Title: Strange Metals From Local Quantum Chaos

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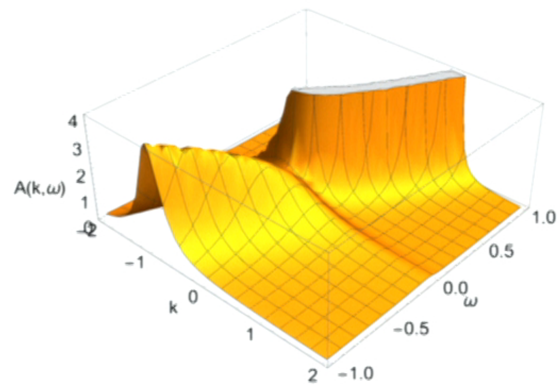
Abstract:

Strange metals from local quantum chaos

John McGreevy (UCSD)

based on work with
Daniel Ben-Zion (UCSD) 1711.02686, PRB
Aavishkar Patel, Subir Sachdev (Harvard), Dan Arovas (UCSD) 1712.05026, PRX



Compressible states of fermions at finite density

The metallic states that we understand well are Fermi liquids.

Landau quasiparticles $\to$ Single-fermion Green function G_R has poles

at $k_\perp \equiv |\vec{k}| - k_F = 0,\ \omega = \omega_\star(k_\perp) \sim 0$: $G_R \sim \dfrac{Z}{\omega - v_F k_\perp + i\Gamma}$

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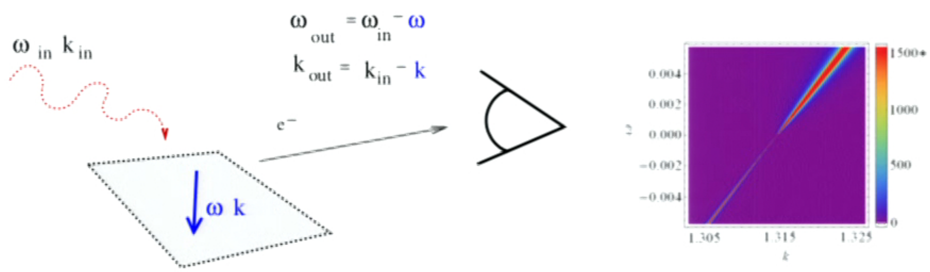
Measurable by angle-resolved photoemission:

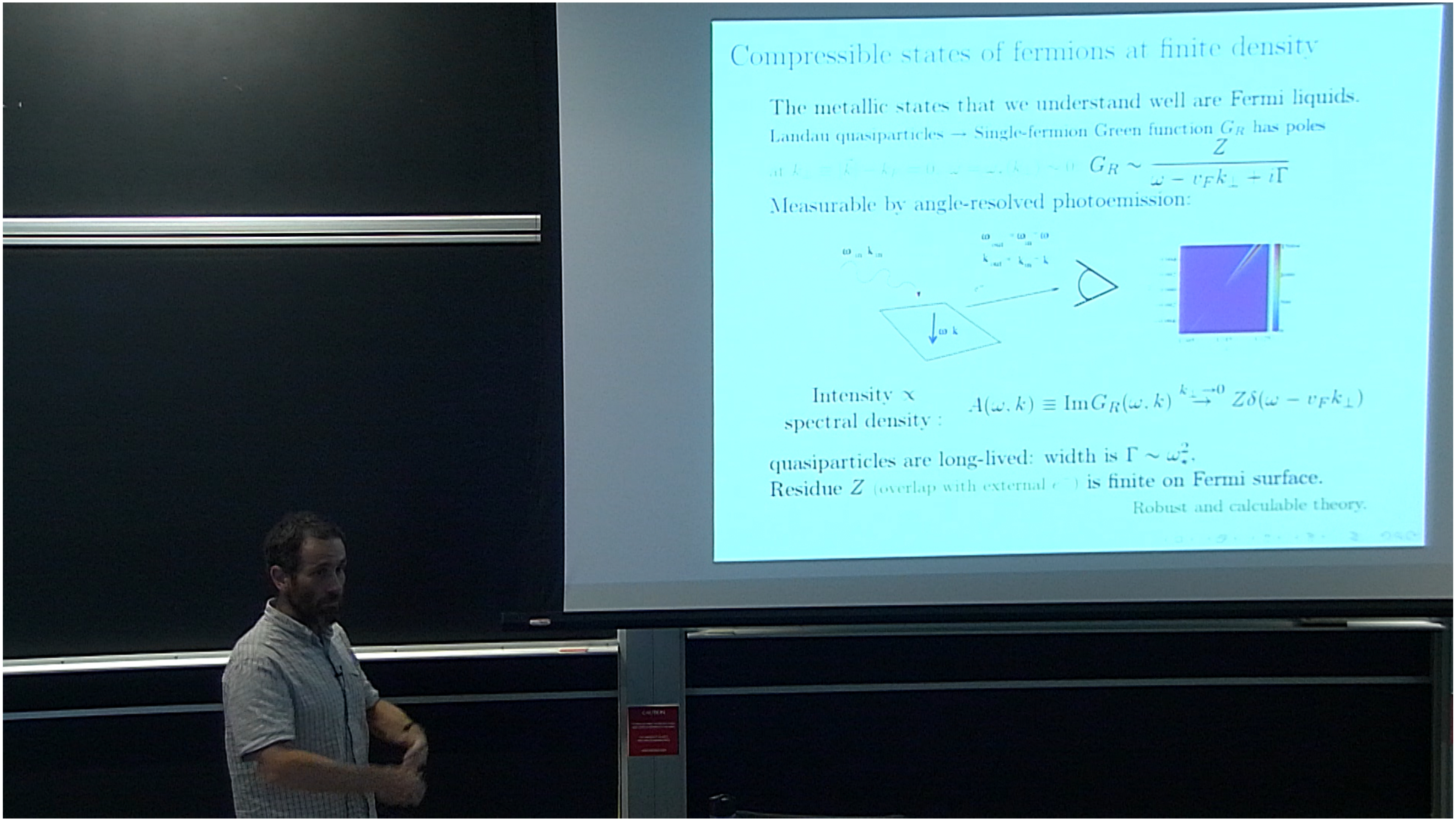


Intensity $\propto$ spectral density : $A(\omega, k) \equiv \mathrm{Im}\, G_R(\omega, k) \overset{k_\perp \to 0}{\rightarrow} Z\delta(\omega - v_F k_\perp)$

quasiparticles are long-lived: width is $\Gamma \sim \omega_\star^2$,

Residue Z (overlap with external e^-) is finite on Fermi surface.

Robust and calculable theory.



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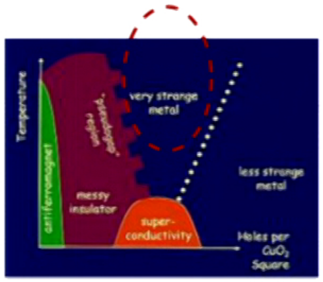
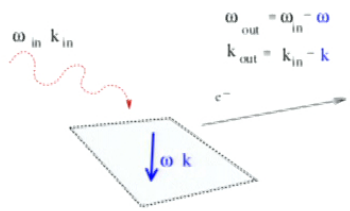
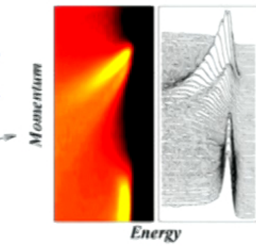
Non-Fermi liquids exist but are mysterious

There are other states with a Fermi surface, but no *pole* in G_R at $\omega = 0$.

e.g.: 'normal' phase of optimally-doped cuprates: ('strange metal')







[Shen et al]

among other anomalies indicating absence of quasiparticles:

ARPES shows gapless modes at finite k (a Fermi surface)

with width $\Gamma(\omega_\star) \sim \omega_\star$, vanishing residue $Z \overset{k_\perp \to 0}{\to} 0$.

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but no long-lived quasiparticles.

