Title: Anomalous Dimensions for Conserved Currents from Holographic Dilatonic Models to Superconductivity

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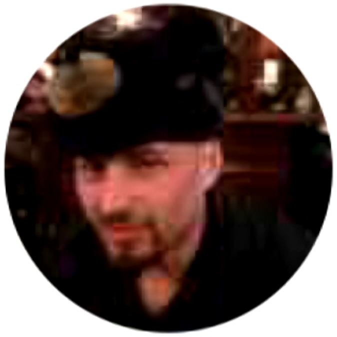
Abstract: It is well known that the dimension of conserved currents is determined simply from dimensional analysis. However, a recent proposal is that what is strange about the conserved currents in the strange metal in the cuprate superconductors is that they carry anomalous dimensions. The basic model invoked to exhibit such behaviour is a holographic dilatonic one in which the field strength couples to the radial coordinate. I will show that the anomalous dimension in such cases arises from a fractional electromagnetism that can be thought of as a general loop-hole in Noether's second theorem. The general mechanism operative is a mass term in the IR that couples to the UV current. Such a mass that couples to the radial component of the gauge field introduces a breaking of U(1) everywhere except at the boundary. I will also show that even the Pippard kernel invoked to explain the Meissner effect in traditional low-temperature superconductors is a special case of the non-local action found here, implying that symmetry breaking is the general mechanism for fractional electromagnetisms. I will also construct the Virasoro algebra for such fractional currents and discuss the general implications for the bulk-boundary construction in holography.

`Anomalous dimensions' for conserved currents from holographic dilatonic models to superconductivity

Thanks to: NSF, EFRC (DOE)

Gabriele La Nave



Kridsangaphong Limtragool

standard electricity and magnetism

$A \to A + \partial\Lambda$ gauge invariance

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$$S = \int d^d x (J_\mu A^\mu + \cdots)$$

standard electricity and magnetism

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gauge invariance

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$$S \to S + \int d^d x \cancel{J_\mu \partial\Lambda}$$

$$\partial_\mu J^\mu = 0$$

Noether's Thm. I

current conservation

standard electricity and magnetism

$$A \to A + \partial\Lambda$$

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$$S = \int d^d x (J_\mu A^\mu + \cdots)$$

$$[A] = 1$$

fixes dimension of current

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$$S \to S + \int d^d \cancel{x J_\mu} \partial\Lambda$$

$$[d^d x J A] = 0$$

$$\partial_\mu J^\mu = 0$$

Noether's Thm. I

current conservation

$$[J] = d - 1$$

Are there exceptions?

Superconductivity for Particular Theorists*)

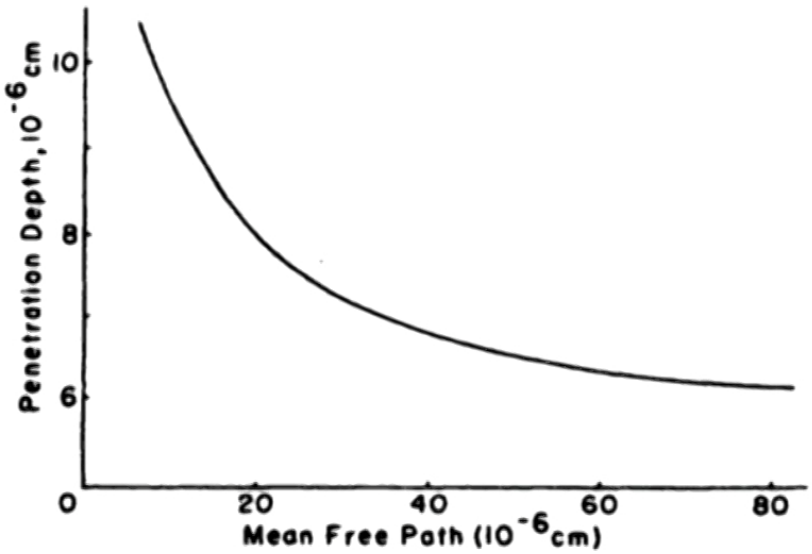
Steven WEINBERG

Theory Group, Department of Physics, University of Texas, Austin, TX 78712

(Received December 10, 1985)

No one did more than Nambu to bring the idea of spontaneously broken symmetries to the attention of elementary particle physicists. And, as he acknowledged in his ground-breaking 1960 article "Axial Current Conservation in Weak Interactions", Nambu was guided in this work by an analogy with the theory of superconductivity, to which Nambu himself had made important contributions. It therefore seems appropriate to honor Nambu on his birthday with a little pedagogical essay on superconductivity, whose inspiration comes from experience with broken symmetries in particle theory. I doubt if anything in this article will be new to the experts, least of all to Nambu, but perhaps it may help others, who like myself are more at home at high energy than at low temperature, to appreciate the lessons of superconductivity.

Pippard's problem



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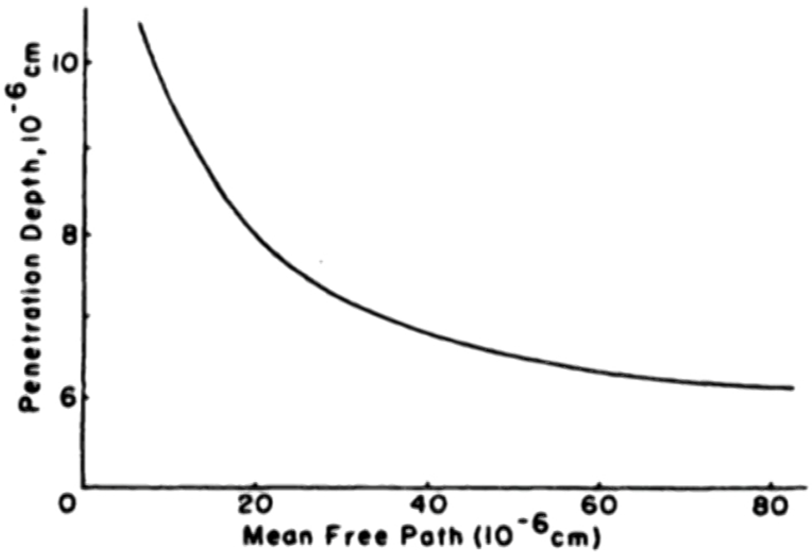
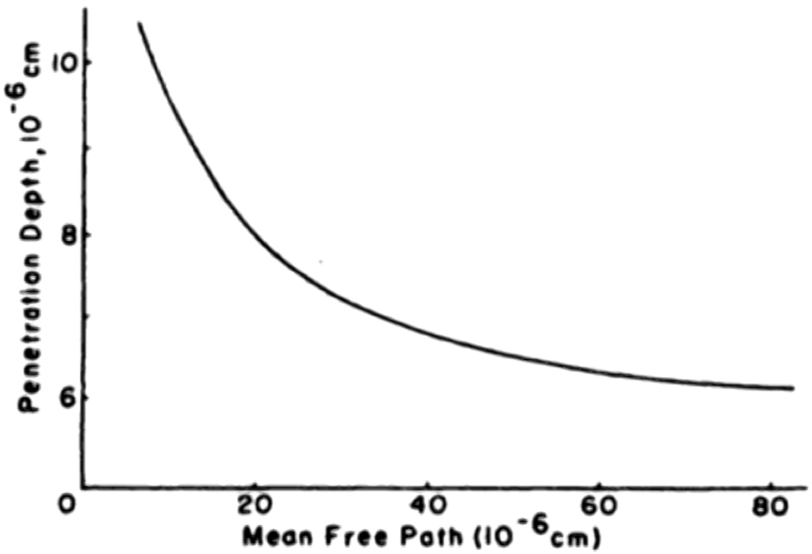


$$J_s \neq \frac{-c}{4\pi\lambda^2} A$$

London Eq.

Pippard's problem



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London Eq.

failure of local London relations

Superconductivity ala Weinberg

$$\boxed{\begin{gathered} U(1) \to \mathbb{Z}_2 \\ A_\mu - \partial_\mu \phi = 0 \end{gathered}}$$

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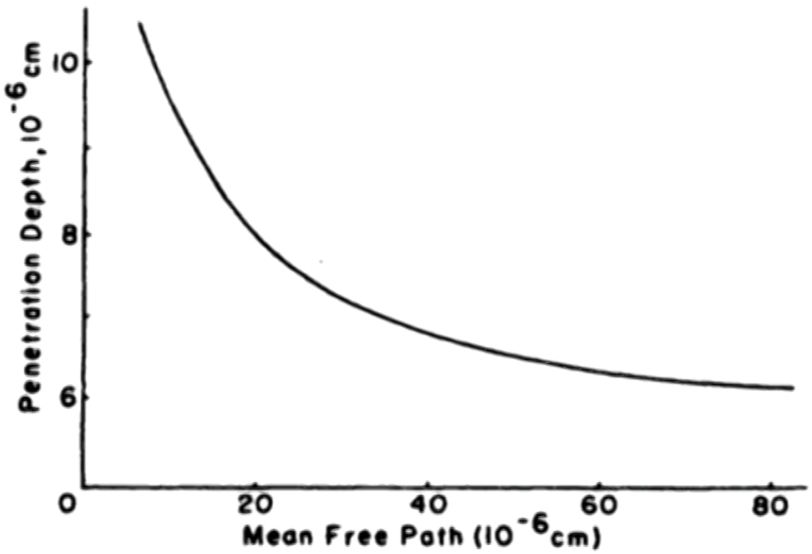

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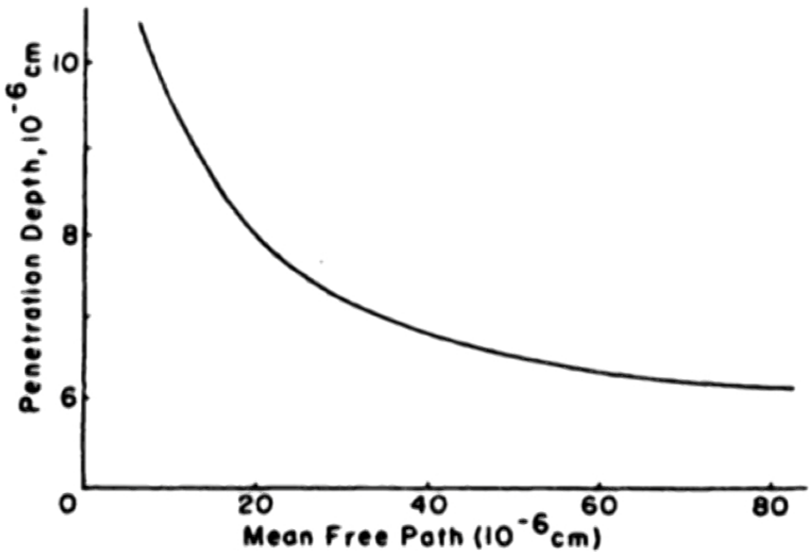

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$U(1)/\mathbb{Z}_2$

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$$\nabla \phi - A = 0$$

stable equilibrium

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stable equilibrium

around minimum

$$L_m = L_{m0} - \frac{1}{2} \int C^{\mu\nu}(\mathbf{x}, \mathbf{x}')(\mathbf{A}_\mu(\mathbf{x}) - \partial_\mu \phi(\mathbf{x}))$$
$$\times (A_\nu(\mathbf{x}') - \partial_\nu \phi(\mathbf{x}'))\mathbf{d^3x'd^3x} + \cdots$$

Pippard Current

$$J_\mu(\mathbf{x}) = \frac{\delta \mathbf{L_m}}{\delta \mathbf{A}_\mu} = -\int \mathbf{C}^{\mu\nu}(\mathbf{x},\mathbf{x}')(\mathbf{A}_\nu(\mathbf{x}') - \partial_\nu \phi(\mathbf{x}'))\mathbf{d^3x}'$$

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Pippard kernel

$$J_s = -\frac{3}{4\pi c \xi_0 \lambda} \int \frac{(\vec{r}-\vec{r}')((\vec{r}-\vec{r}')\cdot \vec{A}(\vec{r}'))e^{-(\vec{r}-\vec{r}')/\xi(\ell)}}{(\vec{r}-\vec{r}')^4} d^3\vec{r}'$$

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non-local

magnetic energies

$$C\xi^3 L^3 A^2 = C\xi^3 L^5 B^2$$

magnetic energies

$$C\xi^3 L^3 A^2 = C\xi^3 L^5 B^2 = C\xi^3 L^2 (\underbrace{L^3 B^2}_{\text{expulsion energy}})$$

Meissner Effect $C\xi^3 L^2 \gg 1$

Units of Current

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$$[J] = d - d_C - d_A$$

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anomalous dimension

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anomalous dimension

Standard Result

$$\int d^3x J_\mu A^\mu$$

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Are there other examples of currents with anomalous dimensions?

