Title: Holographic Solids: Transverse Phonons and Elastic Response

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Abstract:

HOLOGRAPHIC SOLIDS: TRANSVERSE PHONONS AND ELASTIC RESPONSE

Lasma Alberte, ICTP
18 June 2018, Perimeter Institute

based on arXiv:1510.09089, arXiv:1601.03384
with **Matteo Baggioli, Andrey Khmelnitsky and Oriol Pujolas**

and on arXiv:1708.08477 and arXiv:1711.03100 also with
Martin Ammon and Amadaeo Jimenez

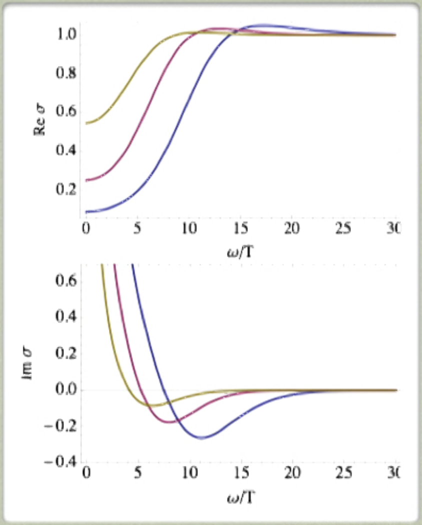
and work in progress also with
Victor Cancer Castillo

use massive gravity to describe strongly correlated materials with momentum dissipation and solid type properties

ADS/CONDENSED MATTER THEORY

Hartnoll (2009)

- linear response via retarded Green's function: $\delta \langle \mathcal{O}_A \rangle = G^R_{O_A O_B} \delta\phi_{B(0)}$
- typical example: electrical conductivity



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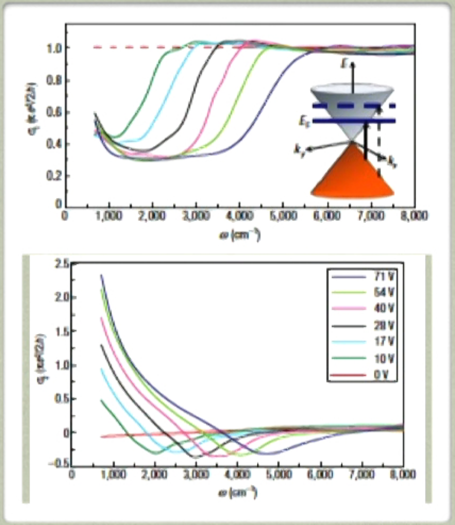

Li et al. (2008)

ADS/CMT RESULT

EXPERIMENT

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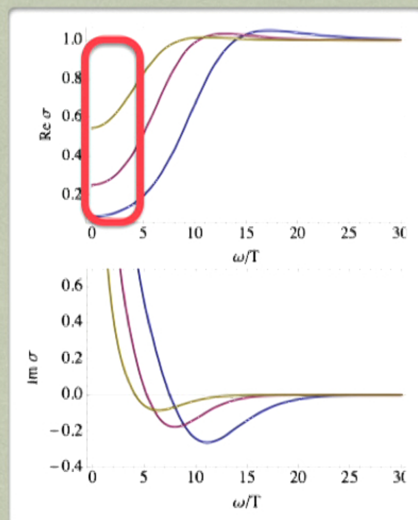

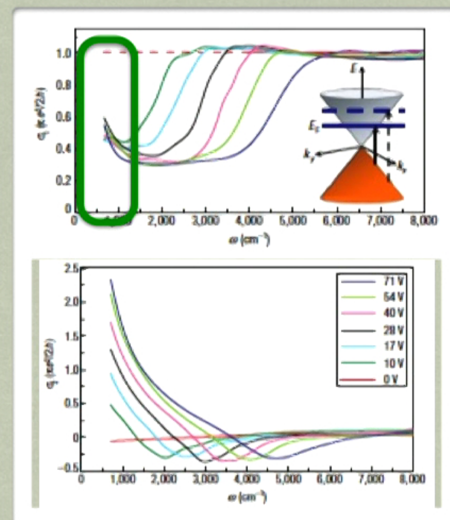

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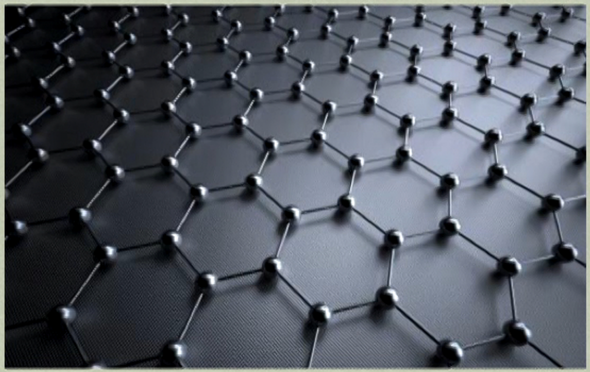
ADS/CMT RESULT

EXPERIMENT

MOMENTUM DISSIPATION

Vegh (2012), Davison (2013), Blake, Tong (2013)

- need to incorporate effects of broken translational invariance
- TRANSLATIONAL INVARIANCE on the boundary follows from the DIFFEOMORPHISM INVARIANCE in the bulk
- break the diffeos — use massive gravity



from massive gravity to elastic response

GRAVITON MASS TERMS

- a unique healthy Lorentz invariant mass term at quadratic level:

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left[-R + \frac{m^2}{4} \left(h^2 - h^{\mu\nu} h_{\mu\nu} \right) \right]$$

Fierz-Pauli (1939)

