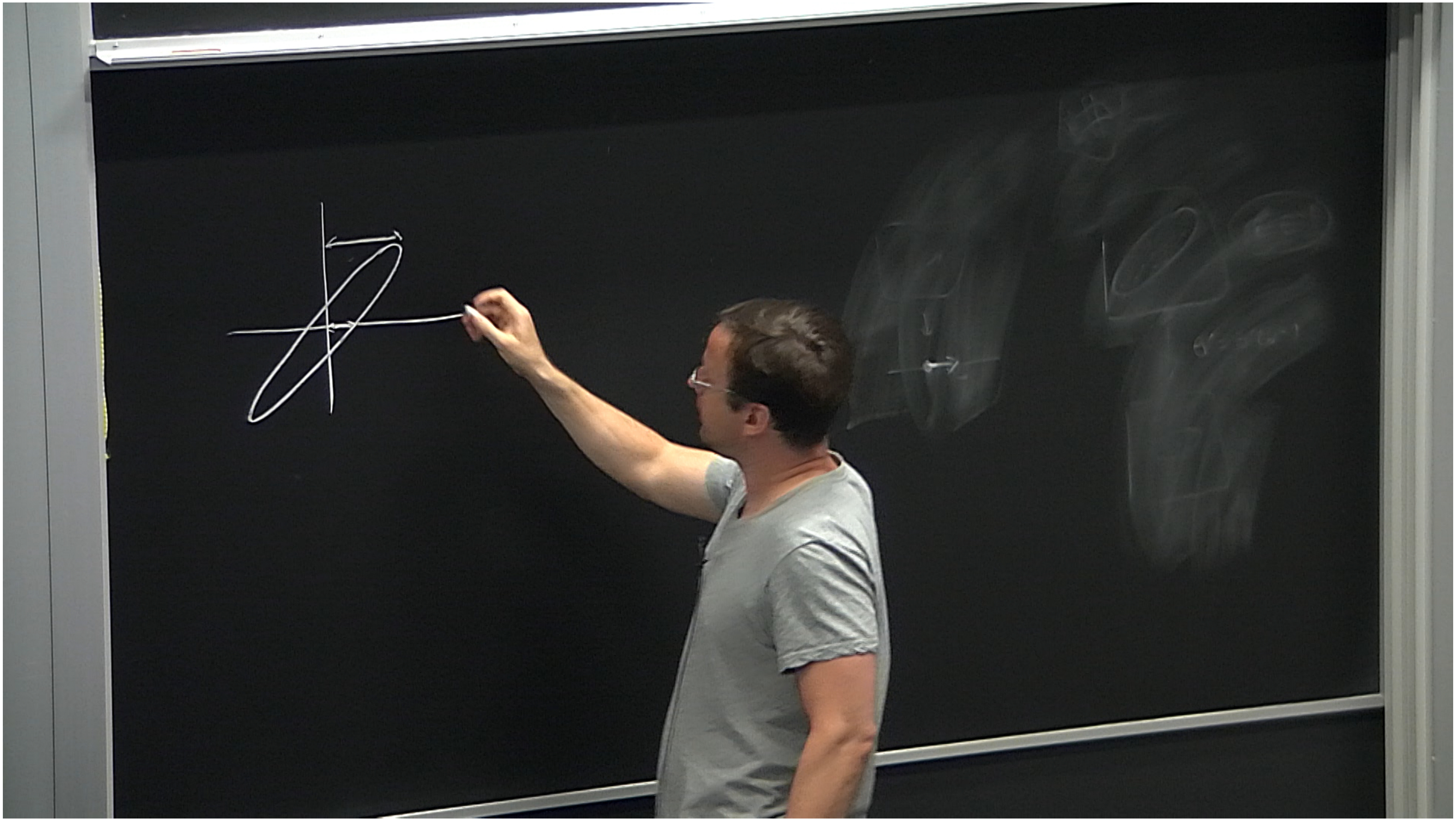

$= \epsilon^{-1}$

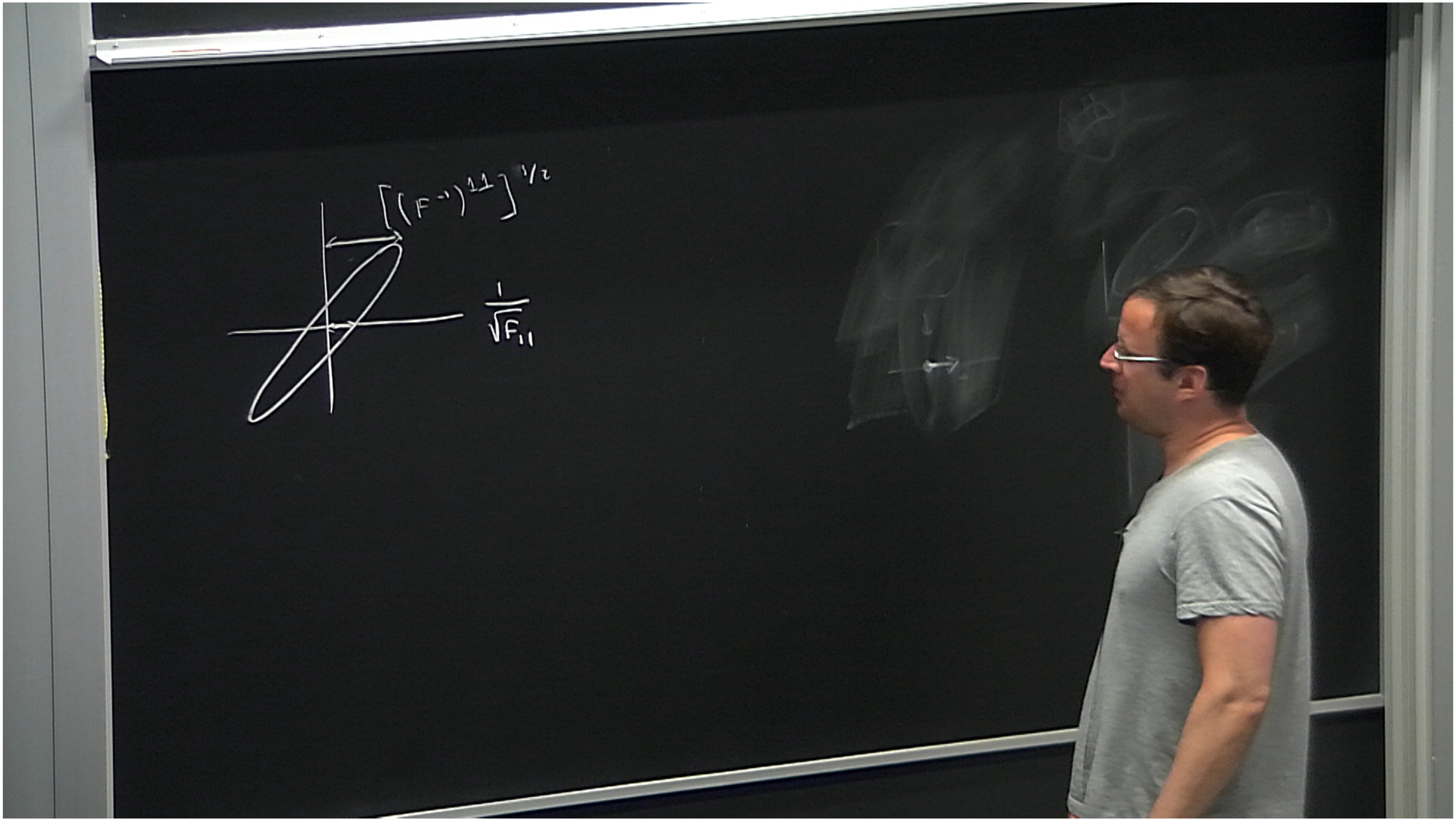

θ′ = θ′(θ^a)

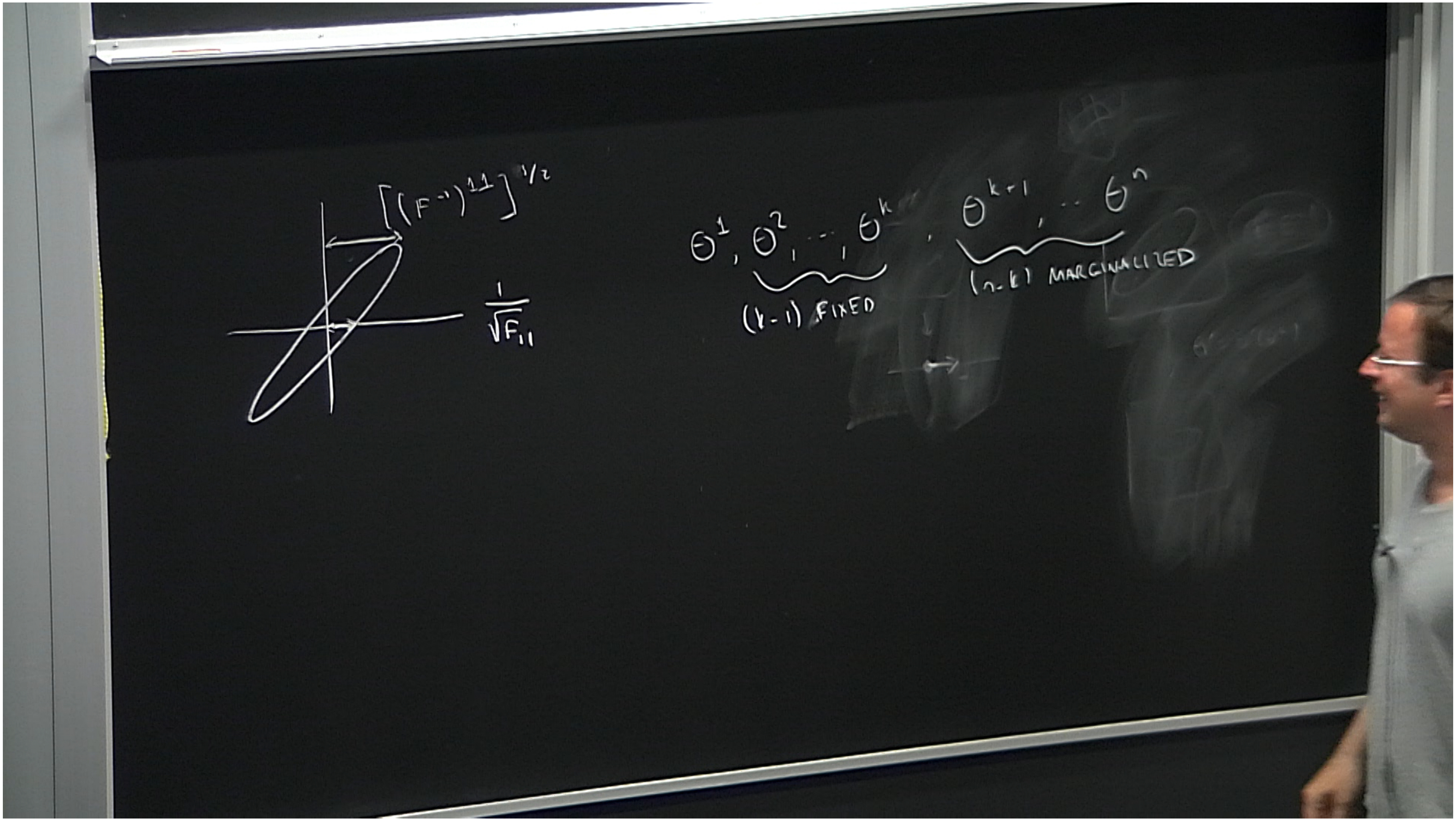

G = F⁻¹
θ' = θ'(θᵃ)

