Title: Quantum scale anomaly in a two-dimensional Fermi gas

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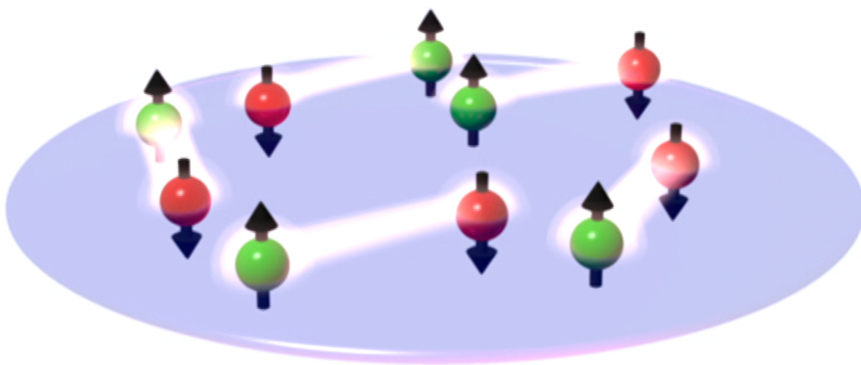
Abstract:



Quantum scale anomaly in a two-dimensional Fermi gas

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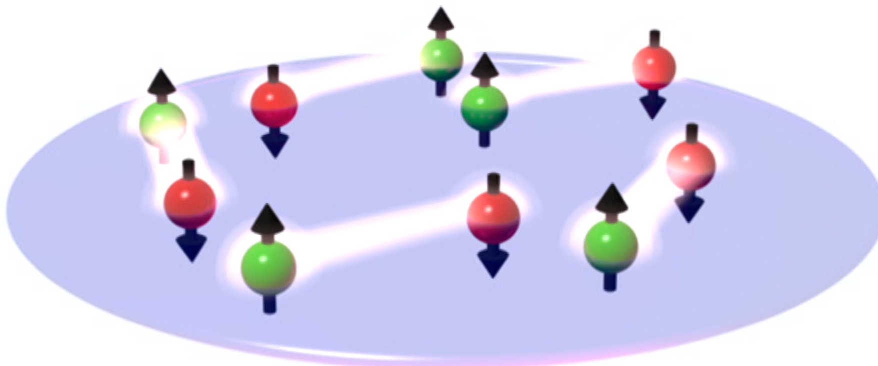
Physics Institute
Heidelberg University

Perimeter Institute
6/21/2018

Ultracold quantum gases

2D Fermi gases



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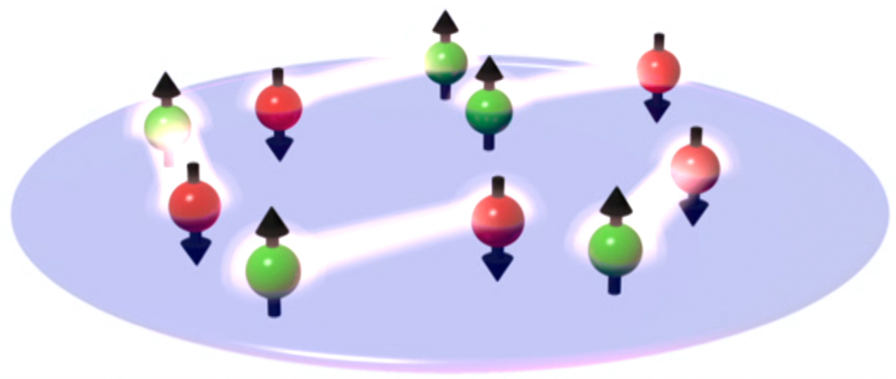


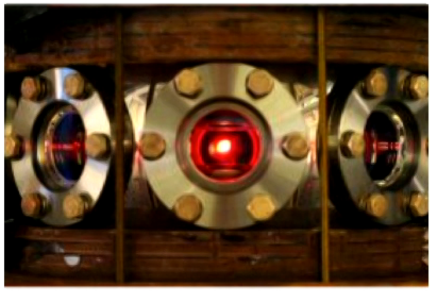
Essential ingredients

- 2-component Fermi gas (labeled ↑ and ↓)
- s-wave contact interaction
- two-dimensional geometry

Ultracold quantum gases

Experimental realization

Ultracold neutral gas



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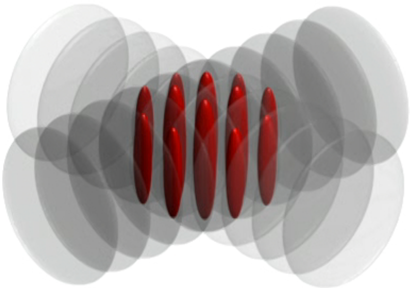
Feshbach resonance

tunable interactions

+

Optical lattices



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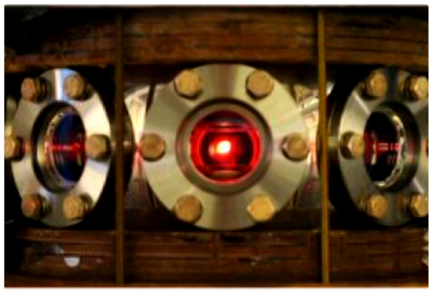

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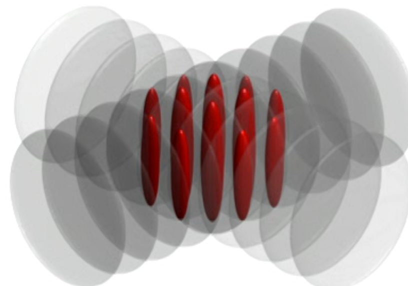

Highly tunable model implementation

Spatially and temporally resolved observables

Scale invariance

Absence of intrinsic length scale

- Physics independent of absolute scale
- Example in statistical physics: fluctuations on all lengths scales at a critical point
- Cold gases: Unitary Fermi gas with diverging scattering length $a \to \infty$
- Universal behaviour

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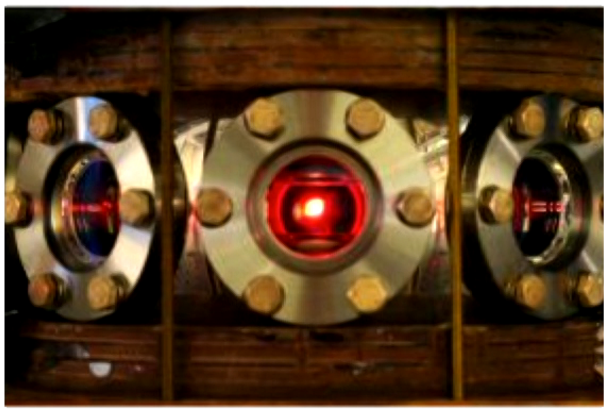
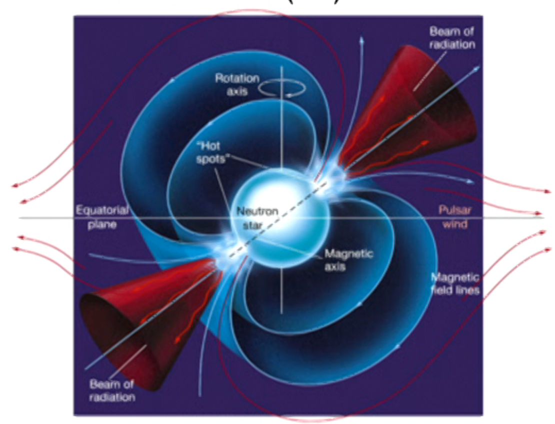
Unitary Fermi gas (µm)



Neutron star (fm)



Ultracold gases & gravitational waves

GW170817 neutron star merger

LIGO collaboration

Ultracold gases & gravitational waves

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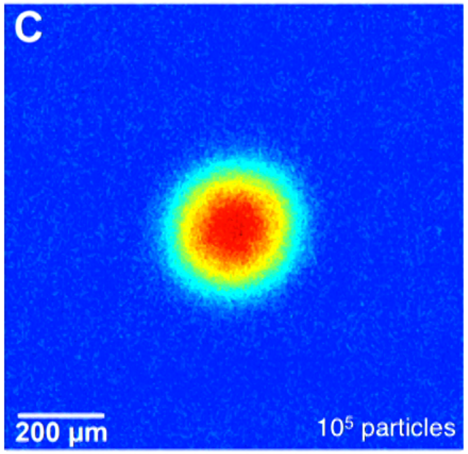

GW170817 neutron star merger

LIGO collaboration

- Excitation spectrum
- Collective modes
- Hydrodynamics

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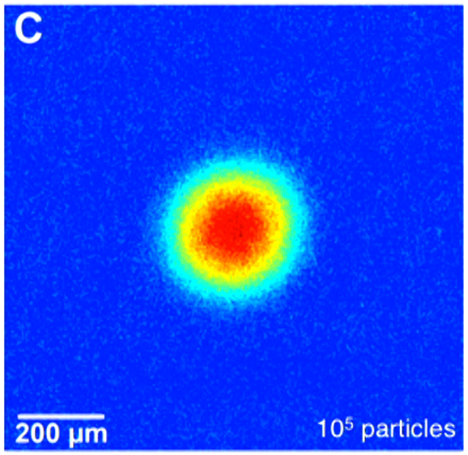

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GW170817 neutron star merger

LIGO collaboration

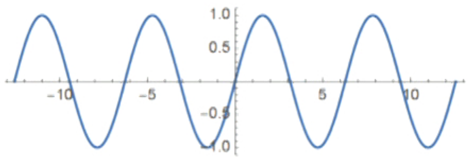
A low energy challenge?

Credit: Aurel Bulgac

Scale invariance

Single particle

$$H = \frac{\boldsymbol{p}^2}{2m}\,, \qquad \psi(\boldsymbol{x}) = e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$



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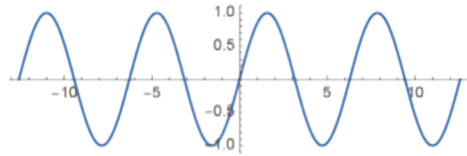
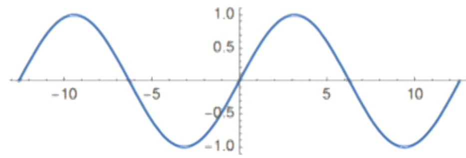


- Rescale coordinates $\boldsymbol{x} \mapsto \lambda\boldsymbol{x}$



- Solution self-similar with $\boldsymbol{k} \mapsto \frac{1}{\lambda}\boldsymbol{k}$:
- Hamiltonian scales as $H \mapsto \frac{1}{\lambda^2} H$

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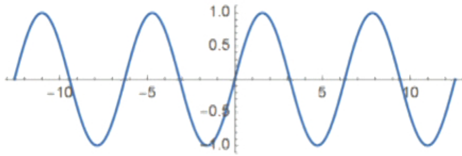

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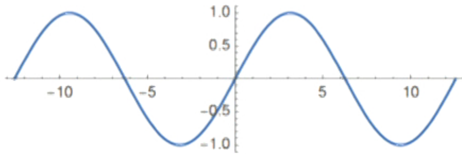

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Energies and eigenstates obtained by simple rescaling

Scale invariance

Interacting system

$$H = \sum_i \frac{\boldsymbol{p}_i^2}{2m} + \frac{1}{2}\sum_{i\neq j} V(\boldsymbol{x}_i - \boldsymbol{x}_j)$$

Candidate: power law

$$V(\boldsymbol{x}) = \frac{g}{|\boldsymbol{x}|^\alpha}\,, \qquad V(\lambda\boldsymbol{x}) = \frac{1}{\lambda^\alpha} V(\boldsymbol{x})$$