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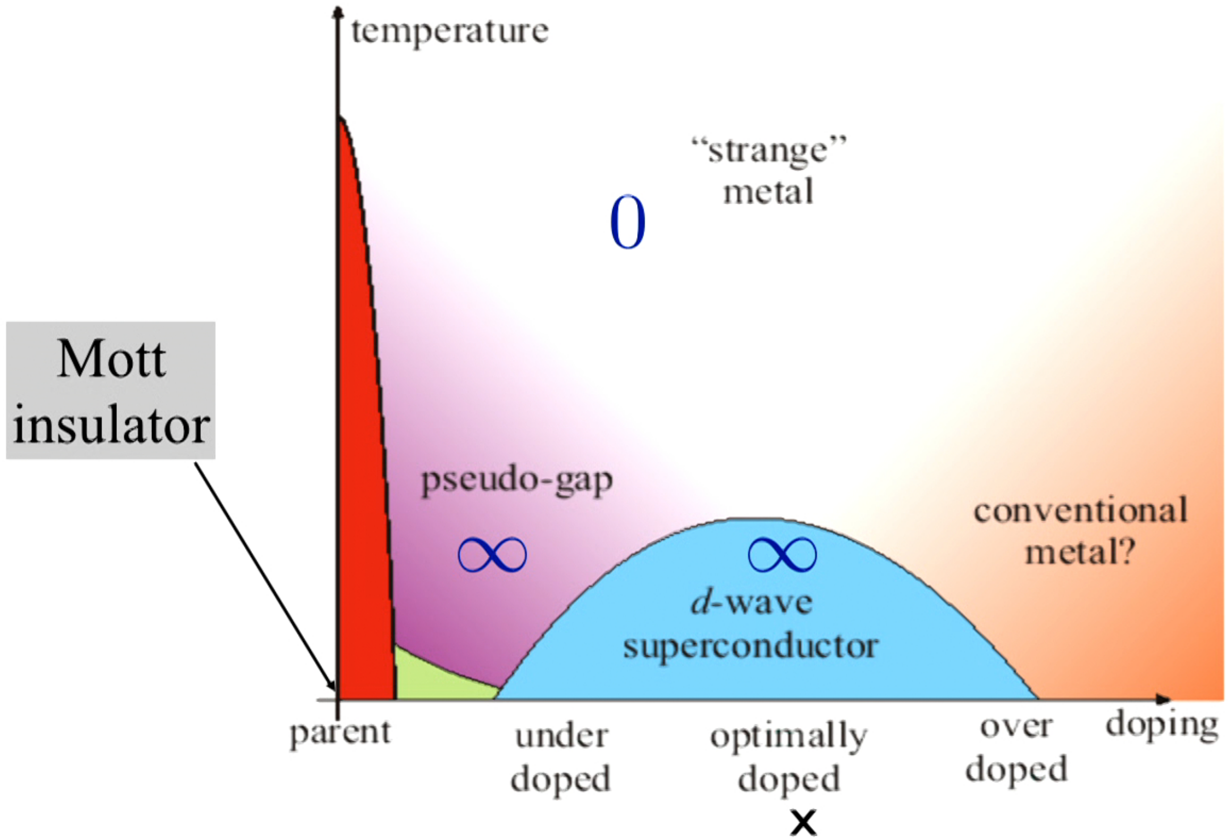
temperature
"strange"
metal
0
Mott
insulator
pseudo-gap
∞
∞
d-wave
superconductor
conventional
metal?
parent
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doped
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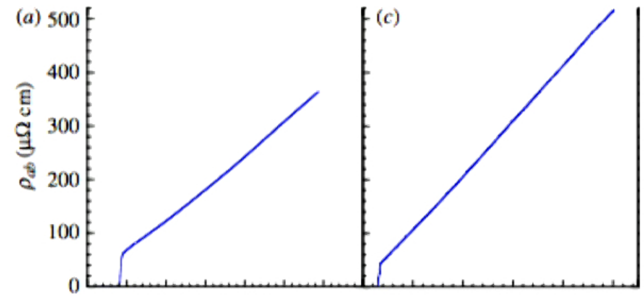
doped
doping
x

strange metal: experimental facts

T-linear resistivity

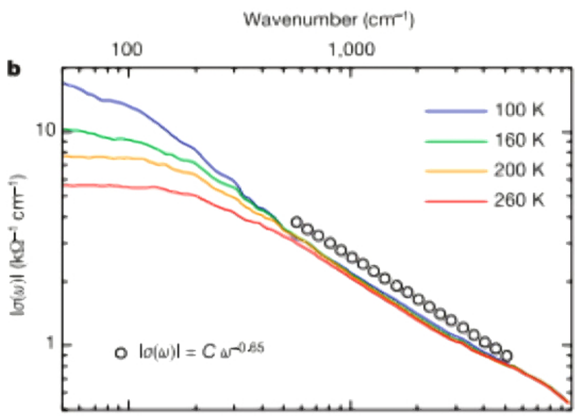


strange metal: experimental facts

Quantum critical behaviour in a high-T_c superconductor

D. van der Marel[1]*, H. J. A. Molegraaf[1]*, J. Zaanen[2], Z. Nussinov[2]*, F. Carbone[1]*, A. Damascelli[3]*, H. Eisaki[3]*, M. Greven[3], P. H. Kes[2] & M. Li[2]

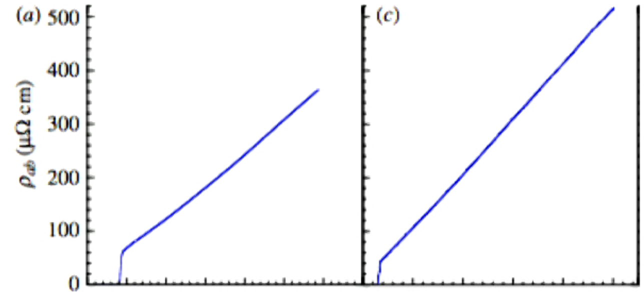
[1] *Materials Science Centre, University of Groningen, 9747 AG Groningen, The Netherlands*
[2] *Leiden Institute of Physics, Leiden University, 2300 RA Leiden, The Netherlands*
[3] *Department of Applied Physics and Stanford Synchrotron Radiation Laboratory, Stanford University, California 94305, USA*



$$\sigma(\omega) = C\omega^{-\frac{2}{3}}$$

$$\frac{n\tau e^2}{m}\frac{1}{1-i\omega\tau}$$

T-linear resistivity



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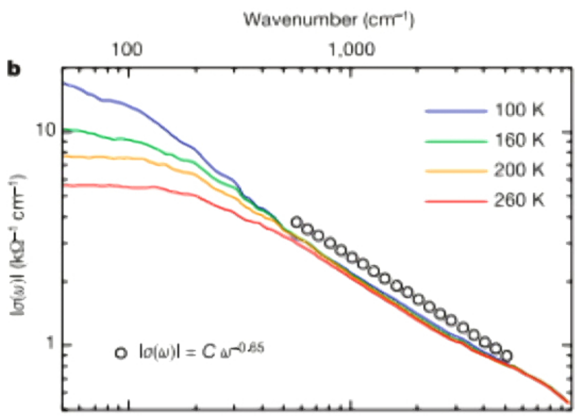

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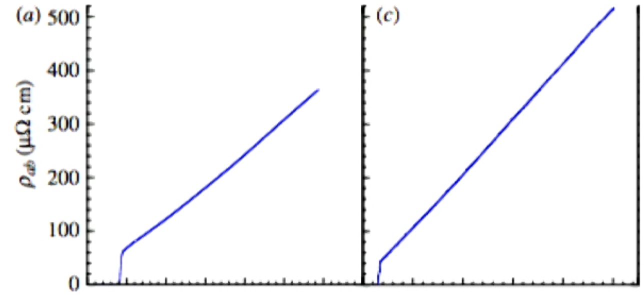
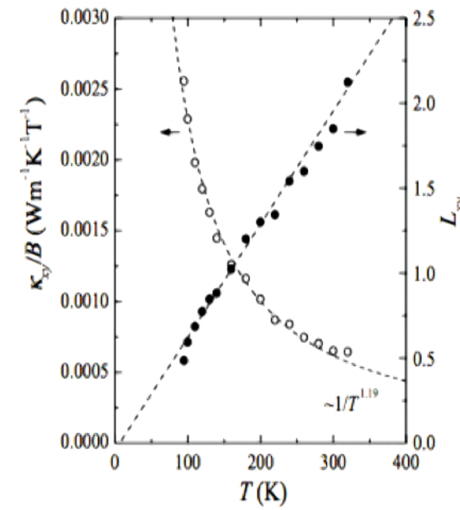
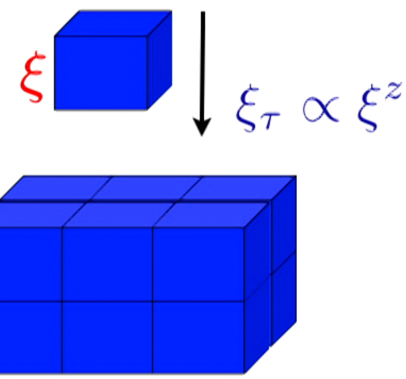


$$L_{xy} = \kappa_{xy}/T\sigma_{xy} \neq \# \propto T$$



why is the problem hard?

single-parameter scaling



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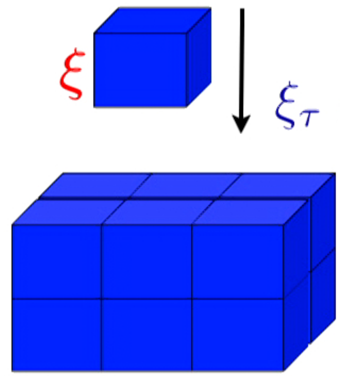


$$\xi_\tau \propto \xi^z$$

$$\frac{\delta^2 \ln Z}{\omega \delta A_\mu \delta A_\mu}$$

$$1$$

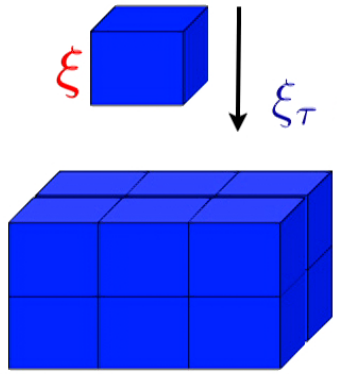
$$\rho \propto T^{(2-d)/z}$$

$$\sigma(\omega, T) \propto \omega^{(d-2)/z}$$

$$C_v \propto T^{d/z}$$

single-parameter scaling

ξ



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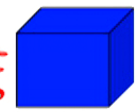
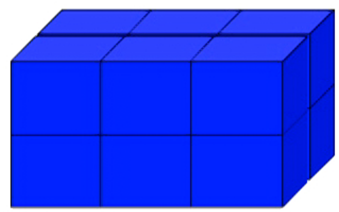
ξ 

$\xi_\tau \propto \xi^z$



$$\frac{\delta^2 \ln Z}{\omega \delta A_\mu \delta A_\mu}$$

$\rho \propto T^{(2-d)/z}$ $\rightarrow 1$

$\sigma(\omega, T) \propto \omega^{(d-2)/z}$ $\longrightarrow -2/3$

$C_v \propto T^{d/z}$

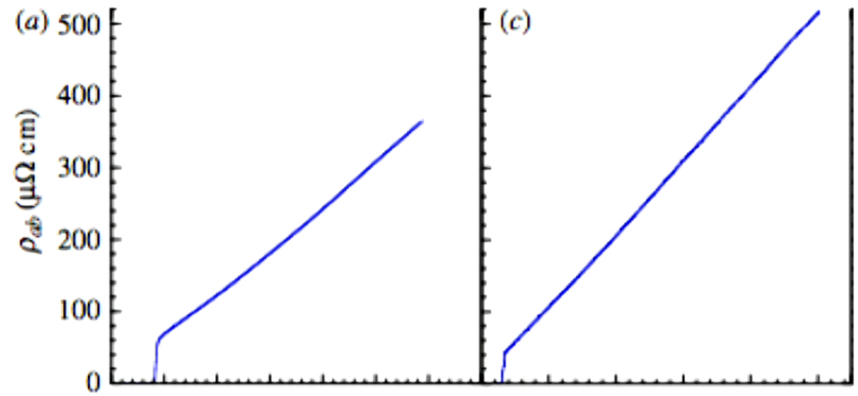
anomalous dimension $2 \rightarrow 2d_A$

strange metal explained!

