Title: Quantum Many-Body Scarring in constrained models

Date: Jun 21, 2018 02:00 PM

URL: http://pirsa.org/18060066

Abstract: <p>Recent quench experiments in a quantum simulator of interacting Rydberg atoms demonstrated surprising long-lived, periodic revivals from certain low entanglement states, while apparently quick thermalization from others. Motivated by these findings, I will in this talk analyze the dynamics of a family of kinetically constrained spin models related to the experiments. By introducing a manifold of locally entangled spins, representable by a low-bond dimension matrix product state (MPS), I will derive "semiclassical" equations of motion for them. I find that they host isolated, unstable periodic orbits, the presence of which captures the long-lived oscillations and gives rise to slow relaxation of local observables from certain initial configurations. This thus represents a form of weak breaking of ergodicity in dynamics. Our results are reminiscent of the phenomenon of quantum scarring in single-particle chaotic systems which is rooted in classical unstable periodic orbits, and complement the explanation of the recurrences given by [Nature Physics (2018), doi:10.1038/s41567-018-0137-5], in terms of motion over special nonergodic many-body eigenstates, suggestively dubbed `quantum many-body scars'. </p>

Quantum simulators & many-body dynamics

Trapped ions, ultracold atoms, superconducting qubits etc

Key features

- Well-isolated
- Control over each atom/ion
- Strong interactions
- Single-site resolution

MENU nature [Bernien et al, Nature (2017)]

Altmetric: 382 Citations: 35 More detail »

Article | Published: 29 November 2017

Probing many-body dynamics on a 51-atom quantum simulator

Hannes Bernien, Sylvain Schwartz, Alexander Keesling, Harry Levine, Ahmed Omran, Hannes Pichler, Soonwon Choi, Alexander S. Zibrov, Manuel Endres, Markus Greiner, Vladan Vuletić & Mikhail D. Lukin

Nature **551**, 579–584 (30 November 2017) Download Citation

MENU nature [Zhang et al, Nature (2017)]

Altmetric: 223 More detail »

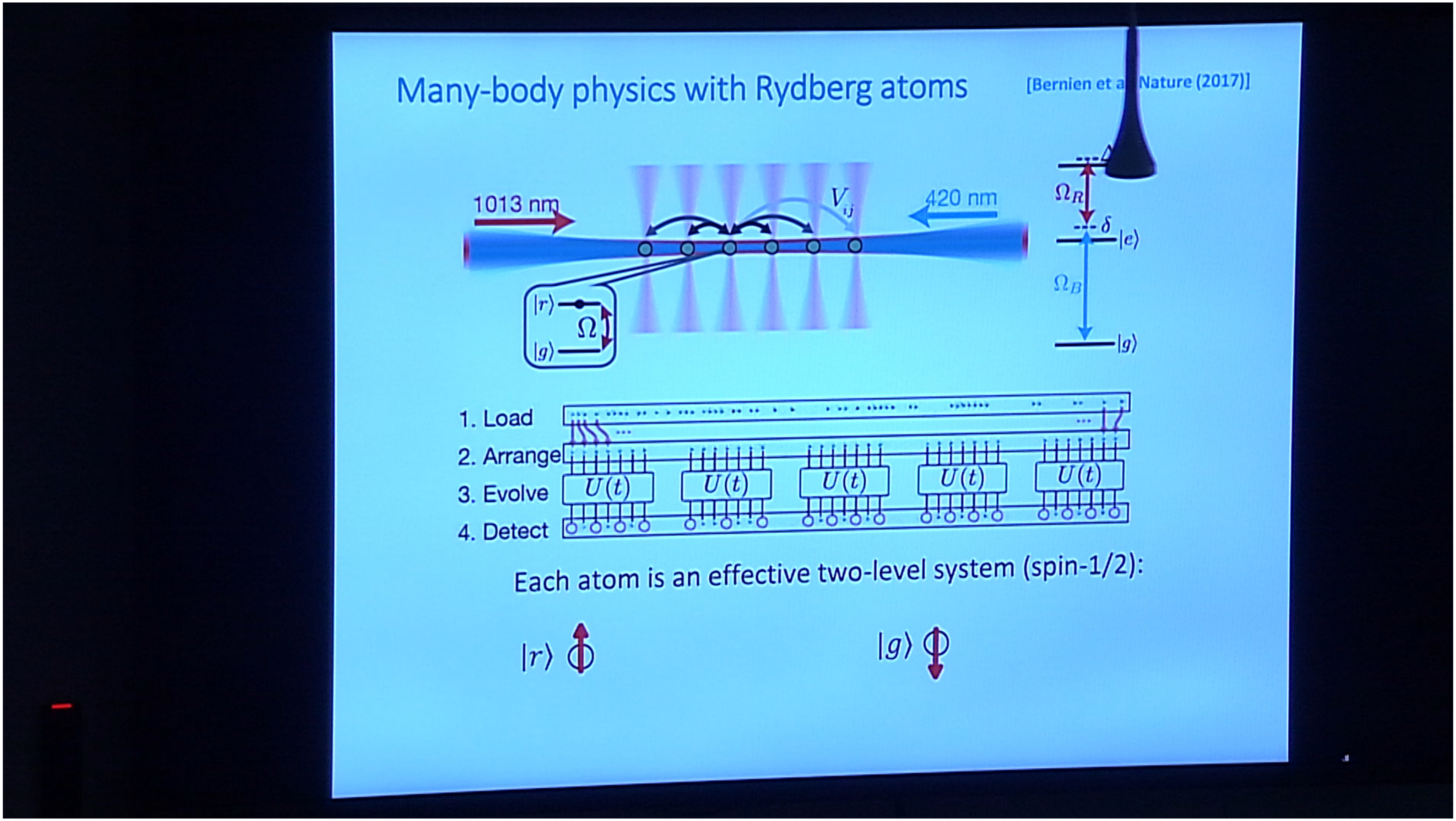
Letter | Published: 29 November 2017

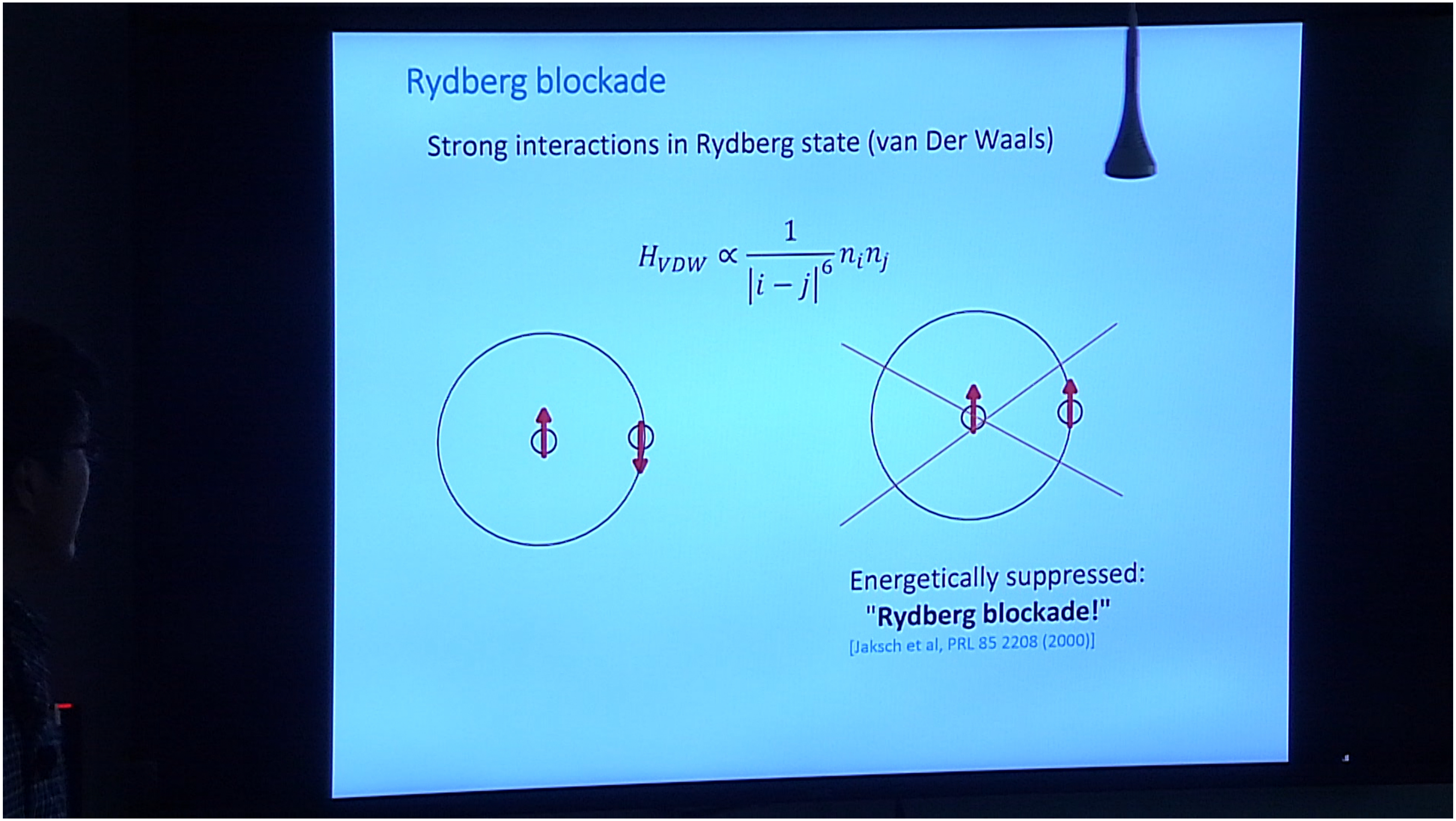
Observation of a many-body dynamical phase transition with a 53-qubit quantum simulator

J. Zhang, G. Pagano, P. W. Hess, A. Kyprianidis, P. Becker, H. Kaplan, A. V. Gorshkov, Z.-X. Gong & C. Monroe

Nature **551**, 601–604 (30 November 2017) Download Citation

Well-suited for probing many-body physics!