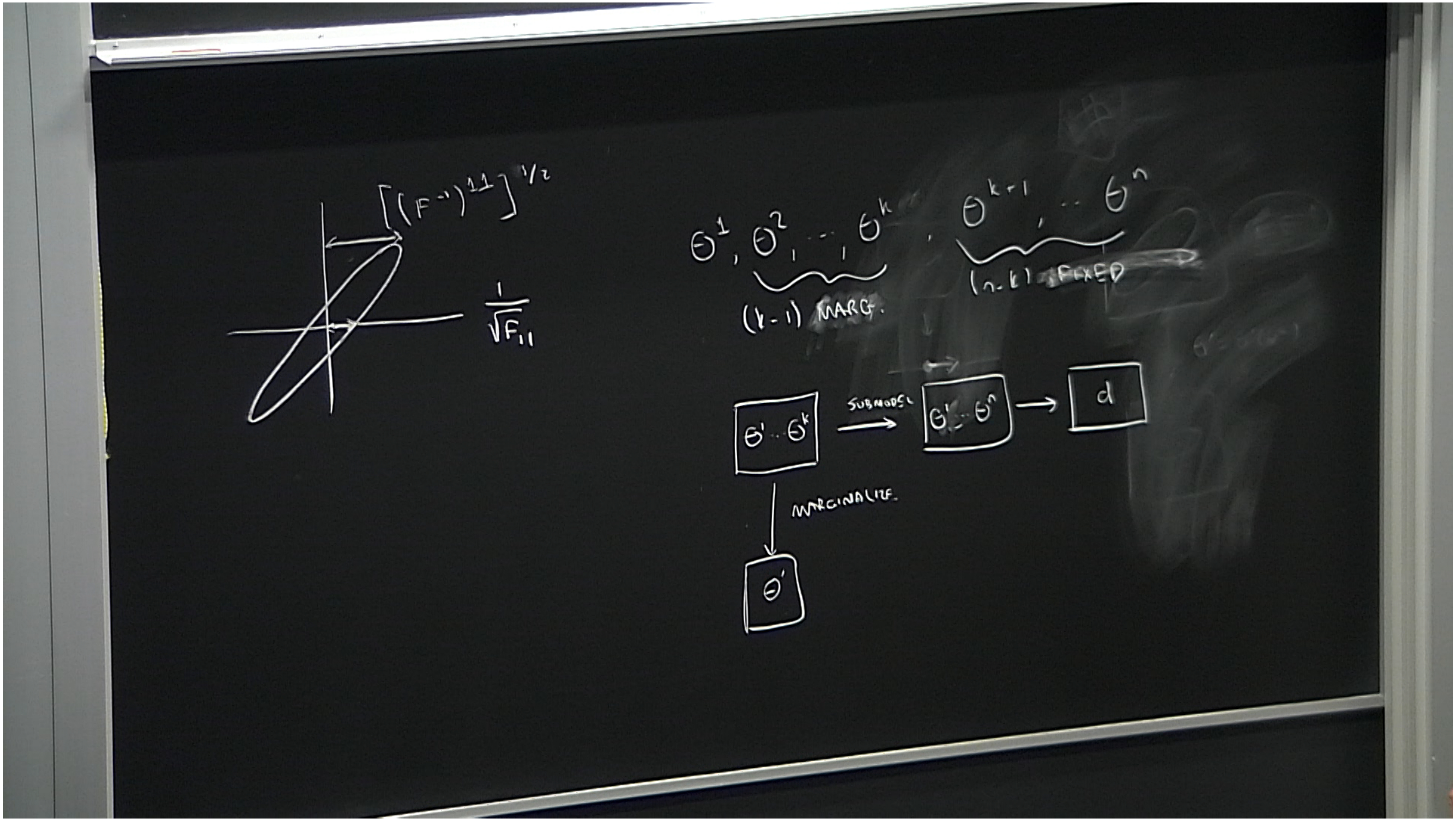


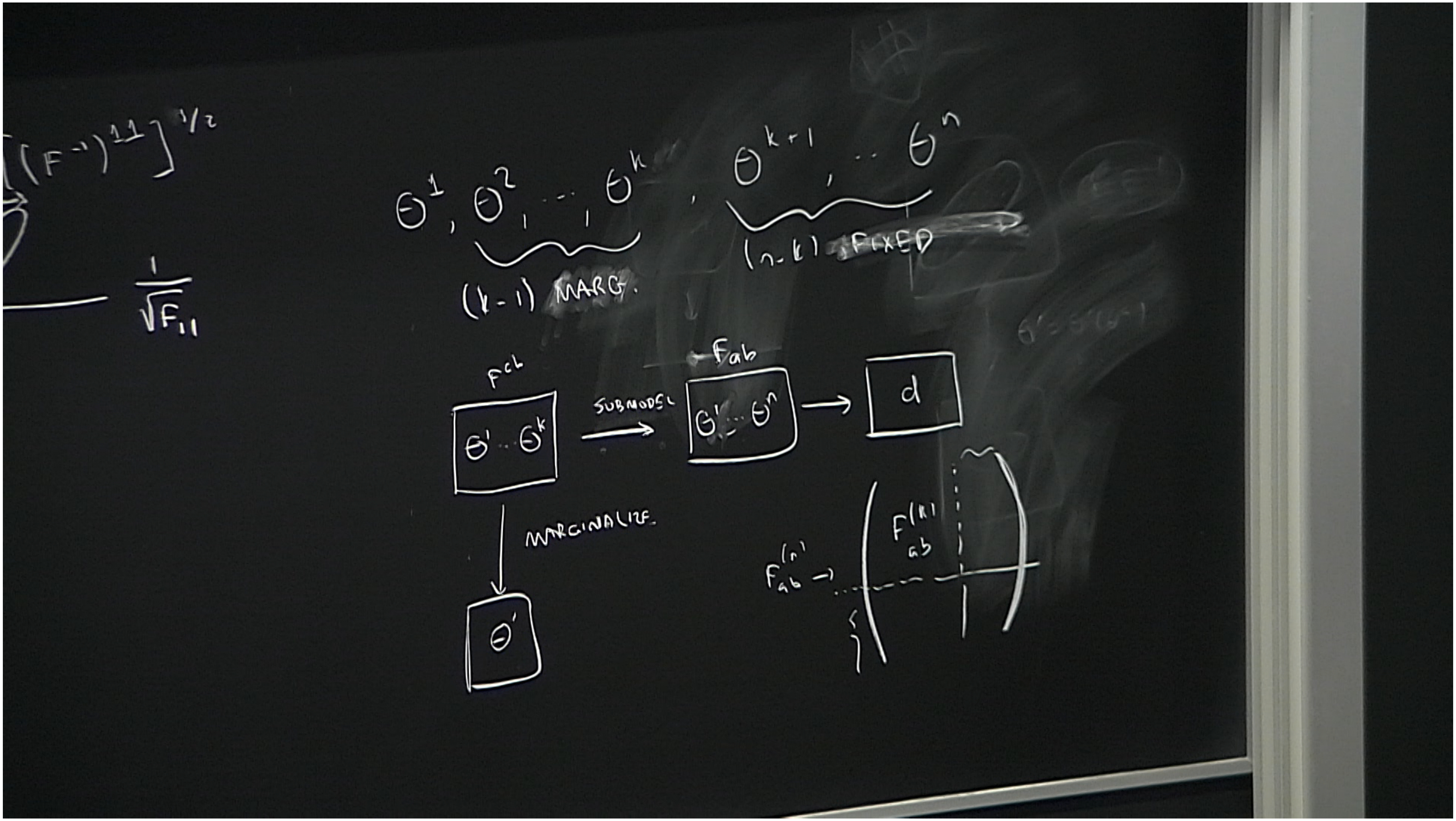
$\theta' = \theta'(\theta^{-})$

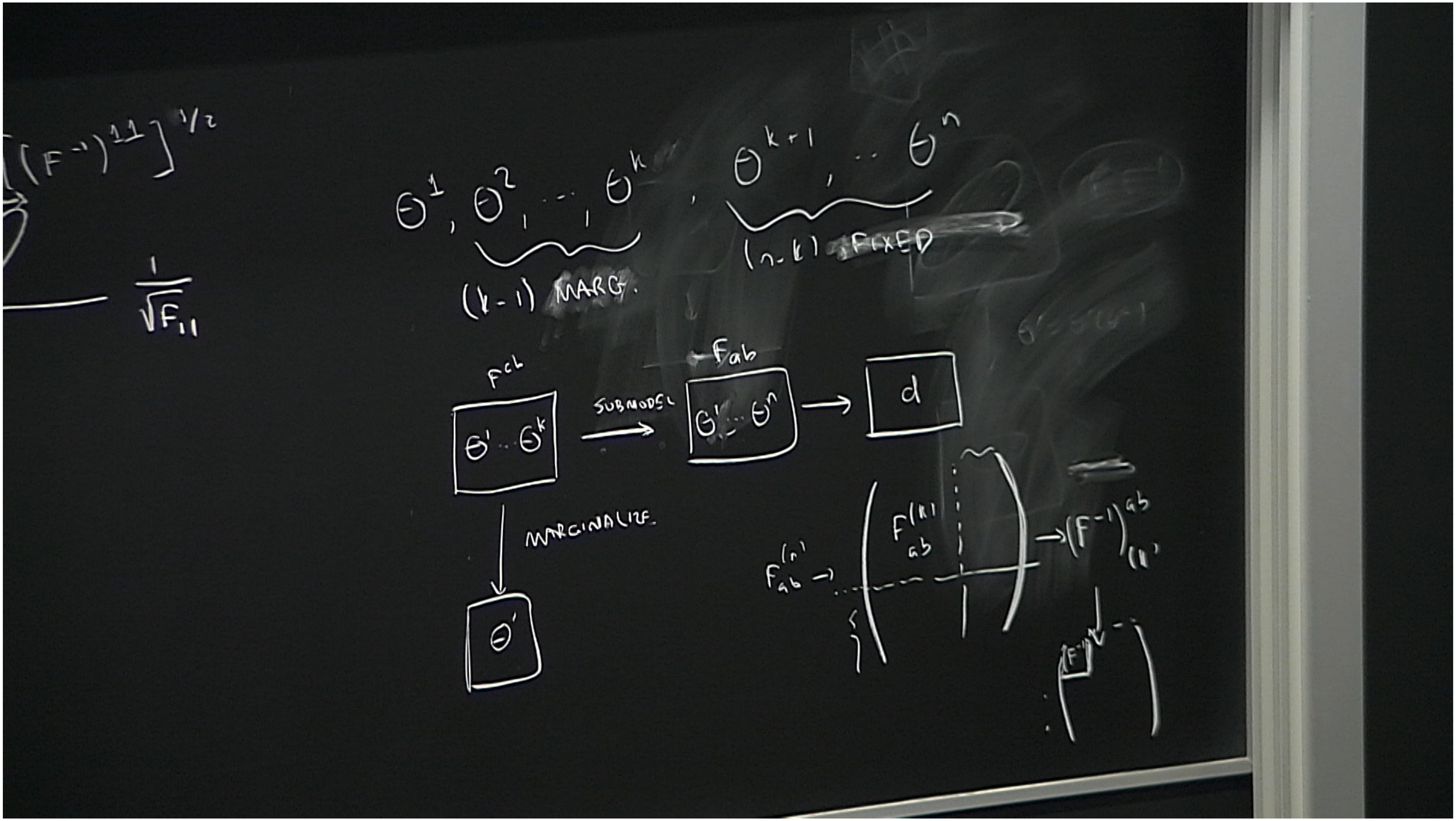


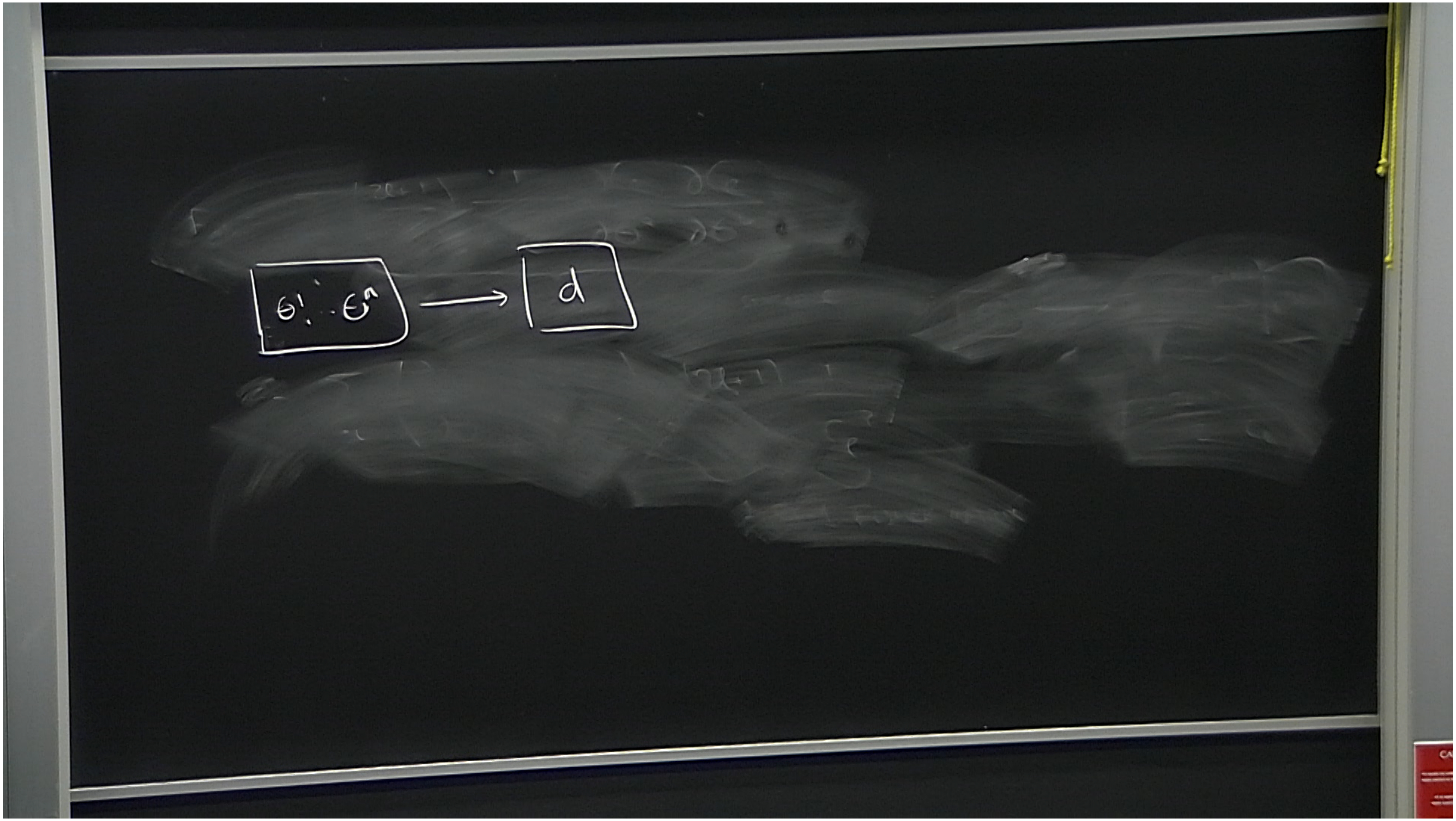

$[(F^{-1})^{11}]^{1/2}$