Hall Angle

$$\cot\theta_H \equiv \frac{\sigma_{xx}}{\sigma_{xy}} \approx T^2$$

T-linear resistivity



Hall Lorenz ratio

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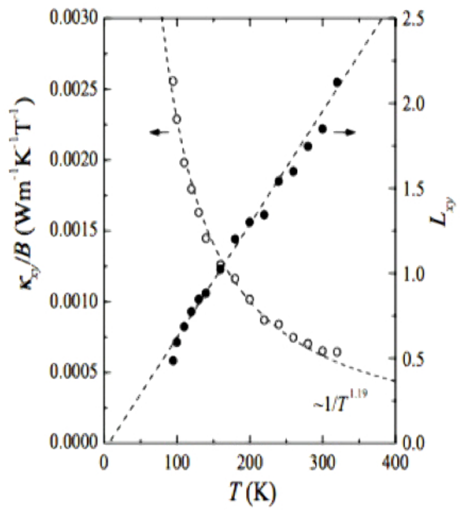

all explained if

$$[J_\mu] = d - \theta + \Phi + z - 1$$

Hartnoll/Karch

$$[A_\mu] = 1 - \Phi$$
$$\Phi = -2/3$$

strange metal

$$[J_\mu] = d - \theta + \Phi + z - 1$$

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$$[E] = 1 + z - \Phi$$

$$[B] = 2 - \Phi$$

strange metal

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$\boxed{\text{note}}$ $\pi r^2 B \neq$ flux

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note $\pi r^2 B \neq$ flux

How is this possible - - if at all?

if

$$[A_\mu] \neq 1$$

if

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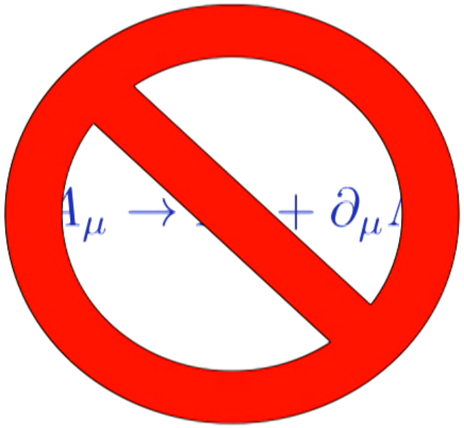
$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

if

$$[A_\mu] \neq 1$$



what is the new gauge principle?

if

$$[A_\mu] \neq 1$$

$$\cancel{A_\mu \to \quad + \partial_\mu}$$

hint

$$\partial_\mu J^\mu = 0$$

hint

$$\partial_\mu J^\mu = 0$$

current conservation

what if

$$[\partial_\mu, \hat{Y}] = 0$$

hint

$$\partial_\mu J^\mu = 0$$

current conservation

what if

$$[\partial_\mu, \hat{Y}] = 0$$

$$\partial_\mu \hat{Y} J^\mu = \partial_\mu \tilde{J}^\mu = 0$$

hint

$$\partial_\mu J^\mu = 0$$

current conservation

what if

$$[\partial_\mu, \hat{Y}] = 0$$

new current

$$[\tilde{J}] = d - 1 - D_Y$$

$$\partial_\mu \hat{Y} J^\mu = \partial_\mu \tilde{J}^\mu = 0$$

possible gauge transformations

$$S = -\frac{1}{4}\int d^d x F^2$$

possible gauge transformations

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$$S = \frac{1}{2}\int \frac{d^d k}{2\pi^d} A_\mu(k)[k^2\eta^{\mu\nu} - k^\mu k^\nu]A_\nu(k)$$

possible gauge transformations

$$S = -\frac{1}{4}\int d^d x F^2$$

$$\Downarrow$$

$$S = \frac{1}{2}\int \frac{d^d k}{2\pi^d} A_\mu(k)[\underbrace{k^2\eta^{\mu\nu} - k^\mu k^\nu}_{M_{\mu\nu}}]A_\nu(k)$$

possible gauge transformations

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$$M_{\mu\nu}k^\nu = 0$$

zero eigenvector

possible gauge transformations

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$$S = \frac{1}{2}\int \frac{d^d k}{2\pi^d} A_\mu(k)[\underbrace{k^2\eta^{\mu\nu} - k^\mu k^\nu}_{M_{\mu\nu}k^\nu = 0}]A_\nu(k)$$

zero eigenvector

$$ik_\mu \to \partial_\nu$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda$$

family of zero eigenvalues

$$M_{\mu\nu} f k^{\nu} = 0$$

family of zero eigenvalues

$$\underbrace{M_{\mu\nu} f k^{\nu}}_{\text{generator of gauge symmetry}} = 0$$

family of zero eigenvalues

$$M_{\mu\nu} \underbrace{f k^{\nu}} = 0$$

generator of gauge symmetry

1.) rotational invariance

family of zero eigenvalues

$$\underbrace{M_{\mu\nu} f k^{\nu}}_{} = 0$$

generator of gauge symmetry

1.) rotational invariance

2.) A is still a 1-form

family of zero eigenvalues

$$\underbrace{M_{\mu\nu} f k^{\nu}}_{} = 0$$

generator of gauge symmetry

1.) rotational invariance

2.) A is still a 1-form

3.) $[f, k_\mu] = 0$

only choice

$$f \equiv f(k^2)$$

$$(\Delta)^{\gamma}$$

$$A_\mu \rightarrow A_\mu + (\Delta)^{\frac{(\gamma-1)}{2}}\partial_\mu \Lambda$$

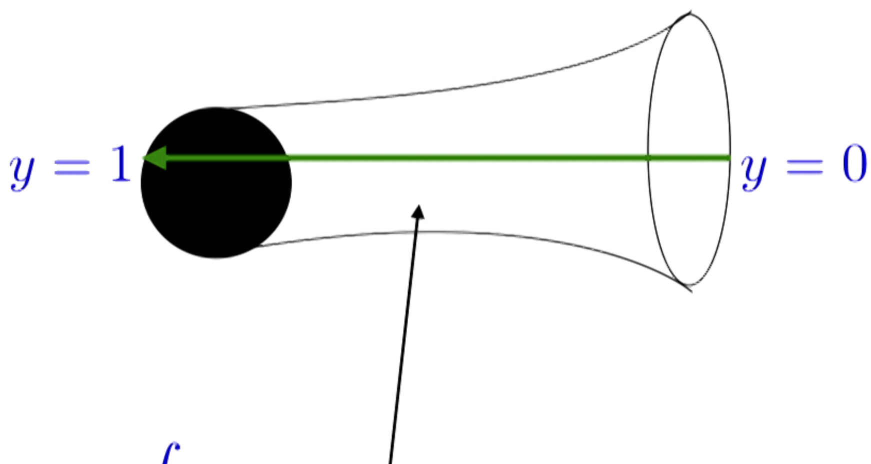
only choice

$$f \equiv f(k^2)$$

$$(\Delta)^{\gamma}$$

$$A_\mu \to A_\mu + (\Delta)^{\frac{(\gamma-1)}{2}} \partial_\mu \Lambda \qquad [A_\mu] = \gamma$$

claim

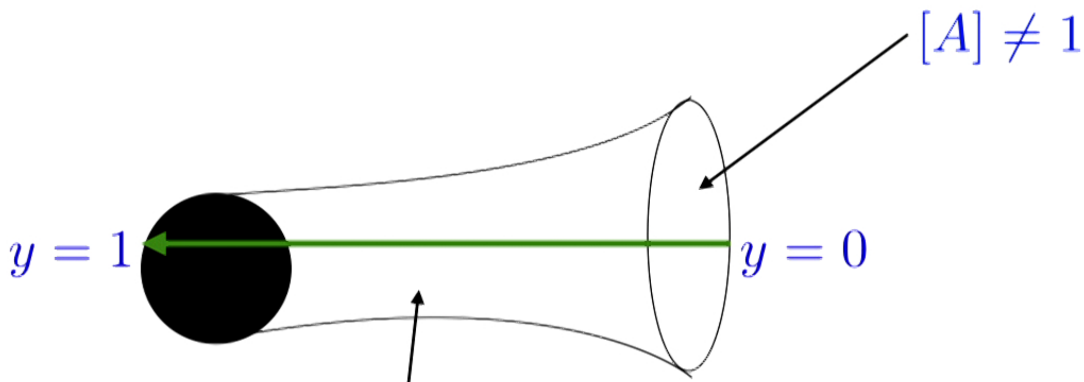


$$S = \int dV_d dy \left(y^a F^2 + \cdots \right)$$

$$F = dA$$

Karch:1405.2926
Gouteraux: 1308.2084

claim



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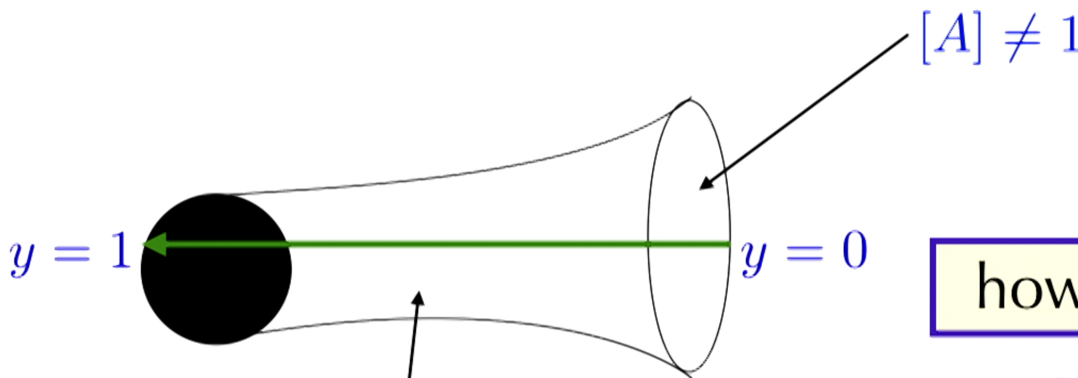

how?

?

