Title: Progress towards a classification of 5d N=1 SCFTs

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Abstract: I describe recent progress in classifying 5d N=1 field theories with interacting UV superconformal fixed points (i.e. 5d SCFTs). In the first part of the talk, I review a newly proposed catalog of candidate (simple) gauge theories which captures theories missed by prior field theoretic classification efforts. In the second part of the talk, I discuss a classification program for rank 1 and 2 5d SCFTs in terms of Calabi-Yau 3-folds, along with prospects for its extension to arbitrary rank. This geometric classification program refines the field theoretic approach by incorporating non-perturbative physics, predicts a number of dualities between 5d gauge theories, and supports the idea that all 5d SCFTs can be obtained by compactifying 6d (1,0) SCFTs on a circle.

Geometric Classification of 5d $\mathcal{N} = 1$ SCFTs

Patrick Jefferson (Harvard University)

June 5, 2018

Introduction

The subject of this talk is recent progress in classifying 5d $\mathcal{N} = 1$ SCFTs in the Coulomb phase

Based primarily on:

- 1705.05836 (PJ, H.C. Kim, C. Vafa, G. Zafrir)
- 1801.04036 (PJ, S. Katz, H.C. Kim, C. Vafa)

Introduction

Part of larger effort to classify $d > 4$ SCFTS using SUSY & geometric singularities in string theory. Inspiration:

- 6d $(2,0)$ SCFTs = ADE singularities [Witten '95]
- 6d $(1,0)$ SCFTs = 7-branes wrapping collapsing configs of $\mathbb{P}^1$'s in F-theory [Heckman-Morrison-Vafa '13] [Bhardwaj '15] [Heckman-Morrison-Rudelius-Vafa '15]

String theory/geometry instrumental for understanding UV completions

5d is a natural next step, related to many (possibly all?) 6d theories via KK compactification

Status of 5d classification problem

A bit of history

Systematic investigation of 5d $\mathcal{N} = 1$ theories initiated by series of papers from '96-'98 [Seiberg] [Morrison-Seiberg] [Douglas-Katz-Vafa] [Ganor-Morrison-Seiberg] [Intriligator-Morrison-Seiberg (IMS)] [Diaconescu-Entin]

Used interplay of geometric engineering, brane constructions and SUSY to identify theories with nontrivial interacting UV fixed points

Two (partial) classifications were proposed:

- Geometric: Rank one theories = contracting del Pezzo surfaces $\mathrm{dP}_{n\leq 8}$
- Gauge theoretic: bounds on matter hypermultiplet representation $\mathbf{R}$ and topological data k for given (simple) gauge algebra $\mathfrak{g}$

Status of 5d classification problem

However, both seem to be incomplete.

Geometric classification for theories of arbitrary rank has not been pursued

Moreover, stringy counterexamples to gauge theory (IMS) classification have been studied:

- Quiver gauge theories
- 5d theories exceeding IMS bounds

How can we explain this?

IMS criteria are too restrictive (sufficient, but not necessary). Possible to relax IMS criteria to necessary (but perhaps not sufficient) criteria, leading to catalog of candidate gauge theories with interacting UV fixed points

Status of 5d classification problem

What approach should be taken to classify 5d fixed points?

Only surefire way to argue existence is to carefully account for non-perturbative physics

UV completions are indispensable, i.e. 5-brane configurations (type IIB) or geometric singularities (M-theory), which exhibit non-perturbative physics

However, geometric classification is difficult compared to gauge theory. . . is this a reasonable strategy?

Goal and outline of talk

I hope to convince you of the following:

- Classification of 5d $\mathcal{N} = 1$ SCFTs (including non-Lag theories) is incomplete
- Gauge theory classification is still an open problem
- Classification of geometric singularities has a hope of being a viable solution

Plan for talk

1. Gauge theory
   1.1 Review of 5d $\mathcal{N} = 1$ gauge theories
   1.2 (Simple) gauge theory classification
2. Geometry
   2.1 M-theory compactifications on CY 3-fold
   2.2 Geometric classification
   2.3 Future directions

Review of 5d $\mathcal{N} = 1$ gauge theories

$\mathcal{N} = 1$ SUSY algebra in 5d consists of 8 supercharges

5d superconformal algebra has bosonic subalgebra $\mathfrak{so}(4,1) \times \mathfrak{su}(2)_R$

Massless fields:

$$\begin{cases} \text{vector multiplet } (A_\mu, \phi; \lambda) \text{ in adjoint of gauge algebra } \mathfrak{g} \\ \text{hypermultiplet } (q^a; \psi) \text{ in rep } \mathbf{R} = \oplus_f \mathbf{R}_f \text{ (w/ } q^a \text{ an } \mathfrak{su}(2)_R \text{ doublet)} \end{cases}$$

Moduli space of vacua:

- Higgs branch $\mathcal{M}_\mathrm{H}$ parametrized by vevs of hypermultiplet scalars
- Coulomb branch $\mathcal{M}_\mathrm{C}$ parametrized by vevs of vector multiplet scalars ϕ