Many-body physics with Rydberg atoms
[Bernien et a Nature (2017)]
1013 nm
420 nm
V_{ij}
Ω_R
δ
$|e\rangle$
Ω_B
$|g\rangle$
$|r\rangle$
Ω
$|g\rangle$
1. Load
2. Arrange
3. Evolve
4. Detect
$U(t)$
Each atom is an effective two-level system (spin-1/2):
$|r\rangle$
$|g\rangle$


Rydberg blockade
Strong interactions in Rydberg state (van Der Waals)
$$H_{VDW} \propto \frac{1}{|i-j|^6} n_i n_j$$
Energetically suppressed:
"Rydberg blockade!"
[Jaksch et al, PRL 85 2208 (2000)]

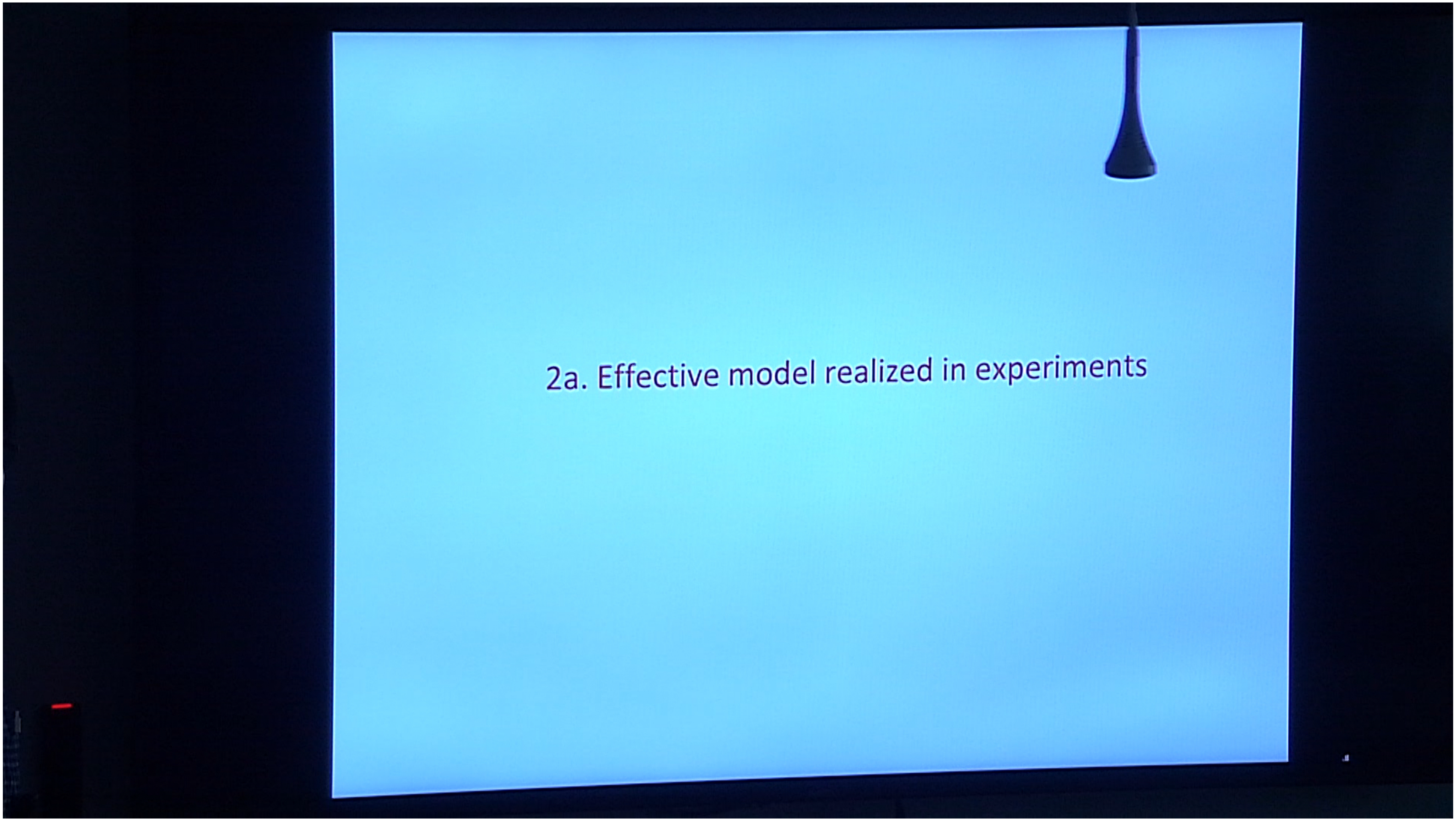
Effective, constrained Hilbert space

Let $= |0\rangle$ and $= |1\rangle$

Hilbert space of L Rydberg atoms?

$|0\rangle \quad |1\rangle$

D = 2


2a. Effective model realized in experiments

Experimental quantum quench

Rydberg blockaded model in constrained space:

$$H = \Omega \sum_i \widetilde{X}_i$$

Equivalent mod[illegible] spin-1/2s:

$$H = \Omega \sum_i P_{i-1} X_i P_{i+1}$$

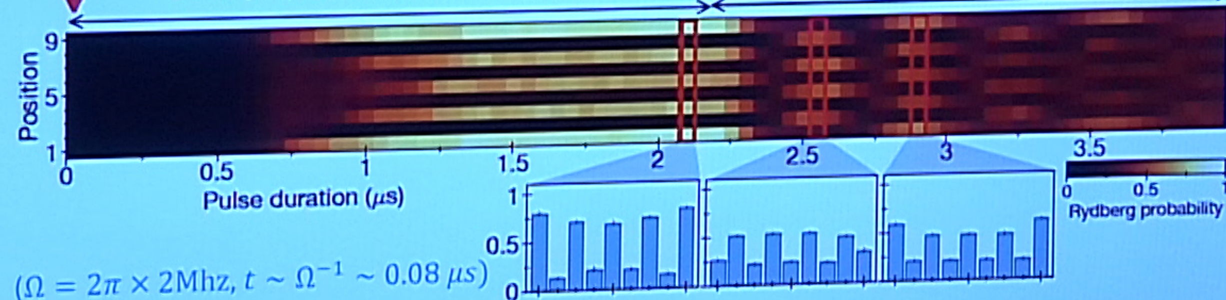
where $P_i = |0\rangle_i \langle 0|_i$

$|000000000\rangle$

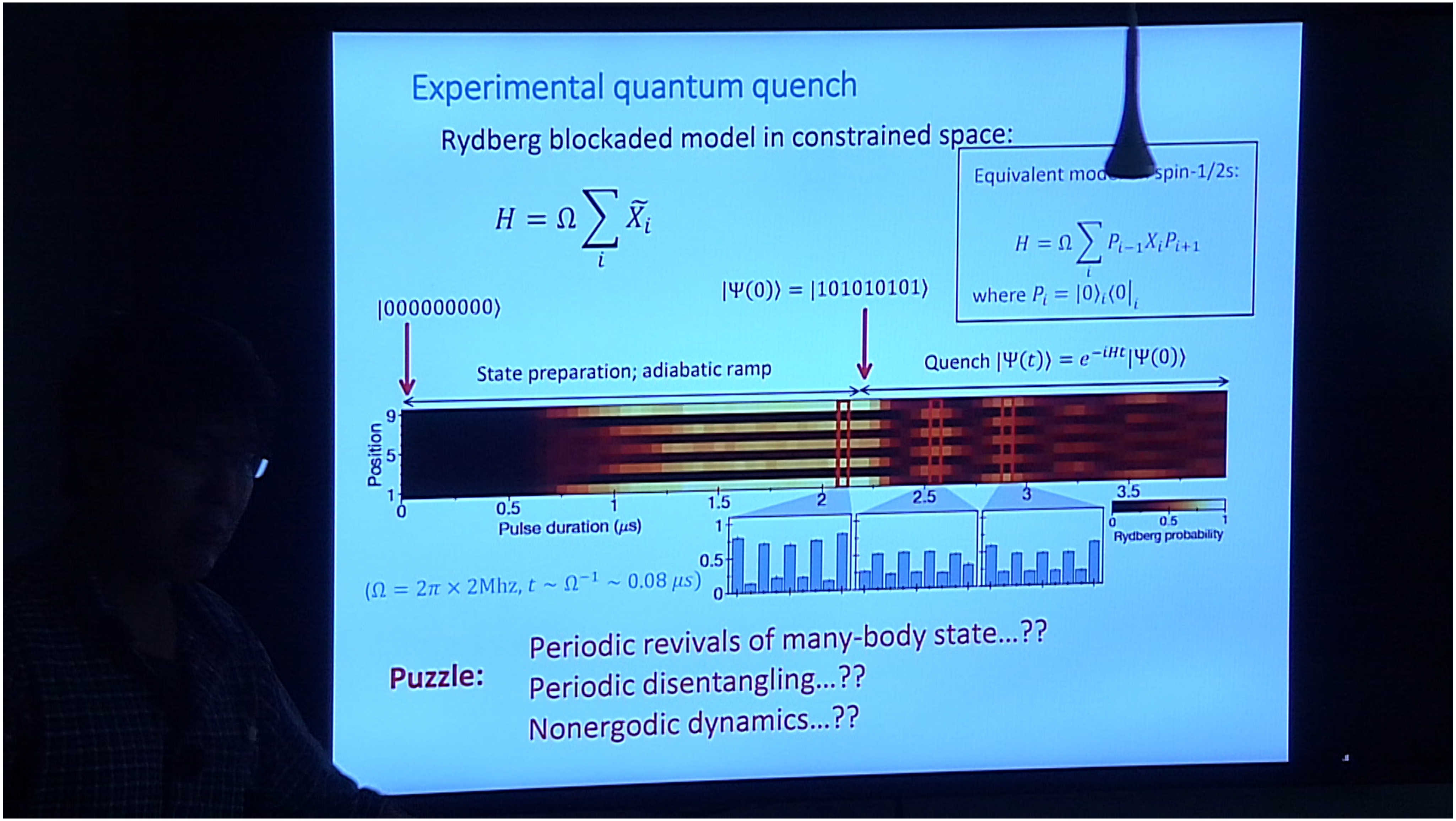
$|\Psi(0)\rangle = |101010101\rangle$

State preparation; adiabatic ramp

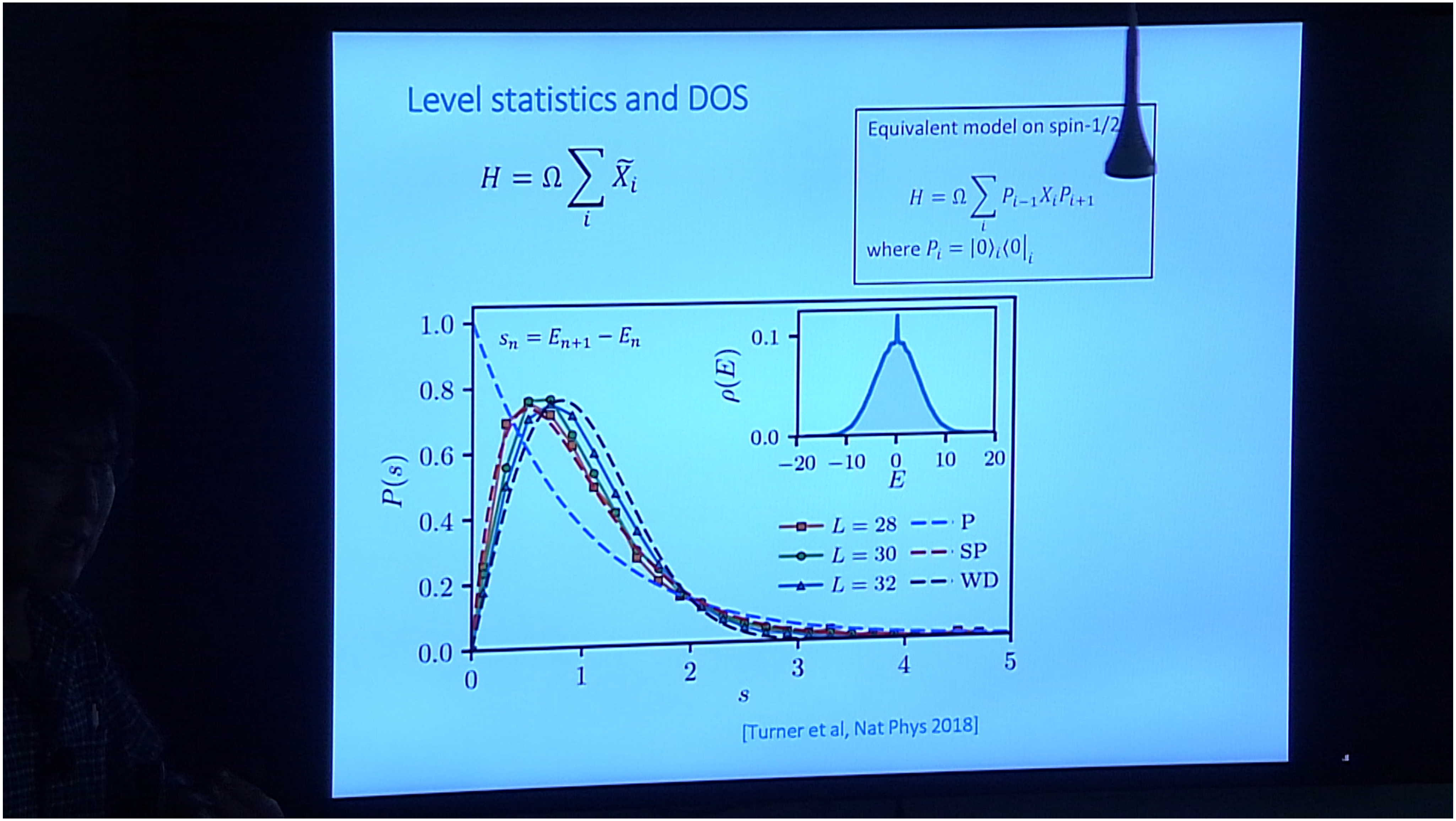
Quench $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$



$(\Omega = 2\pi \times 2\text{Mhz}, t \sim \Omega^{-1} \sim 0.08\ \mu s)$


Experimental quantum quench
Rydberg blockaded model in constrained space:
$H = \Omega \sum_i \widetilde{X}_i$
Equivalent mod… spin-1/2s:
$H = \Omega \sum_i P_{i-1} X_i P_{i+1}$
where $P_i = |0\rangle_i \langle 0|_i$
$|000000000\rangle$
$|\Psi(0)\rangle = |101010101\rangle$
State preparation; adiabatic ramp
Quench $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$
Position
Pulse duration (μs)
Rydberg probability
$(\Omega = 2\pi \times 2\text{Mhz}, t \sim \Omega^{-1} \sim 0.08\ \mu s)$
Puzzle:
Periodic revivals of many-body state…??
Periodic disentangling…??
Nonergodic dynamics…??