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$$H \mapsto \frac{1}{\lambda^2}H_{\mathrm{kin}} + \frac{1}{\lambda^\alpha}H_{\mathrm{int}} \stackrel{!}{=} \frac{1}{\lambda^2}H$$

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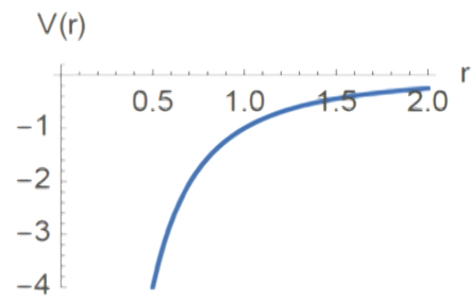

Only scale invariant for **inverse square potential**

Scale invariance

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Candidate: Contact interactions

$$V(\boldsymbol{x}) = g\delta(x)\delta(y)\cdots$$

$$V(\lambda\boldsymbol{x}) = \frac{1}{\lambda^d} V(\boldsymbol{x}) \qquad \text{since} \qquad \delta(\lambda x) = \frac{1}{\lambda}\delta(x)$$

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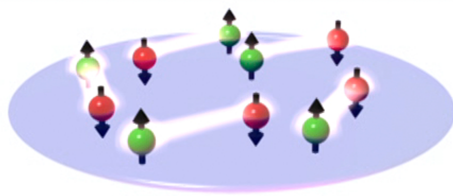
Contact interactions make system **scale invariant** in d=2 for all interactions

Trapped 2D gases

Harmonic radial trapping potential



$$H = \underbrace{\sum_i \frac{\boldsymbol{p}_i^2}{2m} + \frac{1}{2} g \sum_{i \neq j} \delta^{(2)}(\boldsymbol{x}_i - \boldsymbol{x}_j)}_{1/\lambda^2}$$

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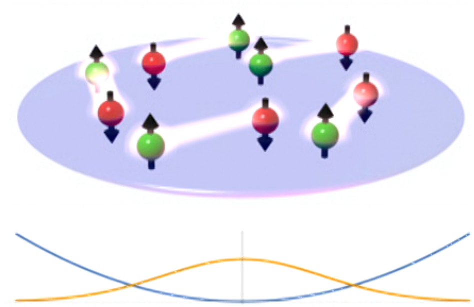

Harmonic trap breaks scale invariance

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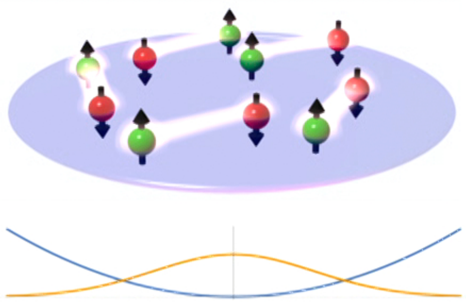

Harmonic trap breaks scale invariance

Pitaevskii-Rosch symmetry

Scale invariance replaced by SO(2,1) scaling symmetry

L. P. Pitaevskii et al. Phys. Rev. A, 55, R 835 (1997)
F. Werner and Y. Castin, Physical Review A 74, 053604 (2006)

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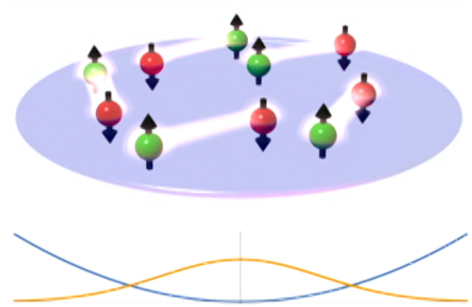
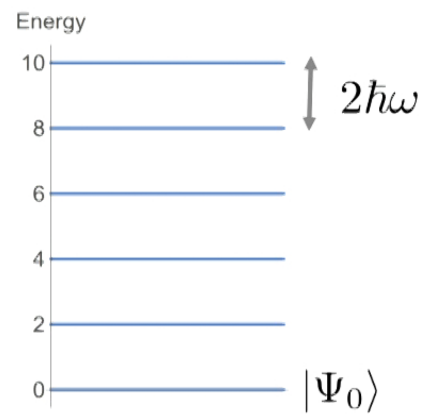


Harmonic trap breaks scale invariance

Pitaevskii-Rosch symmetry

Scale invariance replaced by SO(2,1) scaling symmetry

- Tower of eigenstates generated by symmetry
- Valid for all interaction strengths
- Fixes dynamics



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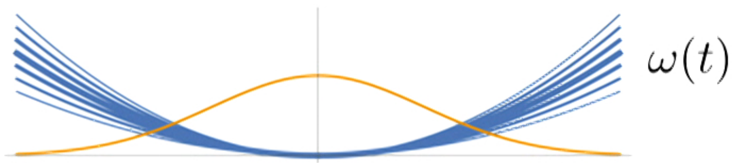
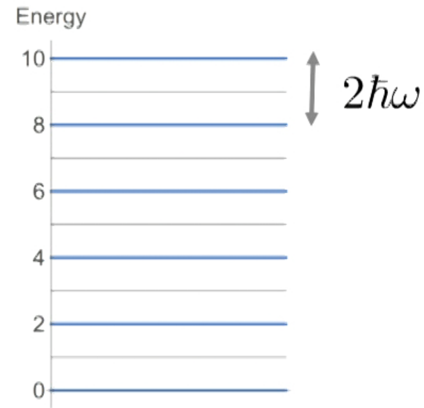
Dynamical scaling symmetry

Single-particle harmonic oscillator

Excitation of Gaussian wavepacket





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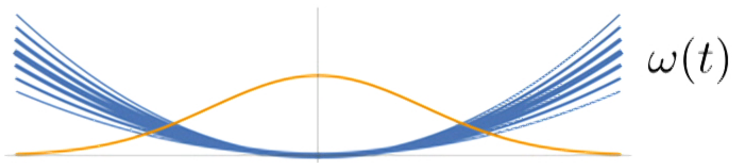
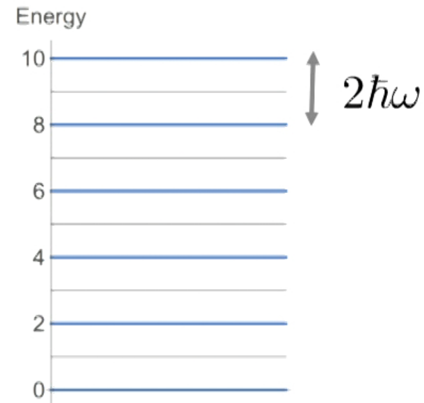
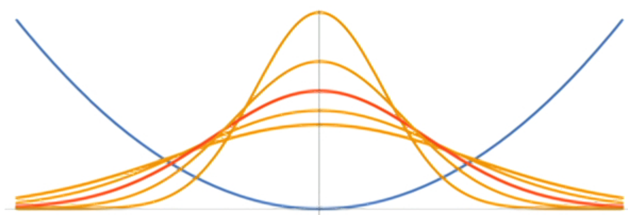




Breathing motion



$$n(x,t) = |\psi(x,t)|^2 = \frac{1}{\lambda} n\Big(\frac{x}{\lambda}, t=0\Big)$$

- Self-similar
- Determined by dynamics of a single scale factor

$$\lambda \rightarrow \lambda(t)$$

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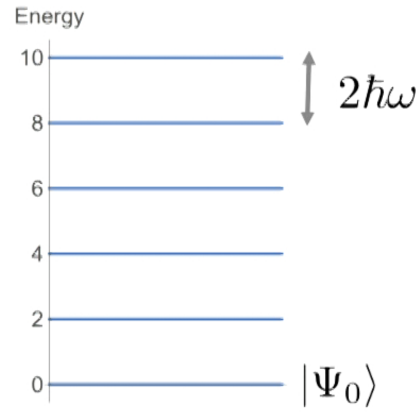
Dynamical scaling symmetry

Applies to a many-body system

N-body system $\mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$

$$\lambda \to \lambda(t)$$

$$\psi(\mathbf{X}, t) = \frac{1}{\lambda^{Nd/2}} \psi\Big(\frac{\mathbf{X}}{\lambda}, t=0\Big) \exp\Big(i \frac{m\dot{\lambda}}{\hbar\lambda} \mathbf{X}^2\Big) e^{i\theta}$$



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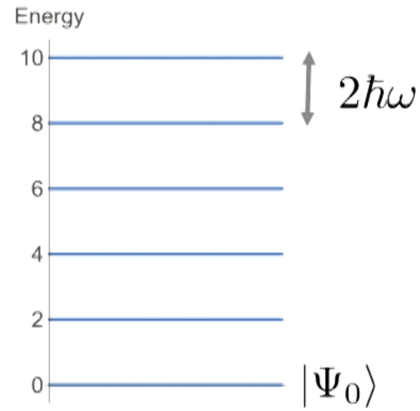
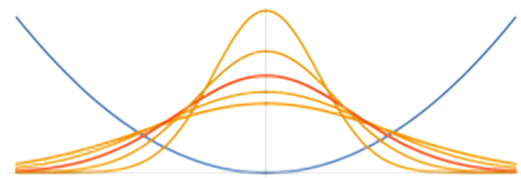


For given initial state, **exact many-body wavefunction known at all times**

- Periodic dynamics completely specified by $\lambda(t)$
- No damping/thermalization
- Period of breathing modes exactly known $\omega_B = 2\omega_R$



Quantum scattering

Regularized δ-potential

Bare δ-potential produces diverging bound state energies

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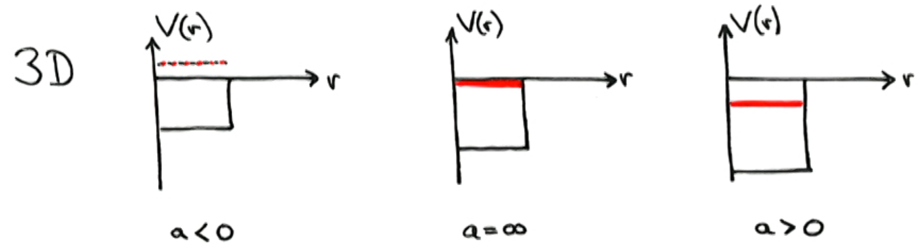
Bare δ-potential produces diverging bound state energies



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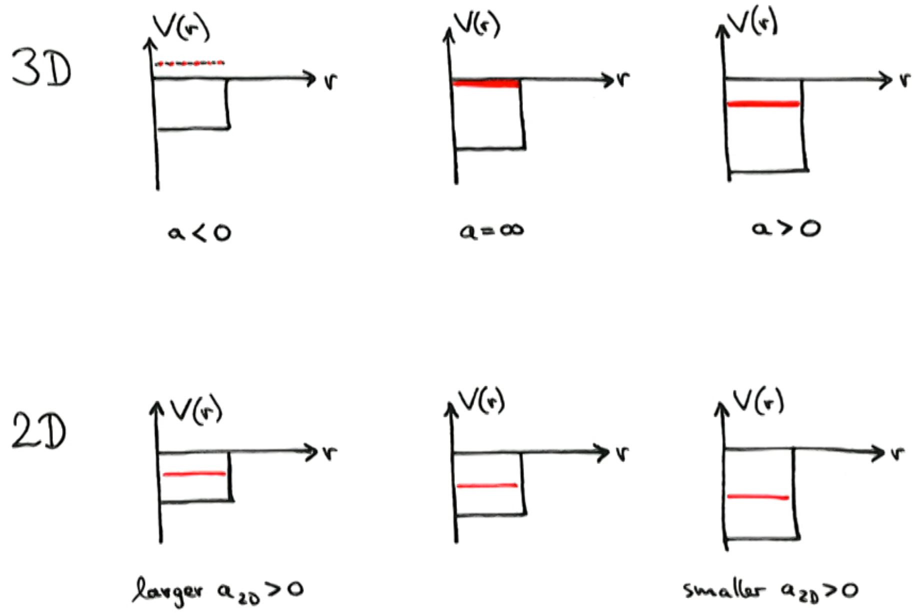


