Title: Emergent Gravity From Relatively Local Hamiltonians

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Abstract:

Emergence of gravity from relatively local Hamiltonians

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Outline

- Quantum RG for emergent space
- Emergent time
  - Set-up
  - Implications
    - Black hole information puzzle
- Summary

AdS/CFT correspondence

[Maldacena; Gubser, Klebanov, Polyakov; Witten]

Non-perturbative dynamics of some quantum field theories can be understood in terms of classical physics of dual gravitational theories

D-dim QFT ⇄ (D+1)-dim quantum gravity

Quantum field theories provide new perspectives on problems in quantum gravity

- A lot of progress has been made in both directions
- However, a microscopic understanding of the correspondence is desired for some problems
  - QFT that can not be easily embedded in string theory
  - Black hole information puzzle

Gravity from QFT

$$Z[g^{(0)}_{\mu\nu}, ..] = \int D_{g^{(0)}}\phi \;\; e^{-S[\phi; g^{(0)}_{\mu\nu}, ...]}$$

- S : single trace action $\quad Tr[\phi \nabla_{\mu_1} \nabla_{\mu_2} ... \phi \nabla_{\nu_1} \nabla_{\nu_2} ... \phi ...]$

- Ward identity :

$$Z[g_{\mu\nu}, ..] = Z[g_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu, ..]$$

Local coarse graining

$$Z[g^{(0)}_{\mu\nu}, ..] = \int D\phi \;\; e^{-S[\phi; g^{(0)}_{\mu\nu} e^{-dz\, n(x)}, ..] - \delta S'[g^{(0)}_{\mu\nu}, T^{\mu\nu}, ..]}$$

[Osborn]

$$= \int D\phi \;\; e^{-S[\phi; g^{(0)}_{\mu\nu}, ..] - \delta S[g^{(0)}_{\mu\nu}, T^{\mu\nu}, ..]}$$

- δS includes multi-trace operators

[Becchi, Giusto, Imbimbo; Heemskerk, Polchinski; Faulkner, Liu, Rangamani;..]

$$\delta S[g_{\mu\nu}, T^{\mu\nu}, ..] = dz \int dx \; n(x) \; \sqrt{g} \left(-\Lambda + \alpha R + \beta_{\mu\nu} T^{\mu\nu} + \frac{G_{\mu\nu\rho\sigma}}{g} T^{\mu\nu} T^{\rho\sigma} + ... \right)$$

- Multi-trace operators can be removed by promoting sources for single-trace operators into dynamical variables [SL]

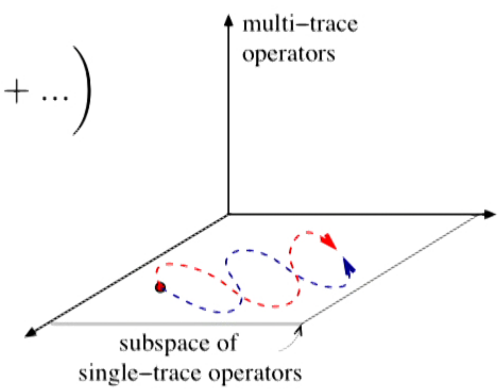
$$Z[g^{(0)}_{\mu\nu}, ..] = \int Dg^{(1)}_{\mu\nu} D\pi^{(1)\mu\nu} \;\; e^{i \int dx \pi^{(1)\mu\nu} (g^{(1)}_{\mu\nu} - g^{(0)}_{\mu\nu}) - \delta S[g^{(1)}_{\mu\nu}, \pi^{(1)\mu\nu}, ..]} \;\; Z[g^{(1)}_{\mu\nu}]$$

Sum over RG paths

- Repetition of the coarse graining leads to a sum over RG paths in the space of single-trace sources
- Beta functions for multi-trace operators in the presence of single-trace sources determines the quantum Hamiltonian for dynamical single-trace sources

$$Z[g^{(0)}_{\mu\nu}, ..] = \int \prod_{l=1}^{\infty} Dg^{(l)}_{\mu\nu} D\pi^{(l)\mu\nu} \; e^{i\sum_l \int dx \pi^{(l)\mu\nu}(g^{(l)}_{\mu\nu} - g^{(l-1)}_{\mu\nu}) - \sum_l dz \int dx n^{(l)}(x) \mathcal{H}[g^{(l-1)}_{\mu\nu}, \pi^{(l)\mu\nu}, ..]} \; Z[g^{(\infty)}_{\mu\nu}]$$

$$\mathcal{H}[g_{\mu\nu}, \pi^{\mu\nu}, ..] = \sqrt{g}\left(-\Lambda + \alpha R + \beta_{\mu\nu}\pi^{\mu\nu} + \frac{G_{\mu\nu\rho\sigma}}{g}\pi^{\mu\nu}\pi^{\rho\sigma} + ...\right)$$



$$g^{(l)}_{\mu\nu}(x) \to g_{\mu\nu}(x, z)$$

Generating function as an overlap between quantum states defined on spacetime

$$Z[g^{(0)}_{\mu\nu}, ..] = \langle 0 | g^{(0)}_{\mu\nu}, .. \rangle$$

UV state $$|g^{(0)}_{\mu\nu}, .. \rangle = \int D_{g^{(0)}}\phi \;\; e^{-S[\phi; g^{(0)}_{\mu\nu}, ..]} \; |\phi\rangle$$

basis states for spacetime configurations

IR state $$|0\rangle \qquad \langle 0 | \phi \rangle = 1$$

$$\hat{\phi}(x)|\phi\rangle = \phi(x)|\phi\rangle$$

- |0⟩ has an infinite norm

x : Euclidean spacetime

Coarse graining

$$Z[g^{(0)}_{\mu\nu}, ..] = \langle 0 \big| e^{-\int_0^z dz' \int dx \;\; n(x,z')\hat{\mathcal{H}}(x)} \big| g^{(0)}_{\mu\nu}, .. \rangle$$