$\frac{1}{\sqrt{F_{11}}}$

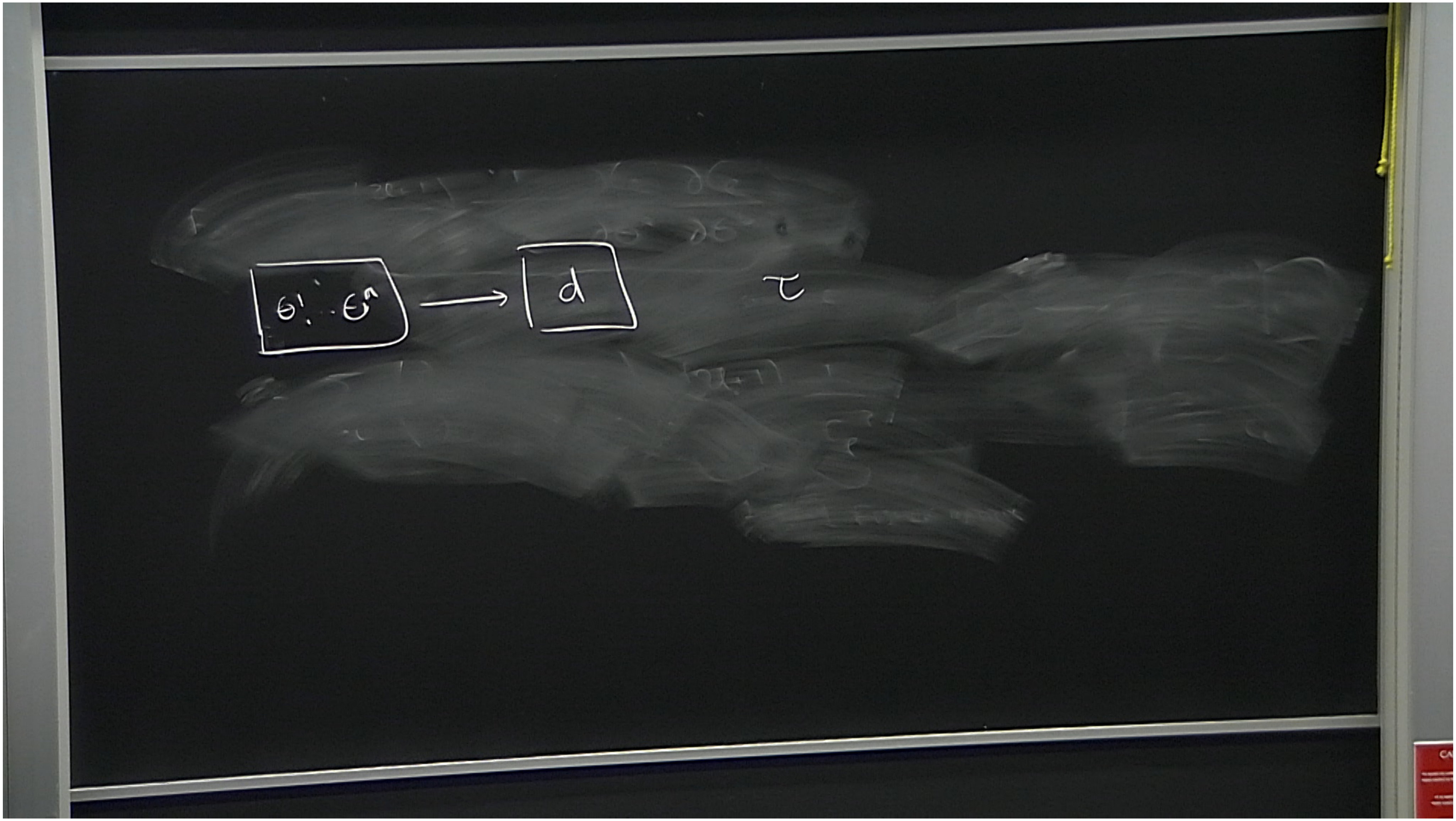

$[(F^{-1})^{11}]^{1/2}$
$\frac{1}{\sqrt{F_{11}}}$
$\theta^1, \theta^2, \dots, \theta^k, \theta^{k+1}, \dots \theta^n$
(k-1) FIXED
(n-k) MARGINALIZED