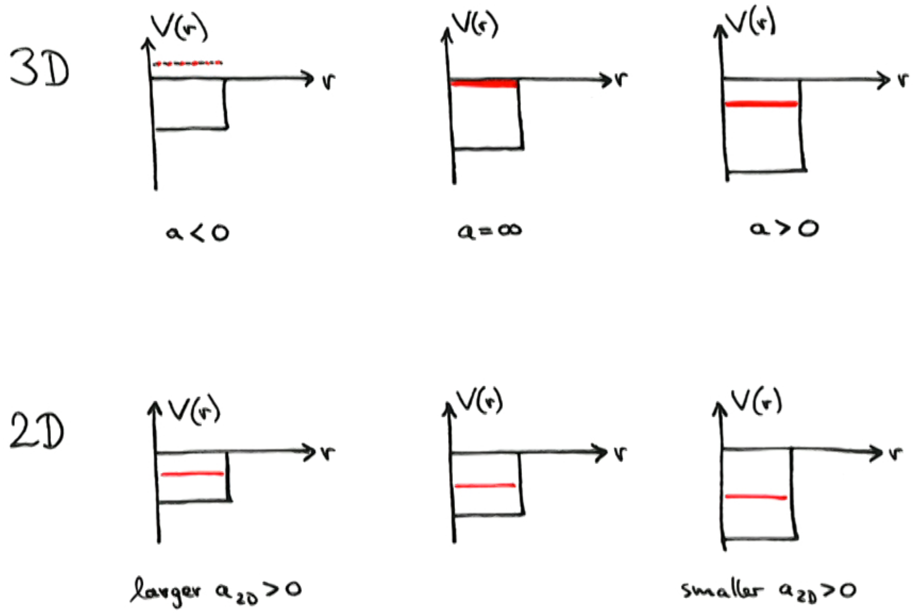
$$\varepsilon_B = \frac{\hbar^2}{m a_{\mathrm{2D}}^2}$$

Bound state of size a_{2D} always exists in two dimensions

Quantum scattering

Regularized δ-potential

Bare δ-potential produces diverging bound state energies



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Scale invariance broken by 2D scattering length

Quantum scattering in 2D

Quantum anomaly

1. Exact symmetry of the classical Hamiltonian
2. Divergence appearing in the quantized theory
3. Violation of the original symmetry in the renormalized theory

2D scattering length gives **quantum scale anomaly**

W. Zwerger: Varenna lecture notes *Strongly Interacting Fermi Gases* (2014)

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2-body scattering

2-body scattering amplitude (cross sections, phase shifts)

$$f(k) = \frac{2\pi}{i\frac{\pi}{2} - \ln(ka_{2D})}$$

Logarithmic “running coupling” with **interaction parameter $\ln(k_F a_{2D})$**

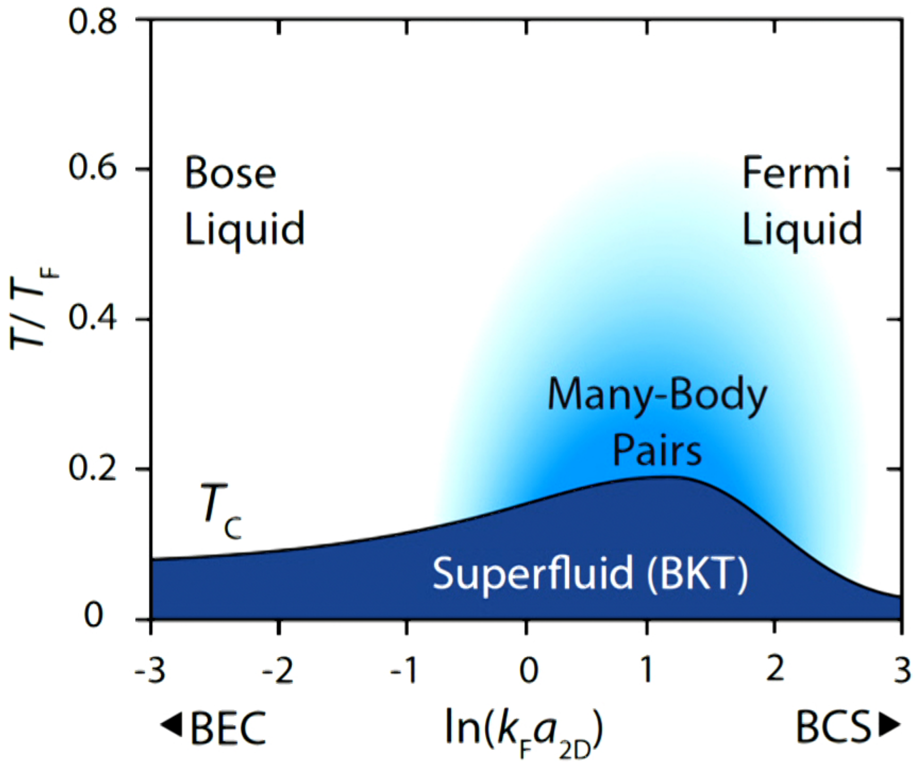
W. Zwerger: Varenna lecture notes *Strongly Interacting Fermi Gases* (2014)

Phase Diagram

2D BEC-BCS Crossover

Interaction parameter $\ln(k_{\mathrm{F}} a_{2\mathrm{D}})$

$$\epsilon_B = E_F e^{-2\ln(k_{\mathrm{F}} a_{2\mathrm{D}})}$$



P.A. Murthy et al., Science 359, 6374 (2018)

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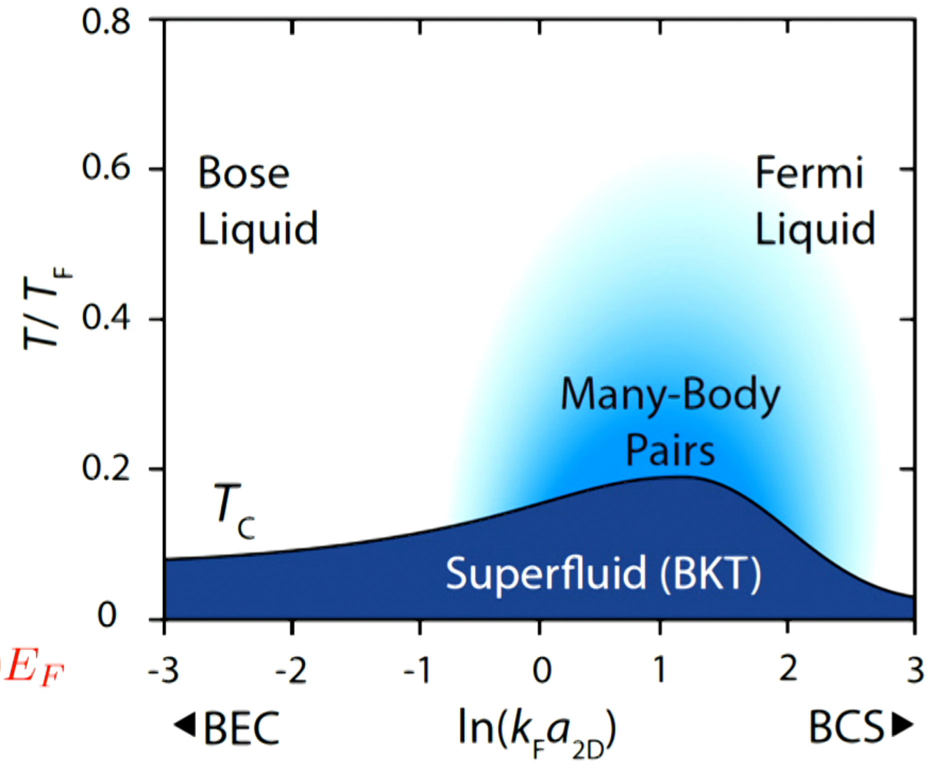

$\epsilon_B \approx 400 E_F$ $\epsilon_B \approx 0.002 E_F$

P.A. Murthy et al., Science 359, 6374 (2018)

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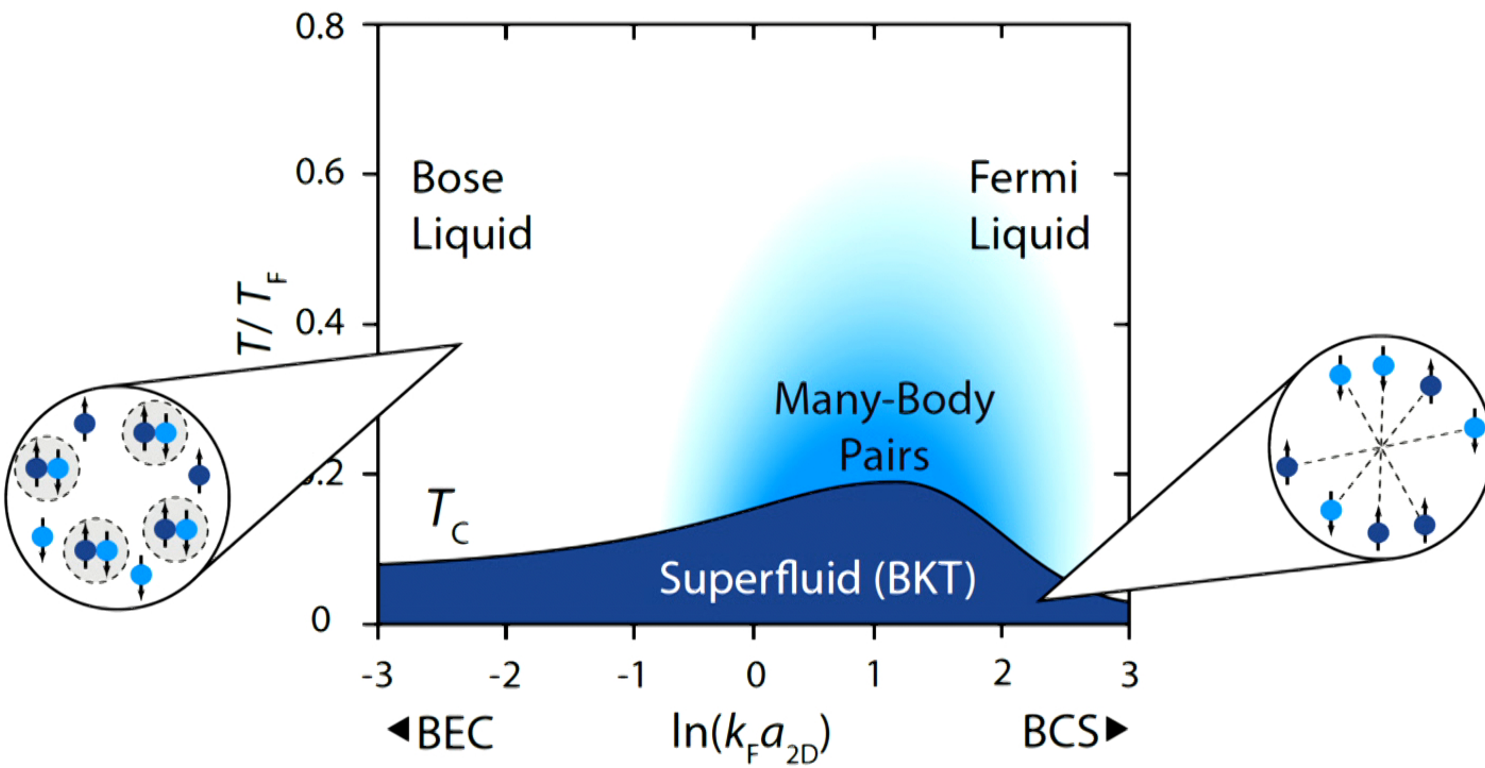

