Title: TBA

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Abstract:

Bootstrapping Scattering Amplitudes

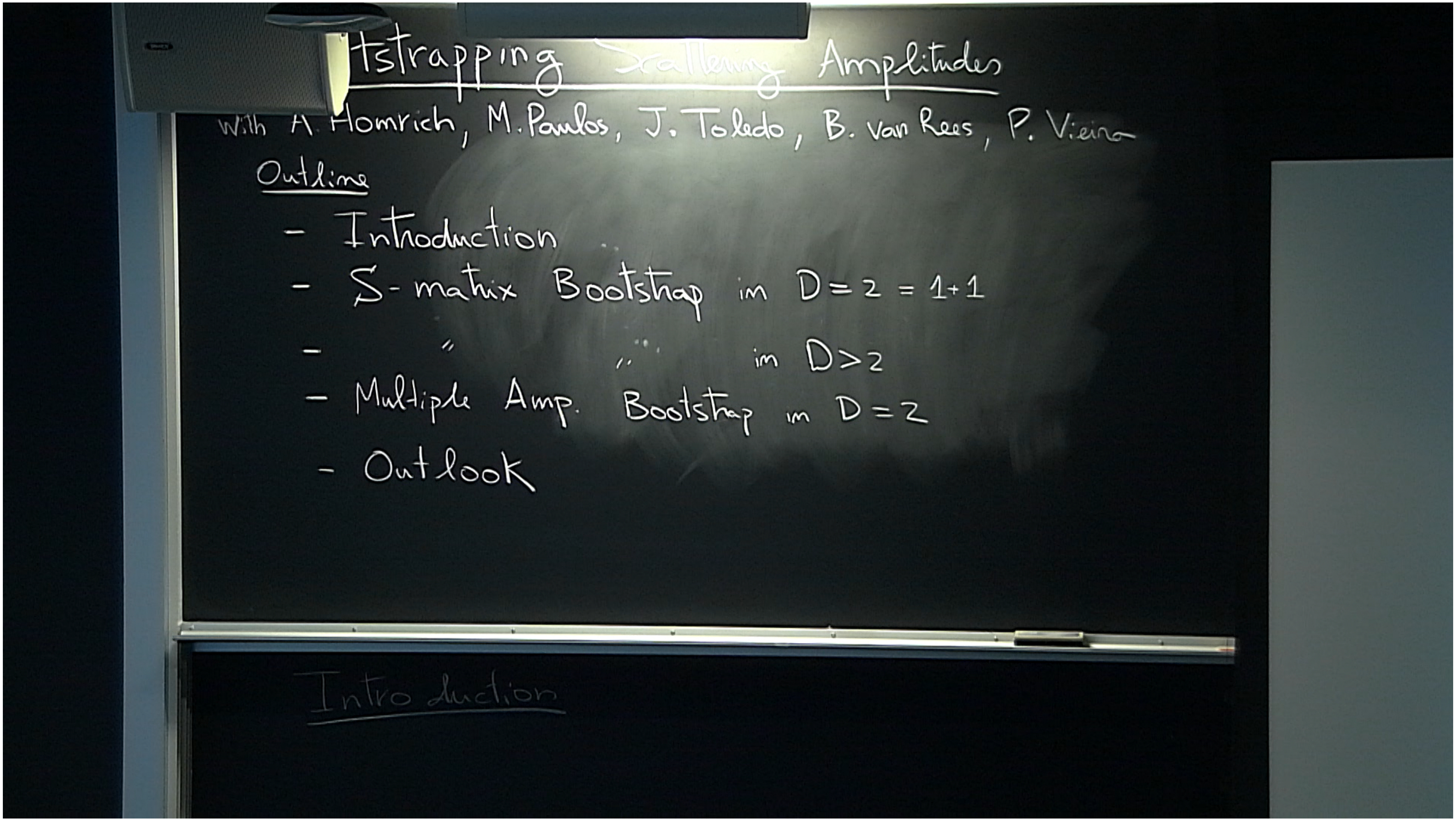
João Penedones

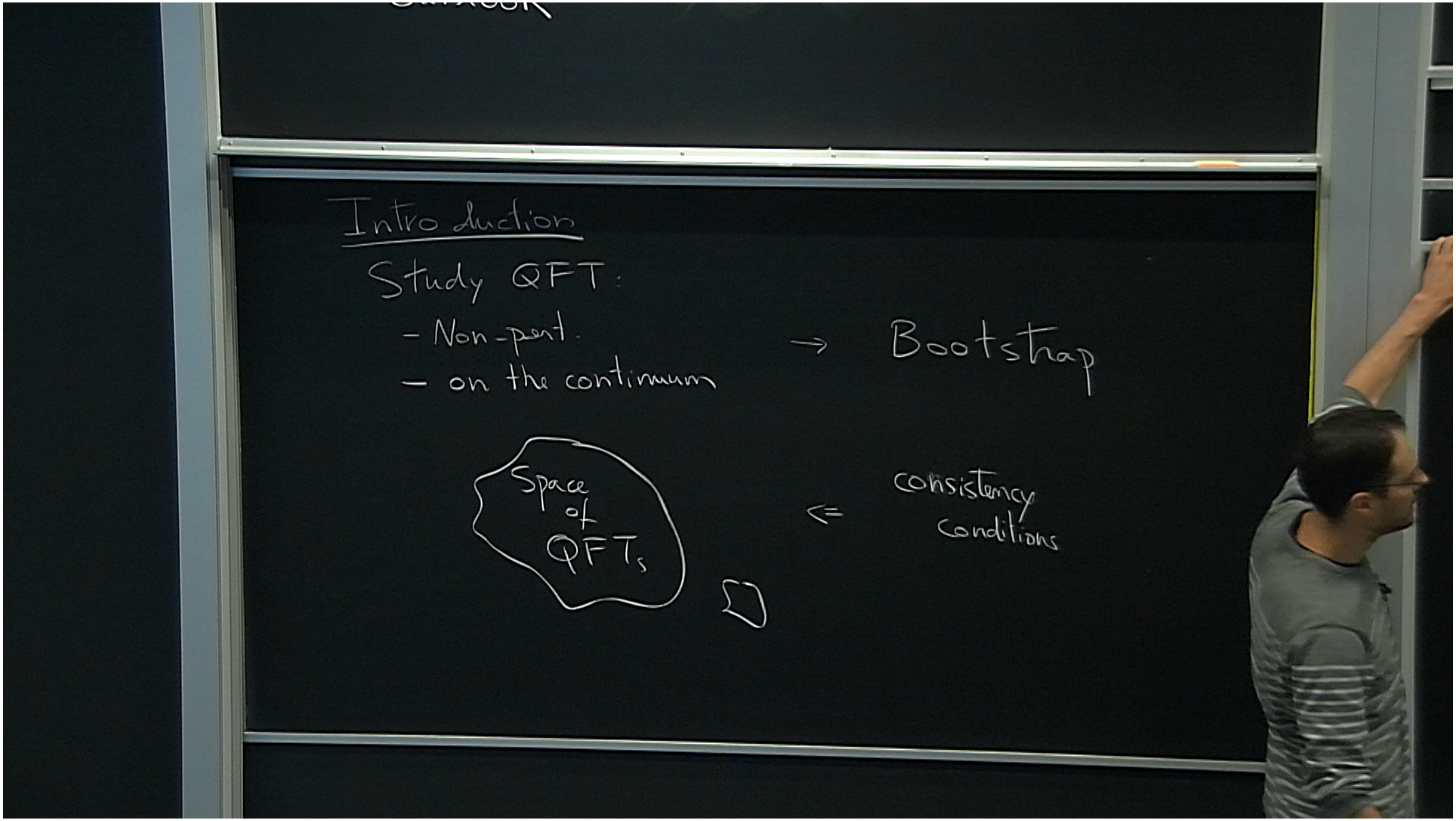
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

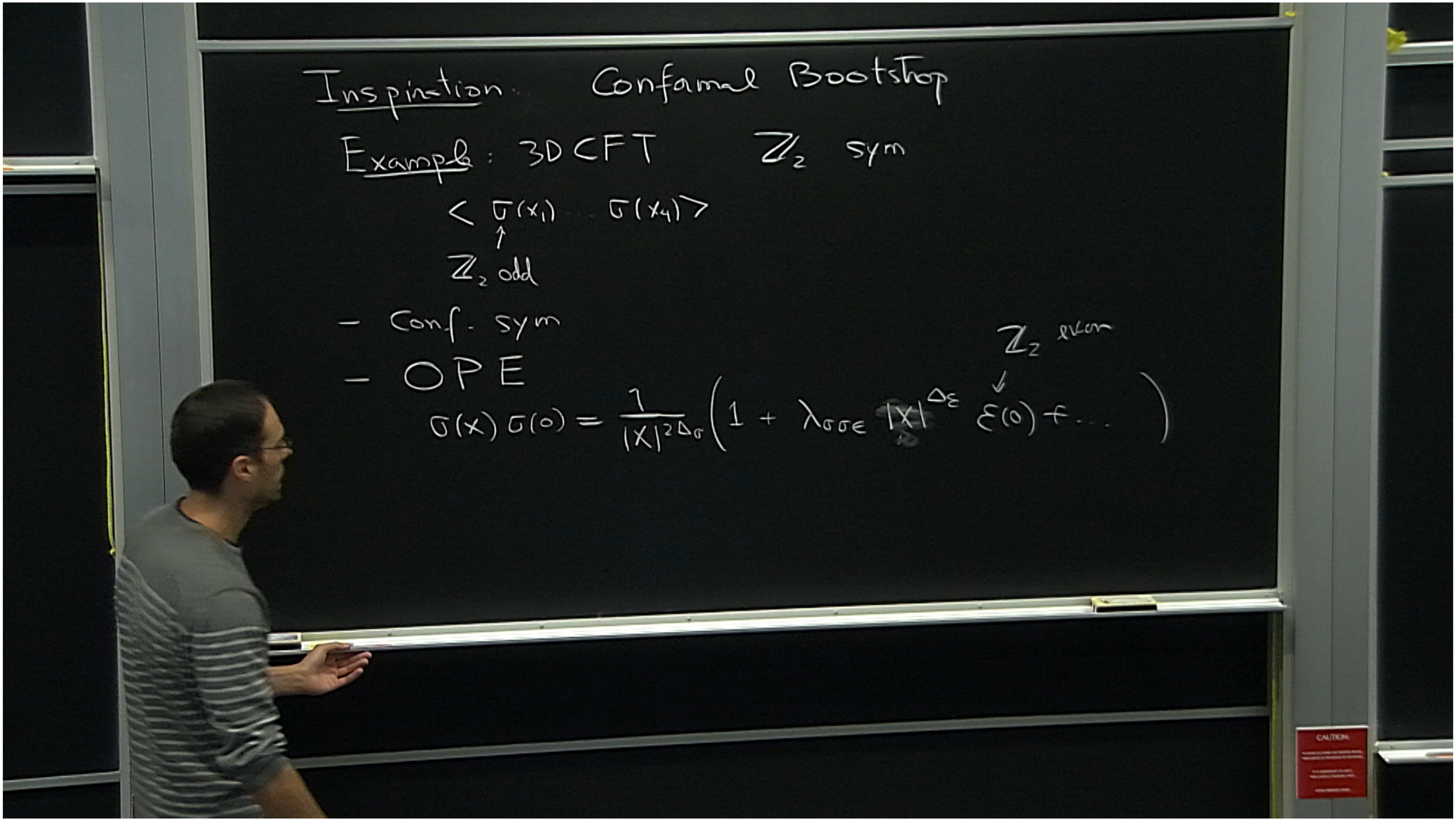
S-matrix Bootstrap I: QFT in AdS [arXiv:1607.06109]
S-matrix Bootstrap II: two-dimensional amplitudes [arXiv:1607.06110]
S-matrix Bootstrap III: higher dimensional amplitudes [arXiv:1708.06765]
S-matrix Bootstrap IV: multiple amplitudes [work in progress]