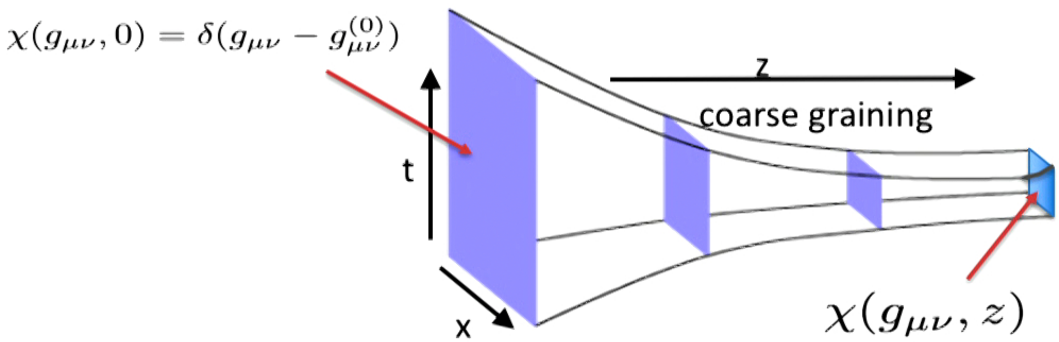
$\hat{\mathcal{H}}(x) = F(\hat{\pi}(x), \hat{\phi}(x), \nabla)$: generator of a local RG transformation acting on the spacetime state of the matte field

$$e^{-dz \int dx \;\; n(x)\hat{\mathcal{H}}(x)} \big| g^{(0)}_{\mu\nu}, .. \rangle = \int D\phi \;\; e^{-S[\phi; g^{(0)}_{\mu\nu}, ..] - \delta S[g^{(0)}_{\mu\nu}, T^{\mu\nu}, ..]} \big| \phi \rangle$$

$\langle 0 \big| \hat{\mathcal{H}}(x) = 0$: the generating function is invariant under coarse graining

- $|0\rangle$ represents a fixed point action under the coarse graining

Radial evolution



- The evolution of the wavefunction for metric is governed by a quantum theory of gravity

$$e^{-\int_0^z dz' \int dx \; n(x,z')\hat{\mathcal{H}}(x)} \left| g^{(0)}_{\mu\nu}, .. \right\rangle = \int dg_{\mu\nu} \left| g_{\mu\nu}, .. \right\rangle \chi(g_{\mu\nu}, ..; z)$$

$$\chi(g_{\mu\nu}, ..; z) = e^{-\int_0^z dz' \int dx \; n(x,z')\mathcal{H}(g_{\mu\nu}, \frac{\delta}{\delta g_{\mu\nu}})} \chi(g_{\mu\nu}, ..; 0)$$

- The wavefunction at UV and at any finite radial slice satisfies neither the momentum constraint nor the Hamiltonian constraint
  - A QFT with shifted sources represents a different QFT
  - There is a non-trivial RG flow
- Only the IR state satisfies the constraints

Emergent time?

- AdS/CFT does not directly apply to our universe
- dS/CFT aims to understand an emergent gravity/time with the guidance from AdS/CFT

  [Strominger; Anninos, Hartman, Strominger; Anninos, Denef, Monten, Sun;..]
- Here, we attempt to take a microscopic approach for emergent gravity/time

Road map

	Emergent space	Emergent time
\`UV' state	Local QFT action defined on a curved spacetime with other sources	Short-range entangled quantum state parameterized by space metric and other collective variables
Bulk evolution	Coarse graining	Unitary evolution
'IR' state	A fixed point action	Topological state with zero energy density
Physical observable (overlap)	Generating function	A wavefunction of the topological state

From emergent space to emergent time

$$Z[g^{(0)}_{\mu\nu}, ..] = \langle 0 | g^{(0)}_{\mu\nu}, .. \rangle$$

$$|g^{(0)}_{\mu\nu}, ..\rangle = \int D_{g^{(0)}}\phi \quad \Psi(\phi; g^{(0)}_{\mu\nu}, ..) \; |\phi\rangle$$

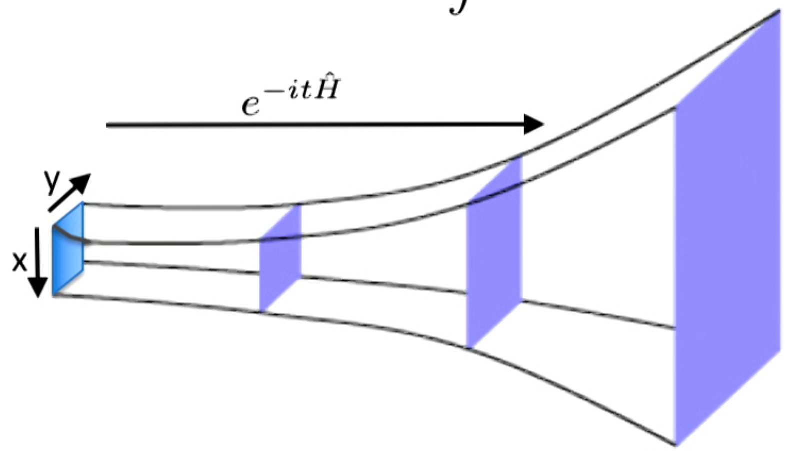
$$|0\rangle = \int Dg \; |g_{\mu\nu}, ..\rangle \; \chi_0(g_{\mu\nu}, ..)$$

From emergent space to emergent time

$$Z[g^{(0)}_{\mu\nu},..] = \langle 0 | e^{-i\int_0^t d\tau \int dx n(x,\tau)\hat{\mathcal{H}}(x)} | g^{(0)}_{\mu\nu},..\rangle$$