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Quantum anomaly

Breathing mode frequency

- SO(2,1) – Symmetry directly leads to breathing mode solution with:

$$\omega_B = 2\omega_R \qquad \Gamma_B = 0$$

L. P. Pitaevskii et al. Phys. Rev. A, 55, R 835 (1997)
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- Coupling to bound state shifts breathing mode frequency

$$\omega_B = 2\omega_R + \delta\omega_A$$

- **Quantum anomaly**: classical symmetry broken after quantization

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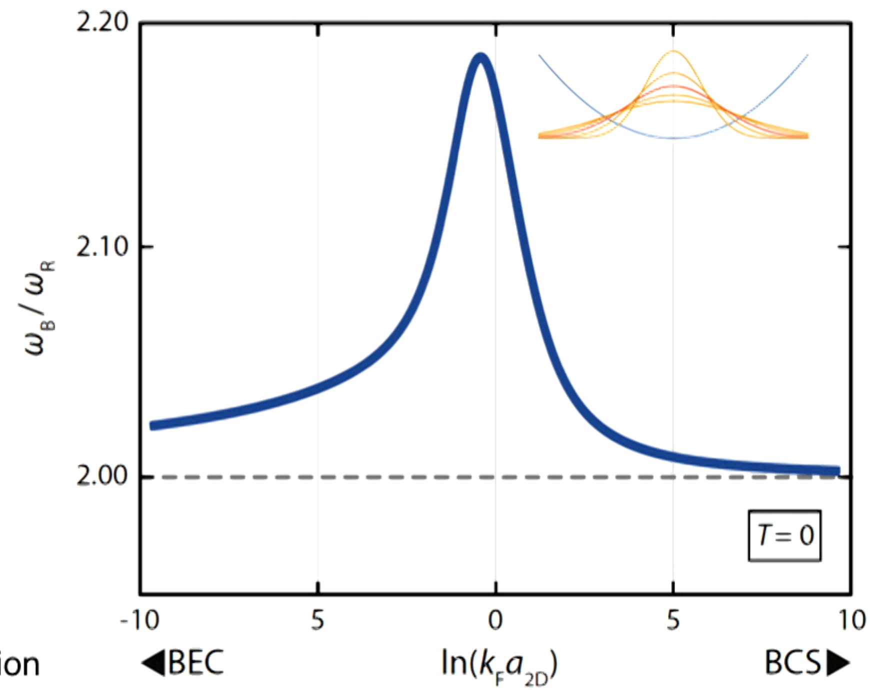

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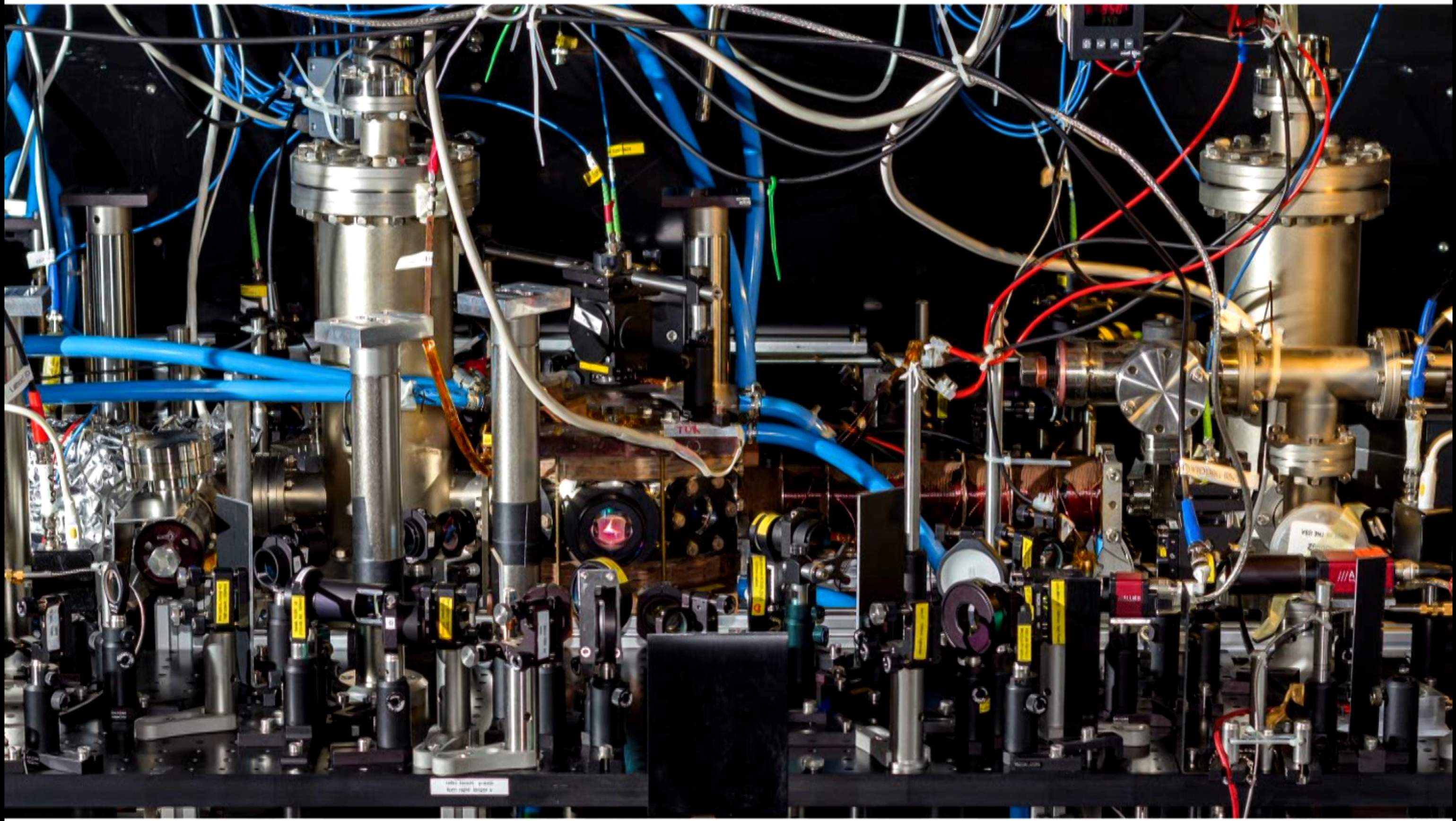
The Experiment



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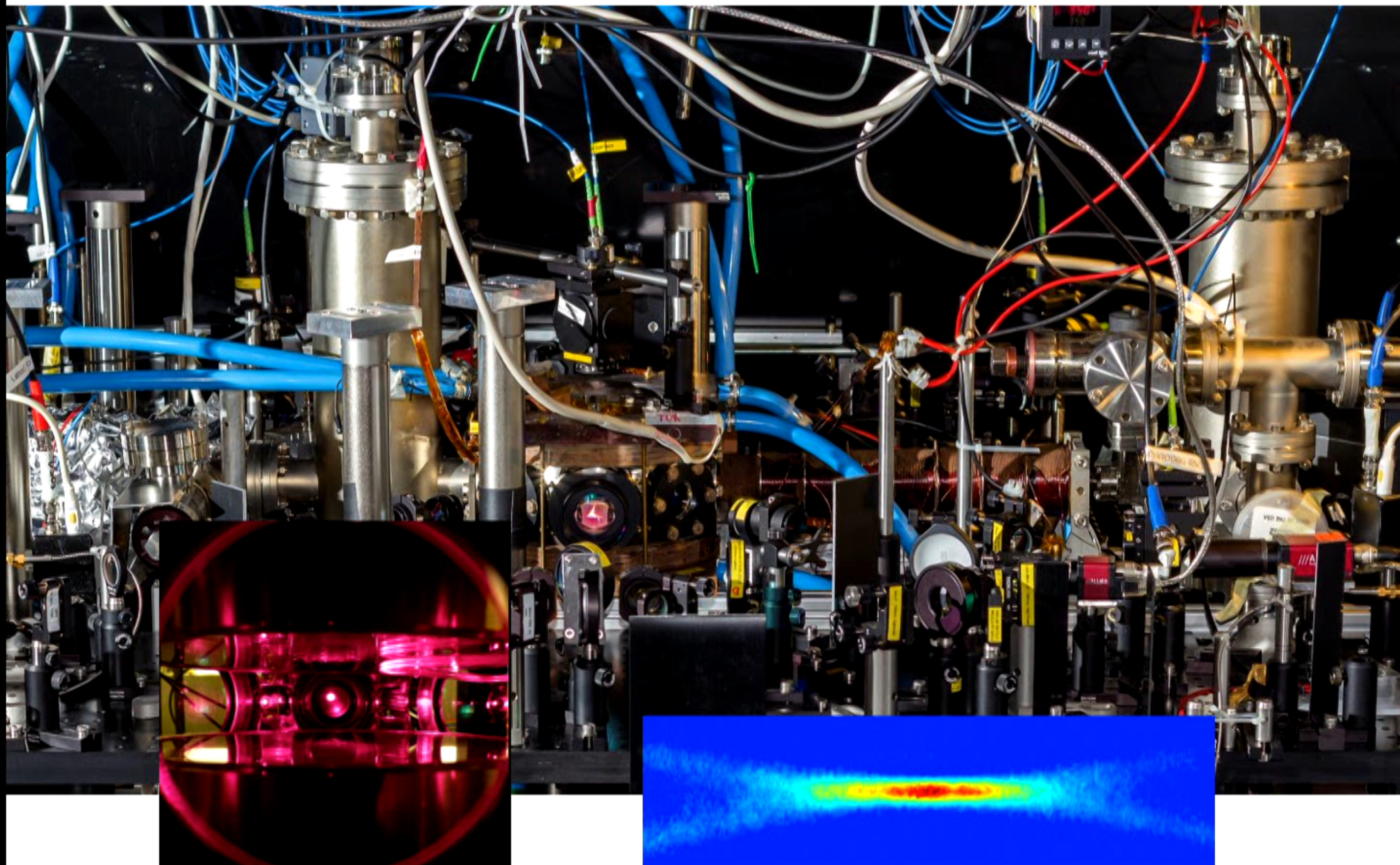
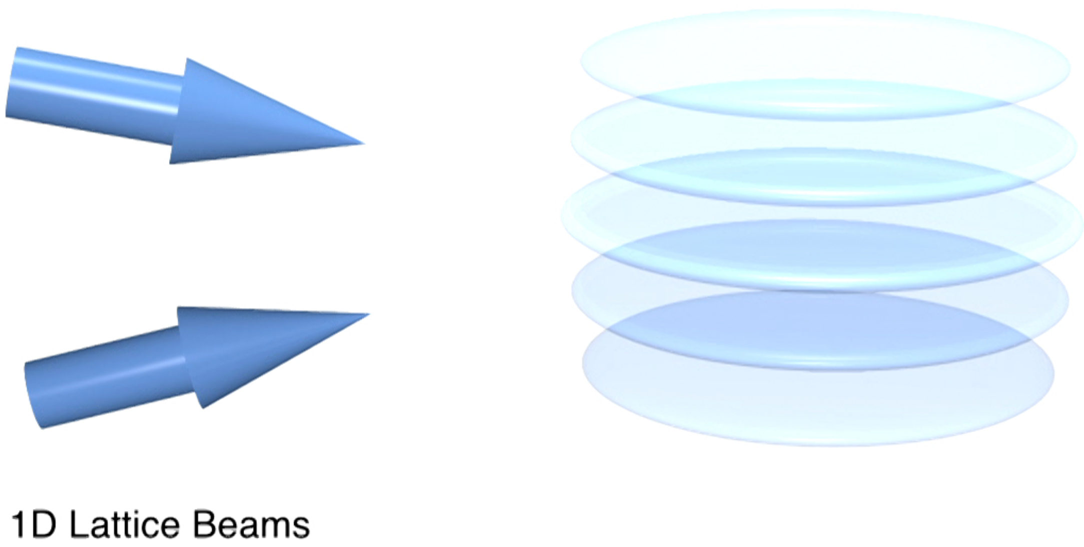