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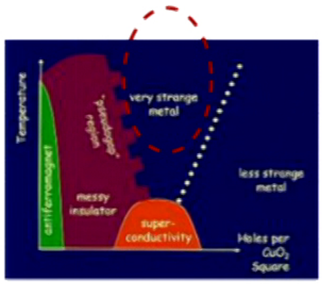

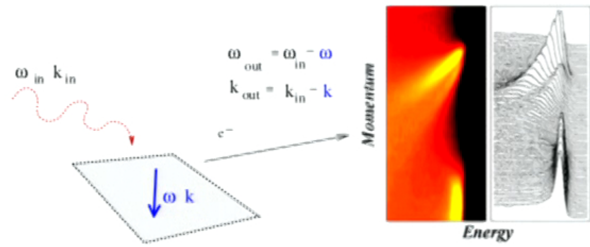

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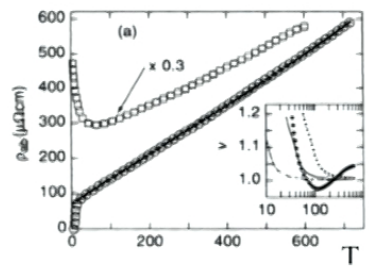
More prominent

mystery of the strange metal phase:

e-e scattering: $\rho \sim T^2$, phonons: $\rho \sim T^5$, ...

no known robust effective theory: $\rho \sim T$.



[S. Martin et al, PRB41, 846 (1990)]

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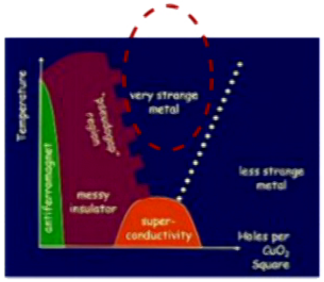
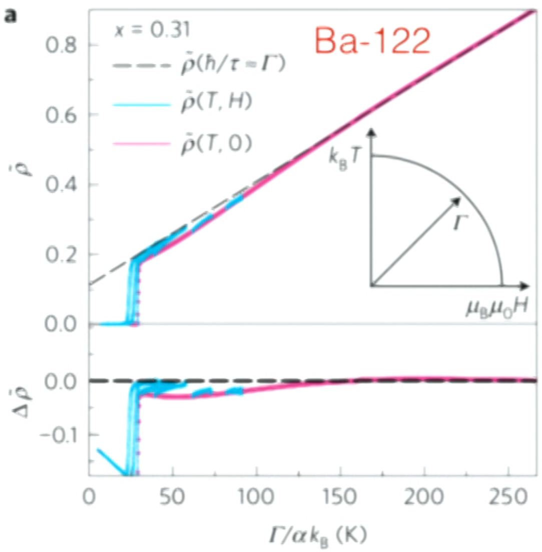


New mystery of the strange metal phase:

Linear-B magnetoresistance, scaling between B, T:



$$\rho(H,T) - \rho(0,0) \propto \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2} \equiv \Gamma$$

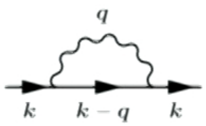
I. M. Hayes et. al., Nat. Phys. 2016

Non-Fermi liquid from non-Holography

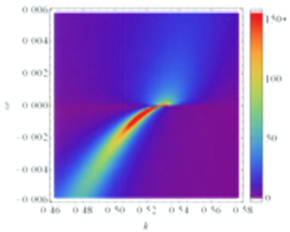
- Luttinger liquid in 1+1 dims. $G^R(k,\omega) \sim (k-\omega)^\alpha$ ✓
- loophole in RG argument for ubiquity of FL:

couple a Landau FL perturbatively to a bosonic mode

(e.g.: magnetic photon, emergent gauge field, critical order parameter...)



→ nonanalytic behavior in $G^R(\omega) \sim \frac{1}{v_F k_\perp + c\omega^{2\nu}}$ at FS: NFL.



[Huge literature: Hertz, Millis, Nayak-Wilczek, Chubukov, S-S Lee, Metlitski-Sachdev, Mross-JM-Liu-Senthil, Kachru-Torroba-Raghu...]

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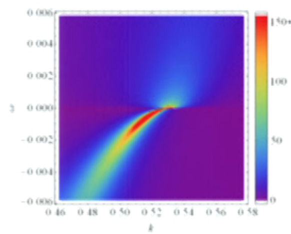

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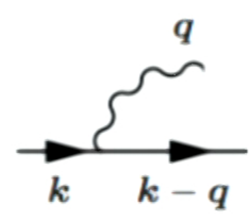
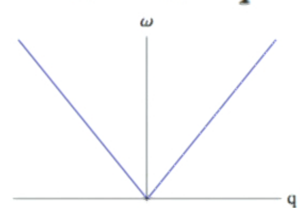
Not strange enough:

These NFLs are not strange metals in terms of transport. $\rho \sim T^{2\nu+2} \gg T$

If the quasiparticle is killed by a boson with $\omega \sim q^z$, $z \sim 1$, small-angle scattering dominates

$\implies$ 'transport lifetime' $\gg$ 'single-particle lifetime'

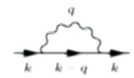




Frameworks for non-Fermi liquid in $d \geq 1$

- a Fermi surface coupled to a critical boson field

$$L = \bar{\psi}\left(\omega - v_F k_\perp\right)\psi + L(a) + \bar{\psi}\psi a \qquad \rightarrow$$

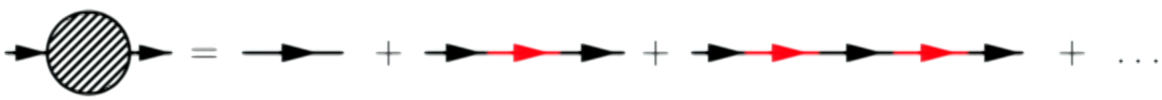


- a Fermi surface mixing with a bath of critical fermionic fluctuations with large dynamical exponent $z \gg 1$

Discovered with AdS/CFT [Faulkner-Liu-JM-Vegh 0907.2694, Faulkner-Polchinski 1001.5049, FLMV+Iqbal 1003.1728]

$$L = \bar{\psi}\left(\omega - v_F k_\perp\right)\psi + L(\chi) + \bar{\psi}\chi + \psi\bar{\chi}$$

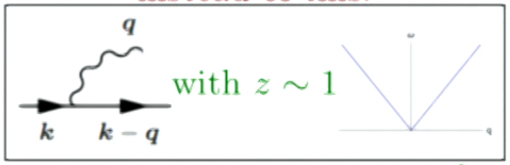
χ: fermionic operator with $\mathcal{G} \equiv \langle \bar{\chi}\chi \rangle = c(k)\omega^{2\nu}$



$$\langle \bar{\psi}\psi \rangle = \frac{1}{\omega - v_F k_\perp - \mathcal{G}} \qquad i.e.,\ \Sigma^\psi \propto \mathcal{G}.$$

Charge transport and momentum sinks

instead of this:



$\downarrow \omega \sim q^z$

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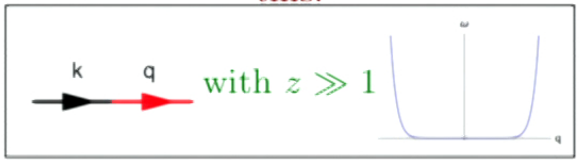

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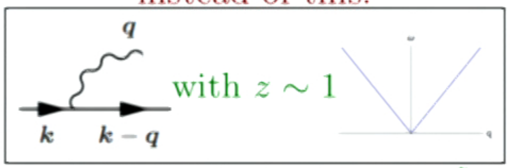

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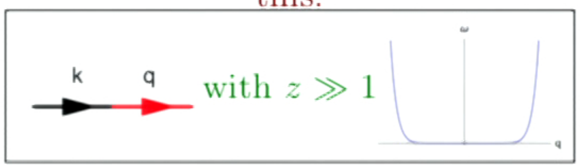


The contribution to the conductivity from the Fermi surface [Faulkner-Iqbal-Liu-JM-Vegh, 1003.1728 and 1306.6396]:

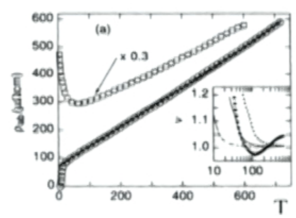
is $\rho_{\rm FS} \sim T^{2\nu}$ when $\Sigma \sim \omega^{2\nu}$.