- changes the gravitational potential

$$\phi = -\frac{4}{3} \frac{G_N M_0}{r} e^{-m_g r}$$

IR suppression

fifth force

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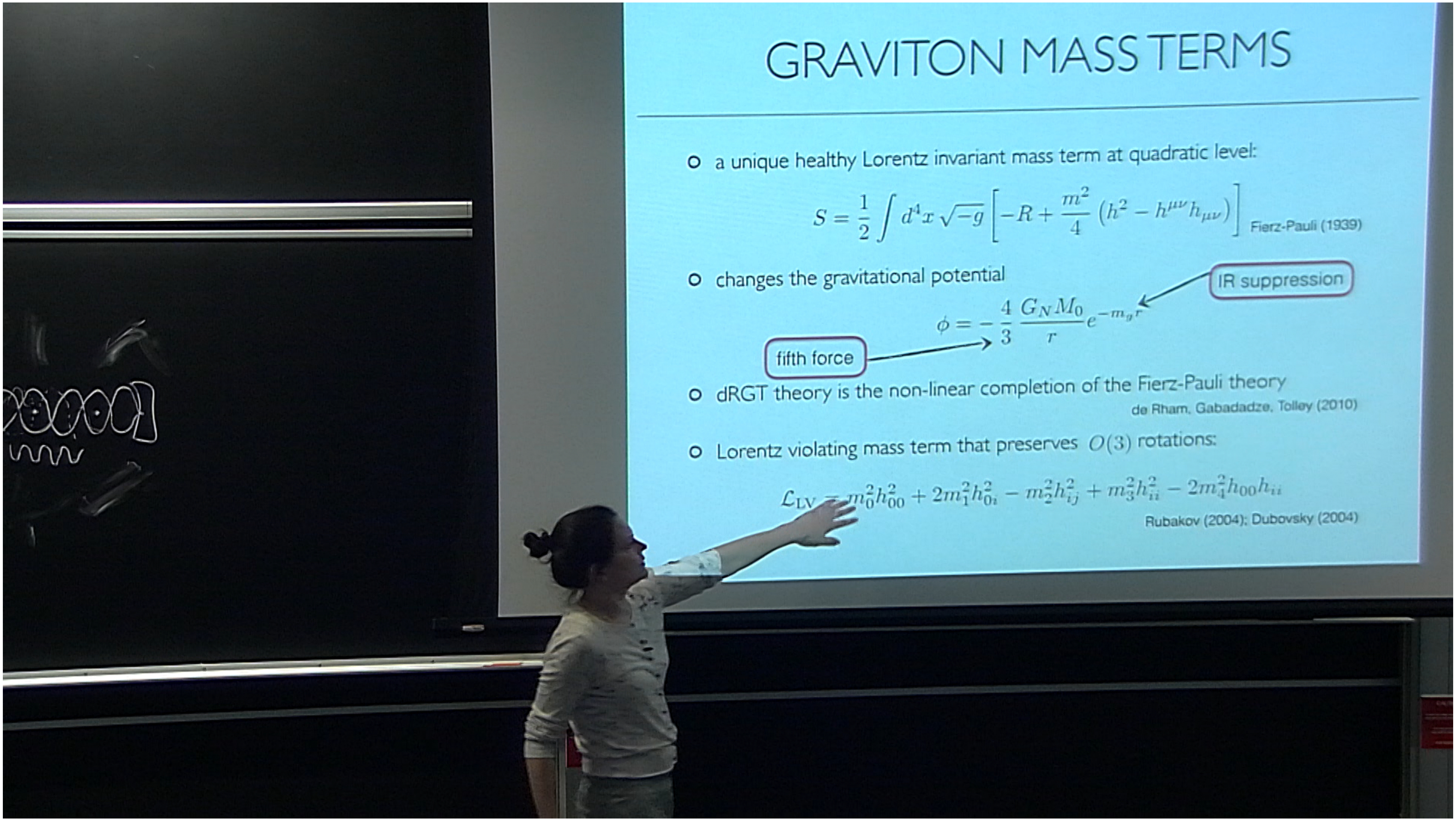
- dRGT theory is the non-linear completion of the Fierz-Pauli theory

de Rham, Gabadadze, Tolley (2010)

- Lorentz violating mass term that preserves $O(3)$ rotations:

$$\mathcal{L}_{\rm LV} = m_0^2 h_{00}^2 + 2m_1^2 h_{0i}^2 - m_2^2 h_{ij}^2 + m_3^2 h_{ii}^2 - 2m_4^2 h_{00}h_{ii}$$

Rubakov (2004); Dubovsky (2004)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STÜCKELBERG SCALARS

$$\mathcal{L} = m^2 h^{\mu\nu} h^{\alpha\beta} \left(f_{\mu\nu} f_{\alpha\beta} - f_{\mu\alpha} f_{\nu\beta} \right)$$

- $f_{\mu\nu}$ — auxiliary reference metric that encodes the symmetries of the graviton mass term:
  - a metric like $f_{\mu\nu} = \mathrm{diag}\,(-a, b, b, b)$ gives an $O(3)$ invariant mass term
- Parametrize the auxiliary metric with scalars: $f_{\mu\nu} = \partial_\mu \phi^A \partial_\nu \phi^B f_{AB}$

On the scalar fields background $\langle \phi^A \rangle = x^\mu \delta^A_\mu$, one recovers $f_{AB} = f_{\mu\nu}\, \delta^\mu_A \delta^\nu_B$

V(X, Z) THEORIES

Baggioli, Pujolas (2015)
LA, Baggioli, Khmelnitsky, Pujolas (2015)

$$ds^2 = L^2 \left(\frac{dr^2}{f(r) r^2} + \frac{-f(r) dt^2 + dx^2 + dy^2}{r^2} \right)$$

t, x^i — coordinates on the boundary

r — the holographic direction

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- in order to only give mass in the transverse directions x^i and to preserve the $t\,,r$ diffeos use TWO SCALARS ϕ^I with $\langle\phi^I\rangle = \delta^I_j\, x^j$
- introduce an internal $O(2)$ symmetry for isotropy

THE MASS LAGRANGIAN IS A FUNCTION OF POWERS OF THE MATRIX $\mathcal{I}^{IJ} = g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J$ WITH ALL THE INDICES CONTRACTED WITH δ_{IJ}

a 2×2 matrix ⟶ two independent contractions: $X = \frac{1}{2}\mathrm{tr}\,\mathcal{I}^{IJ}\,,\quad Z = \det\mathcal{I}^{IJ}$

Final mass Lagrangian: $\mathcal{L}_{\mathrm{mass}} = \sqrt{-g}\,V(X,Z)$

EFTs FOR SOLIDS AND FLUIDS

Leutwyler (1993); Dubovsky et al. (2005, 2012), Nicolis et al. (2013)

- describe the low energy behavior of fluid and solid systems in $d+1$ dimensions by d scalar fields $\phi^I(x^i, t)$
- introduce internal symmetries to recover long scale homogeneity and isotropy
- spontaneously break these by the equilibrium configuration: $\langle \phi^I \rangle = \delta^I_j \, x^j$

THE FIELD EXCITATIONS $\pi^I(x^\mu) =$ THE GOLDSTONE BOSONS

- MASSIVE GRAVITIES written in terms of $\mathcal{I}^{IJ} = g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J$ give

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FLUIDS

volume preserving diffeomorphisms

$$\phi^I \mapsto \psi^I(\phi^J)\,, \qquad \det\left[\frac{\partial\psi^I}{\partial\phi^J}\right] = 1$$

$$\mathcal{L}^{(\text{fluids})} = V_{\text{f}}\left(\det\mathcal{I}^{IJ}\right)$$

SOLIDS

rotations and constant shifts

$$\phi^I \mapsto \psi^I = O^I_J\phi^J + c^I$$

$$\mathcal{L}^{(\text{solids})} = V_{\text{s}}\left(\text{tr}\,\mathcal{I}^{IJ},\, \det\mathcal{I}^{IJ}\right)$$

MASSIVE?

