Title: Relativistic False Vacuum Decay in Analogue Cold Atom Systems

Date: Jun 21, 2018 09:30 AM

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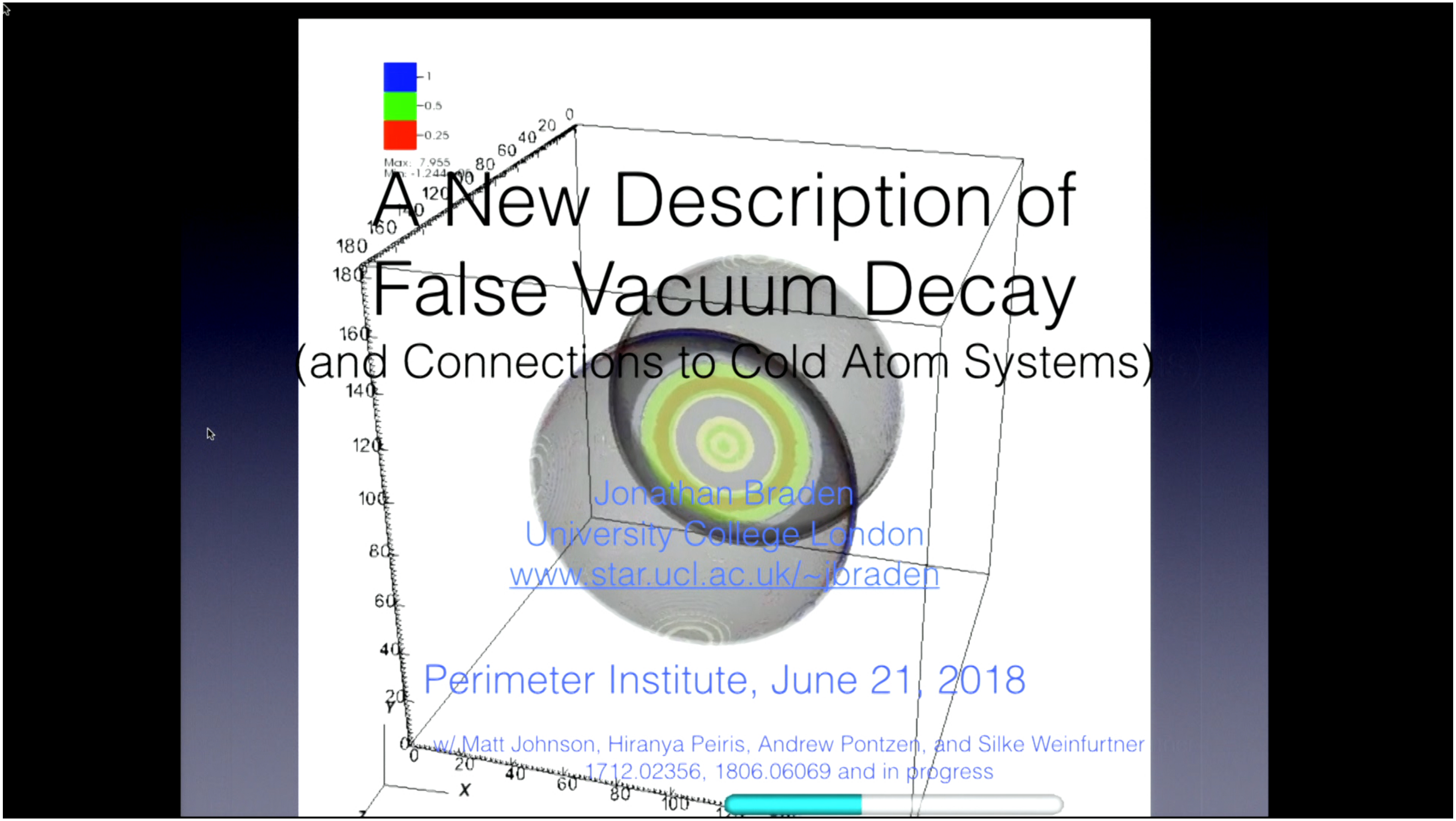
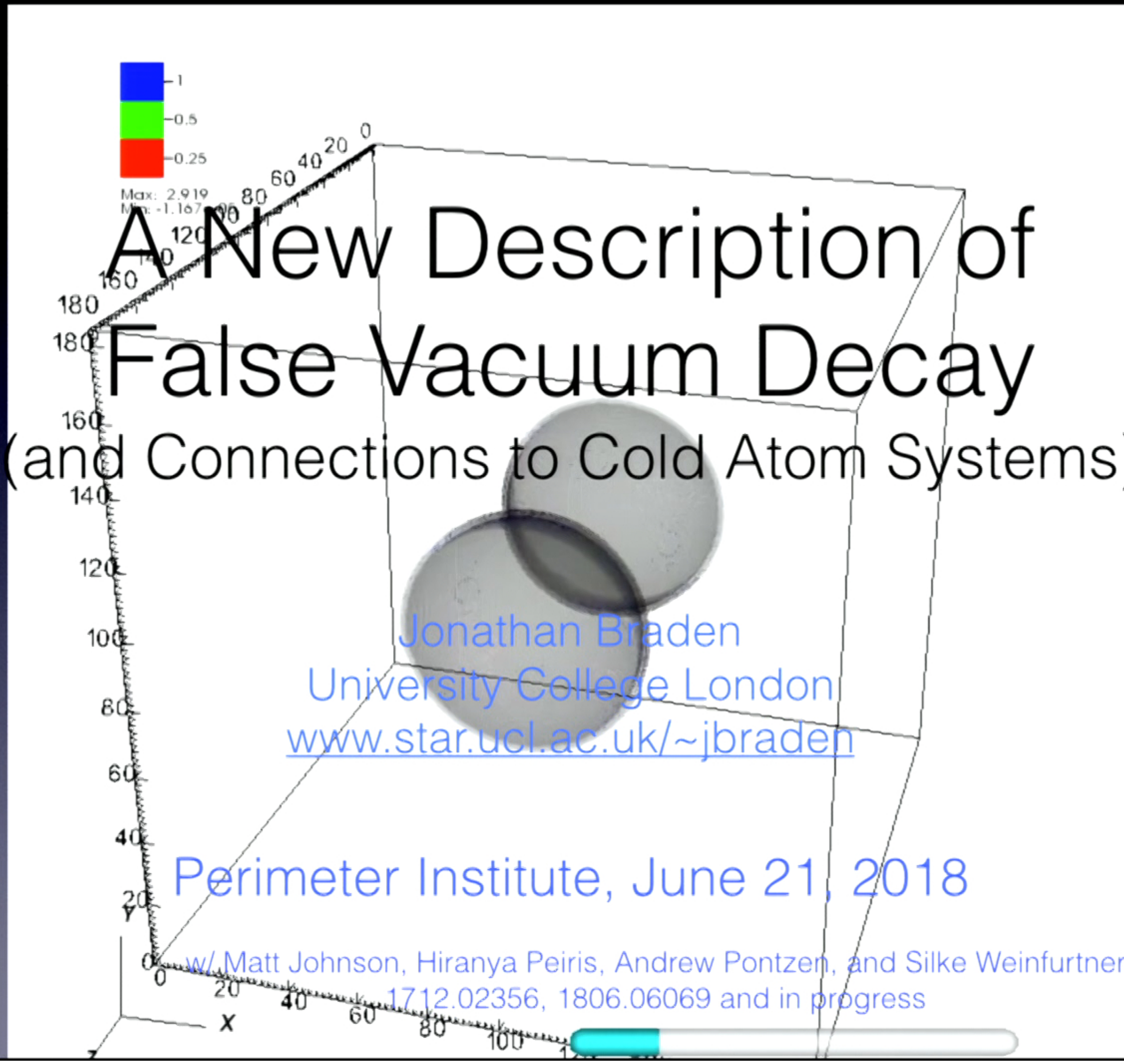
Abstract:


A New Description of
False Vacuum Decay
(and Connections to Cold Atom Systems)
Jonathan Braden
University College London
www.star.ucl.ac.uk/~jbraden
Perimeter Institute, June 21, 2018
w/ Matt Johnson, Hiranya Peiris, Andrew Pontzen, and Silke Weinfurtner
1712.02356, 1806.06069 and in progress



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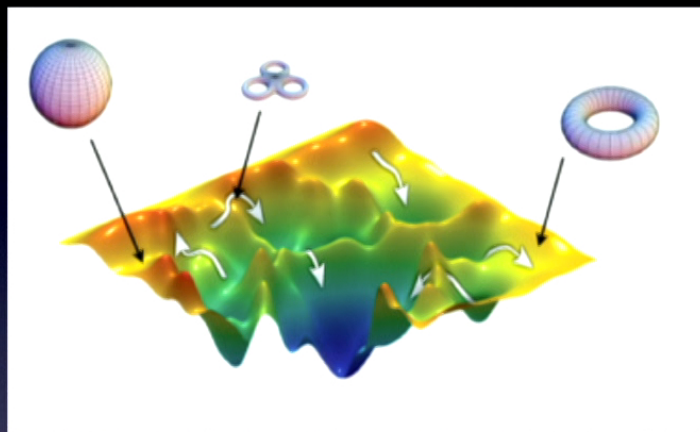
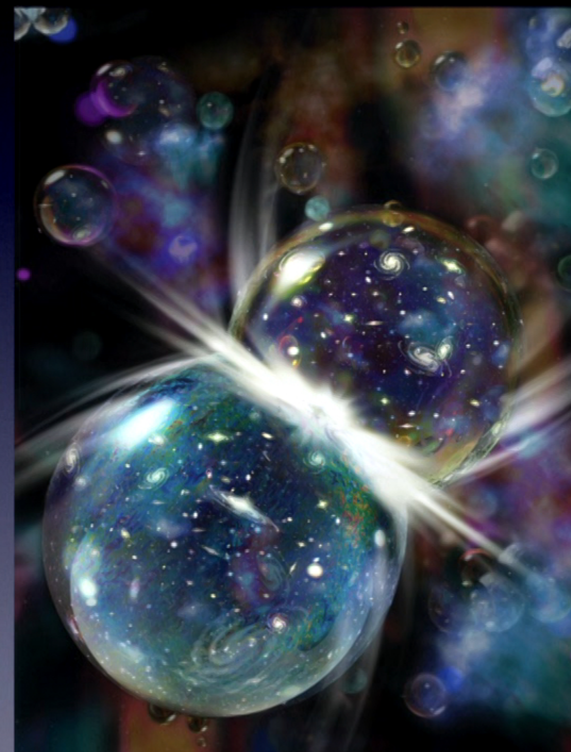
1712.02356 (accepted to JHEP), 1806.06069, and in progress

Overview

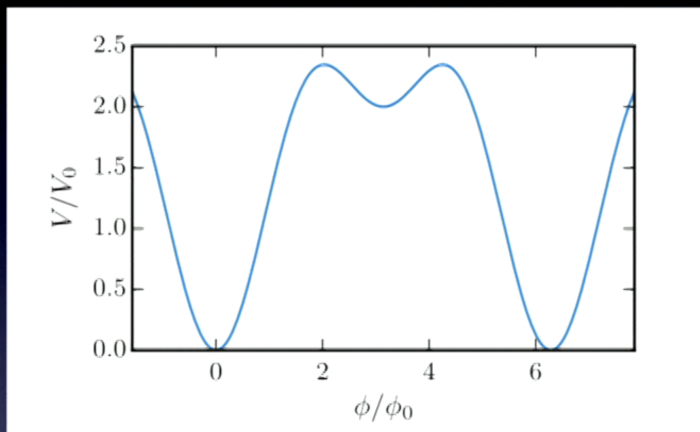
- Review of First-Order Phase Transitions and Standard Formalism (including limitations)
- New Time-Dependent Description of Decay
- Connection to Bose-Einstein Condensates
- Effective Relativistic Dynamics in BECs
- Linear Stability Analysis (Instabilities and How to Tame Them)

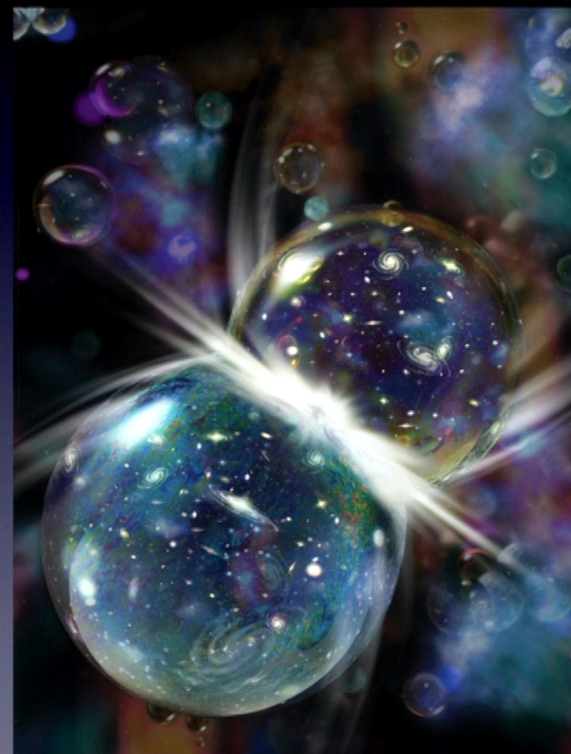
First Order Phase Transitions



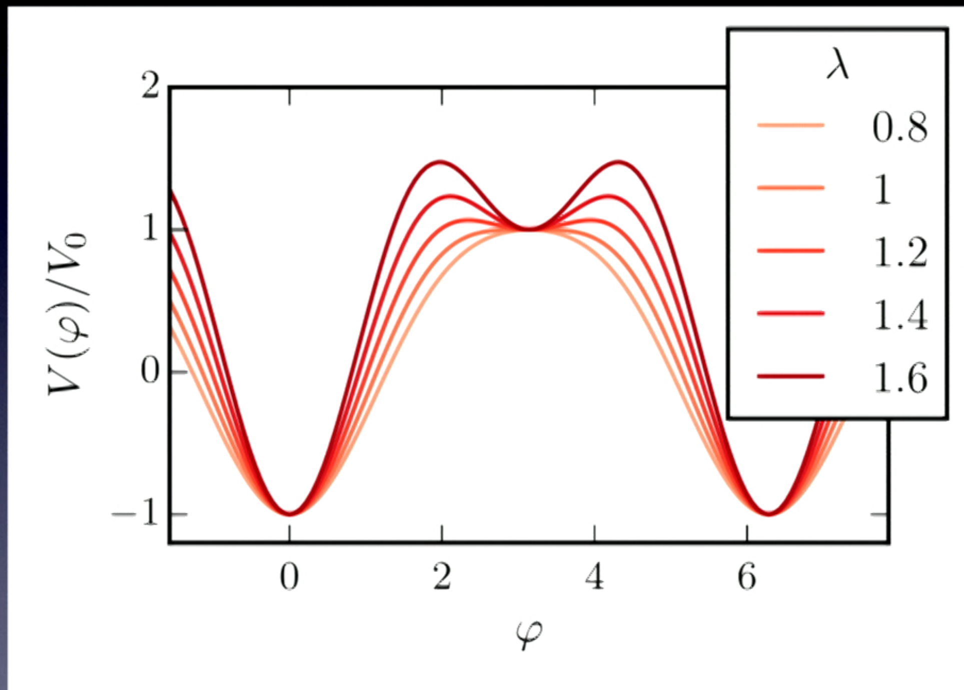


First Order Phase Transitions





Model

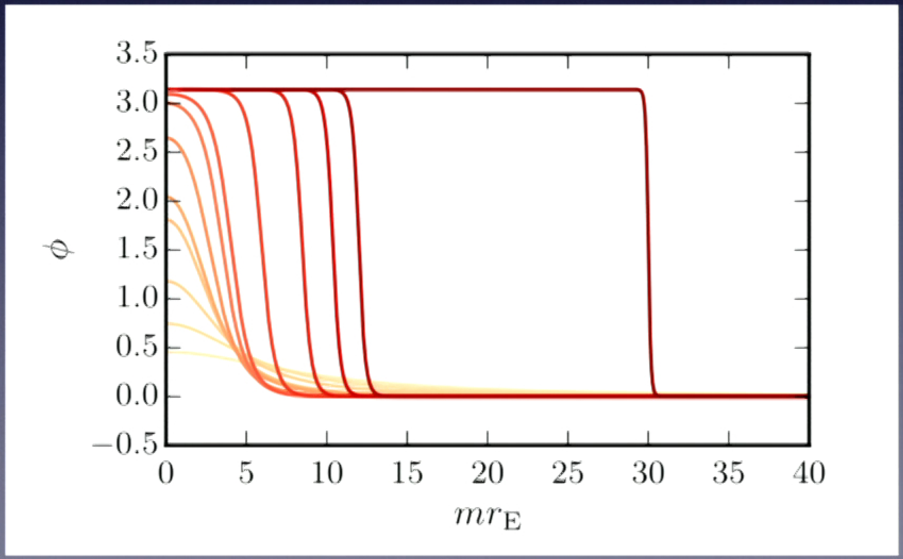


$$V(\phi) = V_0 \left(-\cos\left(\frac{\phi}{\phi_0}\right) + \frac{\lambda^2}{2} \sin^2\left(\frac{\phi}{\phi_0}\right) + 1 \right)$$