Dissipation of current is controlled by the decay of the fermions into the χ DoFs.

$\Longrightarrow$ single-particle lifetime controls transport.

(marginal Fermi liquid: $\nu = \frac{1}{2}^+$ [Varma et al]

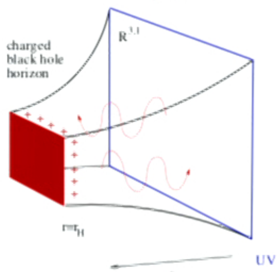
$\Longrightarrow \quad \rho_{FS} \sim T$.)



A few words about the holographic construction

Certain strongly-coupled large-N field theories have a dual description in terms of gravity in extra dimensions.

Anti-de Sitter (AdS$_{d+1}$) spacetime $ds^2 = \frac{dr^2 + dx_\mu dx^\mu}{r^2}$	$\leftrightsquigarrow$	vacuum of conformal field theory
Symmetries of AdS	$\leftrightsquigarrow$	conformal symmetry $\supset x^\mu \to \lambda x^\mu$
Bulk metric $g_{\mu\nu}$	$\leftrightsquigarrow$	$T_{\mu\nu}$ stress tensor
Bulk $\mathsf{U}(1)$ gauge field A_μ	$\leftrightsquigarrow$	J_μ conserved current
Bulk spinor field ψ_α	$\leftrightsquigarrow$	Ψ fermionic operator



$\leftrightsquigarrow$ Turn on a chemical potential to make a finite density of CFT stuff.

A few words about the holographic construction

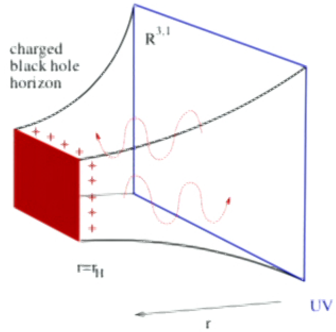
The near-horizon region of the geometry is $AdS_2 \times \mathbb{R}^d$

$$ds^2 = \frac{-dt^2 + d\zeta^2}{\zeta^2} + d\vec{x}^2, \; A = \frac{\mathcal{E}dt}{\zeta}$$

has $\tau + \mathbf{i}\zeta \rightarrow \frac{a(\tau+\mathbf{i}\zeta)+b}{c(\tau+\mathbf{i}\zeta)+d}$ 1+1d conformal symmetry.

This describes a $z = \infty$ fixed point at large N:

many critical dofs which are localized.



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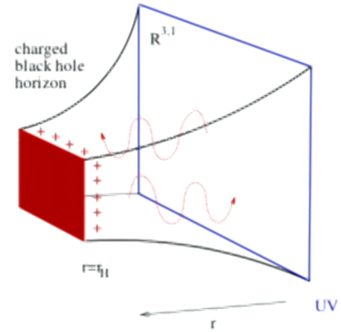

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Shortcomings:

• The Fermi surface degrees of freedom are a small part $(o(N^0))$ of a large (conducting) system $(o(N^2))$.

• Here N^2 is the control parameter which makes gravity classical (and holography useful).

• Understanding their effects on the black hole requires quantum gravity. [Some attempts: Suh-Allais-JM 2012, Allais-JM 2013]

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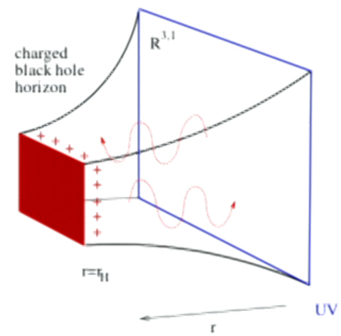

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All we need is a $z = \infty$ fixed point

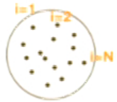
(with fermions, and with U(1) symmetry).

SYK with conserved U(1)

A solvable $z = \infty$ fixed point [Sachdev, Ye, Kitaev]:

$H_{\mathrm{SYK}} = \sum_{ijkl}^{N} J_{ijkl} \chi_i^\dagger \chi_j^\dagger \chi_k \chi_l$.

$\overline{J_{ijkl}} = 0$, $\overline{J_{ijkl}^2} = \frac{J^2}{2N^3}$



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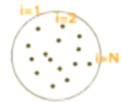

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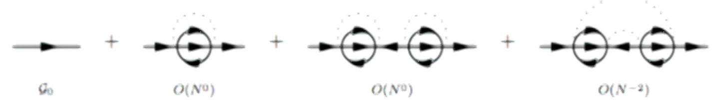
$\mathcal{G} =$ 

Schwinger-Dyson equations:

$$\mathcal{G}^{-1}(\omega) = (\mathbf{i}\omega)^{-1} - \Sigma(\omega) \overset{\omega \ll J}{\rightarrow} \mathcal{G}(\omega)\Sigma(\omega) \approx -1$$

$\Sigma(\tau) =$  $= J^2 \mathcal{G}^2(\tau)\mathcal{G}(-\tau)$

$\implies \mathcal{G}(\omega) \propto (\mathbf{i}\omega)^{-1/2}, \nu(\chi) = -\frac{1}{4}$. A (very) compressible state of fermions at finite density: Low-energy level spacing is e^{-Ns_0} ($s_0 < \ln 2$).

(vs. $1/N$ for a model with quasiparticles, like SYK_2).

- The S-D equations have a low-energy conformal symmetry $\implies$ finite-temperature correlators also determined.

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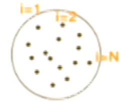

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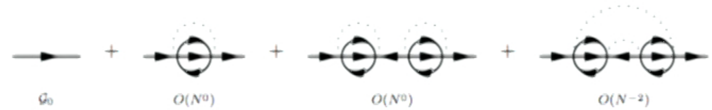
$\mathcal{G} =$ 

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- Also useful is the 'bath field': $\tilde{\chi}_i \equiv J_{ijkl}\chi_j^\dagger \chi_k \chi_l$, which has $\langle \tilde{\chi}^\dagger \tilde{\chi} \rangle \propto (\mathbf{i}\omega)^{+\frac{1}{2}}, \quad \nu(\tilde{\chi}) = +\frac{1}{4}$.

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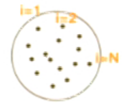

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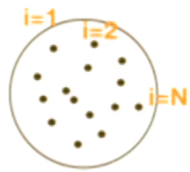
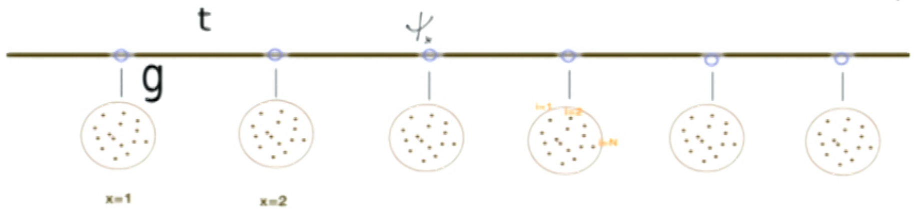
- Duality: this model has many properties in common with gravity (plus electromagnetism) in AdS_2.