Lithium MOT

All-optical forced evaporation

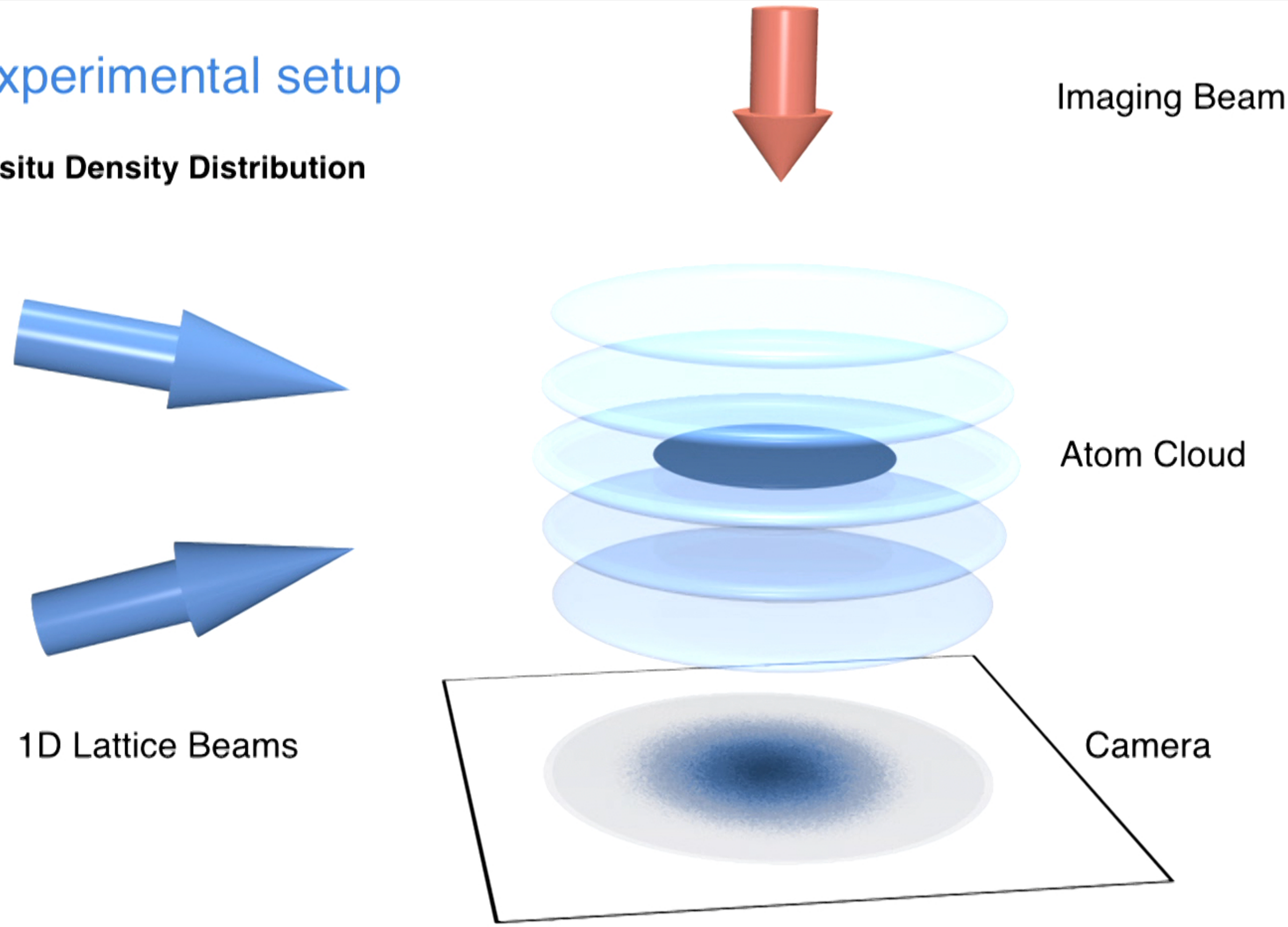
Experimental setup

In-situ Density Distribution

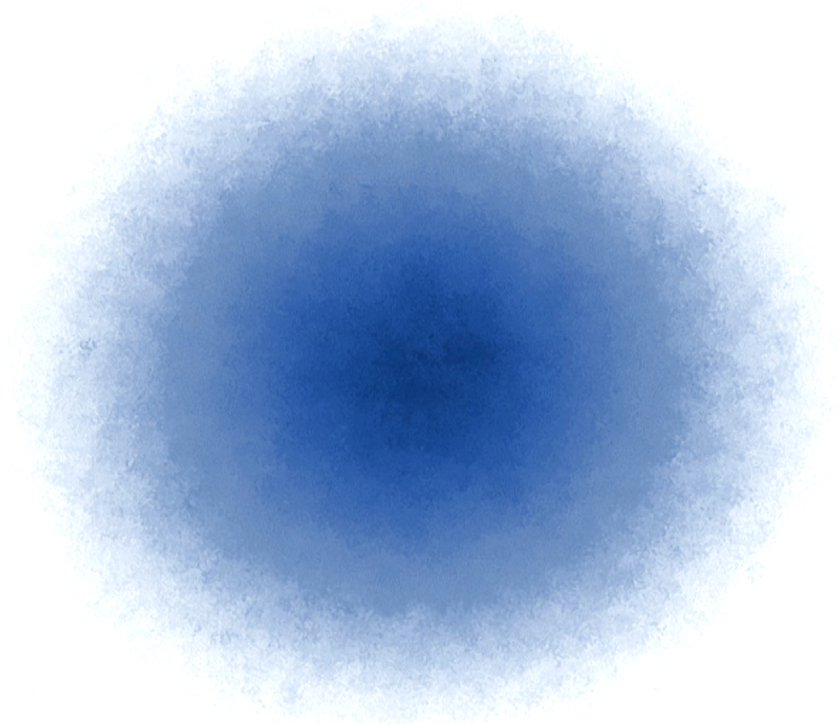


Experimental setup

In-situ Density Distribution

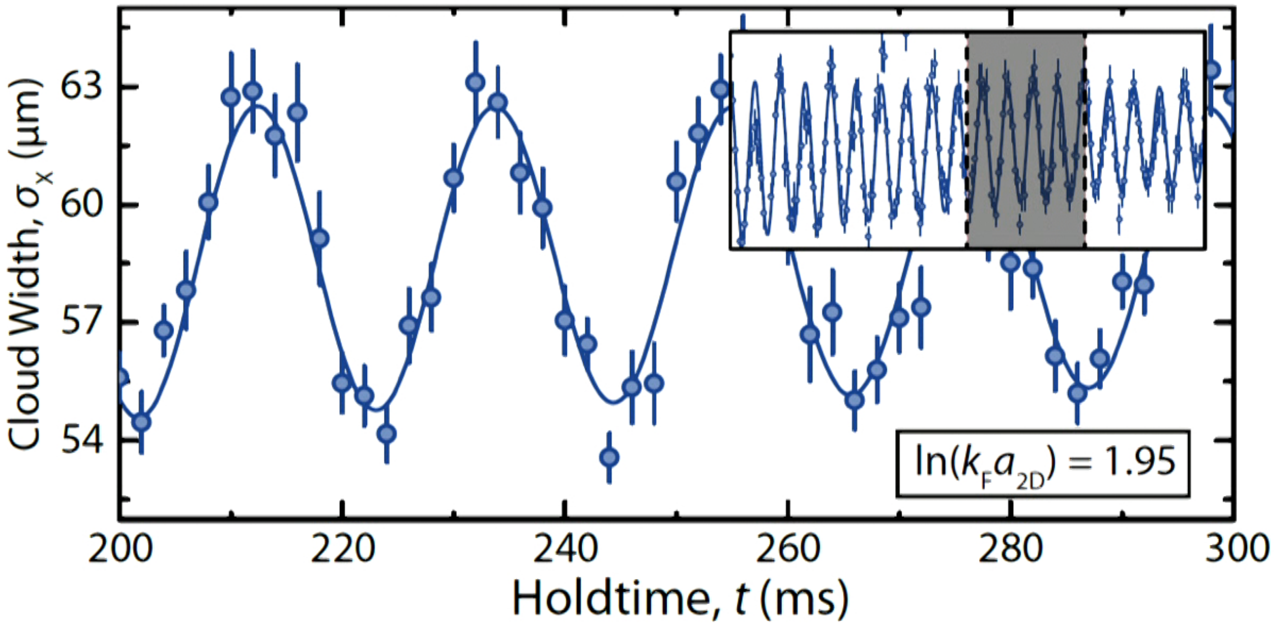


Monopole mode

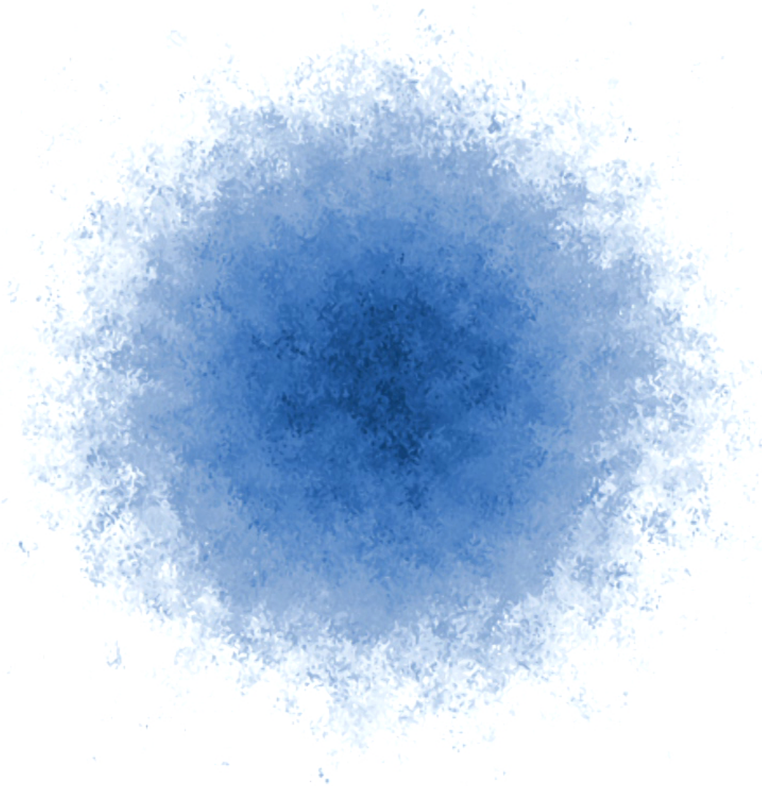


Frequency Measurements

Breathing mode

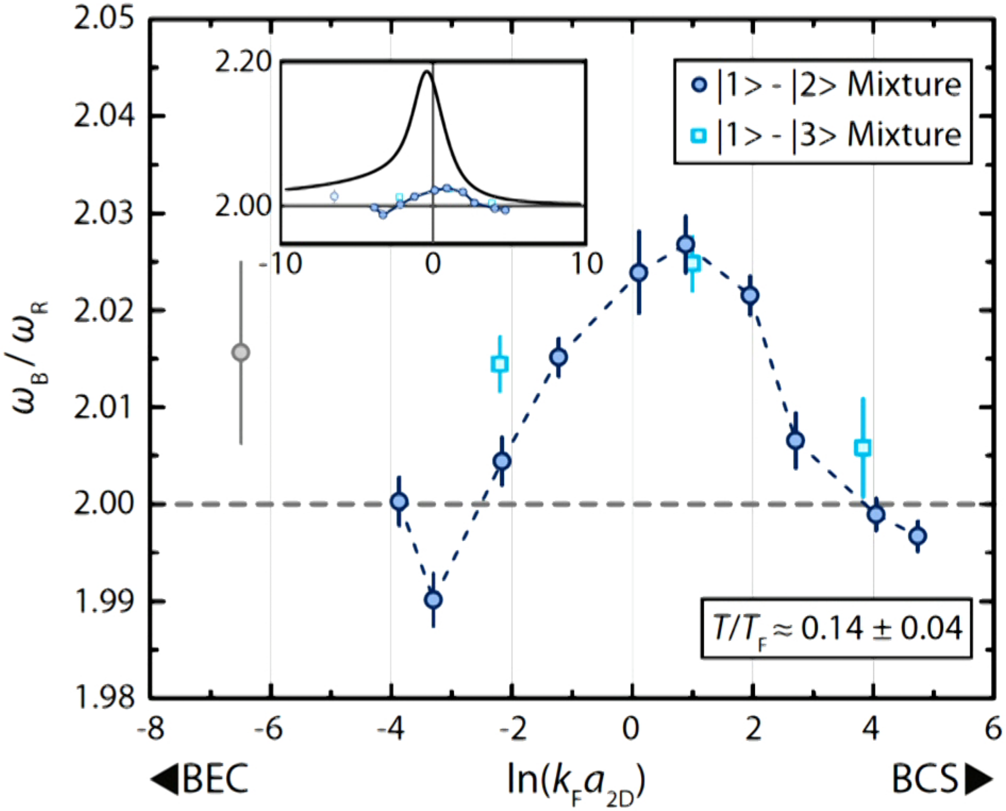


Dipole Mode



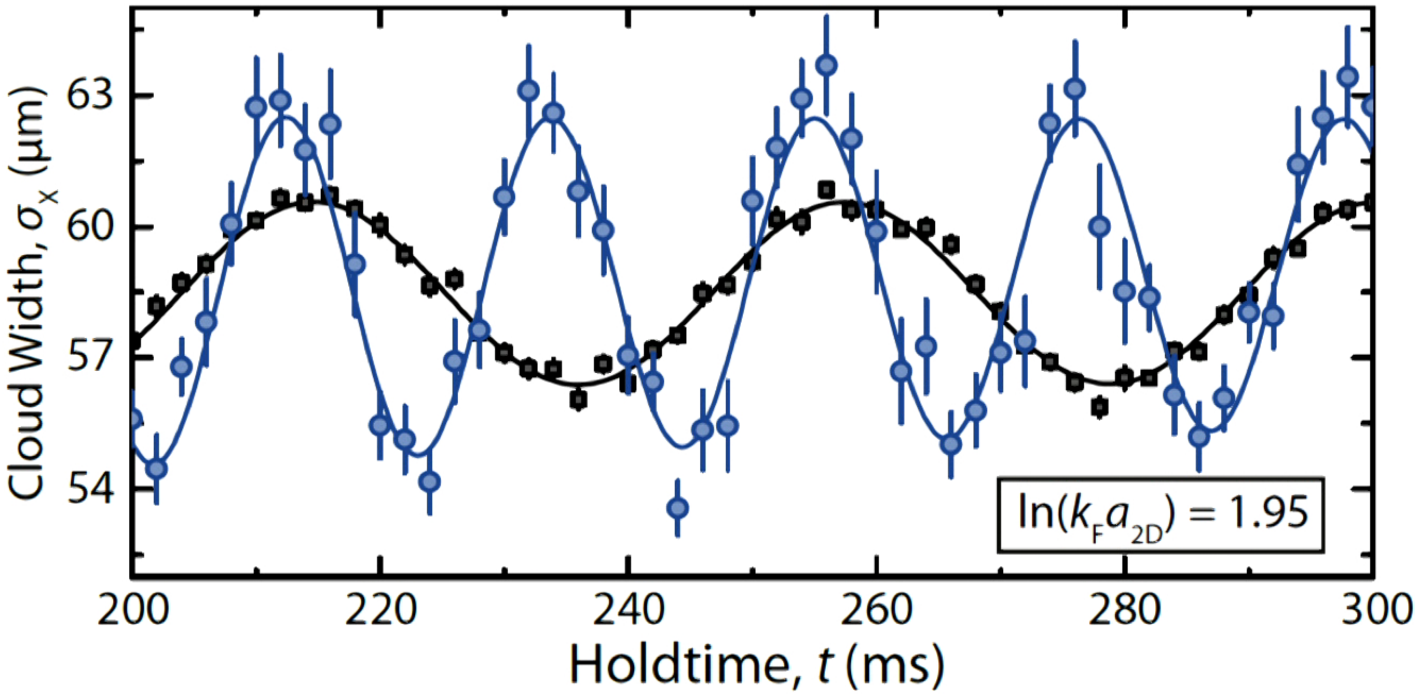
Anomalous Shift

Frequency ratios



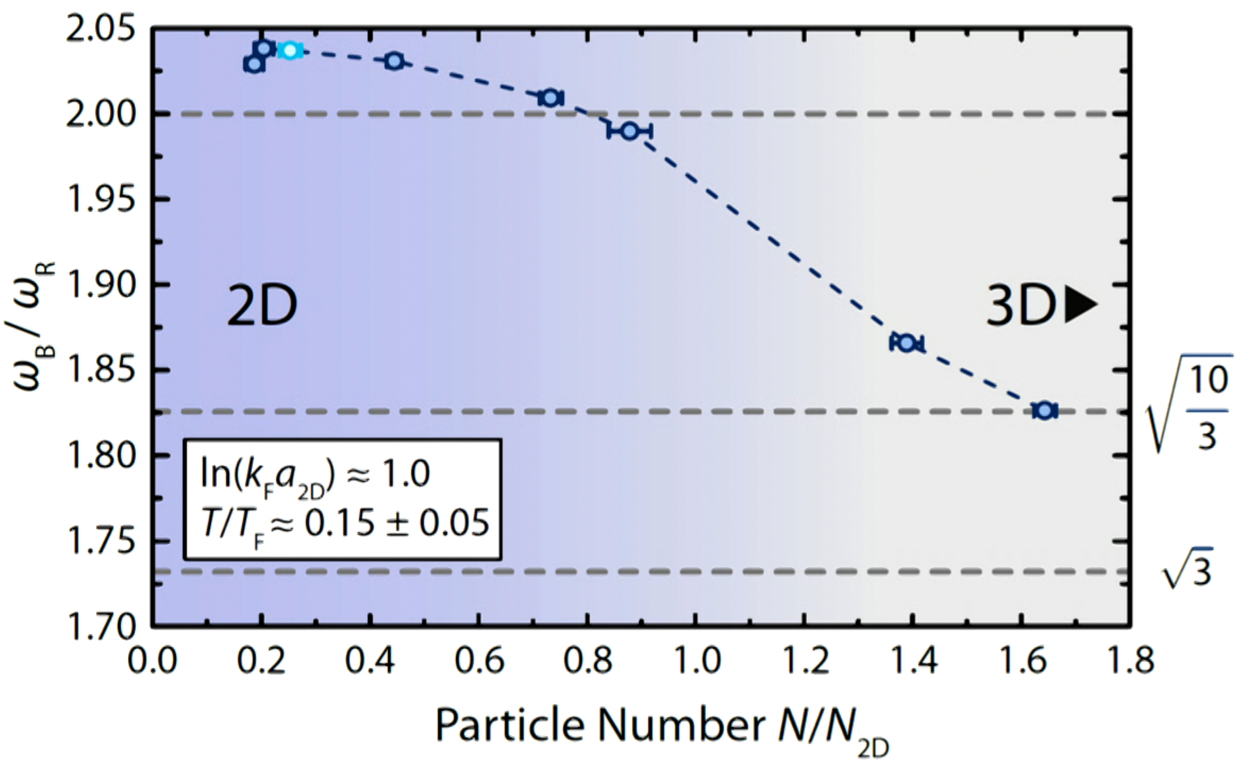
Frequency Measurements

Breathing & Dipole mode



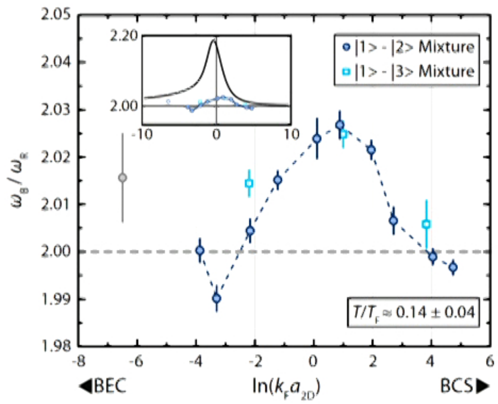