$$\langle 0 | \hat{\mathcal{H}}(x) = 0$$

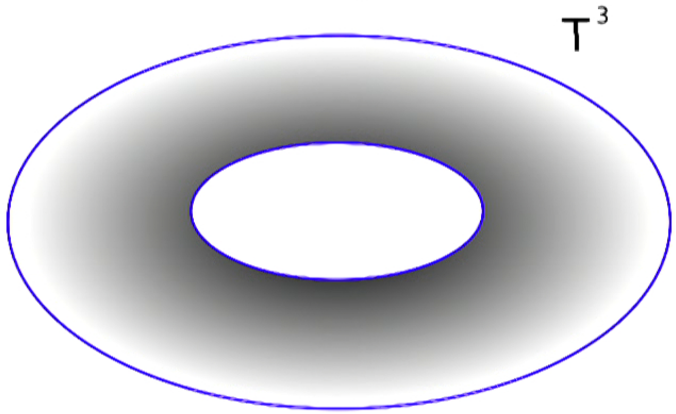
$$e^{-i\int_0^t d\tau \int dx n(x,\tau)\hat{\mathcal{H}}(x)} | g^{(0)}_{\mu\nu},..\rangle = \int Dg_{\mu\nu} \ | g_{\mu\nu},..\rangle \chi(g_{\mu\nu},..;t)$$



$$\chi(g_{\mu\nu},..;t) = e^{-\int_0^t d\tau \int dx \ n(x,\tau)\mathcal{H}(g_{\mu\nu},\frac{\delta}{\delta g_{\mu\nu}})}\chi(g_{\mu\nu},..;0)$$

Example for emergent time

- 3D manifold



- Matrix fields : $\Phi_{ab}(x)$
- Full Hilbert space spanned by $\left\{ \middle| \Phi \right\rangle \}$

$$\hat{\Phi}_{ab} \big| \Phi \big\rangle = \Phi_{ab} \big| \Phi \big\rangle$$

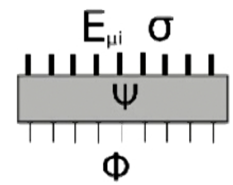
$$\big\langle \Phi' \big| \Phi \big\rangle = \prod_n \delta(\Phi_n'^{\hat{g}} - \Phi_n^{\hat{g}}) \qquad \Phi(x) = \sum_n \Phi_n^{\hat{g}} \, f_n^{\hat{g}}(x)$$

Sub-Hilbert space

A sub-Hilbert space $\mathcal{V}$ is spanned by states parameterized by two singlet collective variables : a triad and a scalar field

$$g_{E,\mu\nu}(x) = E_{\mu i}(x) E^i_\nu(x)$$

$$\left|E,\sigma\right\rangle = \int D^{(\hat{E},\hat{\sigma})}\Phi \left|\Phi\right\rangle \Psi(\Phi;E,\sigma)$$



$$\Psi(\Phi;E,\sigma) = e^{-\int dx \ |E(x)| \ \frac{1}{2} tr[\Phi K_R \Phi] - \frac{1}{2} S_0[E,\sigma]}$$

$$K_R = \exp\left\{ -\int_{l_c^2}^{\infty} \frac{dt}{t} e^{-t\left(-g_E^{\mu\nu}\nabla_\mu^E \nabla_\nu^E + \frac{e^{2\sigma}}{l_c^2}\right)} \right\}$$

Regularized Laplacian defined with $g_{E,\mu\nu}$ and 'mass' e^{σ}/l_c

$$\left\langle E,\sigma \middle| E,\sigma \right\rangle = 1.$$

Inner product

- Invariant under spatial diffeomorphism

$$\langle E', \sigma' | E, \sigma \rangle = \langle \tilde{E}', \tilde{\sigma}' | \tilde{E}, \tilde{\sigma} \rangle$$

$$\tilde{E}_{\mu i}(x) = E_{\mu i}(x) - \nabla_\mu \xi^\nu E_{\nu i},$$
$$\tilde{\sigma}(x) = (1 - \xi^\mu \partial_\mu)\sigma(x),$$

- In the large N limit, the inner product between states with different collective variables vanishes
- The inner product induces a natural measure

$$\int DE_{\mu i} D\sigma \, \langle E', \sigma' | E, \sigma \rangle = 1$$

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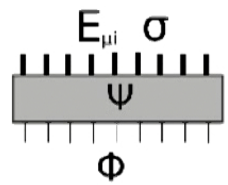

$$\left| E, \sigma \right\rangle = \int D^{(\hat{E},\hat{\sigma})}\Phi \left| \Phi \right\rangle \Psi(\Phi; E, \sigma)$$

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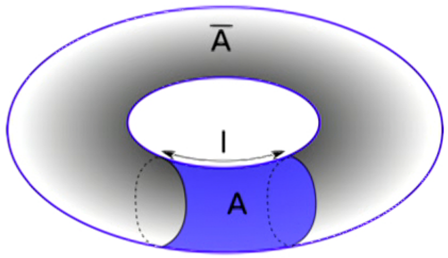
Regularized Laplacian defined with $g_{E,\mu\nu}$ and 'mass' e^{σ}/l_c

$$\left\langle E, \sigma \middle| E, \sigma \right\rangle = 1.$$

Physical meaning of the collective variables

$$g_{E,\mu\nu}(x)$$

- Sets the notion of locality in how matter fields are entangled in space
- Determines the number of degrees of freedom that are entangled
- Von Neumann entanglement entropy of $|E,\sigma\rangle$ obeys the area law measured with the metric $g_{E,\mu\nu}$

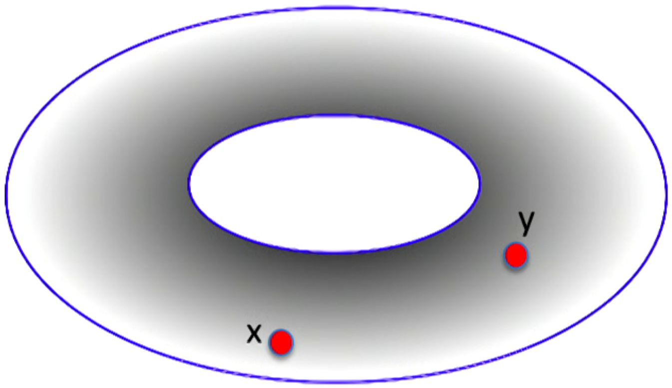


$$S_{\Phi}(A) = \frac{\mathcal{A}_{\partial A}}{4\kappa^2} \qquad \kappa^2 \equiv \frac{4\pi l_c^2}{N^2}$$