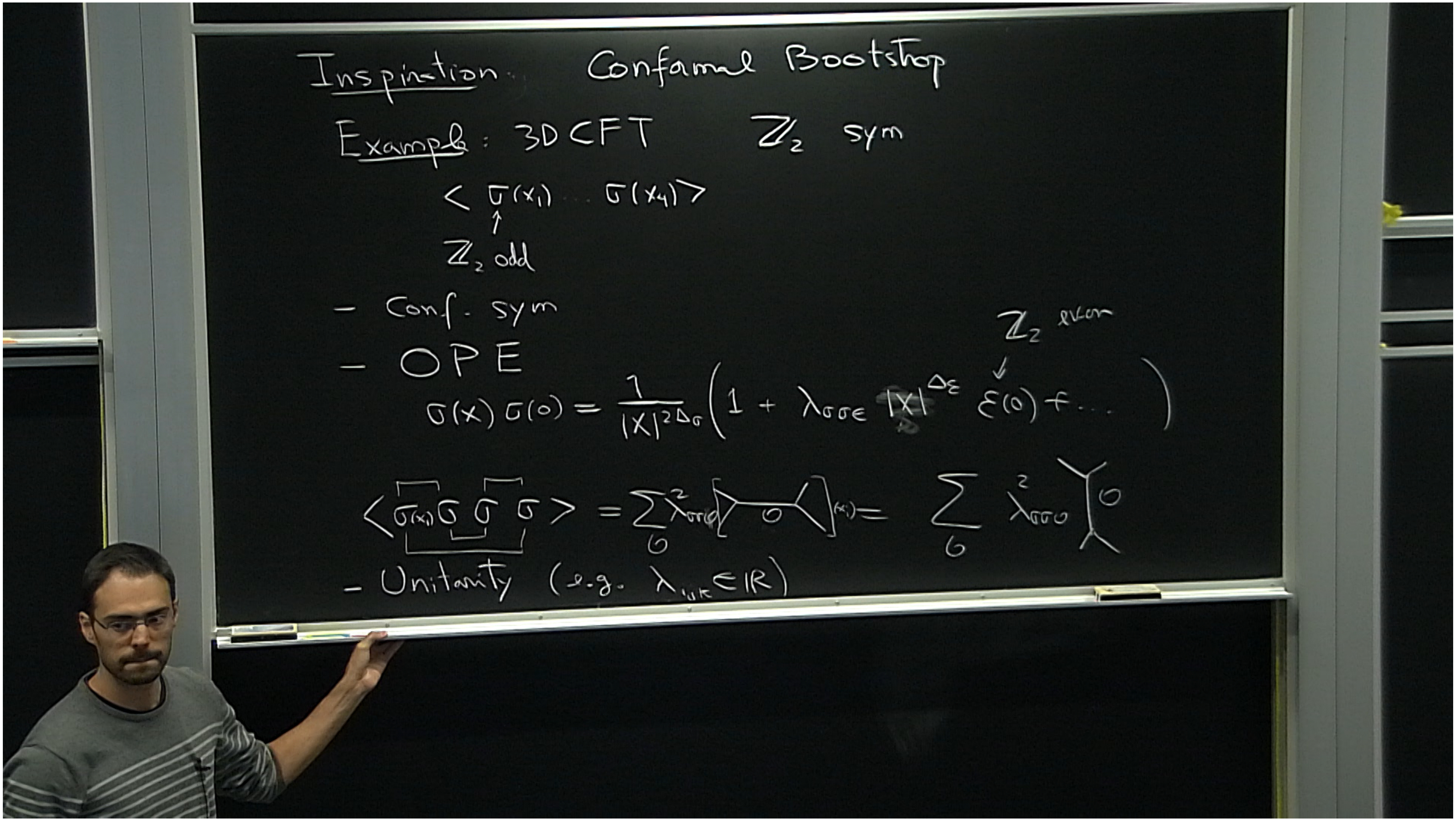
with A. Homrich, M. Paulos, J. Toledo, B. Van Rees, P. Vieira

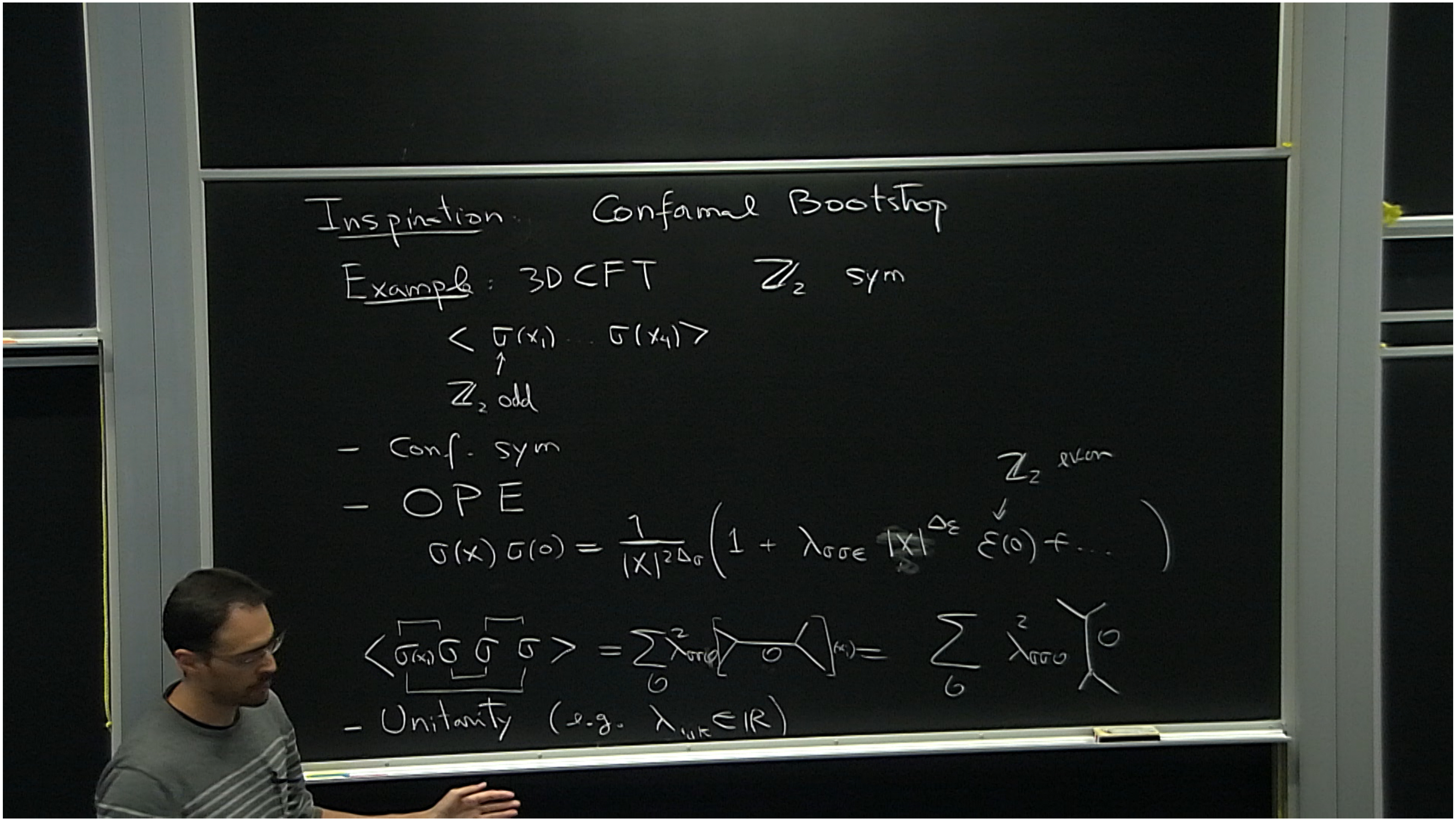
Perimeter Institute, 20th of June, 2018


tstrapping Scattering Amplitudes
With A. Homrich, M. Paulos, J. Toledo, B. van Rees, P. Vieira
Outline
- Introduction
- S-matrix Bootstrap in D = 2 = 1+1
- " " in D > 2
- Multiple Amp. Bootstrap in D = 2
- Outlook
Introduction


Introduction
Study QFT:
- Non-pert.
- on the continuum
→ Bootstrap
Space of QFTs
⇐ consistency conditions


Inspiration: Conformal Bootstrap
Example: 3D CFT Z_2 sym
< σ(x_1) ... σ(x_4) >
Z_2 odd
- Conf. sym
- OPE
Z_2 even
σ(x) σ(0) = 1/|x|^{2Δσ} (1 + λ_{σσε} |x|^{Δε} ε(0) + ...)