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$$F = dA$$

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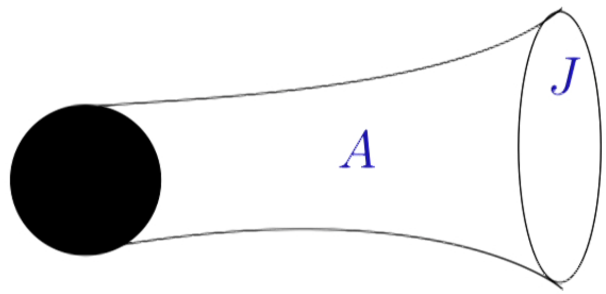
Gouteraux: 1308.2084

if holography is RG then how can it lead to an anomalous dimension?

standard case



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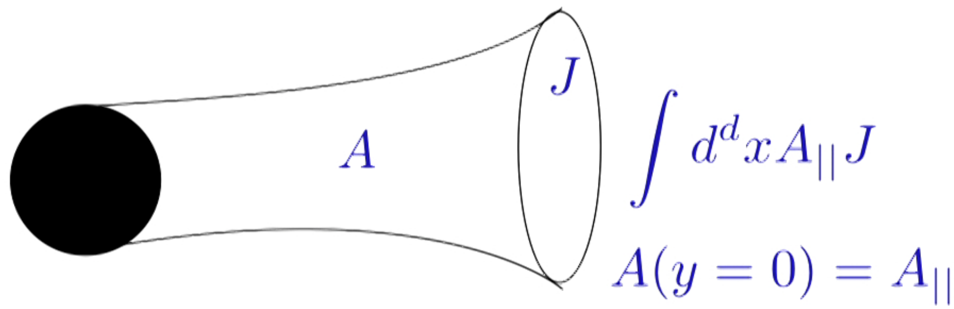

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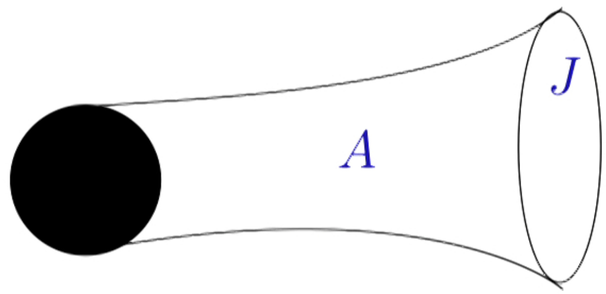

$$\int d^d x A_{||} J$$

$$A(y=0) = A_{||}$$

bc does not satisfy

$$A(y=0) \neq A_{||} + d\Lambda$$

alternatively

$$(A + d\Lambda)_{\partial\Omega} = a + d^{||}\Lambda_{\partial\Omega}$$

alternatively

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boundary theory has non-trivial gauge structure alternatively

$$(A + d\Lambda)_{\partial\Omega} = a + d^{||}\Lambda_{\partial\Omega}$$

boundary theory has non-trivial gauge structure

AdS/ Lifshitz

$$\int dy/y = \infty$$

large gauge transformation

construct boundary
theory explicitly

$$S = \int dV_d dy \left(y^a F^2 + \cdots\right)$$

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$$\Big\downarrow \boxed{\text{eom}}$$

$$d(y^a \star dA) = 0$$

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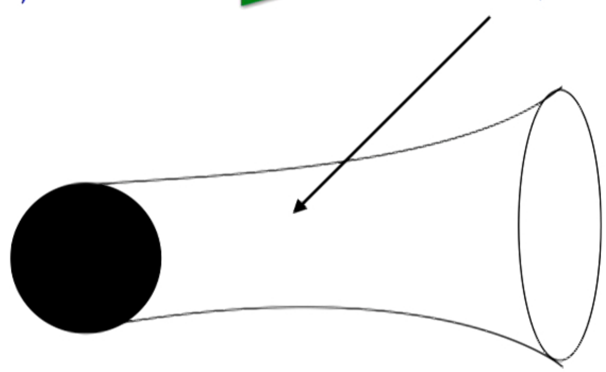

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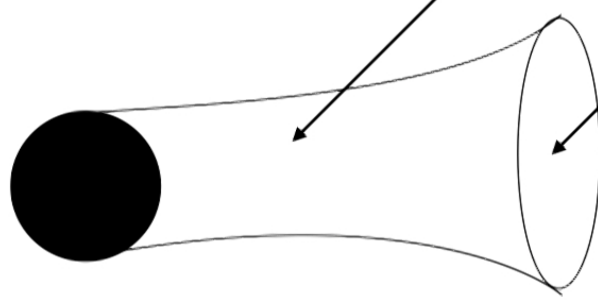
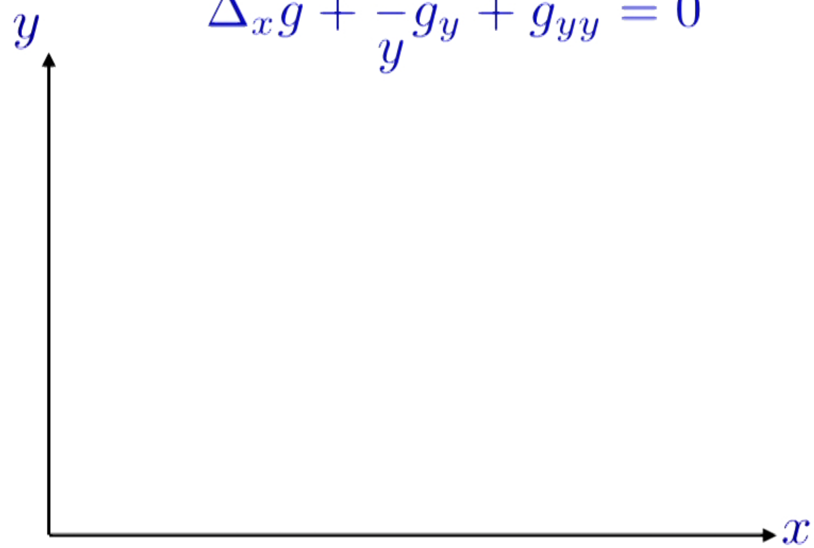


what about the boundary?

Caffarelli-Silvestre extension theorem (2006)

$$g(x, y = 0) = f(x)$$

$$\Delta_x g + \frac{a}{y} g_y + g_{yy} = 0$$



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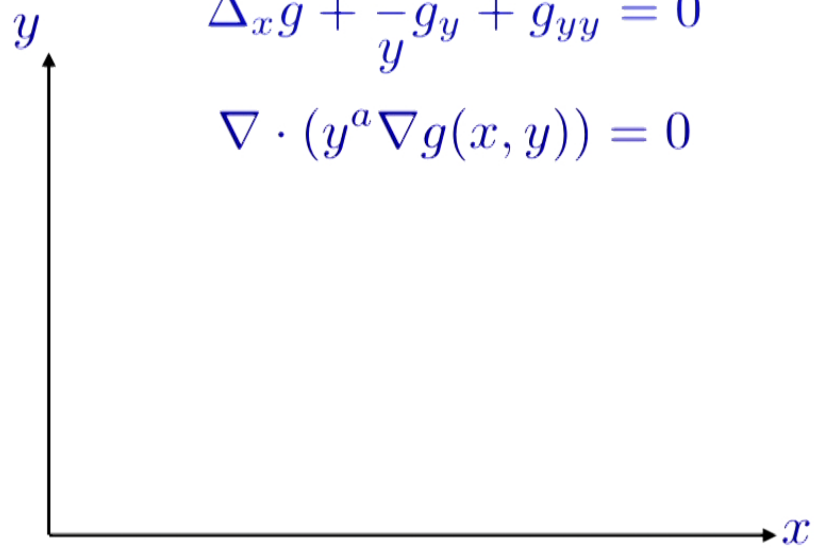

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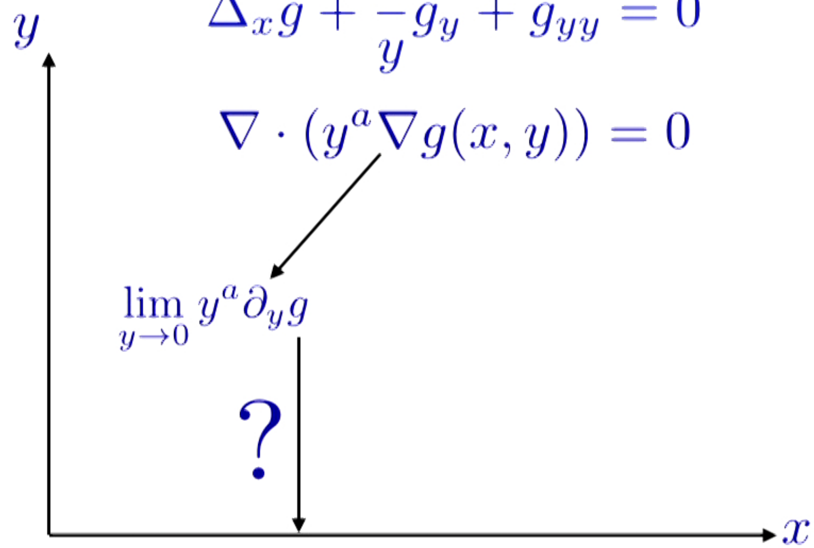

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y

$$\lim_{y \to 0} y^a \partial_y g$$

?

$$C_{d,\gamma}(-\Delta)^\gamma f$$

x

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fractional Laplacian

$$g(z = 0, x) = f(x)$$

$$\gamma = \frac{1-a}{2}$$

closer look

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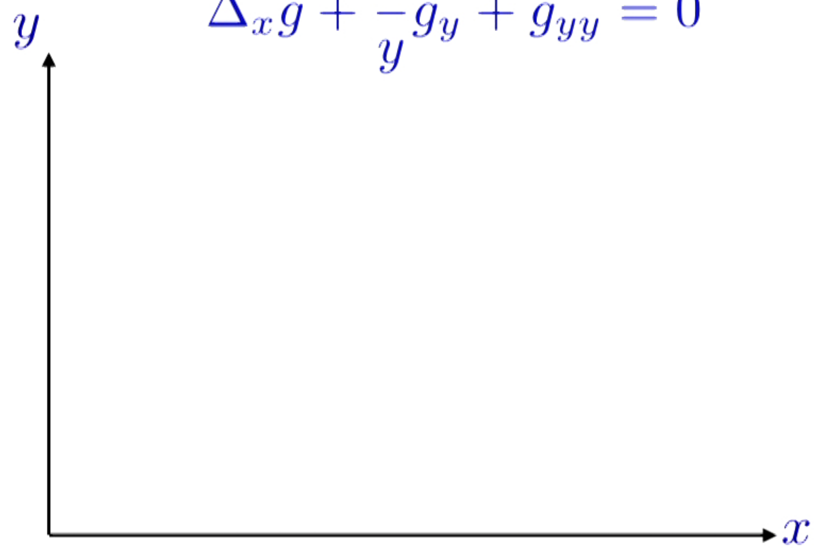

closer look

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scalar field
(use CS theorem)

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similar equations

closer look

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holography

similar equations

generalize CS
theorem to p-forms
GL,PP:1708.00863
(CIMP)

boundary action: fractional Maxwell equations

$$\Delta^{\gamma} A_{\perp} = J$$

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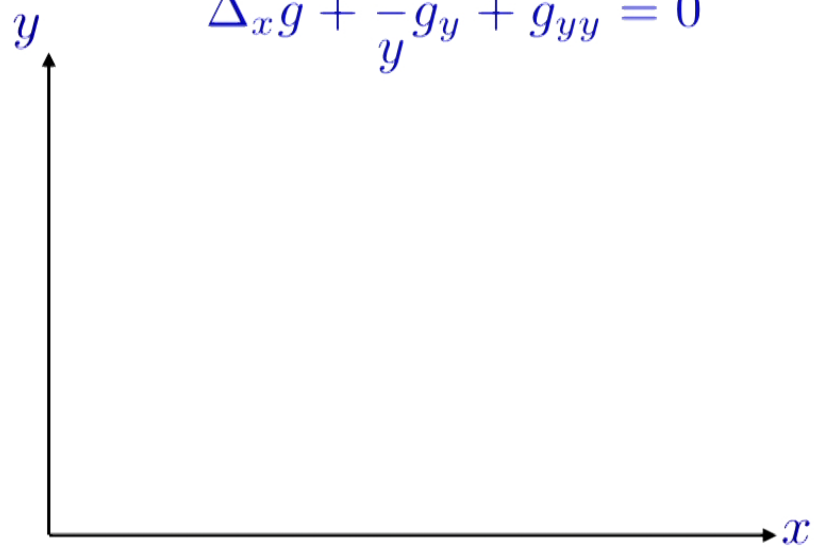