Split the mass term as

$$\mathcal{L}_{\text{mass}} = m^2_{\text{fluids}} U(Z) + m^2_{\text{solids}} V(X, Z)$$

and look at the homogeneous transverse traceless metric perturbation $h = \{h_+, h_\times\}$ on the black brane. The EOM becomes:

$$\left[f\partial_r^2 + \left(f' - 2\frac{f}{r} \right) \partial_r + \left(\frac{\omega^2}{f} - 4m^2_{\text{solids}} M^2(r) \frac{r^2}{L^2} \right) \right] h = 0$$

where $M^2(r) \propto r^\nu$ — a mass function, depending on the potential.

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ONLY THE SOLID-TYPE MASS TERM GIVES MASS TO THE GRAVITON!

'STANDARD' ELASTICITY

- under applied stress material points move: $u_i = x'_i - x_i$
- the corresponding **displacement tensor:** $u_{ij} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i\right)$

related to the **stress tensor** as: $\sigma_{ik}^{(\mathrm{T})} = 2\mu u_{ik}^{(\mathrm{T})}$

elastic waves = propagation of the deformation u_i inside the medium

the non-relativistic sound speed $c_T^2 = \frac{\mu}{\varepsilon}$

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- the analogue of mechanical deformations: $\pi^I = \phi^I - \langle \phi^I \rangle$
- the analogue of deformation tensor: $u^{IJ} = \frac{1}{2}\left(\partial^I \pi^J + \partial^J \pi^I\right)$
- find elasticity as $\delta T^{IJ}_{(\mathrm{T})} = 2\mu\, u^{IJ}_{(\mathrm{T})}$ and the transverse sound:

$c_T^2 = \frac{\mu}{\varepsilon + p}$ **the relativistic sound speed** ✓

WHAT TO EXPECT ON BOUNDARY?

- the standard CM **phonons**
- two types of sound waves: the longitudinal and the **transverse**

$$c_L^2 = \frac{\kappa + \frac{4}{3}\mu}{\varepsilon + p}, \qquad c_T^2 = \frac{\mu}{\varepsilon + p}$$

κ — the bulk modulus describing the change in volume

μ — the shear modulus describing the change in shape

- holographic fluids are well-studied; **holographic solids are new**

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See also: Esposito, Garcia-Saenz, Nicolis, Penco (2017)

ELASTICITY IN ADS/CMT

LA, Baggioli, Khmelnitsky, Pujolas (2015)
LA, Baggioli, Pujolas (2016)

MAKE THE IDENTIFICATION

$$u_{ij} \longleftrightarrow \frac{1}{2} h_{ij}$$

$$\sigma_{ij} \longleftrightarrow \langle T_{ij} \rangle$$

ADS/CMT PRESCRIPTION:

- gives boundary stress-tensor in terms of retarded Green's function

$$\langle T_{ij} \rangle = \mathcal{G}^R_{T_{ij}\,T_{ij}} h_{ij} \longrightarrow G = -\mathrm{Re}\, \mathcal{G}^R_{T_{ij}\,T_{ij}}$$

- analytic expression for $m/T \ll 1$:

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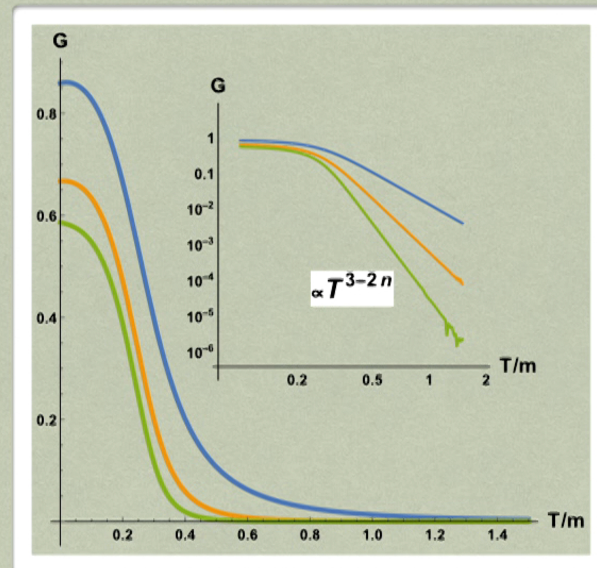
$$V(X) = X^n$$



$$G = \frac{L^2}{2r_h^3} \frac{2m^2}{2n-3} \left(\frac{r_h}{L} \right)^{2n}$$

transverse phonons

GOLDSTONE BOSONS

- **SPONTANEOUS SYMMETRY BREAKING:** a massless Goldstone boson
  - **add explicit breaking:** mass gap $\omega^2 = \omega_0^2 + c_T^2 k^2$
  - **add dissipation:** non-zero decay rate $\omega^2 + i\,\Gamma\,\omega = \omega_0^2 + c_T^2\,k^2$
- due to $T \neq 0$ there is also a tower of thermal excitations in the spectrum
- if Λ is the typical scale of the system then we need

$$\frac{\omega_0}{\Lambda}, \quad \frac{\Gamma}{\Lambda} \ll 1$$

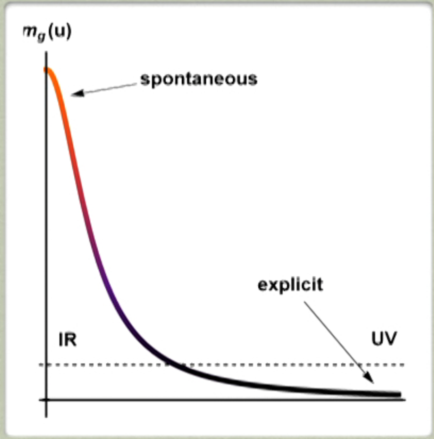
EXPLICIT vs. SPONTANEOUS

HOW TO DISTINGUISH, GIVEN A $V(X) = X^n$

- NAIVELY: graviton mass is the 'order parameter'
- QUANTITATIVELY: source vs. expectation value in the boundary expansion

$$\phi^I(x^\mu) = \phi^I_{(0)}(t, x^i) + \phi^I_{(1)}(t, x^i) r^{5-2n} + \dots$$

By setting $\phi^I = x^I$**, we are 'turning on'** $\phi^I_{(0)}$**;**
the interpretation, however, changes at $n = \frac{5}{2}$ **!**



