

Title: Bounds on Thermalization and Viscosity from the Average Null Energy Condition & Magnetic States with non-trivial Topology

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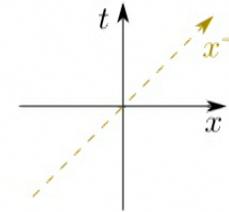
Abstract: Bounds on Thermalization and Viscosity from the Average Null Energy Condition:

I will present implications of the averaged null energy condition for thermal states of relativistic quantum field theories. A key property of such thermal states is the thermalization length. This lengthscale generalizes the notion of a mean free path beyond weak coupling, and allows finite size regions to independently thermalize. Using the eigenstate thermalization hypothesis, we show that thermal fluctuations in finite size `fireballs' can produce states that violate the averaged null energy condition if the thermalization length is too short or if the shear viscosity is too large. These bounds become very weak with a large number  $N$  of degrees of freedom but can constrain real-world systems, such as the quark-gluon plasma.

## STRATEGY IN A NUTSHELL

Relativistic QFTs have  $\mathcal{E} \equiv \int dx^+ T_{++} \geq 0$

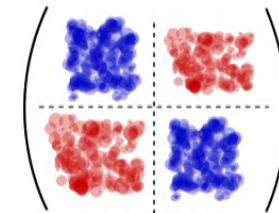
[Faulkner Leigh Parrikar Wang '16, Hartman Kundu Tajdini '16]



This bounds off-diagonal elements  $\langle E|\mathcal{E}|E' \rangle \sim$  fluctuations  
in terms of diagonal elements  $\langle E|\mathcal{E}|E \rangle \sim$  thermal equilibrium

This is made quantitatively precise using the ETH Ansatz

[Deutsch '91, Srednicki '94, Rigol Dunjko Olshanii '08]



For  $E - E'$  small, the fluctuations are universally constrained by finite temperature hydrodynamics

The resulting bound constrains finite temperature properties of QFTs

## Thermalization time and length

The expectation at finite temperature is that correlation functions decay

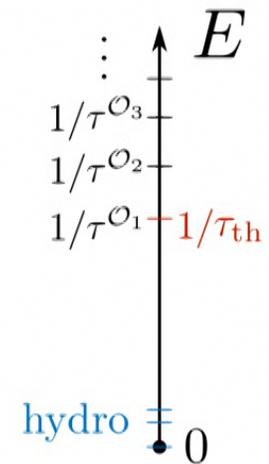
$$\langle \mathcal{O}(t)\mathcal{O} \rangle_\beta \sim e^{-t/\tau_{\text{th}}^\mathcal{O}}$$

except those of conserved densities:  $T^{00}, T^{0i}, J^0, \dots$

Integrating out these ‘thermally gapped’ modes leads to a local effective theory for  $T^{00}, T^{0i}, J^0, \dots$

Hydrodynamics is valid at late times and long distances

$$t \gtrsim \tau_{\text{th}} \equiv \max_{\mathcal{O}} \tau_{\text{th}}^\mathcal{O} \qquad |x| \gtrsim \ell_{\text{th}} \equiv \max_{\mathcal{O}} \ell_{\text{th}}^\mathcal{O}$$



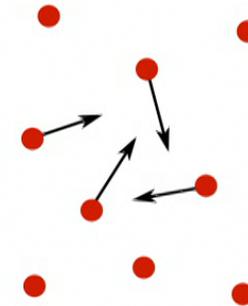
In systems with quasiparticles, the quasiparticle life-time  $\tau_{qp}$  plays an important role in transport

[Drude 1900]

$$\sigma_{dc} = \frac{ne^2}{m} \tau_{qp}$$

At weak coupling (Fermi-Liquid theory)  $\tau_{th} = \tau_{qp} \sim 1/g^2$

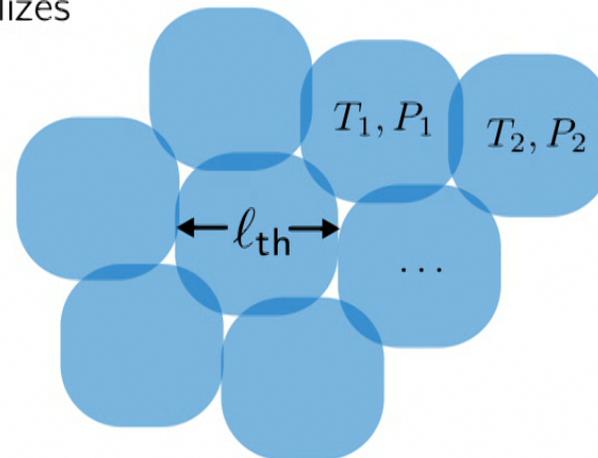
Without quasiparticles,  $\tau_{th}$  generalizes  $\tau_{qp}$



Similarly, thermalization length  $\ell_{th}$  generalizes the notion of mean free path

Smallest blob that can reach local thermal equilibrium

Hydrodynamics describes  $T(x), P(x)$  on length scales  $x \gg \ell_{th}$



$$\left. \begin{array}{l}
 \text{Fermi-Liquid: } \tau_{\text{th}} \sim \frac{1}{\lambda^2} \frac{E_F}{k_B T} \frac{\hbar}{k_B T} \\
 \text{Large } N \text{ vector: } \tau_{\text{th}} \sim N \frac{\hbar}{k_B T} \\
 \epsilon\text{-expansion: } \tau_{\text{th}} \sim \frac{1}{\epsilon^2} \frac{\hbar}{k_B T}
 \end{array} \right\} \gg \frac{\hbar}{k_B T}$$

Large  $N$  matrix models (SYK, holography) typically have  $\tau_{\text{th}} \sim \frac{\hbar}{k_B T}$

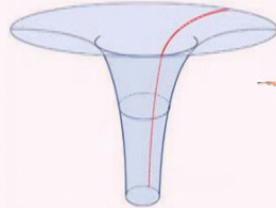
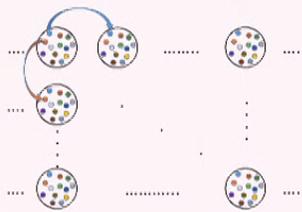
Natural to expect a lower bound

$$\tau_{\text{th}} \geq \# \frac{\hbar}{k_B T}$$

Analogous to chaos bound  $\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}$  [Maldacena Shenker Stanford '15]

where  $\langle [A(t), B]^2 \rangle \sim e^{t/\tau_L}$ , but  $\tau_{\text{th}}$  is relevant for transport!

Large  $N$   
 $\epsilon$  expansion



theory

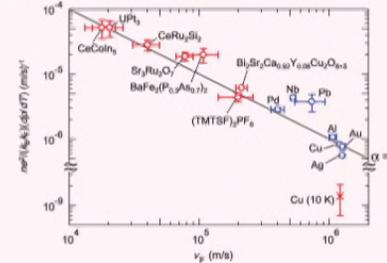


Thermalization

numerics

ED – small systems  
QMC – hard to access real time dynamics  
...

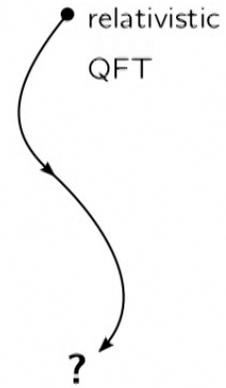
Bad metals  
[Chubukov Sachdev '93, ...  
Mackenzie et al 2013, ...]



Cold atoms, Heavy fermions  
...

experiment

We would like to find bounds on thermalization and transport  
For simplicity, let us consider focus on theories which (at least in the UV) are described by relativistic QFTs



Such theories contain a positive definite operator [Faulkner Leigh Parrikar Wang '16]  
[Hartman Kundu Tajdini '16]

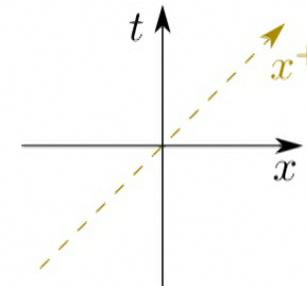
$$\int dx^+ T_{++} \geq 0$$

where  $x^+ = x + t$  and  $T_{++} = T_{tt} + 2T_{tx} + T_{xx}$

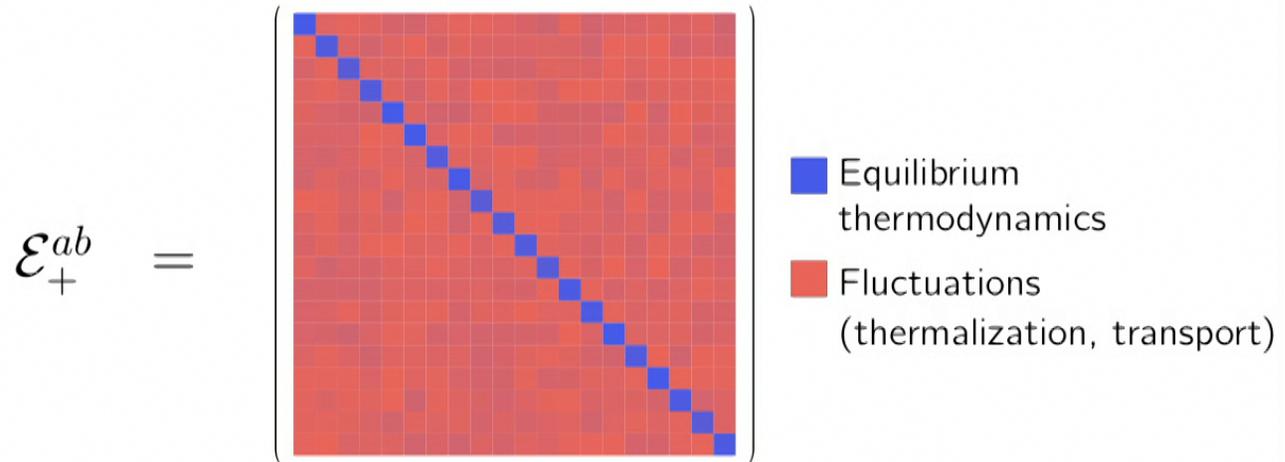
energy density

momentum density

'pressure'



$\mathcal{E}_+^{ab} \equiv \langle a | \int dx^+ T_{++} | b \rangle$  is a positive matrix in any sub-Hilbert space  
**diagonal**  $\geq$  **off-diagonal**



To make this picture quantitative, we will use the ETH Ansatz:

$$\langle a | \mathcal{O} | b \rangle = \delta_{ab} \langle \mathcal{O} \rangle_T + e^{-S/2} \sqrt{G_{\mathcal{O}\mathcal{O}}} R_{ab}$$

[Deutsch '91]

[Srednicki '94]

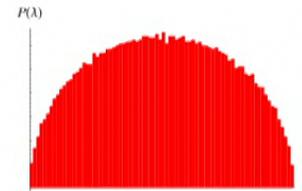
with states in a microcanonical window  $E_a, E_b \in [E - \frac{\Delta E}{2}, E + \frac{\Delta E}{2}]$



ETH Ansatz:  $\langle a|\mathcal{O}|b\rangle = \delta_{ab}\langle\mathcal{O}\rangle_T + e^{-S/2}\sqrt{G_{\mathcal{O}\mathcal{O}}(\omega, k)}R_{ab}$   
 applied to positive  $\mathcal{O} = \int dx^+ T_{++}$ . (with  $\omega = E_b - E_a$  and  $k = k_b - k_a$ )

Eigenvalue repulsion of  $R_{ab}$  can overcome  $e^{-S/2}$  suppression  
 Specifically, a real symmetric matrix  $A_{ab}$  satisfies

$$\lambda_{\max}^2 \geq \frac{1}{N} \sum_{ab} A_{ab}A_{ba}$$



which gives schematically

$$|\langle\mathcal{O}\rangle|^2 \geq \frac{1}{N} \sum_{ab} e^{-S} G_{\mathcal{O}\mathcal{O}}(\omega, k)$$

$$\langle T_{\mu\nu}\rangle = \begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix}$$

so LHS  $\sim L^2$  !

choose narrow window so that  $\omega, k$  small, and  $G_{\mathcal{O}\mathcal{O}}(\omega, k)$  controlled by hydrodynamics

$$|\langle \mathcal{O} \rangle|^2 \geq \frac{1}{N} \sum_{ab} e^{-S} G_{\mathcal{O}\mathcal{O}}(\omega, k) \quad \text{with} \quad \mathcal{O} = \int dx^+ T_{++}$$

The RHS (fluctuations) will be universally fixed by hydrodynamics if

$$\omega \lesssim \frac{2\pi}{\tau_{\text{th}}} \quad k \lesssim \frac{2\pi}{\ell_{\text{th}}}$$

Many things can happen in the IR and  $G_{\mathcal{O}\mathcal{O}}$  will reflect that.  
Generically, bound gives UV constraints on IR physics.