The focus of this talk will be on $\mathcal{M}_\mathrm{C}$

Review of 5d $\mathcal{N} = 1$ gauge theories

Coulomb branch

At a generic point $\phi \in \mathcal{M}_C$, ϕ takes values in the Cartan subalgebra $\mathfrak{t} \subset \mathfrak{g}$, breaking the gauge group G to the stabilizer $U(1)^{r=\text{rk}(G)} \subseteq G$

Consequently, $\mathcal{M}_C$ is a subset of the (dual) fundamental Weyl chamber $\mathbb{R}^r / W_{\mathfrak{g}}$

SUSY protected data: spectrum of massive BPS states:

- Electric particles/gauge instantons, central charge $Z_e = \sum_i n_e^{(i)} \phi^i + \sum_f s_f m_f + s_0 m_0$
- Magnetic strings, central charge $Z_m = \sum_i n_m^{(i)} \phi_{Di}$

Gauge instantons charged under $U(1)$ current $j = \star \operatorname{tr} F \wedge F$, instanton number s_0 (can carry gauge charges as well)

Review of 5d $\mathcal{N} = 1$ gauge theories

What does SUSY buy us?

Constraints of SUSY imply low energy EFT characterized by prepotential which is at most cubic in ϕ:

$$\begin{aligned}\mathcal{F}(\phi) =& \frac{1}{2} m_0 h_{ij} \phi^i \phi^j + \frac{1}{6} k d_{ijk} \phi^i \phi^j \phi^k \\ &+ \frac{1}{12} \left[\sum_{\alpha \in \mathbf{adj}} |\langle \phi, \alpha \rangle|^3 - \sum_f \sum_{w_f \in \mathbf{R}_f} |\langle \phi, w_f \rangle + m_f|^3 \right]\end{aligned}$$

Perturbatively, $\mathcal{F}$ is 1-loop exact

Review of 5d $\mathcal{N} = 1$ gauge theories

Phase structure of $\mathcal{M}_C$

For $\phi \in \mathcal{M}_C$, BPS spectrum generically massive; however, states can become massless/tensionless for special values ϕ_*

Particle masses are linear in $\phi, m_f \implies$ hypermultiplets states massless on interior hyperplanes ("walls") of $\mathcal{M}_C$; monopole strings tensionless at boundaries of $\mathcal{M}_C$

Walls denote sharp phase transitions, characterized by singular behavior (e.g. k_{ijk} jump discontinuously); this is evident from absolute values $|\langle \phi, w_f \rangle + m_f| \subset \mathcal{F}$

Conformal limit at the origin $\phi = m_f = 0, 1/g_0^2 \to 0$, characterized by interacting massless/tensionless states

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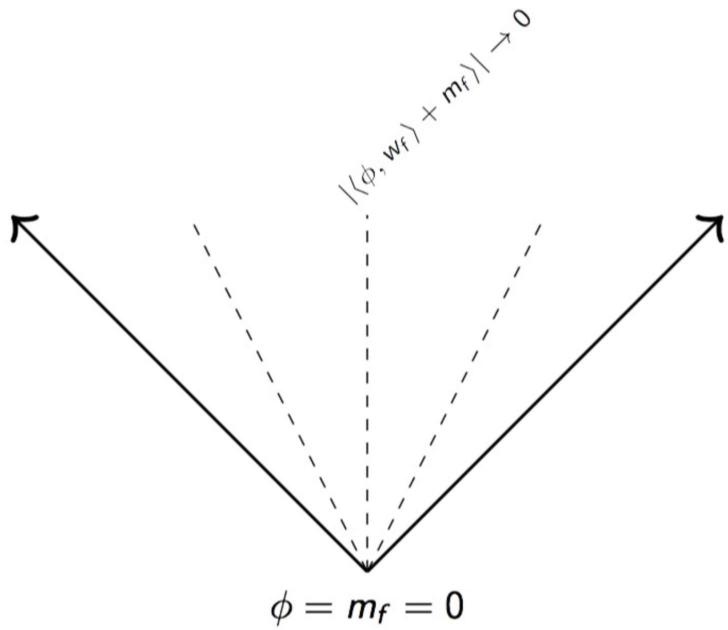
Review of 5d $\mathcal{N} = 1$ gauge theories

Phase structure of $\mathcal{M}_C$, cont ...