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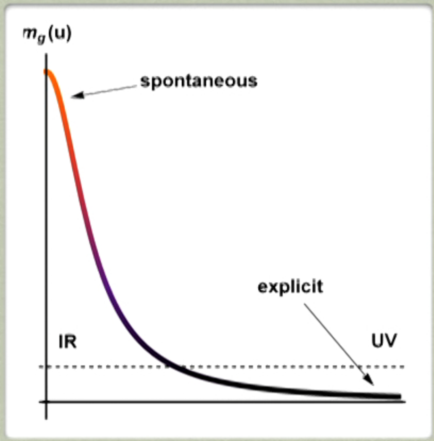

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- EXPLICIT: $V(X) = X$
- EXPLICIT + SPONTANEOUS: $V(X) = X + \beta X^5$
- SPONTANEOUS: $V(X) = X^5$

QUASI-NORMAL MODES IN HOLOGRAPHY

Kovtun, Starinets (2005)

- IN GR: QNMs = solutions to eom's for metric fluctuations around BHs + ingoing wave bdy conditions ⟶ complex eigenfrequencies
- IN DUAL THEORY: complex frequencies = dissipation = poles in the retarded Green's function
- near the AdS bdy $z = \frac{r}{r_h} \to 0:$ $\phi(z) = \mathcal{A}(\omega, k) z^{\Delta_-} + \mathcal{B}(\omega, k) z^{\Delta_+}$

  (leading) (sub-leading)

- prescription for correlators: $\langle \mathcal{O}\mathcal{O} \rangle_R \sim \frac{\mathcal{B}}{\mathcal{A}}$

Dirichlet bdy conditions
$\mathcal{A} = 0$
in the bulk

⟷

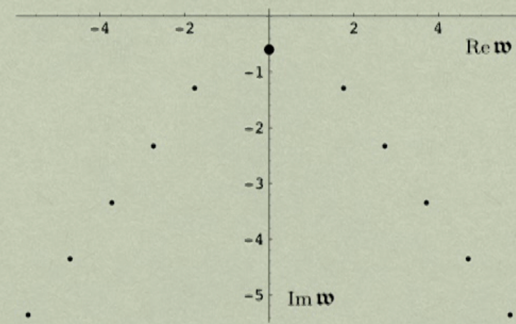
QNMs correspond to the poles in the Green's function in the dual QFT

(STANDARD) MASSLESS CASE

Kovtun, Starinets (2005)

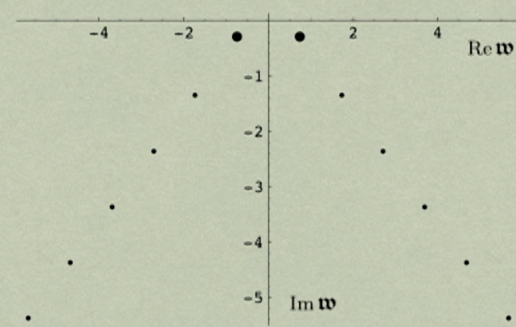
- in hydrodynamic limit: $\frac{\omega}{T} \ll 1, \frac{k}{T} \ll 1$
- scalar channel (tensor): no poles
- shear channel (vector): diffusion of transverse momentum

$$\omega = -i\Gamma k^2$$



- sound channel (scalar, longitudinal): oscillatory momentum relaxation

$$\omega(k) = \pm c_L k - i\Gamma_L k^2$$

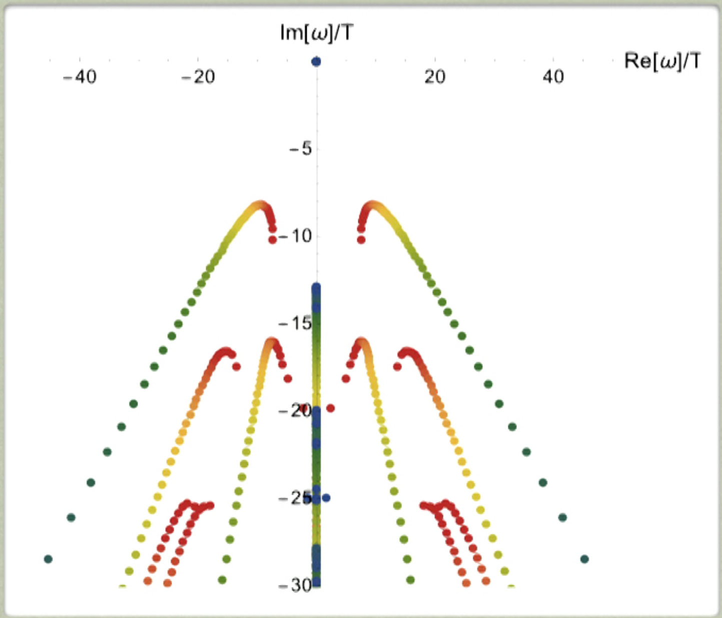


- the higher QNMs at $k = 0$:

$$\omega_n \sim 2\pi n T$$

MASSLESS PHONONS

LA, Ammon, Baggioli, Jimenez, Pujolas (PRL 2018)



($k = 0$, green-red = increasing T/m)

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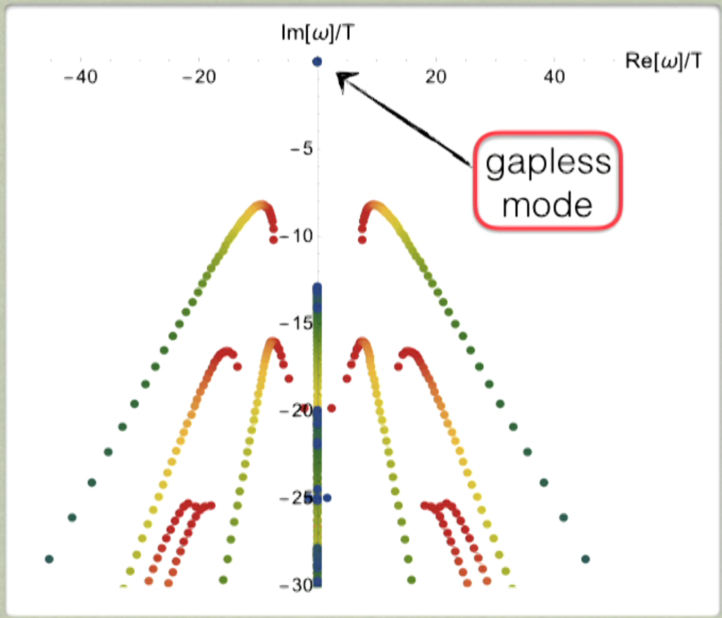