Explicit Symmetry Breaking

Dimensional Crossover



Breathing mode frequency

Summary



- Conclusive observation of anomalous frequency shift
- Effect siginifcantly smaller than expected

Open questions

- Finite temperature
- Presence of third dimension

Finite T

C. Chafin andT. Schäfer Phys. Rev. A 88, 043636 (2013)

B. C. Mulkerin, *et al.*, Phys. Rev. A 97, 053612 (2018)

quasi 2D

K. Merloti *et al.*, Physical Review A 88, 061603 (2013)

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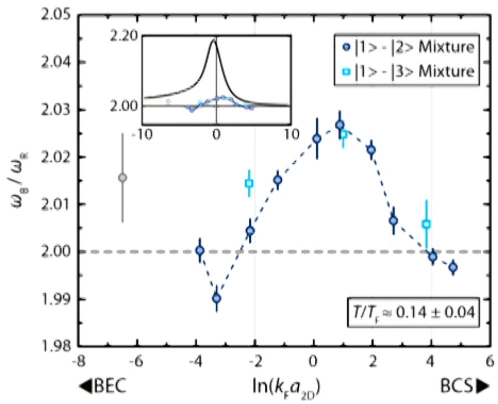

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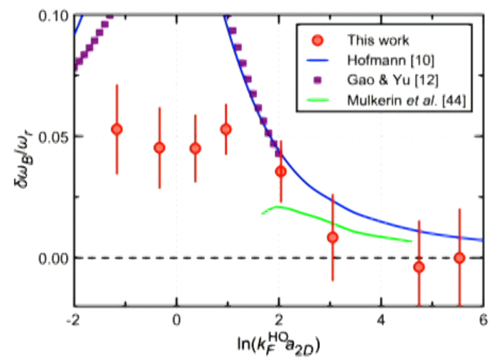