Inspiration: Conformal Bootstrap
Example: 3D CFT $\mathbb{Z}_2$ sym
$\langle \sigma(x_1) \ldots \sigma(x_4) \rangle$
$\mathbb{Z}_2$ odd
- Conf. sym
- OPE
$\mathbb{Z}_2$ even
$\sigma(x)\sigma(0) = \frac{1}{|x|^{2\Delta_\sigma}} \left(1 + \lambda_{\sigma\sigma\epsilon} |x|^{\Delta_\epsilon} \epsilon(0) + \ldots \right)$
$\langle \sigma(x_1) \sigma \sigma \sigma \rangle = \sum_{O} \lambda^2_{\sigma\sigma O} = \sum_{O} \lambda^2_{\sigma\sigma O}$
- Unitarity (e.g. $\lambda_{ijk} \in \mathbb{R}$)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- Conf. sym
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$\langle \sigma(x_1) \sigma \sigma \sigma \rangle = \sum_{\mathcal{O}} \lambda^2_{\sigma\sigma\mathcal{O}} [\ldots](x_i) = \sum_{\mathcal{O}} \lambda^2_{\sigma\sigma\mathcal{O}}$
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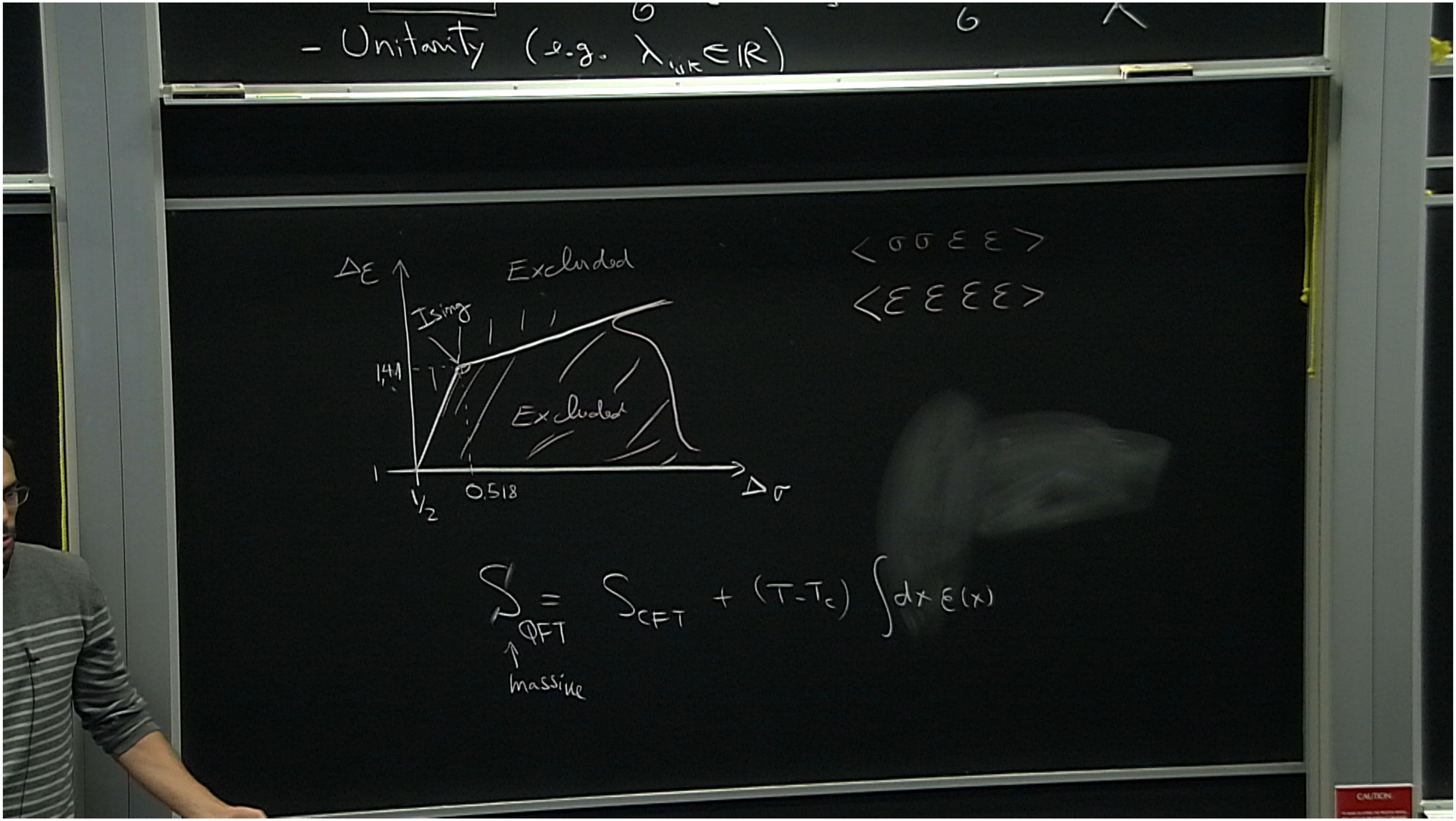
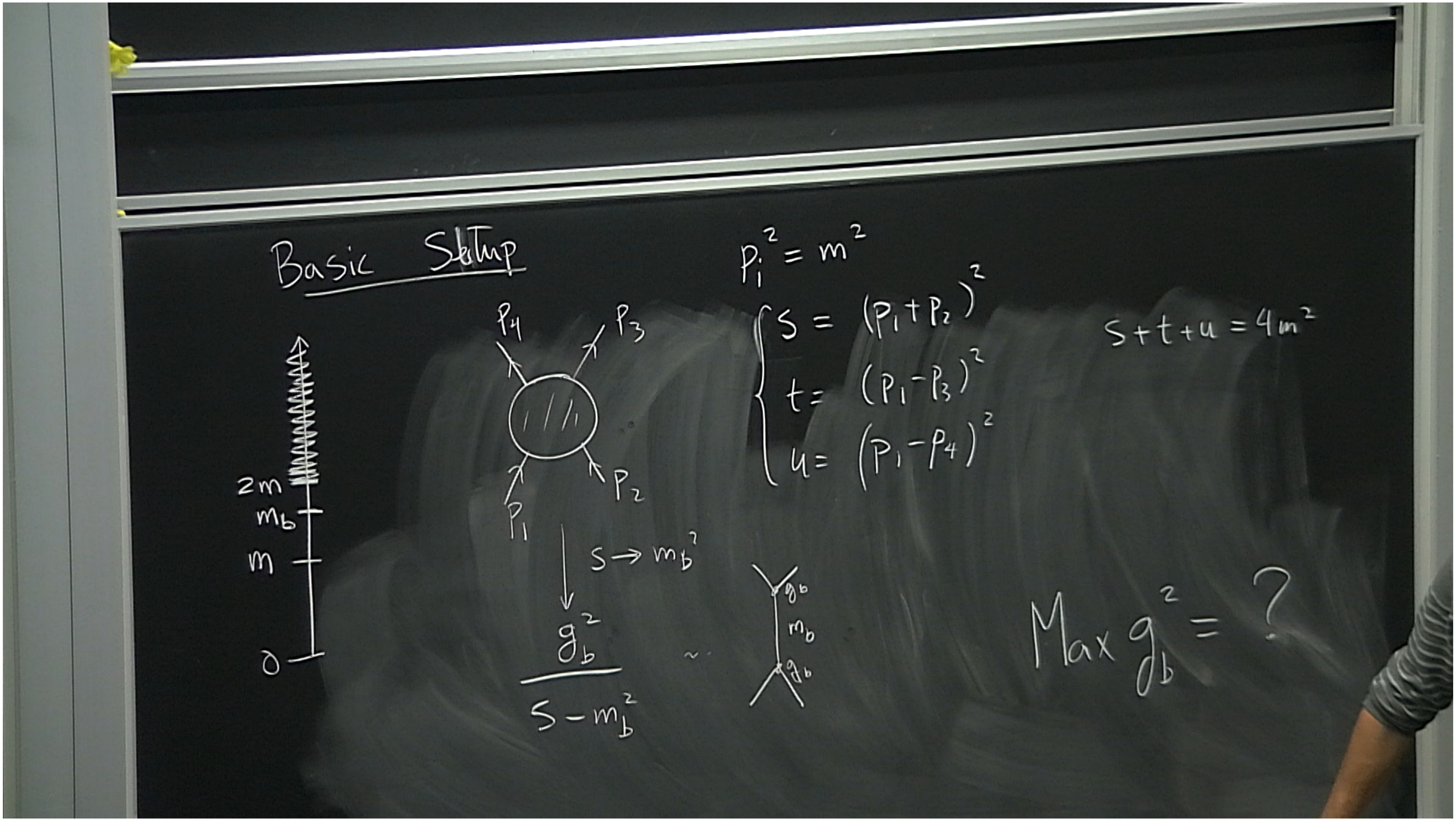
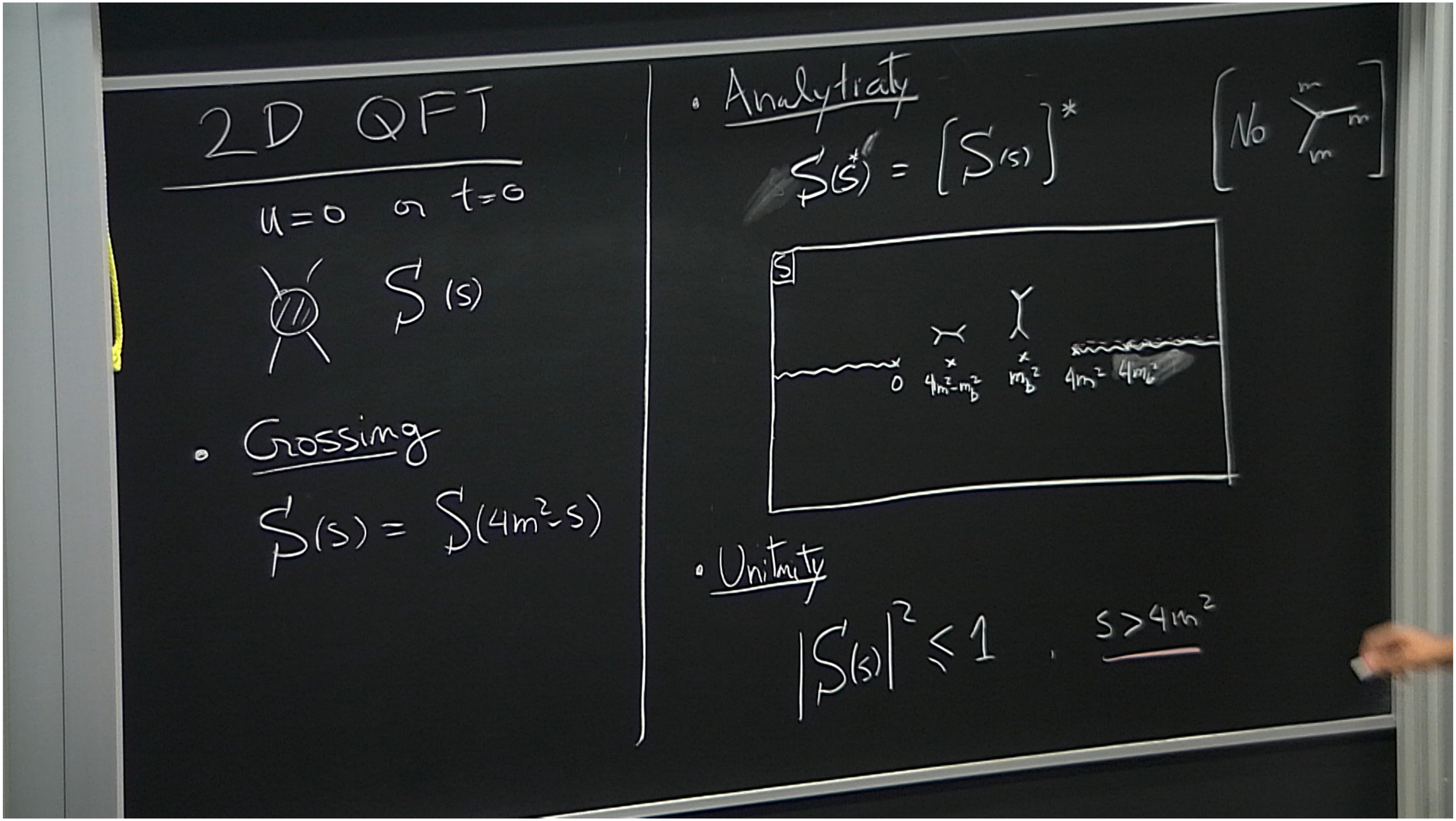
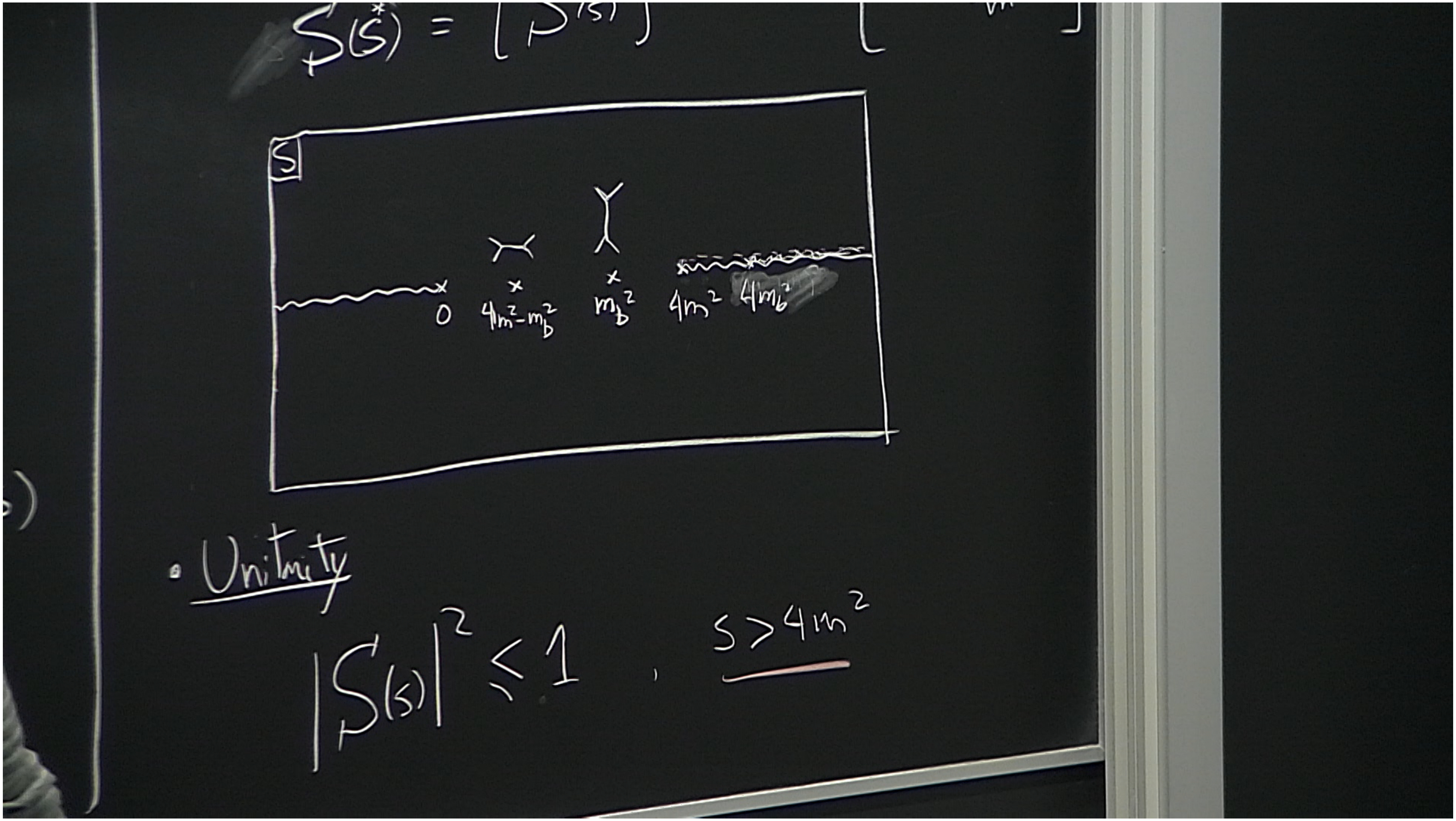
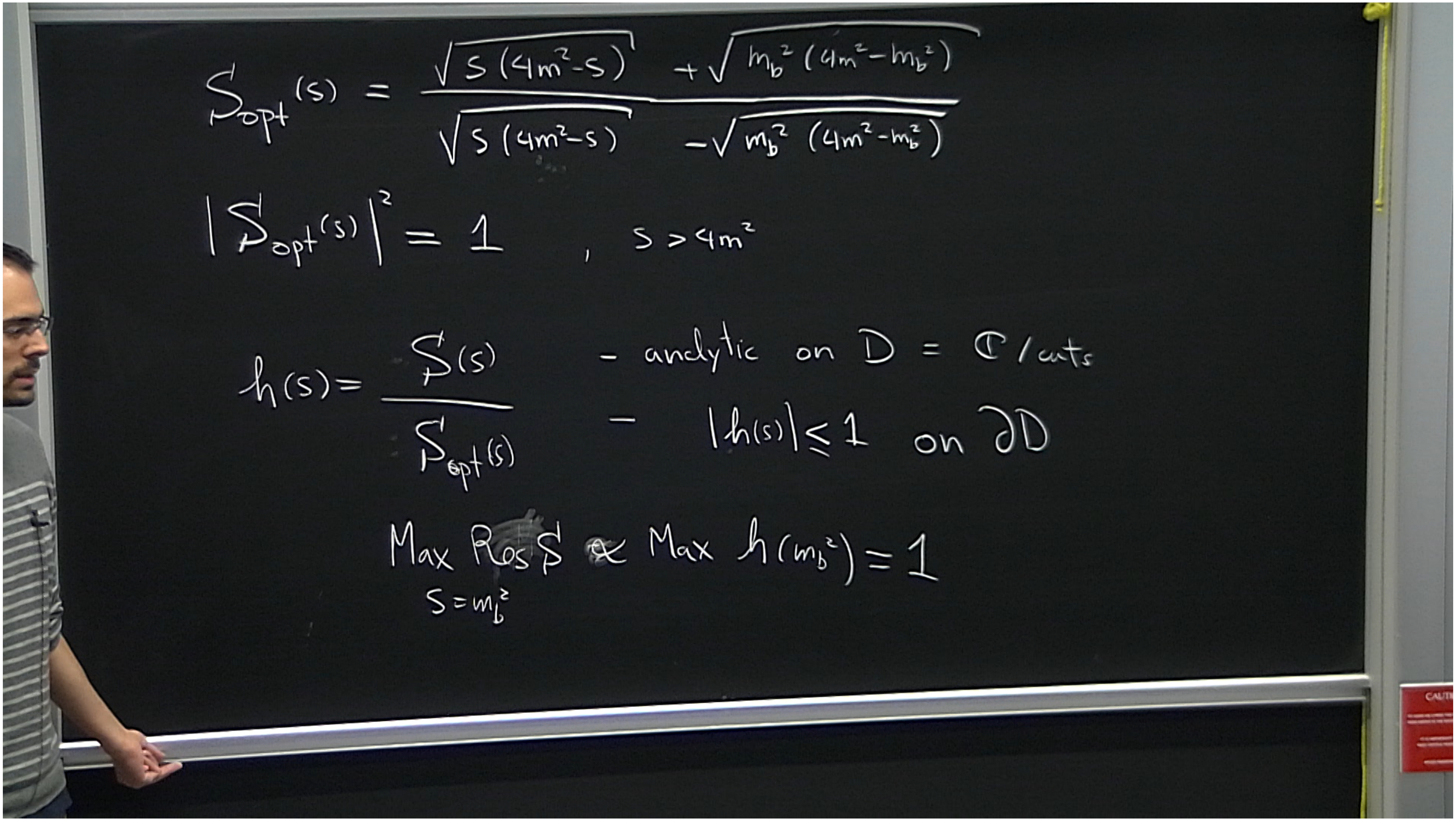