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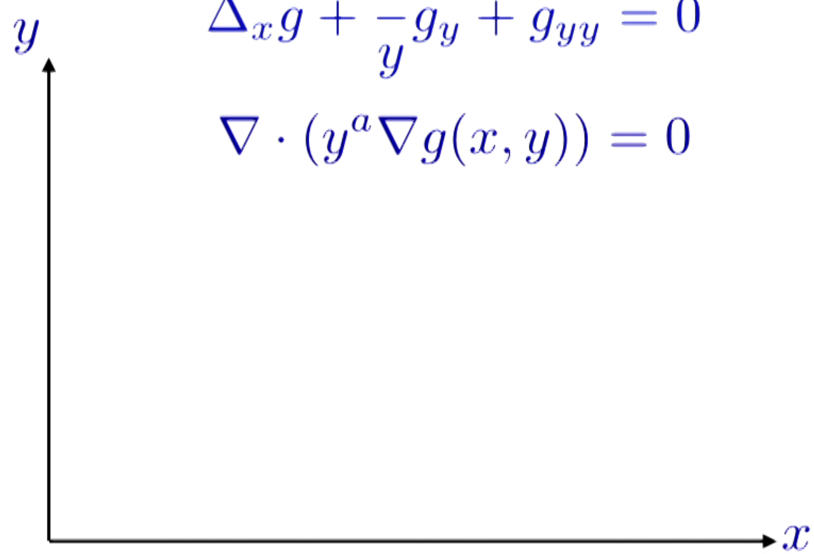

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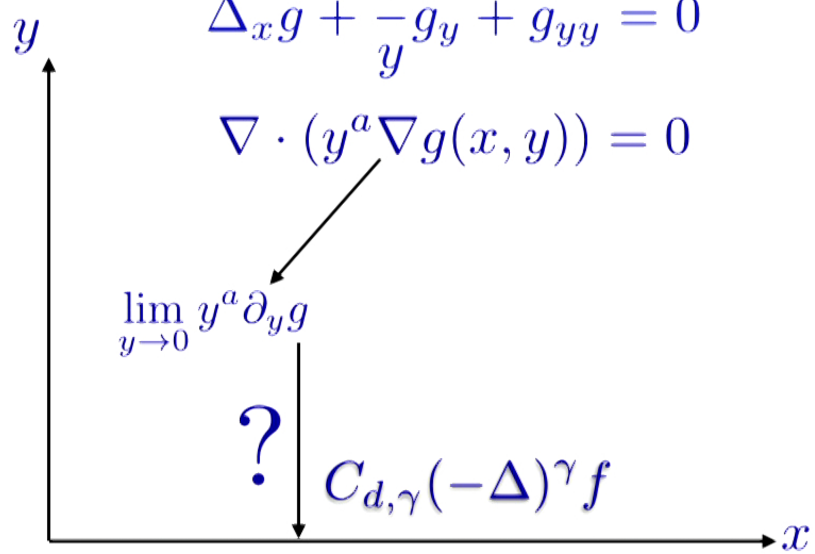

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$$[A] = 1 - a/2$$

fractional differential

$$d_a = \frac{1}{2}(d(d^*d)^{(a-1)/2}\omega + (dd^*)^{(a-1)/2}d\omega)$$

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does not change the order of the form

define

$$F_{ij} = \partial_i^\gamma A_j - \partial_j^\gamma A_i \equiv d_\gamma A = d\Delta^{\frac{\gamma-1}{2}} A,$$

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integrate by parts

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integrate by parts

$$S = \int \frac{1}{2} A_i (-\Delta)^{2\gamma} A^i,$$

non-local boundary action

new gauge transformation

$$A \to A + d_\gamma \Lambda,$$

$$d_\gamma \equiv (\Delta)^{\frac{\gamma-1}{2}} d$$

$$[A] = \gamma$$

boundary lies at infinity (large gauge transformation)

causality

$$\Box^{\gamma} A^{\mu} = 0$$

causality

$$\Box^\gamma A^\mu = 0$$

$$[\Box^\gamma, \Box] = 0$$

share the same eigenfunctions

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share the same eigenfunctions

$$\Box^\gamma \left(e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)} \right) = (k^2 - \omega^2)^\gamma e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}$$

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$$\boxed{\omega = ck}$$

$$\partial_\mu J^\mu = 0$$

current conservation

what if

$$[\partial_\mu, \hat{Y}] = 0$$

answer

?

$$[\partial_\mu, \hat{Y}] = 0$$

answer

?

$$[\partial_\mu, \hat{Y}] = 0$$

$$[d, \Delta^\alpha] = 0$$

$\boxed{\text{answer}}$

$$?$$

$$[\partial_\mu, \hat{Y}] = 0$$

$$[d, \Delta^\alpha] = 0$$

$$\Big\downarrow$$

$$\hat{Y} = \Delta^\alpha$$

$$J \to \Delta^\alpha J \qquad [J] = d - 1 - \alpha$$

current-current correlator

$$C^{ij}(k) \propto (k^2)^\gamma \left(\eta^{ij} - \frac{k^i k^j}{k^2}\right).$$

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$$k^{\gamma-1} k_\mu C^{\mu\nu} = 0$$

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but

$$k^{\gamma-1} k_\mu C^{\mu\nu} = 0 \Rightarrow \partial_\mu (-\Delta)^{\frac{\gamma-1}{2}} C^{\mu\nu} = 0$$

family of zero eigenvalues

$$M_{\mu\nu} f k^{\nu} = 0$$

family of zero eigenvalues

$$M_{\mu\nu} f k^{\nu} = 0$$

most fundamental conservation law

$$\partial^{\mu}(-\nabla^2)^{(\gamma-1)/2} J_{\mu} = 0$$

Noether's Theorems

$$
\begin{aligned}
\sum \psi_i \, \delta u_i = \delta f \; - \\
- \frac{d}{dx} \left\{ \sum \left[\binom{1}{1} \frac{\partial f}{\partial u_i^{(1)}} \delta u_i + \binom{2}{1} \frac{\partial f}{\partial u_i^{(2)}} \delta u_i^{(1)} + \cdots + \binom{\kappa}{1} \frac{\partial f}{\partial u_i^{(\kappa)}} \delta u_i^{(\kappa-1)} \right] \right\} + \\
+ \frac{d^2}{dx^2} \left\{ \sum \left[\binom{2}{2} \frac{\partial f}{\partial u_i^{(2)}} \delta u_i + \binom{3}{2} \frac{\partial f}{\partial u_i^{(3)}} \delta u_i^{(1)} + \cdots + \binom{\kappa}{2} \frac{\partial f}{\partial u_i^{(\kappa)}} \delta u_i^{(\kappa-2)} \right] \right\} + \\
\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
+ (-1)^{\kappa} \frac{d^{\kappa}}{dx^{\kappa}} \left\{ \sum \binom{\kappa}{\kappa} \frac{\partial f}{\partial u_i^{(\kappa)}} \delta u_1 \right\} \qquad (6)
\end{aligned}
$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda + \partial_\mu \partial_\nu G^\nu + \cdots ,$$

Noether's Second Theorem and Ward Identities for Gauge Symmetries

Steven G. Avery[a], Burkhard U. W. Schwab[b]

For simplicity, we focus on the case when the transformation may be written in the form[6]

$$\delta_\lambda \phi = f(\phi)\,\lambda + f^\mu(\phi)\partial_\mu \lambda, \tag{10}$$

but it is straightforward to consider transformations, as Noether did, involving arbitrarily high derivatives of λ. (Although, the authors know of no physically interesting examples.) Let us start with

arxiv:1510.07038

is this just a game?

Yes

Virasoro algebra

$$L_n := -z^{n+1}\frac{\partial}{\partial z}$$

Virasoro algebra

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$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}$$

Witt algebra

central extension

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Witt algebra

central extension

conformal transformations on unit disk

$$\mathcal{V} \to \mathcal{W} \to 1$$

Fractional Virasoro algebra

generators

$$L_n^a = -z^{a(n+1)} \left(\frac{\partial}{\partial z} \right)^a \qquad \bar{L}_n^a := -\bar{z}^{a(n+1)} \left(\frac{\partial}{\partial \bar{z}} \right)^a$$

$$\Delta^{\gamma} f(x) = \int d$$

Fractional Virasoro algebra

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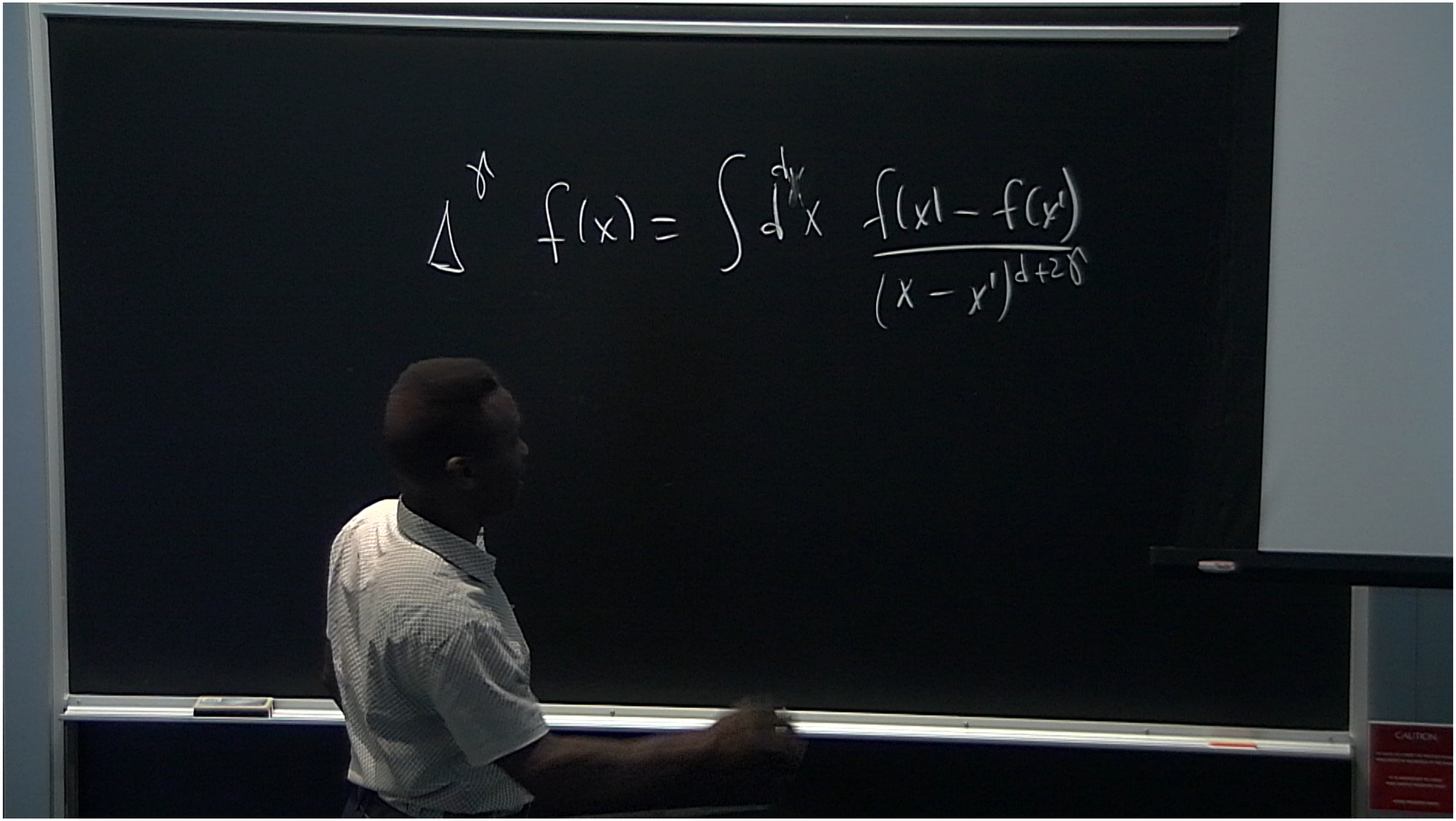
$\Delta^{\gamma} f(x) = \int d^d x \frac{f(x) - f(x')}{(x-x')^{d+2\gamma}}$

$$\Delta^{\gamma} f(x) = \int d^{d}x \, \frac{f(x) - f(x')}{(x - x')^{d+2\gamma}}$$

$$\frac{d^{\alpha}}{dx^{\alpha}} x^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} x^{\beta-\alpha}$$