Standard Description

$$r_{\mathrm{E}}^2 = \tau^2 + \mathbf{r}^2 \qquad \tau = it$$

$$\frac{\partial^2 \phi_{\mathrm{I}}}{\partial r_{\mathrm{E}}^2} + \frac{d}{r_{\mathrm{E}}}\frac{\partial \phi_{\mathrm{I}}}{\partial r_{\mathrm{E}}} - \frac{\partial V}{\partial \phi} = 0 \qquad \frac{\partial \phi_{\mathrm{I}}}{\partial r_{\mathrm{E}}}(0) = 0 \qquad \phi(\infty) = \phi_{\mathrm{fv}}$$



Decay Rates

$$S_{\mathrm{E}}[\phi] = \Omega_{\mathrm{D}} \int dr_{\mathrm{e}} \left(\frac{(\phi')^2}{2} + V(\phi) \right)$$

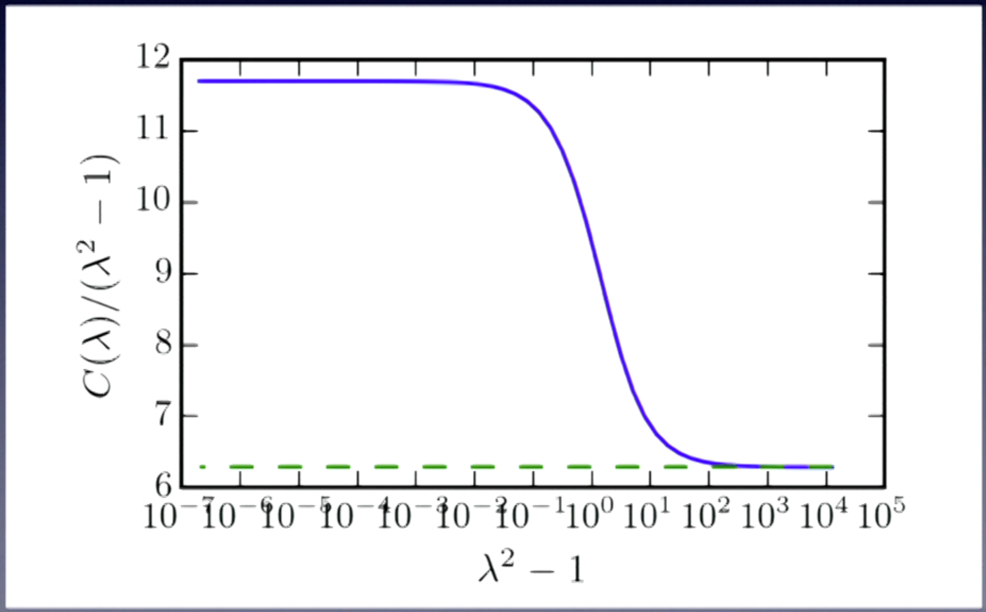
$$S_{\mathrm{I}} = S_{\mathrm{E}}[\phi_{\mathrm{B}}] - S_{\mathrm{E}}[\phi_{\mathrm{fv}}]$$

- Single negative eigenmode

$$\frac{\Gamma}{V} = \left(\frac{S_{\mathrm{I}}}{2\pi} \right)^{D/2} \sqrt{\frac{\det \delta^2 S_{\mathrm{E}}[\phi_{\mathrm{fv}}]}{\det' \delta^2 S_{\mathrm{E}}[\phi_{\mathrm{B}}]}} e^{-S_{\mathrm{I}}} \left(1 + \mathcal{O}(\hbar)\right)$$

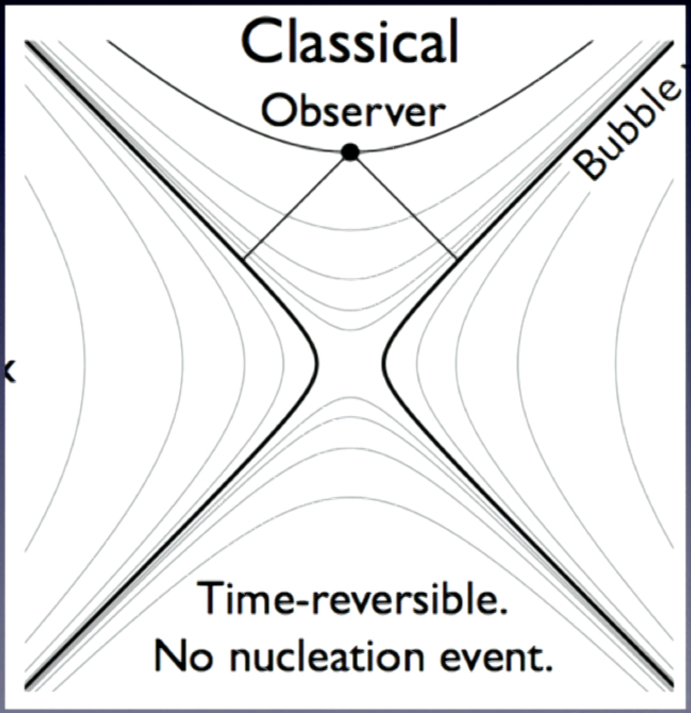
Nucleation Rates

$$\frac{\Gamma}{V} \approx g(\lambda)\left[m_{\text{eff}}^2\right]^{\frac{D}{2}}\left(\frac{S_{\text{I}}}{2\pi}\right)^{\frac{D}{2}} e^{-S_{\text{I}}}$$



Real-Time View

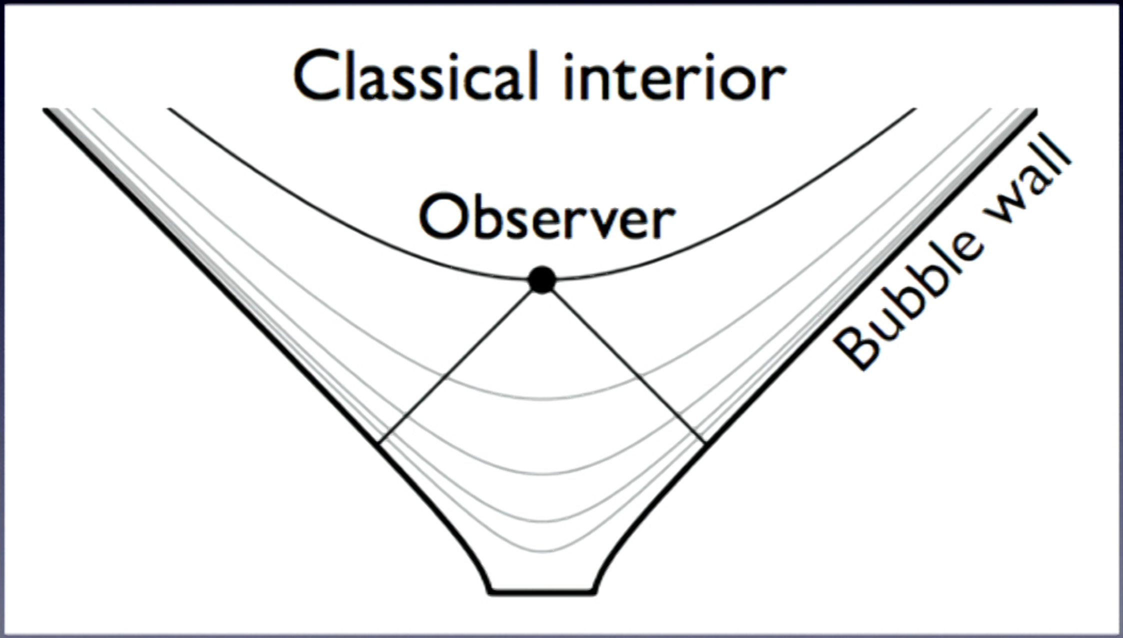
$$\phi(x,t) = \phi_{\mathrm{I}}(\sqrt{x^2 - t^2})$$



[Figure courtesy of Andrew Pontzen]

Ad-Hoc Nucleation

$$\phi(\mathbf{x}, t = 0) = \phi_{\mathrm{I}}(|\mathbf{x}|)$$

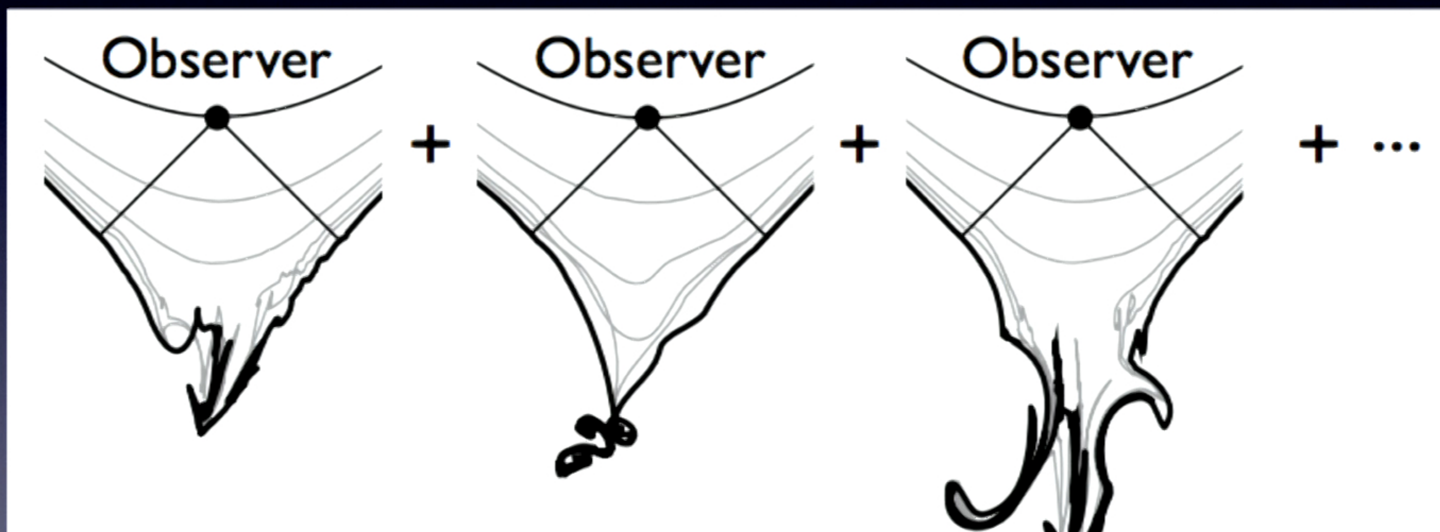


[Figure courtesy of Andrew Pontzen]

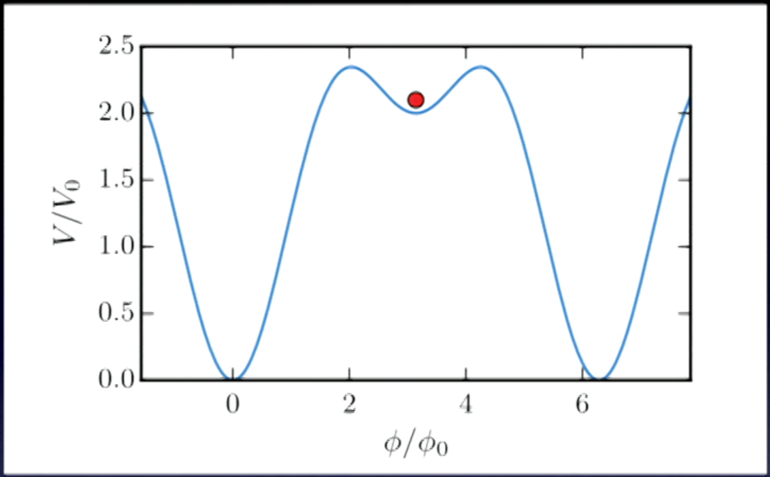
Some Questions

- QFT exponentially complex. Full sims infeasible
- Large fluctuation limit?
- Time-dependent description of nucleation (i.e. how do bubbles appear, impurities)
- Is there a bubble precursor?
- Correlations between bubbles?

Full Evolution?