2b. Numerical checks on model


Level statistics and DOS
$H = \Omega \sum_i \widetilde{X}_i$
Equivalent model on spin-1/2
$H = \Omega \sum_i P_{i-1} X_i P_{i+1}$
where $P_i = |0\rangle_i \langle 0|_i$
$s_n = E_{n+1} - E_n$
$P(s)$
s
$\rho(E)$
E
$L = 28$
$L = 30$
$L = 32$
P
SP
WD
[Turner et al, Nat Phys 2018]

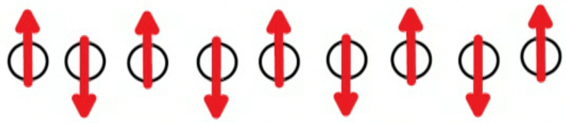
Numerics: quantum quench

$$H = \Omega \sum_i \widetilde{X}_i$$

Equivalent model on spin-1/2s:

$$H = \Omega \sum_i P_{i-1} X_i P_{i+1}$$

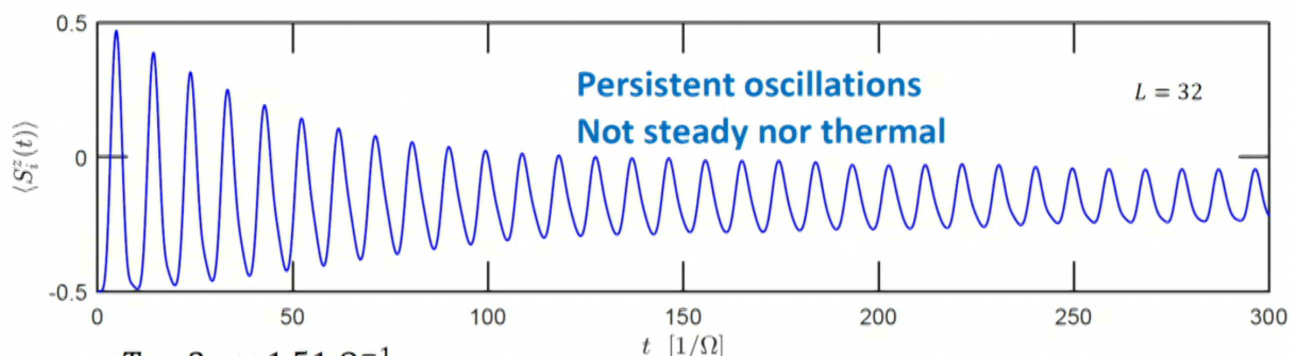
where $P_i = |0\rangle_i \langle 0|_i$



Initial State: $|Z_2\rangle = \otimes_i^L |1\rangle_{2i-1} |0\rangle_{2i}$
$= |1010101010 \ldots\rangle$

Energy: $\langle Z_2 | H | Z_2 \rangle = 0$

Eff Temp: $\beta = \frac{1}{k_B T} = 0$



$T = 2\pi \times 1.51\ \Omega^{-1}$

Not equal to Rabi oscillation period of free spins

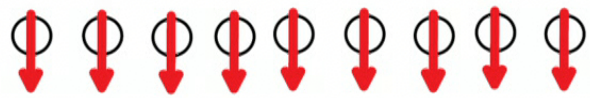
Numerics: quantum quench

$$H = \Omega \sum_i \widetilde{X}_i$$

Equivalent model on spin-1/2s:

$$H = \Omega \sum_i P_{i-1} X_i P_{i+1}$$

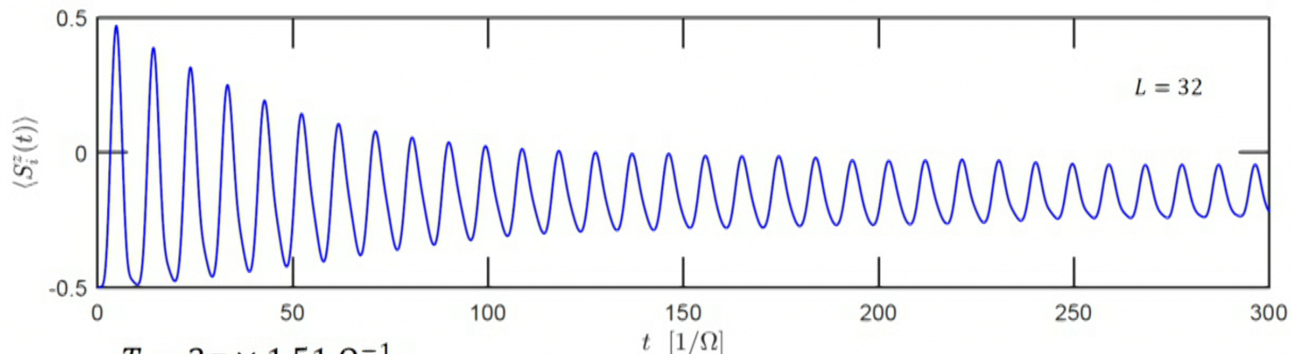
where $P_i = |0\rangle_i \langle 0|_i$



Initial State: $|\mathbf{0}\rangle = \otimes_i^L |0\rangle_i = |0000000 \ldots\rangle$

Energy: $\langle \mathbf{0} | H | \mathbf{0} \rangle = 0$

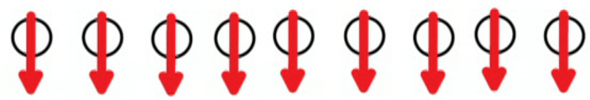
Eff Temp: $\beta = \frac{1}{k_B T} = 0$



$T = 2\pi \times 1.51\ \Omega^{-1}$

Numerics: quantum quench

$$H = \Omega \sum_i \tilde{X}_i$$



Initial State: $|\mathbf{0}\rangle = \otimes_i^L |0\rangle_i$
$= |0000000 \ldots\rangle$

Equivalent model on spin-1/2s:

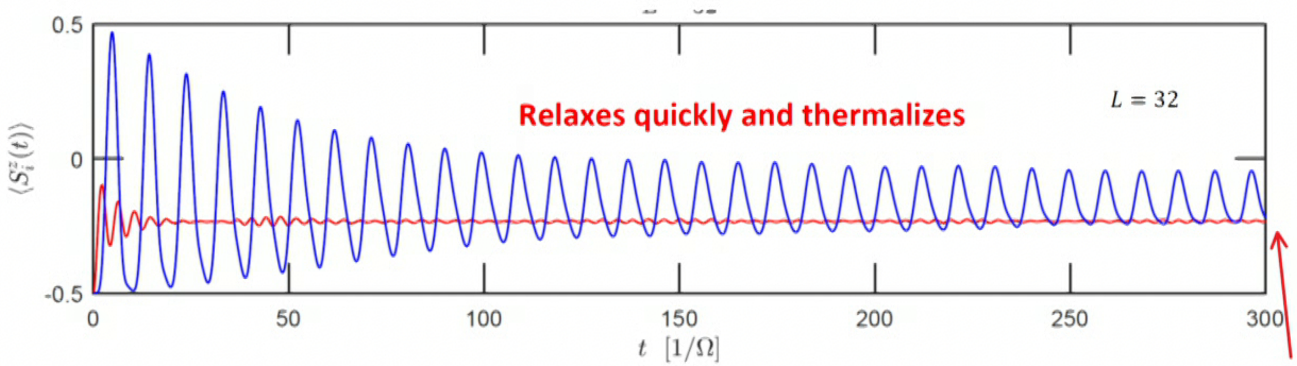
$$H = \Omega \sum_i P_{i-1} X_i P_{i+1}$$

where $P_i = |0\rangle_i \langle 0|_i$

Thermodynamically indistinguishable

Energy: $\langle \mathbf{0}|H|\mathbf{0}\rangle = 0$

Eff Temp: $\beta = \frac{1}{k_B T} = 0$

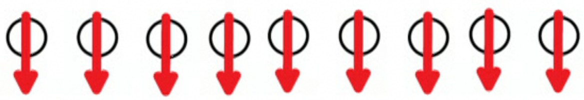


Generic behavior for random $|10100010 \cdots 010\rangle$ state

Agrees with ETH predictions

Numerics: quantum quench

$$H = \Omega \sum_i \widetilde{X}_i$$



Initial State: $|\mathbf{0}\rangle = \otimes_i^L |0\rangle_i$
$= |0000000 \ldots\rangle$

Equivalent model on spin-1/2s:

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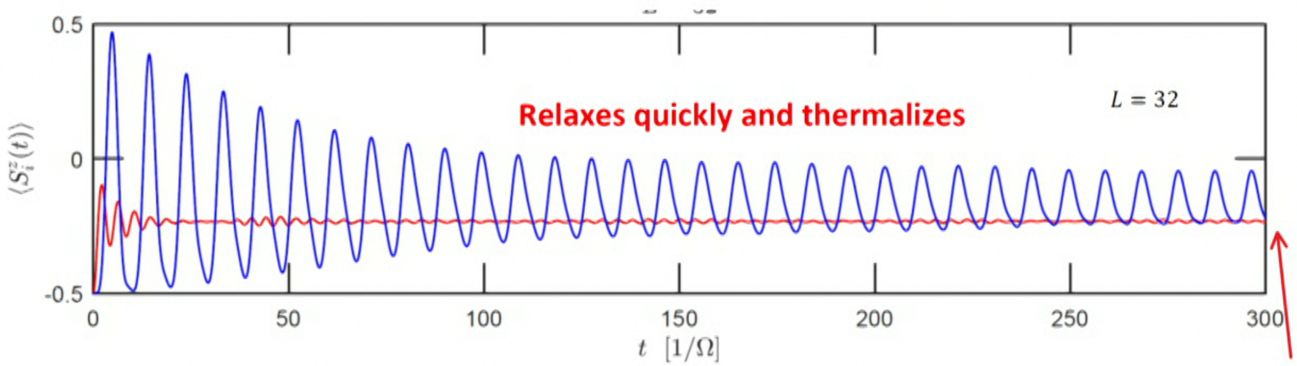
Thermodynamically indistinguishable