- Unitarity (e.g. λ_ijk ∈ ℝ)
Δε
Excluded
Ising
1.41
Excluded
1
1/2
0.518
Δσ
<σσεε>
<εεεε>
S_QFT = S_CFT + (T-T_c) ∫dx ε(x)
massive


Basic Setup
$p_i^2 = m^2$
$s = (p_1 + p_2)^2$
$t = (p_1 - p_3)^2$
$u = (p_1 - p_4)^2$
$s + t + u = 4m^2$
p_1 p_2 p_3 p_4
2m
m_b
m
0
$s \to m_b^2$
$\frac{g_b^2}{s - m_b^2}$
g_b
m_b
g_b
Max $g_b^2 = ?$

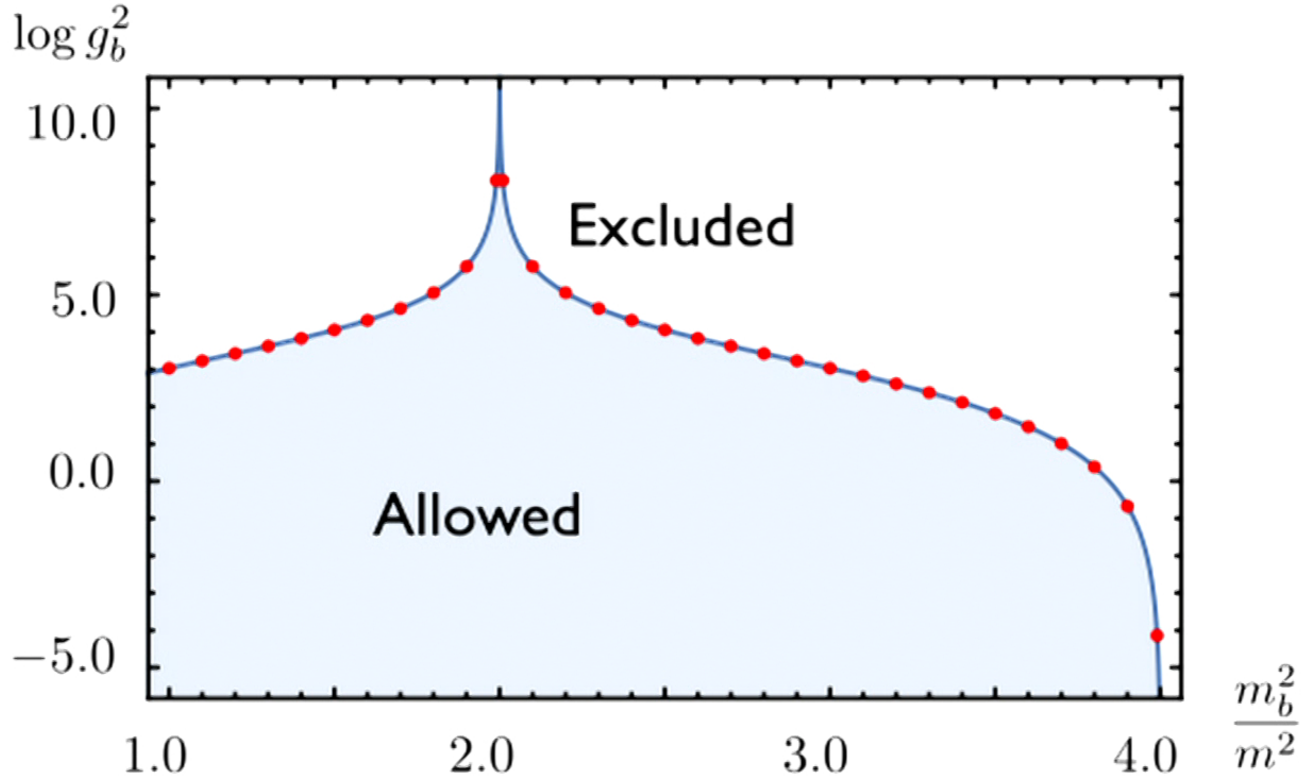

2D QFT
u=0 or t=0
S(s)
Crossing
S(s) = S(4m²-s)
Analyticity
S(s*) = [S(s)]*
No
m
m
m
S
0
4m²-m_b²
m_b²
4m²
4m²
Unitarity
|S(s)|² ≤ 1 , s>4m²


S
0
4m²-m_b²
m_b²
4m²
4m_b²
Unitarity
|S(s)|² ≤ 1 , s > 4m²


$S_{opt}(s) = \frac{\sqrt{s(4m^2-s)} + \sqrt{m_b^2(4m^2-m_b^2)}}{\sqrt{s(4m^2-s)} - \sqrt{m_b^2(4m^2-m_b^2)}}$
$|S_{opt}(s)|^2 = 1 \quad , \quad s > 4m^2$
$h(s) = \frac{S(s)}{S_{opt}(s)}$
- analytic on $D = \mathbb{C}$ / cuts
- $|h(s)| \leq 1$ on ∂D
$\mathrm{Max}_{s=m_b^2} \mathrm{Res}\, S \;\&\; \mathrm{Max}\, h(m_b^2) = 1$

Maximum cubic coupling

$$S_{opt}(s) = \frac{\sqrt{s(4m^2 - s)} + \sqrt{m_b^2(4m^2 - m_b^2)}}{\sqrt{s(4m^2 - s)} - \sqrt{m_b^2(4m^2 - m_b^2)}}$$