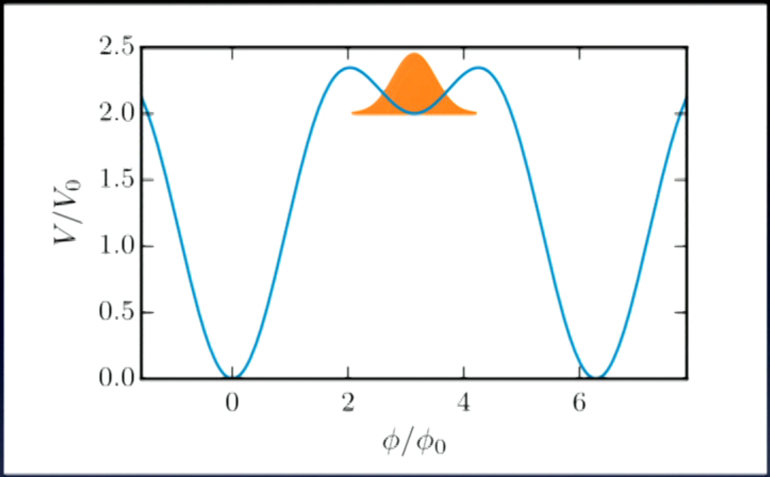


[Figure courtesy of Andrew Pontzen]



$$\phi = \phi_{\mathrm{fv}}$$

$$\Pi = 0$$



$$\phi = \phi_{\rm fv} + \delta\hat{\phi}(\mathbf{x}, t)$$

$$\Pi = 0 + \delta\hat{\Pi}(\mathbf{x}, t)$$

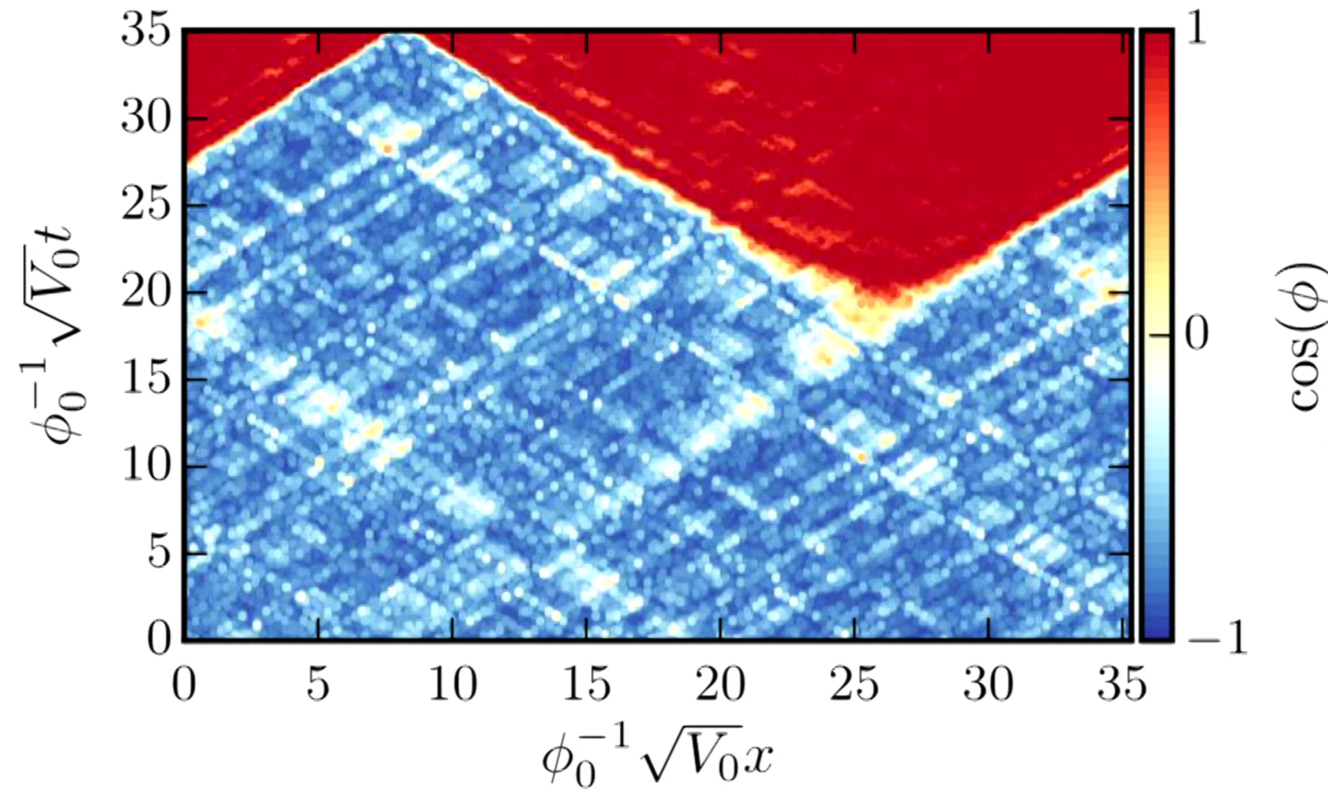
$$\langle \delta\tilde{\phi}_k \delta\tilde{\phi}^*_p \rangle = \frac{1}{2\omega_k}\delta(k-p) \qquad \langle \delta\tilde{\Pi}_k \delta\tilde{\Pi}^*_p \rangle = \frac{\omega_k}{2}\delta(k-p)$$

$$\hat{\delta\phi}_k = \frac{\hat{\alpha}_k}{\sqrt{2\omega}} \frac{1}{\sqrt{L}}$$

$$\langle |\hat{\alpha}_k^* \alpha_k|^2 \rangle = 1$$

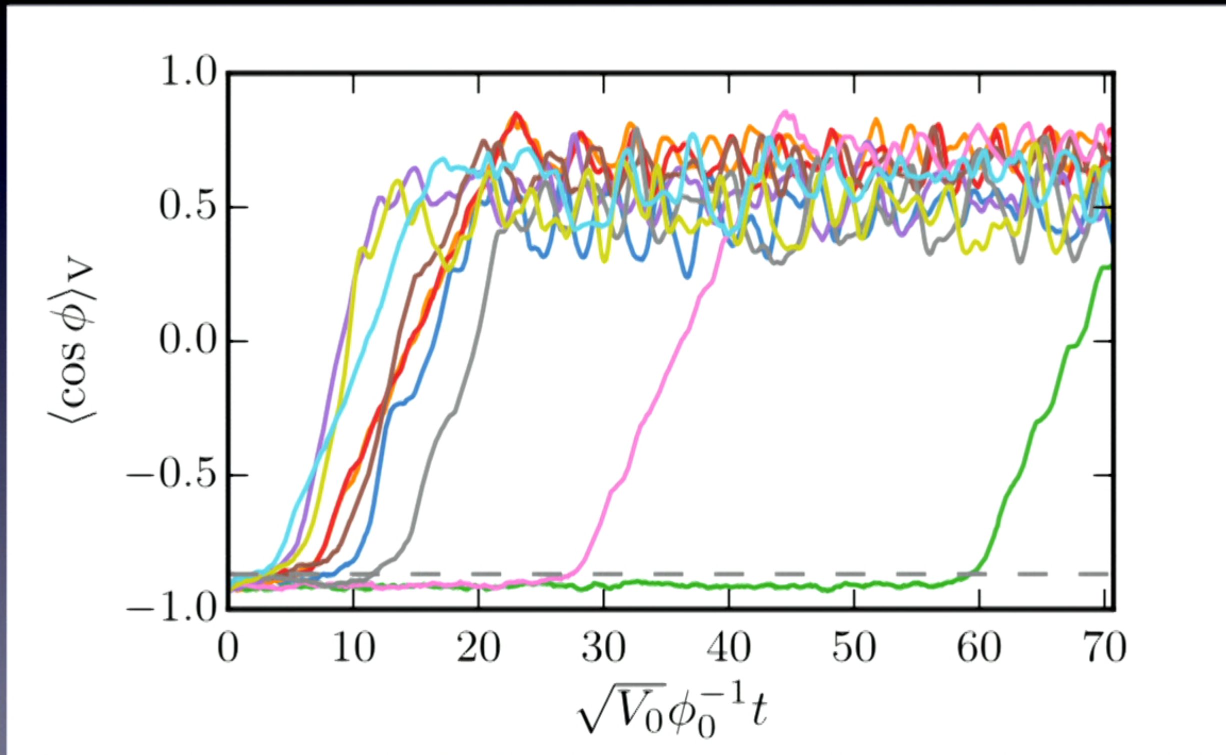
$\hat{\alpha}_k$ is a complex GRD

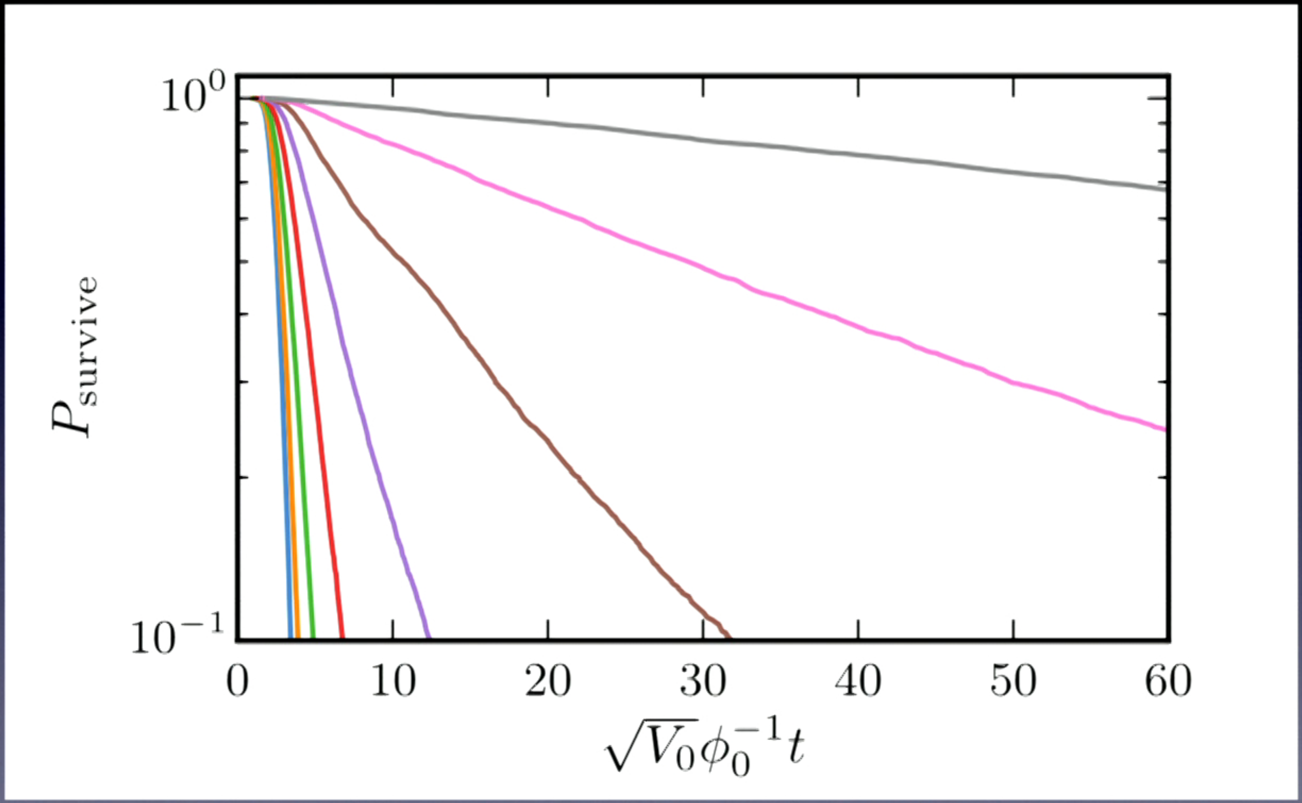
$$\ddot{\phi} - \nabla^2\phi + V'(\phi) = 0$$



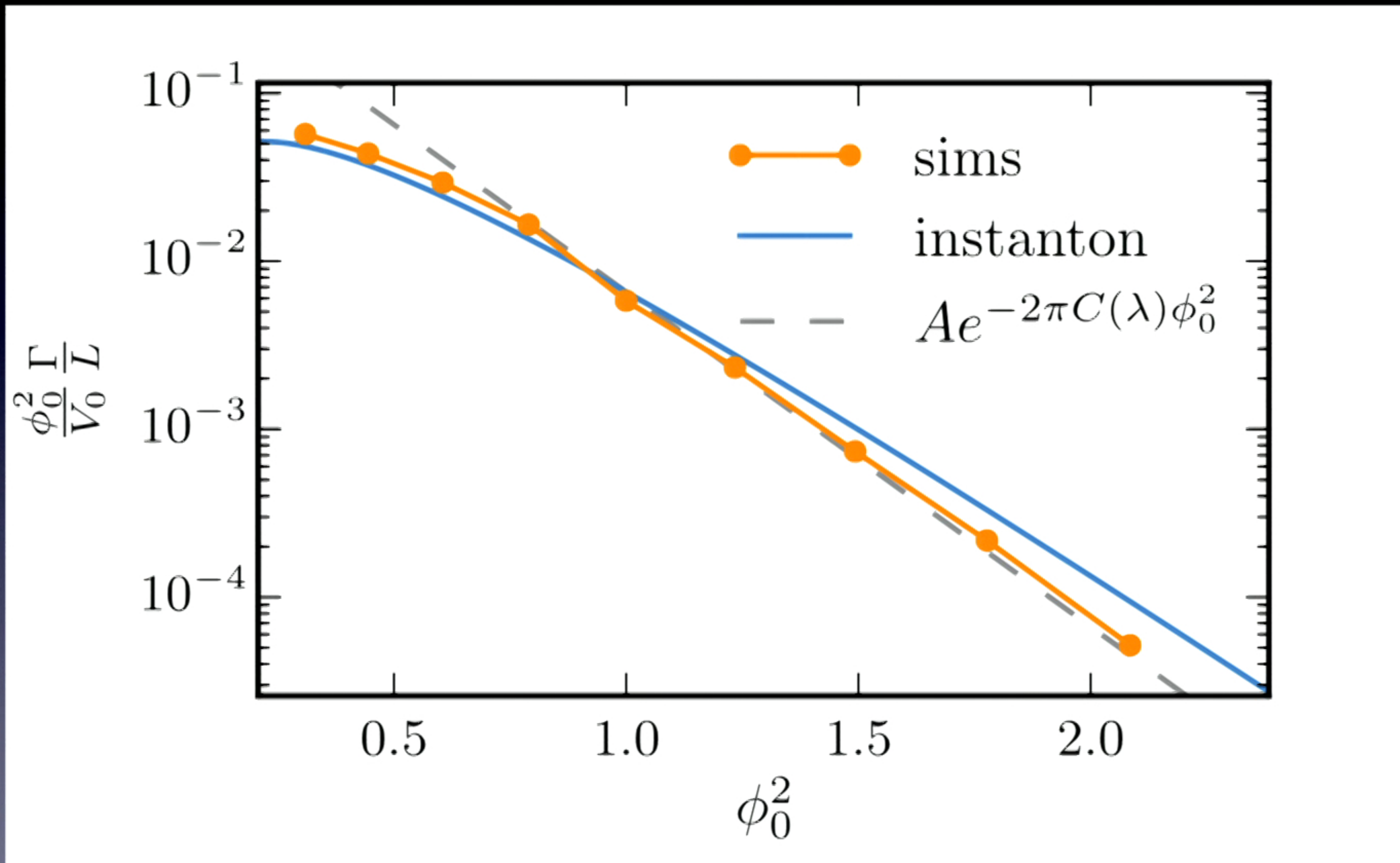
Classically-Allowed Decay

Defining Decays





$$P_{\text{survive}} \sim e^{-\Gamma(t-t_0)}$$



[JB, Johnson, Peiris, Pontzen, Weinfurtner, 1806.06069]

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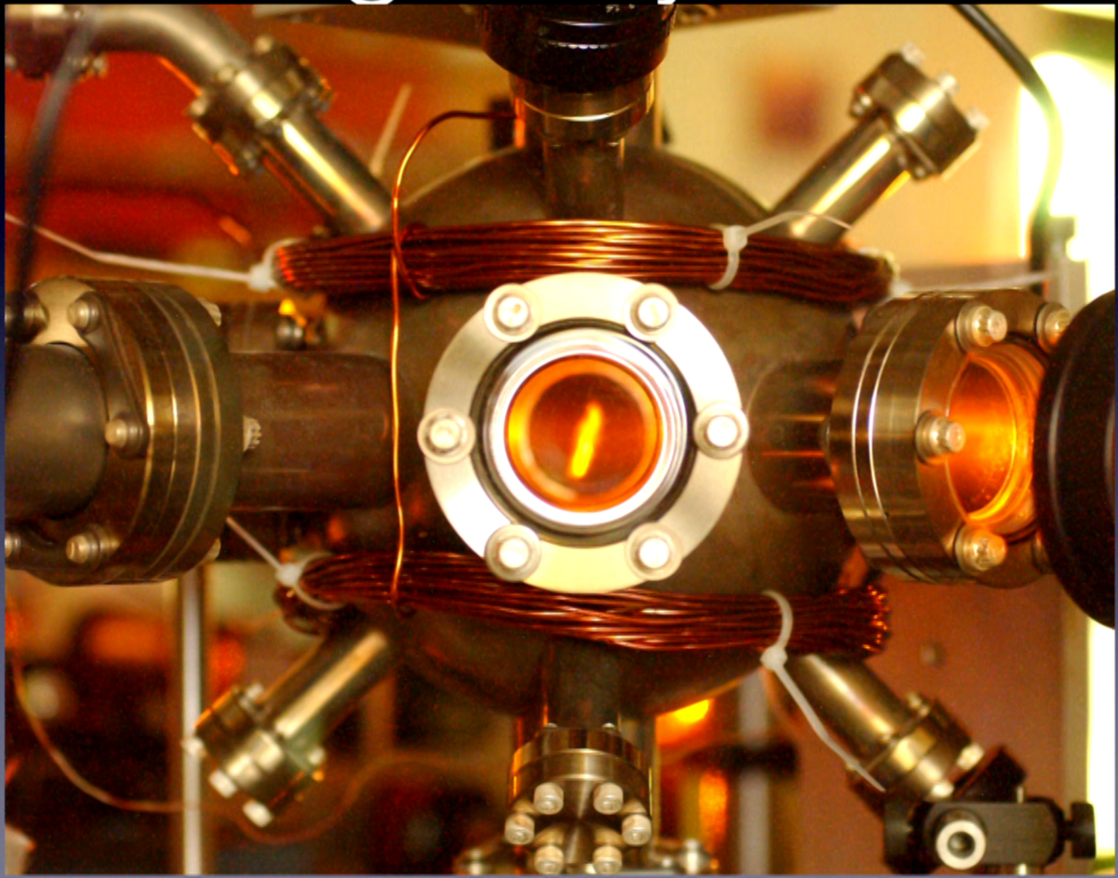
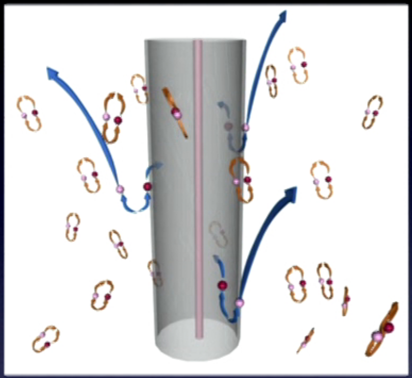
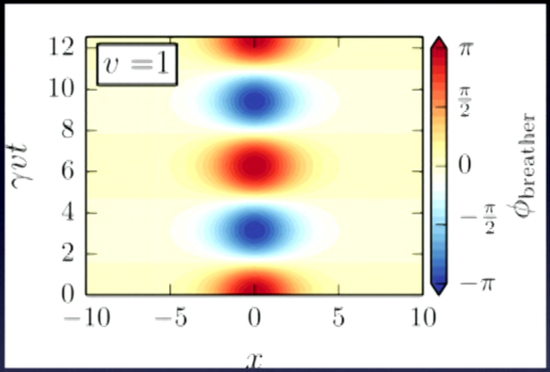
Analogue Systems?