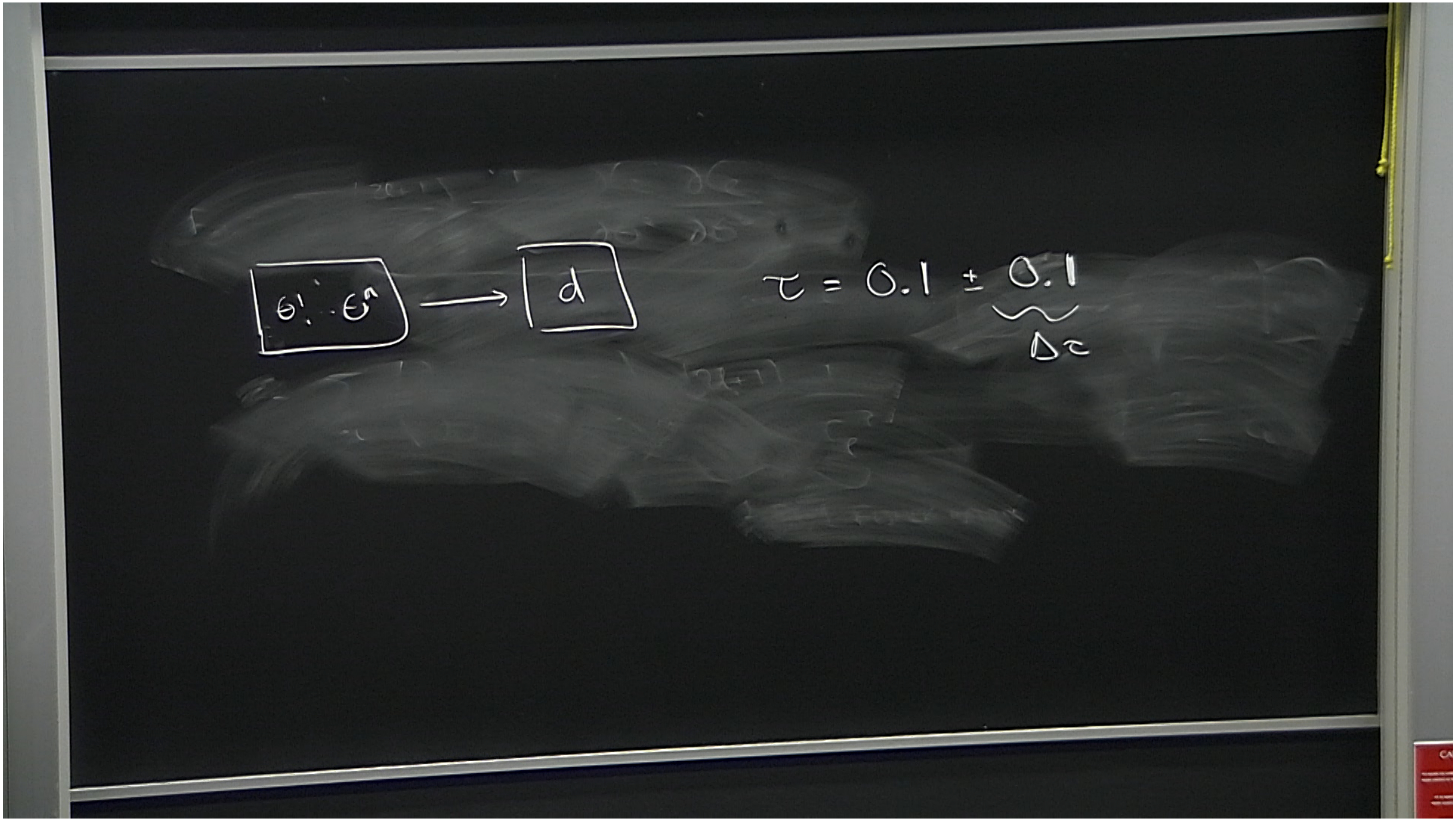

[(F⁻¹)¹¹]^{1/2}
1/√F₁₁
(k-1) MARG.
(n-k) FIXED
SUBMODEL
MARGINALIZE
d

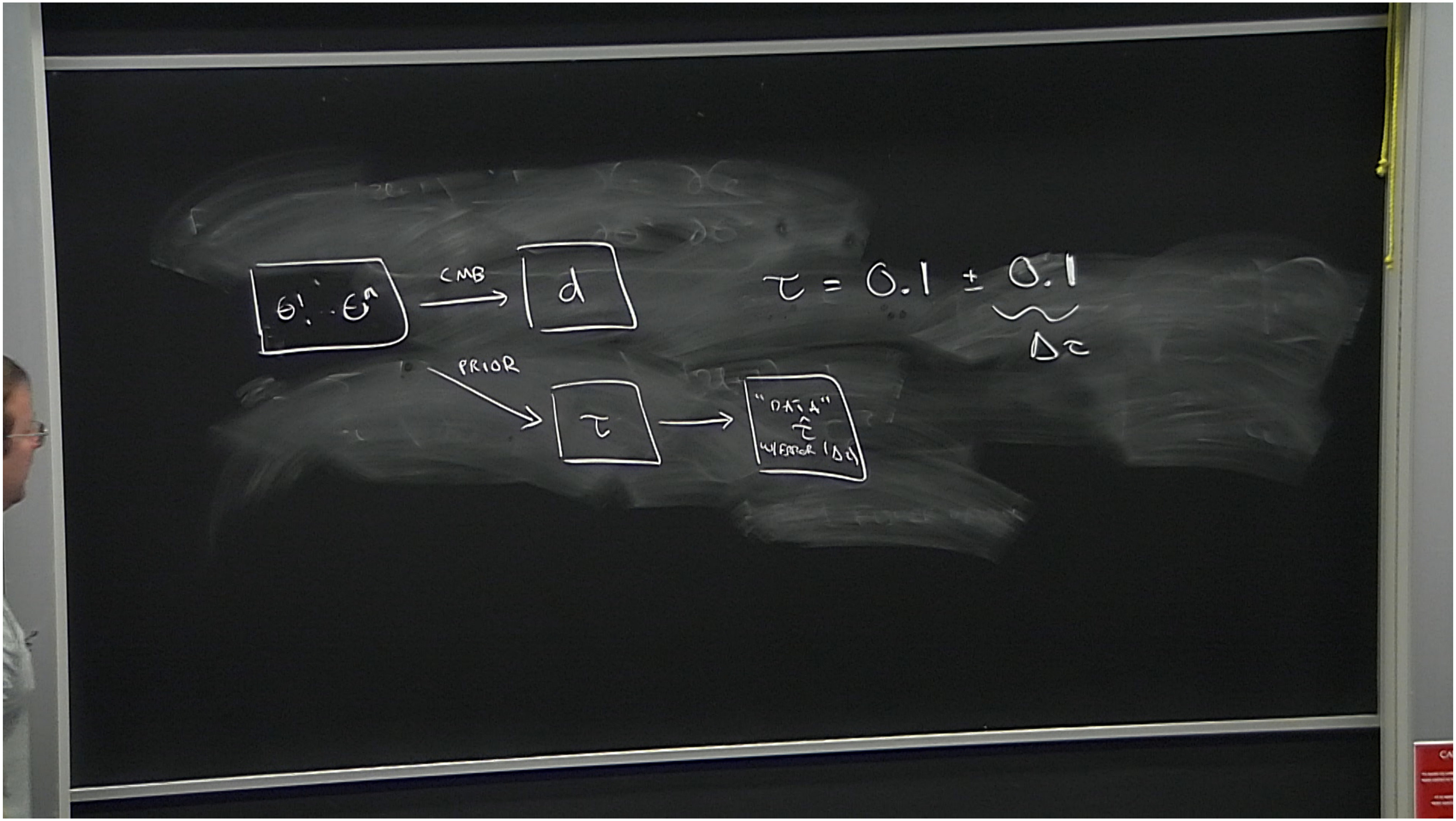

(k-1) MARG.
(n-k) FIXED
SUBMODEL
MARGINALIZE

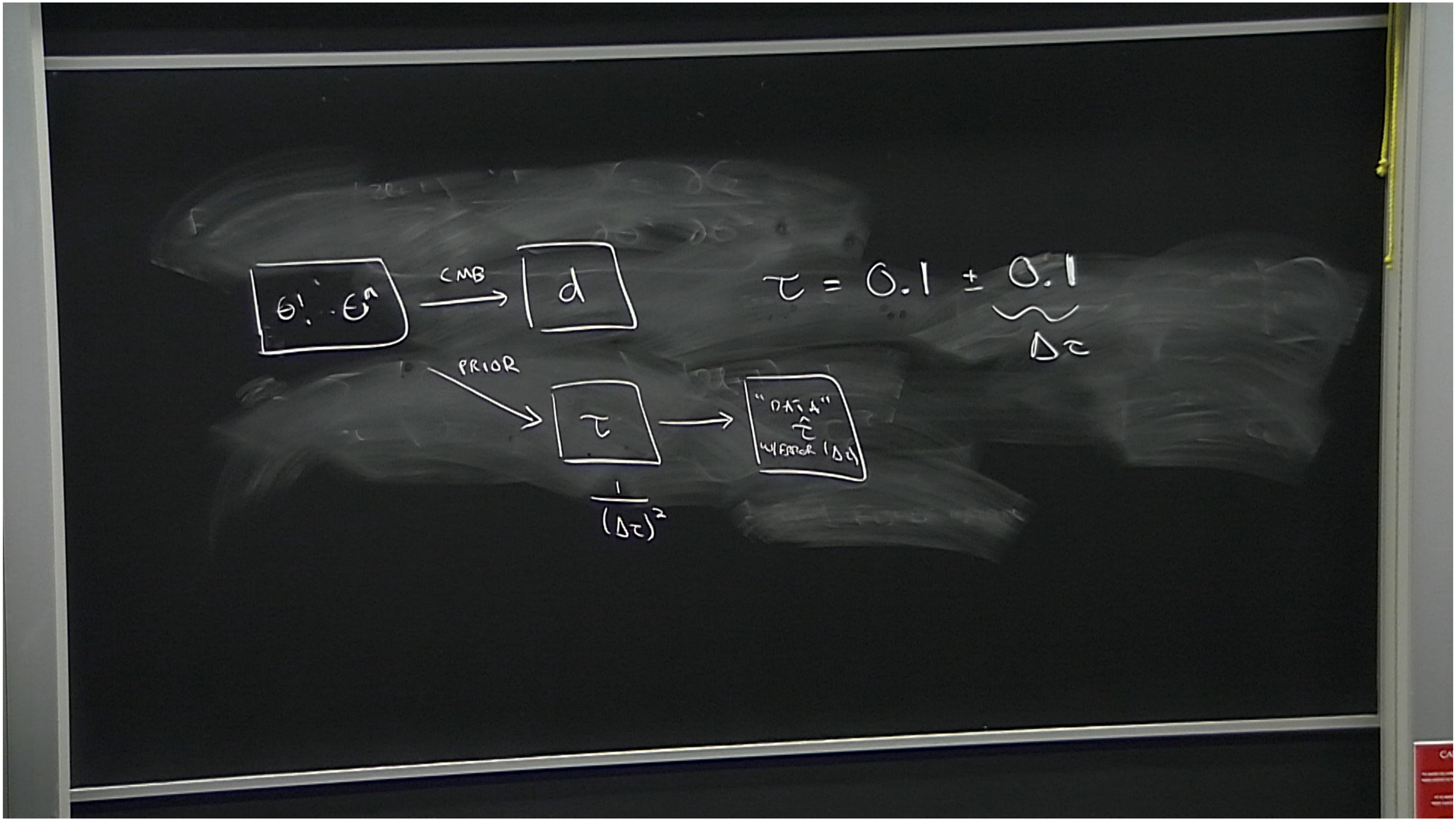

(k-1) MARG.
(n-k) FIXED
SUBMODEL
MARGINALIZE
d

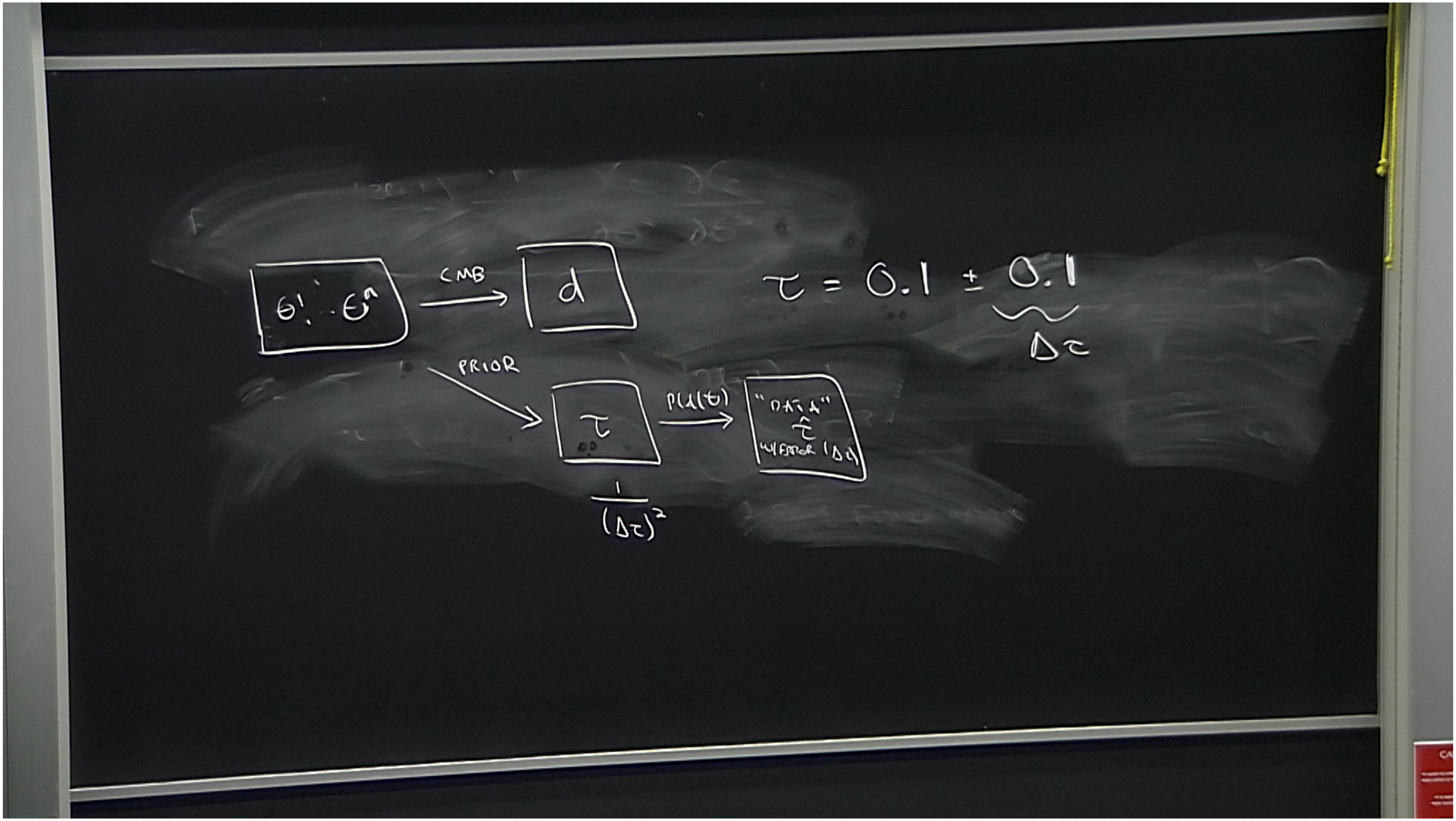


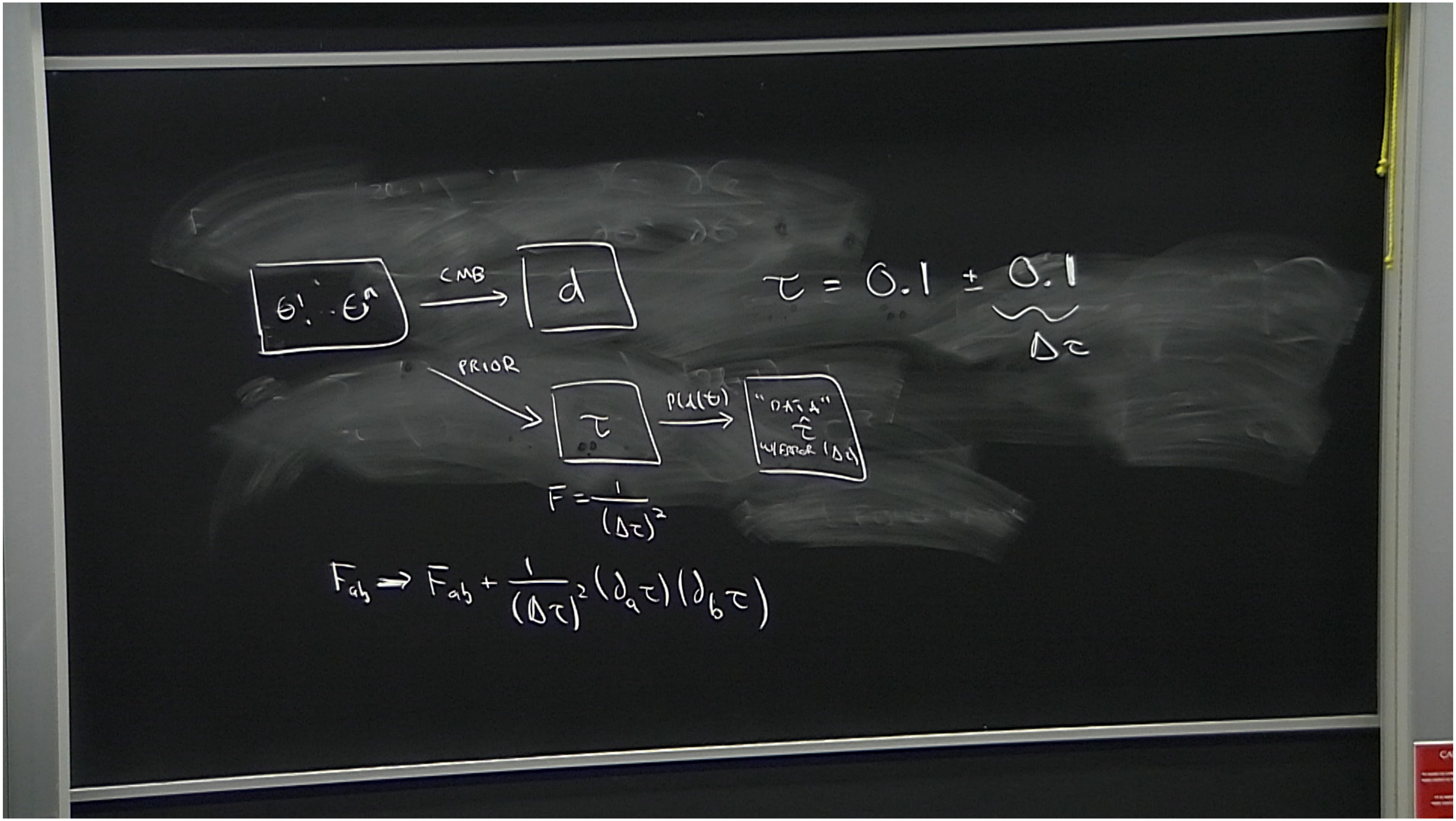

d
τ

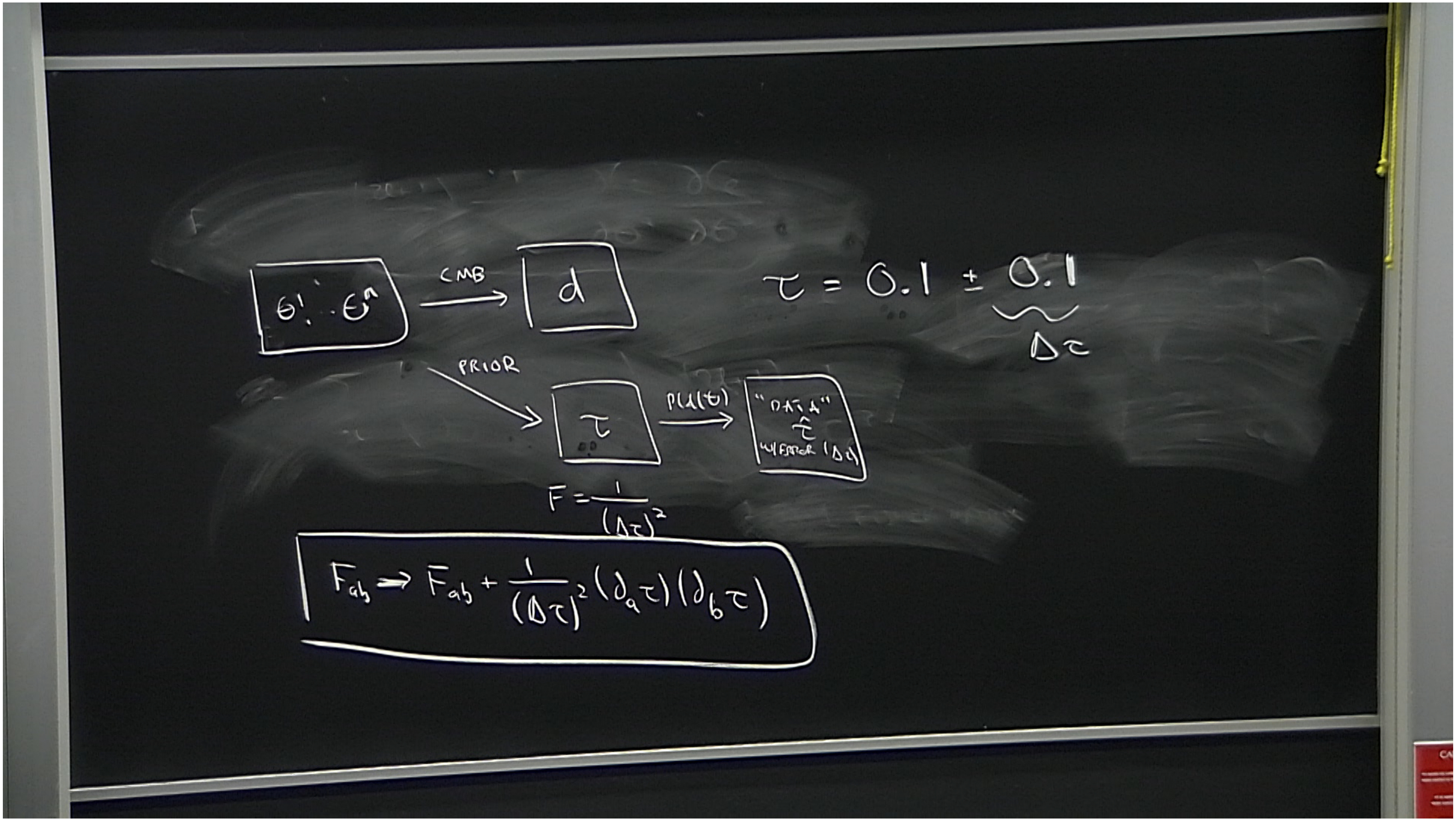

$\theta^1 \cdots \theta^n$
d
$\tau = 0.1 \pm 0.1$
$\Delta\tau$

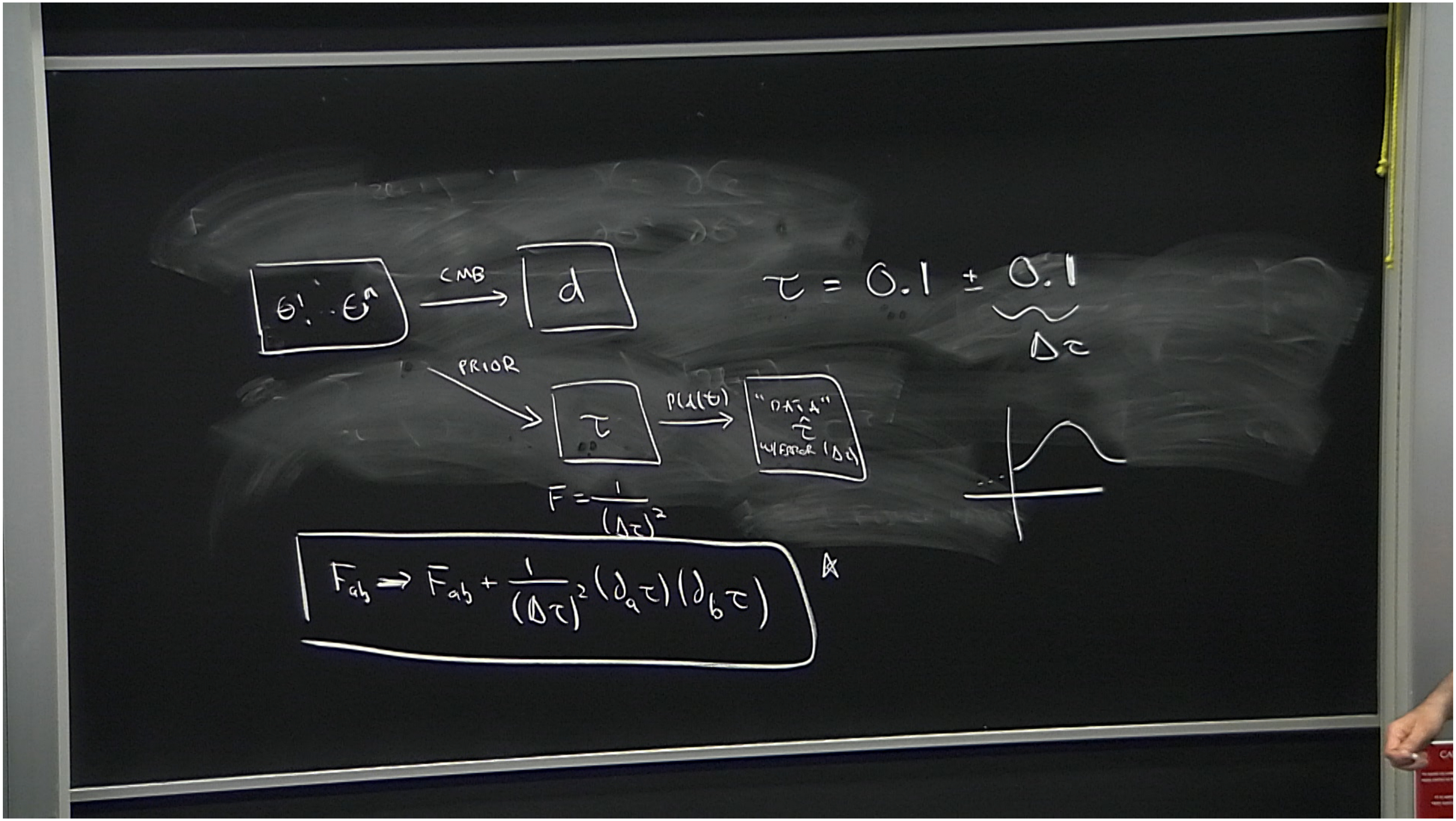

CMB
d
PRIOR
τ
"DATA"
W/ ERROR (Δτ)
τ = 0.1 ± 0.1
Δτ

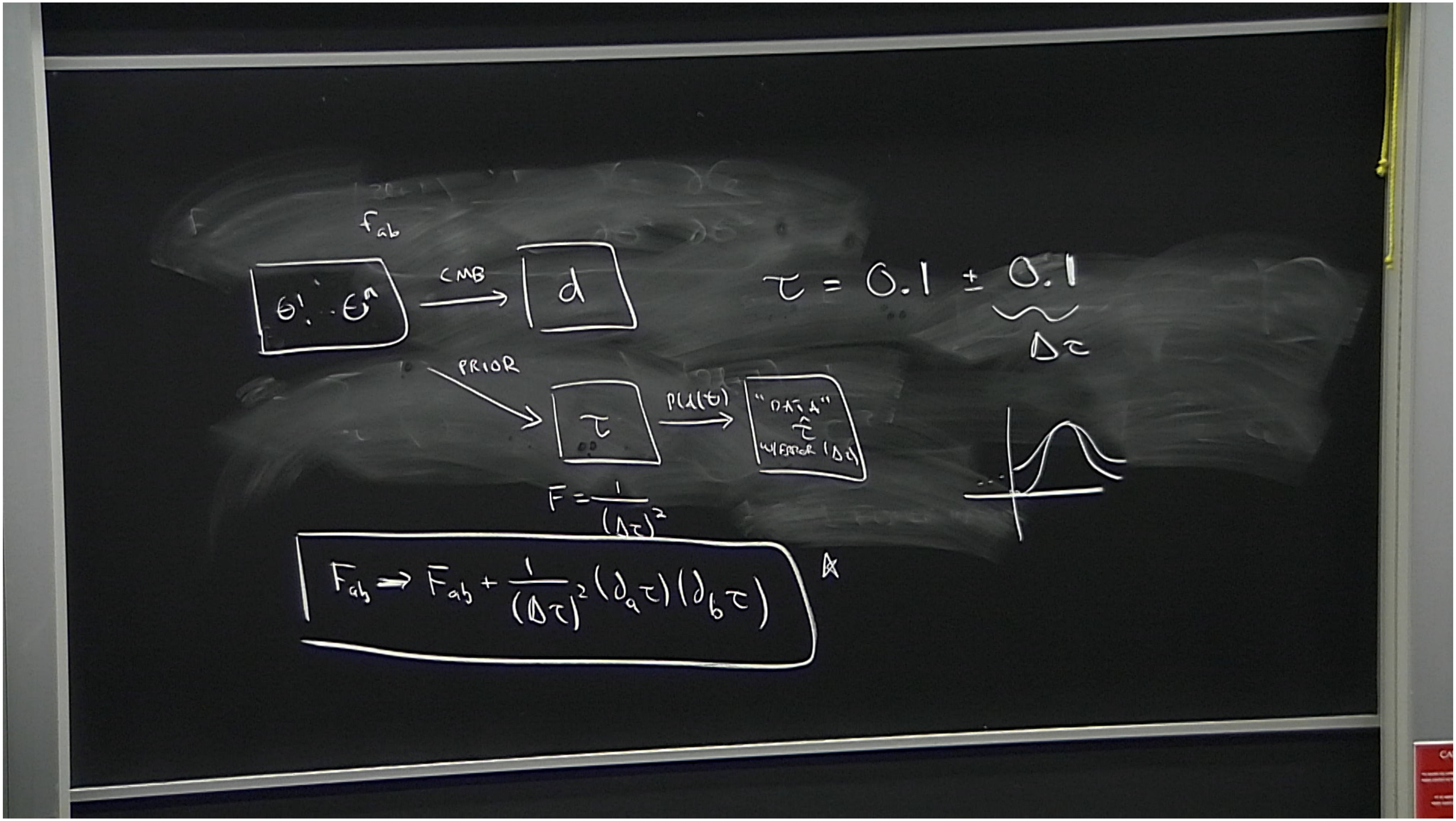

CMB
d
PRIOR
τ
"DATA"
$\hat{\tau}$
w/ ERROR (Δτ)
$\frac{1}{(\Delta\tau)^2}$
τ = 0.1 ± 0.1
Δτ


CMB
d
PRIOR
τ
P(d|τ)
"DATA"
$\hat{\tau}$
w/ ERROR (Δτ)
$\frac{1}{(\Delta\tau)^2}$
τ = 0.1 ± 0.1
Δτ


θ¹ ... θⁿ
CMB
d
PRIOR
τ
P(d|θ)
"DATA"
τ̂
w/ ERROR (Δτ)
τ = 0.1 ± 0.1
Δτ
F = 1/(Δτ)²
F_ab ⇒ F_ab + 1/(Δτ)² (∂_a τ)(∂_b τ)


$\theta^1 \cdots \theta^n$
CMB
d
PRIOR
τ
P(d|θ)
"DATA"
$\hat{\tau}$
w/ ERROR ($\Delta\tau$)
$\tau = 0.1 \pm 0.1$
$\Delta\tau$
$F = \frac{1}{(\Delta\tau)^2}$
$F_{ab} \Rightarrow F_{ab} + \frac{1}{(\Delta\tau)^2}(\partial_a\tau)(\partial_b\tau)$


θ¹ ... θⁿ
CMB
d
PRIOR
τ
P(d|θ)
"DATA"
τ̂
w/ ERROR (Δτ)