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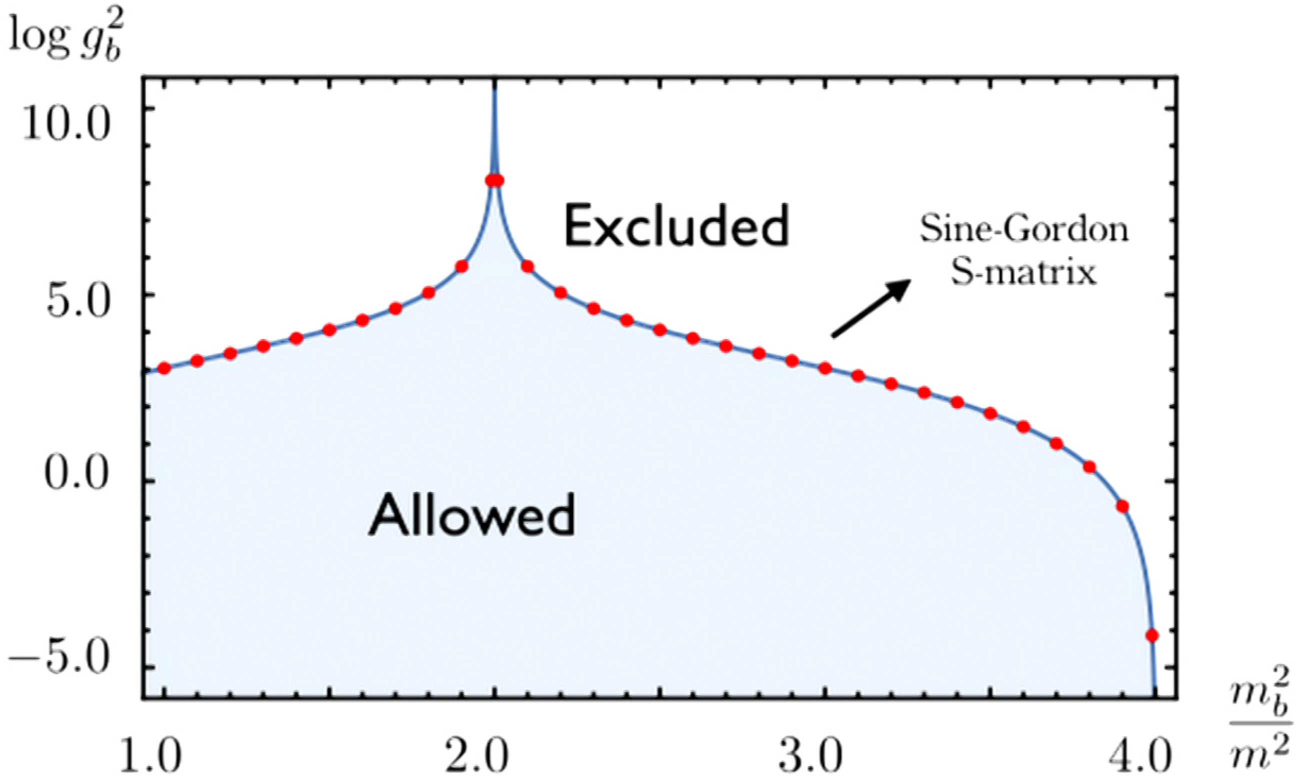
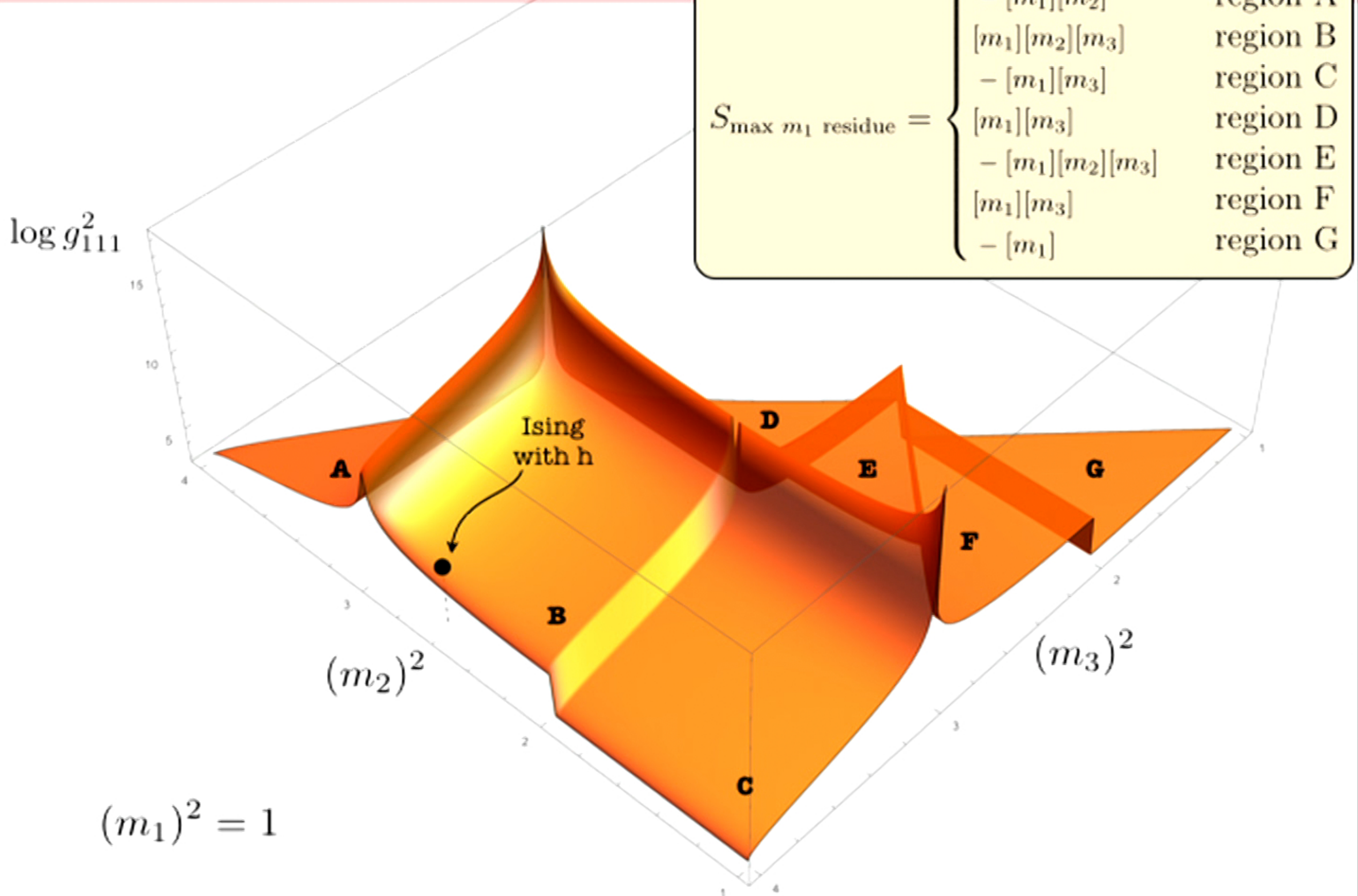
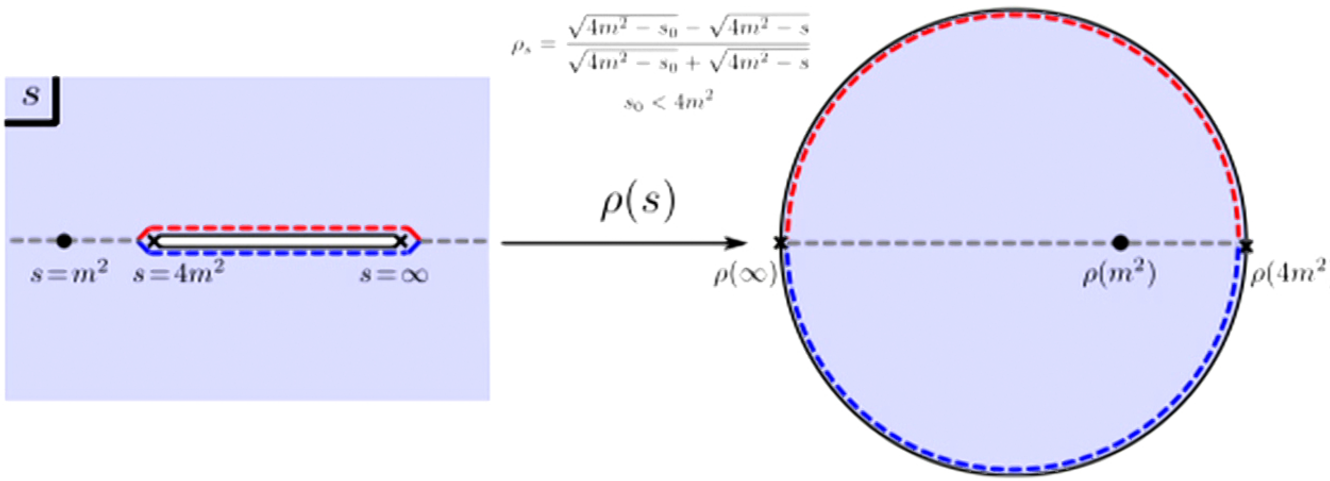


3 stable particles

$$S_{\text{max } m_1 \text{ residue}} = \begin{cases} -[m_1][m_2] & \text{region A} \\ [m_1][m_2][m_3] & \text{region B} \\ -[m_1][m_3] & \text{region C} \\ [m_1][m_3] & \text{region D} \\ -[m_1][m_2][m_3] & \text{region E} \\ [m_1][m_3] & \text{region F} \\ -[m_1] & \text{region G} \end{cases}$$



Numerical approach



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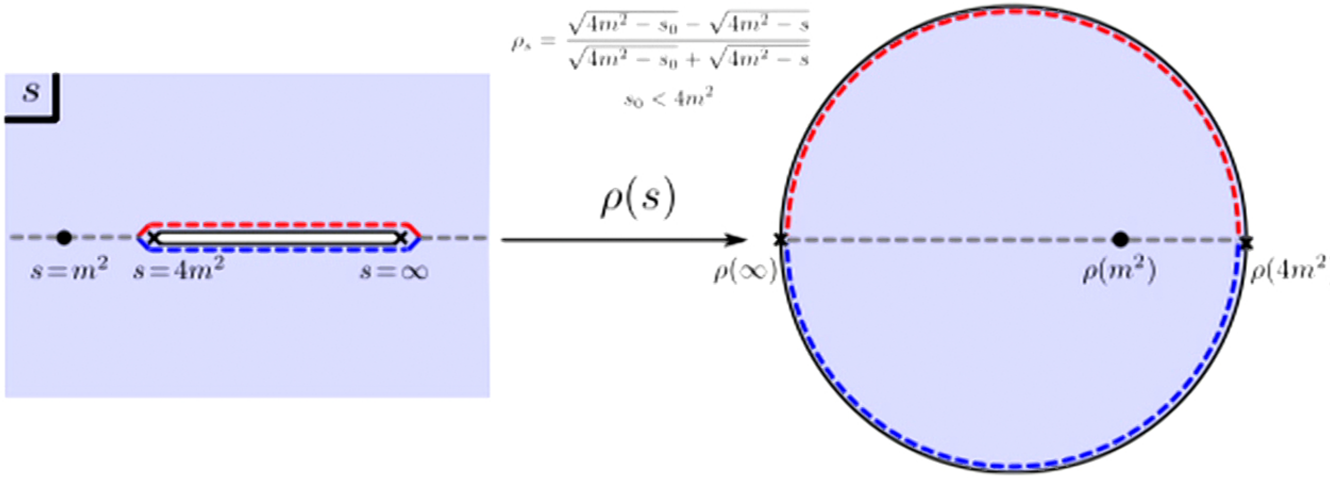


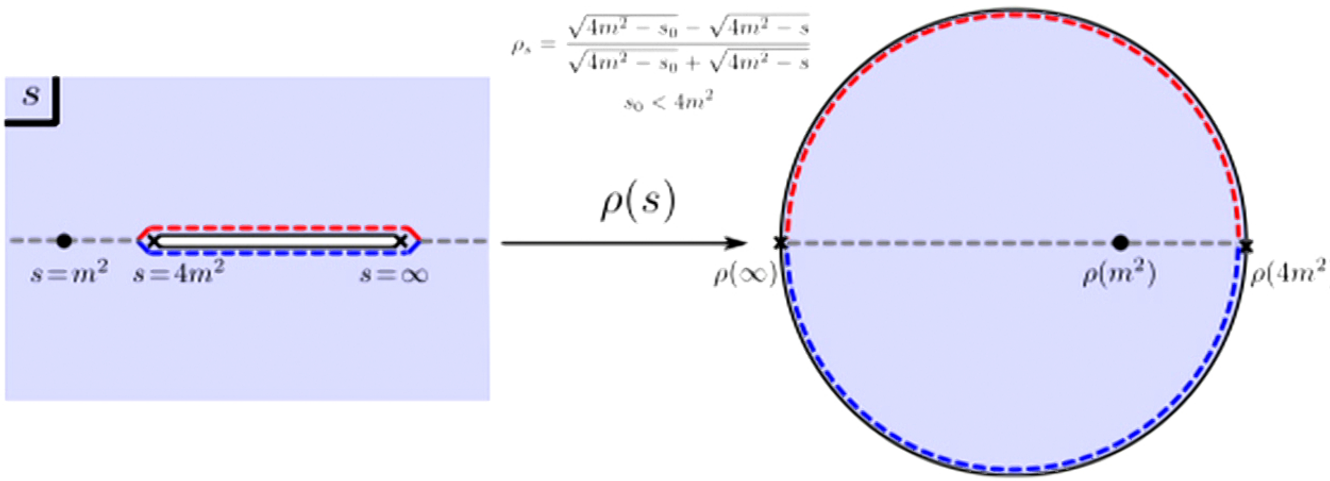
Ansatz:

$$S_{ext}(s,t) = \frac{g_b^2}{s - m_b^2} + \frac{g_b^2}{t - m_b^2} + \sum_{a,b=0} c_{(ab)} \rho_s^a \rho_t^b$$

Numerical approach

$$\rho_s = \frac{\sqrt{4m^2 - s_0} - \sqrt{4m^2 - s}}{\sqrt{4m^2 - s_0} + \sqrt{4m^2 - s}}$$

$$s_0 < 4m^2$$



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Crossing symmetry and analyticity are automatic.