Using SYK clusters to kill the quasiparticles and take their momentum

One SYK cluster: 

$\overset{\sim}{\leftrightarrow}$ AdS_2: 

To mimic $AdS_2 \times \mathbb{R}^d$, consider a d-dim'l lattice of SYK models:



$\overset{?}{\leftrightarrow} AdS_2 \times \mathbb{R}^d$

$$H_0 = \sum_{\langle xy \rangle \in \text{lattice}} t \left(\psi_x^\dagger \psi_y + hc \right) + \sum_{x \in \text{lattice}} H_{SYK}(\chi_{xi}, J_{ijkl}^x)$$

$$H = H_0 + H_{\text{int}}$$

Couple SYK clusters to Fermi surface

- [D. Ben-Zion, JM, 1711.02686]: couple by hybridization

$$H_{\text{int}} = \sum_{x,i} g_{xi} \psi_x^\dagger \chi_{xi} + h.c.$$

by random gs ($\overline{g_{ix}} = 0$, $\overline{g_{ix} g_{jy}} = \delta_{ij} \delta_{xy} g^2/N$)
$\longrightarrow$ Evidence for finite-g, N fixed point, 'strange semiconductor' with $\rho(T) \sim T^{-1/2}$.

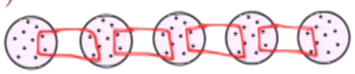
- [A. Patel, JM, D. Arovas, S. Sachdev, 1712.05026, D. Chowdhury, Y. Werman, E. Berg, T. Senthil, 1801.06178]: couple by density-density interaction

$$H_{\text{int}} = \sum_{x,i} g_{xabij} \psi_{xa}^\dagger \psi_{xb} \chi_{xi}^\dagger \chi_{xj} + h.c.$$

by random gs ($\overline{g_{xabij}} = 0$, $\overline{g_{xabij} g_{x'a'b'i'j'}} = \delta_{xabij,x'a'b'i'j'} g^2/N$)
$\longrightarrow$ Controlled (intermediate-temperature) marginal fermi liquid, $\rho(T) \sim T$, realistic magnetoresistance.

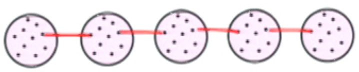
Pause to advertise related work

- [Gu-Qi-Stanford]: a chain of SYK clusters with 4-fermion couplings (no hybridization, no Fermi surface)



- [Banerjee-Altman]: add all-to-all quadratic fermions to SYK (no locality)

- [Song-Jian-Balents]: a chain of SYK clusters with quadratic couplings (no Fermi surface)



Large-N analysis

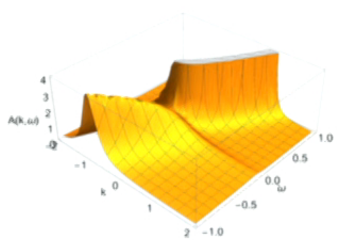
$= \frac{1}{\omega - v_F k_\perp}$, $= \langle \chi_x^\dagger \chi_y \rangle$, = disorder contraction

Full ψ propagator:

$\mathcal{O}(N^{-1})$

$\implies$ the ψ self-energy is $\Sigma(\omega, k) = \mathcal{G}(\omega)$
(just as in the holographic model).

$$G_\psi(\omega, k) \overset{\text{small } \omega}{=} \frac{1}{\omega - v_F k_\perp - \mathcal{G}(\omega)}$$



This has $\nu = -\frac{1}{4}$:

$\mathcal{G}(\omega) \sim \omega^{-\frac{1}{2}}$.

$\implies \rho(T) \sim \frac{1}{\sqrt{T}}$.

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$\longrightarrow = \frac{1}{\omega - v_F k_\perp}$, $\quad$ $\color{red}{\longrightarrow} = \langle \chi_x^\dagger \chi_y \rangle$, $\quad$ ⌒ = disorder contraction

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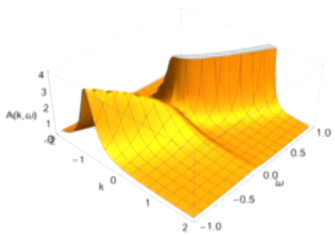
For more general q in $H(\chi) = J_{i_1 \cdots i_q} \chi_{i_1}^\dagger \cdots \chi_{i_q}$, we'd have $\nu(q) = \frac{1-q}{2q}$.

Coupling to bath field would give $\tilde{\nu}(q) = -\frac{1}{2} + \frac{3}{q} \overset{q \to 4}{\to} + \frac{1}{4}$.



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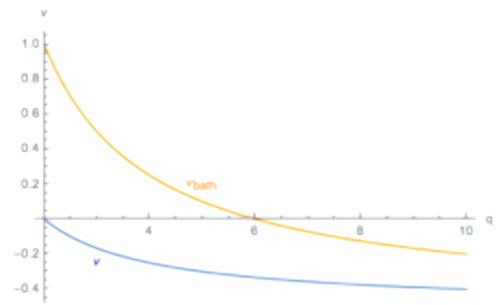
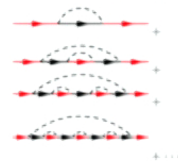


Does the Fermi surface destroy the clusters?

$$\overline{g_{ix}} = 0, \quad \overline{g_{ix} g_{jy}} = \delta_{ij} \delta_{xy} g^2/N.$$

The 'SYK-on' propagator $\mathcal{G}$ looks like:

Leading $1/N$ contributions to $\mathcal{G}_{xy}$:



are still local (on average), and are less singular than $\omega^{-1/2}$.

$$\Longrightarrow \quad z = \infty \text{ behavior survives.}$$

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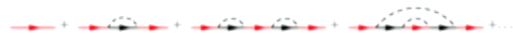

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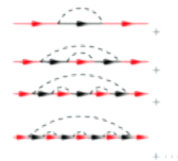

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$\Longrightarrow \quad z = \infty$ behavior survives.

Replica analysis reproduces diagrammatic results:

$$\overline{Z^n} = \int [d\mathcal{G} d\Sigma d\rho d\sigma] e^{-NS[\mathcal{G},\Sigma,\rho,\sigma]}$$

$$\frac{\delta S}{\delta\{\mathcal{G},\Sigma,\rho,\sigma\}} = 0 \quad \Longrightarrow$$

$$\Sigma = -J^2|\mathcal{G}|^2\mathcal{G}, \quad \mathcal{G} = -\frac{1}{\partial_t - \Sigma - G_\psi/N}, \quad G_\psi = -\frac{1}{G_{\psi 0}^{-1} - \mathcal{G}}.$$

RG analysis of impurity problem

Weak coupling: Consider a single SYK cluster coupled to FS, $g \ll t, J$. Following Kondo literature [Affleck] only s-wave couples:

$$H_{FS} = \frac{v_F}{2\pi} \int_0^\infty dr \left(\psi_L^\dagger \partial_r \psi_L - \psi_R^\dagger \partial_r \psi_R \right) \quad \Longrightarrow [\psi_{L/R}] = \frac{1}{2}.$$

$$\Delta H = g \psi_L^\dagger(0) \chi, \qquad \Delta \tilde{H} = \tilde{g} \psi_L^\dagger(0) \tilde{\chi}.$$

$\tilde{\chi}_i \equiv J_{ijkl} \chi_j^\dagger \chi_k \chi_l$. $\chi \equiv g_i \chi_i / g$.

Coupling to χ:
$[\int \; \psi^\dagger \chi] = -1 + \frac{1}{2} + \frac{1}{4} = -\frac{1}{4}$
is relevant.

Coupling to bath field:
$[\int dt \; \psi^\dagger \tilde{\chi}] = -1 + \frac{1}{2} + \frac{3}{4} = \frac{1}{4}$
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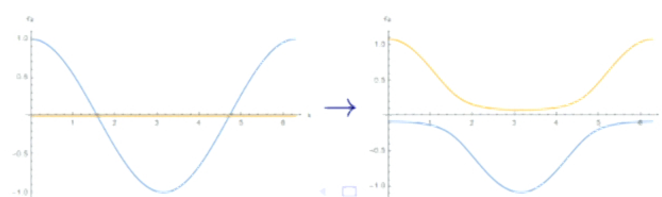
is irrelevant.