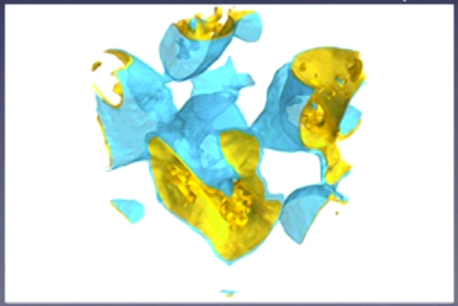
Analogue Systems



Hawking Radiation
(Linear, Quantum)

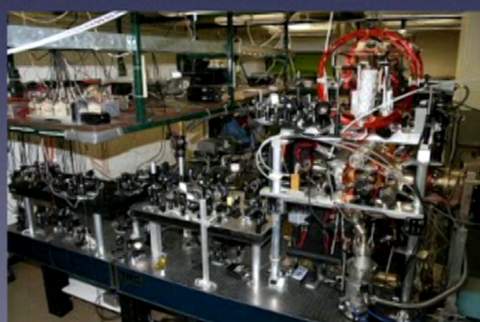


Solitons
(Nonlinear, Classical)

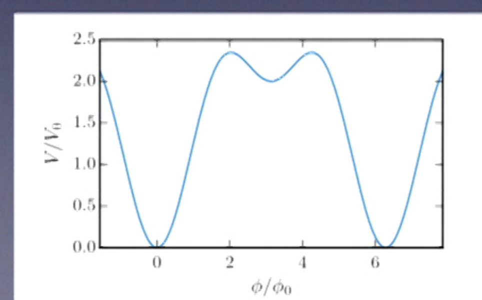


Bubble Nucleation
(Nonlinear, Quantum)

$$\mathcal{L} = \rho_i \dot{\phi}_i - \frac{\hbar^2}{2m_i}|\nabla\psi_i|^2 - \frac{g_{ij}}{2}|\psi_i|^2|\psi_j|^2 + \frac{\nu_{ij}}{2}\left(\psi_i\psi_j^* + \psi_j\psi_i^*\right)$$

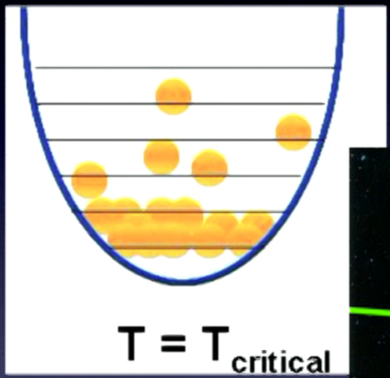


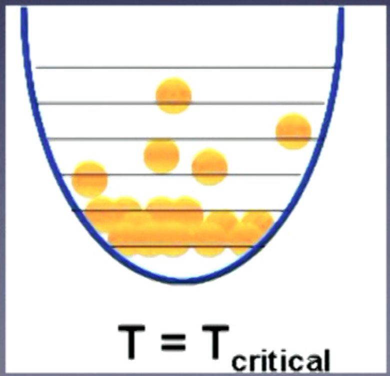
$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

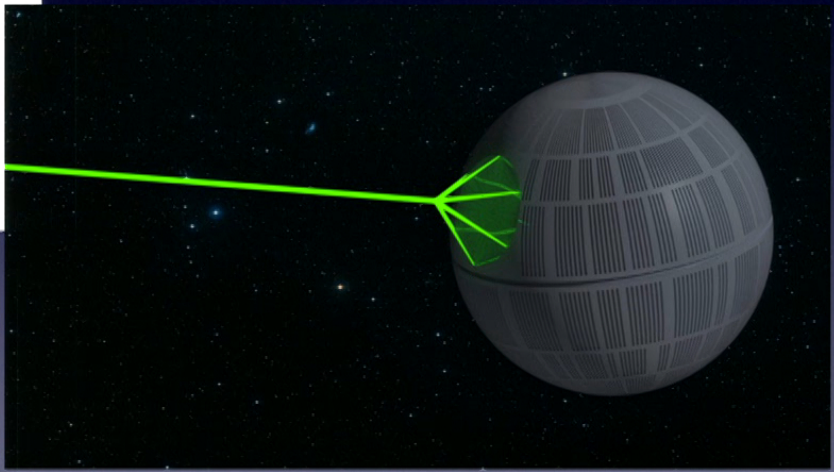


Coupled Condensates

[Fialko et al]







Modelling BEC Dynamics

$$\hat{\Psi}_i = \psi_i + \Delta\hat{\psi}_i$$

$$i\hbar\dot{\psi}_i = \left(-\delta_{ij}\frac{\hbar^2}{2m_i}\nabla^2 + V(\mathbf{x}) + g_{ij}|\psi_j|^2\right)\psi_i - \nu_{ij}\psi_j$$

KE PE S-wave Propagator

$$\mathcal{H} = \frac{\hbar^2}{2m_i}|\nabla\psi_i|^2 + V(x)|\psi_i|^2 + \frac{g_{ij}}{2}|\psi_i|^2|\psi_j|^2 + \frac{\nu_{ij}}{2}\left(\psi_i\psi_j^* + \psi_j\psi_i^*\right)$$

BECs and Relativity

[JB,Johnson, Peiris, Weinfurtner, 1712.02356]

$$\psi_i = \sqrt{\rho_i} e^{-i\phi_i}$$

Can. Momentum Can. Position

$$\mathcal{H} = \frac{\hbar^2}{8m_i}\rho_i(\nabla \ln \rho_i)^2 + \frac{\hbar^2}{2m_i}\rho_i(\nabla\phi_i)^2 + \frac{g_{ij}}{2}\rho_i\rho_j - \nu_{ij}\sqrt{\rho_i\rho_j}\cos(\phi_j - \phi_i)$$

Assumptions

$$\rho_i(x,t) = n_i + \delta\rho_i(x,t)$$

1) Homogeneous 2) Small

Modelling BEC Dynamics

$$i\hbar\dot{\psi}_1 = \left(-\frac{\hbar^2}{2m_1}\nabla^2 + g_{11}|\psi_1|^2 + g_c|\psi_2|^2\right)\psi_1 - \nu\psi_2$$

$$i\hbar\dot{\psi}_2 = \left(-\frac{\hbar^2}{2m_2}\nabla^2 + g_{22}|\psi_2|^2 + g_c|\psi_1|^2\right)\psi_2 - \nu\psi_1$$

$$g_{11} = g + \frac{\delta g}{2} \qquad g_{22} = g - \frac{\delta g}{2}$$

Simplified Case

$$m_1 = m_1 = m \qquad \delta g = 0$$

Small Density Fluctuations

$$Z = \int d\psi_i d\psi_i^* e^{i\int \mathcal{L}}$$

$$\alpha = \frac{\phi_1 + \phi_2}{2} \qquad \phi = \phi_2 - \phi_1 \qquad \delta\rho = \delta\rho_1 + \delta\rho_2 \qquad \Delta\rho = \frac{\rho_2 - \rho_1}{2}$$

$$\mathcal{L} = -\frac{\hbar^2 n}{m}(\nabla\alpha)^2 \boxed{- \frac{\hbar^2 n}{4m}(\nabla\phi)^2 + 2\nu n \cos\phi}$$

$$+ (\dot\alpha + J_\delta)\,\delta\rho + \left(\frac{\dot\phi}{2} + J_\Delta\right)\Delta\rho$$

$$+ \frac{1}{2}\begin{pmatrix} \delta\rho & \Delta\rho \end{pmatrix} C_{\delta\rho}^{-1} \begin{pmatrix} \delta\rho & \Delta\rho \end{pmatrix}^T + \mathcal{O}(\delta\rho^3, \Delta\rho^3)$$

Limit of Small Number Fluctuations

A convenient variable is

$$\phi = \phi_2 - \phi_1$$

Integrate out fluctuations in number density

$$Z_{\text{eff}} \propto \int d\phi e^{i\mathcal{L}_{\text{eff}}}$$

$$\mathcal{L}_{\text{eff}} \sim G(\phi)\frac{\dot{\phi}^2}{2} - c_s^2 \frac{(\nabla\phi)^2}{2} + \nu\Lambda\cos\phi + \ldots$$

Limit of Small Number Fluctuation

Sine-Gordon Model

A convenient variable is

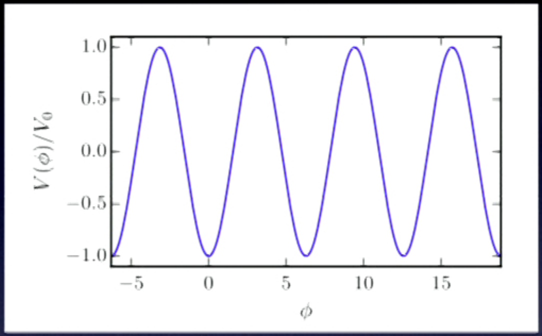
$$\phi = \quad \phi_1$$

Integrate out flu in number density

$$\propto \int d\phi e^{i\mathcal{L}_{\rm eff}}$$

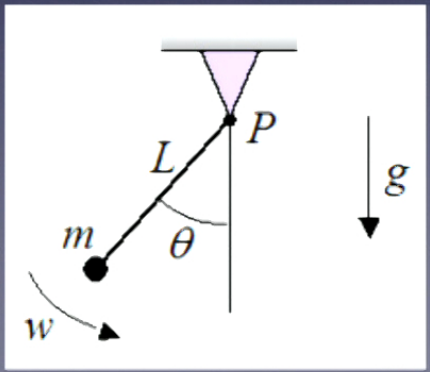
$$\mathcal{L}_{\rm eff} \quad G(\phi)\frac{\dot{\phi}^2}{2} - c_s^2\frac{(\nabla\phi)^2}{2} + \nu\Lambda\cos\phi + \ldots$$

SG and the Pendulum



Where's My False Vacuum!