Unitarity gives quadratic constraints:

$$\left| S_{ext}(s, 4m^2 - s) \right|^2 \leq 1 \,, \qquad s > 4m^2$$

Numerical approach

Ansatz:

$$S_{ext}(s,t) = \frac{g_b^2}{s-m_b^2} + \frac{g_b^2}{t-m_b^2} + \sum_{a,b=0} c_{(ab)} \rho_s^a \rho_t^b$$

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Truncate to finite number of variables and **quadratic constraints**

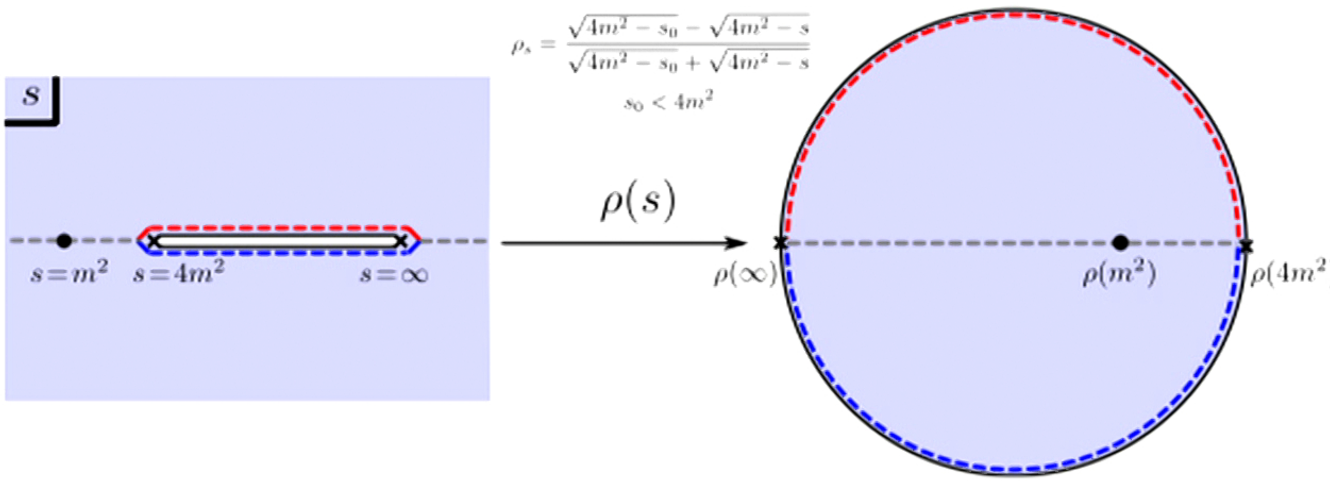
$a + b \leq N_{\max}$ ↓ $\{g_b^2, c_{(ab)}\}$

↓ at $s = s_1, s_2, \ldots, s_M$

Numerical approach

$$\rho_s = \frac{\sqrt{4m^2 - s_0} - \sqrt{4m^2 - s}}{\sqrt{4m^2 - s_0} + \sqrt{4m^2 - s}}$$

$$s_0 < 4m^2$$



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$a + b \leq N_{\max}$ ↓ $\{g_b^2, c_{(ab)}\}$ ↓ at $s = s_1, s_2, \ldots, s_M$

[Simmons-Duffin '15]

Use semidefinite programming (SDPB) to maximize g_b^2 subject to these constraints. This reproduces the analytic solution as $N_{\max} \to \infty$

2 to 2 Scattering Amplitude

$$\langle \mathbf{p}_3, \mathbf{p}_4 | S | \mathbf{p}_1, \mathbf{p}_2 \rangle = \mathbb{1} + i(2\pi)^{d+1} \delta^{(d+1)}(p_1 + p_2 - p_3 - p_4)\, T(s,t,u)$$

Crossing symmetry & Analyticity:

$$T(s,t,u) = \frac{g_b^2}{s - m_b^2} + \frac{g_b^2}{t - m_b^2} + \frac{g_b^2}{u - m_b^2} + \sum_{a,b,c=0} \alpha_{(abc)} \rho_s^a \rho_t^b \rho_u^c$$

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Partial waves:

Gegenbauer polynomial
$x = \cos\theta$

$$S_\ell(s) = 1 + i \frac{(s - 4m^2)^{\frac{d-2}{2}}}{\sqrt{s}} \int_{-1}^{1} dx (1 - x^2)^{\frac{d-3}{2}} P_\ell^{(d)}(x)\, T(s,t,u)\Big|_{\substack{t \to -\frac{1-x}{2}(s-4m^2) \\ u \to -\frac{1+x}{2}(s-4m^2)}}$$

Unitarity: $|S_\ell(s)|^2 \leq 1\,, \qquad s > 4m^2\,, \quad \ell = 0, 2, 4, \ldots$

2 to 2 Scattering Amplitude

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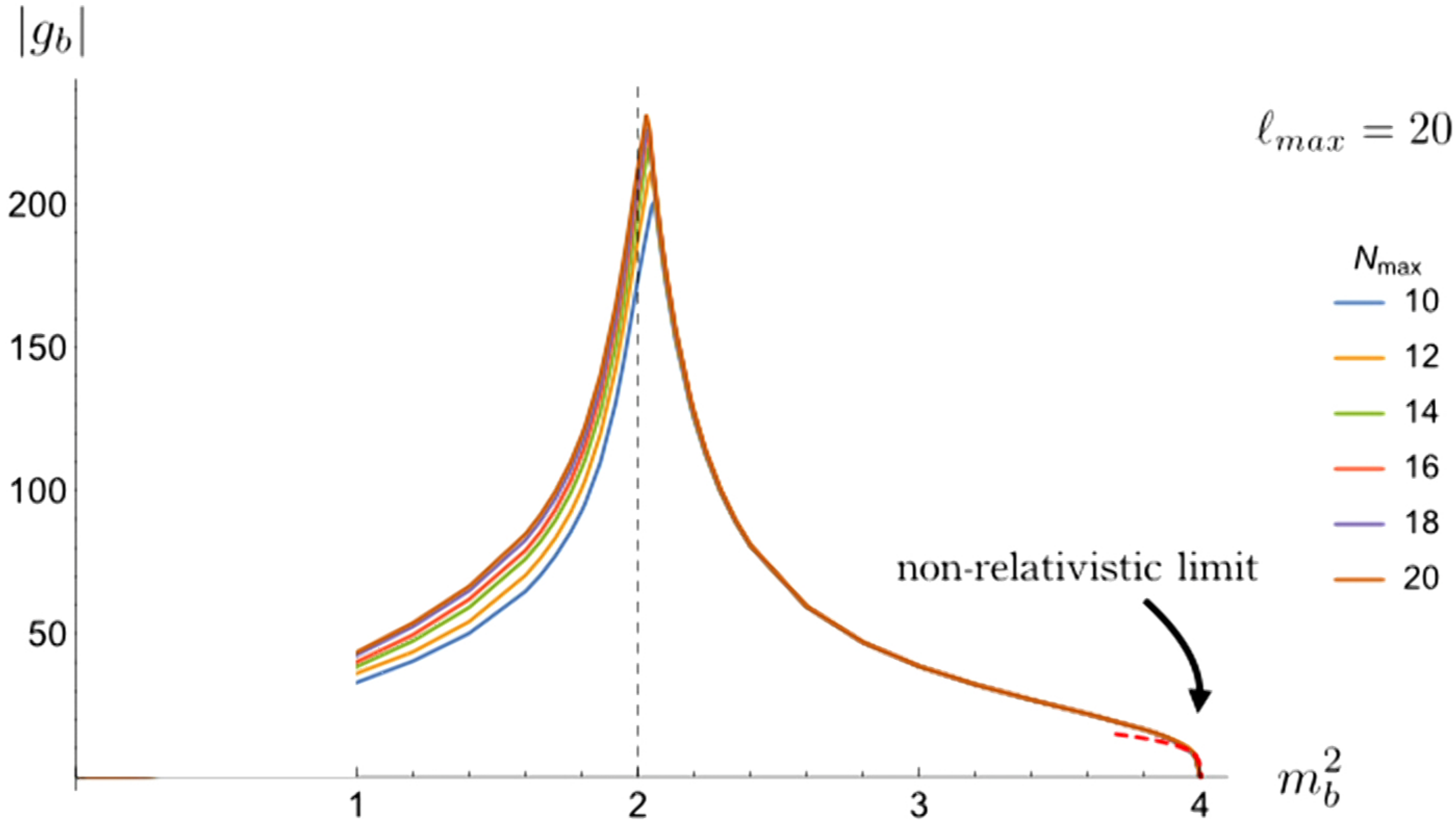
Unitarity: $|S_\ell(s)|^2 \leq 1\,, \qquad s > 4m^2\,, \quad \ell = 0, 2, 4, \ldots \ell_{\max}$

$\Rightarrow$ Quadratic constraints on the variables $\{g_b^2, \alpha_{(abc)}\}$

$a + b + c \leq N_{\max}$

Maximal cubic coupling in 3+1 QFT



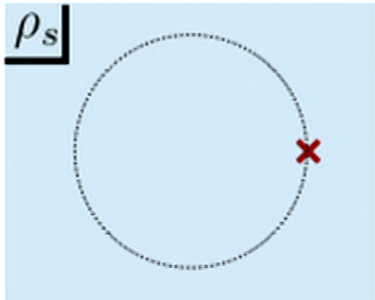
Maximal quartic coupling

Ansatz with **no poles**. Maximize $\lambda = \frac{1}{32\pi} T(s = t = u = \frac{4}{3} m^2)$
$(\text{e.g.} \quad \pi^0 \pi^0 \to \pi^0 \pi^0)$

Maximal quartic coupling

Improved ansatz with threshold bound state:

$$T(s,t,u) = \beta\left(\frac{1}{\rho_s - 1} + \frac{1}{\rho_t - 1} + \frac{1}{\rho_u - 1}\right) + \sum_{\substack{a,b,c=0 \\ a+b+c \leq N_{\max}}} \alpha_{(abc)} \rho_s^a \rho_t^b \rho_u^c$$