τ = 0.1 ± 0.1
Δτ
F = 1/(Δτ)²
F_ab ⇒ F_ab + 1/(Δτ)² (∂_a τ)(∂_b τ)


F_ab
θ¹ … θⁿ
CMB
d
τ = 0.1 ± 0.1
Δτ
PRIOR
τ
P(d|θ)
"DATA"
τ̂
w/ ERROR (Δτ)
F = 1/(Δτ)²
F_ab ⇒ F_ab + 1/(Δτ)² (∂_a τ)(∂_b τ)

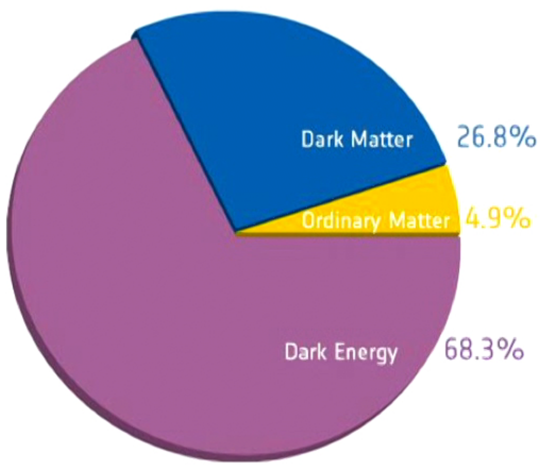
Part 4: observational cosmology,
past, present and future

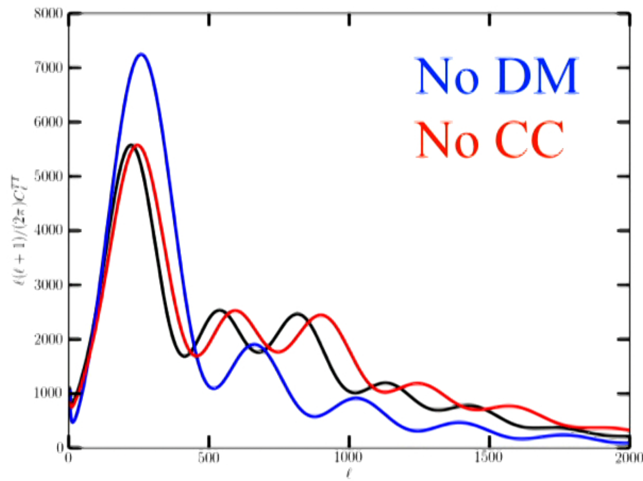
In the last few decades, observational cosmology has made amazing progress:

- Sandage (1970): "Cosmology is a search for two numbers" [$H = \dot{a}/a$ and $q = -\ddot{a}a/\dot{a}^2$]
- Peebles: "I did not continue (with computation of CMB anisotropy), in part because I had trouble imagining that such tiny disturbances could be observed" [1992 (COBE)]
- Sunyaev: "I did not think that the acoustic oscillation would ever be observed" [2000 (multiple experiments)]
- Mukhanov: "I thought it would take 1000 years to detect the logarithmic dependence of the power spectrum" [2006 (WMAP)]

(some quotes taken from a talk by Enrico Pajer)

Good: cosmology provides a real glimpse of physics beyond the (particle physics!) standard model





Planck: Cold dark matter detected at 80σ

Cosmological constant detected at 75σ

Bad: only measure a small number of parameters, hard to test competing hypotheses, or narrow down to a specific model.