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06, ...]

Focus on simple case: **Lorentz** and **translation** invariant IR

$$\langle T^{\mu\nu} \rangle = \epsilon u^\mu u^\nu + P \Delta^{\mu\nu} - \eta \Delta^{\mu\alpha} \nabla_{\langle\alpha} u_{\beta\rangle} \Delta^{\beta\nu} - \zeta \Delta^{\mu\nu} \nabla_\alpha u^\alpha + \mathcal{O}(\ell_{\text{th}}^2 \nabla^2)$$

with  $\Delta_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ .

Solve hydro eom for  $u^\mu$  and  $\epsilon$  to get  $\langle T^{\mu\nu}(g) \rangle \rightsquigarrow G_{TT} \sim \frac{\delta \langle T \rangle}{\delta g}$ .

$$\text{sound:} \quad \omega = \pm c_s k - \frac{i}{2} \Gamma_s k^2 + \dots \quad \Gamma_s = \frac{\zeta + \frac{4}{3}\eta}{sT}$$

$$\text{diffusion:} \quad \omega = -i D_\perp k^2 + \dots \quad D_\perp = \frac{\eta}{sT}$$

The bound:

$$(\varepsilon + P)^2 \geq \frac{64\pi^3}{L^3 \sinh \frac{\Delta\omega}{2T}} \int_{-\Delta\omega}^{\Delta\omega} d\omega \frac{\sin^2 \frac{L\omega}{2}}{L^2 \omega^2} \frac{\sinh \frac{1}{2T} (\Delta\omega - |\omega|)}{\sinh \frac{\omega}{2T}} \text{Im } G_{T_{++}T_{++}}^R(\omega, k_{\min})$$

$|\langle \mathcal{O} \rangle|^2$       size of window      from  $dx^+$  integral      fluctuation-dissipation      fixed by hydro

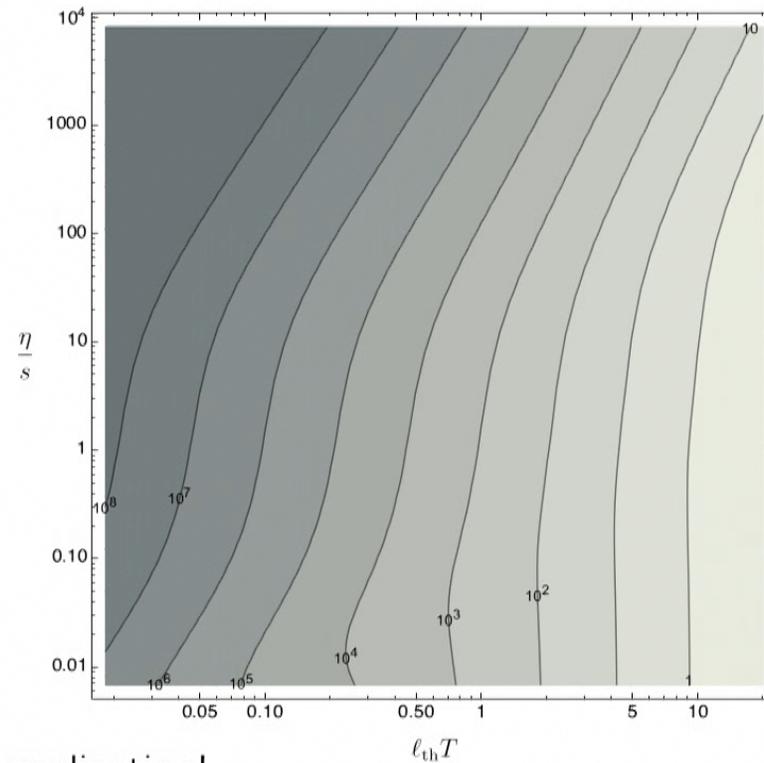
Writing  $s = s_o T^3$ , it has the form

$$s_o \geq \mathcal{F} \left( \frac{\eta}{s}, \frac{\zeta}{s}, c_s, \ell_{\text{th}} T, \tau_{\text{th}} T \right)$$

for  $N \rightarrow \infty$ ,  $s_o \sim N^2$  bound becomes weak.

(free scalar:  $s_o = \frac{4\pi^2}{45}$ . QGP at large  $T$ :  $s_o \simeq 20$ )

Exclusion plot  $s_o \geq \mathcal{F} \left( \frac{\eta}{s}, \frac{\zeta}{s}, c_s, \ell_{\text{th}} T, \tau_{\text{th}} T \right)$



Bound on thermalization!

$$s \ell_{\text{th}}^3 \geq 4\pi^3 \int_0^\infty \frac{y(c_1 + c_2 y^2)}{(e^y - 1)(c_3 + c_4 y^2)} dy$$

**Caveat:**

A bound of the form  $s\ell_{\text{th}}^3 \gtrsim 1$  better be true for our derivation to work!

But:

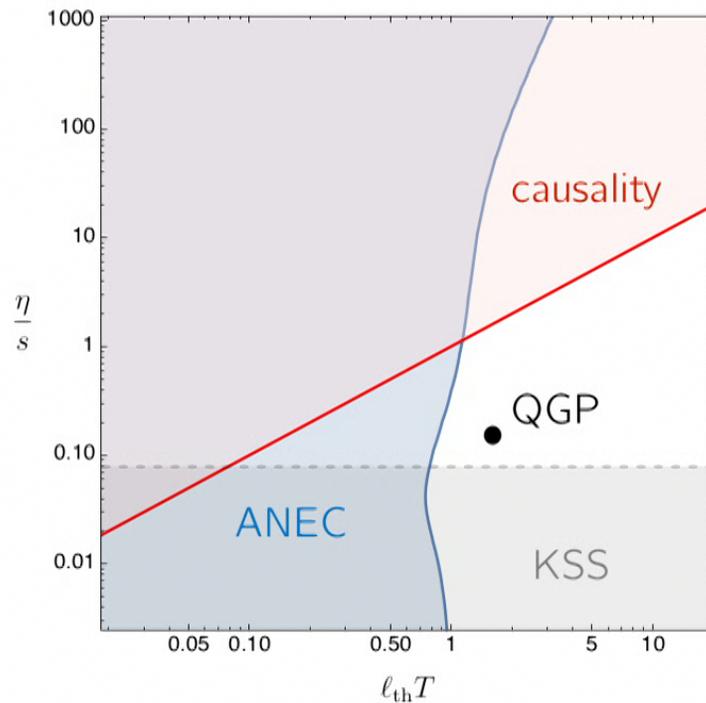
$$s\ell_{\text{th}}^3 \geq 4\pi^3 \int_0^\infty \frac{y(c_1 + c_2 y^2)}{(e^y - 1)(c_3 + c_4 y^2)} dy \sim 500 \text{ for QGP}$$

Bound becomes parametrically strong in certain limits, e.g.

$$\begin{aligned} \text{small speed of sound:} & \quad s\ell_{\text{th}}^3 \gtrsim \frac{\pi^4}{4} \frac{1}{c_s^2} \\ \text{large viscosity:} & \quad s\ell_{\text{th}}^3 \gtrsim \frac{32\pi^3}{9} \frac{\eta}{s} \end{aligned}$$

**Applications:** ideally many cond mat systems

For now, available example with all the symmetries: QGP



causality bound:

$$D_{\perp} \leq c \ell_{\text{th}}$$

[Hartman Hartnoll Mahajan '17]

ANEC bound:

$$\ell_{\text{th}} T \gtrsim 1$$

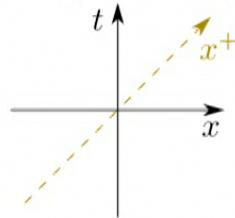
KSS conjecture:

$$\eta/s \geq 1/4\pi$$

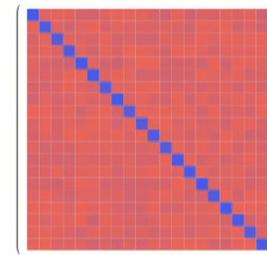
[Kovtun Son Starinets '04]

**Summary:**

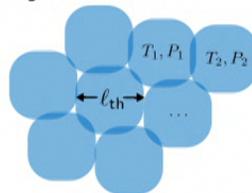
(1) relativistic QFTs have a positive-definite operator  $\int dx^+ T_{++}$



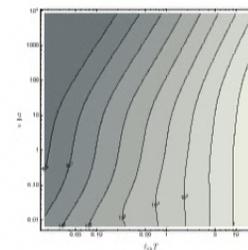
(2) ETH makes precise  $\text{diagonal} \geq \text{off-diagonal}$



(3) off-diagonal terms universally controlled by hydrodynamics



(4) certain parameter regions violate ANEC!



## Outlook:

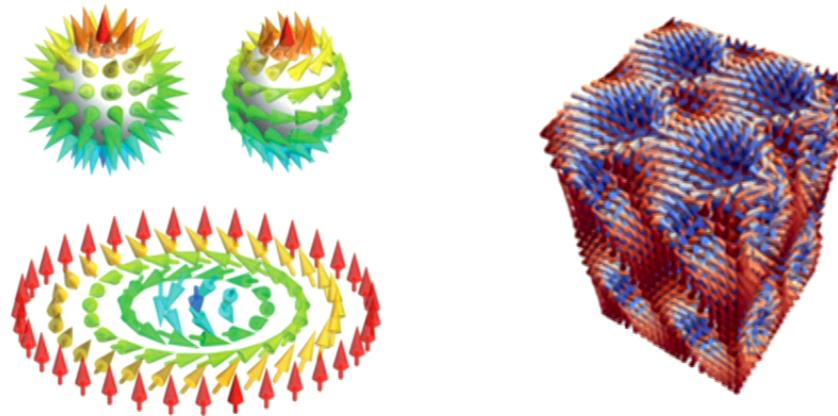
- Should be possible to prove weaker statement  $s\ell_{\text{th}}^3 \geq 1$  w/o ANEC
- IR Green's function controlled by symmetries. Other choices?
- Needed finite  $L$  to regulate divergence. Cleaner QNEC proof?
- Reminiscent of numerical bootstrap – how far does the analogy go?