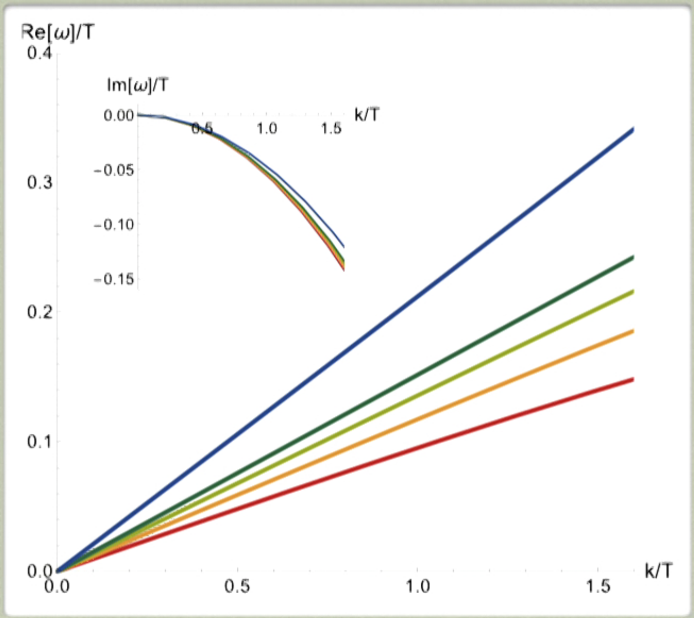

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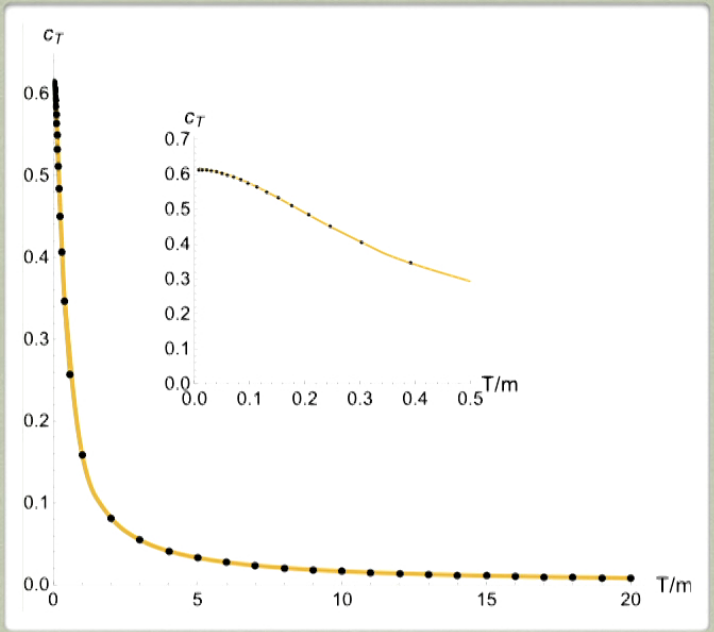
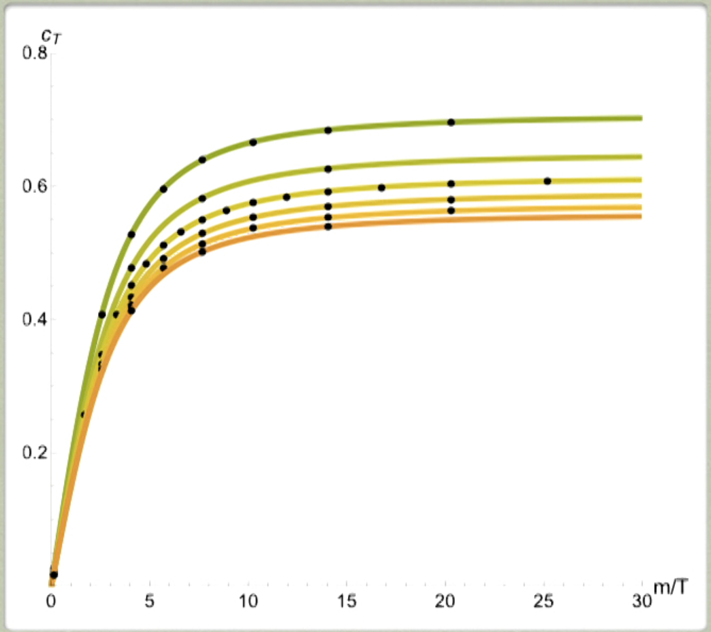
SOUND SPEED

LA, Ammon, Baggioli, Jimenez, Pujolas (PRL 2018)



$(n = 5)$



$(n = 3, 4, 5, 6, 7, 8)$

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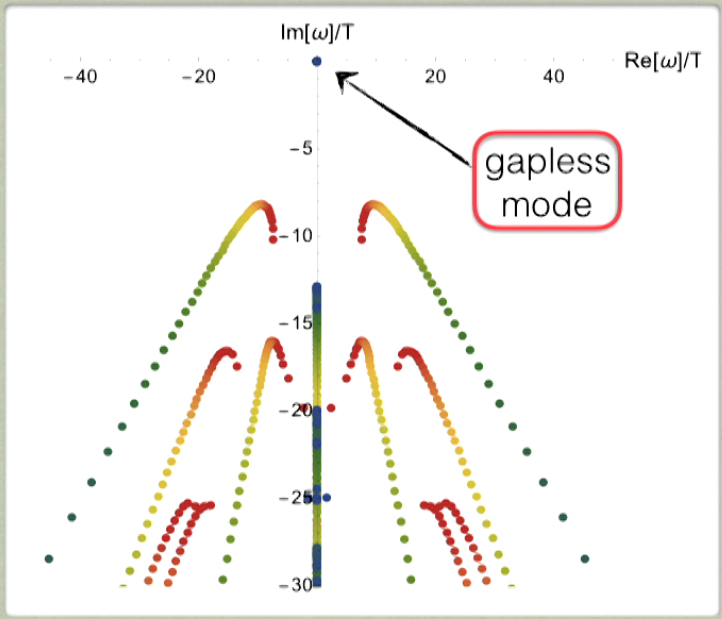