Energy: $\langle \mathbf{0} | H | \mathbf{0} \rangle = 0$

Eff Temp: $\beta = \frac{1}{k_B T} = 0$

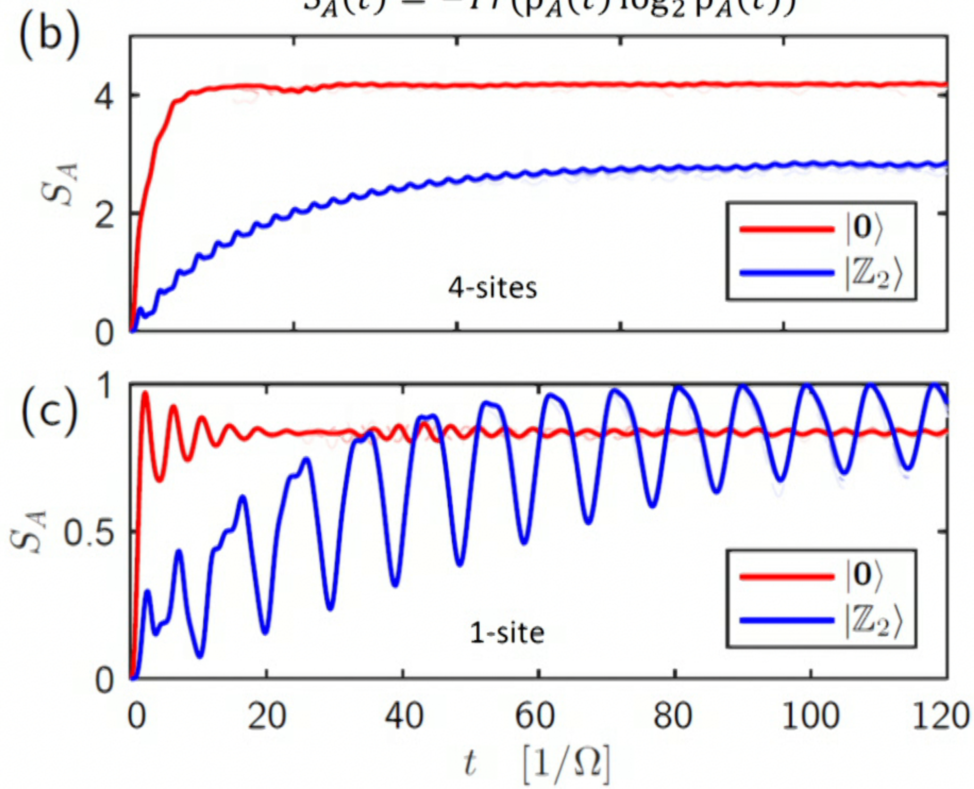


Generic behavior for random $|10100010 \cdots 010\rangle$ state

Agrees with ETH predictions

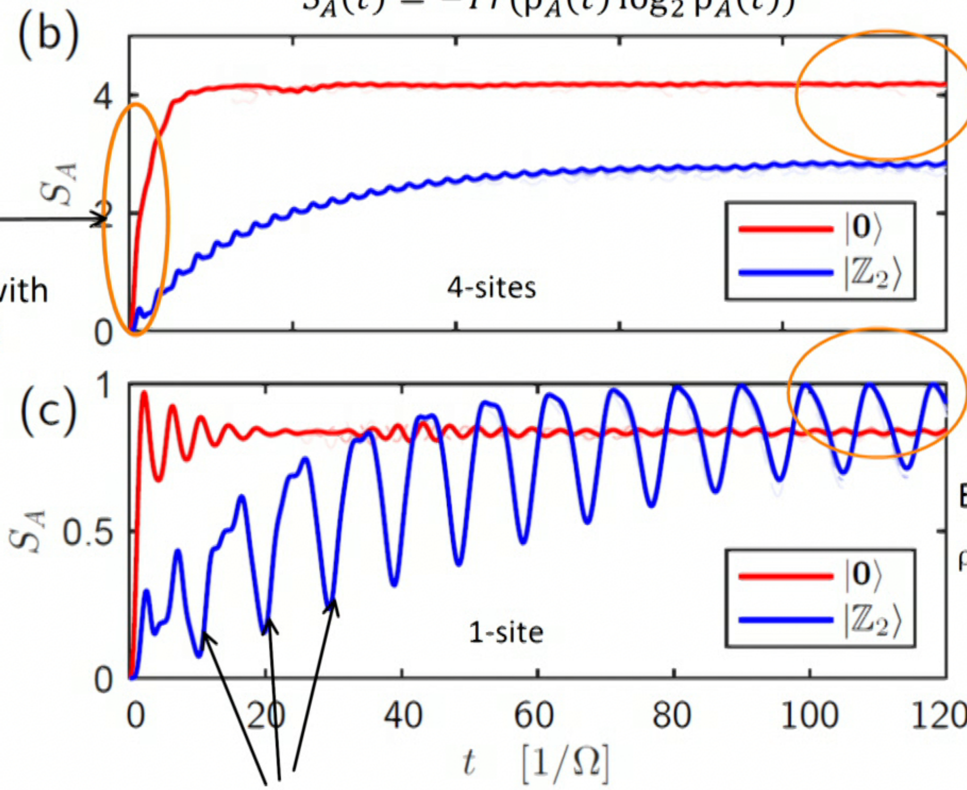
More on dynamics... Entanglement growth

$S_A(t) = -Tr(\rho_A(t) \log_2 \rho_A(t))$



More on dynamics... Entanglement growth

$S_A(t) = -Tr(\rho_A(t)\log_2 \rho_A(t))$



Early time:
Linear growth with different slopes

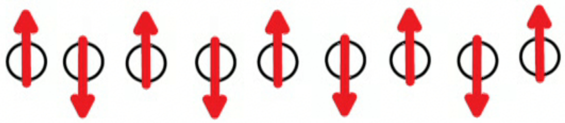
Late times:
Saturation to page bound (top)

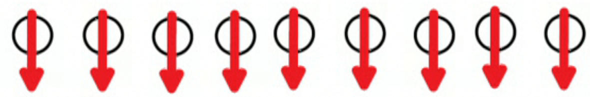
Expected:
$\rho_A = \frac{1}{Z}(P_0 + \frac{\varphi}{1+\varphi}P_1)$

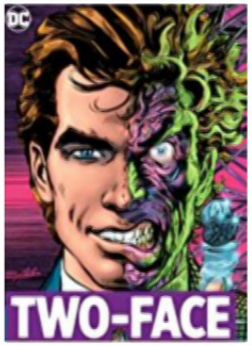
Periodic disentangling of single spin;
second law of thermodynamics "violated"

Two different behaviors in the same ergodic many-body system

$$H = \Omega \sum_i \widetilde{X}_i$$







$$|Z_2\rangle = |101010\cdots\rangle$$

- Periodic revivals
- No or extremely slow equilibriation or thermalization
- **Weak ergodicity breaking in dynamics**

$$|\mathbf{0}\rangle = |000000\cdots\rangle$$

- Generic behavior
- Quickly thermalizing
- **Ergodic**

Q:

Can we capture periodic revivals in a simple, effective fashion, i.e. some "semiclassical" state?

Experimental quantum quench

Rydberg blockaded model in constrained space:

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Equivalent model on spin-1/2s:

$$H = \Omega \sum_i P_{i-1} X_i P_{i+1}$$

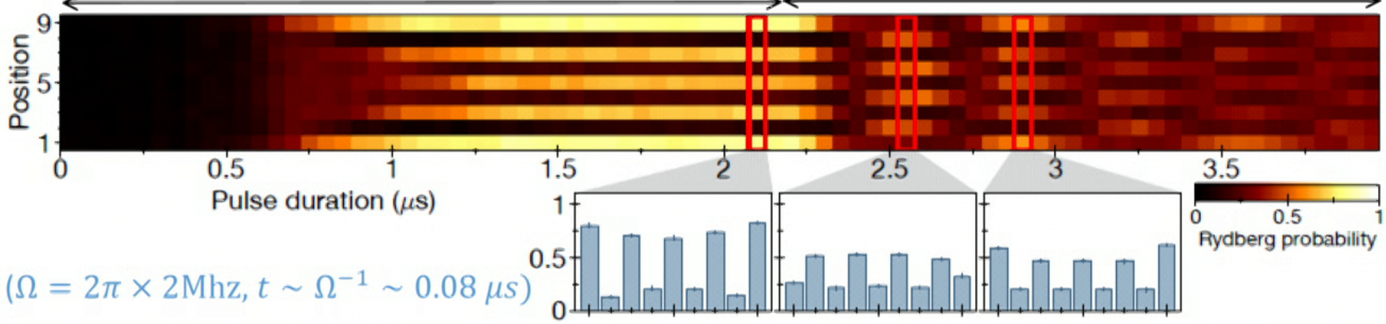
where $P_i = |0\rangle_i \langle 0|_i$

$|000000000\rangle$

$|\Psi(0)\rangle = |101010101\rangle$

State preparation; adiabatic ramp

Quench $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$



$(\Omega = 2\pi \times 2\text{Mhz}, t \sim \Omega^{-1} \sim 0.08\ \mu s)$

Puzzle:

Periodic revivals of many-body state…??

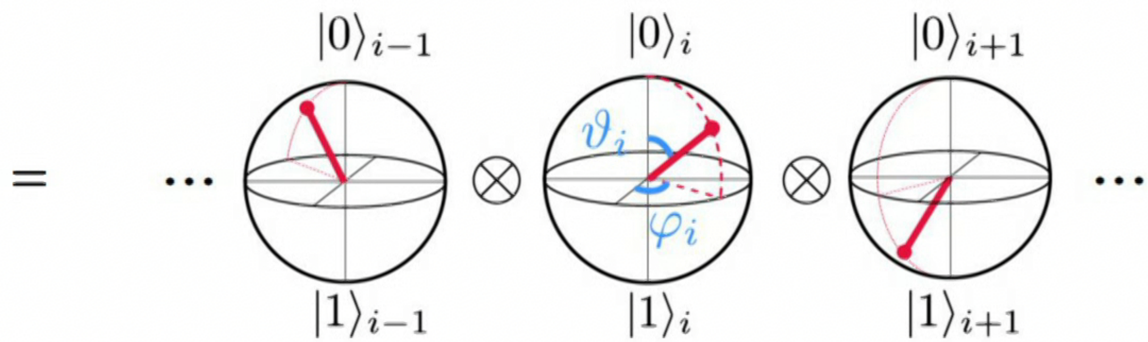
Periodic disentangling…??

Nonergodic dynamics…??