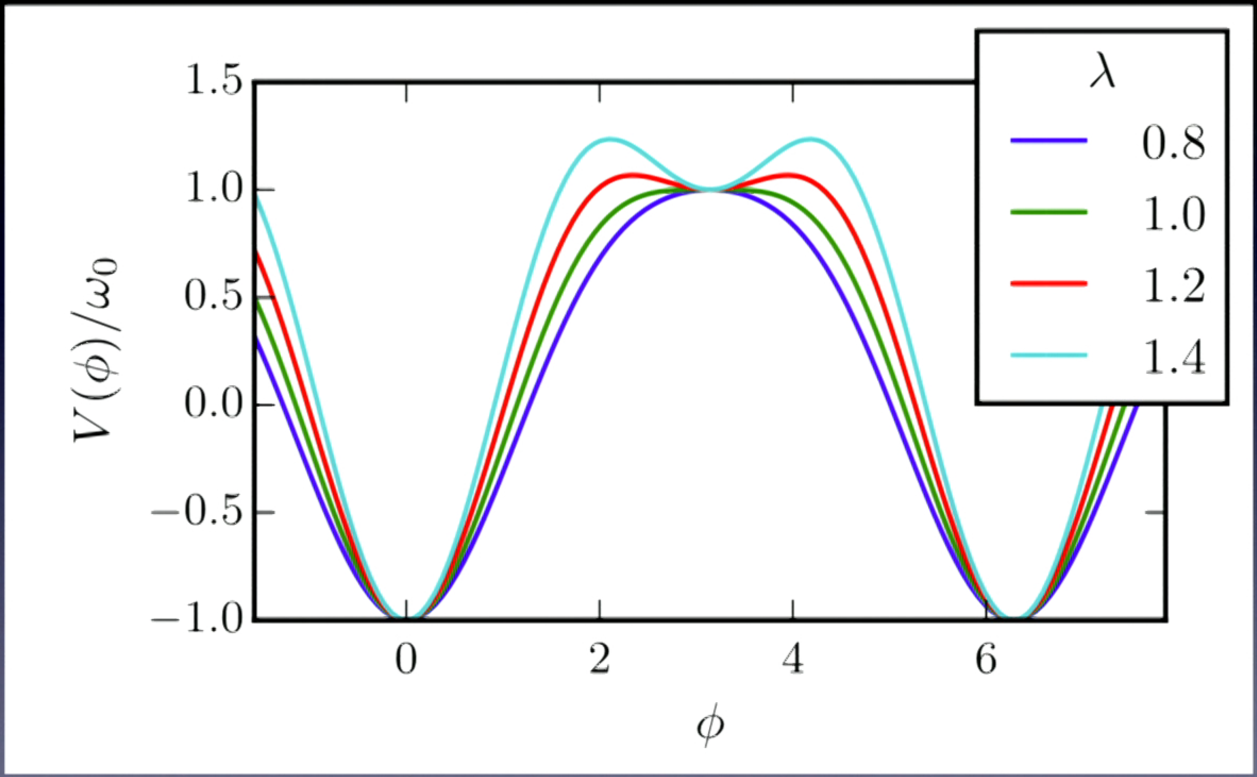
Homogeneous Limit:
Rigid Pendulum



Modulate Transition Rate

$$\nu = \nu_0 + \delta\hbar\omega\cos(\omega t)$$

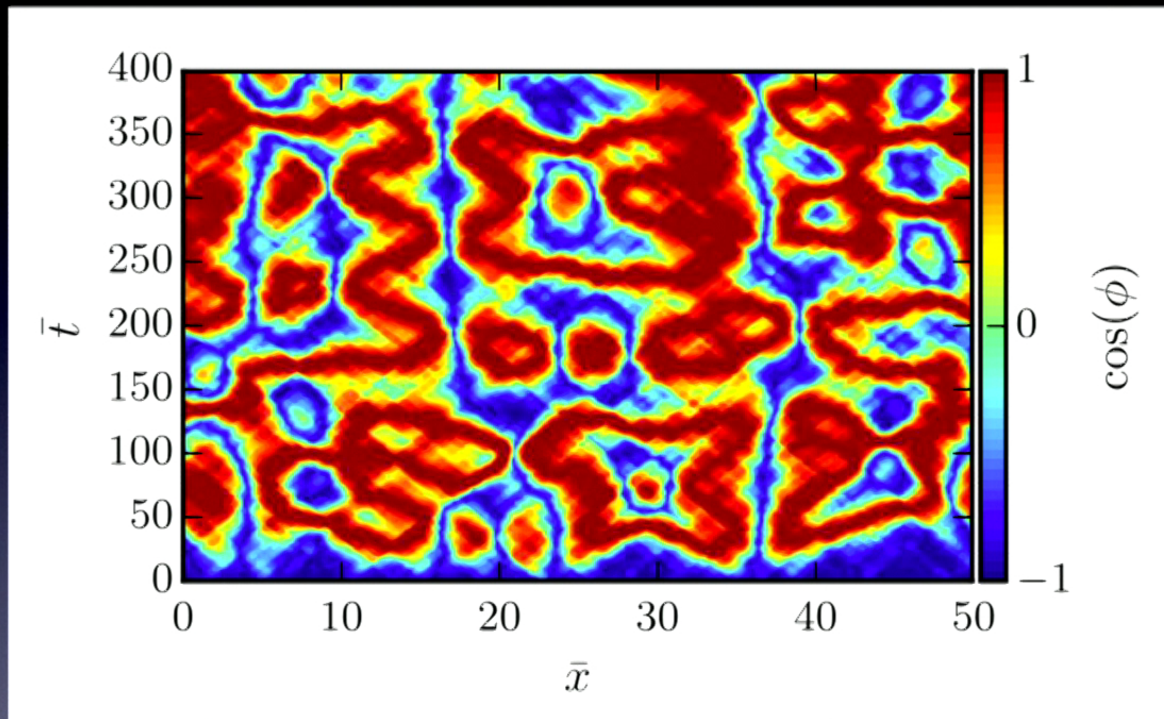
Time Averaged Potential



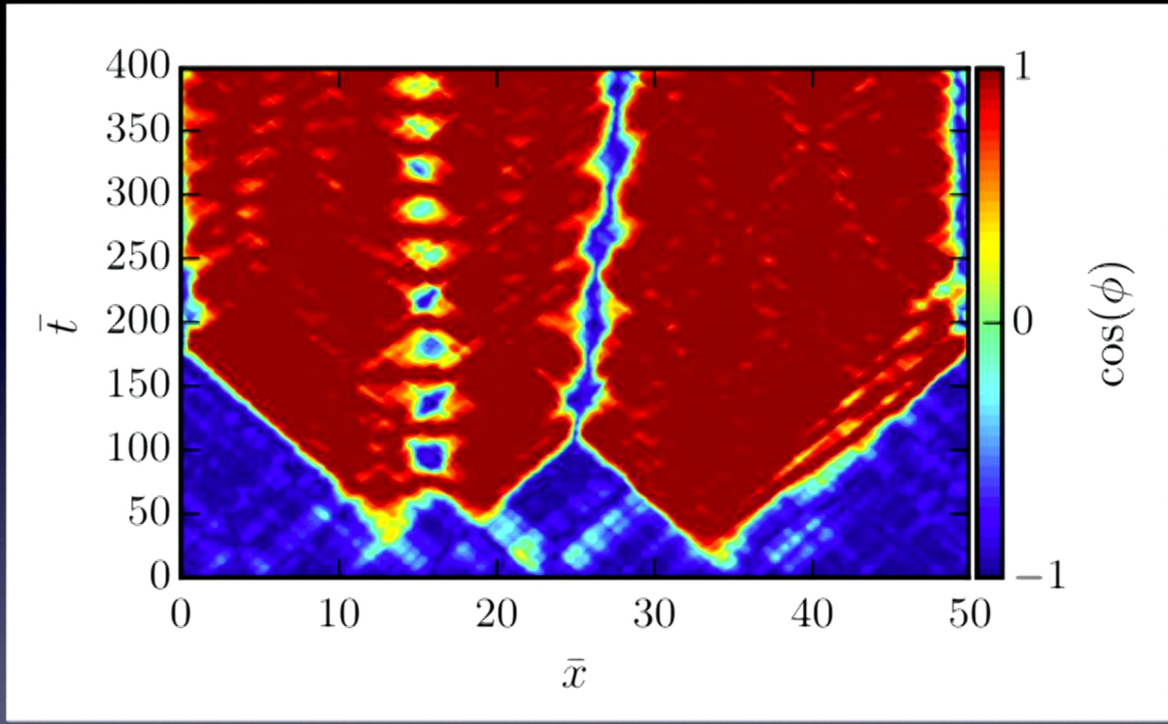
$$\nu = \nu_0 + \delta\hbar\omega\cos(\omega t)$$

$$i\hbar\dot{\psi}_i = \left(-\delta_{ij}\frac{\hbar^2}{2m_i}\nabla^2 + g_{ij}|\psi_j|^2\right)\psi_i - \nu_{ij}\psi_j$$

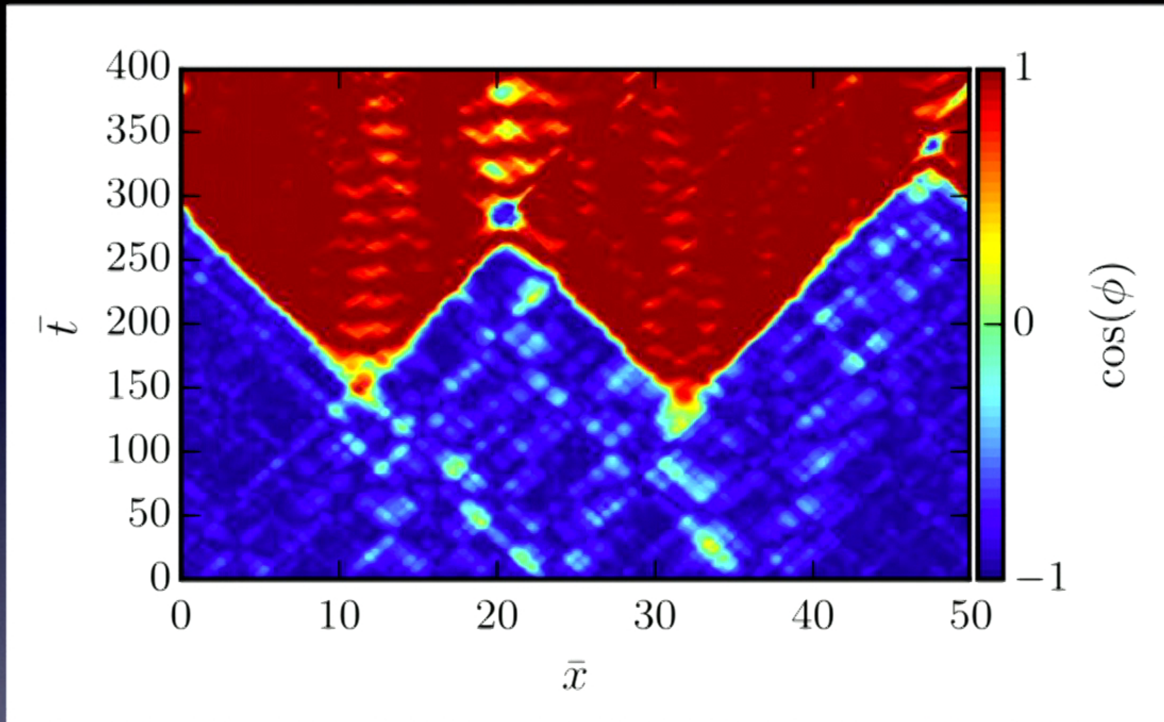
$$\phi = \phi_2 - \phi_1$$



Spinodal Instability

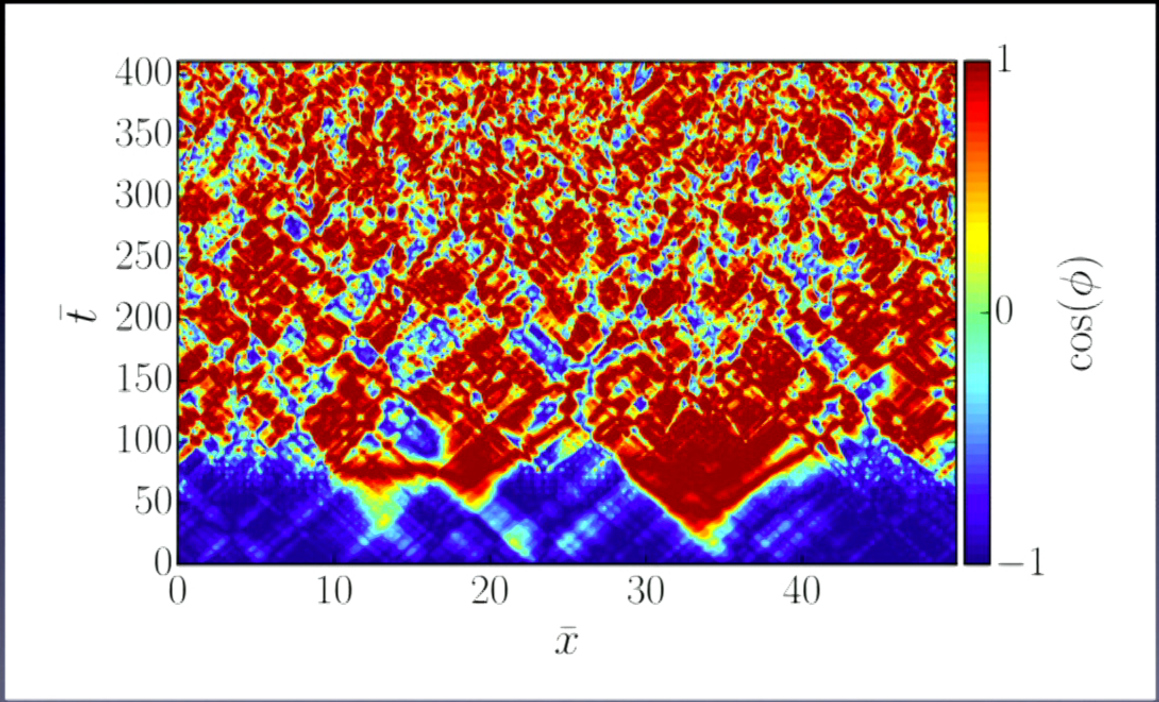


Rapid Nucleation



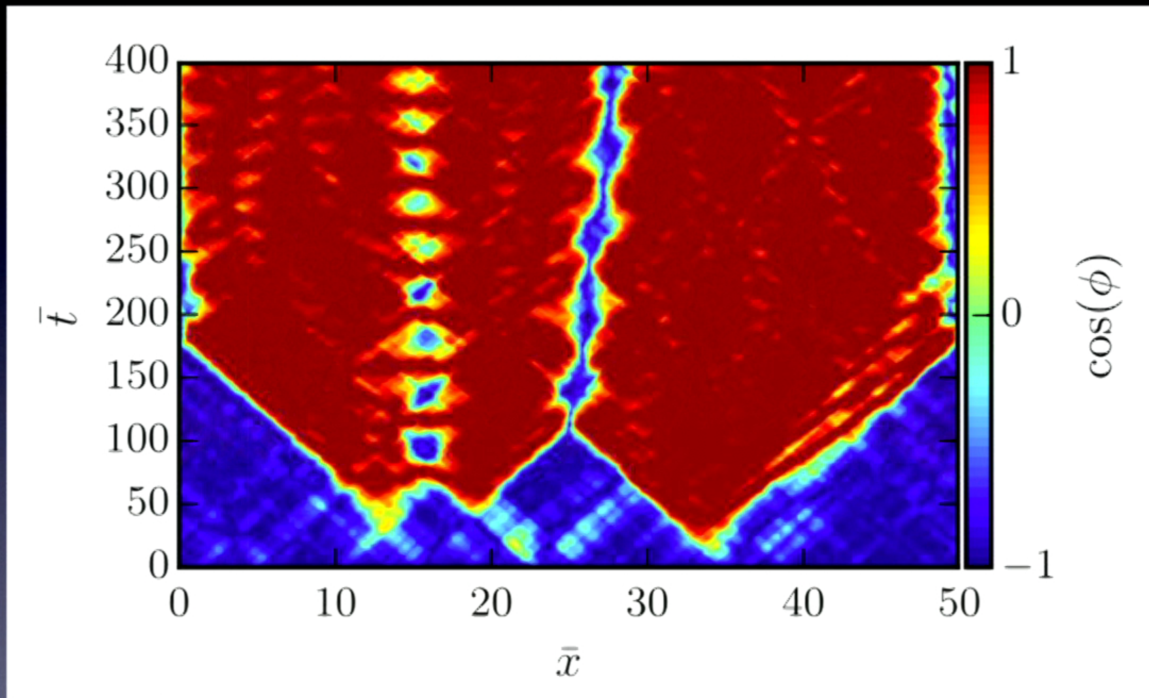
Slower Nucleation

Decreasing $\bar{\omega}$, decreasing dx



Reveals Additional Instabilities

Decreasing $\bar{\omega}$, decreasing dx



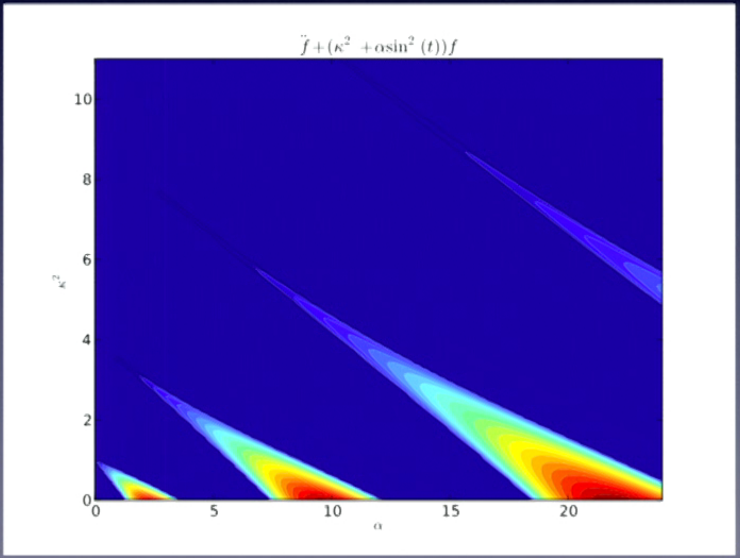
Reveals Additional Instabilities

Floquet Theory of Linear Instabilities

Floquet Theory: Basics

$$\frac{d\mathbf{y}}{dt} = M(t)\mathbf{y} \qquad M(t+T) = M(t)$$

$$\mathbf{y} = \mathbf{P}(t)e^{\mu t}$$



T-indep Dispersion

$\delta g = 0$ $\qquad m_1 = m_1 = m$

Total Phase and Density

$$\hbar^2\omega_k^2 = \frac{\hbar^2 k^2}{2m}\left(\frac{\hbar^2 k^2}{2m} + 2n(g+g_c)\right)$$

Relative Phase and Density

$$\hbar^2\omega_k^2 = \left(\frac{\hbar^2 k^2}{2m} + 2\nu\cos\bar{\phi}\right)\left(\frac{\hbar^2 k^2}{2m} + 2n(g-g_c) + 2\nu\cos\bar{\phi}\right)$$