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$$[L_n, L_m](z^{ak}) = \left(\frac{\Gamma(a(k+n)+1)}{\Gamma(a(k-1+n)+1)} - \frac{\Gamma(a(k+m)+1)}{\Gamma(a(k-1+m)+1)} \right) L_{n+m}(z^{ak})$$

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$$= (A_{n,m}^a(k) \otimes L_{n+m})(z^{ak})$$

$$\boxed{[L_m^a, L_n^a] = A_{m,n} L_{m+n}^a + \delta_{m,n} h(n) c Z^a}$$

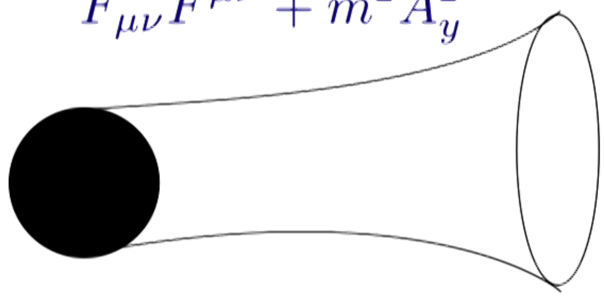
algebra for conformal non-local actions

$$Z_\star^2(\mathcal{W}_a, \mathcal{H}) / B_\star^2(\mathcal{W}_a, \mathcal{H})$$

is there a hidden broken symmetry?

application: gauge fields with anomalous dimensions

$$F_{\mu\nu}F^{\mu\nu} + m^2 A_y^2$$



$$\gamma = \sqrt{d^2 + m^2 - 1}/2$$

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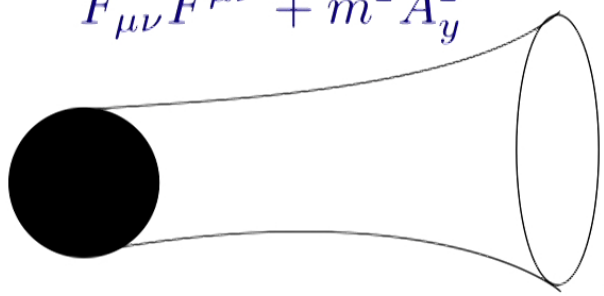

$$\boxed{A_\mu^\perp (-\Delta)^\gamma A^{\perp\mu}}$$

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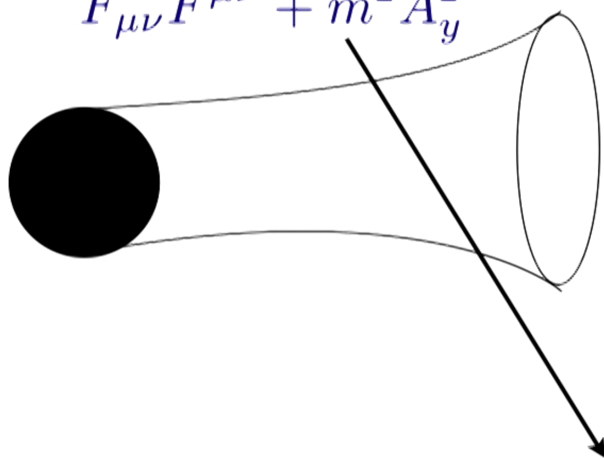

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dynamical `Higgs' mode

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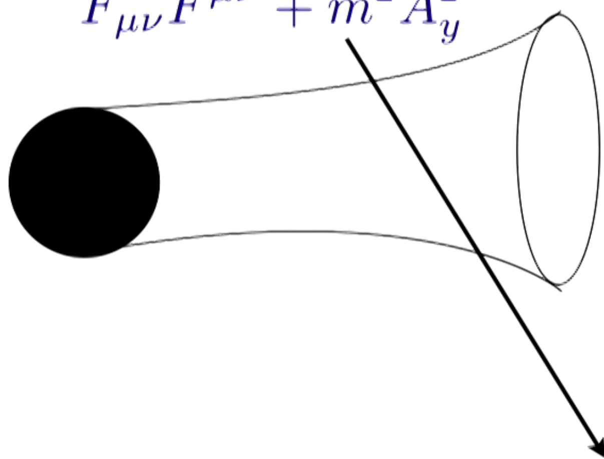

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dynamical `Higgs' mode

additional length scale

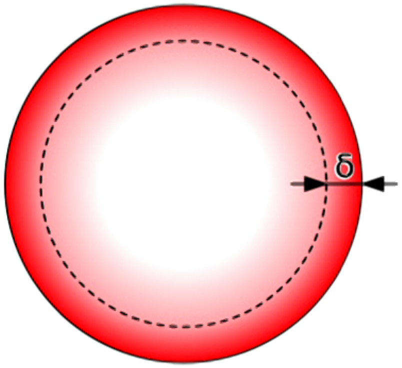
$m_{\mathrm{IR}} \longrightarrow J_{\mathrm{UV}}$

non-local E&M

broken symmetry in higher dimension

experiments?

skin effect



$$\delta = \sqrt{\frac{2\rho}{\omega\mu}}$$

new result

$$\Box^{\frac{\gamma-1}{2}}\left(\nabla\times\vec{B}-\frac{1}{v^2}\frac{\partial\vec{E}}{\partial t}\right)=\mu\vec{J}$$

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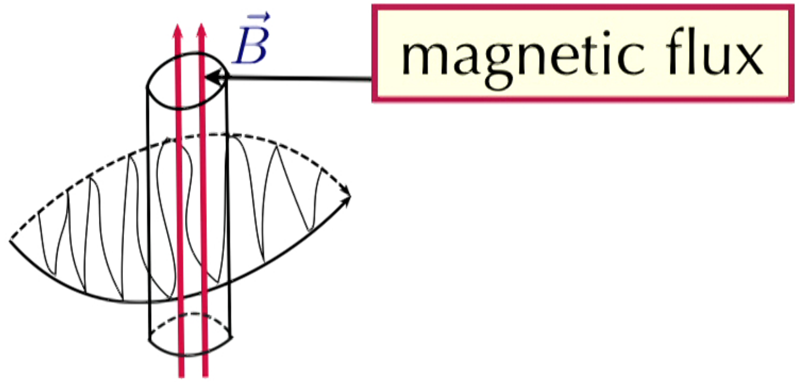
$$\delta = 1/k_2 = \left(\frac{\epsilon v^2}{\omega\sigma}\right)^{\frac{1}{2(\gamma+1)}}\frac{1}{\sin\left(\frac{\pi}{2(\gamma+1)}+\frac{2\pi n}{\gamma+1}\right)}$$



$\pi r^2 B$

new result

$$\Box^{\frac{\gamma-1}{2}}\left(\nabla \times \vec{B} - \frac{1}{v^2}\frac{\partial \vec{E}}{\partial t}\right) = \mu \vec{J}$$

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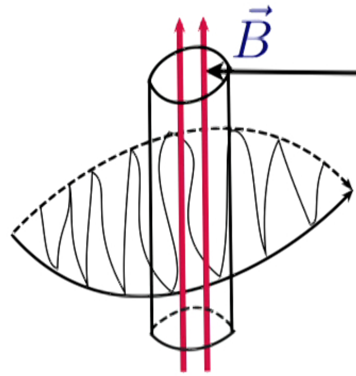
$$\Box^{\frac{\gamma-1}{2}}\left(\nabla\times\vec{B}-\frac{1}{v^2}\frac{\partial\vec{E}}{\partial t}\right)=\mu\vec{J}$$

$$\delta=1/k_2=\left(\frac{\epsilon v^2}{\omega\sigma}\right)^{\frac{1}{2(\gamma+1)}}\frac{1}{\sin\left(\frac{\pi}{2(\gamma+1)}+\frac{2\pi n}{\gamma+1}\right)}$$



magnetic flux

$\pi r^2 B$

should be dimensionless

$$[B] = 2 - \Phi = 2 + 2/3 \neq 2$$

what's the resolution?

quantization of charge

$$d\Box^{\frac{\gamma-1}{2}}(\star d\Box^{\frac{\gamma-1}{2}}A) = \star J$$

quantization of charge

$$d\Box^{\frac{\gamma-1}{2}}(\star d\Box^{\frac{\gamma-1}{2}}A) = \star J$$

$$\tilde{A} = \Delta^{\frac{\gamma-1}{2}}A$$

$$\int_\Sigma d_\gamma A = \oint_{\partial\Sigma}\tilde{A}$$

correct dimensionless quantity

$$D_i \equiv \partial_i - i\frac{e}{\hbar}a_i$$

correct dimensionless quantity

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fictitious gauge field

$$a_i \equiv [\partial_i, I_i^{\alpha}{}_{\alpha} A_i] = \partial_i I_i^{\alpha}{}_{\alpha} A_i$$

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$$D_i \equiv \partial_i - i\frac{e}{\hbar}a_i$$

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$$a_i \equiv [\partial_i, I_i^{\alpha}{}_{\alpha}A_i] = \partial_i I_i^{\alpha}{}_{\alpha}A_i$$

$${}_{\alpha}A \rightarrow_{\alpha} A + d_{\alpha}\Lambda$$

$$a_{\mu} \rightarrow a_{\mu} + \partial_{\mu}\Lambda$$

correct dimensionless quantity

$$D_i \equiv \partial_i - i\frac{e}{\hbar}a_i$$

fictitious gauge field

$$a_i \equiv [\partial_i, I_i^{\alpha}{}_{\alpha}A_i] = \partial_i I_i^{\alpha}{}_{\alpha}A_i$$

$${}_{\alpha}A \rightarrow_{\alpha} A + d_{\alpha}\Lambda$$

$$a_\mu \rightarrow a_\mu + \partial_\mu \Lambda$$

$$\Delta\phi = \frac{e}{\hbar}\oint \vec{a}(\vec{r})\cdot d\vec{l}.$$

$$a_i \equiv [\partial_i, I_i^{\alpha}{}_{\alpha} A_i] = \partial_i I_i^{\alpha}{}_{\alpha} A_i$$

$$\Delta^{-\alpha}$$

$$a_i \equiv [\partial_i, I^{\alpha}_{i}{}_{\alpha} A_i] = \partial_i I^{\alpha}_{i}{}_{\alpha} A_i$$

$$\Delta^{-\alpha}$$

$$a_i \equiv [\partial_i, I_i^{\alpha}{}_{\alpha} A_i] = \partial_i I_i^{\alpha}{}_{\alpha} A_i$$

$$\Delta^{-\alpha}$$

what's the relationship?

$$\oint_{\partial\Sigma} a \qquad\qquad \oint_{\partial\Sigma} \Delta^{-\alpha} A$$

$$a_i \equiv [\partial_i, I_i^{\alpha}{}_{\alpha} A_i] = \partial_i I_i^{\alpha}{}_{\alpha} A_i$$