Six-parameter standard model:

$\rho_\Lambda = (2.56 \pm 0.04) \times 10^{-47}$ GeV^4	Dark energy density (c.c.)
$\Omega_b = 0.0486 \pm 0.0007$	Baryonic matter abundance
$\Omega_c = 0.267 \pm 0.009$	Cold dark matter abundance
$\Delta_\zeta^2 = (2.11 \pm 0.05) \times 10^{-9}$	Initial power spectrum amplitude
$n_s = 0.967 \pm 0.004$	Spectral index
$\tau = 0.058 \pm 0.012$	CMB optical depth

Extensions:

- Non-Gaussian initial conditions ($f_{NL} = 2.5 \pm 5.7$)
- Non-minimal neutrino mass ($m_\nu < 0.23$ eV at 95% CL)
- Extra neutrino species or other light relics ($N_{eff} = 3.04 \pm 0.18$)
- Nonzero spatial curvature ($\Omega_K = 0.000 \pm 0.005$)
- Time-dependent dark energy density ($w = -1.02 \pm 0.08$)
- Cosmological gravity waves ($r < 0.12$ at 95% CL)

+ many more!

Forecasts suggest that there is still a lot of room to shrink error bars in future experiments.

Should we take futuristic forecasts seriously? Historical exercise: Tegmark (1999) forecasts for Planck, very futuristic at the time.

Parameter	Forecasted uncertainty (1999)	Reported uncertainty (2015)
ρ_b	0.94%	0.72%
$(\rho_b + \rho_c)$	1.6%	0.9%
n_s	0.0076	0.0048
$\Lambda/\rho_{\rm tot}$	0.022	0.0087

Planck somewhat outperformed its forecasts.