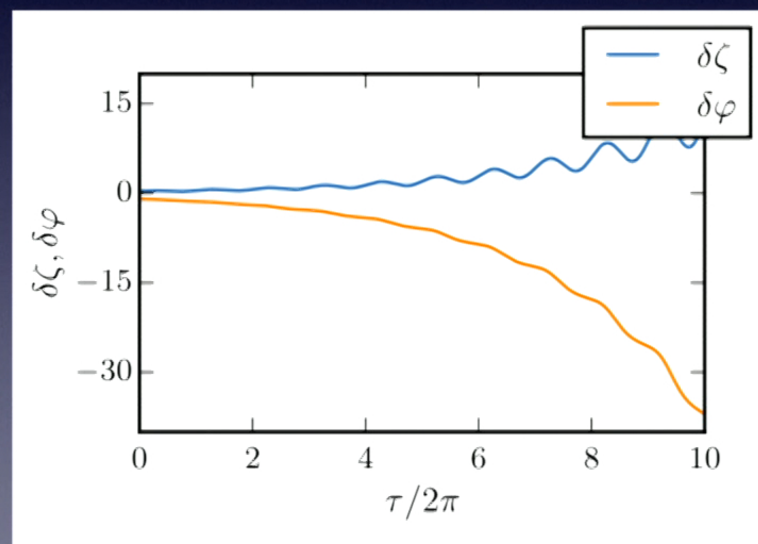
Stable vs Unstable Behaviour

$$\frac{d}{d\tau}\begin{pmatrix} \frac{\delta_2-\delta_1}{2} \\ \delta\phi_2-\delta\phi_1 \end{pmatrix} = \begin{pmatrix} 0 & ab+d\cos\tau \\ -b(a+1)-d\cos\tau & 0 \end{pmatrix}\begin{pmatrix} \frac{\delta_2-\delta_1}{2} \\ \delta\phi_2-\delta\phi_1 \end{pmatrix}$$

$$\mathbf{y} = \mathbf{P}(t)e^{\mu t}$$

$$\delta g = 0$$

$$m_1 = m_1 = m$$



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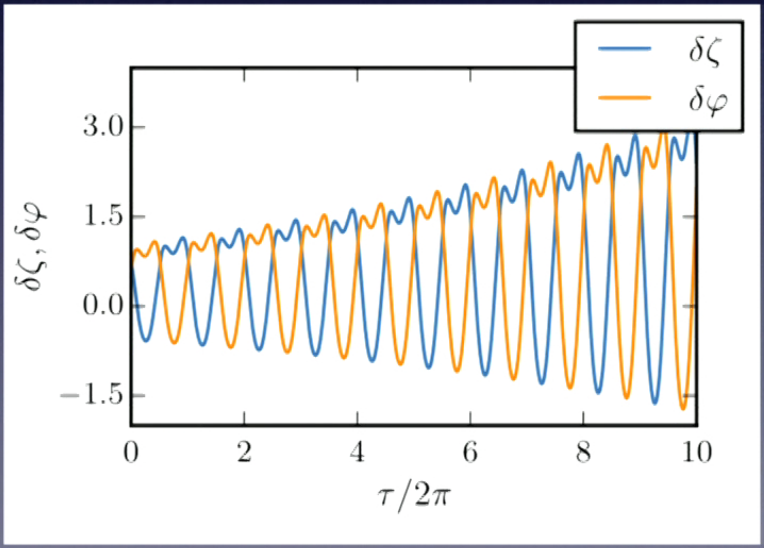

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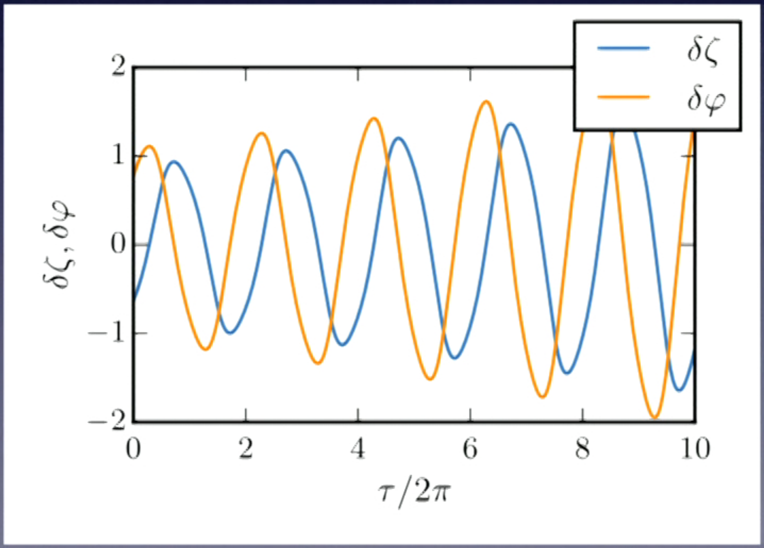

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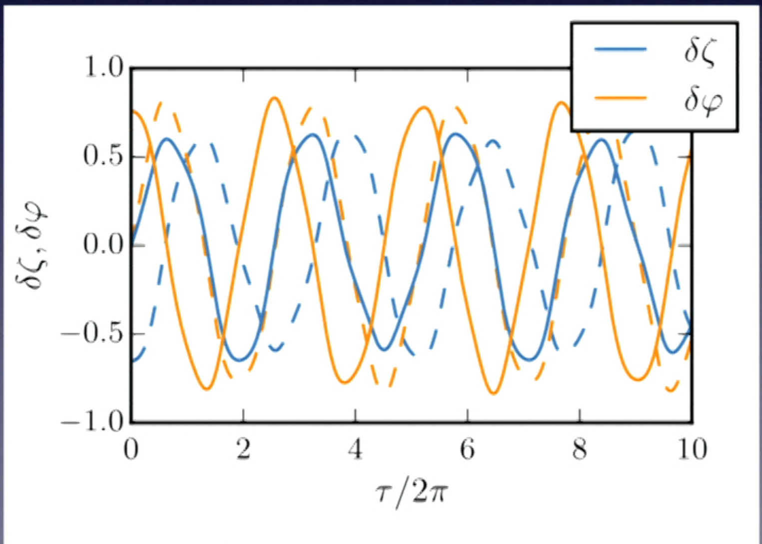
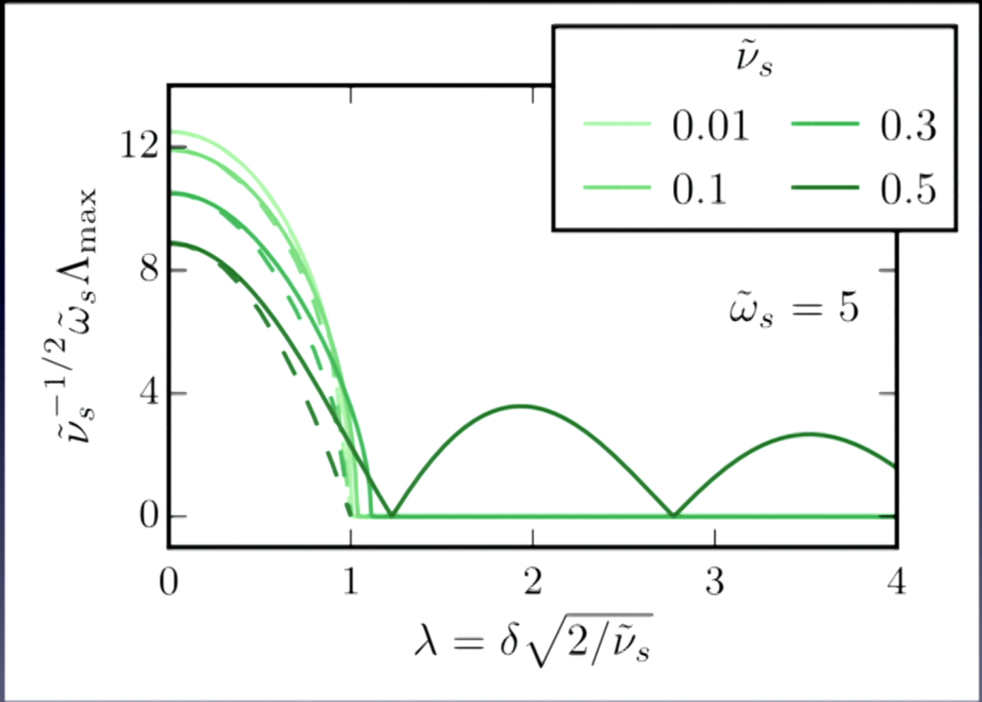
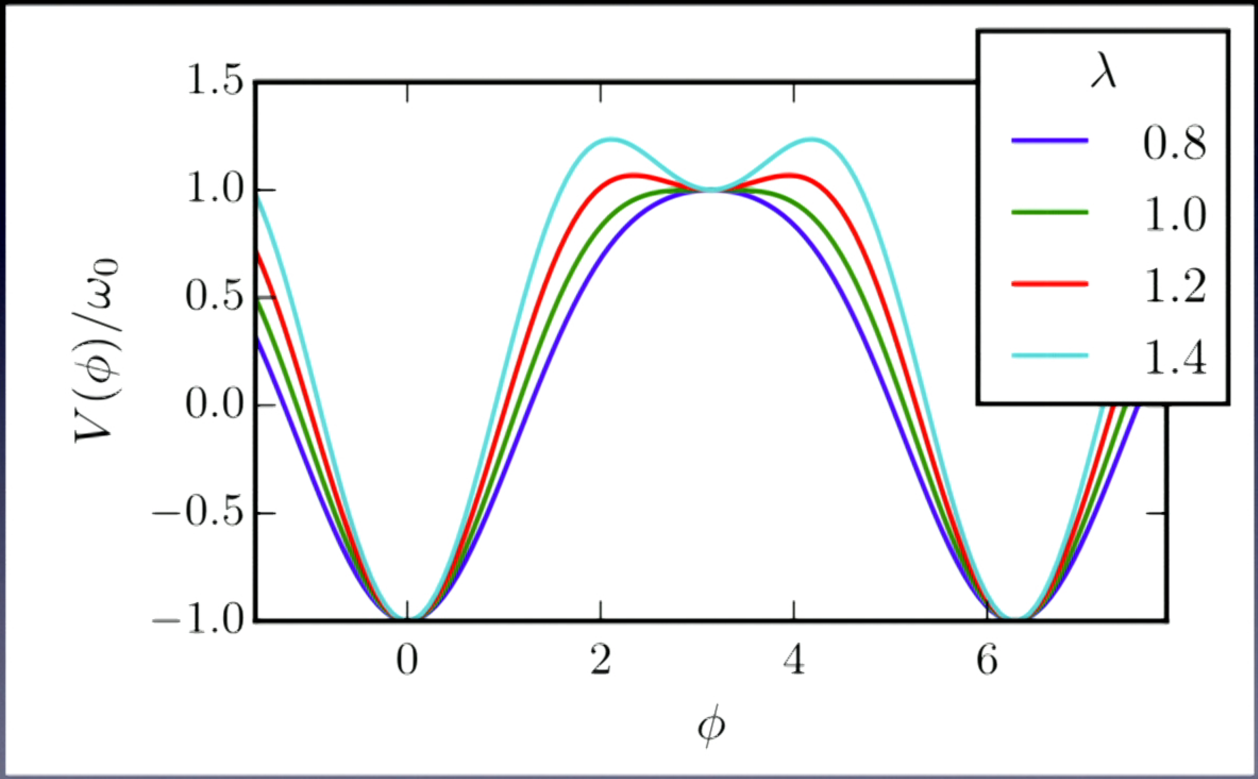