Physical meaning of the collective variables

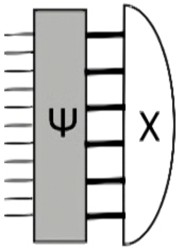
$$\sigma(x)$$

- Determines the range of mutual information in $|E,\sigma\rangle$



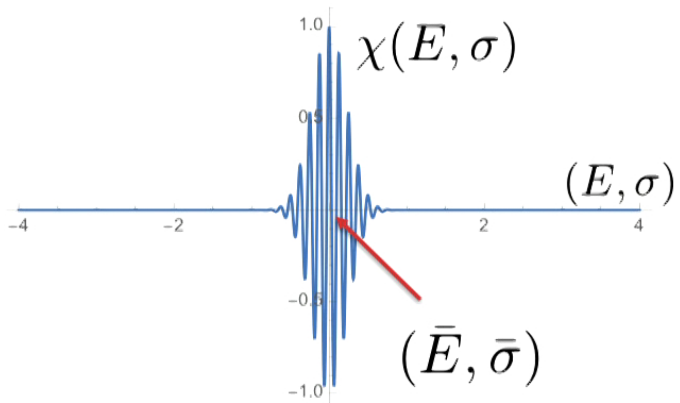
$$I(x,y) \sim e^{-\int_x^y \frac{e^{\sigma}}{l_c} ds}$$

General states



$$|\chi\rangle = \int DED\sigma \, |E,\sigma\rangle \chi(E,\sigma)$$

- For semi-classical states, total EE is given by the sum of two contributions



$$S(A) \approx S_{\Phi}(A) + S_{E,\sigma}(A)$$

Color EE

- $O(N^2)$
- Generated by matter fields fluctuating on a classical background metric
- Area law (measured with respect to the classical metric)

Singlet EE

- O(1)
- Generated by correlations between fluctuations of the collective variables
- Can obey volume law if the collective variables have long-range correlations

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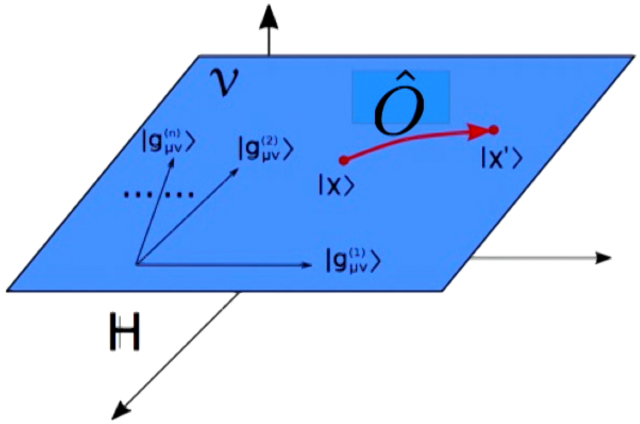
Endomorphism

$$\hat{O} : \mathcal{V} \to \mathcal{V}$$



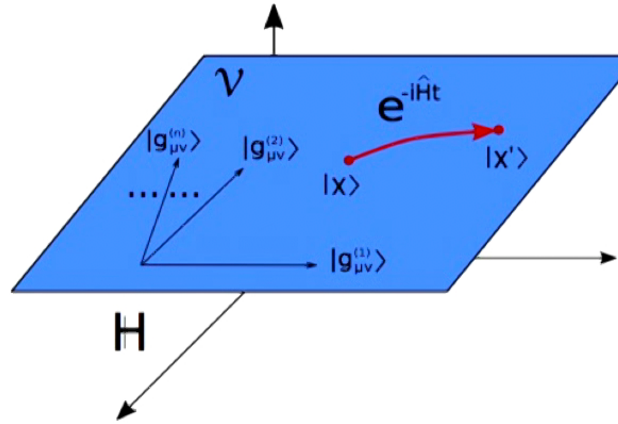
- Operators that map the sub-Hilbert space into the sub-Hilbert space
- Momentum operator generates diffeomorphism for the collective variables

$$\hat{\mathcal{H}}_\mu(x) \;=\; -\frac{1}{2}\left[\left(\nabla_\mu \hat{\Phi}_{ab}(x)\right)\hat{\pi}_{ba}(x) + \hat{\pi}_{ba}(x)\left(\nabla_\mu \hat{\Phi}_{ab}(x)\right)\right]$$

$$\int dx\, n^\mu(x)\hat{\mathcal{H}}_\mu(x)\left[\int DEDσ\, |E,\sigma\rangle \chi(E,\sigma)\right]$$
$$= \int DED\sigma\, |E,\sigma\rangle \left[\int dx\, n^\mu(x)\mathcal{H}^{E,\sigma}_\mu(x)\chi(E,\sigma)\right]$$

$$\mathcal{H}^{E,\sigma}_\mu(x) = -iE_{\mu i}\nabla_\nu \frac{\delta}{\delta E_{\nu i}} + i(\nabla_\mu \sigma)\frac{\delta}{\delta\sigma(x)}.$$

Hamiltonian flow



$$e^{-i\int_0^t d\tau \int dx n(x,\tau)\hat{\mathcal{H}}(x)} \int DEDσ\ \big|E,\sigma\big\rangle \chi(E,\sigma;0)$$

$$= \int DED\sigma\ \big|E,\sigma\big\rangle \chi(E,\sigma;t)$$

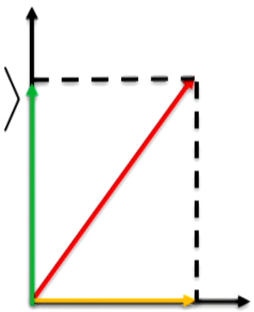
$$\chi(E,\sigma;t) = e^{-\int_0^t d\tau \int dx\ n(x,\tau)\mathcal{H}(E,\sigma,\frac{\delta}{\delta E},\frac{\delta}{\delta\sigma})}\chi(E,\sigma;0)$$

- There exists a Hamiltonian for the matter field whose flow stays within the sub-Hilbert space, and induces the Wheeler-DeWitt Hamiltonian for the collective variables