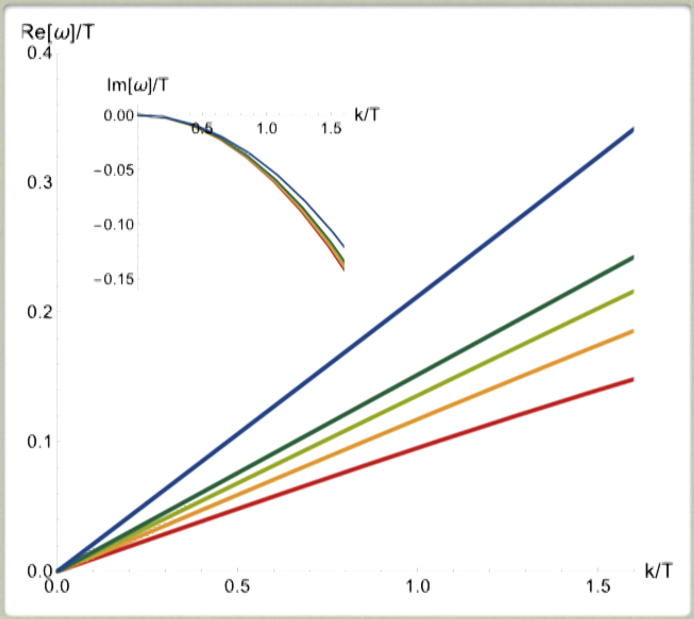

($k = 0$, green-red = increasing T/m)

Massless, well-separated mode with linear dispersion relation—
transverse phonon!

SOUND SPEED

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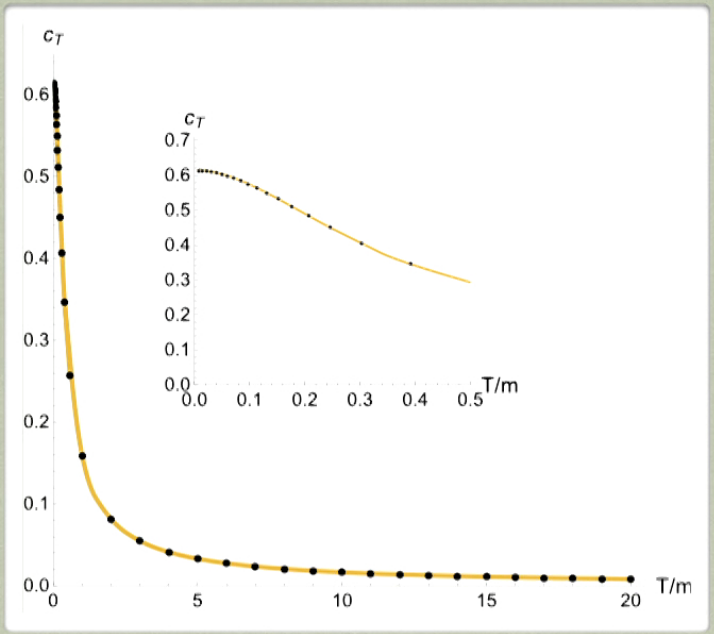

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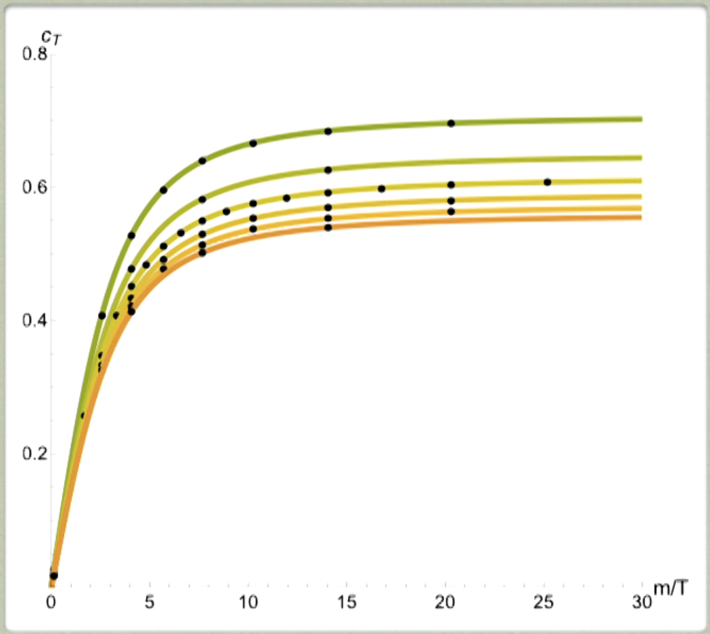

$(n = 3, 4, 5, 6, 7, 8)$

non-linear elastic response

FINITE DEFORMATIONS

LA, Baggioli, Cancer Castillo, Pujolas (to appear)

- Pick a background configuration **AWAY FROM EQUILIBRIUM:**

$$\bar{\phi}^I = O^I_J x^J \,, \qquad O^I_J \neq \delta^I_J$$

- WHEN is this perturbatively stable?

$$\phi^I = \bar{\phi}^I + \pi^I \,, \qquad \pi^I_L = O^{IK}\partial_K \pi_L \,, \quad \pi^I_T = \varepsilon^{IJ} O^K_J \partial_K \pi^T$$

(longitudinal) (transverse)

- check at the level of quadratic action…

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(longitudinal) (transverse)

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$$\delta S_2 = \int d^3x \Big[N_T \dot{\pi}_T^2 + N_L \dot{\pi}_L^2 + 2N_{TL}\dot{\pi}_T\dot{\pi}_L - c_T^2(\partial_x \pi_T)^2 - c_L^2(\partial_x \pi_L)^2 - 2c_{TL}^2 \partial_x \pi_T \partial_x \pi_L \Big]$$

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(longitudinal) (transverse)

- check at the level of quadratic action…

$$\delta S_2 = \int d^3x \left[N_T \dot{\pi}_T^2 + N_L \dot{\pi}_L^2 + 2N_{TL}\dot{\pi}_T\dot{\pi}_L - c_T^2(\partial_x \pi_T)^2 - c_L^2(\partial_x\pi_L)^2 - 2c_{TL}^2 \partial_x\pi_T\partial_x\pi_L \right]$$