$k = 0?$

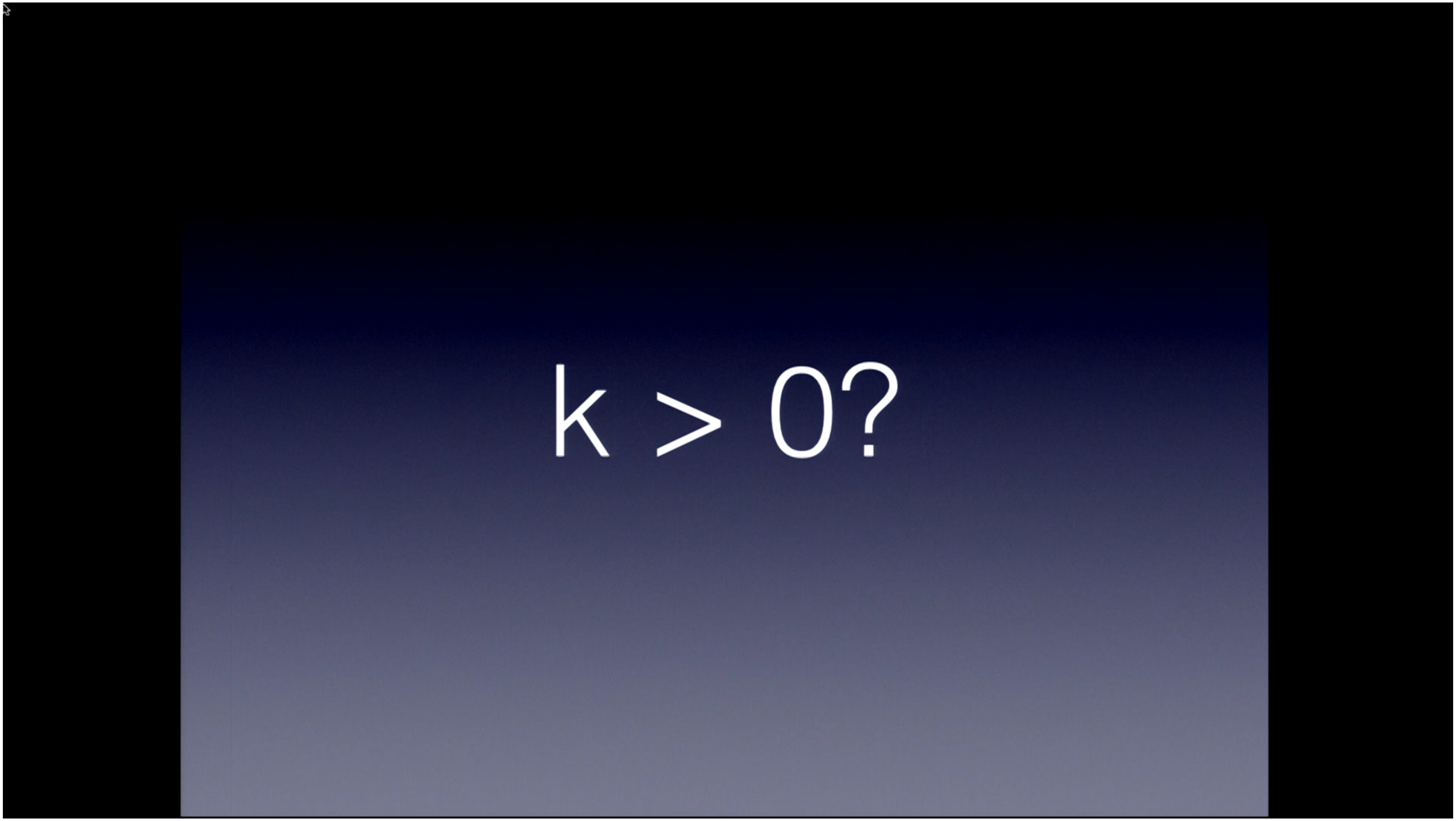


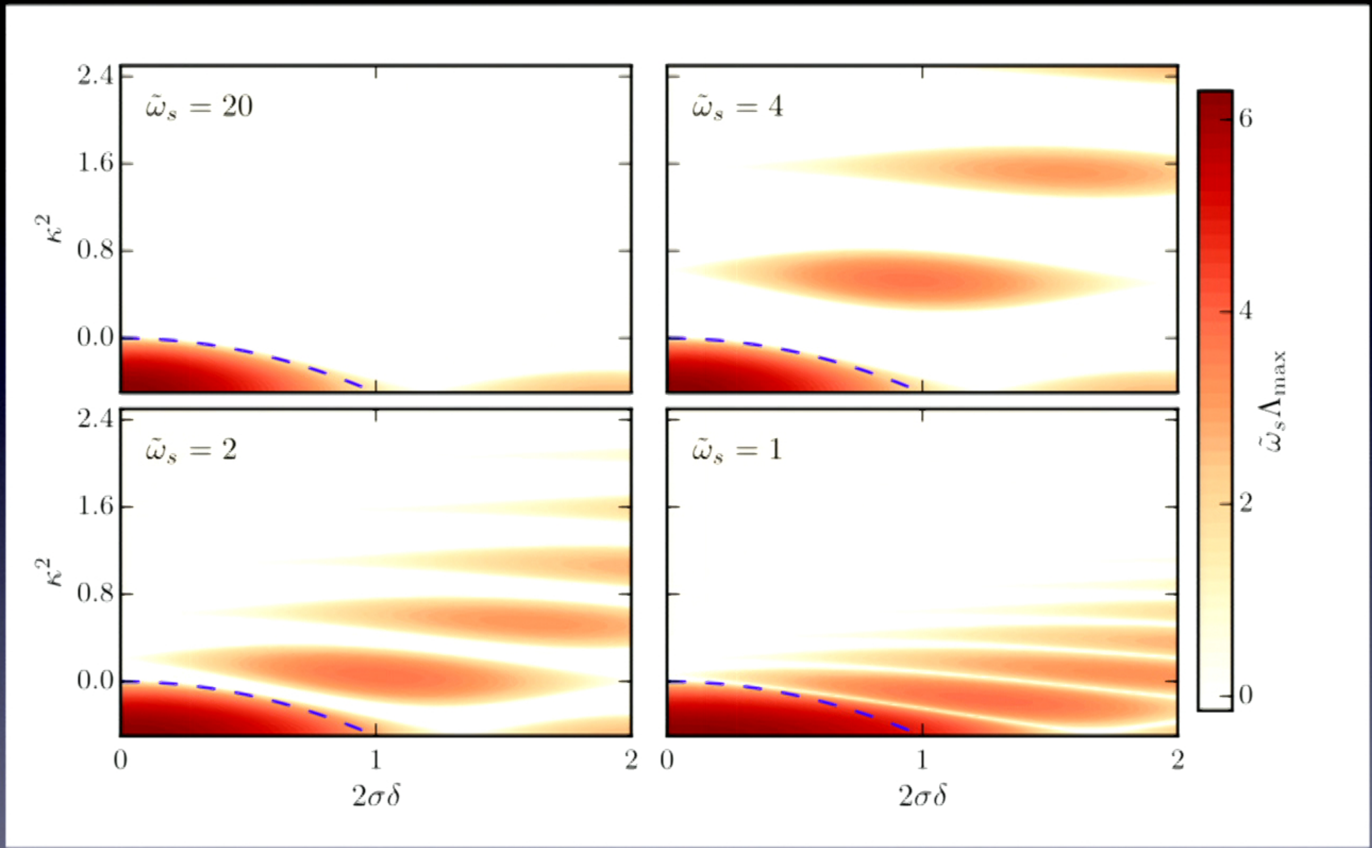
Stabilisation of Zero Mode

$$V(\phi) = V_0\left(-\cos\phi + \frac{\lambda^2}{2}\sin^2\phi\right)$$

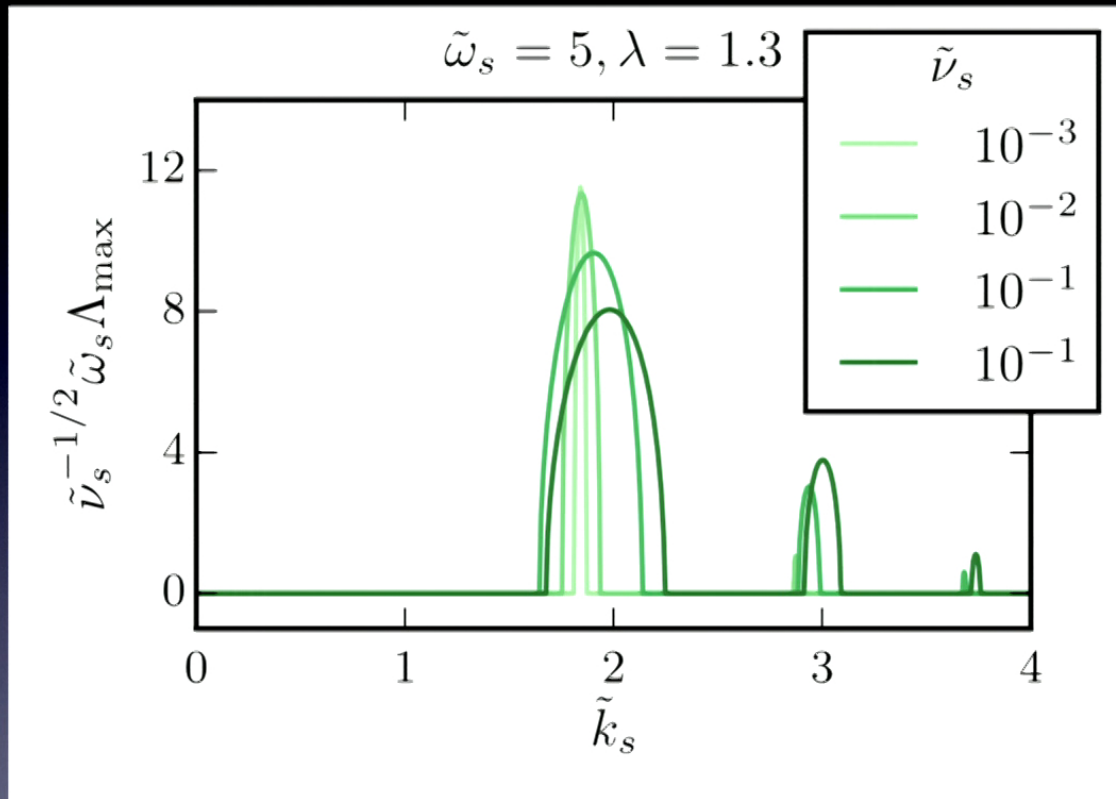


Stability: $\lambda \geq 1$

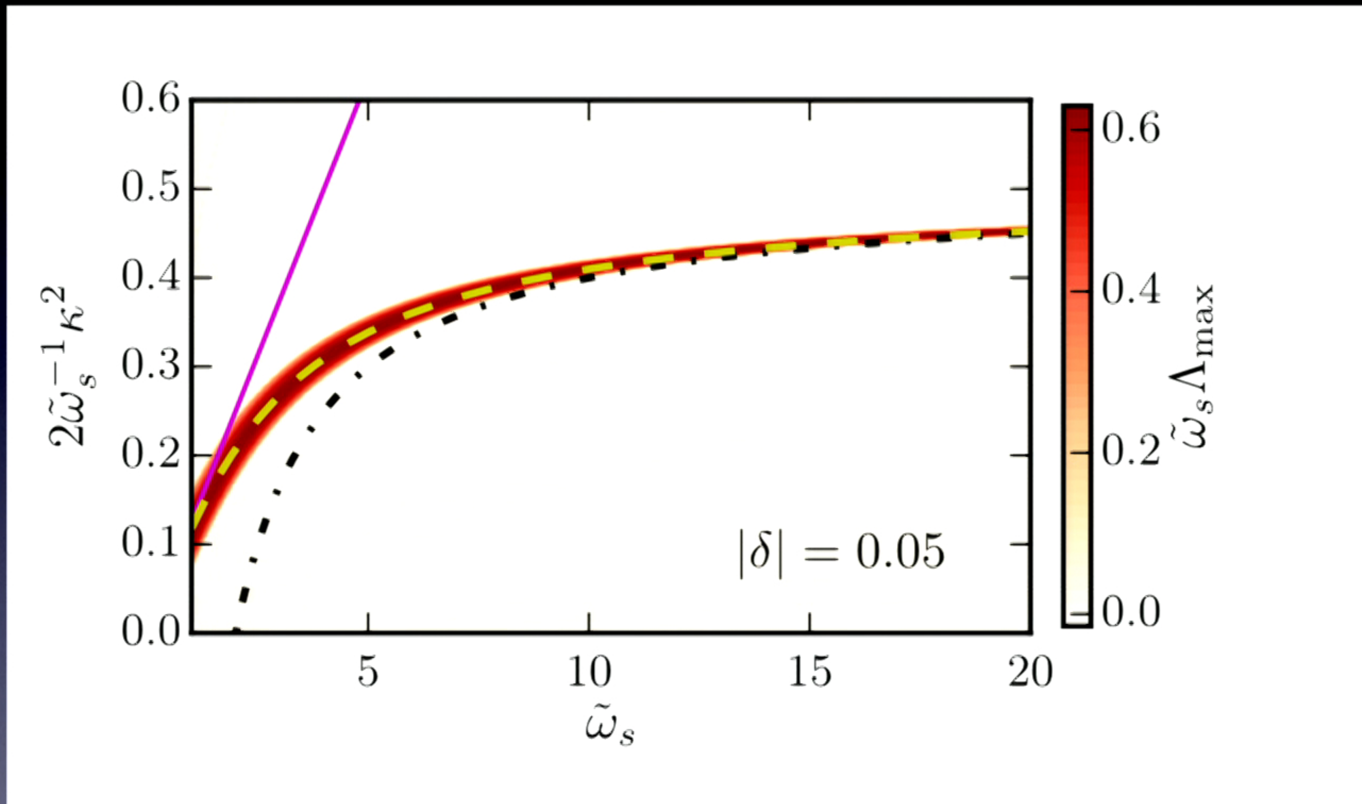

k > 0?



$$\kappa^2 = \frac{\hbar^2 k^2}{4g\bar{n}m} - \frac{\nu}{g\bar{n}} \qquad \tilde{\omega}_s = \frac{\hbar\omega}{g\bar{n}} \qquad \delta = \lambda\sqrt{\frac{\nu}{2g\bar{n}}}$$

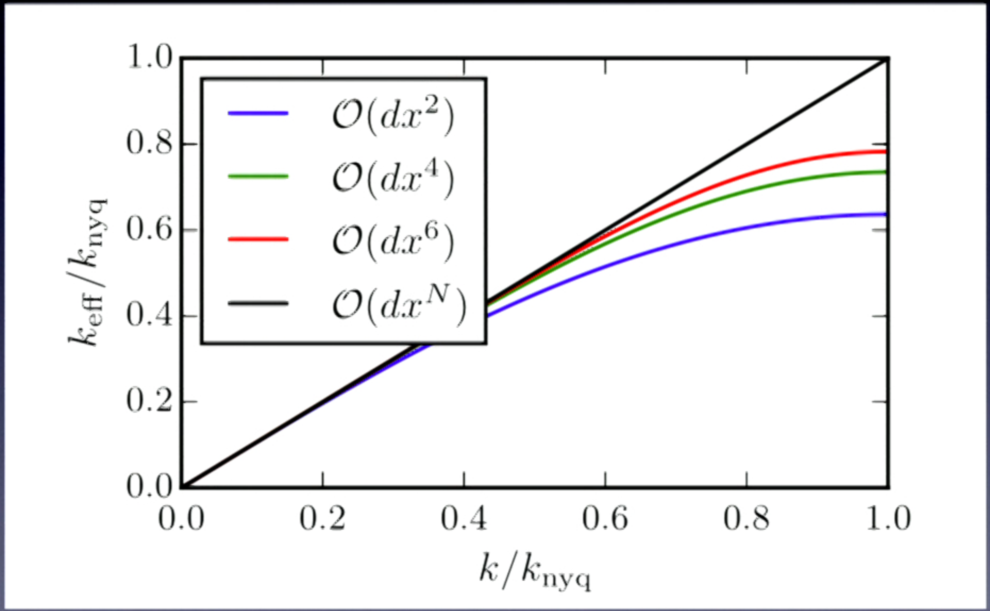


$$\nu = \nu_0 + \delta\hbar\omega\cos(\omega t)$$



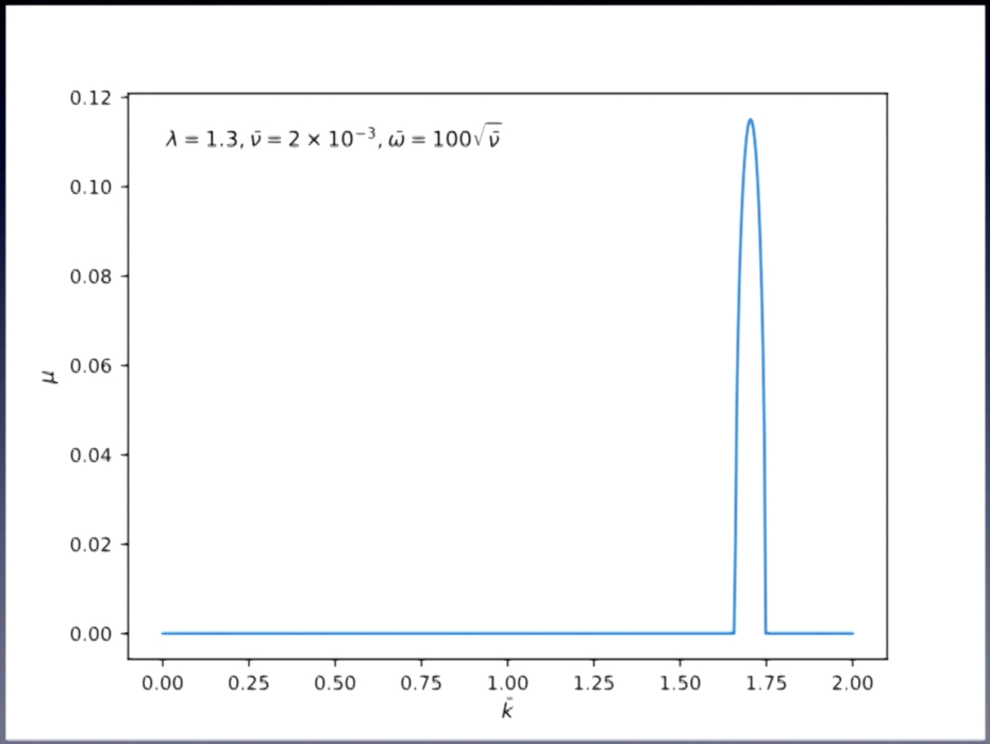
Increasing $\bar{\omega}$ pushes instability to higher $\bar{k}$

Are They Floquet Modes?

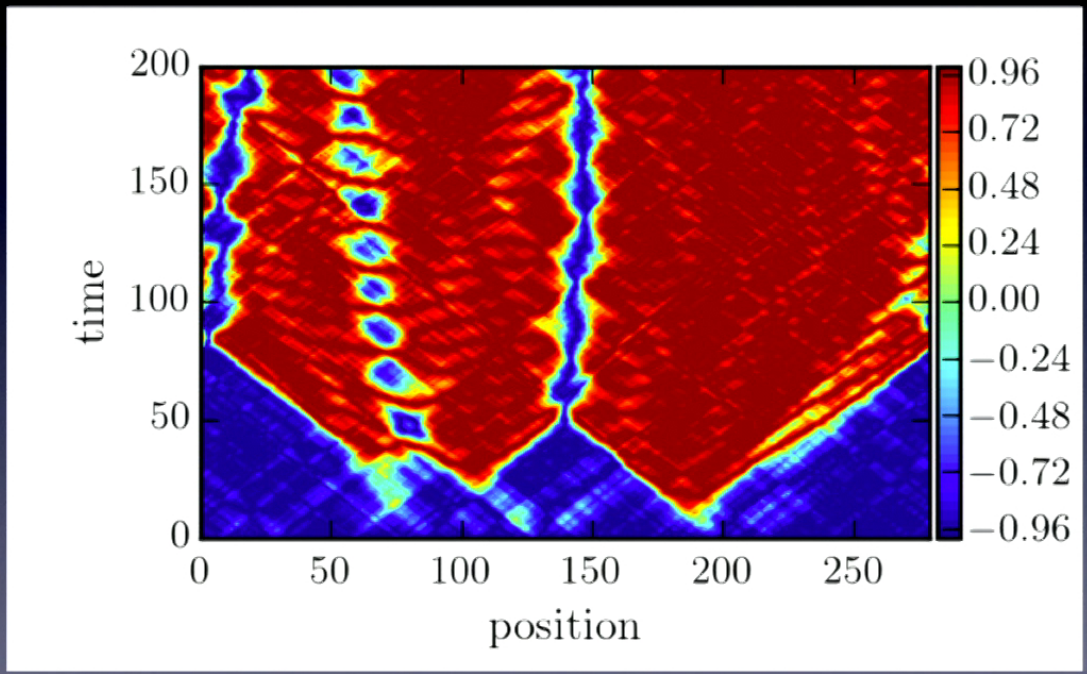


$$-\nabla^2 \to k_{\mathrm{eff}}^2$$

Are They Floquet Modes?