`State-dependent' Hamiltonian

$\beta\left|g_{E',\mu\nu},..\right\rangle$

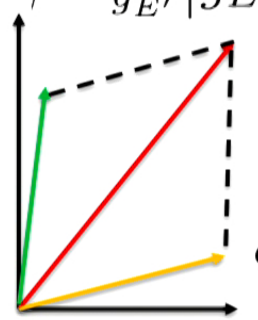
$\hat{H}$



$\alpha\left|g_{E,\mu\nu},..\right\rangle$

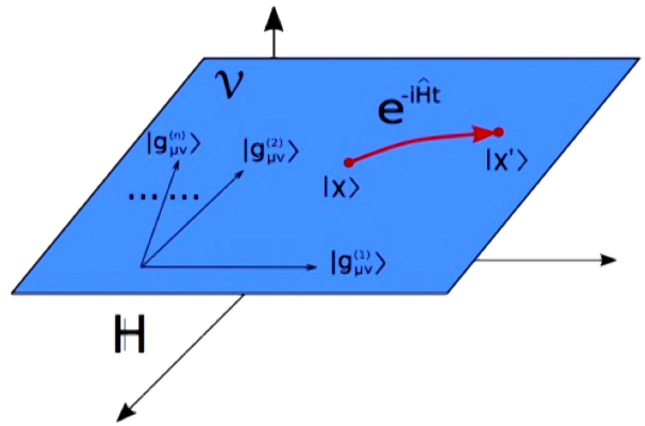
$\beta\hat{H}_{g_{E'}}\left|g_{E',\mu\nu},..\right\rangle$

=



$\alpha\hat{H}_{g_E}\left|g_{E,\mu\nu},..\right\rangle$

Hamiltonian flow



$$e^{-i\int_0^t d\tau \int dx n(x,\tau)\hat{\mathcal{H}}(x)} \int DEDσ\ \big|E,\sigma\big\rangle \chi(E,\sigma;0)$$

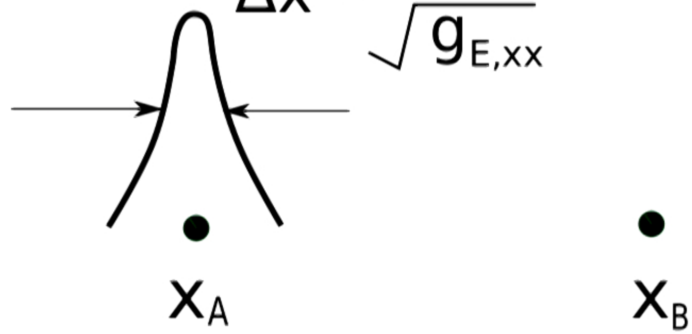
$$= \int DED\sigma\ \big|E,\sigma\big\rangle \chi(E,\sigma;t)$$

$$\chi(E,\sigma;t) = e^{-\int_0^t d\tau \int dx\ \ n(x,\tau)\mathcal{H}(E,\sigma,\frac{\delta}{\delta E},\frac{\delta}{\delta\sigma})}\chi(E,\sigma;0)$$

- There exists a Hamiltonian for the matter field whose flow stays within the sub-Hilbert space, and induces the Wheeler-DeWitt Hamiltonian for the collective variables

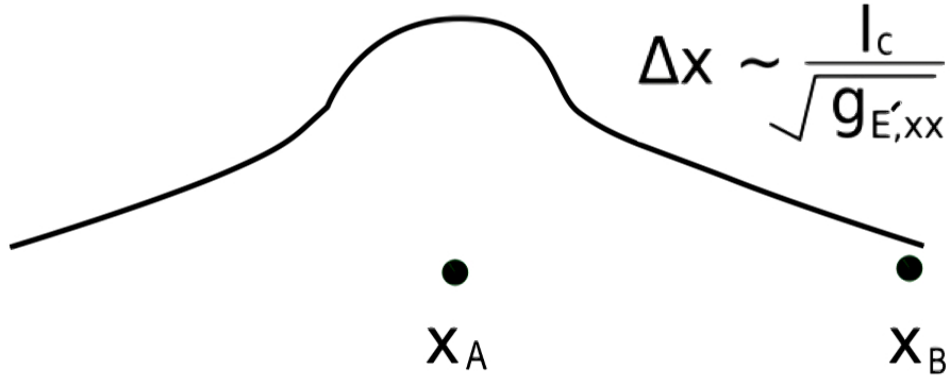
Relatively local Hamiltonian

$$\Delta x \sim \frac{l_c}{\sqrt{g_{E,xx}}}$$



For states with large proper distance between A and B

Relatively local Hamiltonian



$$\Delta x \sim \frac{l_c}{\sqrt{g_{E,xx}}}$$

For states with small proper distance between A and B

- The strength of the coupling between two points in space is determined by the state on which the Hamiltonian acts
- The notion of locality in the Hamiltonian is determined by states

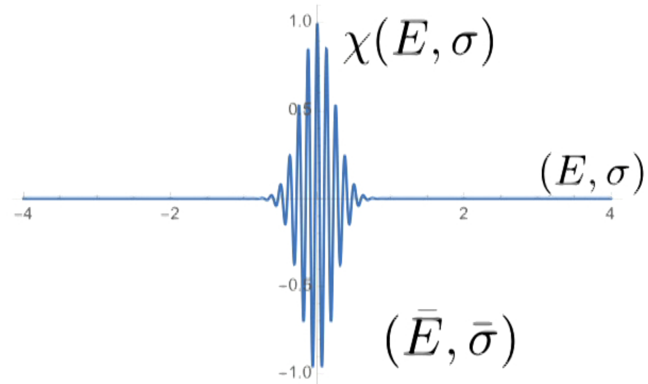
Time evolution

$$e^{-i\int_0^t d\tau \int dx n(x,\tau)\hat{\mathcal{H}}(x)} \int DEDσ \,\left|E,\sigma\right\rangle \chi(E,\sigma;0)$$

- Time evolution generated by the matter Hamiltonian is described by a time evolution of the collective variables governed by the Wheeler De Witt Hamiltonian in a fixed gauge