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# Magnetic States with non-trivial Topology

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Wolfgang Simeth

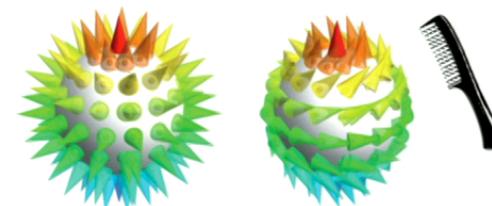
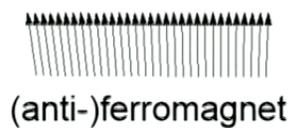
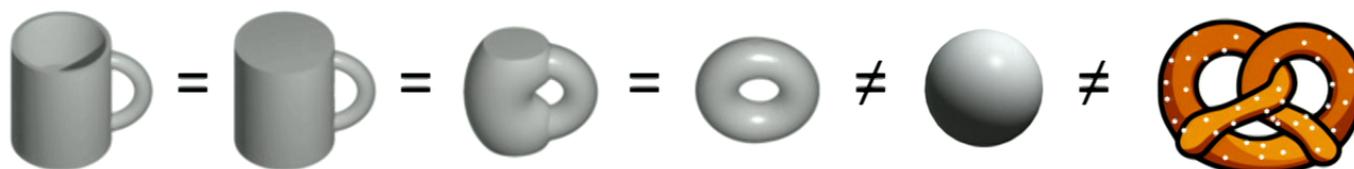


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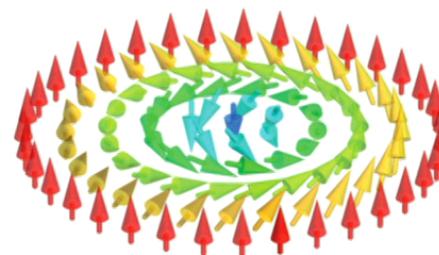




# Topology and Homotopies



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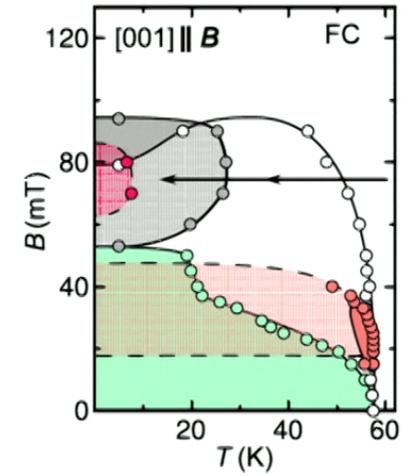
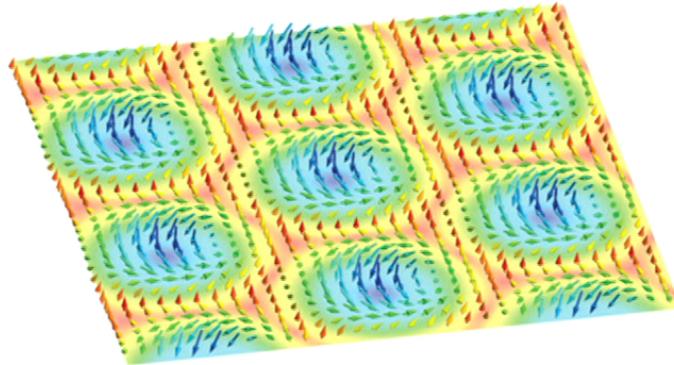


magnetic skyrmion

C. Pfleiderer, Nature Phys. 7, 673 (2011)

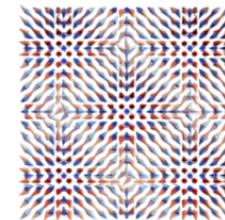
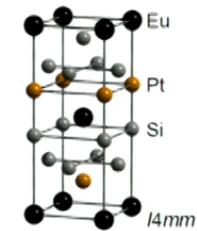
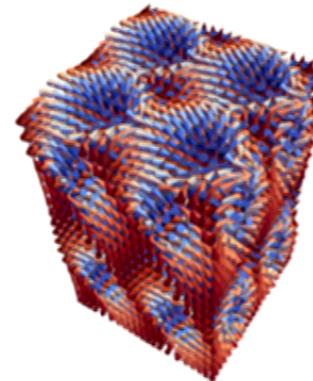


# Outline



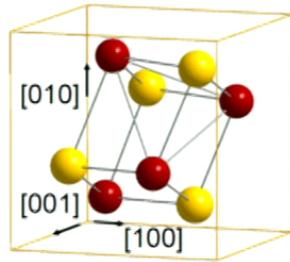
$\pi_m(S^n)$

	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$	$\pi_7$	$\pi_8$	$\pi_9$	$\pi_{10}$	$\pi_{11}$	$\pi_{12}$	$\pi_{13}$	$\pi_{14}$	$\pi_{15}$
$S^0$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$S^1$	$z$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$S^2$	0	$z$	$z$	$z_2$	$z_2$	$z_{12}$	$z_2$	$z_2$	$z_3$	$z_{15}$	$z_2$	$z_2^2$	$z_{12} \times z_2$	$z_{84} \times z_2^2$	$z_2^2$
$S^3$	0	0	$z$	$z_2$	$z_2$	$z_{12}$	$z_2$	$z_2$	$z_3$	$z_{15}$	$z_2$	$z_2^2$	$z_{12} \times z_2$	$z_{84} \times z_2^2$	$z_2^2$
$S^4$	0	0	0	$z$	$z_2$	$z_2$	$z \times z_{12}$	$z_2^2$	$z_2^2$	$z_{24} \times z_3$	$z_{15}$	$z_2$	$z_2^3$	$z_{120} \times z_{12} \times z_2$	$z_{84} \times z_2^5$
$S^5$	0	0	0	0	$z$	$z_2$	$z_2$	$z_{24}$	$z_2$	$z_2$	$z_2$	$z_{30}$	$z_2$	$z_2^3$	$z_{72} \times z_2$
$S^6$	0	0	0	0	0	$z$	$z_2$	$z_2$	$z_{24}$	0	$z$	$z_2$	$z_{60}$	$z_{24} \times z_2$	$z_2^3$
$S^7$	0	0	0	0	0	0	$z$	$z_2$	$z_2$	$z_{24}$	0	0	$z_2$	$z_{120}$	$z_2^3$
$S^8$	0	0	0	0	0	0	0	$z$	$z_2$	$z_2$	$z_{24}$	0	0	$z_2$	$z \times z_{120}$



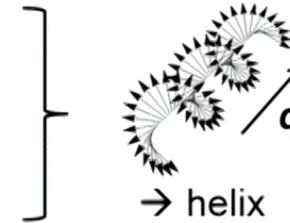


# Skyrmions in Helimagnets



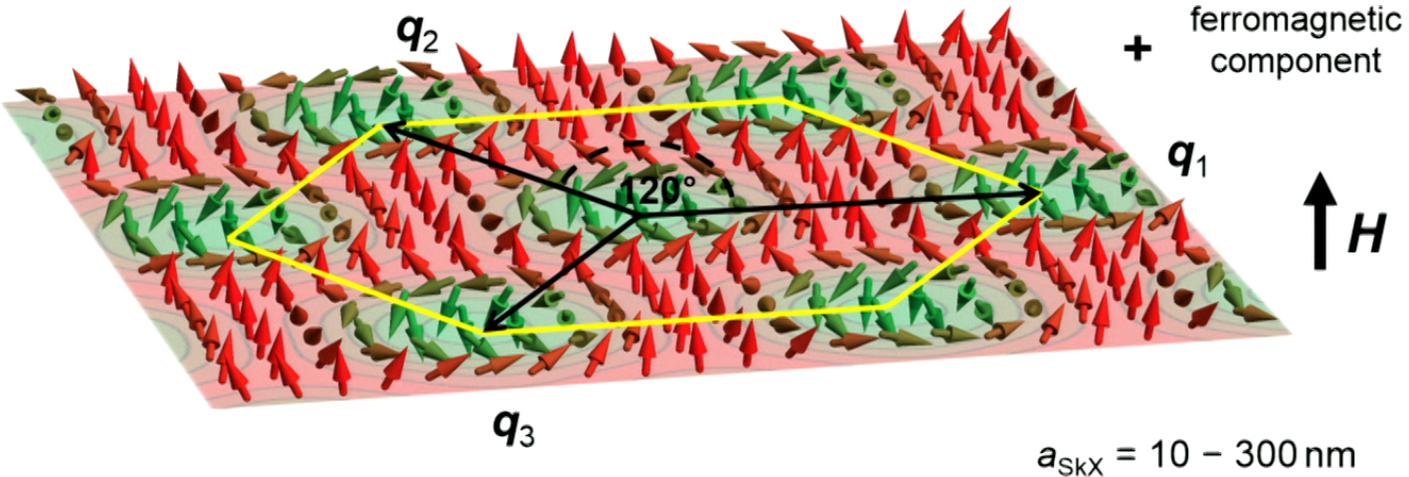
● TM  
● Si/Ge

- Heisenberg exchange:  
 $H \sim S_i \cdot S_j$
- Dzyaloshinski-Moriya interaction:  
 $H \sim S_i \times S_j$



Skyrmion lattice in a helimagnet:

T. Schwarze et al., Nature Mater. 14, 478 (2015)

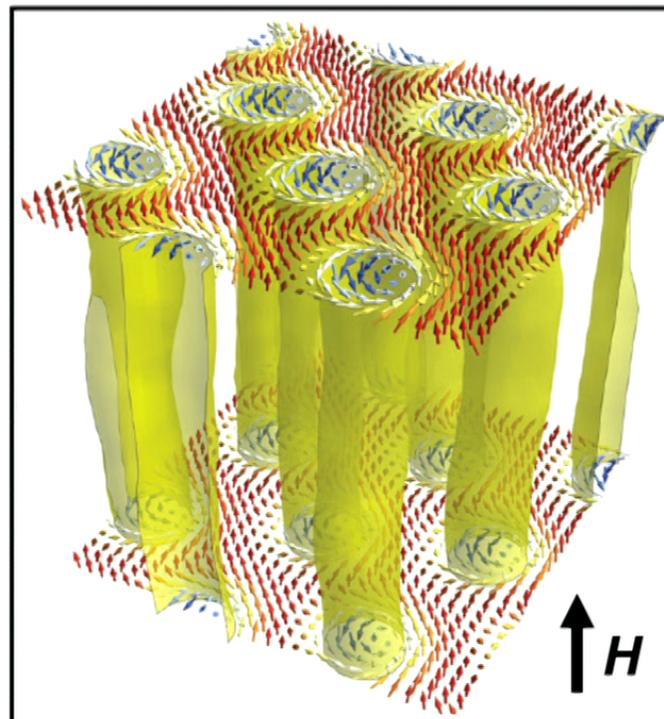




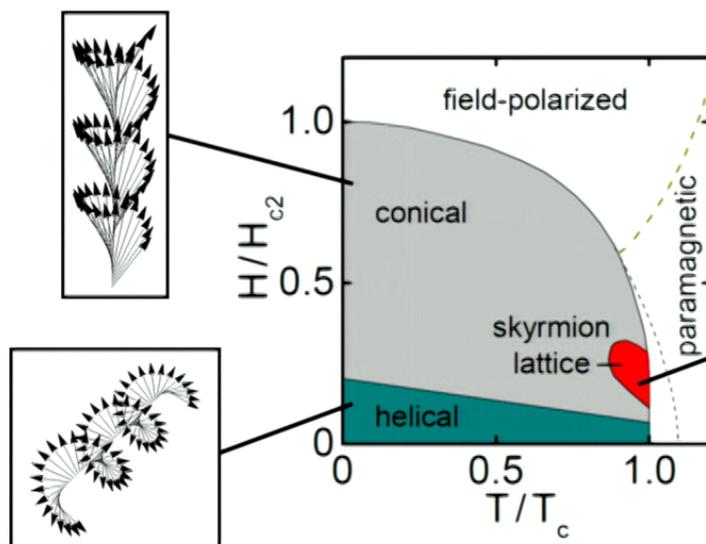
# Skyrmions in Cubic Chiral Magnets



material	$\lambda_h$ (nm)	$T_c$ (K)	$\mu_0 H_{c2}$ (T)
MnSi	18	29	0.6
$Fe_{1-x}Co_xSi$	30 - 300	< 50	< 0.15
FeGe	70	278	0.12
$Cu_2OSeO_3$	62	58	0.08
$Co_{1-x}Zn_{1-y}Mn_{x+y}$	120 - 180	< 450	~0.1