$\mathcal{M}_C$ has the structure of a fan consisting of several cones



Classifying UV fixed points

Criteria for existence (IMS)

Effective coupling $\tau_{ij}(\phi) = m_0 h_{ij} + \partial_i \partial_j \mathcal{F}(\phi)$ must be positive on $\mathcal{M}_C = \mathbb{R}^r / W_{\mathfrak{g}}$ as $m_0 = 1/g_0^2 \to 0$ [Seiberg] [IMS]

Equivalently, $\mathcal{F}$ is convex on $\mathcal{M}_C$

Matter hypermultiplet contribution negative $\Longrightarrow$ more matter makes $\mathcal{F}$ "less convex"

Convexity therefore bounds "size" of the matter representation $\mathbf{R} = \oplus_f \mathbf{R}_f$ (i.e. multiplicities N_f and types of representatations f)

This leads to constraints on $(\mathfrak{g}, N_f, k)$ admitting UV fixed points. E.g.:

- $\mathfrak{su}(N)$, $N_{\mathbf{F}} + 2|k| \leq 2N$
- $\mathfrak{su}(N)$, $N_{\wedge^2 \mathbf{F}} = 1$, $N_{\mathbf{F}} + 2|k| \leq 8 - N$
- No quiver gauge theories

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Classifying UV fixed points

Disagreement with stringy constructions

Counter-examples:

- quiver gauge theories (e.g. $\mathfrak{su}(2) + \mathfrak{su}(2)$ w/ one bifund. $\leftrightarrow \mathbb{F}_0 \cup \mathrm{Bl}_2^* \mathbb{F}_0$ singularity)
- brane systems beyond IMS bounds, e.g. $\mathfrak{su}(3)$ w/ $N_\mathsf{F} = 10$ [Yonekura '15] [Hayashi-Kim-Lee-Taki-Yagi '15] [Zafrir '15]

Source of the discrepancy? Assuming $\mathcal{M}_\mathsf{C} = \mathbb{R}^r / W(\mathfrak{g})$ is too restrictive

Resolution: only assume Coulomb branch is a proper subset $\mathcal{M}_\mathsf{C} \subset \mathbb{R}^r / W_\mathfrak{g}$, as EFT could be unphysical for some regions of $\mathbb{R}^r / W_\mathfrak{g}$

Extra restrictions on $\mathcal{M}_C$ come from non-perturbative states/instantons; perturbatively we do not know how to compute their masses, thus cannot see how they affect $\mathcal{M}_\mathsf{C}$

Classifying UV fixed points

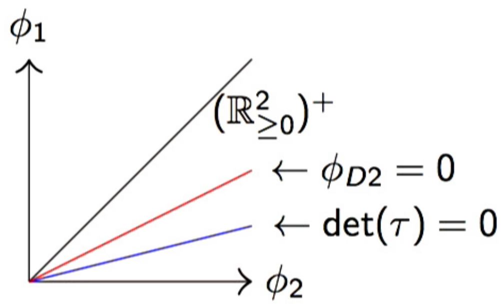
Example: $\mathfrak{su}(2)_\pi + \mathfrak{su}(2)'_\pi$ w/ $N_{(\mathbf{F},\mathbf{F}')} = 1$ [Aharony-Hanany '97]

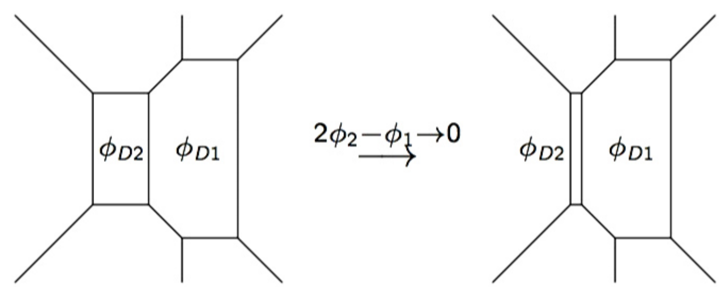
Non-perturbative construction using geometric singularity (equivalently, (p, q) 5-brane web in IIB)

$\mathbb{R}^2/W_{\mathfrak{su}(2)+\mathfrak{su}(2)} = \mathbb{R}^2_{\geq 0}$. Consider the phase $(\mathbb{R}^2_{\geq 0})^+ = \{\phi_1 \geq \phi_2 \geq 0\}$ (shaded region)

IMS criteria excludes this theory because τ_{ij} develops zero eigenvalue on (blue line, $\phi_2/\phi_1 \approx 1/4$)

However even before τ_{ij} degenerates, one encounters a massless instanton



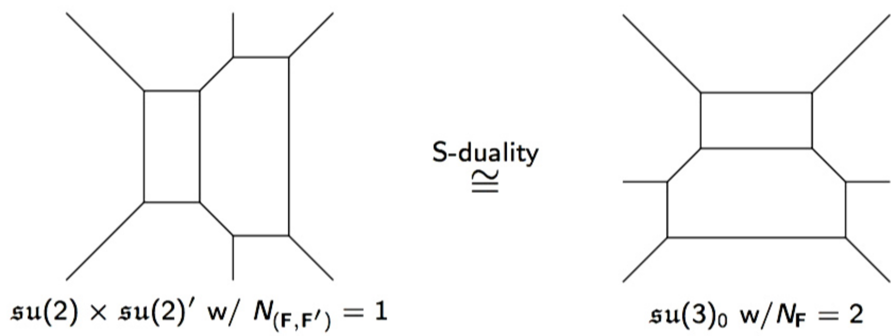


Classifying UV fixed points

Example: $\mathfrak{su}(2) + \mathfrak{su}(2)'$, cont...