Vale group, Swinburne University

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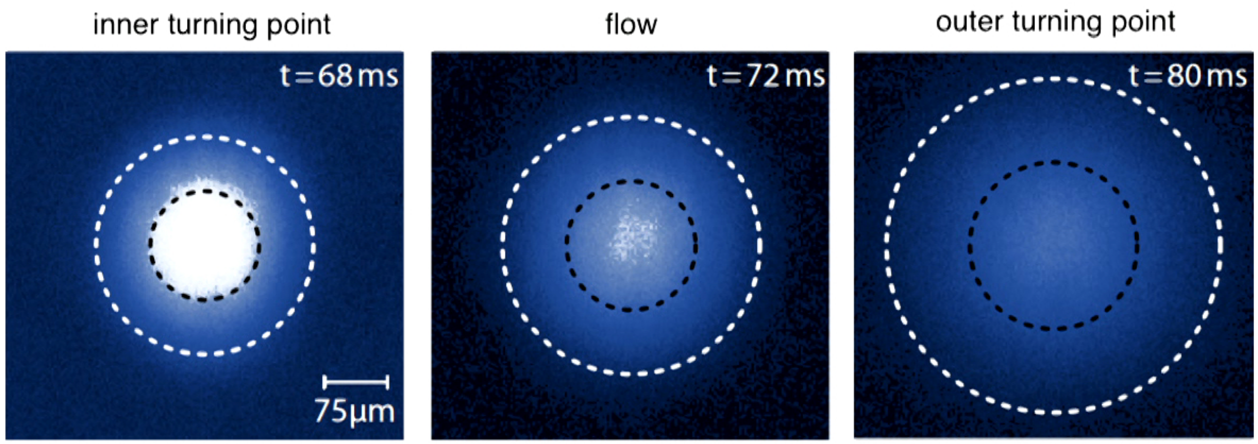
Evolution of the many-body wavefunction

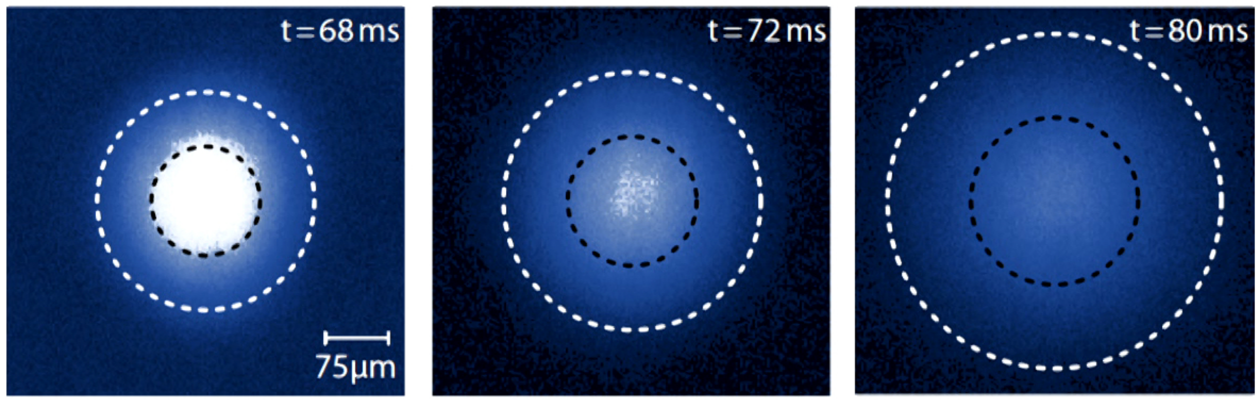
Scale invariant scenario



Evolution of the many-body wavefunction

Scale invariant scenario

inner turning point | flow | outer turning point

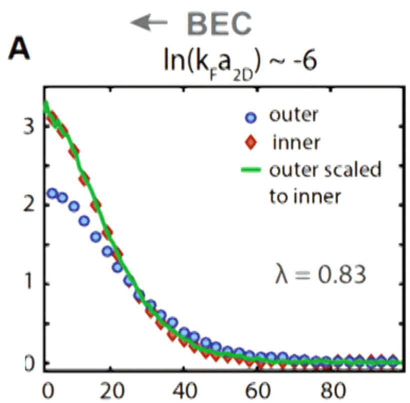


in-situ:	$\rho(x, t=0)$	$\lambda^{-2}\rho(x/\lambda, t=0)$	$\lambda^{-2}\rho(x/\lambda, t=0)$
k-space:	$n(k, t=0)$	~	$\lambda^{2} n(\lambda k, t=0)$

In-situ and momentum space

Tuning interactions

In-situ



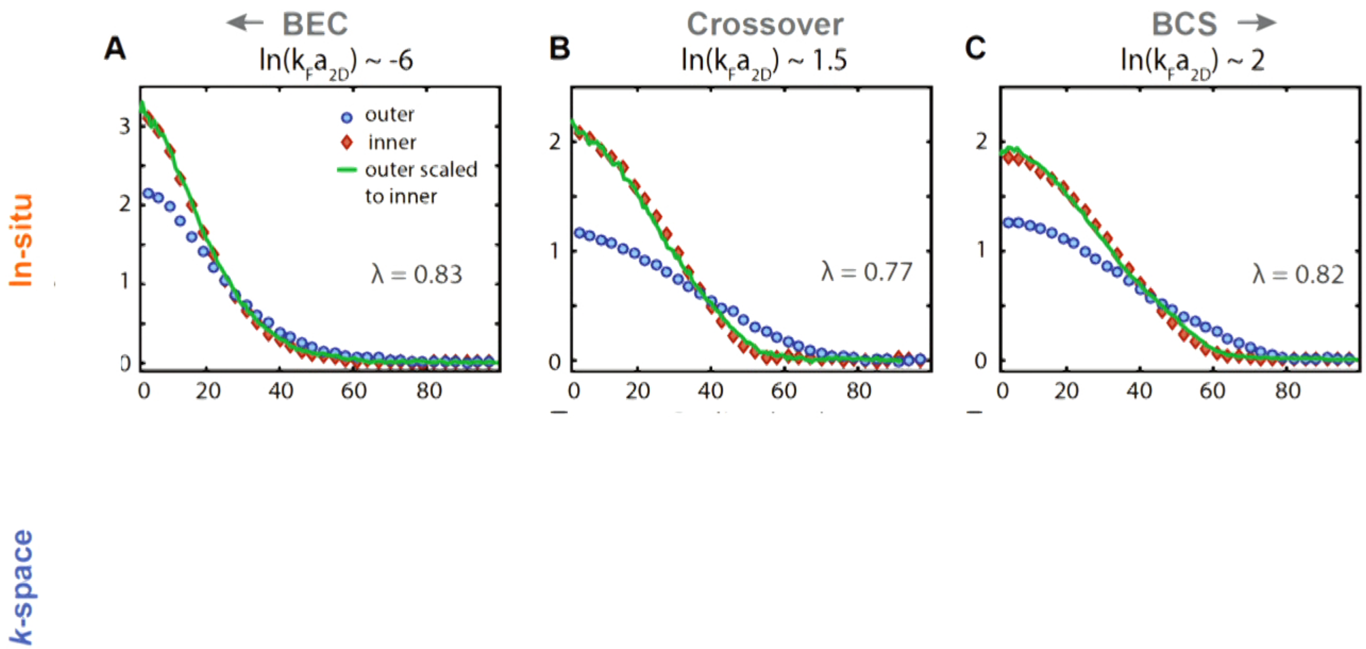
- BEC side (weakly interacting dimers)
- Compare density profile at inner and outer TP
- Perfect scaling

k-space

P. A. Murthy, N. Defenu *et al.*, arXiv:1805.04734 (2018)

In-situ and momentum space

Tuning interactions



P. A. Murthy, N. Defenu *et al.*, arXiv:1805.04734 (2018)

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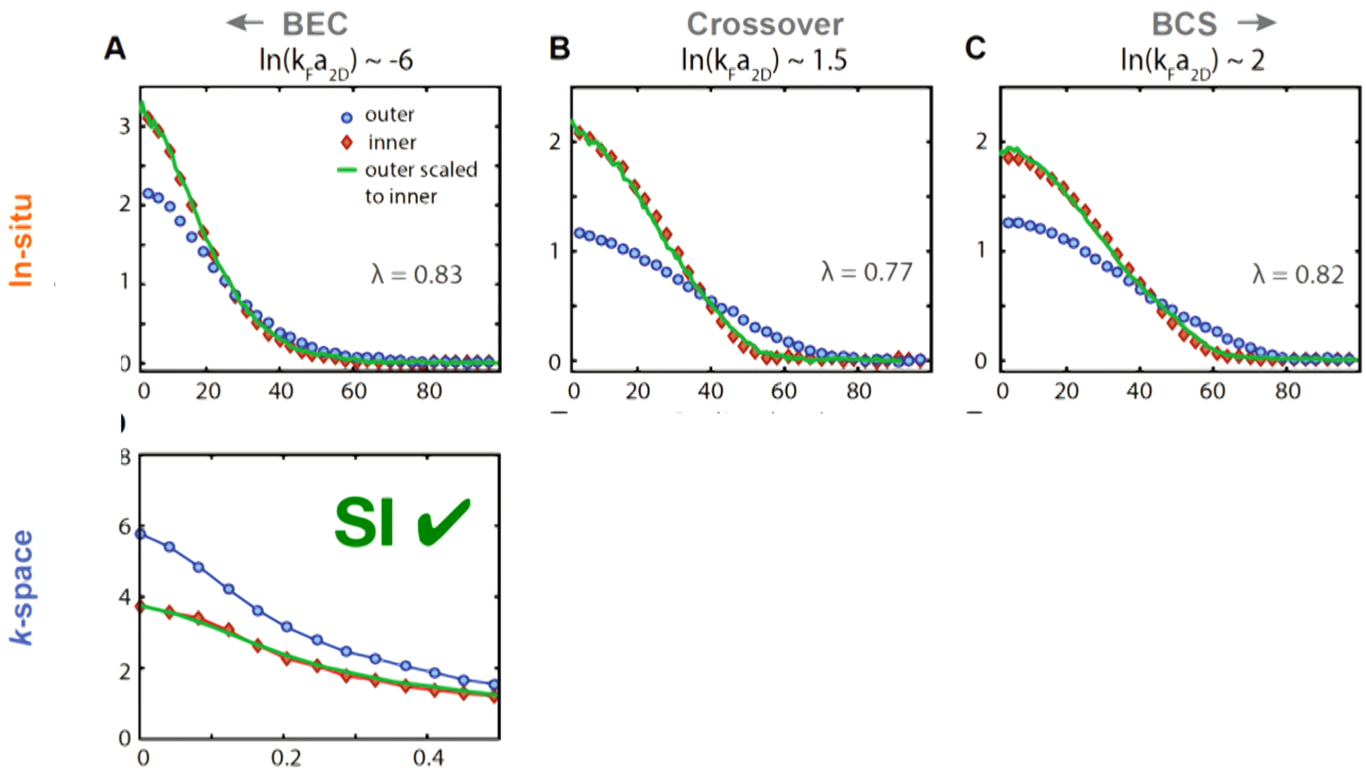