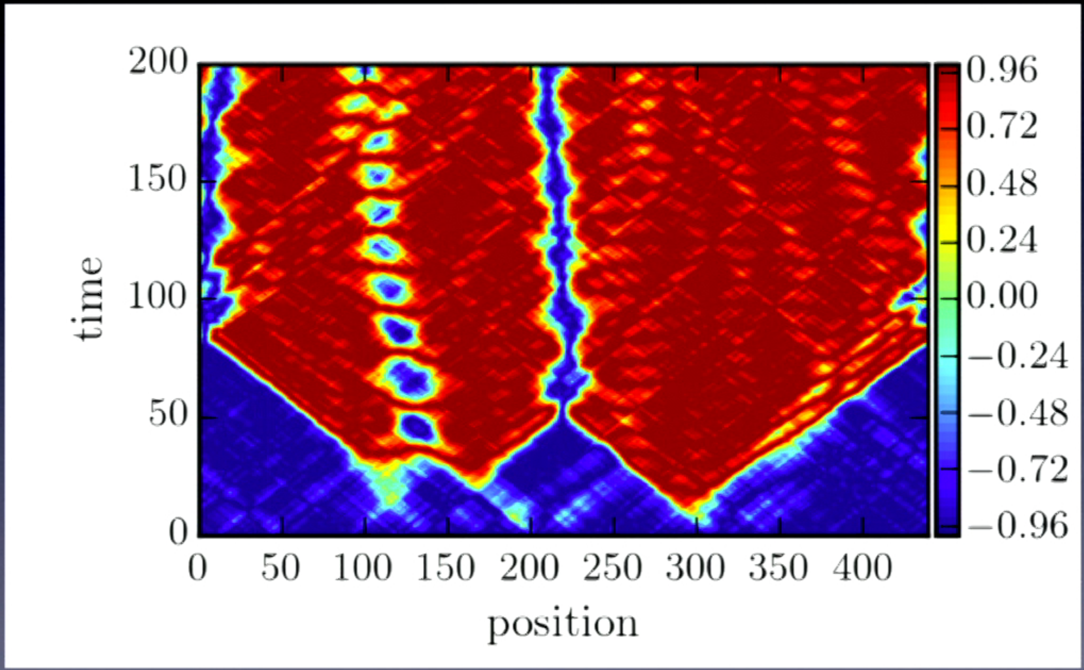


Spectral



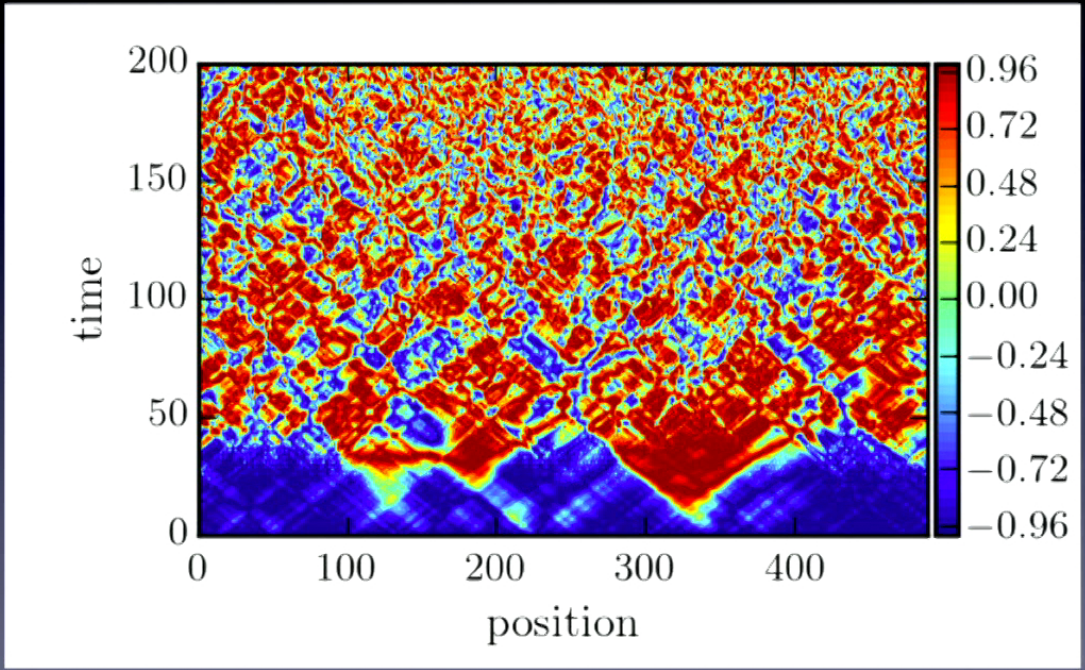
$$\bar{k}_{nyq} < \bar{k}_{\text{unstable}}$$

Discrete

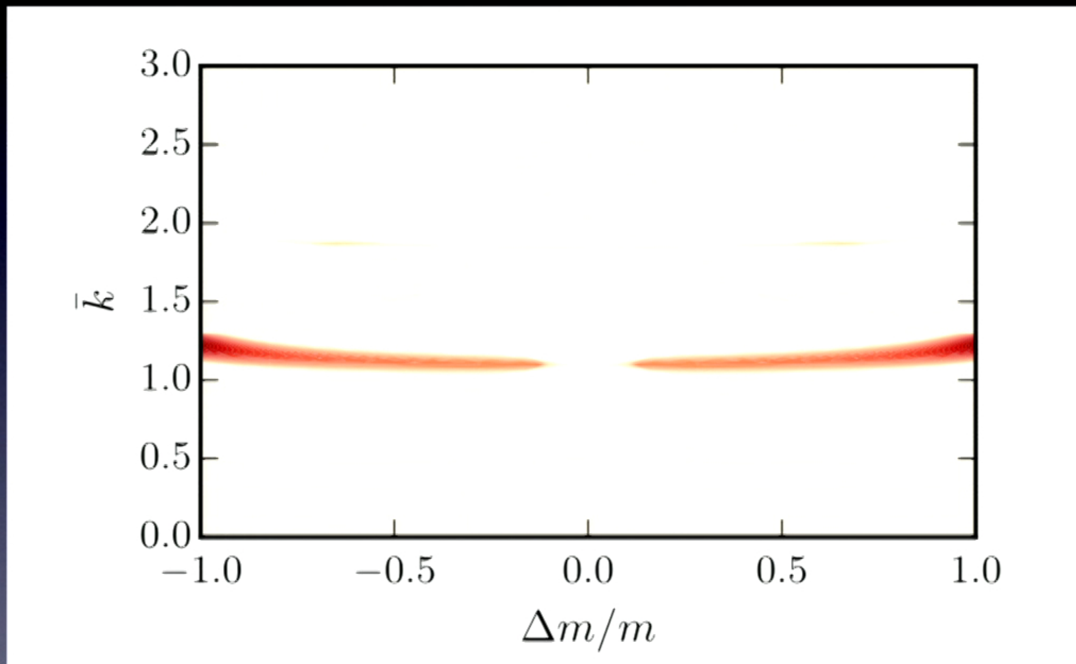


$$\bar{k}_{nyq} < \frac{\pi}{2}\bar{k}_{\text{unstable}}$$

Discrete Unstable



$$\bar{k}_{nyq} > \frac{\pi}{2}\bar{k}_{\text{unstable}}$$



Second Growing Mode

Validity of GPE

$$i\hbar\dot{\psi}_i = \left(-\delta_{ij}\frac{\hbar^2}{2m_i}\nabla^2 + V(\mathbf{x}) + g_{ij}|\psi_j|^2\right)\psi_i - \nu_{ij}\psi_j$$

$$g = \frac{4\pi\hbar^2 a_s}{m_i}$$

1. Dilute Gas: $na_s^3 \ll 1$
2. Don't Probe Potential: $\bar{k} \lesssim \pi(na_s^3)^{-1/2}$

- Expected corrections: viscosity and noise

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Implications

$$i\hbar\dot{\psi}_i = \left(\boxed{-\delta_{ij}\frac{\hbar^2}{2m_i}\nabla^2} + \boxed{V(\mathbf{x})} + \boxed{g_{ij}|\psi_j|^2}\right)\psi_i - \boxed{\nu_{ij}\psi_j}$$

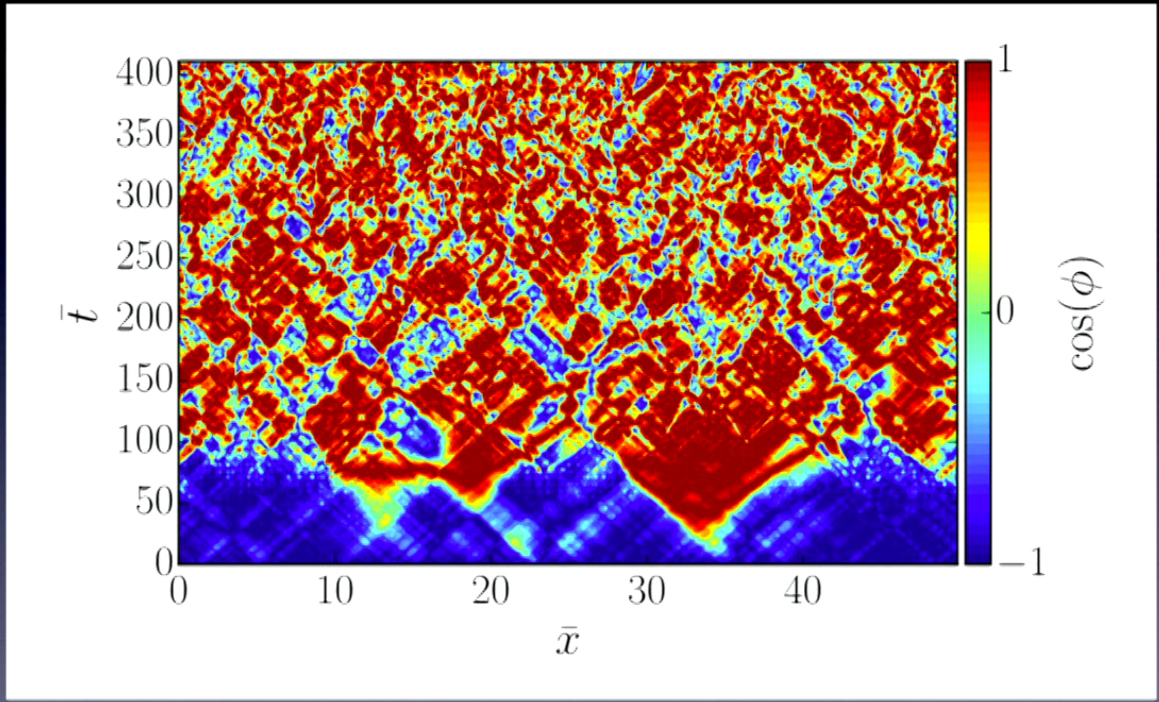
Atom Properties

Laser / Trap Properties

$$\nu = \nu_0 + \delta\hbar\omega\cos(\omega t)$$

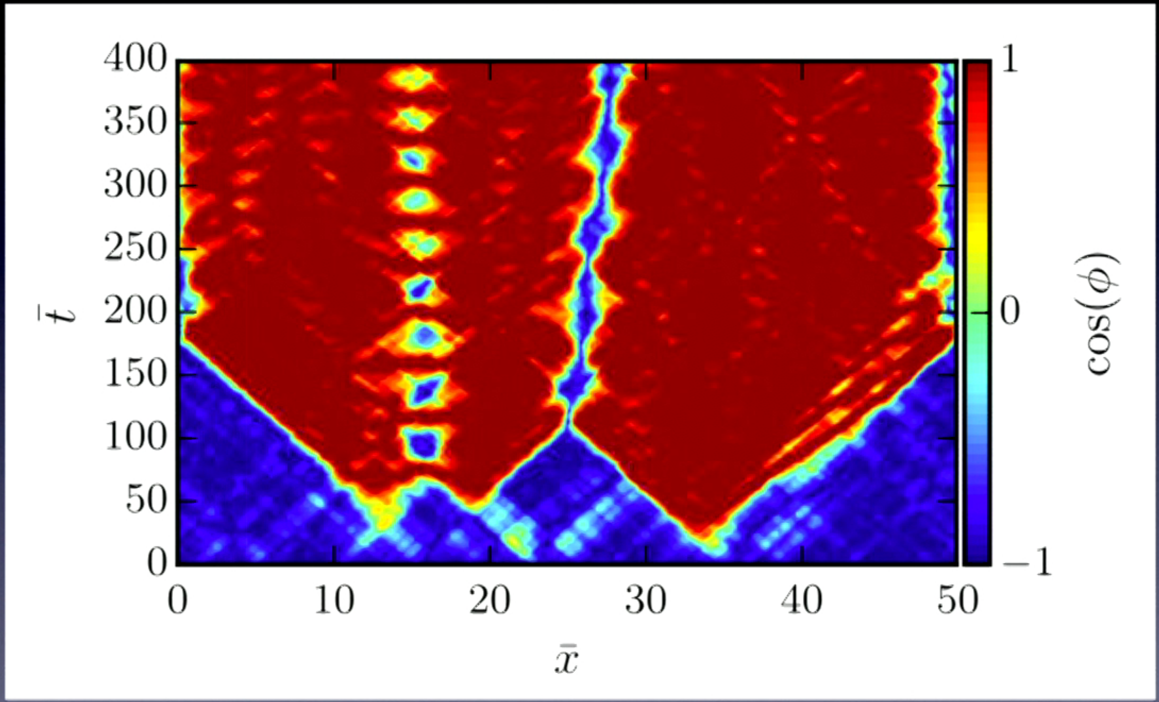
TUNABLE!

Increasing $\bar{\omega}$



Removes Instabilities

Increasing $\bar{\omega}$



Removes Instabilities

$$m = Z m_{\rm proton} \qquad a_{\rm s} = r a_{\rm Bohr}$$

Healing Length

S-wave Scattering

$$10^{15}\left(\frac{\bar{n} a_{\rm s}^3}{Z r^2}\right)\,\mathrm{Hz} \lesssim \omega \lesssim 10^{15}\left(\frac{1}{Z r^2}\right)\,\mathrm{Hz}$$

Conclusions / Future

- New real-time picture of false vacuum decay
- Good agreement with tunnelling calculation
- Novel opportunity to test nonperturbative dynamics of relativistic QFT in the lab - generic effective dynamics
- But beware of dragons (new instabilities)
- Instability length scale can be tuned by varying modulation frequency

Current / Future Work

- Wider ranger of field values, lattice renormalisation
- Non-vacuum states, understanding loop corrections, higher dimensional simulations
- Mean profile and fluctuations of bubble precursor
- Detailed comparison between pure scalar and cold atom simulations