"Semiclassical" description

For a many-body spin system, the semiclassical limit is taken by considering spin coherent states

$$|\psi(\boldsymbol{\vartheta},\boldsymbol{\varphi})\rangle = \bigotimes_i^L e^{-i\vartheta_i S_i^z} e^{-i\varphi_i S_i^x} |0\rangle_i$$

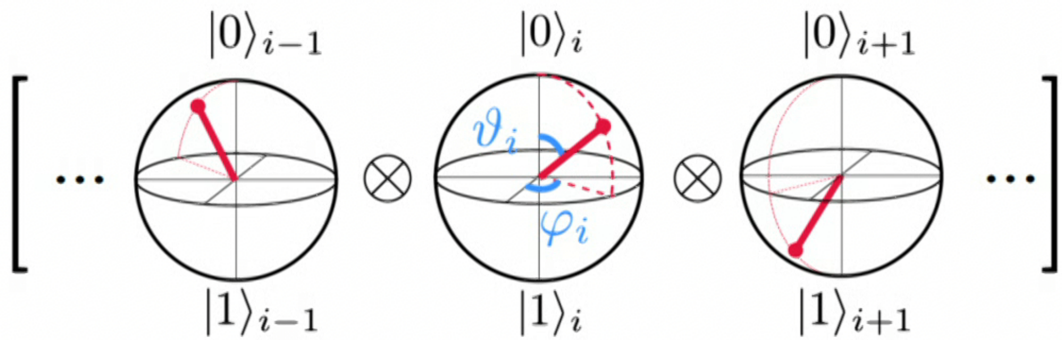


"Semiclassical" description

Solution: consider projecting out $|11\rangle$:

$$|\psi(\boldsymbol{\vartheta},\boldsymbol{\varphi})\rangle = \mathcal{P} \otimes_i^L e^{-i\vartheta_i S_i^z} e^{-i\varphi_i S_i^x} |0\rangle_i$$

$$= : \mathcal{P}$$



"Gutzwiller projection", Naturally introduces entanglement

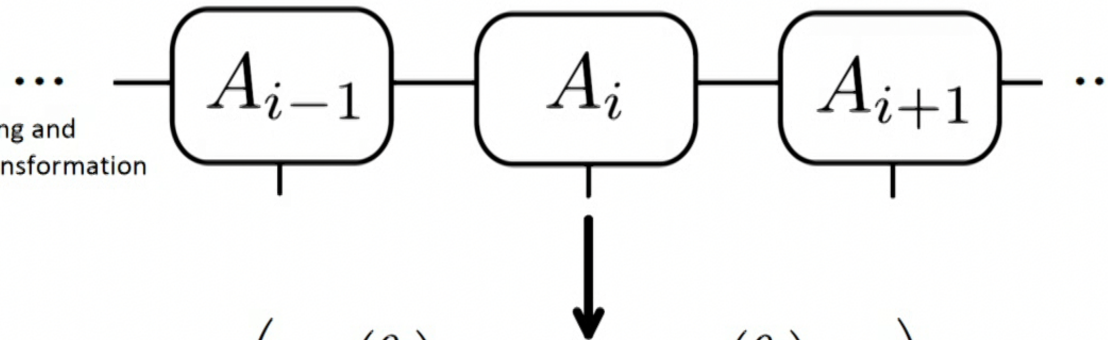
"Semiclassical" description

Solution: consider projecting out $|11\rangle$:

$$|\psi(\boldsymbol{\vartheta},\boldsymbol{\varphi})\rangle = \mathcal{P} \otimes_i^L e^{-i\vartheta_i S_i^z} e^{-i\varphi_i S_i^x} |0\rangle_i$$

=

After normalizing and using gauge transformation



Explicit bond-2 MPS with norm 1

$$\begin{pmatrix} \cos\left(\frac{\theta_i}{2}\right)|0\rangle_i & e^{i\phi_i}\sin\left(\frac{\theta_i}{2}\right)|1\rangle_i \\ |0\rangle_i & 0 \end{pmatrix}$$

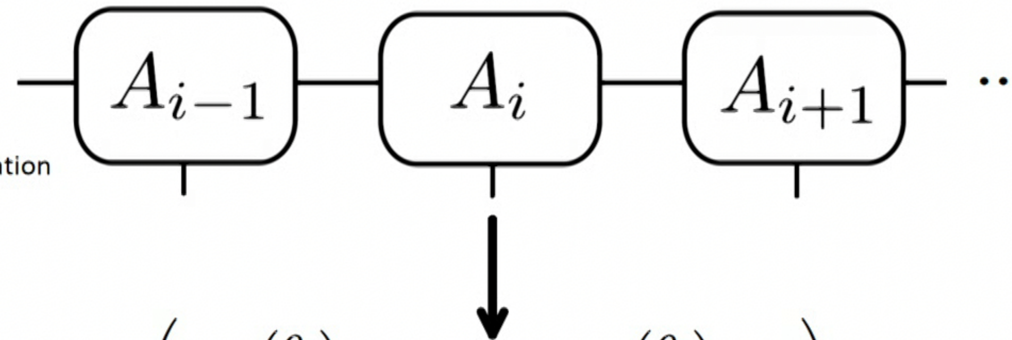
"Semiclassical" description

Solution: consider projecting out $|11\rangle$:

$$|\psi(\boldsymbol{\vartheta},\boldsymbol{\varphi})\rangle = \mathcal{P} \otimes_i^L e^{-i\vartheta_i S_i^z} e^{-i\varphi_i S_i^x} |0\rangle_i$$

=

After normalizing and using gauge transformation



Explicit bond-2 MPS with norm 1

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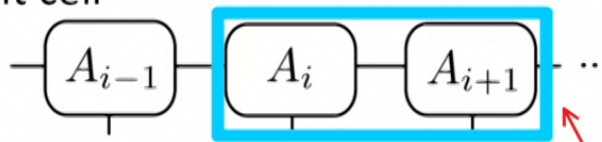
"Semiclassical" description

Consider 2-site unit cell

$|\psi(\boldsymbol{\theta},\boldsymbol{\phi})\rangle = \cdots$



$$A_i = \begin{pmatrix} \cos\left(\frac{\theta_i}{2}\right)|0\rangle_i & e^{i\phi_i}\sin\left(\frac{\theta_i}{2}\right)|1\rangle_i \\ |0\rangle_i & 0 \end{pmatrix}$$

Captures $|Z_2\rangle$ and $|\mathbf{0}\rangle$ states

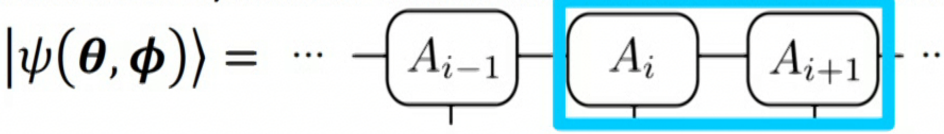
$\phi_{2i+1} = 0, \phi_{2i} = 0$

$\theta_{2i+1} = 0, \theta_{2i} = 0$

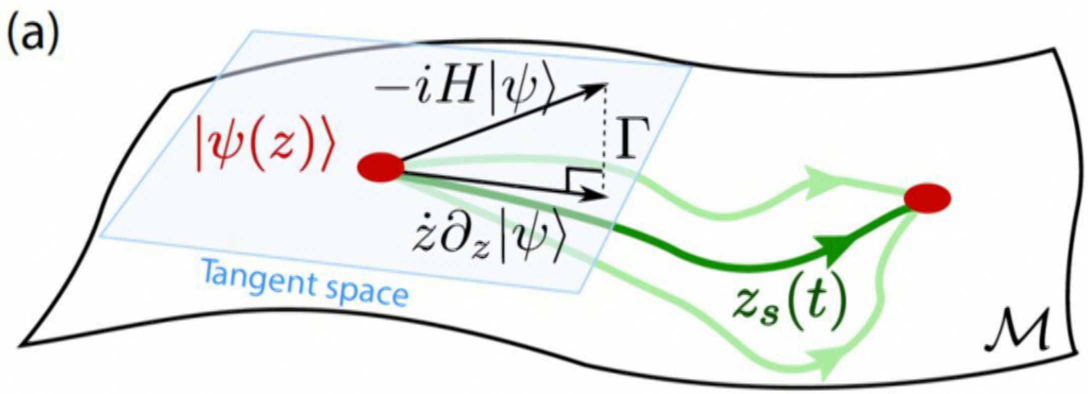
$$Tr\left[\cdots \begin{pmatrix} |0\rangle_i & 0 \\ |0\rangle_i & 0 \end{pmatrix} \begin{pmatrix} |0\rangle_{i+1} & 0 \\ |0\rangle_{i+1} & 0 \end{pmatrix} \begin{pmatrix} |0\rangle_{i+2} & 0 \\ |0\rangle_{i+2} & 0 \end{pmatrix} \begin{pmatrix} |0\rangle_{i+3} & 0 \\ |0\rangle_{i+3} & 0 \end{pmatrix} \cdots \right] = |0000\cdots\rangle = |\mathbf{0}\rangle$$

Time dependent variational principle (TDVP)

Find best dynamics within this variational manifold



$$A_i = \begin{pmatrix} \cos\left(\frac{\theta_i}{2}\right)|0\rangle_i & e^{i\phi_i}\sin\left(\frac{\theta_i}{2}\right)|1\rangle_i \\ |0\rangle_i & 0 \end{pmatrix}$$

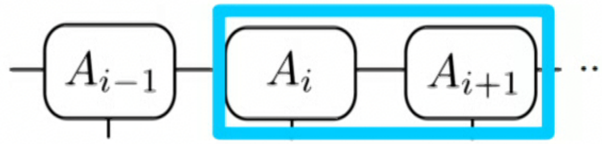


Find $\dot{z}$ that minimizes instantaneous $||\dot{z}\,\partial_z|\psi\rangle + iH|\psi\rangle||$

TDVP for MPS: [Hageman et al, PRL 107 070601 (2011)]

"Semiclassical" equations of motion

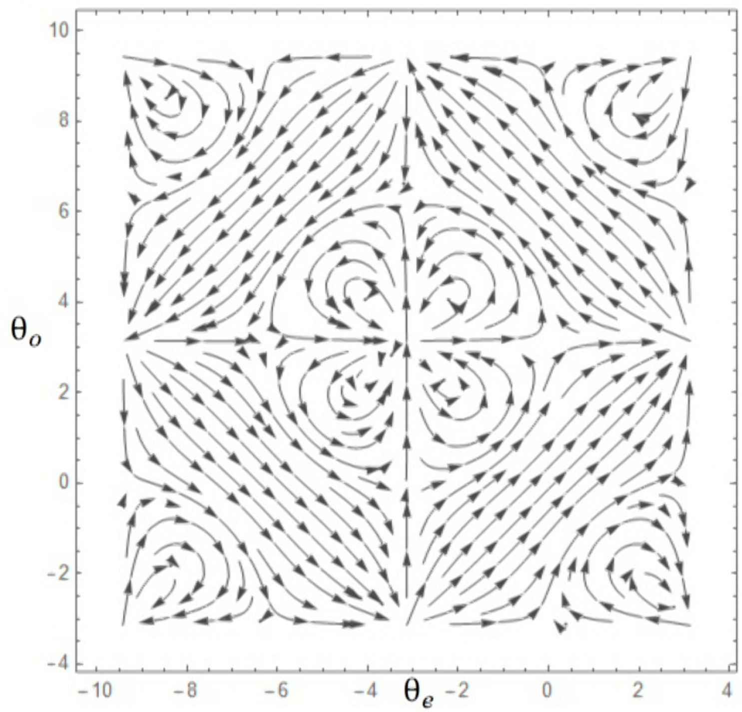
$|\psi(\boldsymbol{\theta}, \boldsymbol{\phi})\rangle =$



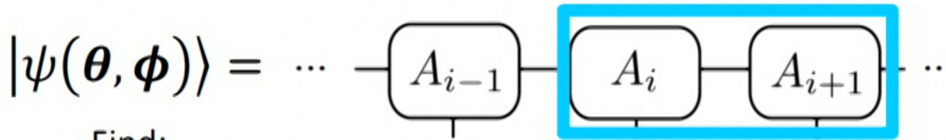
Find:

$\dot{\phi}_e(t) = 0, \qquad \dot{\phi}_o(t) = 0$

$\dot{\theta}_e(t) = -2\Omega\left(\cos\left(\frac{\theta_o}{2}\right) + \cos\left(\frac{\theta_o}{2}\right)\sin\left(\frac{\theta_o}{2}\right)\sin\left(\frac{\theta_e}{2}\right)\cos^2\left(\frac{\theta_e}{2}\right)\right) / \cos^2\left(\frac{\theta_o}{2}\right), \quad \dot{\theta}_o(t) = \cdots(\theta_o \leftrightarrow \theta_e)\cdots$