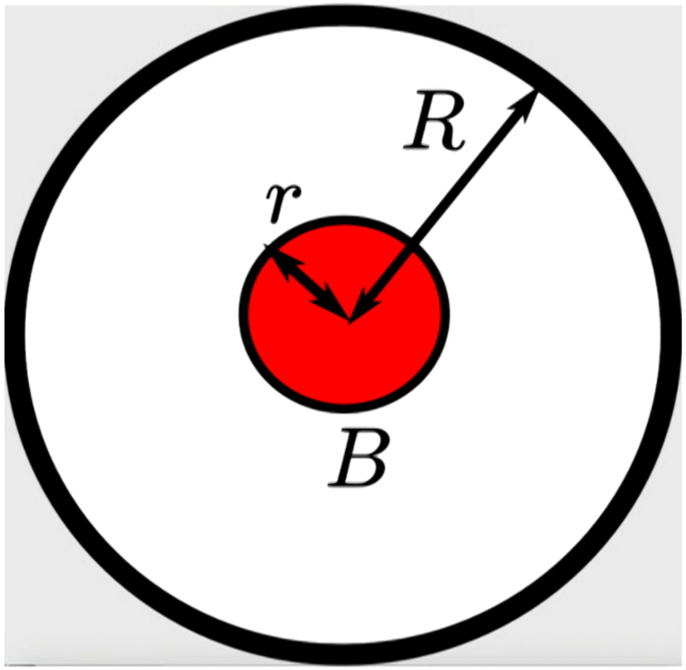
$$\Delta^{-\alpha}$$

what's the relationship?

$$\oint_{\partial\Sigma} a \qquad\qquad \oint_{\partial\Sigma} \Delta^{-\alpha} A$$

$$\oint_{\partial\Sigma} a = \frac{1}{\Gamma(3/2-\gamma)} \oint_{\partial\Sigma} A$$

not an integer



$$\Delta\phi_{\mathrm{D}} = \frac{e}{\hbar}\pi r^2 B R^{2\alpha-2}\left(\frac{\sqrt{\pi}2^{1-\alpha}\Gamma(2-\alpha)\Gamma(1-\frac{\alpha}{2})}{\Gamma(\alpha)\Gamma(\frac{3}{2}-\frac{\alpha}{2})}\sin^2\frac{\pi\alpha}{2}\,{}_2F_1(1-\alpha,2-\alpha;2;\frac{r^2}{R^2})\right)$$

$$a_i \equiv [\partial_i, I_i^{\alpha}{}_{\alpha} A_i] = \partial_i I_i^{\alpha}{}_{\alpha} A_i$$

$$a_i \equiv [\partial_i, I_i^{\alpha}{}_{\alpha} A_i] = \partial_i I_i^{\alpha}{}_{\alpha} A_i$$

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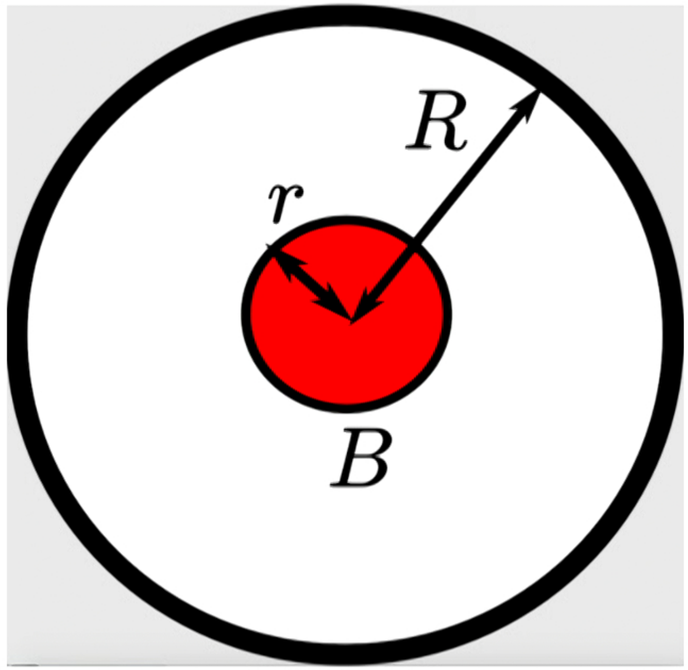
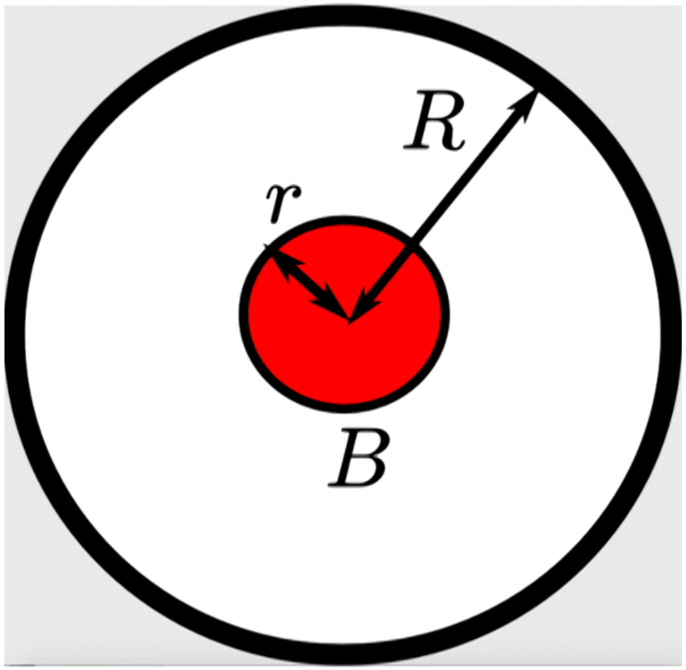


$$\Delta\phi_{\mathrm{D}} = \frac{e}{\hbar}\pi r^2 B R^{2\alpha-2}\left(\frac{\sqrt{\pi}2^{1-\alpha}\Gamma(2-\alpha)\Gamma(1-\frac{\alpha}{2})}{\Gamma(\alpha)\Gamma(\frac{3}{2}-\frac{\alpha}{2})}\sin^2\frac{\pi\alpha}{2}\,{}_2F_1(1-\alpha,2-\alpha;2;\frac{r^2}{R^2})\right)$$



$$\Delta\phi_{\mathrm{D}} = \frac{e}{\hbar}\pi r^2 B \boxed{R^{2\alpha-2}\left(\frac{\sqrt{\pi}2^{1-\alpha}\Gamma(2-\alpha)\Gamma(1-\frac{\alpha}{2})}{\Gamma(\alpha)\Gamma(\frac{3}{2}-\frac{\alpha}{2})}\sin^2\frac{\pi\alpha}{2}\,{}_2F_1(1-\alpha,2-\alpha;2;\frac{r^2}{R^2})\right)}$$

$\boxed{\text{is the correction large?}}$

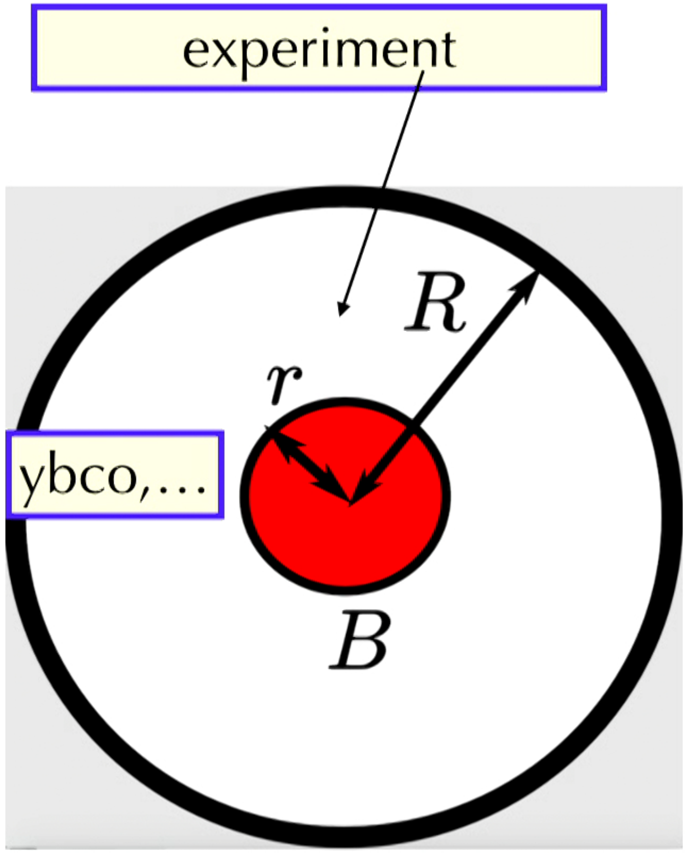
$$\alpha = 1 + 2/3 = 5/3$$

is the correction large?

$$\alpha = 1 + 2/3 = 5/3$$

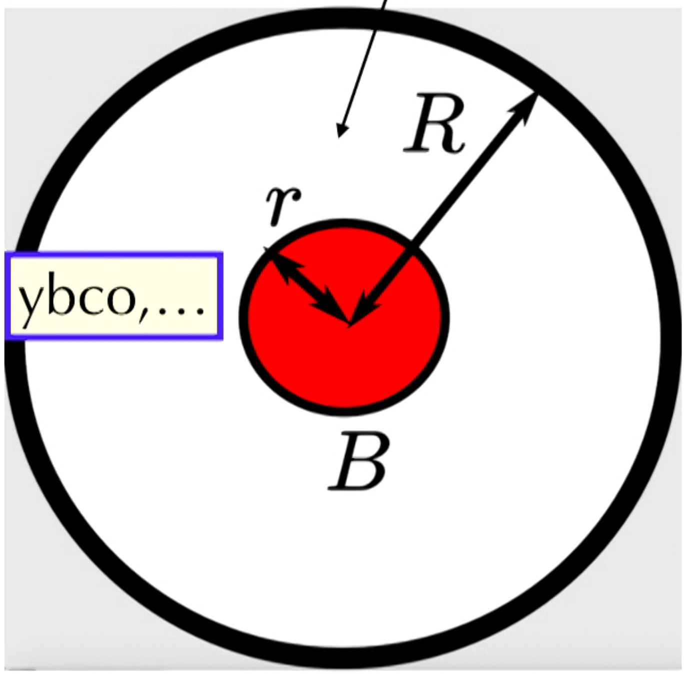
$$\Delta\Phi_R = \frac{eB\ell^2}{\hbar} \boxed{L^{-5/3}/(0.43)^2}$$

yes!