Strong coupling: At large enough g ($g \gg t, J$), this is a highly-underscreened Anderson model: ψ_x and $\chi_x \equiv \frac{1}{g}\sum_i g_i \chi_{ix}$ pair up, $N \to N-1$.

$$H = g\sum_x \psi_x^\dagger \chi_x + h.c.$$



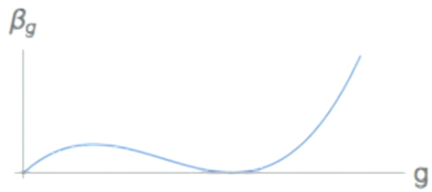
Topology of coupling space

$$H_{\text{int}} = \sum g\psi^{\dagger}\chi + h.c.$$

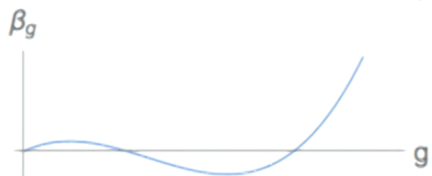
Possibilities for beta function (arrows toward IR):

1)

2)



3)



If we find a fixed point, it is stable.

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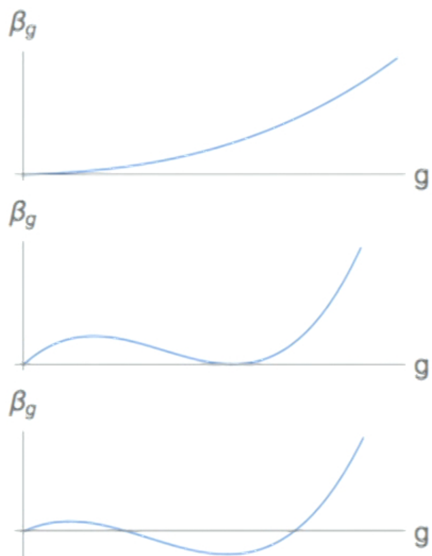

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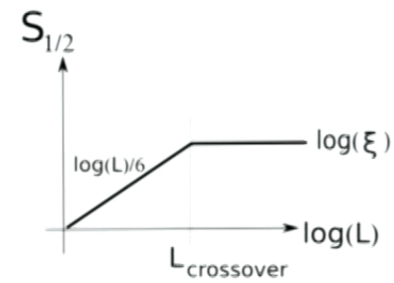
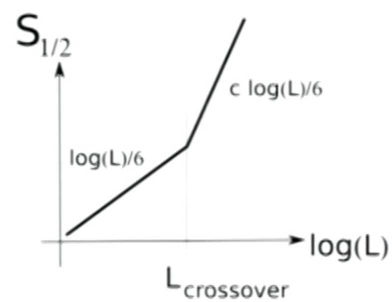
$\leftrightarrow$ $\leftrightarrow$

Consequences for entanglement entropy of half-chain at small g_0:

1)

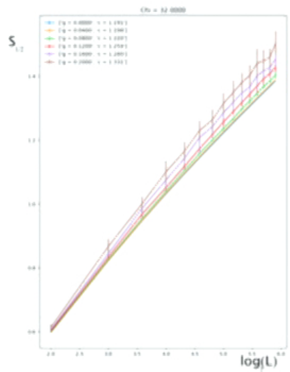


3)



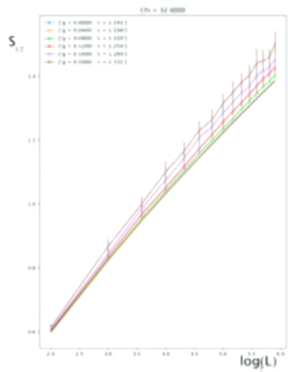
Expect: $L_{\text{crossover}} \sim (g_0 N)^{-\frac{1}{4}}$.

Numerical results



(1) Half-chain entanglement entropy grows faster with L than free-fermion answer!

Numerical results



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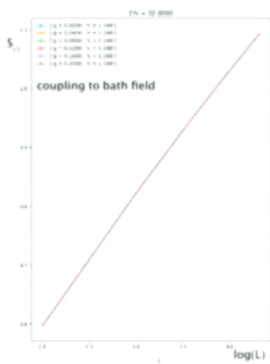
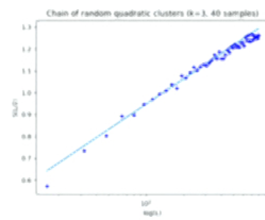


(2) Coupling to bath field $\tilde{g}\psi\tilde{\chi}$ is irrelevant – same as free fermion answer.



(3) Growth doesn't happen for quadratic clusters (SYK_2)

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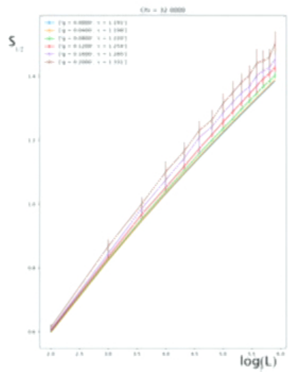

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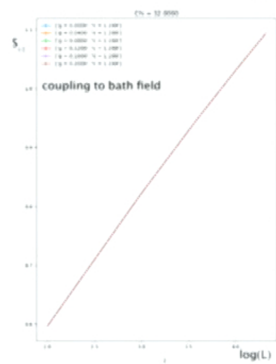

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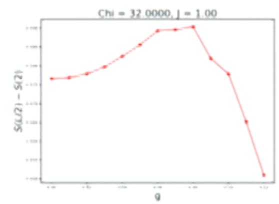
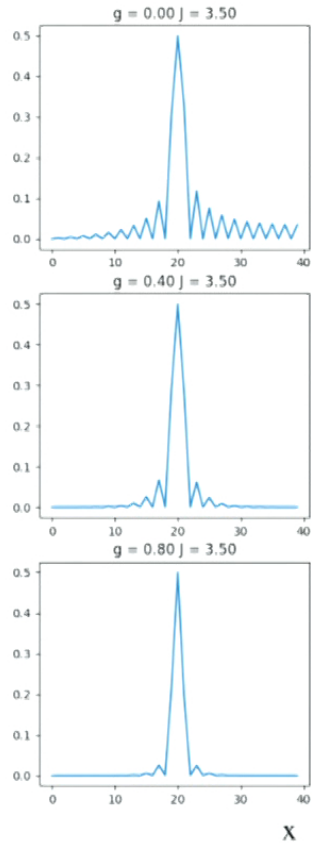


(4) At large g, entanglement is destroyed.

Correlation functions

$|\langle \psi_x^\dagger \psi_{L/2} \rangle|$



$\sim \frac{|\sin 2k_F(x-L/2)|}{|x-L/2|^\alpha}$

$\alpha < 1$: exponent is not free fermion value.

At large g, everybody is localized (anti-Kondo phase).

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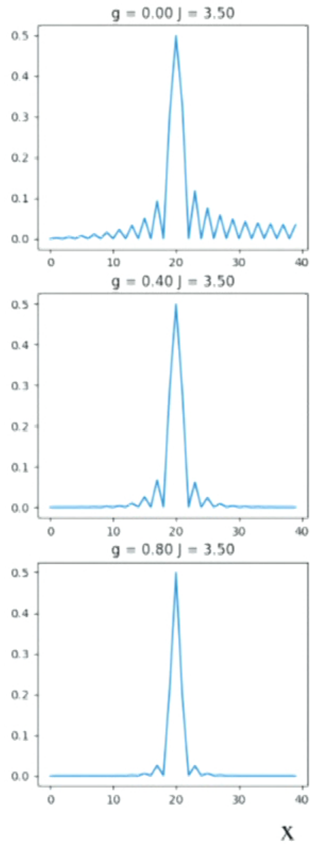

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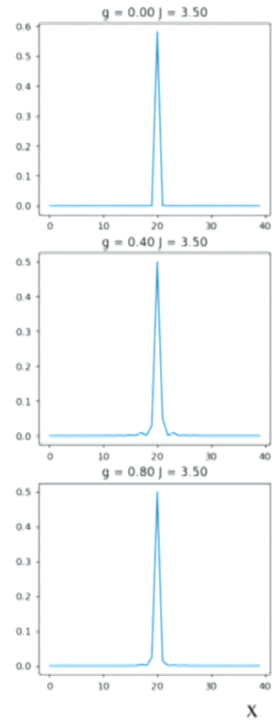
$\sum_i |\langle \chi_{x,i}^\dagger \chi_{L/2,i} \rangle|$



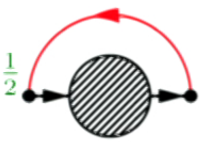
χ are still localized.