P. Milde et al., Science 340, 1076 (2013)





# Stabilization in MnSi



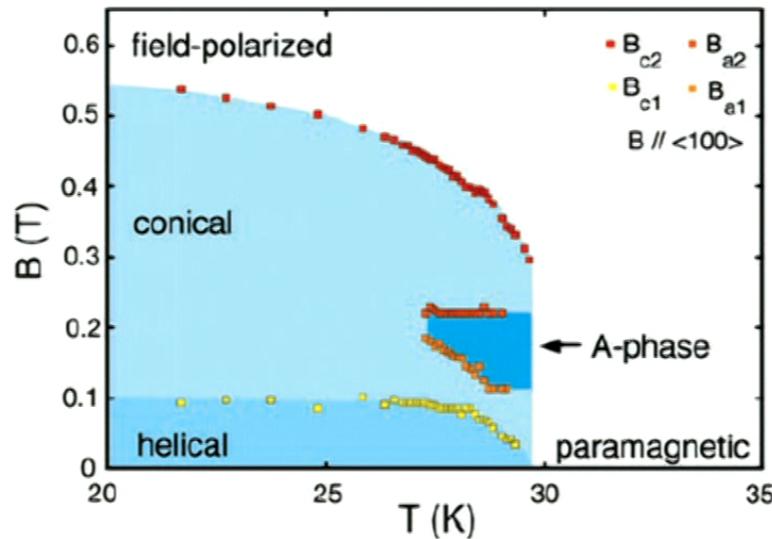
Free Energy:

$$F[\mathbf{M}] = \int d^3r [r_0 \mathbf{M}^2 + J(\nabla \mathbf{M})^2 + 2D\mathbf{M} \cdot (\nabla \times \mathbf{M}) + U\mathbf{M}^4 - \mathbf{B} \cdot \mathbf{M}]$$

$$\mathbf{M}(\mathbf{r}) \approx \mathbf{M}_f + \sum_{i=1}^3 \mathbf{M}_{\mathbf{Q}_i}^h(\mathbf{r} + \Delta \mathbf{r}_i)$$

$$\mathbf{M}_{\mathbf{Q}_i}^h(\mathbf{r}) = A[\mathbf{n}_{i1} \cos(\mathbf{Q}_i \cdot \mathbf{r}) + \mathbf{n}_{i2} \sin(\mathbf{Q}_i \cdot \mathbf{r})]$$

**SkX3**



- Stabilization à la order by disorder:

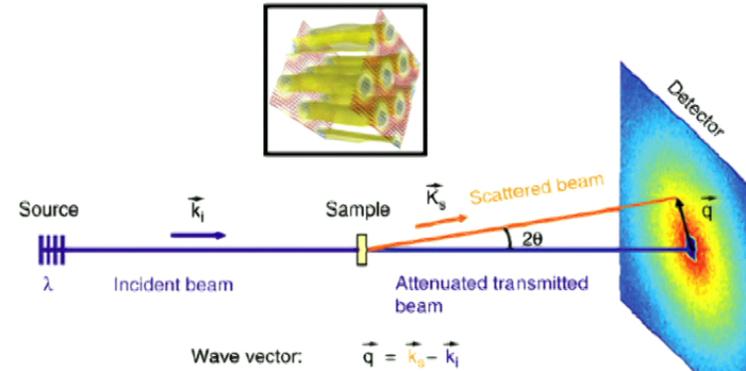
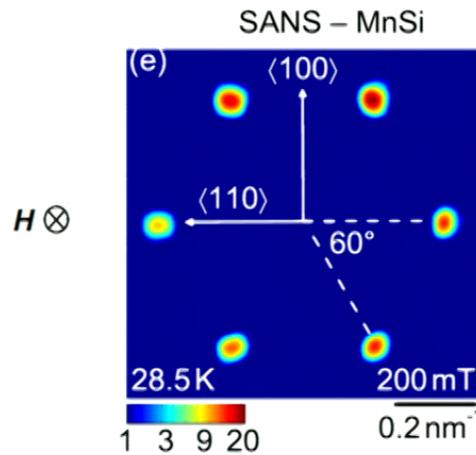
$$G \approx F[\mathbf{M}_0] + \frac{1}{2} \log \det \left( \frac{\delta^2 F}{\delta \mathbf{M} \delta \mathbf{M}} \right) \Big|_{\mathbf{M}_0}$$

- At larger temperatures only
- Isotropic w.r.t. field direction

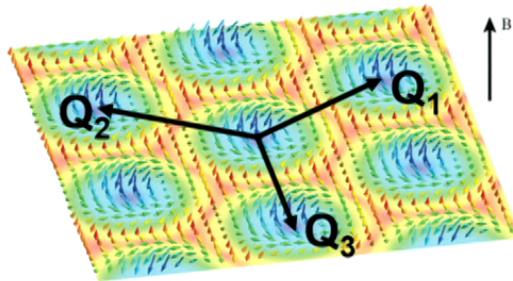
S. Mühlbauer *et al.*, Science **323**, 915 (2009)



# Multi-k and Topological Winding



Skyrmion Lattice as triple-k state:

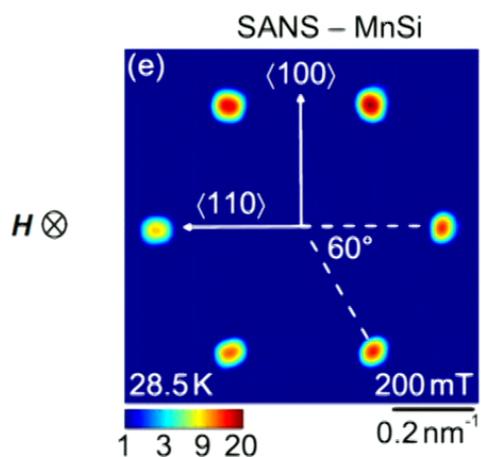


$$\mathbf{M}(r) \approx \mathbf{M}_f + \sum_{i=1}^3 \mathbf{M}_{\mathbf{Q}_i}^h(\mathbf{r} + \Delta \mathbf{r}_i)$$

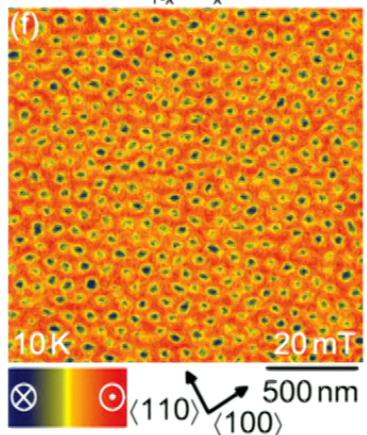
$$\mathbf{M}_{\mathbf{Q}_i}^h(\mathbf{r}) = A[\mathbf{n}_{i1} \cos(\mathbf{Q}_i \cdot \mathbf{r}) + \mathbf{n}_{i2} \sin(\mathbf{Q}_i \cdot \mathbf{r})]$$



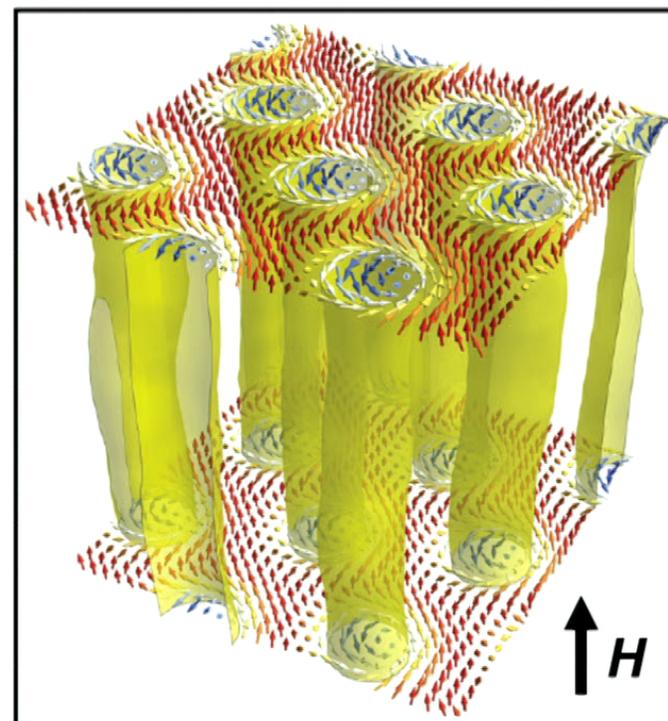
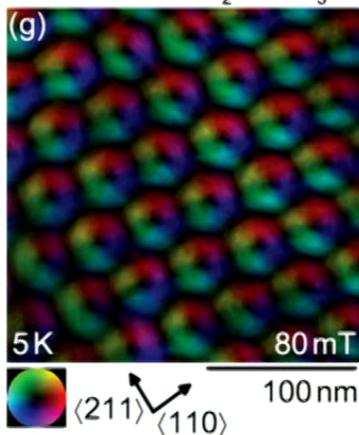
# Skyrmions in Helimagnets



MFM – Fe<sub>1-x</sub>Co<sub>x</sub>Si (x=0.5)



LF-TEM – Cu<sub>2</sub>OSeO<sub>3</sub>



P. Milde *et al.*, Science 340, 1076 (2013)

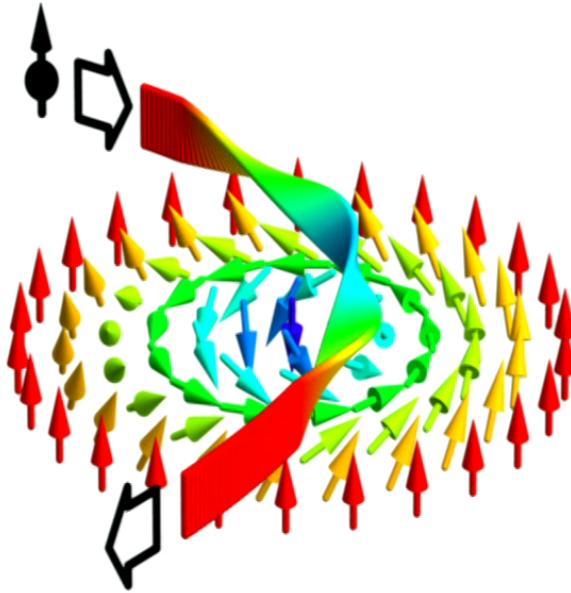
S. Mühlbauer *et al.*, Science 323, 915 (2009)  
X. Yu *et al.*, Nature 465, 901 (2010)  
T. Adams *et al.*, Phys. Rev. Lett. 107, 217206 (2011)  
S. Seki *et al.*, Science 336, 198 (2012)  
P. Milde *et al.*, Science 340, 1076 (2013)