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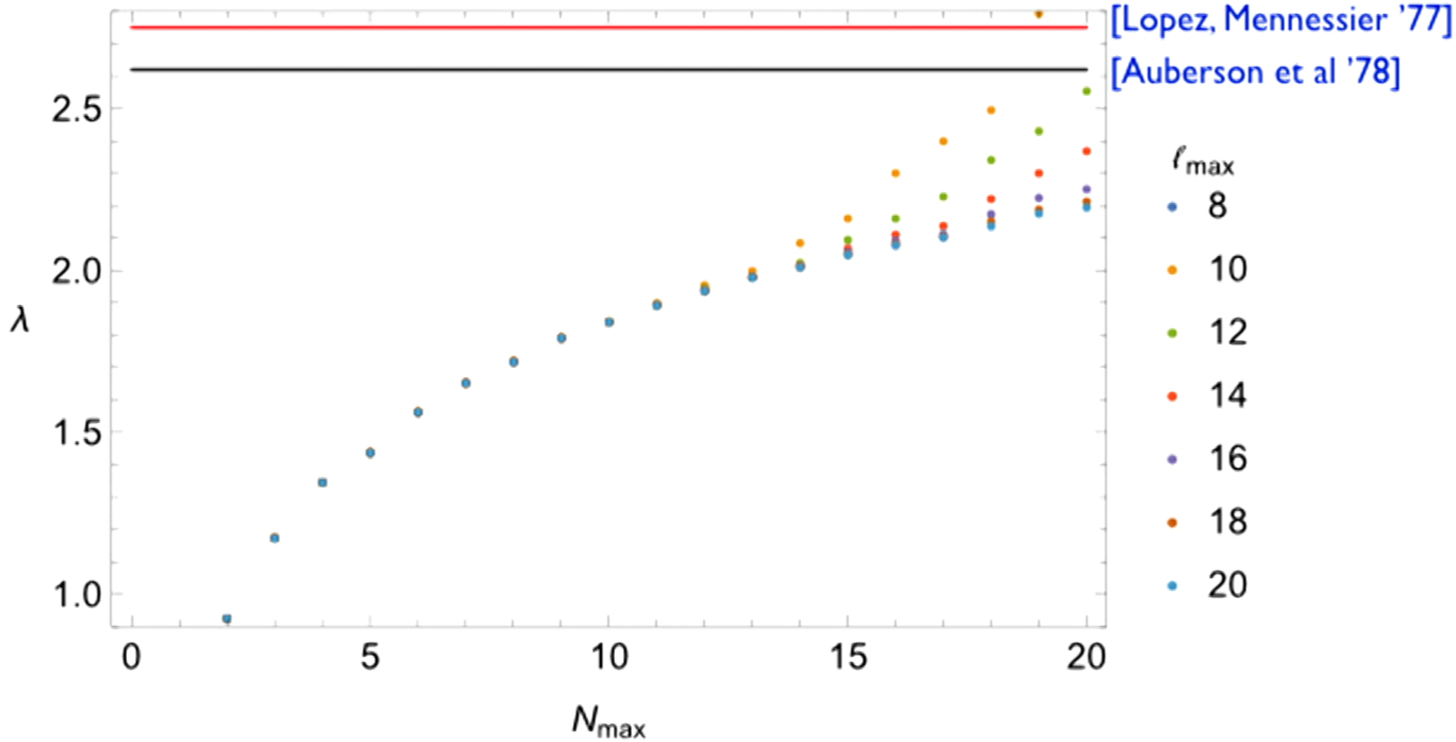

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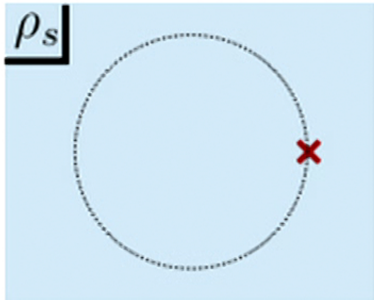
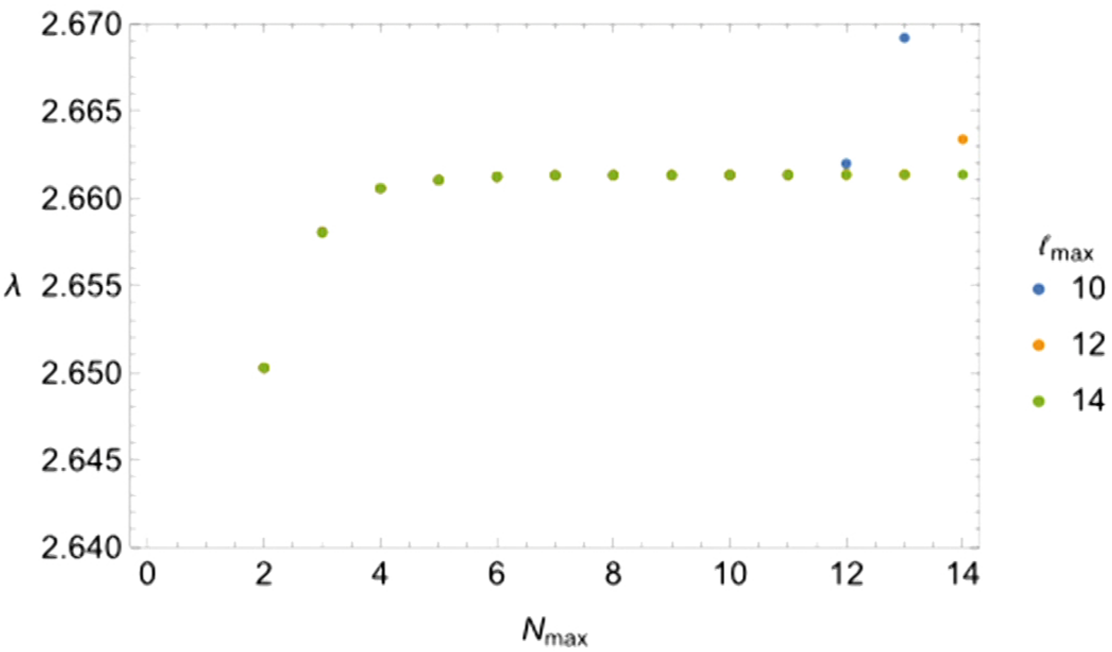
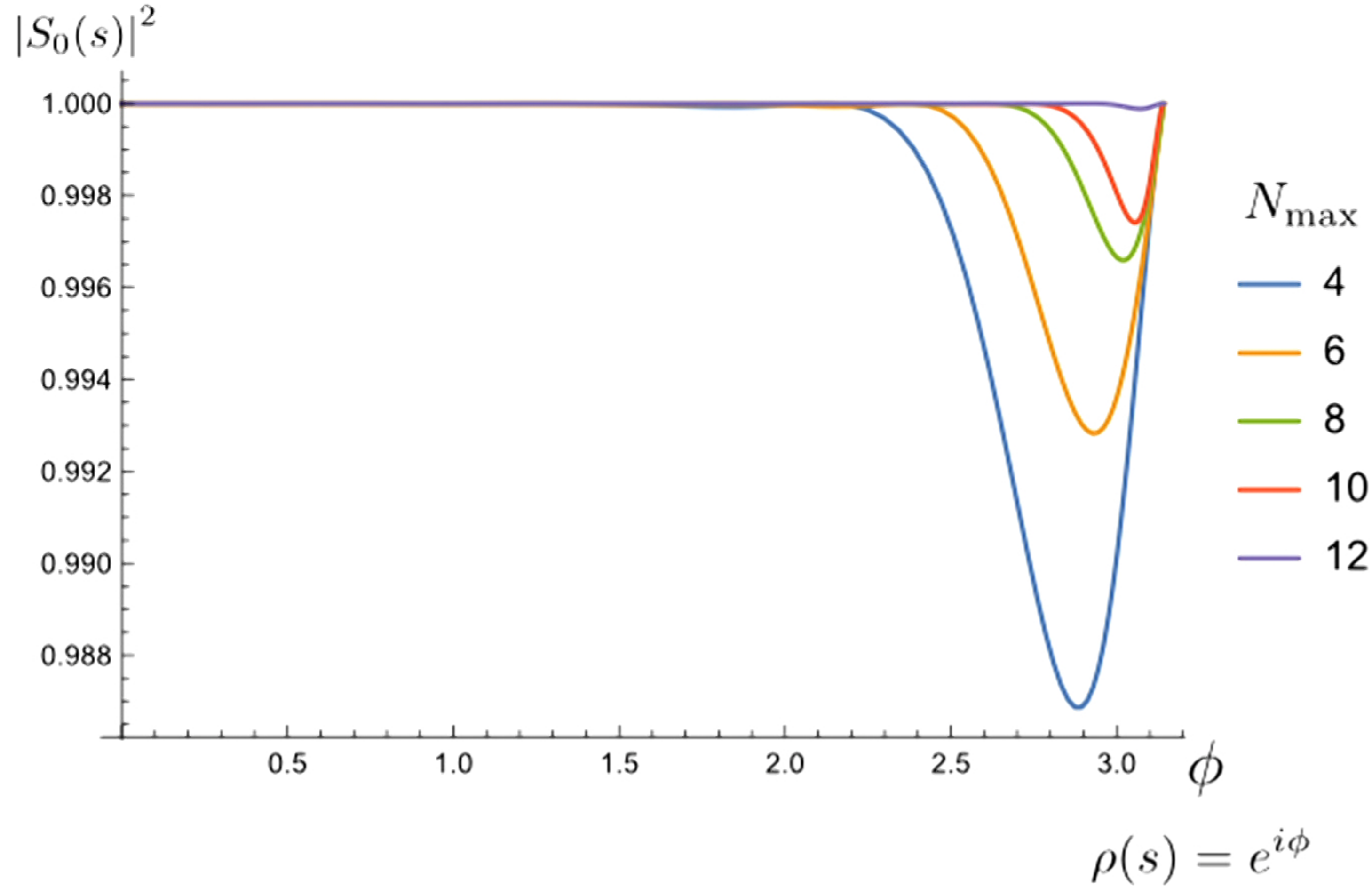




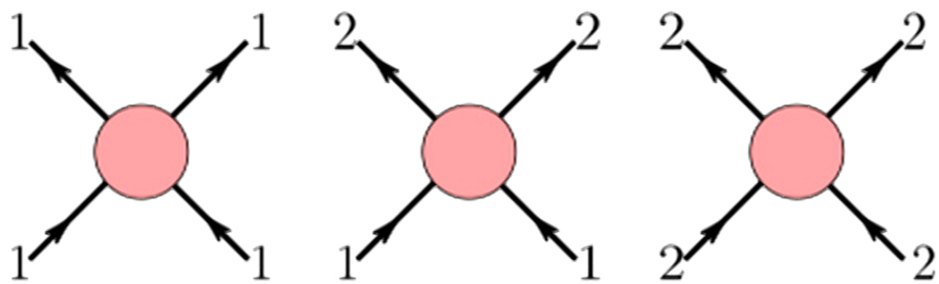
No particle production?