At large g, everybody is localized (anti-Kondo phase).

Conclusions on hybridization coupling

- $\exists$ an interesting NFL fixed point.
- It's not Lorentz invariant.
- Numerical evidence is in 1d, but it's not a Luttinger liquid: $c \neq 1$.
- Can access perturbatively by $q = 2 + \epsilon$

$(H(\chi) = J_{i_1 \cdots i_q} \chi^\dagger_{i_1} \cdots \chi_{i_q})$.

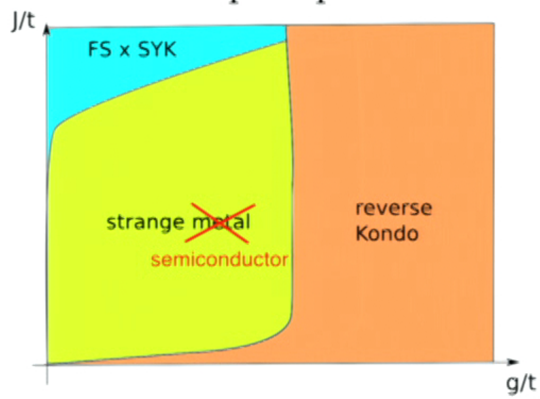
$\delta g^2 = -\frac{1}{2}$  $\to \beta_{g^2} \simeq \epsilon g^2 - \frac{c v_F}{J k_F^{d-1}} g^4$

- It has a Fermi surface

(singularity of G_R at $\omega \to 0, k \to k_F$)

but it's not metallic! $\rho(T) \sim T^{-1/2}$.

Cartoon map of phases:



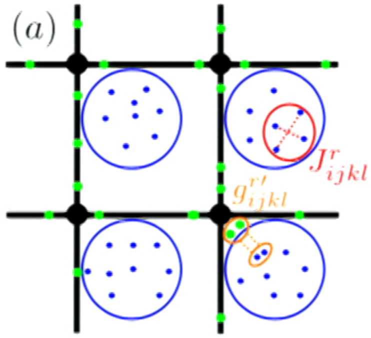
(Warning: this is a cartoon.)

Density-density coupling

[Aavishkar Patel, JM, D. Arovas, S. Sachdev, 1712.05026

D. Chowdhury, Y. Werman, E. Berg, T. Senthil, 1801.06178]

Demanding an IR fixed point is asking too much.



$$H_{\rm int} = \sum_x \sum_{i,j=1}^{N} \sum_{a,b=1}^{M} g_{xabij} \psi^\dagger_{xa} \psi_{xb} \chi^\dagger_{xi} \chi_{xj} + h.c.$$

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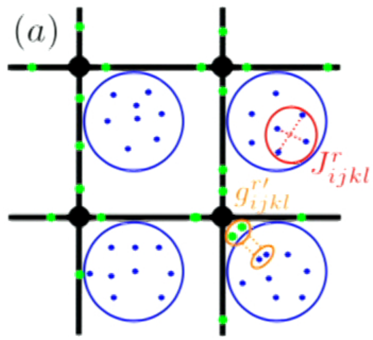

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$$(\overline{g_{xabij}} = 0,\ \overline{g_{xabij} g_{x'a'b'i'j'}} = \delta_{xabij,x'a'b'i'j'} g^2/N)$$

Large N, M Schwinger-Dyson equations are:

$$\Sigma_{\tau-\tau'} = -J^2 \mathcal{G}^2_{\tau-\tau'} \mathcal{G}_{\tau'-\tau} - \frac{M}{N} g^2 \mathcal{G}_{\tau-\tau'} G^\psi_{\tau-\tau'} G^\psi_{\tau'-\tau}, \quad \mathcal{G}(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)},$$

$$\Sigma^\psi_{\tau-\tau'} = -g^2 G^\psi_{\tau-\tau'} G_{\tau-\tau'} G_{\tau'-\tau},$$

ψ, χ coupled only by local Green's function of itinerant fermions:

$$G^\psi(\mathbf{i}\omega_n) \equiv \int đ^d p G^\psi(\mathbf{i}\omega_n, p) = \int \frac{d^d p}{(2\pi)^d} \frac{1}{\mathbf{i}\omega_n - \epsilon_k + \mu_\psi - \Sigma^\psi(\mathbf{i}\omega_n)} \simeq -\frac{\mathbf{i}}{2} \nu(0) \mathrm{sgn}(\omega_n)$$

($\nu(0) \equiv$ dos at FS)

Fate of conduction electrons

The effect on the itinerant fermions is then

$$\Sigma^{\psi}(\omega, q) =$$

$$\sim g^2 \int đ\omega_{1,2} \frac{\mathrm{sgn}(\omega_1)}{|\omega_1|^{1/2}} \frac{\mathrm{sgn}(\omega_2)}{|\omega_2|^{1/2}} G^{\psi}(\omega + \omega_1 + \omega_2)$$

$$\sim g^2 \nu(0) \left(\omega \log \omega/\Lambda - \mathbf{i}\pi\omega\right)$$

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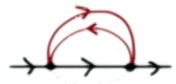
$$\Sigma^\psi(\omega, q) = \quad$$  $$\sim g^2 \int đ\omega_{1,2} \frac{\mathrm{sgn}(\omega_1)}{|\omega_1|^{1/2}} \frac{\mathrm{sgn}(\omega_2)}{|\omega_2|^{1/2}} G^\psi(\omega + \omega_1 + \omega_2)$$

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$$\Sigma^\psi(i\omega_n, q) = \frac{ig^2\nu(0)T}{2J\cosh^{1/2}(2\pi\mathcal{E})\pi^{3/2}} \left(\frac{\omega_n}{T} \ln\left(\frac{2\pi T e^{\gamma_E - 1}}{J} \right) + \frac{\omega_n}{T} \psi\left(\frac{\omega_n}{2\pi T} \right) + \pi \right)$$

$\longrightarrow$ single-particle decay rate = transport scattering rate:

$$\gamma \equiv -2\mathrm{Im}\,\Sigma^\psi_R(\omega = 0) = \frac{g^2\nu(0)T}{J\sqrt{\pi\cosh(2\pi\mathcal{E})}}.$$

($\mathcal{E}$ measures filling.)

Precedent for this mechanism:

[Varma et al 89] $\mathrm{Im}\,\chi(\omega, q) =$

Im  $\sim \tanh \frac{\omega}{2T}$.

Large N, M with $M/N \ll 1$ controls back-reaction on SYK clusters.

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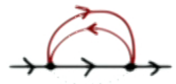
$$\Sigma^{\psi}(\omega, q) = $$  $$\sim g^2 \int đ\omega_{1,2} \frac{\mathrm{sgn}(\omega_1)}{|\omega_1|^{1/2}} \frac{\mathrm{sgn}(\omega_2)}{|\omega_2|^{1/2}} G^{\psi}(\omega + \omega_1 + \omega_2)$$

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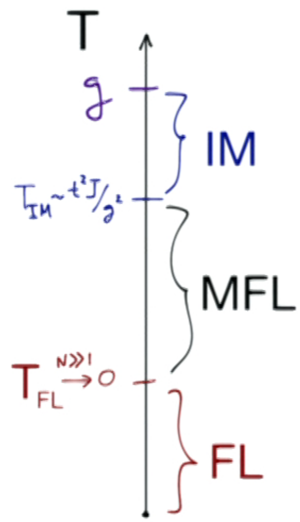
Large N, M with $M/N \ll 1$ controls back-reaction on SYK clusters.