- **No ghosts:** positive kinetic eigenvalues
- **No gradient instability:** after Fourier transform, demand the positivity of

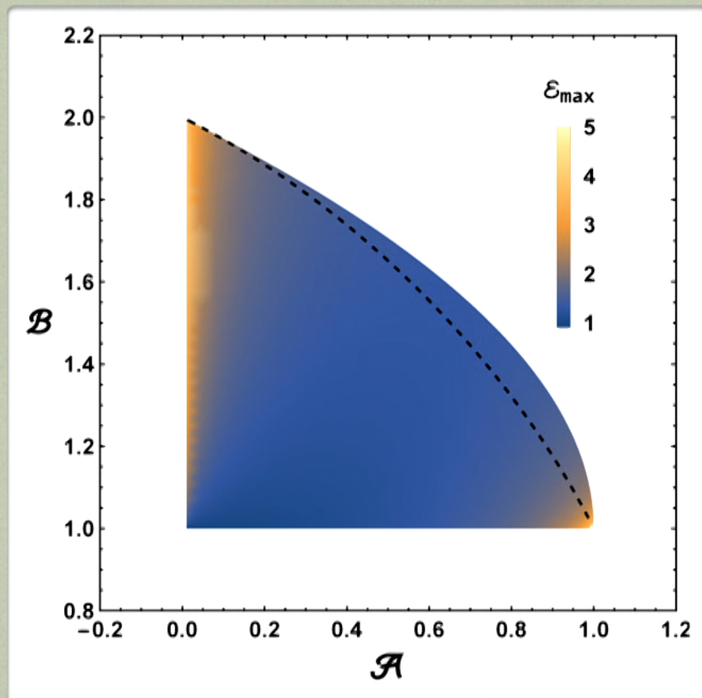
$$\omega_\pm^2 = c_\pm^2 k^2\,, \qquad c_\pm^2 \geq 0$$

- **No superluminal propagation:** $c_\pm^2 \leq 1$

THE POWER OF CONSTRAINTS

LA, Baggioli, Cancer Castillo, Pujolas (to appear)

TAKE A BENCHMARK MODEL: $V(X,Z) = \rho_{\rm eq}\, X^A\, Z^{(B-A)/2}$



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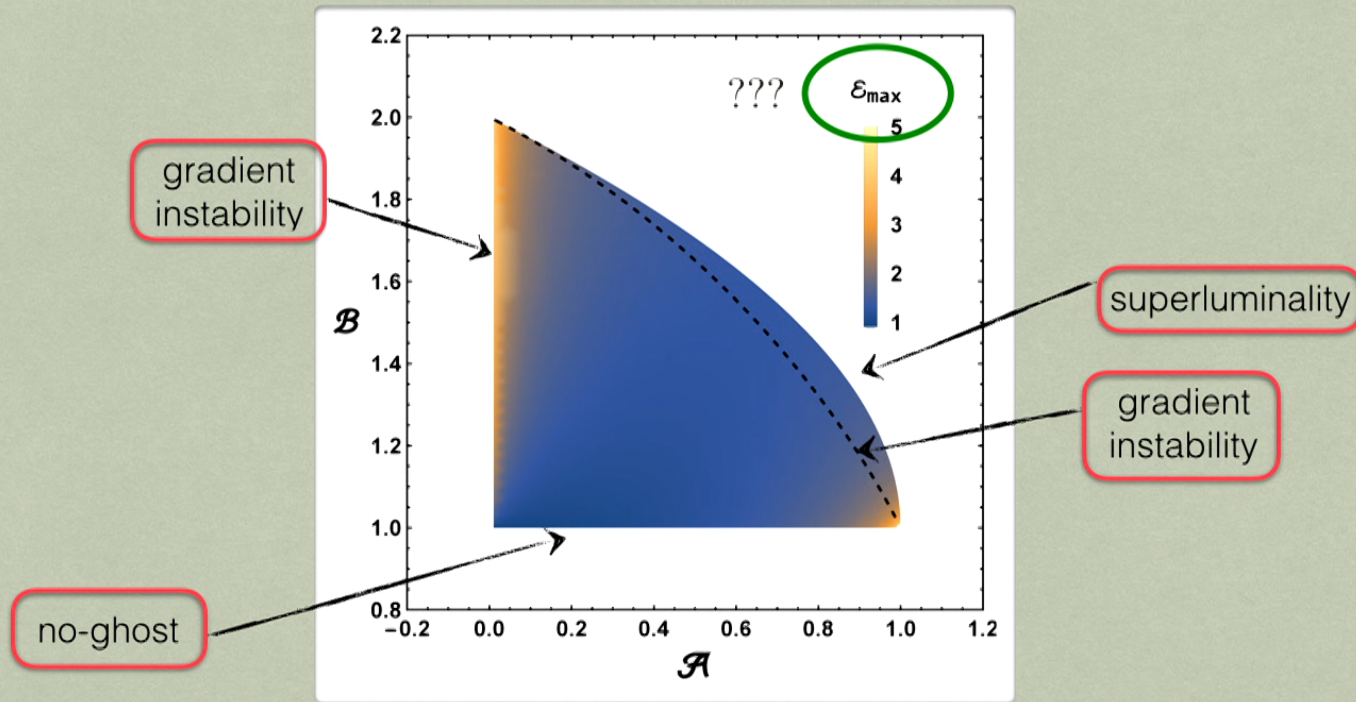

BULK AND SHEAR STRESS

LA, Baggioli, Cancer Castillo, Pujolas (to appear)

- **FOR SMALL DEFORMATIONS:**

$$T_{ij}^{\rm lin} = (p + K\varepsilon_{kk})\,\delta_{ij} + 2G\left(\varepsilon_{ij} - \frac{1}{2}\delta_{ij}\varepsilon_{kk}\right)$$

(bulk modulus) (shear modulus)

- consider a configuration

$$\bar{\phi}^I = \begin{pmatrix}\phi^x\\ \phi^y\end{pmatrix} = \alpha\begin{pmatrix}\sqrt{1+\frac{\varepsilon^2}{4}} & \frac{\varepsilon}{2}\\ \frac{\varepsilon}{2} & \sqrt{1+\frac{\varepsilon^2}{4}}\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$$