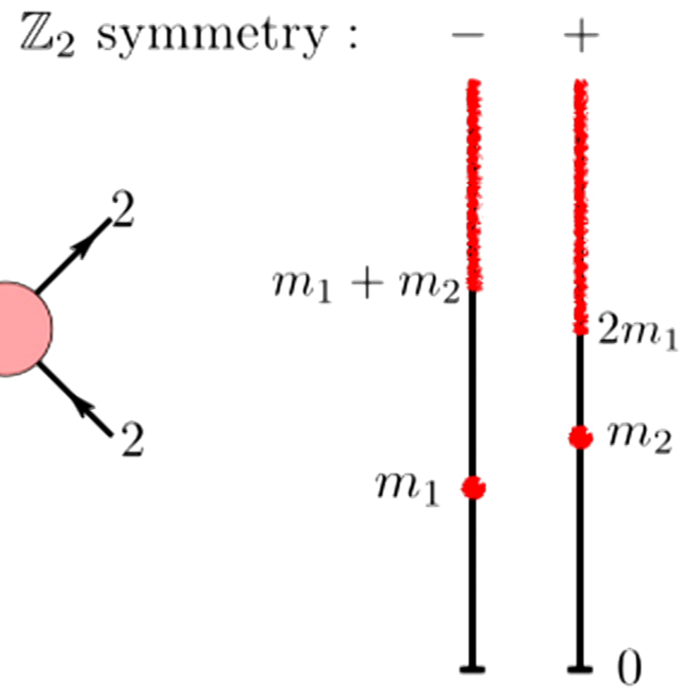
Multiple Amplitudes Bootstrap in 2D QFT

2 to 2 Scattering Amplitudes

Example: two stable particles

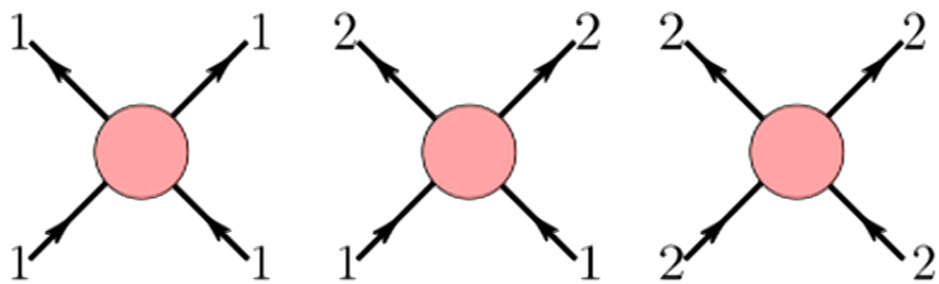


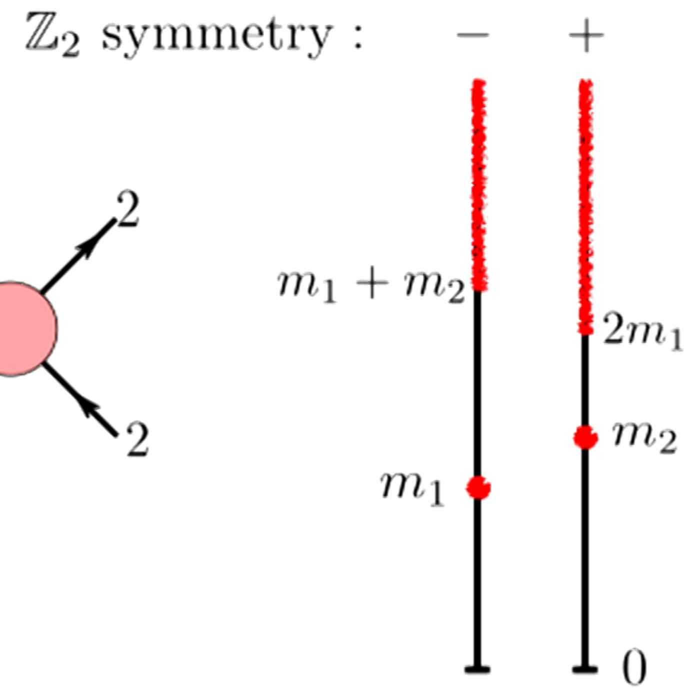
Maximize g_{112}^2



2 to 2 Scattering Amplitudes

Example: two stable particles





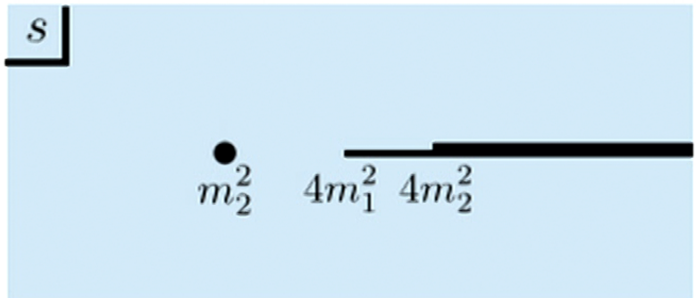
Maximize g_{112}^2

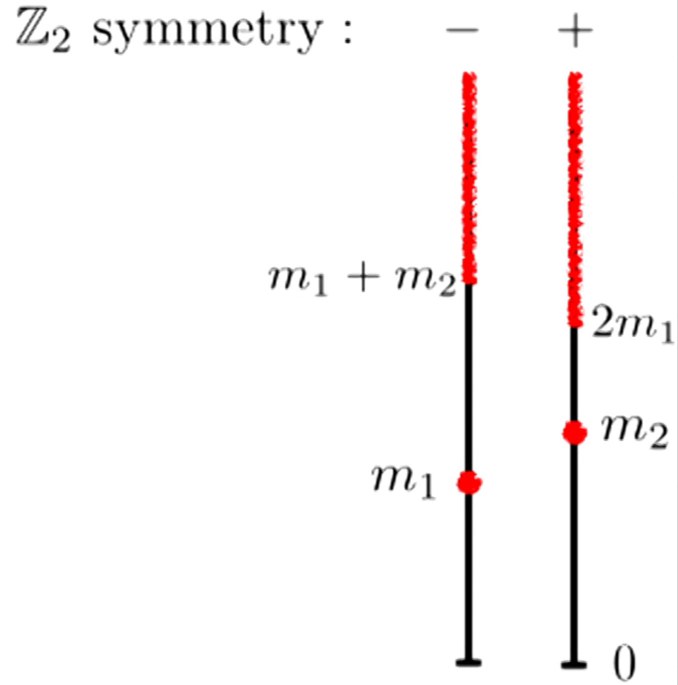
Unitarity: $|S_{11\to11}|^2 + |S_{11\to22}|^2 \leq 1$

Not zero in optimal solution

Extended Unitarity

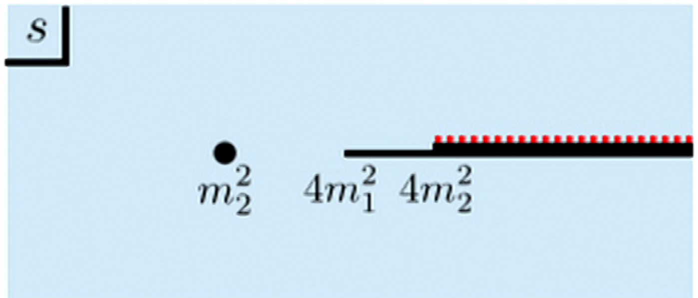
Analyticity:

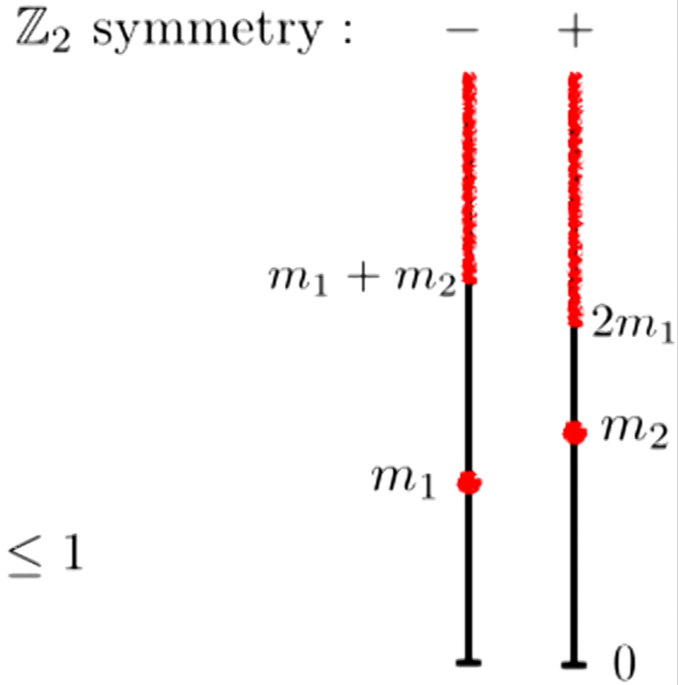




Extended Unitarity

Analyticity:



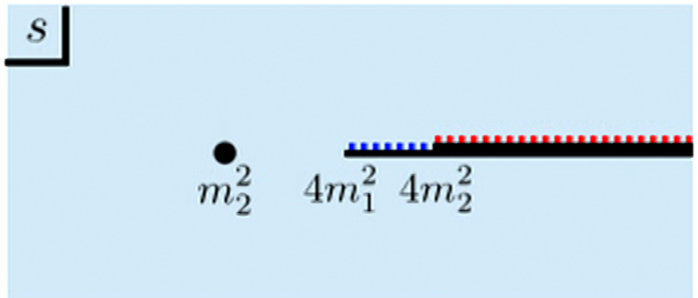


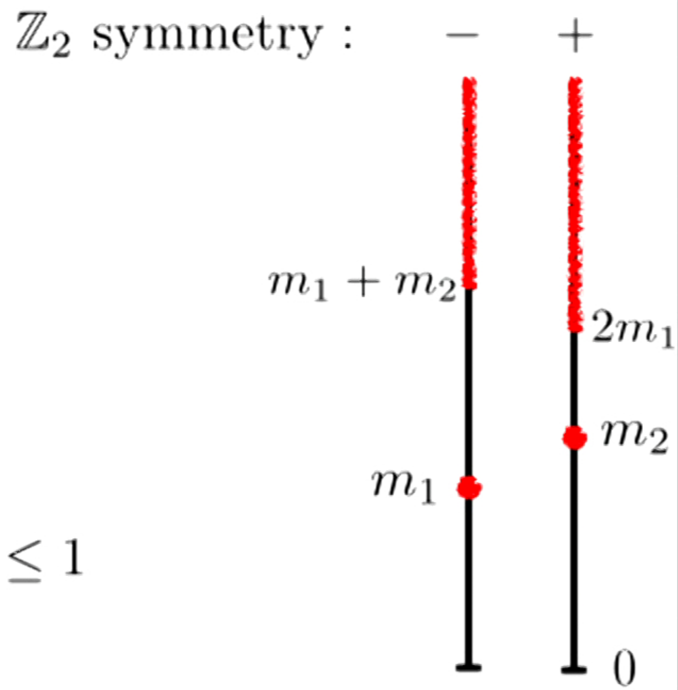
Unitarity: $|S_{22\to11}|^2 + |S_{22\to22}|^2 \leq 1$

$s \geq 4m_2^2$

Extended Unitarity

Analyticity:





Unitarity: $|S_{22\to 11}|^2 + |S_{22\to 22}|^2 \leq 1$

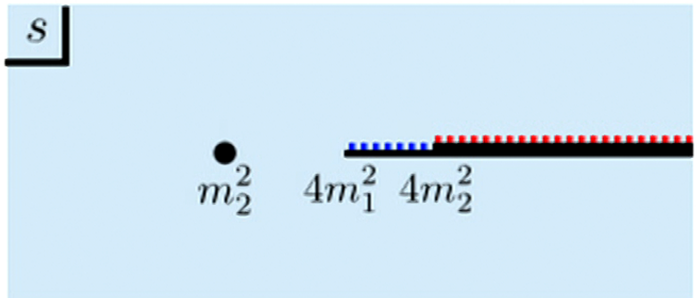
$s \geq 4m_2^2$

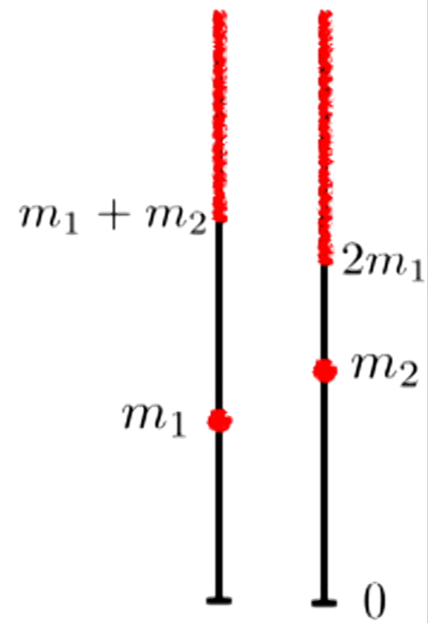
Extended Unitarity: $2\mathrm{Im}T_{22\to 22} = \dfrac{|T_{22\to 11}|^2}{2\sqrt{s(s-4m_1^2)}}$

$4m_1^2 \leq s \leq 4m_2^2$

Extended Unitarity

Analyticity:





Unitarity: $|S_{22\to11}|^2 + |S_{22\to22}|^2 \leq 1$

$s \geq 4m_2^2$

Extended Unitarity: $4m_1^2 \leq s \leq 4m_2^2$

$$2\text{Im}T_{22\to22} = \frac{|T_{22\to11}|^2}{2\sqrt{s(s-4m_1^2)}}$$

3-state Potts model saturates the bound for $m_2 = m_1$ and $\frac{g_{222}}{g_{112}} = -1$

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- Particles with flavour (global symmetries) [He, Irrgang, Kruczenski '18] [Cordova, Vieira '18] [Paulos, Zheng '18]
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