So, we can conclude that $(\mathcal{M}_{\mathsf{C}}^{\mathfrak{su}(2)\times\mathfrak{su}(2)'})^{+} = \{\phi_1 \geq \phi_2 \geq \phi_1/2\}$

In fact, S-duality shows this theory is dual to $\mathfrak{su}(3)$ with $k = 0$ and $N_{\mathsf{F}} = 2$:



The non-perturbative state with mass $2\phi_2 - \phi_1$ is a W-boson of $\mathfrak{su}(3)$!

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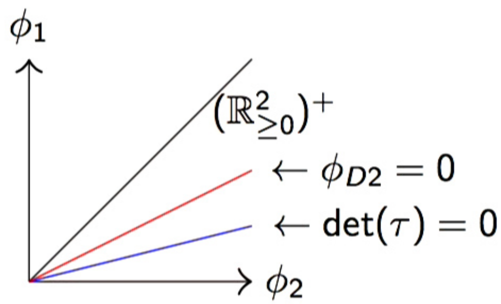

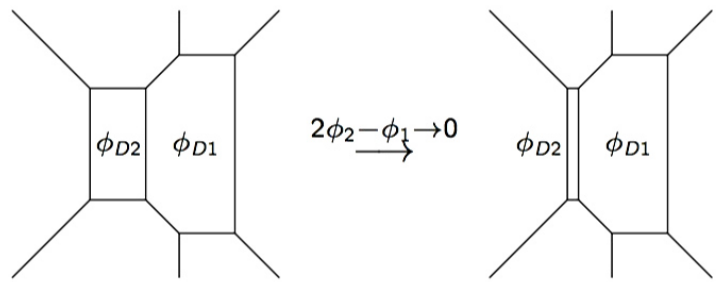

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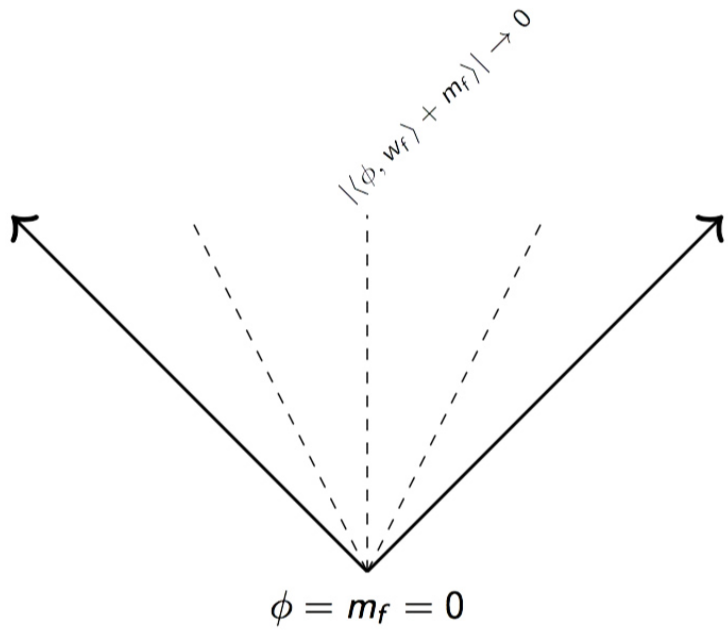

Classifying UV fixed points

What have we learned?

$\mathcal{M}_C$ may be a subset of a Weyl chamber, corrected by non-perturbative states

How to modify gauge theory approach?

Relax assumption $\mathcal{M}_C = \mathbb{R}^r / W_{\mathfrak{g}}$

Determine $\mathcal{M}_C$ using following constraints:

1. All BPS states (including monopole strings) have positive tension at generic $\phi \in \mathcal{M}_C$
2. τ_{ij} is positive definite on interior

We cannot compute all BPS masses, but can compute string tensions ϕ_{Di}; accurate because string tensions "know" about instanton masses (e.g. often $\phi_{Di} \sim Z_e Z_{e'}$; in $\mathfrak{su}(2) + \mathfrak{su}(2)'$ example, $\phi_{D2} = 12\phi_2(2\phi_2 - \phi_1)$.)

Classifying UV fixed points

Example: $\mathfrak{su}(3)_k + N_\mathsf{F}\mathbf{F} + N_\mathsf{Sym}\mathbf{Sym}$

N_Sym	N_F	$\lvert k \rvert$	prior construction
1	0	$\frac{3}{2}$	?
1	1	0	[Yonekura '15]
0	10	0	[Hayashi-Kim-Lee-Yagi '15] [Yonekura '15]
0	9	$\frac{3}{2}$	?
0	6	4	?
0	3	$\frac{13}{2}$	?
0	0	9	?

Table: "Extremal" $\mathfrak{su}(3)_k$ gauge theories with N_Sym symmetric, N_F fundamental matters, and classical Chern Simons level k.