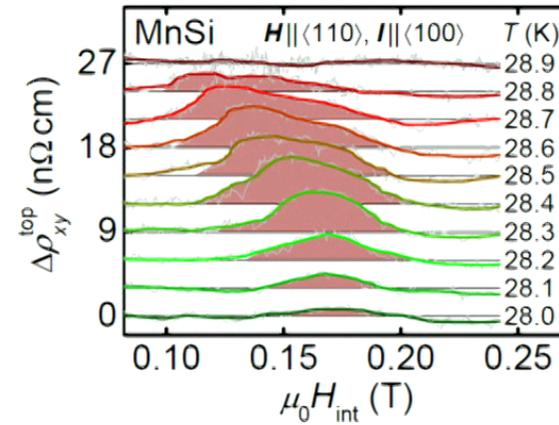
# Topological Hall Effect



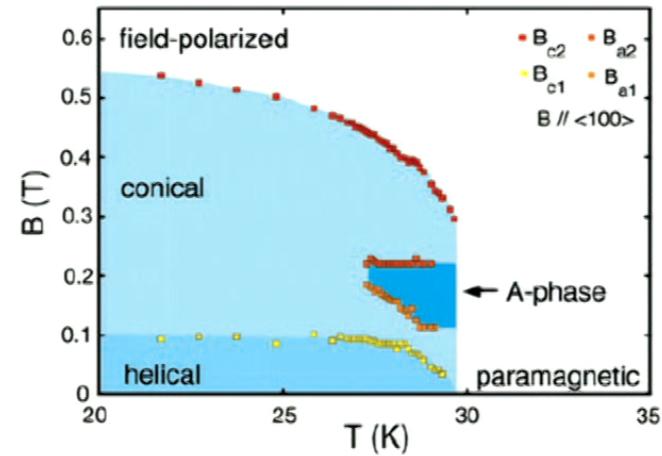
C. Pfleiderer & A. Rosch, Nature 465, 880 (2010)



- Extra force on the electrons

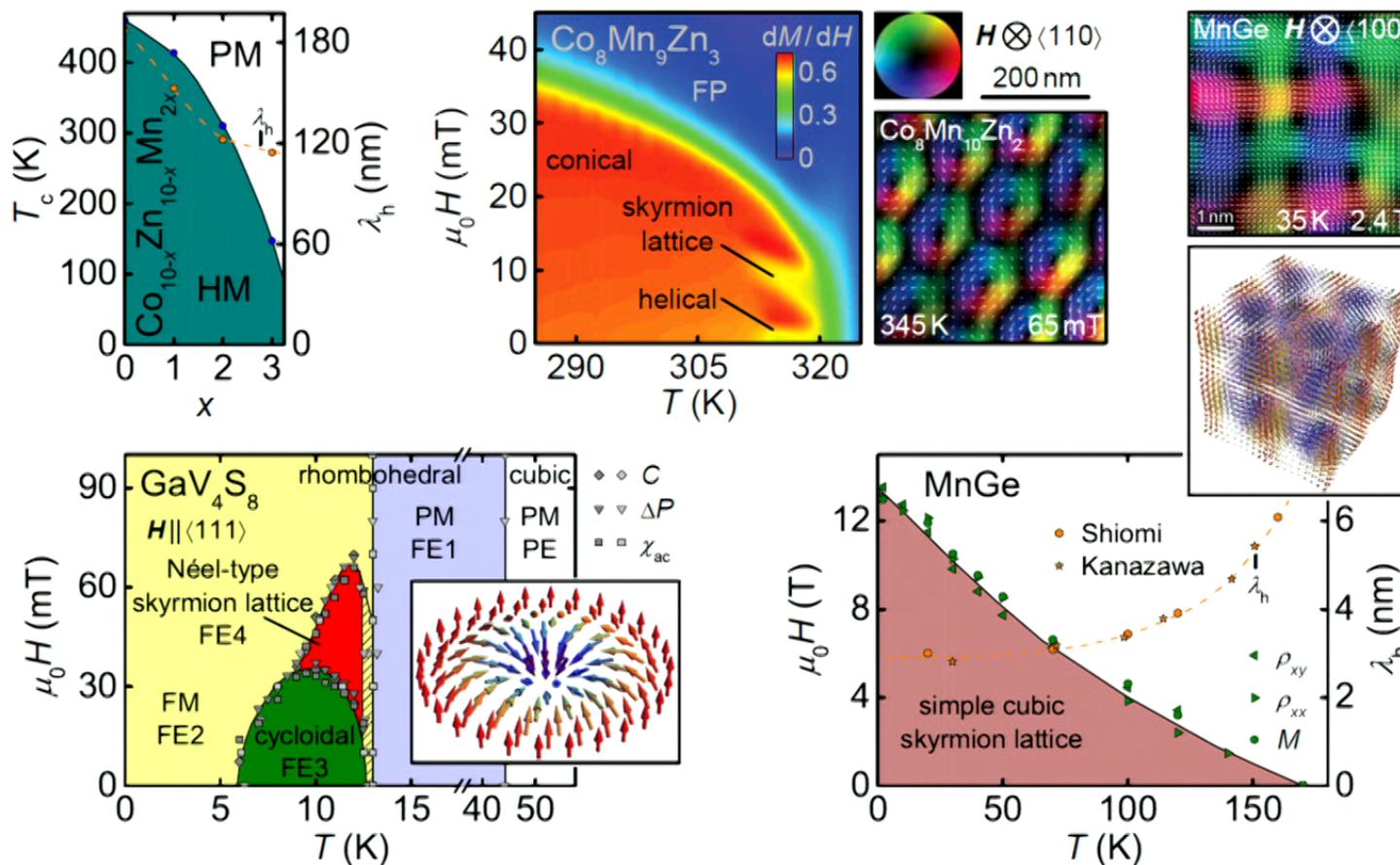


A. Neubauer et al., Phys. Rev. Lett. 102, 186602 (2009)  
R. Ritz et al., Phys. Rev. B 87, 134424 (2013)  
C. Franz et al., Phys. Rev. Lett. 112, 186601 (2014)





# Skyrmions in Bulk Compounds



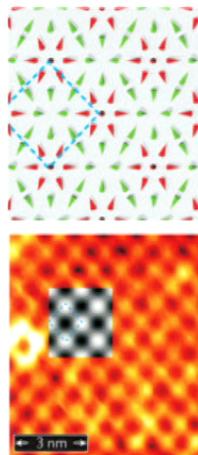
Y. Tokunaga *et al.*, Nat. Commun. 6, 7638 (2015)  
I. Kézsmárki *et al.*, Nature Mater. 14, 1116 (2015)  
T. Tanigaki *et al.*, Nano Lett. 15, 5438 (2015)



# Skyrmions in thin films & monolayers

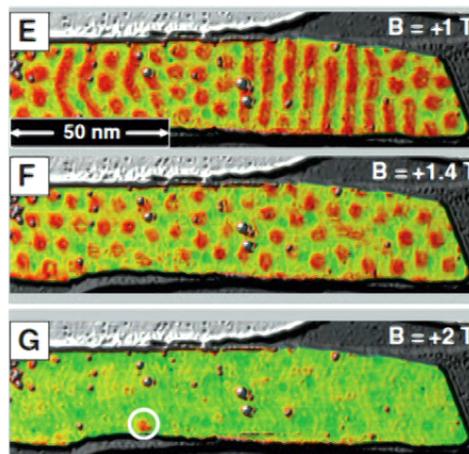


Fe/Ir(111)



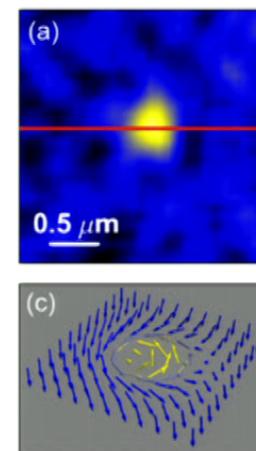
S. Heinze *et al.*, Nat. Phys. 7, 713 (2011)

Pd/Fe/Ir(111)



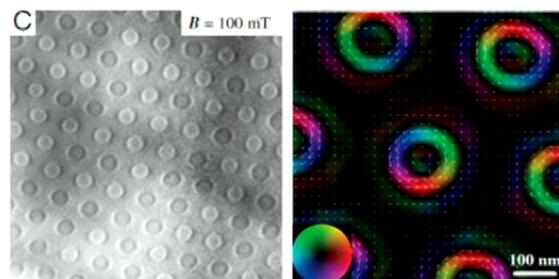
N. Romming *et al.*, Science 341, 636 (2013)

TbFeCo/Si<sub>3</sub>N<sub>4</sub>/AlTi/glass



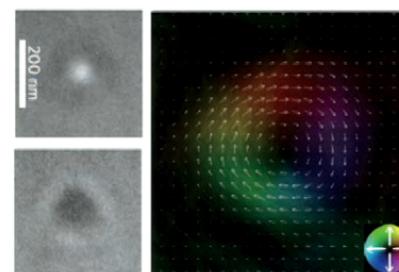
M. Finazzi *et al.*, Phys. Rev. Lett. 110, 177205 (2013)

BaFe<sub>12-x-0.05</sub>Sc<sub>x</sub>Mg<sub>0.05</sub>O<sub>19</sub>, x = 0.16



X. Yu *et al.*, Proc. Natl. Acad. Sci. USA 109, 8856 (2012)

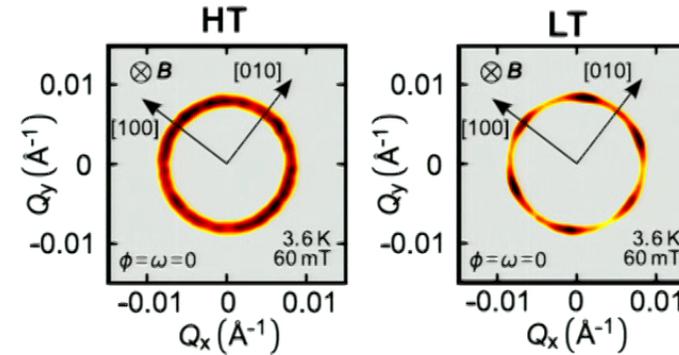
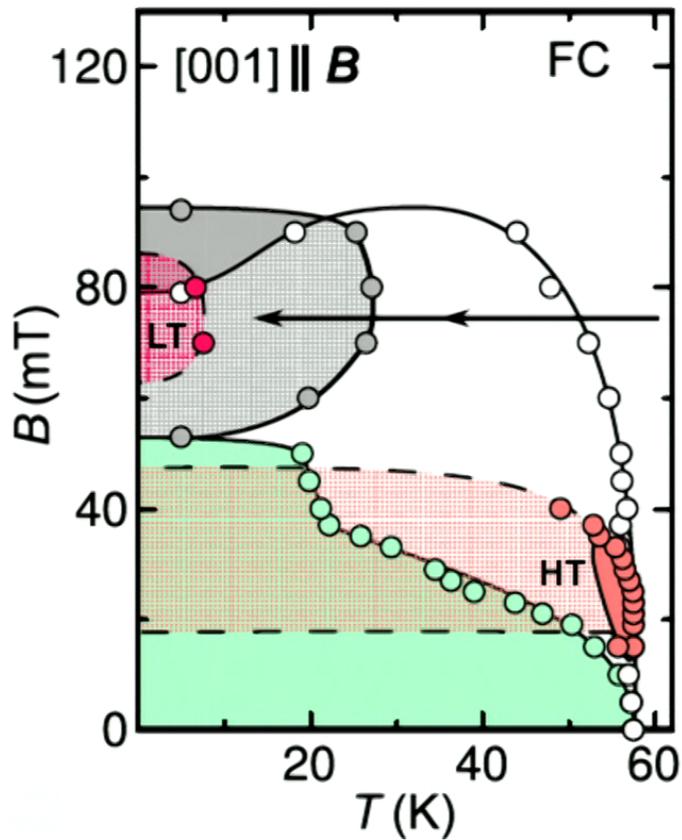
La<sub>0.5</sub>Ba<sub>0.5</sub>MnO<sub>3</sub>