Semi-classical states

- Normalizable states with well defined `coordinates' and `momenta'



$$\pi^{\mu i}(x) = i\kappa^2 \frac{\delta}{\delta E_{\mu i}(x)}, \quad \pi_\sigma(x) = i\kappa^2 \frac{\delta}{\delta \sigma(x)}$$

$$\frac{1}{\sqrt{|\bar{g}|}}\left(\bar{\pi}^{\mu\nu}\bar{\pi}_{\mu\nu} - \frac{1}{2}(\bar{\pi}^{\mu}_{\mu})^2 + \frac{1}{2F(\bar{\sigma})}\bar{\pi}_\sigma^2\right) + \sqrt{|\bar{g}|}\left(-\bar{R} + \frac{F(\bar{\sigma})}{2}\bar{g}^{\mu\nu}\nabla_\mu\bar{\sigma}\nabla_\nu\bar{\sigma} + V(\bar{\sigma})\right) = 0.$$

$$2\nabla_\nu \bar{\pi}^{\mu\nu} - (\nabla^\mu \bar{\sigma})\bar{\pi}_\sigma = 0$$

Black hole formation and evaporation

$$e^{-i\int_0^t d\tau \int dx n(x,\tau)\hat{\mathcal{H}}(x)} \int DEDσ \; |E,\sigma\rangle \chi(E,\sigma;0)$$

- One can choose an initial state that describes a spherically symmetric collapsing mass shell
- In the large N limit, the saddle point solution describes a formation of a black hole
- Suppose a BH with $r_H >> l_c$ is formed. Right after the BH formation, it has large color EE

$$S_\Phi \sim \frac{r_H^2}{\kappa^2}, \quad S_{E,\sigma} \sim O(1)$$

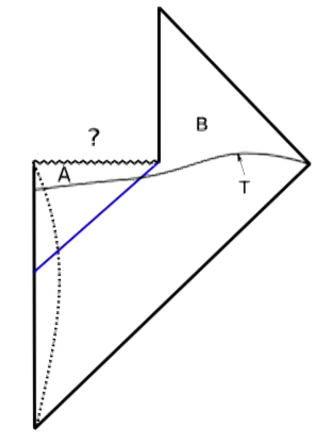
BH evaporation = Entanglement neutralization

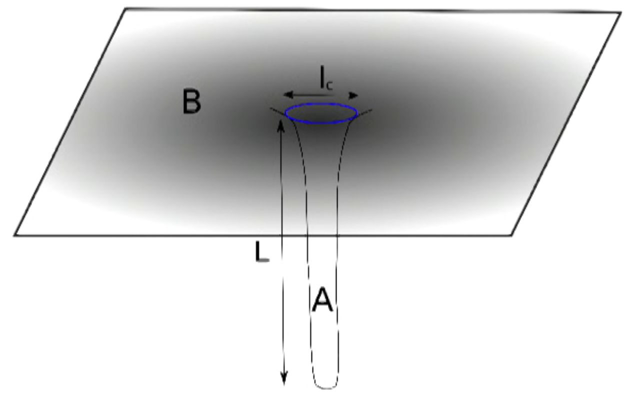
- 1/N corrections generate Hawking radiation
- The local field theory description remains valid until the horizon size becomes l_c
- The Hawking radiation is emitted in the singlet sector
  - N^2 degrees of freedom are subject to strong interaction

[cf. Hubeny, Marolf, Rangamani]

- Entanglement is gradually transferred from the color sector to the singlet sector