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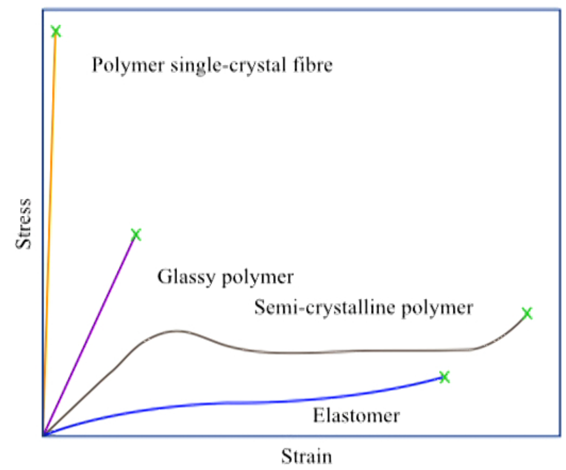
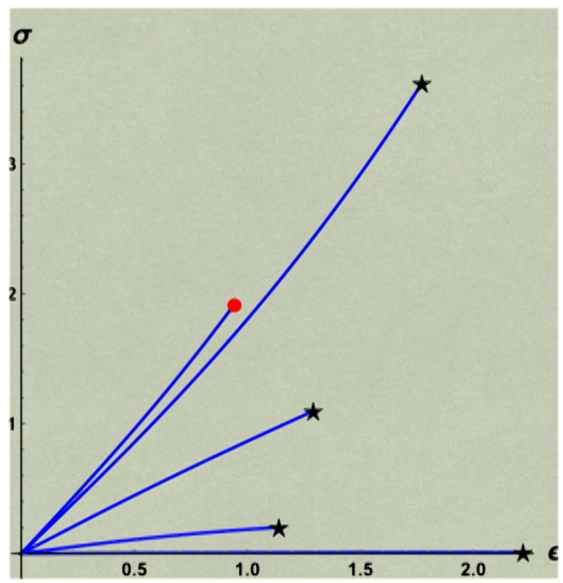
- NON-LINEAR bulk stress: $\delta T_{ii}(\alpha) = \rho_{\text{eq}}\, 2^{A+1}\,(B-1)\left[\alpha^{2B}-1\right]$
- NON-LINEAR shear stress: $\sigma(\varepsilon) = \rho_{\text{eq}}\, A\,\varepsilon\,\sqrt{\varepsilon^2+4}\,\left(\varepsilon^2+2\right)^{A-1}$
- ACOUSTOELASTIC effects: $c_S^2 = c_S^2(\alpha,\varepsilon)$

STRESS-STRAIN CURVES



Schematic stress-strain curves of different types of polymers, drawn approximately to scale.



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UNIVERSALITY?

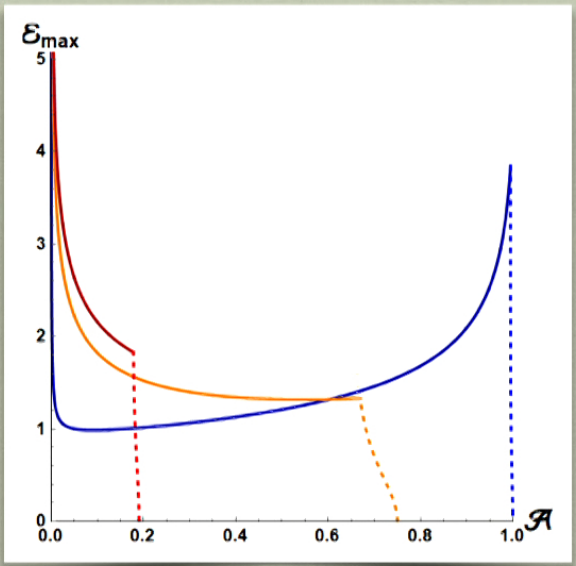
Maximum strain in the small A limit

$$\varepsilon_{\max} = \sqrt{2}\left(\frac{B-1}{A}\right)^{1/4}$$

↔

Scaling of the non-linear stress in the large strain limit

$$\sigma(\varepsilon) \sim \varepsilon^{2A}, \quad \delta T_{ii}(\alpha) \sim \alpha^{2B}$$



FINALLY

- Massive gravity on AdS black branes describe solids and fluids
- Solid-type theories are viscoelastic — both, viscosity and elasticity are non-zero
- There are dynamical transverse modes in the boundary theories dual to the solid type bulk theories
- The transverse sound speed is related to elasticity as expected from standard elasticity theory—these lowest energy modes are thus the Goldstones of the spontaneously broken translational invariance.
- EFT methods useful for non-linear elastic response: acoustoelastic effects, maximal strain etc.

ELASTICITY IN ADS/CMT

LA, Baggioli, Khmelnitsky, Pujolas (2015)
LA, Baggioli, Pujolas (2016)

MAKE THE IDENTIFICATION

$$u_{ij} \longleftrightarrow \frac{1}{2} h_{ij}$$

$$\sigma_{ij} \longleftrightarrow \langle T_{ij} \rangle$$

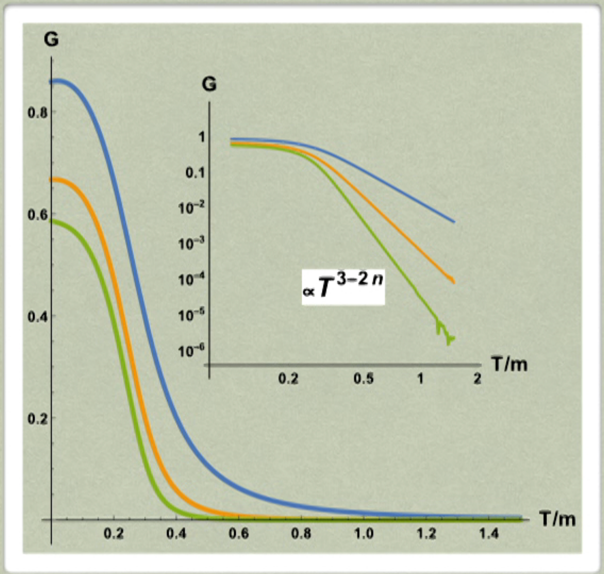
ADS/CMT PRESCRIPTION:

- gives boundary stress-tensor in terms of retarded Green's function

$$\langle T_{ij} \rangle = \mathcal{G}^R_{T_{ij}\,T_{ij}} h_{ij} \longrightarrow G = -\mathrm{Re}\,\mathcal{G}^R_{T_{ij}\,T_{ij}}$$

- analytic expression for $m/T \ll 1$:

$$V(X) = X^n$$



$$G = \frac{L^2}{2r_h^3} \frac{2m^2}{2n-3} \left(\frac{r_h}{L}\right)^{2n}$$