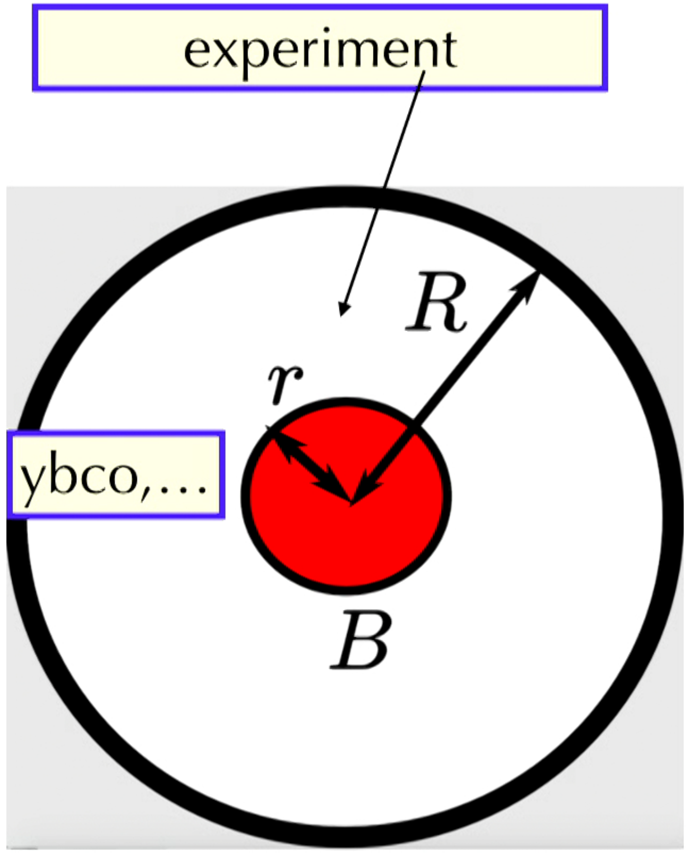

experiment
R
r
ybco,...
B

experiment

Planckian dissipation



$$\tau = \frac{\hbar}{k_B T}$$



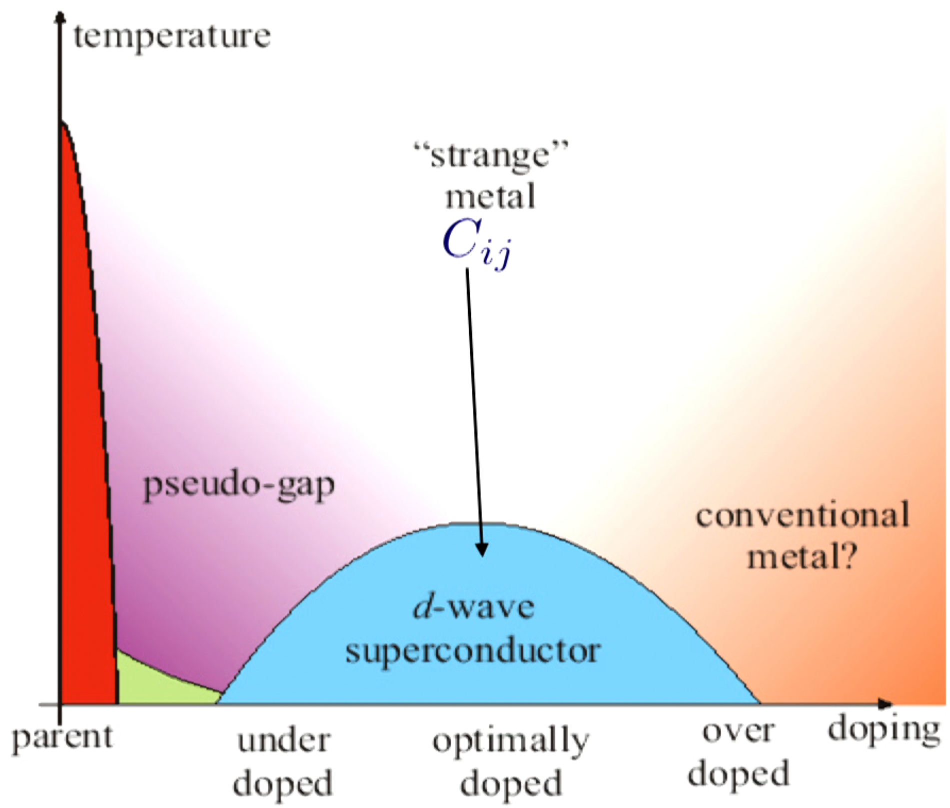
Planckian dissipation

$$\tau = \frac{\hbar}{k_B T}$$

$$\tau \approx 10^{-14} s$$

Table 11.1

Element	77 K	273 K
Li	$7.3 \times 10^{-14} s$	$8.8 \times 10^{-15} s$
Na	$1.7 \times 10^{-13} s$	$3.2 \times 10^{-14} s$
K	$1.8 \times 10^{-13} s$	$4.1 \times 10^{-14} s$
Rb	$1.4 \times 10^{-13} s$	$2.8 \times 10^{-14} s$
Cs	$8.6 \times 10^{-14} s$	$2.1 \times 10^{-14} s$

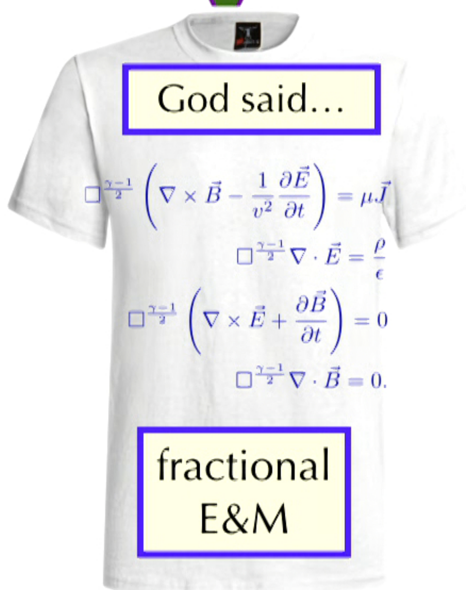

temperature
"strange"
metal
C_{ij}
pseudo-gap
conventional
metal?
d-wave
superconductor
parent
under
doped
optimally
doped
over
doped
doping

if in the strange metal

$[A_\mu] = d_A \neq 1$

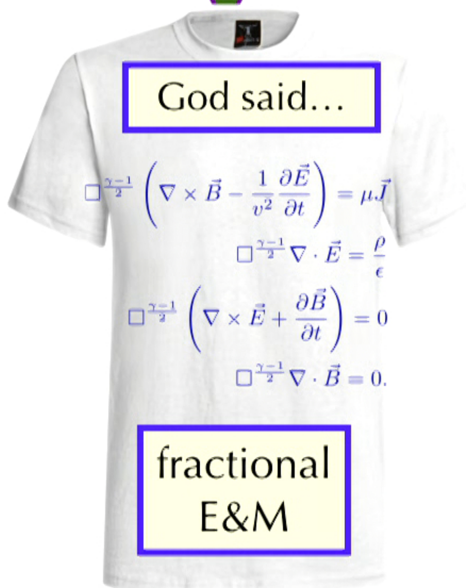
if in the strange metal

$$[A_\mu] = d_A \neq 1$$



if in the strange metal

$$[A_\mu] = d_A \neq 1$$

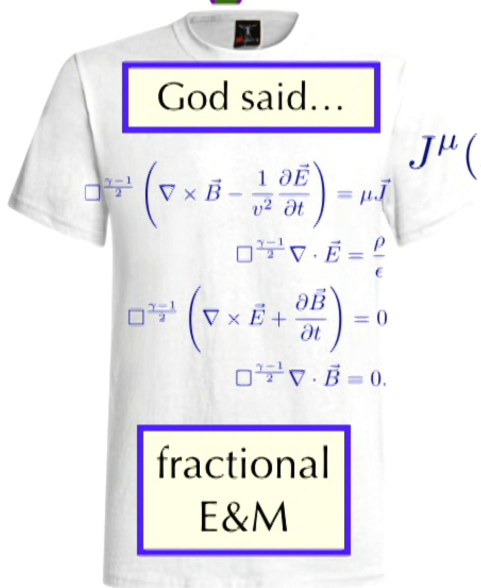


$$\boxed{\omega = ck}$$

if in the strange metal

$$[A_\mu] = d_A \neq 1$$

Pippard Kernel



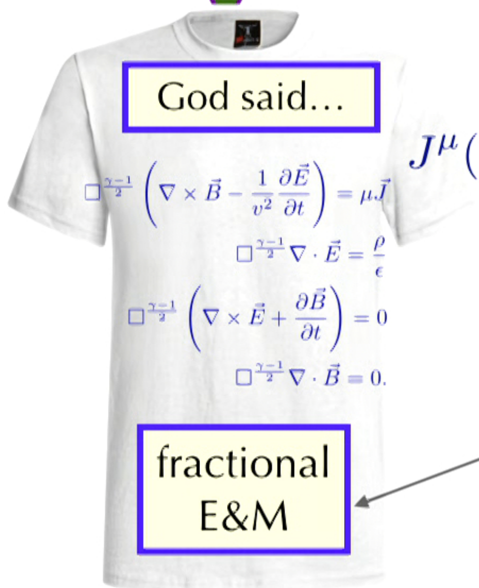
$$J^\mu(x) = -\int d^d x' C_{\mu\nu}(|x - x'|) A^\nu$$

$$\boxed{\omega = ck}$$

if in the strange metal

$[A_\mu] = d_A \neq 1$

Pippard Kernel



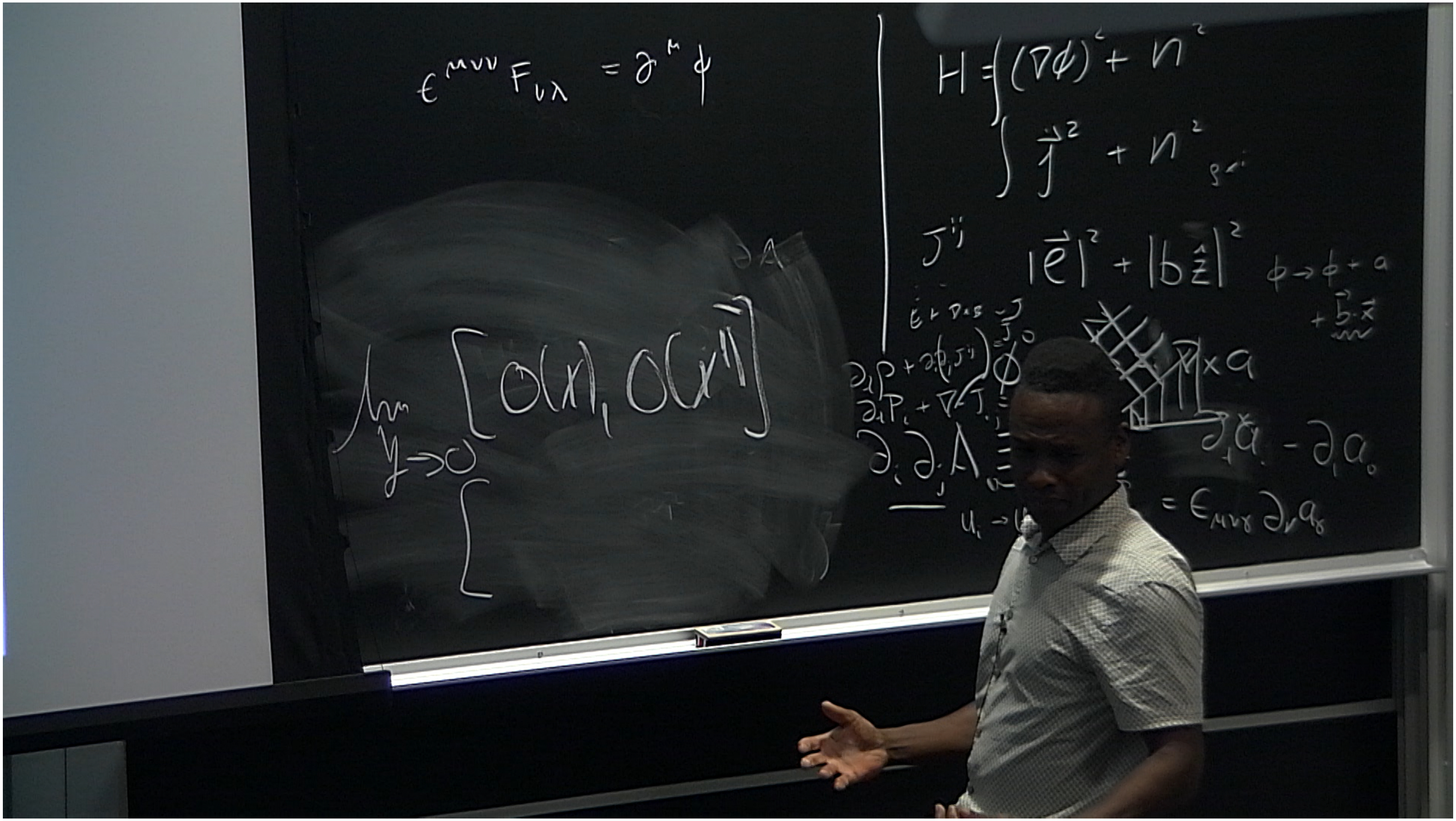
$$J^\mu(x) = -\int d^d x' C_{\mu\nu}(|x-x'|)A^\nu$$

$[J] \neq d-1$

$[A] \neq 1$

in SC!

$\boxed{\omega = ck}$


ϵ^μνν F_νλ = ∂^μ φ
H = ∫ (∇φ)² + n²
∫ j² + n²
|e|² + |b ẑ|²
φ → φ + a + b·x
lim y→0 [O(x), O(x')]
∂₁a₀ − ∂₀a₁
= ϵ_μνγ ∂_ν a_γ