Above theories expected to have 6d fixed points

5d fixed point theories obtained by integrating out massive matters, reducing numbers of flavors and shifting k

Classifying UV fixed points

New gauge theory classification

Use above strategy to classify (simple) gauge theories, possibly overcounting

New classification recovers IMS theories, along with all known stringy examples beyond IMS bounds! [PJ-Kim-Vafa-Zafrir '17]

Consists of standard and exotic theories:

- Standard theories, classical $\mathfrak{g}$ predictable patterns for arbitrary r (e.g. pure $\mathfrak{su}(N)_k$ satisfies $|k| \leq \frac{N^2}{N-2}$)
- Exotic theories, $r \leq 8$, require case-by-case analysis

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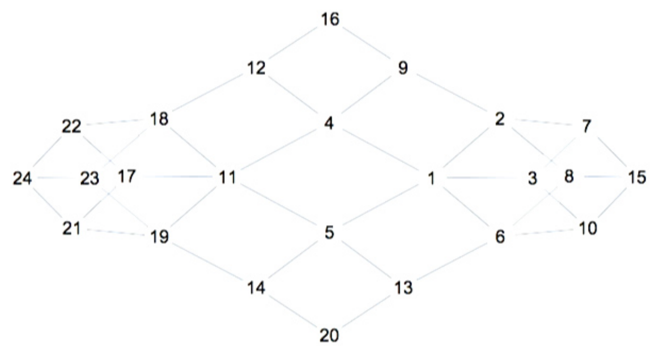
Classifying UV fixed points

But . . . too early to declare victory

General comment, gauge theory description limited; e.g. $\mathfrak{su}(2)_\pi + \mathfrak{su}(2)_\pi$ description only applies to phases 4 and 5 of full Coulomb branch $\mathcal{M}_C$:



Instantons could exclude some candidate gauge theories

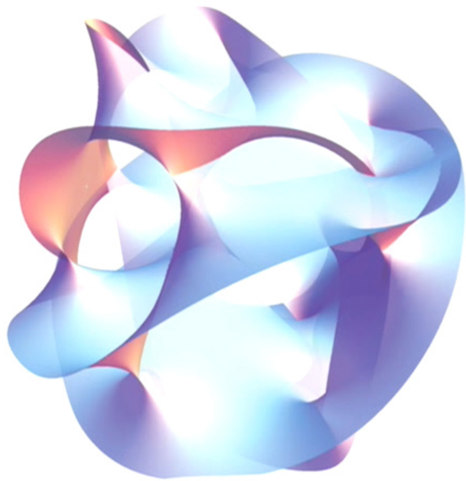
Non-gauge theory phases?

Focus on stringy constructions, which exhibit the full BPS spectrum

Part 2: Geometry

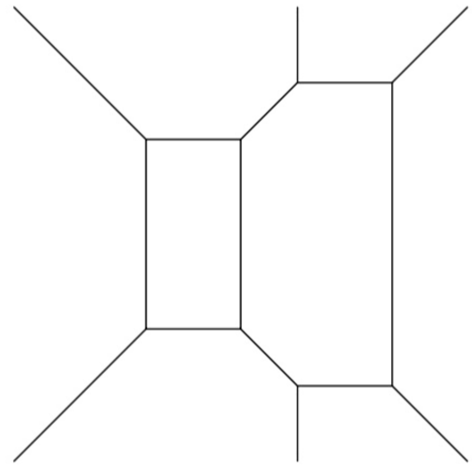
Stringy constructions

M-theory on $\mathbb{R}^{1,5} \times$ (local) CY_3



U-duality
$\cong$

type IIB w/ 5-brane webs



(low energy)

5d field theory

Review of M-theory on CY 3-folds

Geometric engineering

Possible to "engineer" 5d $\mathcal{N} = 1$ SCFT by compactifying M-theory on singular CY 3-fold X [Witten '96] [(many others)]

Massive BPS spectrum

Consider M-theory on a smooth 3-fold Y degenerating to singularity X

BPS spectrum:

- Particles $\leftrightarrow$ M2-branes wrapping holomorphic 2-cycles (complex curves)
- Strings $\leftrightarrow$ M5-branes wrapping holomorphic 4-cycles (divisors)

Masses (tensions) $\leftrightarrow$ volumes of 2-cycles (4-cycles)

Review of M-theory on CY 3-folds

BPS data in terms of triple intersection numbers

Given a basis of (1,1)-forms $D_{i=1,\ldots,h^{1,1}(X)-1}$ dual to 4-cycles in Y,

$$\mathcal{K}(Y/X) = \{\text{Kähler forms } J = \varphi^i D_i : \int_C J > 0 \text{ for all cx curves } C\}$$

Algebro-geometric setting: volumes of classes D corresponding to (complex) p-cycles are

$$\text{vol}(D) = \frac{1}{p!}\int_D J^p = \frac{1}{p!} D \cdot J^p \quad (\cdot \text{ intersection product})$$