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In-situ and momentum space

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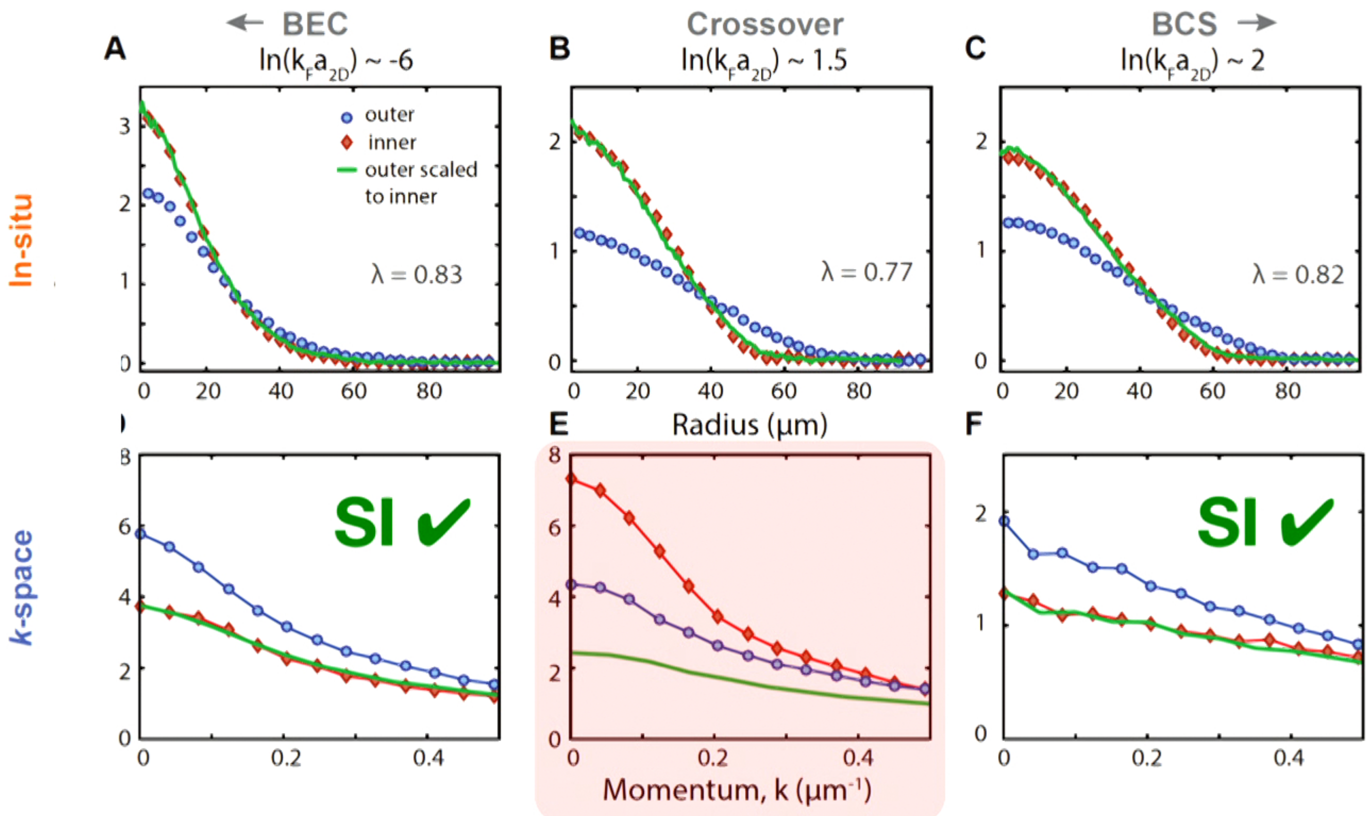


P. A. Murthy, N. Defenu *et al.*, arXiv:1805.04734 (2018)

Scale invariance and coherence

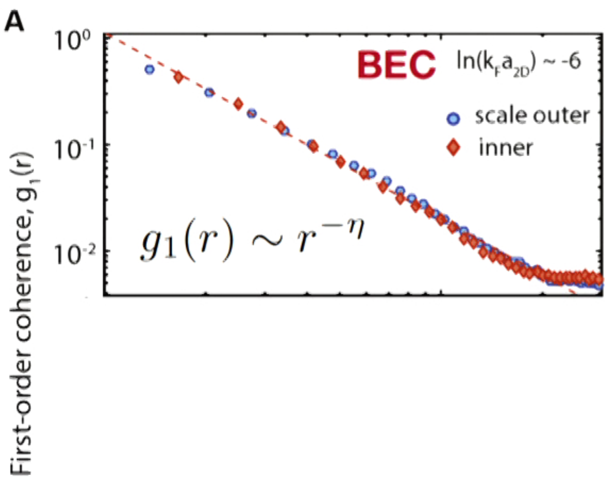
First-order coherence function

$$g_1(\boldsymbol{r}) = \int n(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \mathrm{d}^2 k$$

- Momentum distribution encodes coherence properties
- Power-law decay (BKT type superfluid in 2D)

- Check scale invariance in exponent

A



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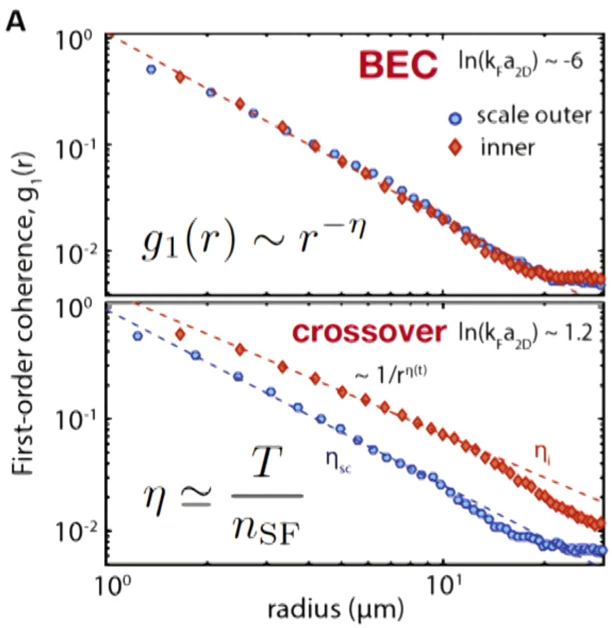

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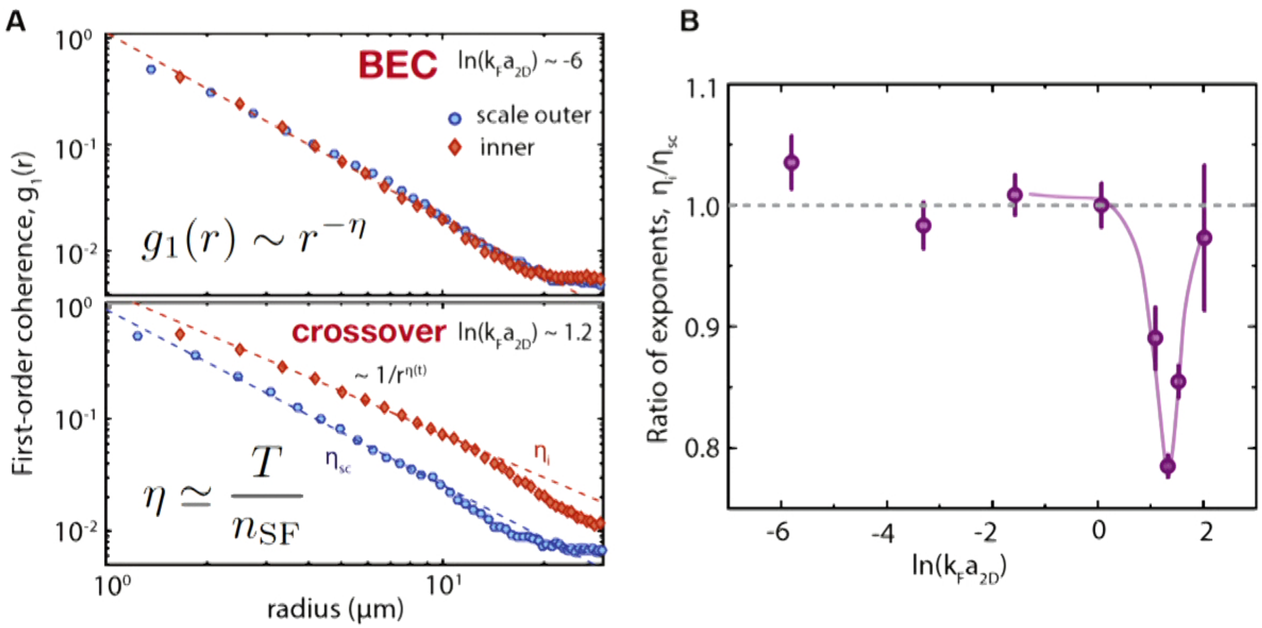
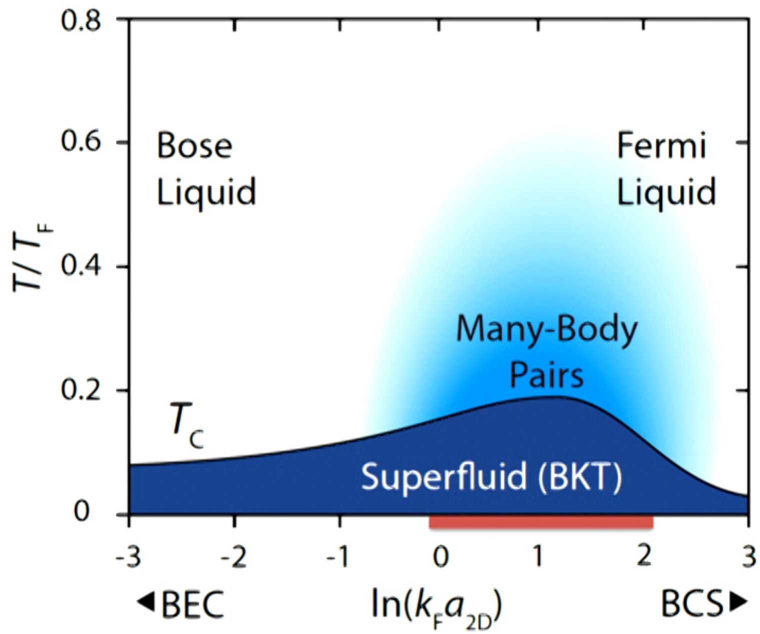


Scale invariance violation

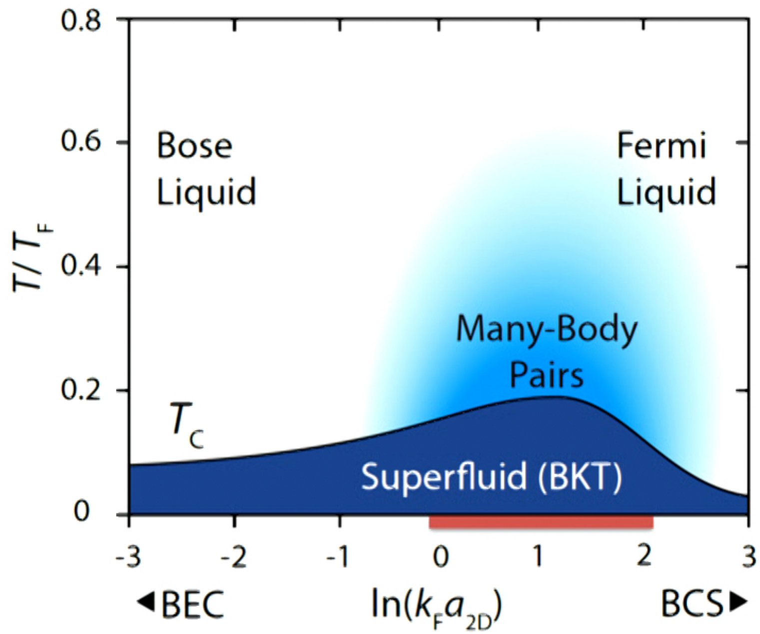
BEC-BCS crossover



Strongly correlated region

Scale invariance violation

BEC-BCS crossover

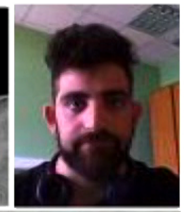


Strongly correlated region

- SI dynamics in BEC and BCS limits
- Frequency shift of breathing mode
- Violation of scale invariance in k-space
- Modification of pairs in normal state (RF spectroscopy)

Acknowledgements

The team



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Thank you for your attention!

Center for Quantum Dynamics

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MARIE CURIE ACTIONS