Entanglement neutralization

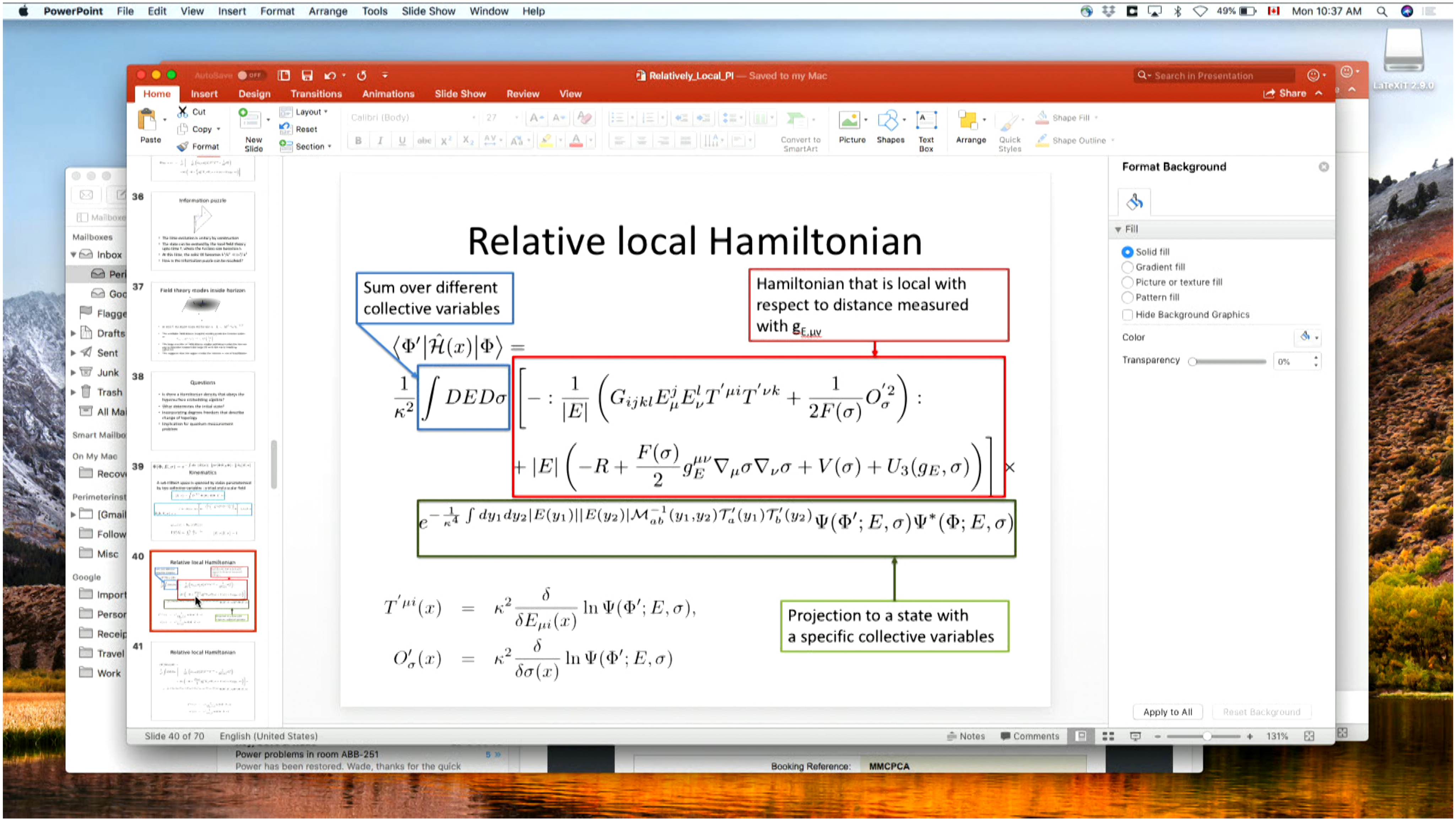


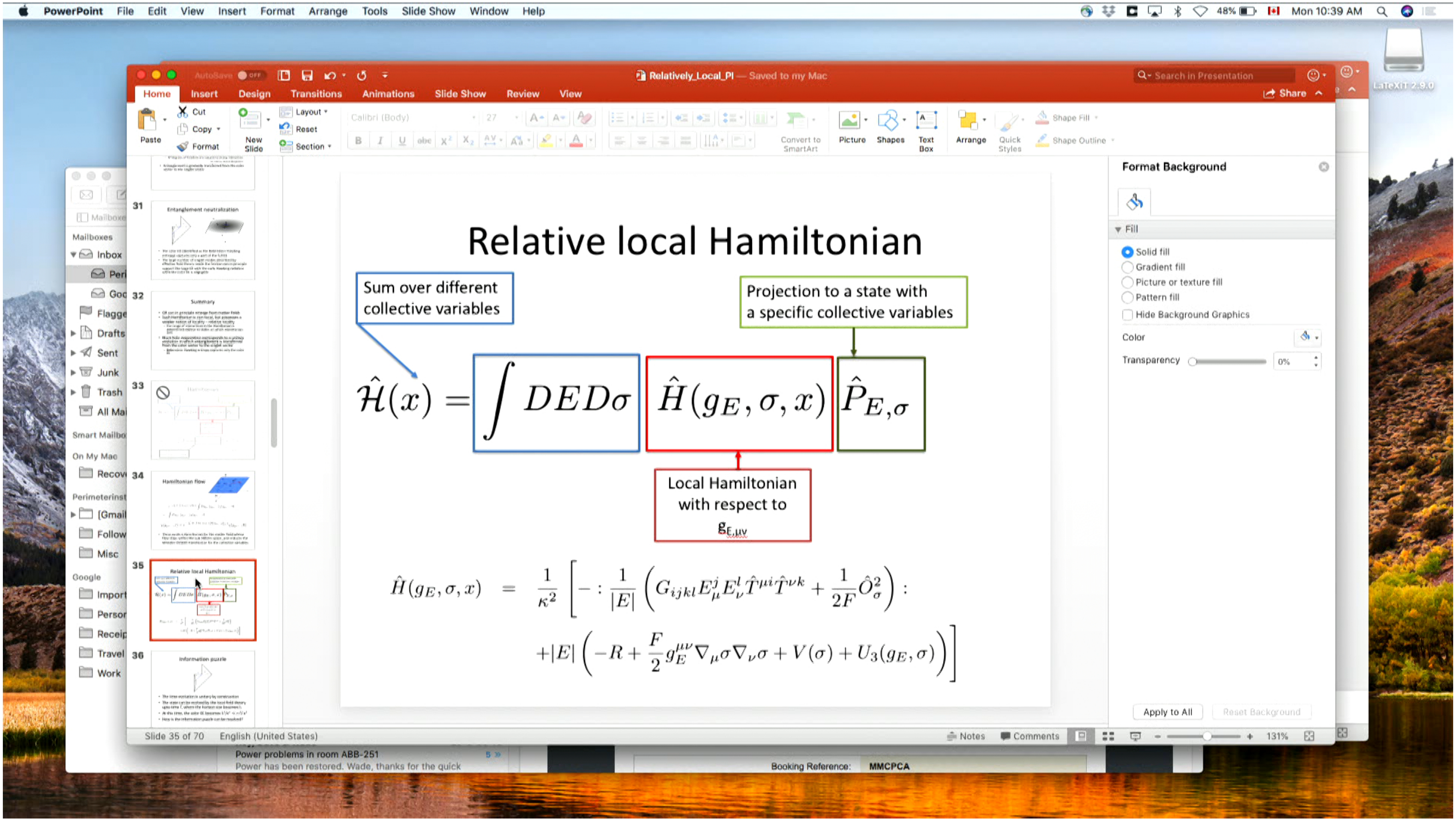


- The color EE (identified as the Bekenstein-Hawking entropy) captures only a part of the full EE
- The large number of singlet modes described by effective field theory inside the horizon can in principle support the large EE with the early Hawking radiation while the color EE is negligible

Summary

- GR can in principle emerge from matter fields
- Such Hamiltonian is non-local, but possesses a weaker notion of locality – relative locality
  - the range of interactions in the Hamiltonian is determined relative to states on which Hamiltonian acts
- Black hole evaporation corresponds to a unitary evolution in which entanglement is transferred from the color sector to the singlet sector
  - Bekenstein-Hawking entropy captures only the color EE