M. Nagao *et al.*, Nature Nano. 8, 325 (2013)



# Two Skyrmion States in $\text{Cu}_2\text{OSeO}_3$



- Thermodynamically separated
- Stabilized by cubic anisotropies:

$$F_a = -K \int d^3r (M_x^4 + M_y^4 + M_z^4)$$

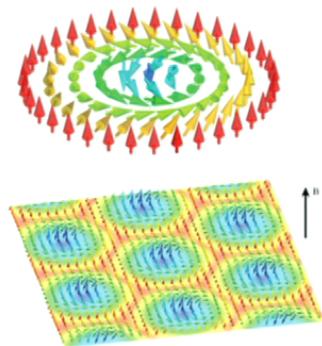
A. Chacon *et al.*, *accepted in Nature Phys.* (2018)



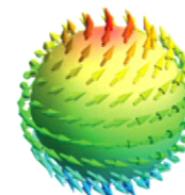
# Topological Classification



$$m : \mathbb{R}^2 \cup \{\infty\} \rightarrow S^2$$



$$m : S^2 \rightarrow S^2$$



$$\mathcal{P} : \mathbb{R}^2 \cup \{\infty\} \cong S^2$$

$$\int_{UC} n \cdot (\partial_x n \times \partial_y n) dA = -1$$

**Winding Number**

$$\Rightarrow 0 \neq [m] \in \pi_2(S^2) = \mathbb{Z}$$

$\pi_m(S^n)$  Homotopy Classes of Maps:  $f : S^m \rightarrow S^n$

	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$	$\pi_7$	$\pi_8$	$\pi_9$	$\pi_{10}$	$\pi_{11}$	$\pi_{12}$	$\pi_{13}$	$\pi_{14}$	$\pi_{15}$
$S^0$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$S^1$	$\mathbb{Z}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$S^2$	0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{12}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_{15}$	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_{12} \times \mathbb{Z}_2$	$\mathbb{Z}_{64} \times \mathbb{Z}_2^2$	$\mathbb{Z}_2^2$
$S^3$	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{12}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_{15}$	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_{12} \times \mathbb{Z}_2$	$\mathbb{Z}_{64} \times \mathbb{Z}_2^2$	$\mathbb{Z}_2^2$
$S^4$	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_{12}$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_{24} \times \mathbb{Z}_3$	$\mathbb{Z}_{15}$	$\mathbb{Z}_2$	$\mathbb{Z}_2^3$	$\mathbb{Z}_{120} \times \mathbb{Z}_{12} \times \mathbb{Z}_2$	$\mathbb{Z}_{64} \times \mathbb{Z}_2^5$
$S^5$	0	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{30}$	$\mathbb{Z}_2$	$\mathbb{Z}_2^3$	$\mathbb{Z}_{72} \times \mathbb{Z}_2$
$S^6$	0	0	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_{60}$	$\mathbb{Z}_{24} \times \mathbb{Z}_2$	$\mathbb{Z}_2^3$
$S^7$	0	0	0	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_{120}$	$\mathbb{Z}_2^3$
$S^8$	0	0	0	0	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	0	0	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_{120}$

Conventional Helix:  $\pi_1(S^1)$

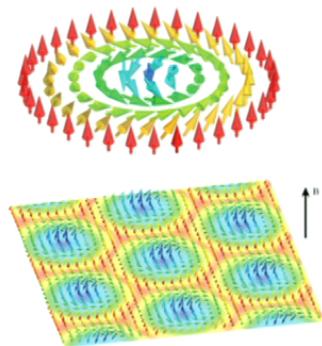
Magnetic Skyrmions:  $\pi_2(S^2)$



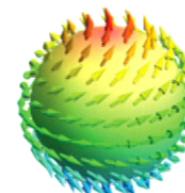
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	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$	$\pi_7$	$\pi_8$	$\pi_9$	$\pi_{10}$	$\pi_{11}$	$\pi_{12}$	$\pi_{13}$	$\pi_{14}$	$\pi_{15}$
$S^0$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$S^1$	$\mathbb{Z}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$S^2$	0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{12}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_{15}$	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_{12} \times \mathbb{Z}_2$	$\mathbb{Z}_{64} \times \mathbb{Z}_2^2$	$\mathbb{Z}_2^2$
$S^3$	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{12}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_3$	$\mathbb{Z}_{15}$	$\mathbb{Z}_2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_{12} \times \mathbb{Z}_2$	$\mathbb{Z}_{64} \times \mathbb{Z}_2^2$	$\mathbb{Z}_2^2$
$S^4$	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_{12}$	$\mathbb{Z}_2^2$	$\mathbb{Z}_2^2$	$\mathbb{Z}_{24} \times \mathbb{Z}_3$	$\mathbb{Z}_{15}$	$\mathbb{Z}_2$	$\mathbb{Z}_2^3$	$\mathbb{Z}_{120} \times \mathbb{Z}_{12} \times \mathbb{Z}_2$	$\mathbb{Z}_{64} \times \mathbb{Z}_2^5$
$S^5$	0	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{30}$	$\mathbb{Z}_2$	$\mathbb{Z}_2^3$	$\mathbb{Z}_{72} \times \mathbb{Z}_2$
$S^6$	0	0	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_{60}$	$\mathbb{Z}_{24} \times \mathbb{Z}_2$	$\mathbb{Z}_2^3$
$S^7$	0	0	0	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_{120}$	$\mathbb{Z}_2^3$
$S^8$	0	0	0	0	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}_{24}$	0	0	$\mathbb{Z}_2$	$\mathbb{Z} \times \mathbb{Z}_{120}$

**Conventional Helix:**  $\pi_1(S^1)$

**Magnetic Skyrmions:**  $\pi_2(S^2)$

**Nuclear Skyrmions:**  $\pi_3(S^3)$



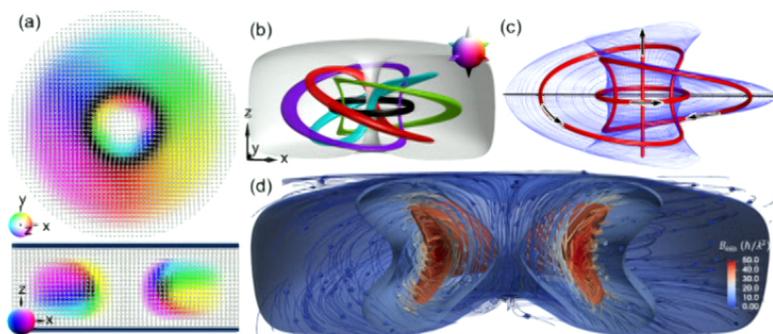
T. H. R. Skyrme, Proc. R. Soc. Lond. A **260**, 127 (1961)  
 T. H. R. Skyrme, Proc. R. Soc. Lond. A **262**, 237 (1961)  
 T. H. R. Skyrme, Nucl. Phys. **31**, 556 (1962)



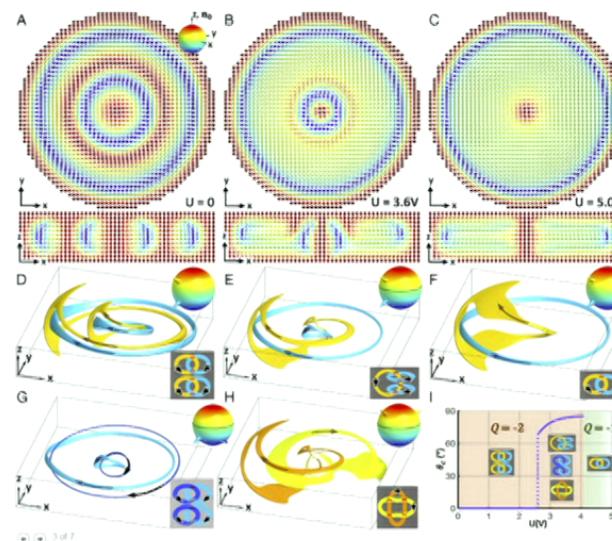
# Search for Three Dimensional Solitons



- Existence of three dimensional magnetic solitons:  $\mathbf{n} : S^3 \rightarrow S^2$  ?



<https://arxiv.org/pdf/1806.00453.pdf>



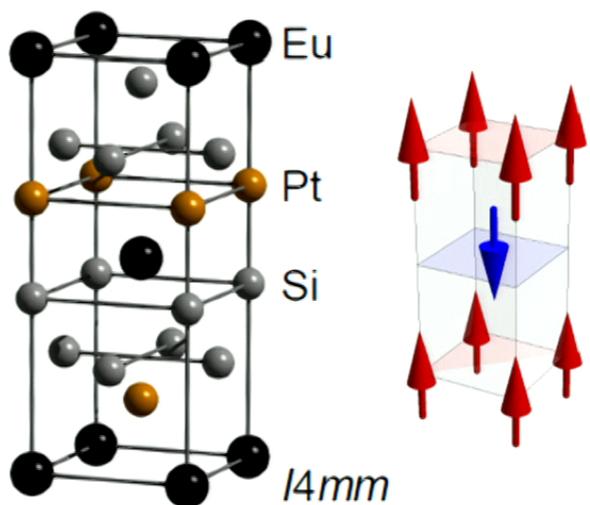
$$Q = \frac{1}{64\pi^2} \int_{\mathbb{R}^3} d^3r \epsilon^{ijk} A_i F_{jk}$$

Q. Liu, P. J. Ackerman T. C. Lubensky and I. I. Smalyukh, PNAS 38, 10479 (2018).

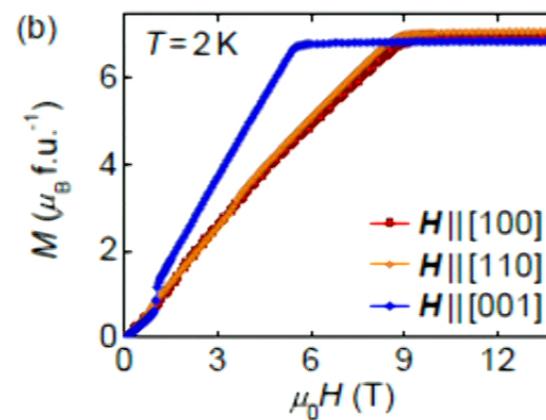
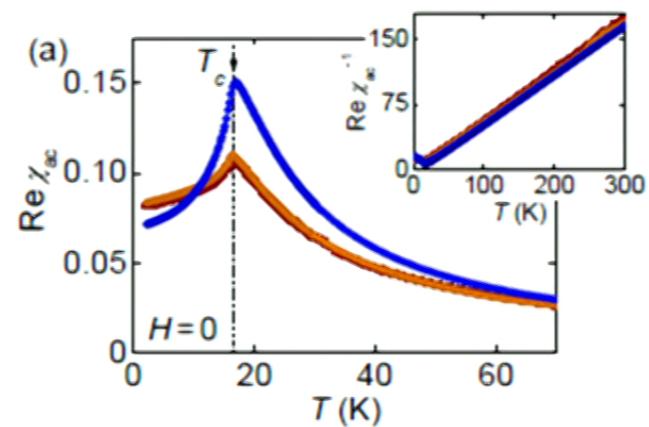
- Strong restrictions by Derrick's Theorem  
G. H. Derrick, Journal of Mathematical Physics 5, 1252 (1964).