"Semiclassical" equations of motion



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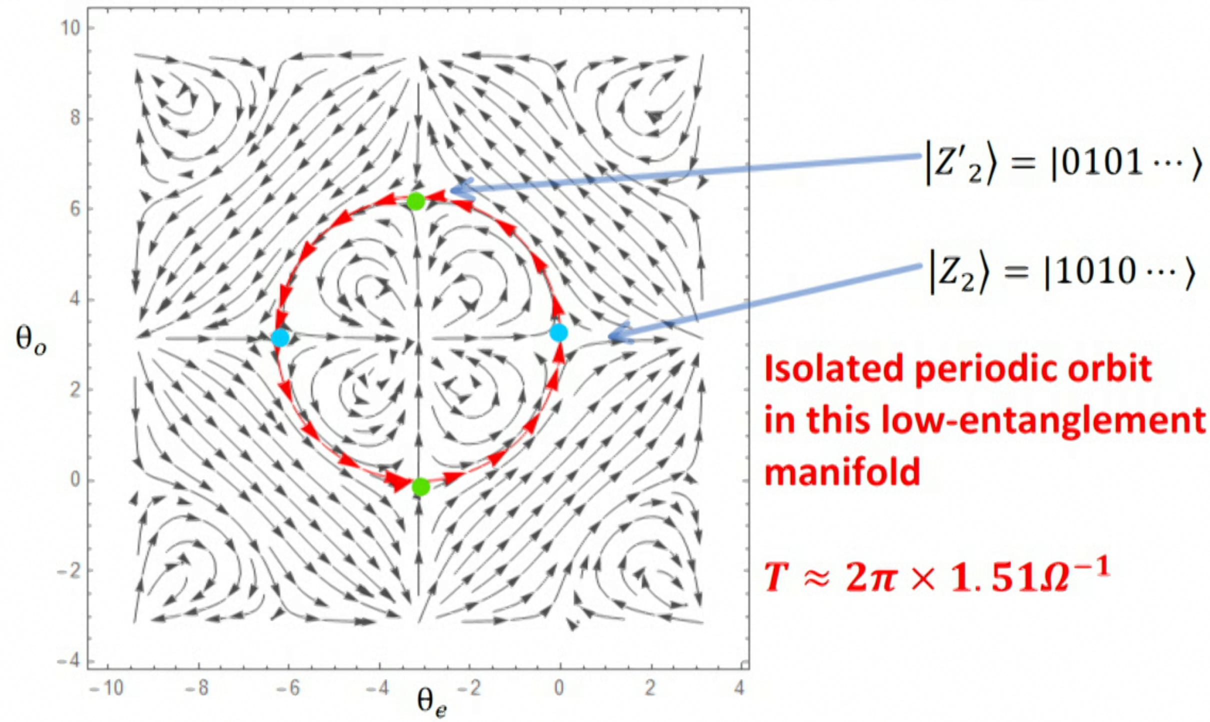


Isolated periodic orbit in this low-entanglement manifold

$T \approx 2\pi \times 1.51\Omega^{-1}$

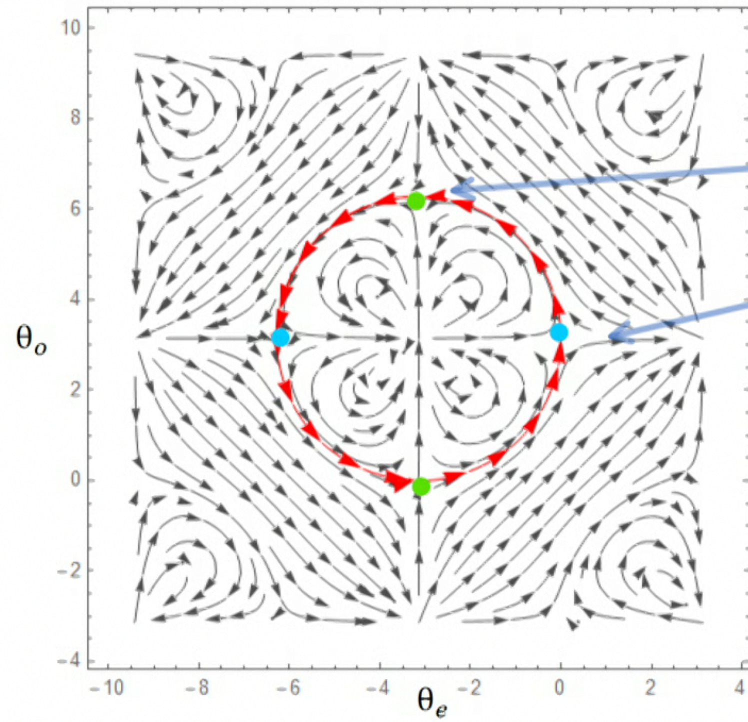
"Semiclassical" equations of motion

$$|\psi(\boldsymbol{\theta},\boldsymbol{\phi})\rangle = \cdots - A_{i-1} - A_i - A_{i+1} - \cdots$$

Find:

$\dot{\phi}_e(t) = 0, \qquad \dot{\phi}_o(t) = 0$

$\dot{\theta}_e(t) = -2\Omega\left(\cos\left(\frac{\theta_o}{2}\right) + \cos\left(\frac{\theta_o}{2}\right)\sin\left(\frac{\theta_o}{2}\right)\sin\left(\frac{\theta_e}{2}\right)\cos^2\left(\frac{\theta_e}{2}\right)\right)/\cos^2\left(\frac{\theta_o}{2}\right), \quad \dot{\theta}_o(t) = \cdots(\theta_o \leftrightarrow \theta_e)\cdots$



$|Z'_2\rangle = |0101\cdots\rangle$

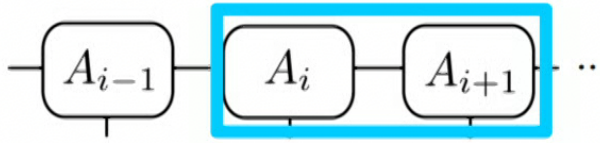
$|Z_2\rangle = |1010\cdots\rangle$

Isolated periodic orbit in this low-entanglement manifold

$T \approx 2\pi \times 1.51\Omega^{-1}$

"Semiclassical" equations of motion + error

$$|\psi(\boldsymbol{\theta}, \boldsymbol{\phi})\rangle = \cdots$$



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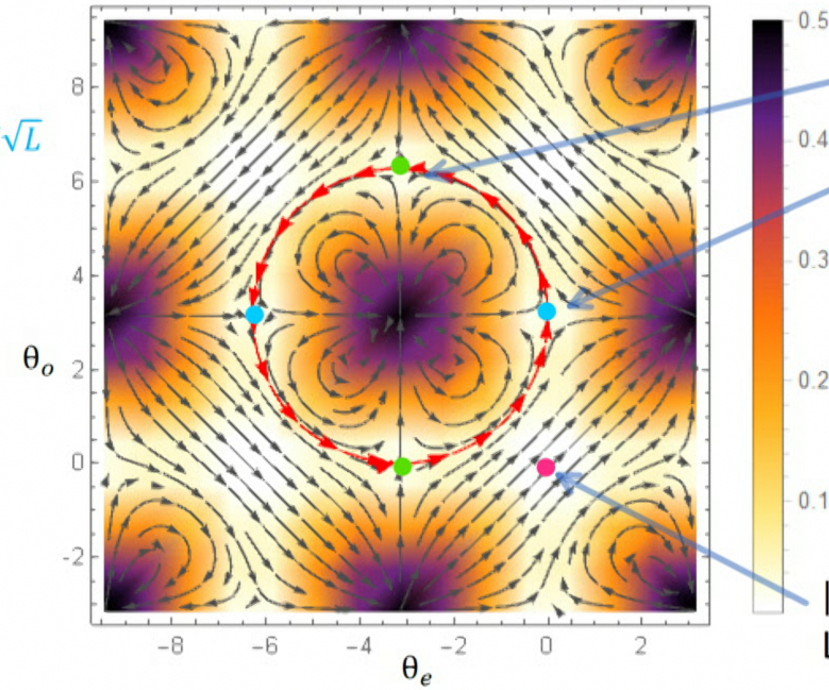
$$\dot{\phi}_e(t) = 0, \qquad \dot{\phi}_o(t) = 0$$

$$\dot{\theta}_e(t) = -2\Omega\left(\cos\left(\frac{\theta_o}{2}\right) + \cos\left(\frac{\theta_o}{2}\right)\sin\left(\frac{\theta_o}{2}\right)\sin\left(\frac{\theta_e}{2}\right)\cos^2\left(\frac{\theta_e}{2}\right)\right) / \cos^2\left(\frac{\theta_o}{2}\right), \quad \dot{\theta}_o(t) = \cdots(\theta_o \leftrightarrow \theta_e)\cdots$$

Error in TDVP:

$\gamma(z) =$
$\left|\left|\dot{z}\,\partial_z|\psi\rangle + iH|\psi\rangle\right|\right|/\sqrt{L}$



$|Z'_2\rangle = |0101\cdots\rangle$

$|Z_2\rangle = |1010\cdots\rangle$

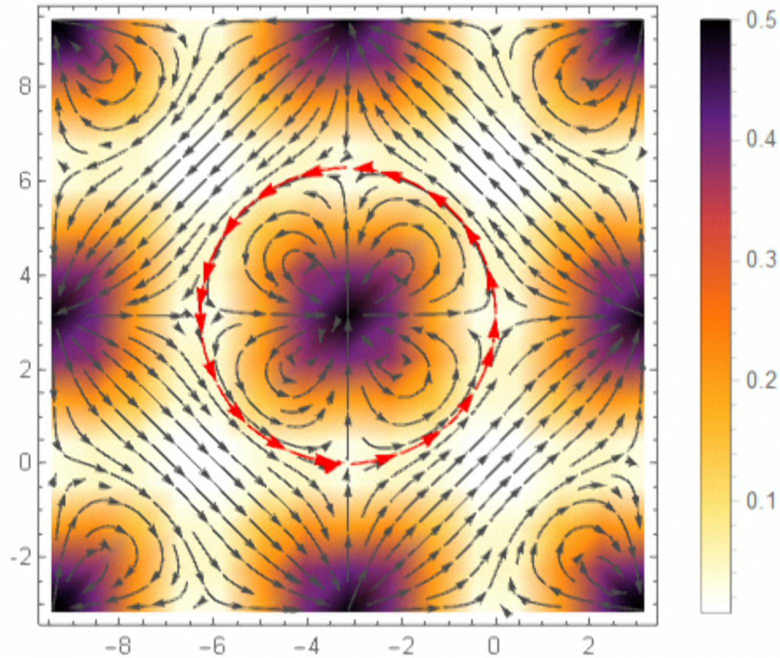
Isolated periodic orbit, small error!

$\epsilon = \int \gamma(z)dt \approx 0.21$

$|\mathbf{0}\rangle = |0000\cdots\rangle$
Leaks out...

"Quantum Newton's Cradle"?

$|Z_2\rangle = |1010\cdots\rangle$ $\quad$ $|Z'_2\rangle = |0101\cdots\rangle$

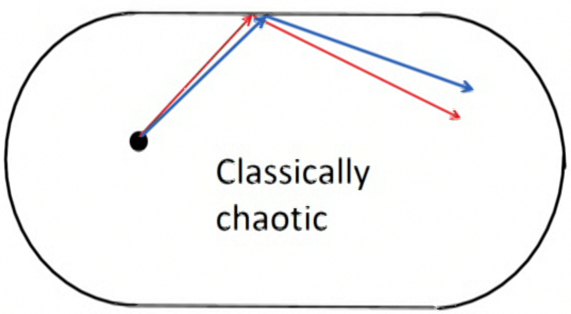


Connection to Quantum Scars?