Relative local Hamiltonian
Sum over different collective variables
Hamiltonian that is local with respect to distance measured with $g_{E,\mu\nu}$
$\langle \Phi' | \hat{\mathcal{H}}(x) | \Phi \rangle = \frac{1}{\kappa^2} \int DED\sigma \left[-: \frac{1}{|E|} \left(G_{ijkl} E^j_\mu E^l_\nu T'^{\mu i} T'^{\nu k} + \frac{1}{2F(\sigma)} O'^2_\sigma \right) : + |E| \left(-R + \frac{F(\sigma)}{2} g^{\mu\nu}_E \nabla_\mu \sigma \nabla_\nu \sigma + V(\sigma) + U_3(g_E, \sigma) \right) \right] \times$
$e^{-\frac{1}{\kappa^4} \int dy_1 dy_2 |E(y_1)||E(y_2)| \mathcal{M}^{-1}_{ab}(y_1,y_2) \mathcal{T}'_a(y_1) \mathcal{T}'_b(y_2)} \Psi(\Phi'; E, \sigma) \Psi^*(\Phi; E, \sigma)$
$T'^{\mu i}(x) = \kappa^2 \frac{\delta}{\delta E_{\mu i}(x)} \ln \Psi(\Phi'; E, \sigma),$
$O'_\sigma(x) = \kappa^2 \frac{\delta}{\delta \sigma(x)} \ln \Psi(\Phi'; E, \sigma)$
Projection to a state with a specific collective variables


Relative local Hamiltonian
Sum over different collective variables
Projection to a state with a specific collective variables
$\hat{\mathcal{H}}(x) = \int DED\sigma \; \hat{H}(g_E, \sigma, x) \; \hat{P}_{E,\sigma}$
Local Hamiltonian with respect to $g_{E,\mu\nu}$
$\hat{H}(g_E,\sigma,x) = \frac{1}{\kappa^2}\left[-:\frac{1}{|E|}\left(G_{ijkl}E^j_\mu E^l_\nu \hat{T}^{\mu i}\hat{T}^{\nu k} + \frac{1}{2F}\hat{O}^2_\sigma \right): + |E|\left(-R + \frac{F}{2} g^{\mu\nu}_E \nabla_\mu\sigma\nabla_\nu\sigma + V(\sigma) + U_3(g_E,\sigma) \right) \right]$
Relatively_Local_PI — Saved to my Mac
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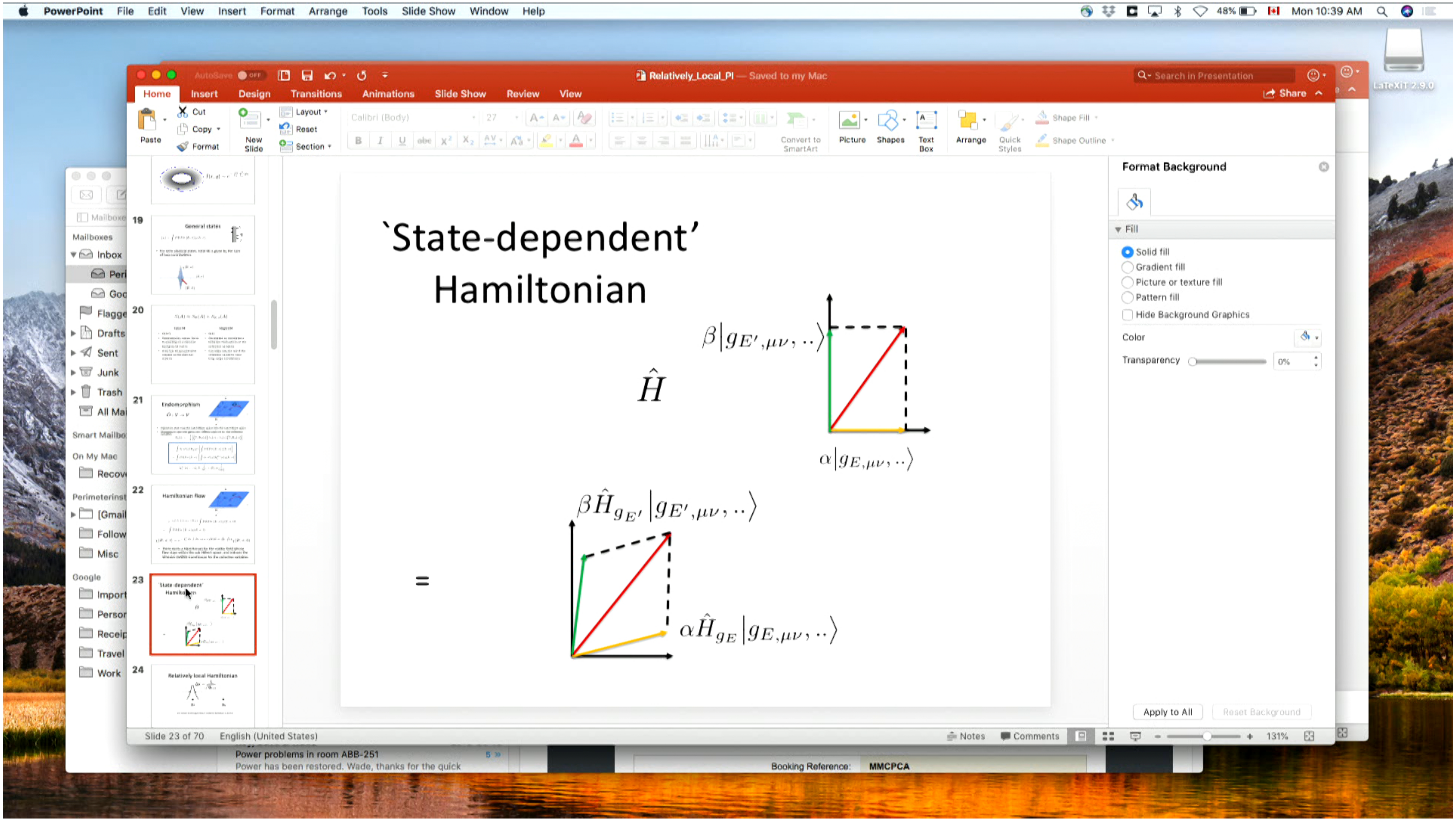
`State-dependent'
Hamiltonian
$\hat{H}$
$\beta|g_{E',\mu\nu},..\rangle$
$\alpha|g_{E,\mu\nu},..\rangle$
$\beta\hat{H}_{g_{E'}}|g_{E',\mu\nu},..\rangle$
=
$\alpha\hat{H}_{g_E}|g_{E,\mu\nu},..\rangle$
Relatively_Local_PI — Saved to my Mac
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English (United States)