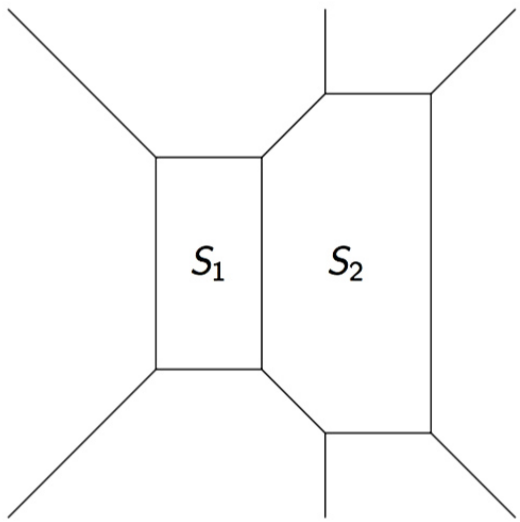
All EFT data can be expressed in terms of triple intersection numbers $D_i \cdot D_j \cdot D_k \overset{\text{roughly}}{=} (\#\text{points w/ mult.})$; in particular,

$$\mathcal{F}(\phi, m) = \frac{1}{3!} J \cdot J \cdot J = \frac{1}{3!}\varphi^i \varphi^j \varphi^k D_i \cdot D_j \cdot D_k$$

Review of M-theory on CY 3-folds

In practice, more intuitive to with the 5-brane diagrams, keeping in mind correspondence with geometry

Compact faces S_i are 4-cycles, compact line segments are 2-cycles



Review of M-theory on CY 3-folds

Kähler cone phase structure

Given a smooth 3-fold Y, the conformal limit is singular limit $Y \to X$ where all 2-cycles and 4-cycles collapse to zero volume

Several (topologically distinct) Y may map to same singularity X and form an equivalence class; $Y \to X$, $Y' \to X$ implies Y and Y' are related by flop transitions

Coulomb branch identified as closure of set of all resolutions preserving CY structure (Kähler deformations) Y_i of X, the extended Kähler cone:

$$\mathcal{M}_C \leftrightarrow \overline{\cup \mathcal{K}(Y_i/X)}$$

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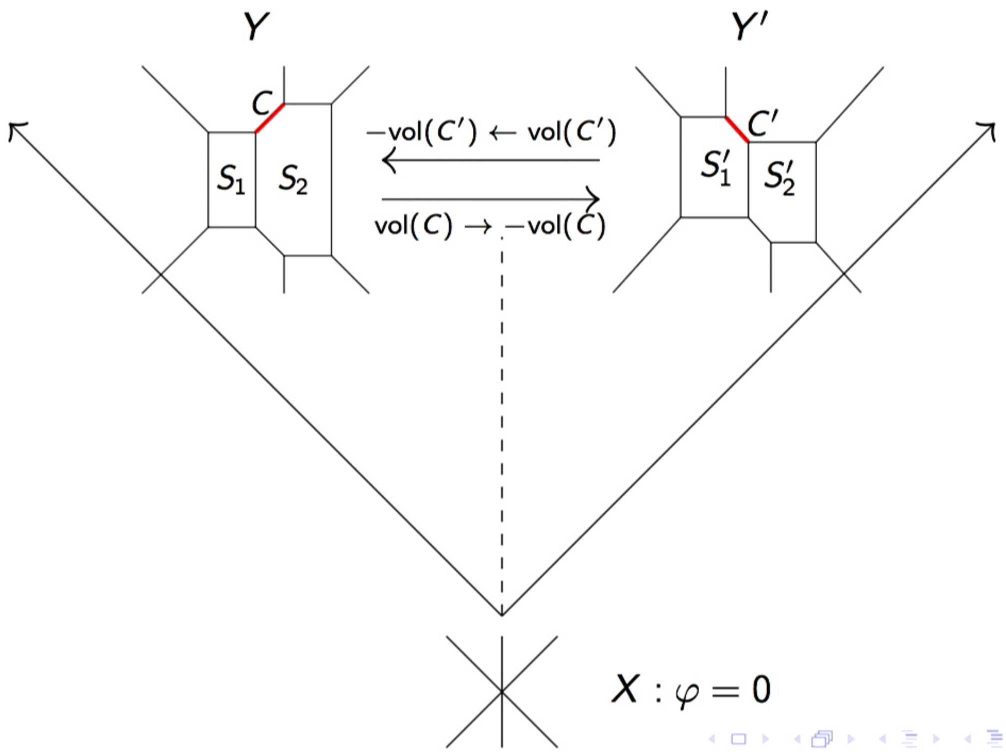
Review of M-theory on CY 3-folds
Kähler cone phase structure, cont ...

Phase transitions in Coulomb branch are flop transitions:



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Geometric classification program

How can we classify 5d theories using geometry?