CMB currently dominates constraints on 6-parameter model space

	CMB alone	CMB + baryon acoustic oscillations + type IA supernovae + direct H_0 measurements
DM density $\rho_{c,0}$	0.2618 ± 0.0087	0.2589 ± 0.0063 $(\times\rho_{tot})$
Bary. density $\rho_{b,0}$	0.04884 ± 0.00085	0.04860 ± 0.00070 $(\times\rho_{tot})$
Cosm. constant Λ	2.543 ± 0.071	2.567 ± 0.051 $(\times 10^{-47}\ \mathrm{GeV}^4)$
Amplitude A_ζ	2.130 ± 0.053	2.142 ± 0.049
Spectral index n_s	0.9653 ± 0.0048	0.9667 ± 0.0040
Optical depth τ	0.063 ± 0.014	0.066 ± 0.012

Not true in extensions of standard model! Non-CMB datasets are important when more parameters are added.

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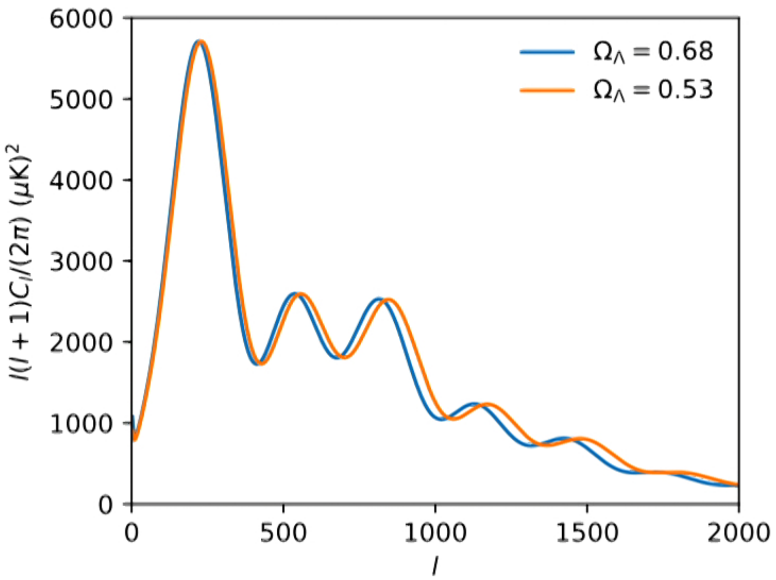
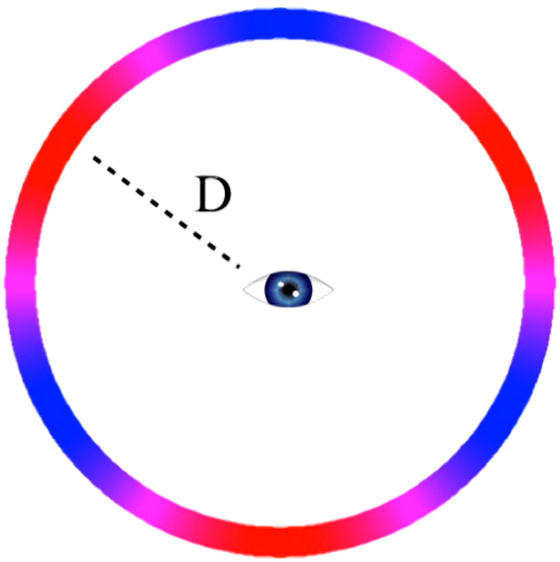
+ many more!

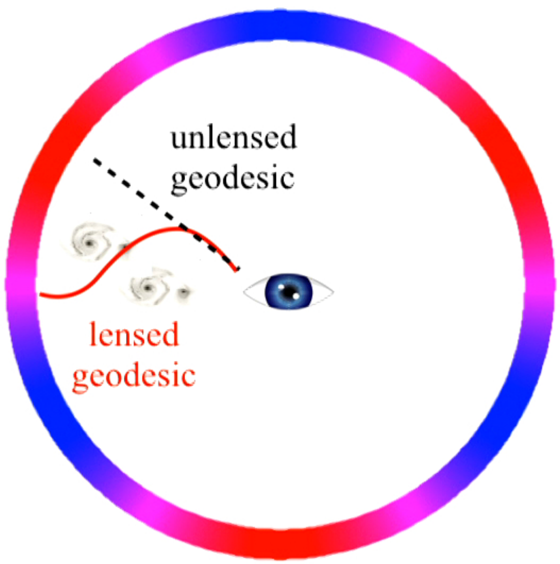
CMB temperature observations can constrain the cosmological constant Λ, even though Λ is negligible when the CMB is formed!

How? By changing the distance D at which the CMB surface is observed.





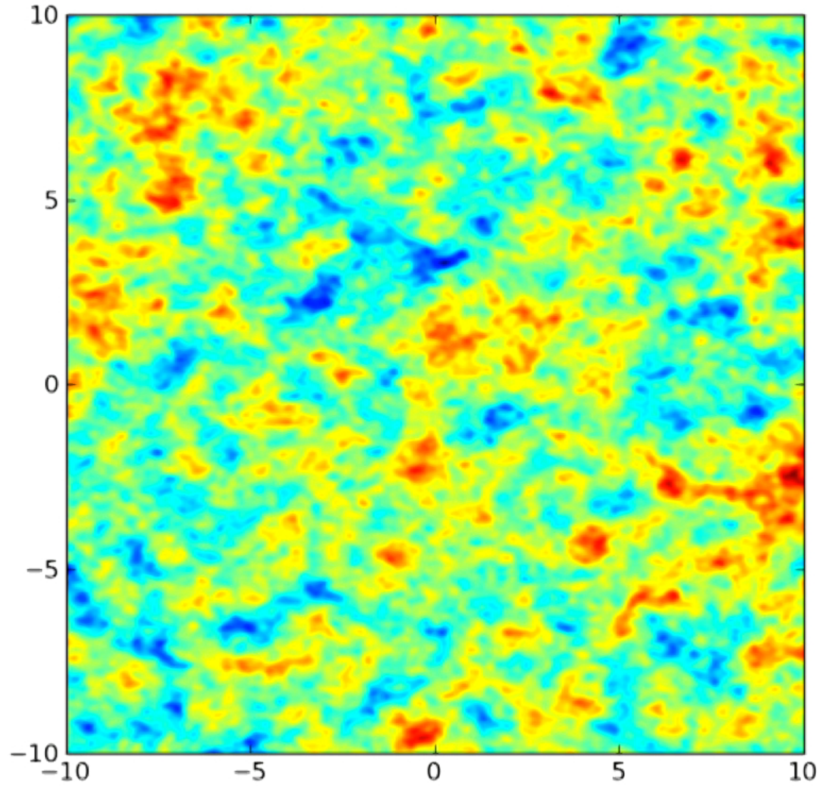
Planck can tell the difference between Λ and spatial curvature. The distance degeneracy is broken by CMB lensing, which I'll describe in the next few slides.



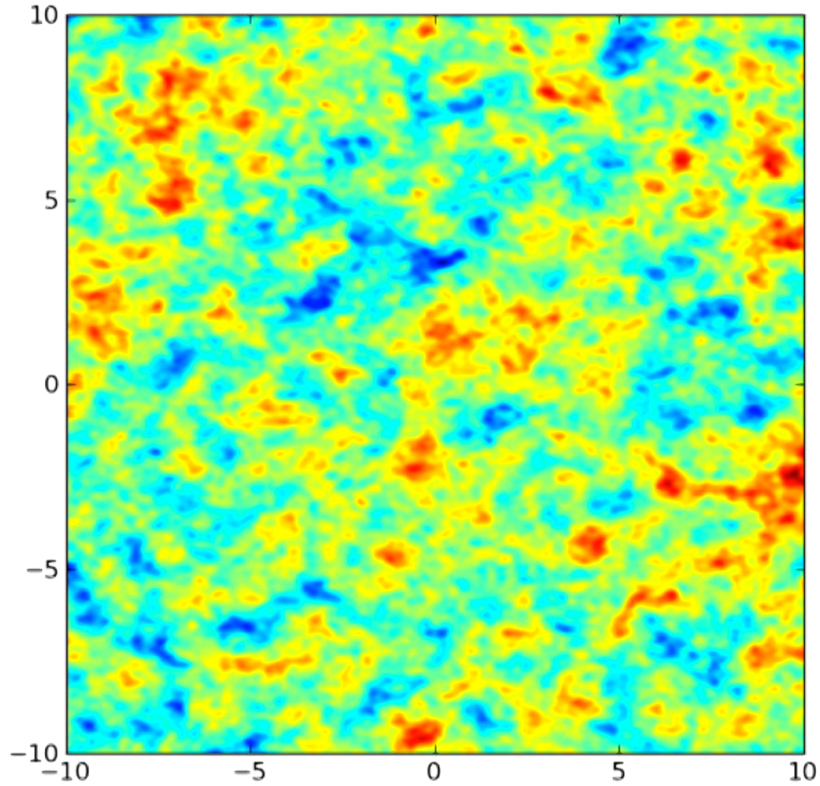
Lensing moves existing temperature fluctuations around, but does not generate new anisotropy (lensing conserves surface brightness)

Shown exaggerated here! Actual lensing deflections are a few arcminutes.