Isolated, closed orbit describing dynamics in a low entanglement manifold is a sign of weak ergodicity breaking in dynamics

Reminiscent of quantum scarring in single-particle chaotic systems

A quick introduction…



e.g. Bunimovich stadium billiard

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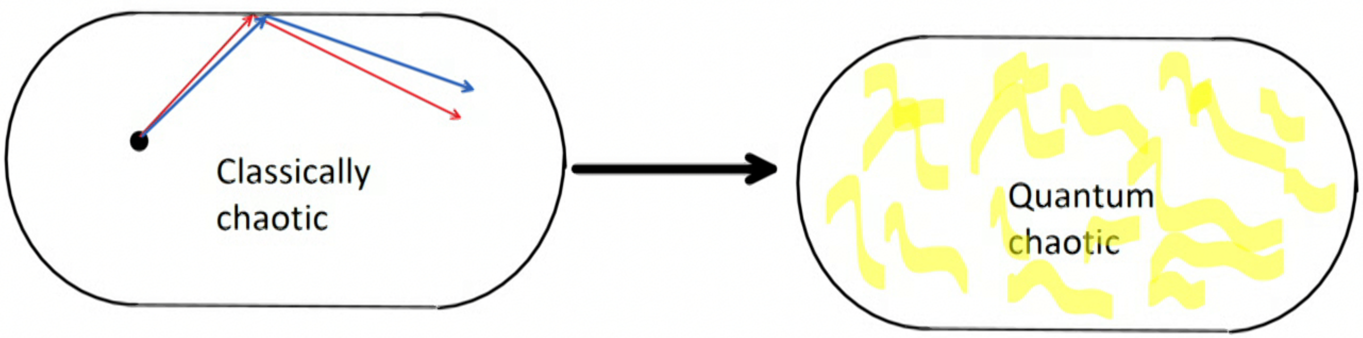

$|\psi(x)|^2$ ~uniformly distributed (ergodic);
c.f. **Berry's conjecture** [J Phys. A. 10, 2083 (1977)]

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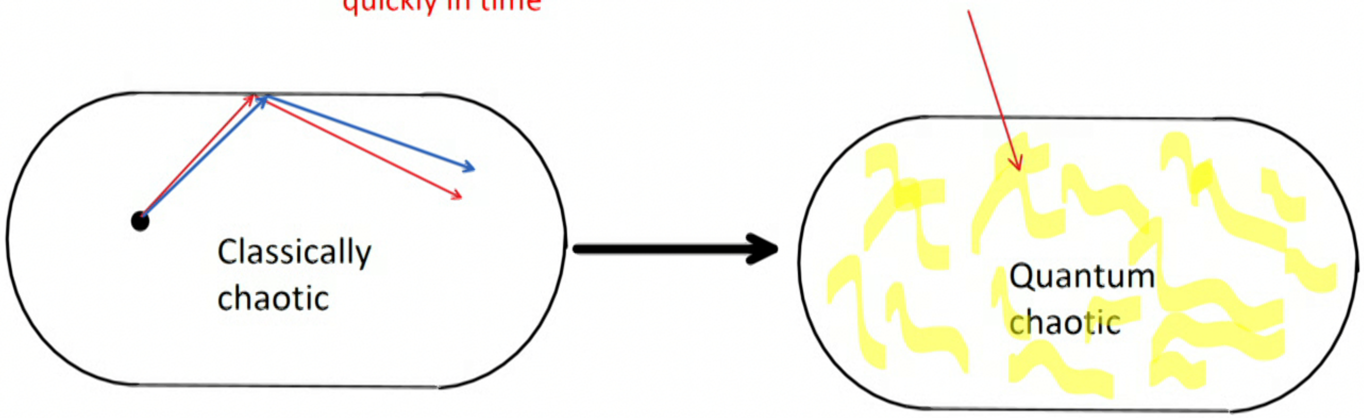

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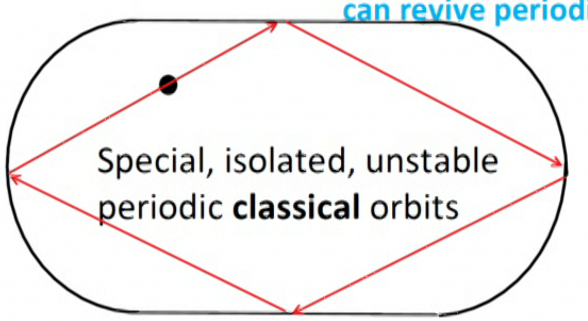
Yes, if it is launched from on a special trajectory; quantum particle can revive periodically before dispersing



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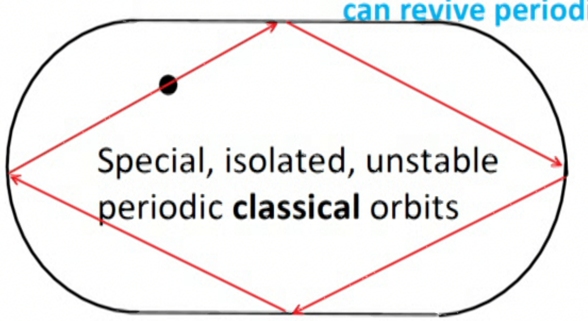

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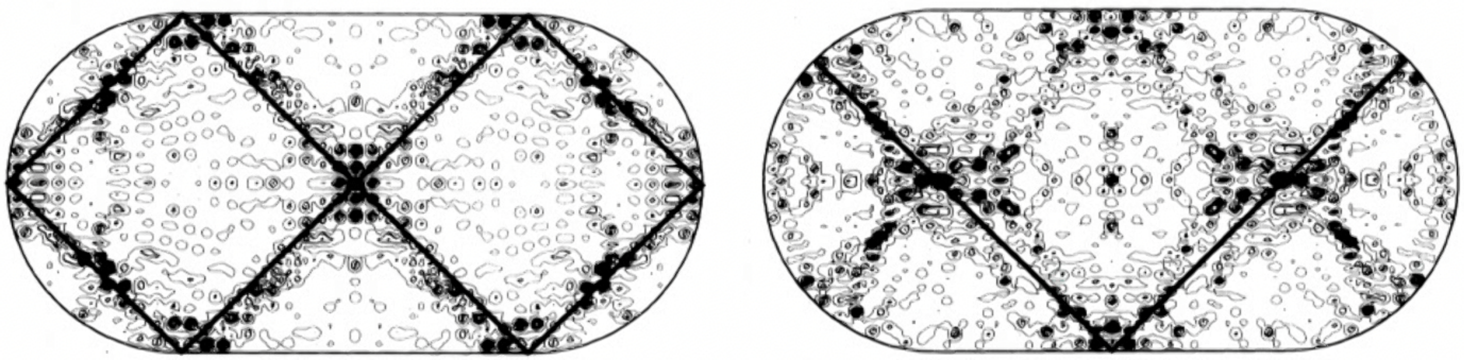
Closed periodic classical orbits →
"Scarred", nonergodic wavefunctions that concentrate around them + nonergodic dynamics



Intensity plot $|\psi(x)|^2$

e.g. Bunimovich stadium billiard

[Heller, PRL 53, 1515 (1984)]

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Closed periodic classical orbits →
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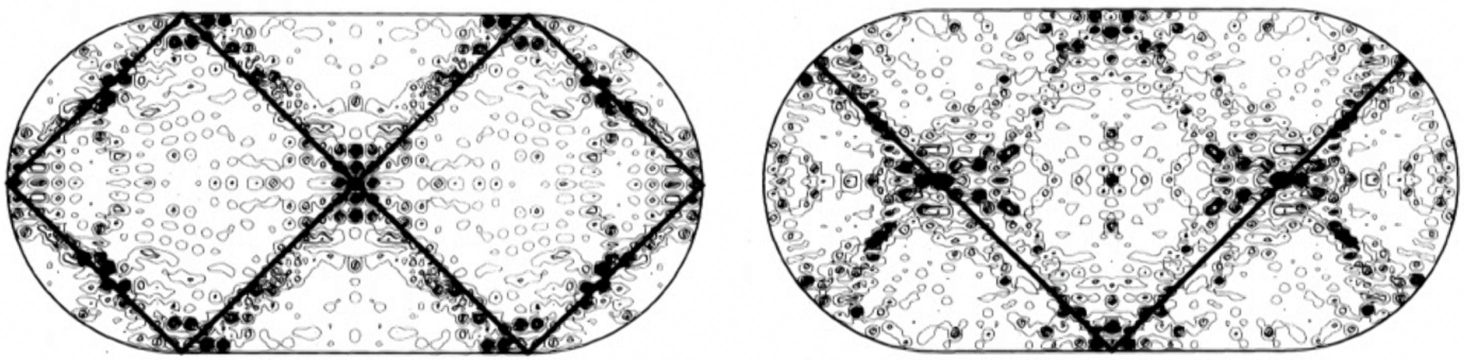
Our case: Many-Body generalization of phenomenon...?



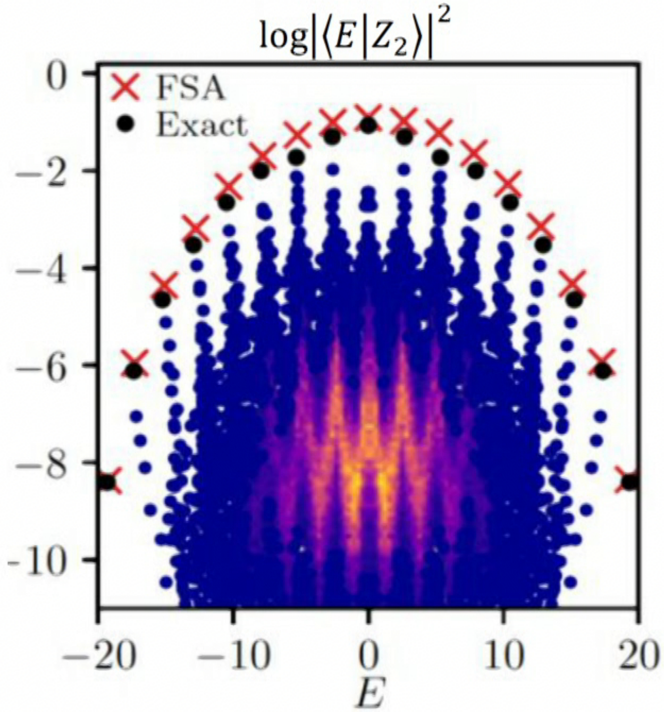
Intensity plot $|\psi(x)|^2$

[Heller, PRL 53, 1515 (1984)]

e.g. Bunimovich stadium billiard

Implications on Many-Body eigenstates?