# EuPtSi3

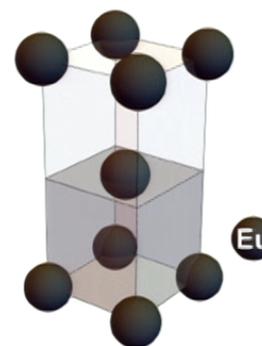
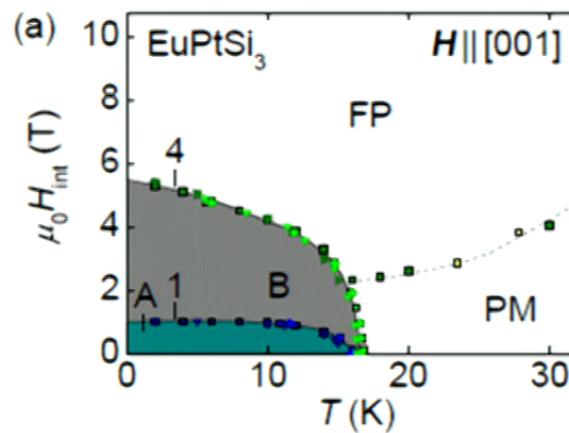
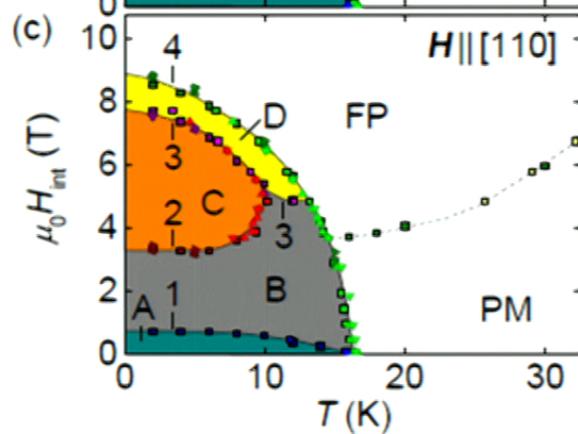
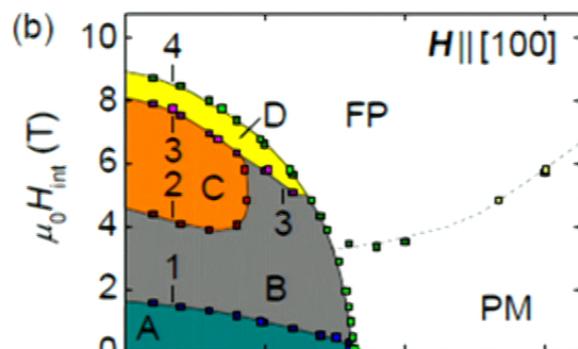


- Non centro-symmetric
- Antiferromagnetic  $\sim [111]$
- Curie Weiss:  $\theta_W > 0$





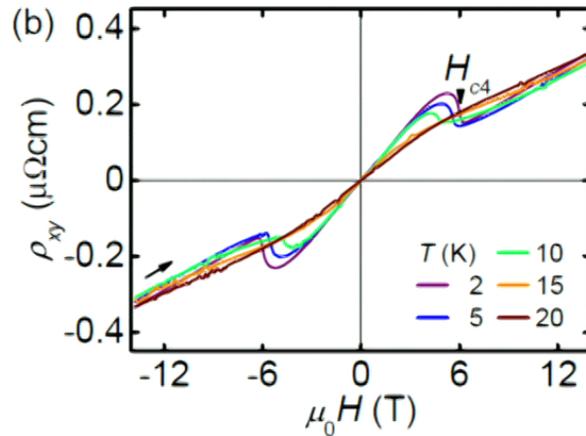
# Magnetic Phase Diagrams



tetragonal anisotropies

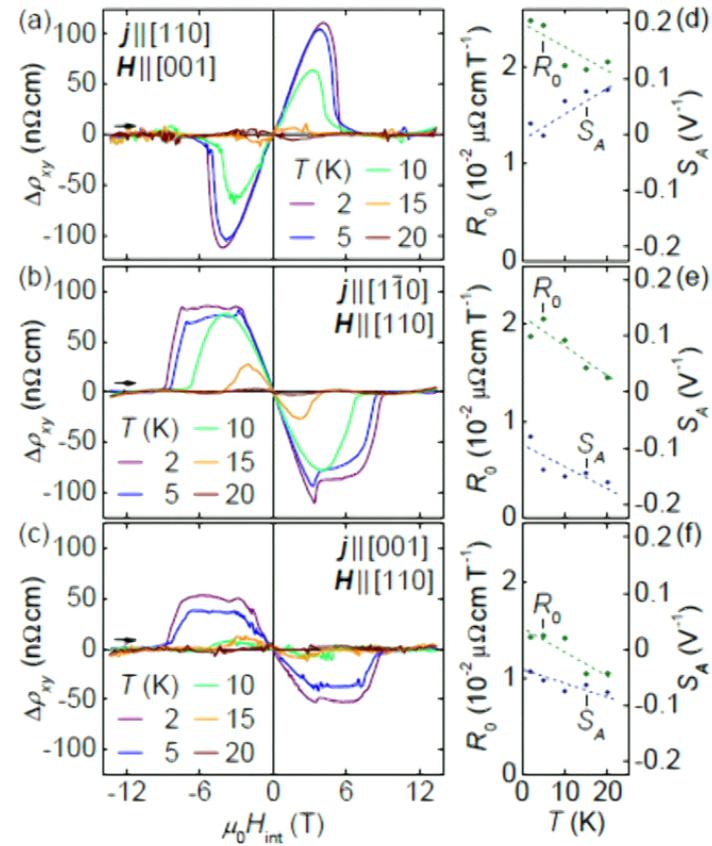


# Transport Properties



$$\rho_{xy}(H) = R_0 \mu_0 H + S_A \rho_{xx}^2(H) M(H) + \Delta \rho_{xy}(H)$$

non trivial Berry phases



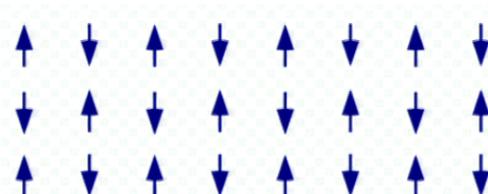


# Berry Phases



Which magnets make for an anomalous Hall signal?

- $\rho_{xy}(H) = R_0\mu_0 H + S_A \rho_{xx}^2(H)M(H)$   
E. M. Pugh and T. W. Lippert, Phys. Rev. **42**, 709 (1932).



Modern approach with **Berry Phases**:

Vector Potential:  $\mathbf{A}_n(\mathbf{k}) := \langle u_n | \nabla | u_n \rangle$

Berry Curvature:  $\Omega_n(\mathbf{k}) := \nabla_{\mathbf{k}} \times \mathbf{A}_n(\mathbf{k})$

Hall conductivity:  $\sigma_{xy} = \int d\mathbf{k} f(\epsilon) \Omega^z(\mathbf{k})$



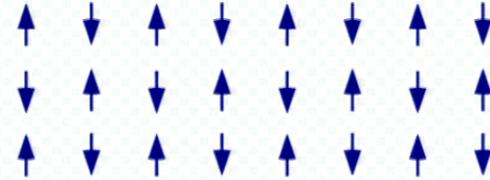


# Berry Phases



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E. M. Pugh and T. W. Lippert, Phys. Rev. **42**, 709 (1932).

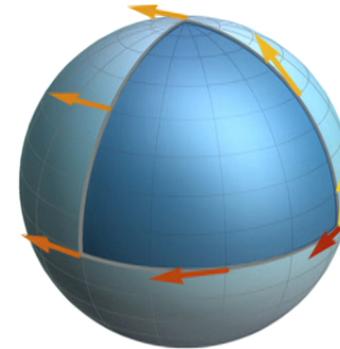


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$$\text{Hall conductivity: } \sigma_{xy} = \int d\mathbf{k} f(\epsilon) \Omega^z(\mathbf{k})$$



Semiclassical picture:

$$\Omega_{ij} = \frac{\partial A_j}{\partial x_i} - \frac{\partial A_i}{\partial x_j} = \hbar \hat{\mathbf{n}} \cdot \left( \frac{\partial}{\partial x_i} \hat{\mathbf{n}} \times \frac{\partial}{\partial x_j} \hat{\mathbf{n}} \right)$$

**real space**

$i = 1, 2, 3$

(~THE)

**reciprocal space**

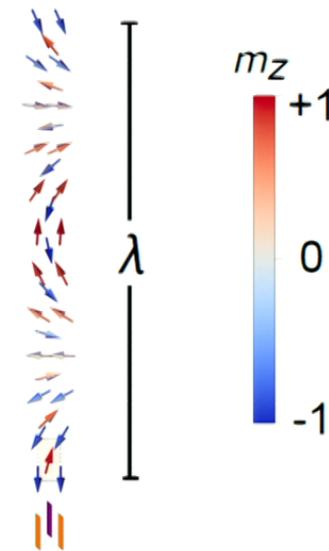
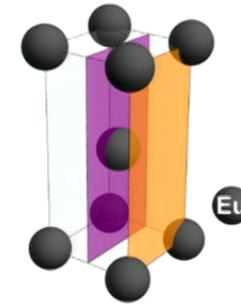
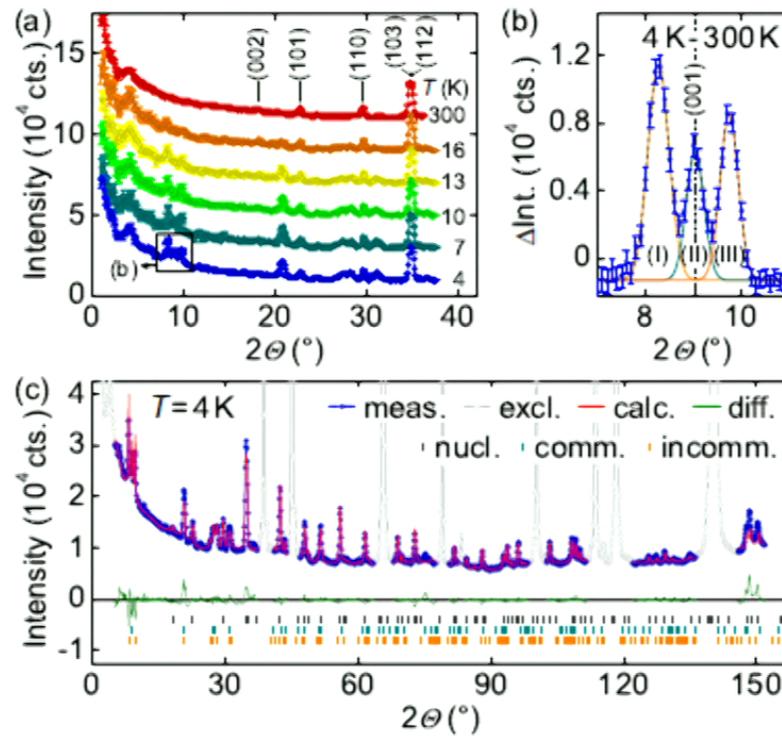
$i = 4, 5, 6$