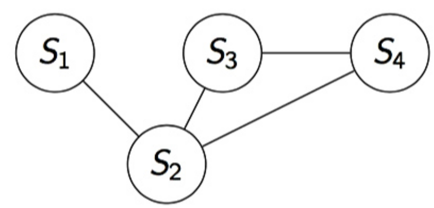
Classifying smooth 3-folds or singularities is a hard problem

Assumption: Y realized as neighborhood of a singular compact complex surface $S = \cup S_i$ intersecting pairwise transversally, and maps to canonical singularity X, which has nice properties

Simplification: physical equivalence classes (i.e. CFT fixed points up to decoupled free states) suggests geometric notion of physical equivalence classes of singularities, $X \sim X'$, roughly up to rank preserving cx structure deformations (including HW transitions)

Problem reduces to graph-theoretic problem:

$$\begin{cases} r = \#\text{ of nodes} \\ m = \sum m(S_i) - \#\text{ of edges} \\ \text{normal bundle of } S_i \cap S_j \text{ satisfies CY cond.} \end{cases}$$



Geometric classification program

Classification algorithm

To make this into an algorithm, we need to know

1. Allowed nodes, i.e. compact Kähler surfaces S_i
2. Allowed graphs, i.e. intersection configurations $S_i \cap S_j$
3. If geometry defines a canonical singularity

Point 1 has a partial answer: X canonical $\overset{\text{(by thm)}}{\Longrightarrow}$ S_i must be rational or ruled surfaces ($\mathbb{P}^2$ or locally $C_g \times \mathbb{P}^1$ and blowups)

Point 2 is a bit subtle, answer only known case-by-case for $r \leq 2$.

Point 3 has a robust check—must be able to shrink all cycles S_i, C_j to a point, or starting from the singularity, bring S_i, C_j to positive volume in Y

Geometric classification program

Checking positive volume

Expand class of J in terms of classes of S_i, $J = \phi^i S_i$

Imposing $J \cdot C = \text{vol}(C) \geq 0$ for all $C = n^j C_j$ equivalent to imposing positivity on generators C_j of cone of effective curves for all surfaces S_i

By a theorem (Kleiman), $J \cdot C > 0$ guarantees that volumes of all cycles will be positive!

All triple intersections can be computed in S_i—suppose $C \in S_i$ and let $C_{ij} = S_i \cap S_j$, then using $S_i \cdot S_j \cdot S_k = (S_i \cdot S_j)_{S_k}$,

$$J \cdot C = (K \cdot C)_{S_i} + \sum_{j \neq i} (C_{ij} \cdot C)_{S_i}$$

where K is the canonical class of S_i

Results

Consistency check: rank one surfaces ($r = 1$)

Graphs are just a single node:

We have[1] $J = \phi S$, hence

$$\mathrm{vol}(C) = J \cdot C = \phi S \cdot (-C) = -\phi (K \cdot C)_S \geq 0$$

$\implies$ S is a del Pezzo surface $dP_{p\leq 8} = Bl_{p\leq 8}\mathbb{P}^2$, gauge description $\mathfrak{su}(2)$ w/ $N_\mathsf{F} = p - 1$ or $\mathbb{F}_0 = \mathbb{P}^1 \times \mathbb{P}^1$

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[1] A subtle point is that if we want J to have positive coefficients ϕ, we must study the *negative* of the Kähler cone, $-\mathcal{K}$.

Results

New: rank two surfaces ($r = 2$)

Graphs are now non-trivial! Multiple gluings? Self-gluings?

Using physical equivalence we argue

where $S_1 \cap S_2 \cong \mathbb{P}^1$, and that all geometries take the form

$$S = S_1 \cup S_2 = \begin{cases} \mathrm{Bl}_{p_1}\mathbb{F}_n \cup \mathrm{dP}_{p_2} \\ \mathrm{Bl}_{p_1}\mathbb{F}_n \cup \mathbb{F}_0 \end{cases}$$

where $\mathbb{F}_n := \mathbb{P}[\mathcal{O} + \mathcal{O}(-n)] \overset{\text{locally}}{\sim} \mathbb{P}^1 \times \mathbb{P}^1$

Parameters (p_1, n, p_2) bounded from above by imposing positivity and excluding 3-folds expected to correspond to 5d KK theories w/ 6d fixed points

Geometric classification program

Classification algorithm

To make this into an algorithm, we need to know

1. Allowed nodes, i.e. compact Kähler surfaces S_i
2. Allowed graphs, i.e. intersection configurations $S_i \cap S_j$
3. If geometry defines a canonical singularity

Point 1 has a partial answer: X canonical $\overset{\text{(by thm)}}{\Longrightarrow}$ S_i must be rational or ruled surfaces ($\mathbb{P}^2$ or locally $C_g \times \mathbb{P}^1$ and blowups)

Point 2 is a bit subtle, answer only known case-by-case for $r \leq 2$.