Consider overlap of $|Z_2\rangle$ with energy eigenbasis $|E\rangle$



[Turner et al, Nat Phys (2018)]

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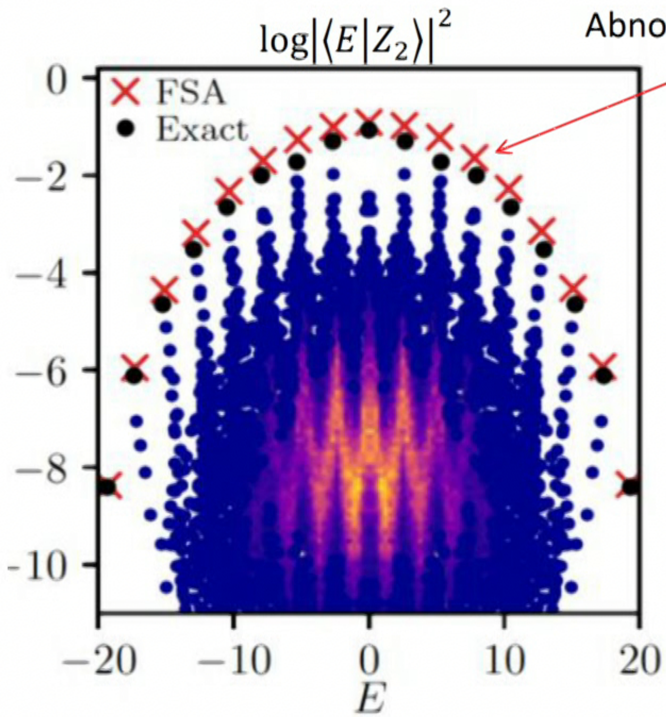
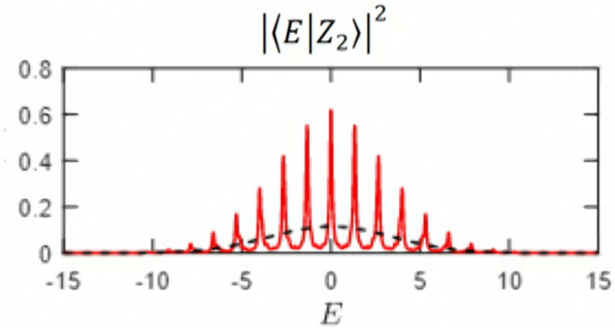
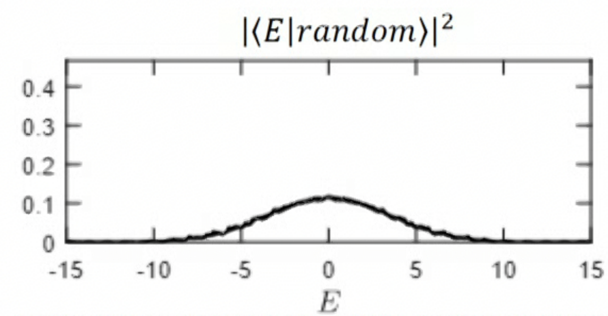


Abnormally high overlap; "Many-body scarred" states?

[Turner et al, Nat Phys (2018)]

→Smear out overlap (broaden δ −function)



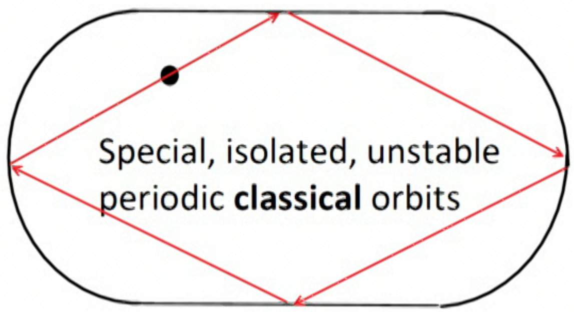


Note: this is expected from a thermalizing system

Implications on Many-Body eigenstates?

Can we use presence of orbit to quantifiably constrain nature of many-body eigenstates?

Single-particle



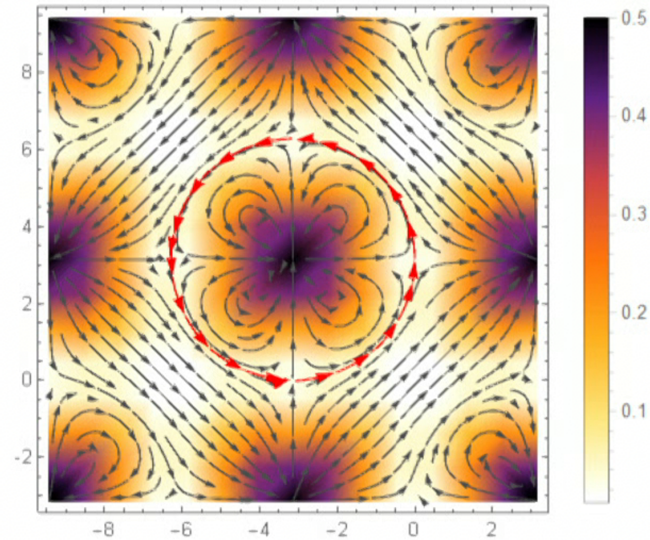
[Heller, PRL 53, 1515 (1984)]

Weakly unstable classical orbits
(λ Lpaunov exponent)

$\lambda T \ll 1$

gives rise to enhancement of intensity of wavefunctions above Berry's conjecture by a factor of $\frac{1}{\lambda T}$ about orbits

Many-body



Weakly unstable classical orbits
(γ TDVP error)

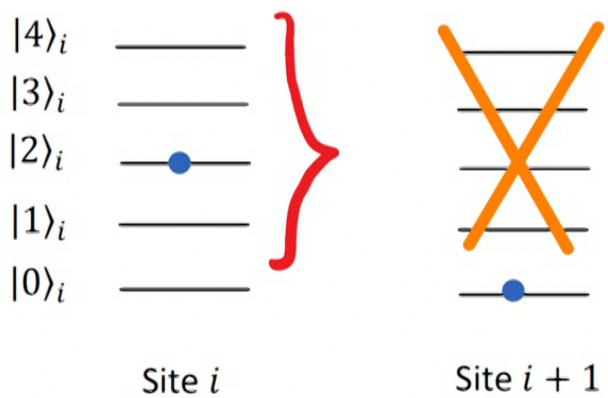
$\int \gamma \, dt \approx 0.21 < 1$

Enhancement of certain eigenstates by a factor... ??

Higher spin constrained models

$$H = \Omega \sum_i \widetilde{S_i^x}$$

$|0\rangle, |1\rangle, \cdots, |2s\rangle$



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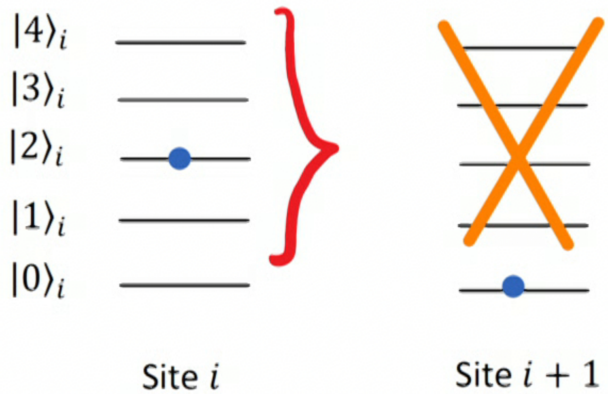

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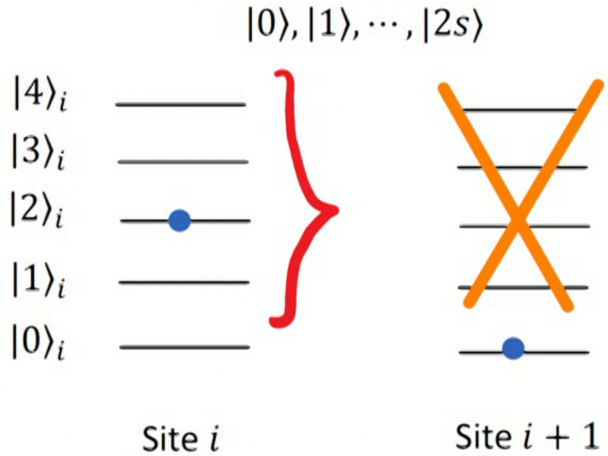


$$|\psi(\boldsymbol{\theta}, \boldsymbol{\phi})\rangle =$$

$$A_i = \begin{pmatrix} P_i |(\theta,\phi)\rangle_i & Q_i |(\theta,\phi)\rangle_i \\ |0\rangle_i & 0 \end{pmatrix}$$

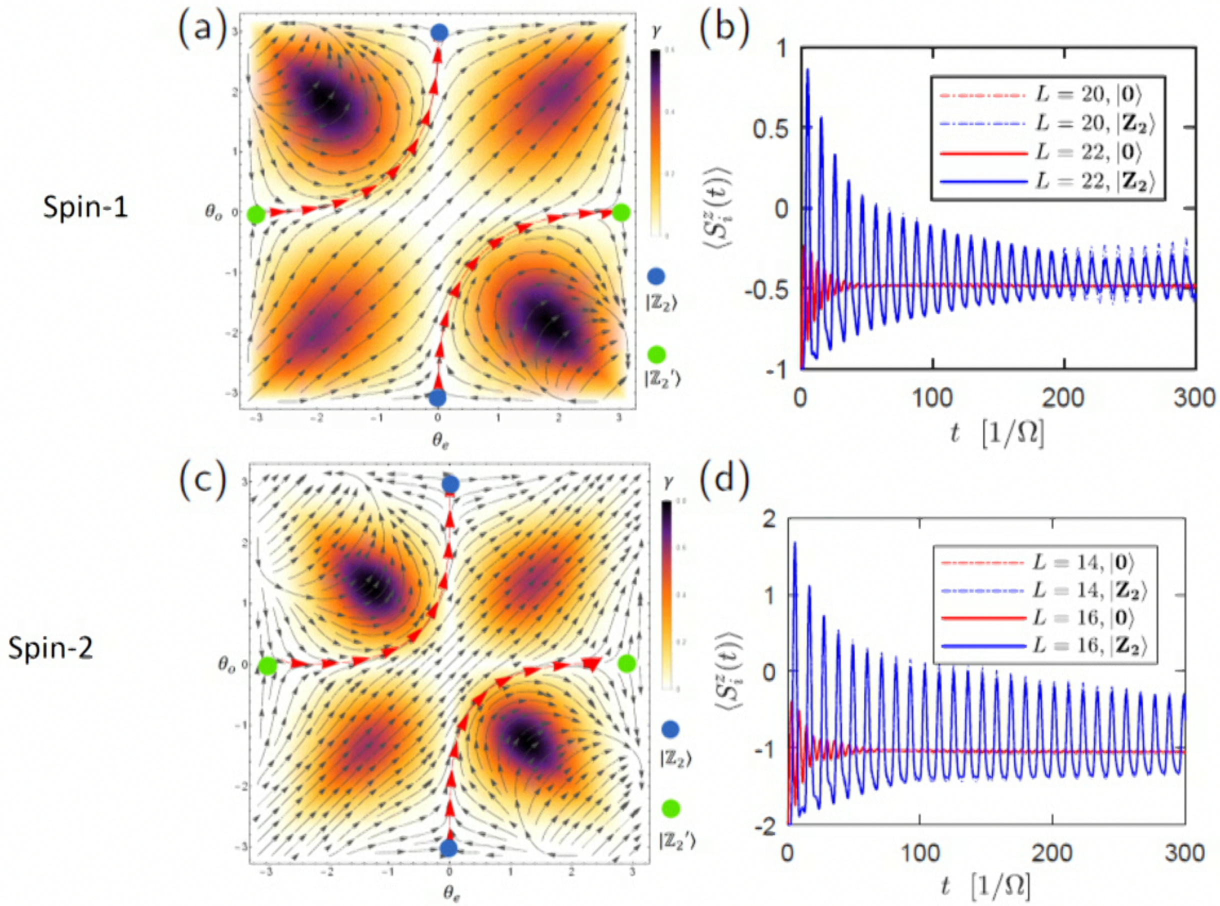
$$P_i = |0\rangle_i \langle 0|_i$$

$$Q_i = 1 - P_i$$

$$|(\theta,\phi)\rangle_i = e^{-i\,\phi_i S_i^z} e^{-i\,\theta_i S_i^x} |0\rangle_i$$

Higher spin coherent state

Higher spin constrained models



Discussion

1. "Semiclassical" description of constrained models require entanglement (here, representable by bond-2 MPS)

2. Found isolated unstable orbits in a simple description of many-body dynamics capturing persistent oscillations, representing non-ergodic dynamics for some states and not others
   a. However, physical reason orbits/oscillations still unclear --- role of constraints??
   b. Features of the model being possibly close to integrability (WIP Khemani, Chandran et al)

3. Possible connection to quantum scars
   a. More work needs to be done to make generalization rigorous
   b. Need to understand nature of many-body eigenstates and stability of phenomenon

4. Connection to scrambling in black holes? Information retrieval?

The end! Thank you!

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