**mixed**

R. Ritz et al., Phys. Rev. B **87**, 134424 (2013)



# Neutron Powder Diffraction



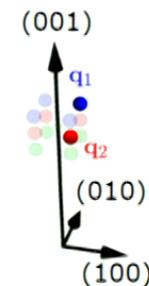
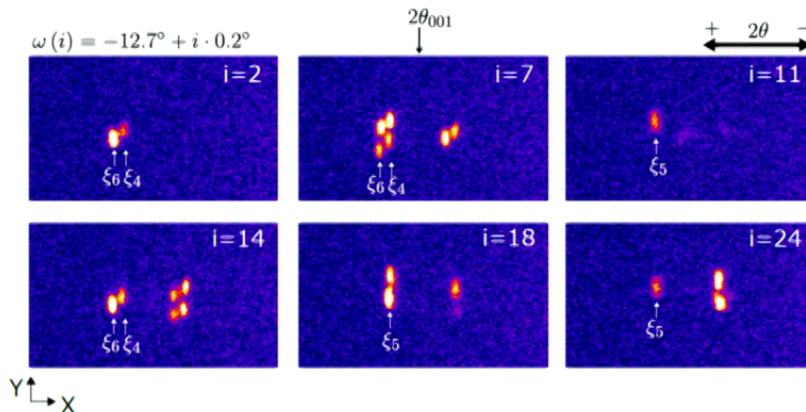
- Cycloidal structures with long wavelength



# Single Crystal Neutron Diffraction

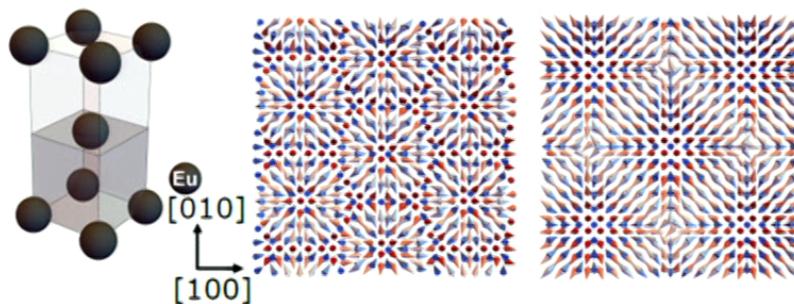


$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{el}} = (\gamma_0)^2 N \left\{ \frac{1}{2} g F(\kappa) \right\}^2 \exp(-2W) \sum_{\alpha, \beta} (\delta_{\alpha\beta} - \hat{\kappa}_\alpha \hat{\kappa}_\beta) \sum_{\mathbf{R}_1, \mathbf{R}_2} \exp(i\kappa \cdot \mathbf{R}) \langle S_{\mathbf{R}_1}^\alpha \rangle \langle S_{\mathbf{R}_2}^\beta \rangle$$

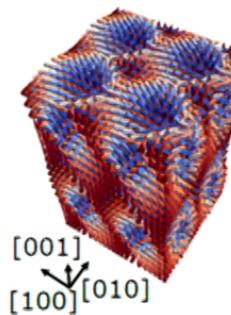


$$m(\mathbf{R}) = \sum_{\mathbf{k}, \nu} A_\nu^{\mathbf{k}} \cdot \Psi_\nu^{\mathbf{k}} \cos(\mathbf{k}\mathbf{R} + \delta_\nu^{\mathbf{k}})$$

pronounced multi-k Signatures



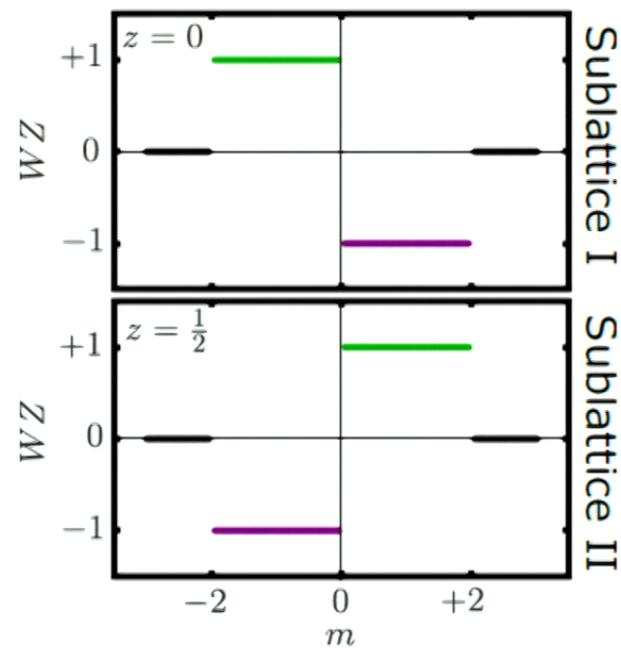
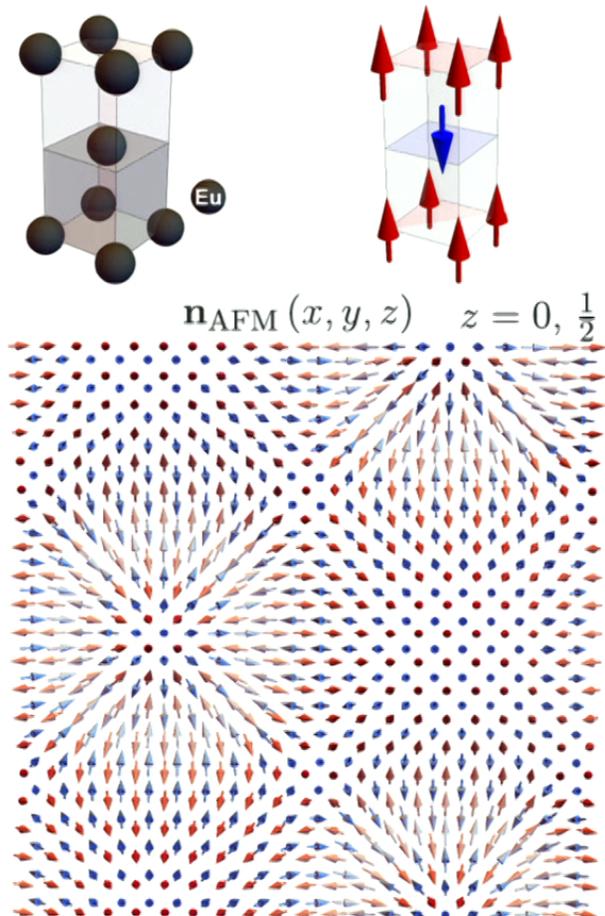
Antiferromagnetic Skyrmion



Antiferromagnetic Hedgehog-Antihedgehog Lattice



# Antiferromagnetic Skyrmions



no THE in Antiferromagnetic Skyrmions

cf. G. Göbel et al., Phys. Rev. B **96**, 060406 (2017)



# Search for Three Dimensional Solitons

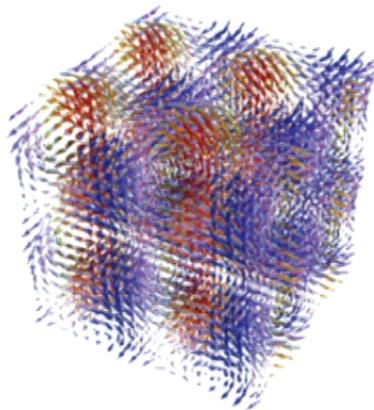


- Magnetic structure with non-coplanar modulations:

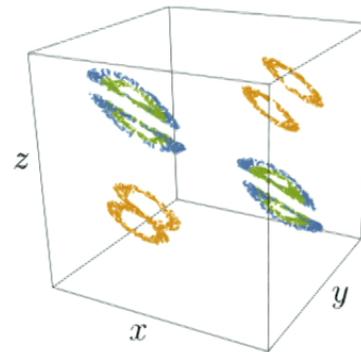
$$\mathbf{m} = \hat{e}_z \cdot \cos(\pi(x+y)) + \hat{e}_x \cdot \cos(\pi(y+z)) + \hat{e}_y \cdot \cos(\pi(x+z)) + \mathbf{fm}$$

- Directorfield:  $\mathbf{n} = \frac{\mathbf{m}}{|\mathbf{m}|}$        $\mathbf{n} : S^3 \rightarrow S^2$

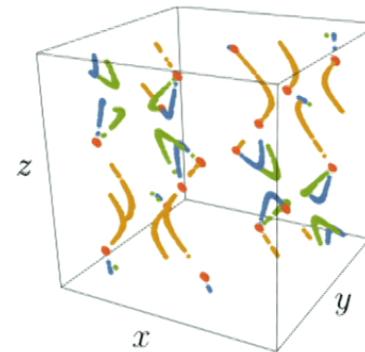
## Preimage method



$$\mathbf{fm} = (0, 0, 4)$$



$$\mathbf{fm} = (0, 0, 0)$$

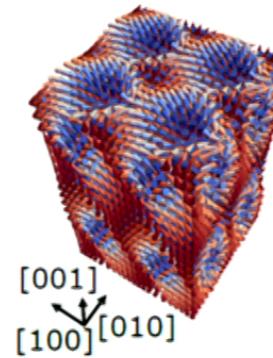
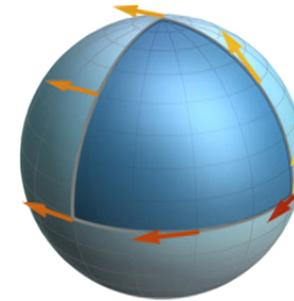
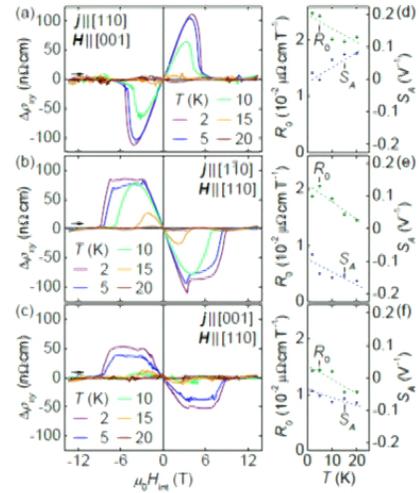
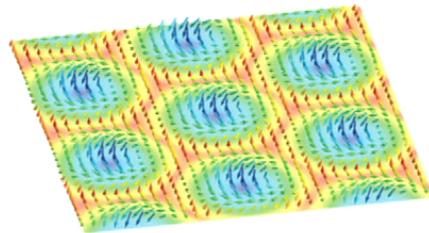
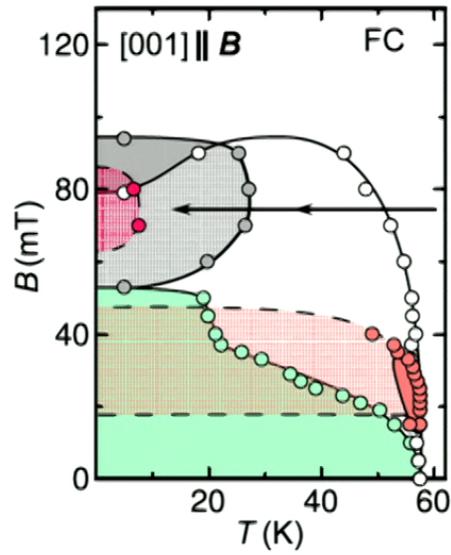


$$\begin{array}{ll} \mathbf{n}^{-1}((0, 0, 0)) & \mathbf{n}^{-1}(p_1) \\ \mathbf{n}^{-1}(p_2) & \mathbf{n}^{-1}(p_3) \end{array}$$

$$x, y, z \in [-1, 1]$$



# Summary





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