With finite bandwidth, three phases (for $g \gg \sqrt{tJ}$):



Incoherent metal: one big SYK cluster, no FS [qv Song-Jian-Balents, Parcollet-Georges 98].

Marginal fermi liquid: $\Sigma \sim \omega \ln \omega$.

Fermi liquid: at finite N, g is an irrelevant perturbation, goes away in IR.

Transport in a single domain

Both IM and MFL have $\rho(T) \sim T$:

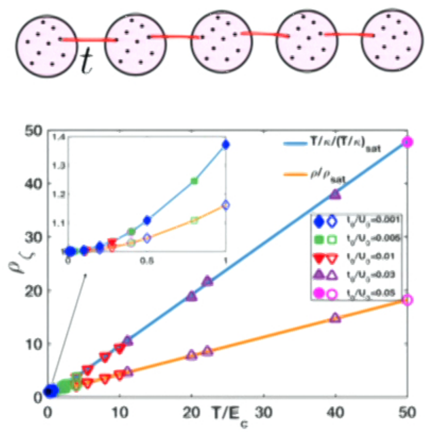
$$\sigma_0^{\mathrm{MFL}} = M \frac{v_F^2 \nu(0)}{16T} \int_{-\infty}^{\infty} \frac{dE_1}{2\pi} \mathrm{sech}^2\left(\frac{E_1}{2T}\right) \frac{1}{|\mathrm{Im}\Sigma_R^c(E_1)|}$$
$$= 0.120251 \times M T^{-1} J \times \left(\frac{v_F^2}{g^2}\right) \cosh^{1/2}(2\pi\mathcal{E}).$$

Both violate Wiedemann-Franz law:

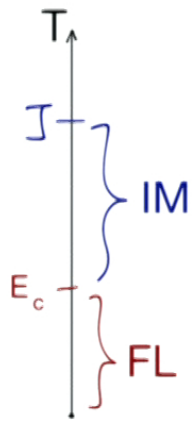
$$L^{\mathrm{MFL}} = \frac{\kappa_0^{\mathrm{MFL}}}{\sigma_0^{\mathrm{MFL}} T} = \frac{\int_{-\infty}^{\infty} \frac{dE_1}{2\pi} E_1^2 \mathrm{sech}^2\left(\frac{E_1}{2}\right) \frac{1}{|\mathrm{Im}[E_1 \psi(-iE_1/(2\pi)) + i\pi]|}}{\int_{-\infty}^{\infty} \frac{dE_1}{2\pi} \mathrm{sech}^2\left(\frac{E_1}{2}\right) \frac{1}{|\mathrm{Im}[E_1 \psi(-iE_1/(2\pi)) + i\pi]|}}$$
$$= 0.713063 \times L_0 < L_0 \equiv \frac{\pi^2}{3}$$

$(L^{IM} = \frac{\pi^2}{8})$ [Song-Jian-Balents, PRL 119, 216601 (2017)]

More on Incoherent Metal



[Song-Jian-Balents, PRL 119, 216601 (2017)]

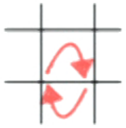


$E_c = t^2/J$

$T < E_c : \rho = A + B\left(\frac{T}{E_c}\right)^2, s \sim s_0\left(\frac{T}{E_c}\right)$. FL

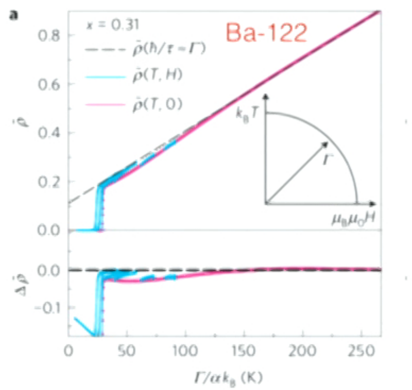
$E_c < T < g : \rho = \frac{h}{e^2}\left(\frac{T}{E_c}\right)^2, s = s_0$. IM

From hopping conductivity:



$\sigma^{\mathrm{IM}} \sim \frac{t^2}{JT} = \frac{E_c}{T}$

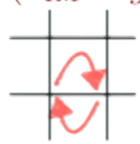
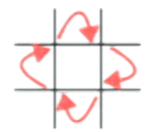
Magnetotransport is very different



$\rho(H,T) - \rho(0,0) \propto \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2} \equiv \Gamma$

I. M. Hayes et. al., Nat. Phys. 2016

IM has no FS and (hence) negligible magnetoresistance: perturbation theory in hopping is valid exactly in IM regime: $t/(J_{\rm IM}T)^{1/2} \ll 1$, $(J_{\rm IM} \equiv g^2/J)$.

 $\sigma_{xx}^{\rm IM} \sim \dfrac{t^2}{J_{IM}T}$  $\sigma_{xy}^{\rm IM} \sim \dfrac{t^4 \sin \mathcal{B}}{(J_{\rm IM}T)^2}$.

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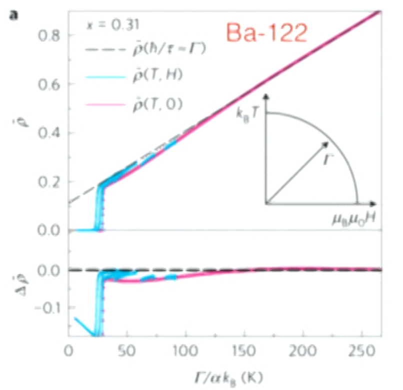

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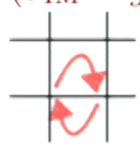
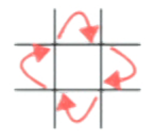
 $\sigma_{xx}^{\rm IM} \sim \dfrac{t^2}{J_{IM} T}$  $\sigma_{xy}^{\rm IM} \sim \dfrac{t^4 \sin \mathcal{B}}{(J_{\rm IM} T)^2}$.

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In MFL: exact quantum Boltzmann equation at large M, N

$$(1 - \partial_\omega \mathrm{Re}(\Sigma^\psi))\partial_t \delta n(t,k,\omega) + v_F \hat{k} \cdot \vec{E}(t) n_f'(\omega) + v_F(\hat{k} \times \mathcal{B}\hat{z}) \cdot \nabla_k \delta n(t,k,\omega) = 2\delta n(t,k,\omega) \mathrm{Im}\,(\Sigma^\psi(\omega))$$

$$\sigma_{(L,H)}^{\rm MFL} = -M \frac{v_F^2 \nu(0)}{16T} \int_{-\infty}^{\infty} \frac{dE_1}{2\pi} \mathrm{sech}^2\left(\frac{E_1}{2T}\right) \frac{\left(\mathrm{Im}[\Sigma_R^c(E_1)], (v_F/(2k_F))\mathcal{B}\right)}{\mathrm{Im}[\Sigma_R^c(E_1)]^2 + (v_F/(2k_F))^2 \mathcal{B}^2},$$

$$\sigma_L^{\rm MFL} \sim T^{-1} s_L((v_F/k_F)(\mathcal{B}/T)), \quad \sigma_H^{\rm MFL} \sim -\mathcal{B} T^{-2} s_H((v_F/k_F)(\mathcal{B}/T)).$$

$$s_{L,H}(x \to \infty) \propto 1/x^2, \quad s_{L,H}(x \to 0) \propto x^0.$$

Macroscopic disorder

Suppose μ varies from region to region.