Point 3 has a robust check—must be able to shrink all cycles S_i, C_j to a point, or starting from the singularity, bring S_i, C_j to positive volume in Y

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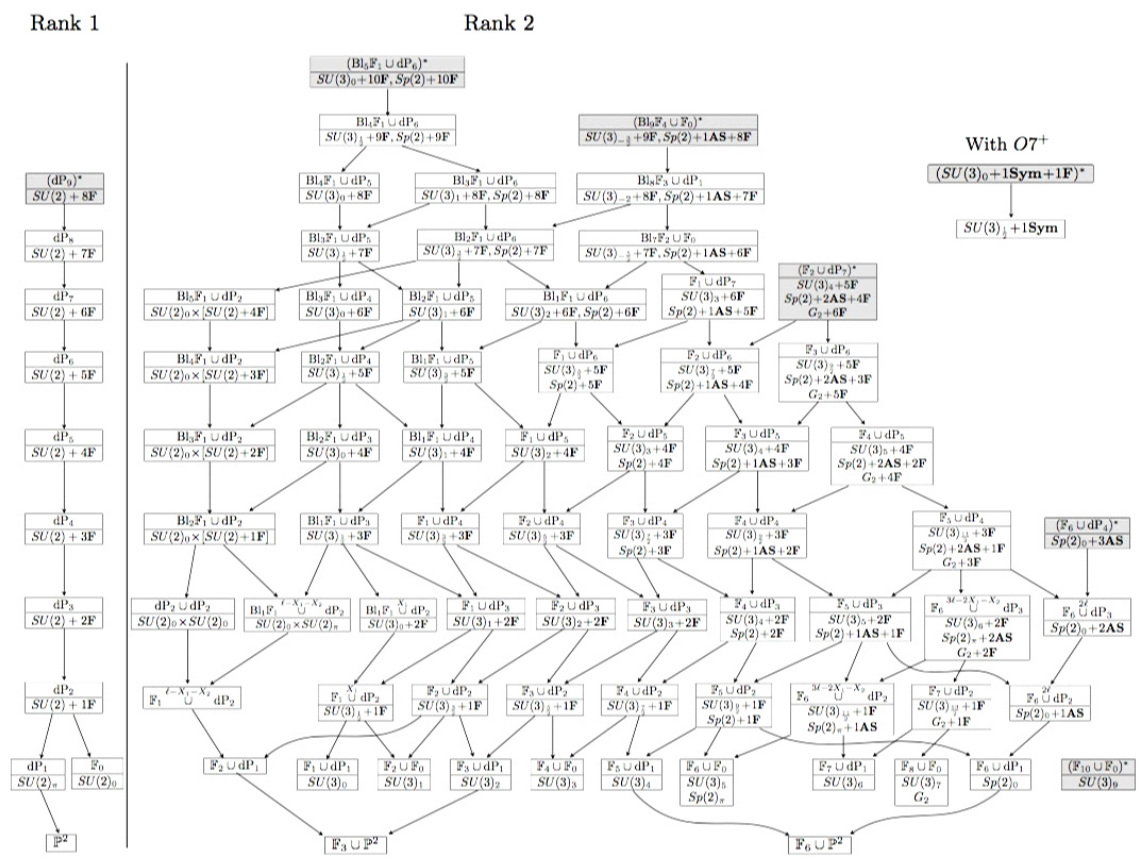

Figure: RG-flow diagram of rank 1 and 2 SCFTs

Results

Interesting conclusions

Many dualities among $\mathfrak{su}(3), \mathfrak{sp}(2), G_2$ theories

Nearly all candidate rank 2 theories were validated, with two exceptions:

- Gauge theory analysis produces bound $|k| \leq \frac{N^2}{N-2}$ for $\mathfrak{su}(N)_k$, but $\mathfrak{su}(3)_{k=\pm 8}$ is excluded
- $\mathfrak{su}(3)_{\frac{1}{2}} + 1\mathbf{Sym}$ related to 6d singularity involving $O7^+$

Family of RG flows encoded in geometry

All rank 2 theories have a 5-brane web description

All rank 1 and 2 5d theories descend from KK compactification of some 6d (1,0) theory!

- Rank 1 theories have a single "parent" theory
- Rank 2 theories have five parents

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Summary and future directions

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Classification of 5d gauge theories with UV fixed points is still an open problem, needs non-perturbative input

Geometric classification is a solution, potentially reduces to a graph theoretic problem where nodes are Kähler surfaces and edges are intersections

Rank 1 and 2 can be obtained systematically

Rank 2 classification confirms nearly all theories predicted by gauge theory analysis

Summary and future directions

Future directions

Higher rank, $r > 2$? (Which nodes and graphs are allowed? Loops now included, e.g. consider 5d T_5 theory given by $\mathbb{C}^3/\mathbb{Z}_5 \times \mathbb{Z}_5$)

Geometric computation of flavor symmetry enhancements due to $U(1)_I$? (e.g. $T_3 = \mathbb{C}^3/\mathbb{Z}_3 \times \mathbb{Z}_3$ has manifest $F = SU(3)^3$ but is equiv to dP_6 for which $F = E_6$ can be computed geometrically)

Top down approach: compactify 6d (1,0) theory on S^1 and then integrate out matter (i.e. blowdowns)

Thank you!