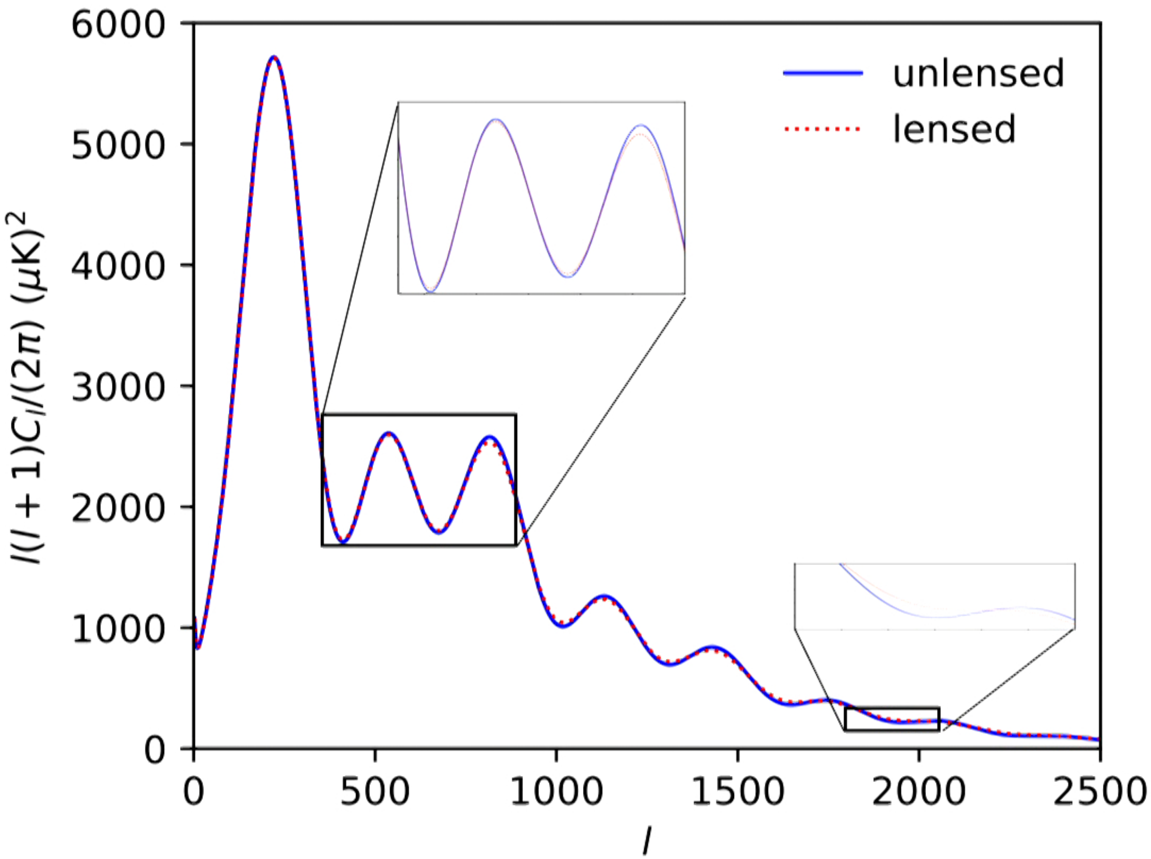
Unlensed vs lensed CMB



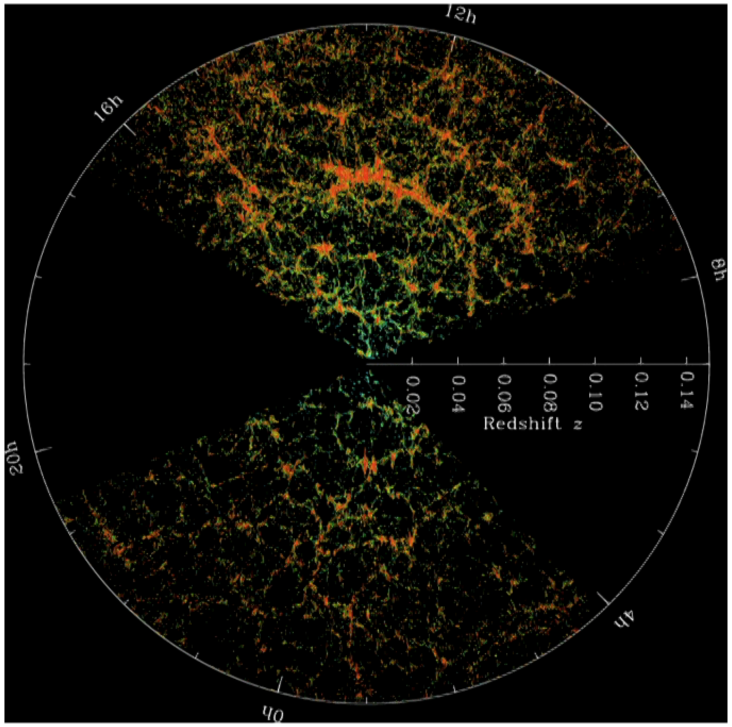
Unlensed vs lensed CMB



Effect of CMB lensing on the temperature power spectrum.
Lensing smooths peaks and adds power on small scales (high l).
Breaks the distance degeneracy (plot forthcoming in a few slides).

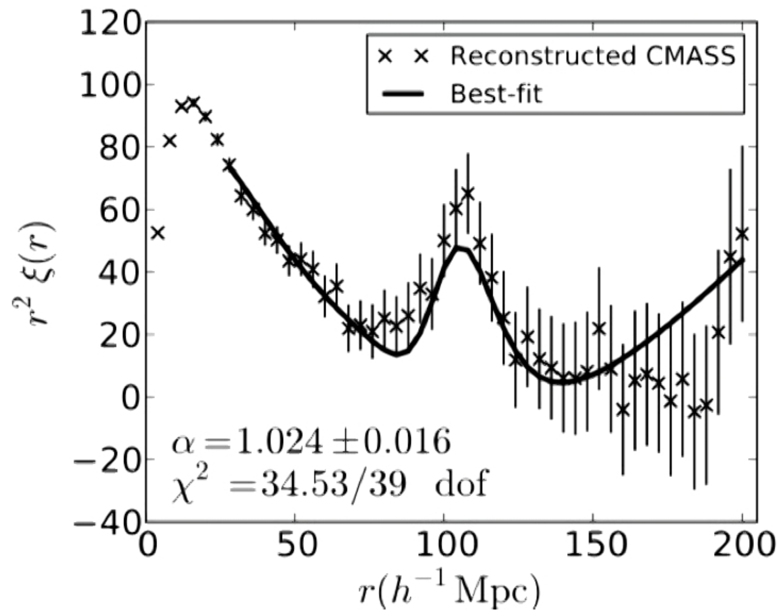


A galaxy survey measures the number density of galaxies throughout the universe. (A 3D field, since redshifts and angular locations are measured.)



The 3D power spectrum of the galaxy density field can be used to constrain cosmological parameters. (In galaxy surveys, the correlation function ζ(r) is usually used instead of the power spectrum P(k)).

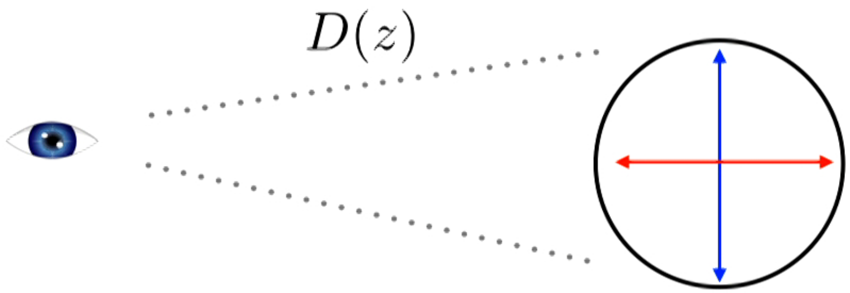
Like the CMB, the correlation function ζ(r) contains an acoustic feature which can be used as a "standard ruler".



SDSS (2012)

More precisely, there are two standard rulers:

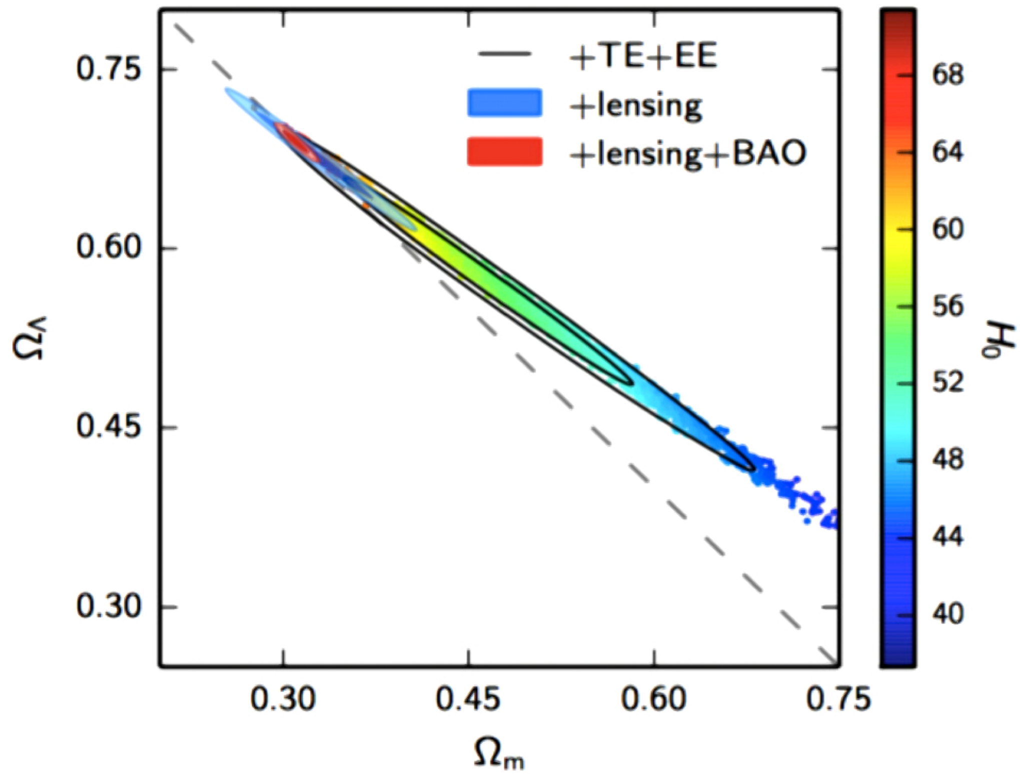
- "Transverse" observation constrains D(z)
- "Radial" observation constrains H(z)



Furthermore, can measure D(z) and H(z) as functions of z!
(Unlike the CMB, where there is only one source redshift.)

Galaxy BAO measurements are very powerful for constraining expansion history and breaking degeneracies between parameters.

Black ellipses: unlensed CMB
Blue ellipses: lensed CMB
Red ellipses: lensed CMB + BAO



Planck 2015

Current constraints on expansion history:

$\Omega_\Lambda = 0.691 \pm 0.006$ cosmological constant

$\Omega_K = 0.000 \pm 0.005$ spatial curvature

$w = -1.02 \pm 0.08$ dark energy equation of state

The last parameter (w) parameterizes the time dependence of dark energy density as $\rho \sim a^{-3(1+w)}$, i.e. $w = -1$ corresponds to a cosmological constant.

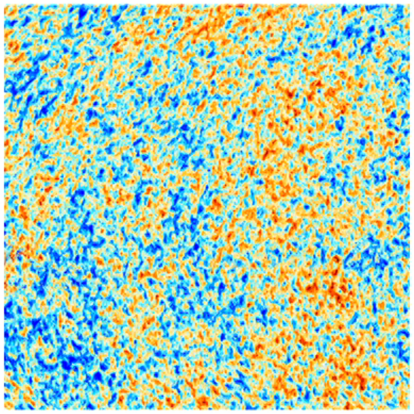
i.e. the DE energy density at z=1 is uncertain by 24% (at 1σ)!

Good news: these constraints will improve by an order of magnitude in the not-too-distant future (mainly from better BAO data).

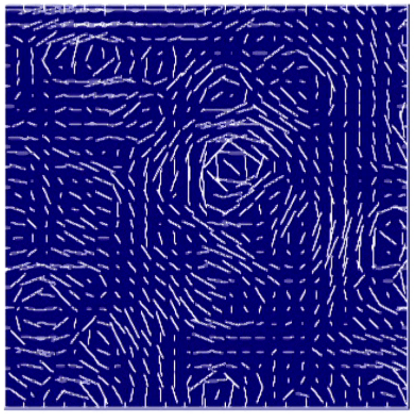
Bad news: there is no natural threshold for these parameters, so this is a fishing expedition (as far as I know!)

CMB polarization

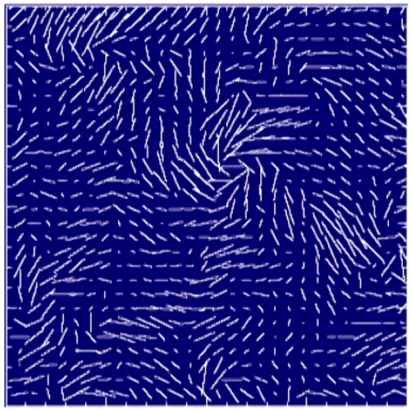
Temperature



E-mode linear polarization



B-mode linear polarization



E/B decomposition of linear polarization (traceless symmetric tensor) is similar to gradient/curl decomposition of vector field.

B-modes are different

Theorem: (scalar sources) + (linear perturbation theory)
$\Rightarrow$ (no B-modes are generated)

Rephrased: B-modes only arise from

- Primordial gravity waves (non-scalar sources)
- Second-order effects (largest by far is CMB lensing)

Some models of inflation predict primordial gravity waves.

$$ds^2 = -dt^2 + a(t)^2 e^{2\zeta(x)}(\delta_{ij} + h_{ij}(x))$$

Parameterized by "tensor-to-scalar" ratio $r = \frac{P_h(k)}{P_\zeta(k)}$

Current upper limit: r < 0.12 (95% CL), mainly from CMB temperature.

CMB temperature and polarization power spectra