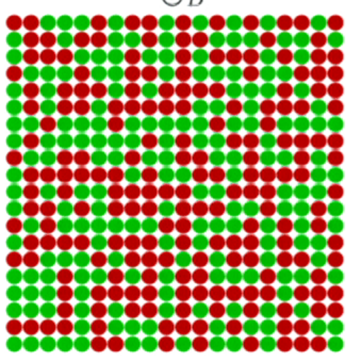
$\vec{\nabla} \cdot \vec{J}(x) = 0, \vec{J}(x) = \sigma(x) \cdot \vec{E}(x), \vec{E}(x) = -\vec{\nabla}\Phi(x).$

Effective medium theory

[Stroud 75, Parish-Littlewood]

Simple case: two types of domains, approximately equal area fractions:

$\odot B$

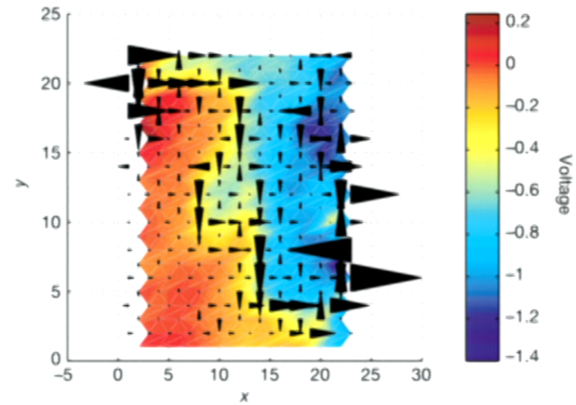


(a)

$\sigma_L^{\text{MFL}} \sim \frac{T}{B^2}, \sigma_H^{\text{MFL}} \sim \frac{1}{B} \overset{\text{EMT}}{\Longrightarrow} \rho_L \sim B$ for equal-areas.

Moreover, $\boxed{\rho_L \sim \sqrt{c_1 T^2 + c_2 B^2}}$

Mechanism:



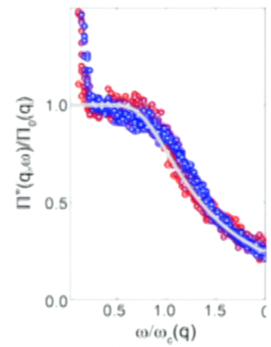
[from Parish-Littlewood 03]

Local Hall resistivity lengthens current path $\propto B$.

Some questions we can now ask

- Plasmon spectrum of BSCCO recently measured by EELS [Mitrano et al 1708.01929]. Apparent agreement with MFL form of $\mathrm{Im}\,\chi(\omega, q)$. Can we say more about plasmon damping in the solvable MFL? About the doping dependence of χ?

- Acoustic damping in MFL?



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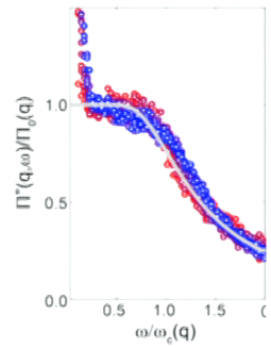


[from Mitrano et al 1708.01929]

- Acoustic damping in MFL?

- Is my title accurate?

Two aspects of SYK:

Maximal chaos: $\langle|\{\chi^\dagger(t), \chi(0)\}|^2\rangle \sim e^{\lambda_L t}$, $\lambda_L = \pi T$

– near the middle of the spectrum.

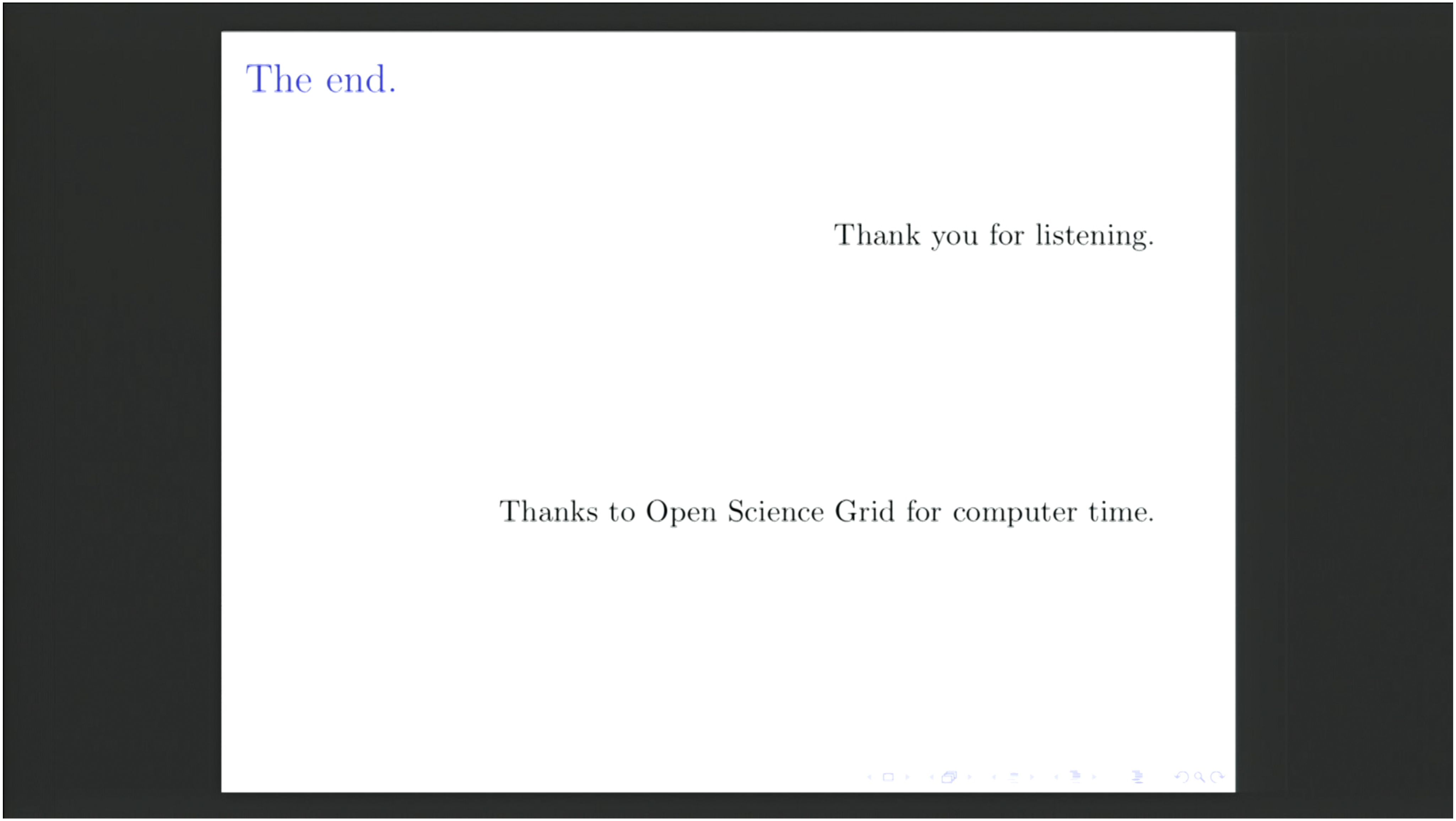
$z = \infty$ local criticality: $\mathcal{G}(\omega) \sim \omega^{2\nu}$

– near the groundstate.

Q: Can we have one without the other?

A [V. Rosenhaus]: Probably not.

Maximal chaos follows from (nearly) CFT_1.


The end.
Thank you for listening.
Thanks to Open Science Grid for computer time.

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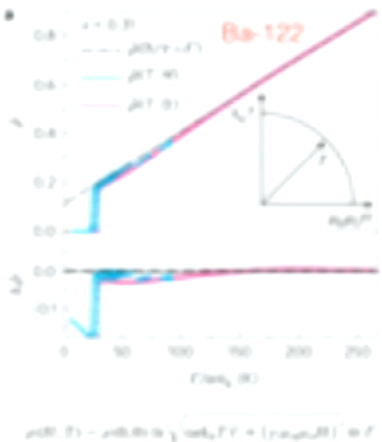

[1] M. Hayes et al., Nat. Phys. 2016

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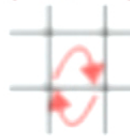
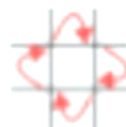
 $\sigma_{xx}^{\rm IM} \sim \frac{t^2}{J_{IM}T}$  $\sigma_{xy}^{\rm IM} \sim \frac{t^4 \sin \mathcal{B}}{(J_{\rm IM}T)^2}$.

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So far, ρ_L saturates at large $\mathcal{B}$.

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