Title: Beta functions at large N_f

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Abstract: Including a large number of vector-like fermions can be used to generate fixed points for the RG flows of gauge theories. Recently this has been used as a foundation for constructing UV safe models. The talk will focus on the machinery behind the large N_f computations extended to generic gauge-Yukawa theories. For semi-simple gauge theories the phase diagram shows the persistence that the UV fixed point of simple gauge theories.

Beta functions at large N_f

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CP^3 Origins

Cosmology & Particle Physics

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Outline

- Large N_f expansion applied to beta-functions in
  - Gauge-fermion theories
  - Semi-simple gauge theories
  - Gauge-Yukawa theories
- Computational framework
- Application to asymptotic safety

Based on:

Oleg Antipin, Nicola Andrea Dondi, Francesco Sannino, AET, and Zhi-Wei Wang [arXiv:1803.09770], to appear in PRD

Motivation

- Model building tool for constructing new UV complete BSM models

  See talk by Zhi-Wei Wang
  See talk by Steven Abel — S. Abel and F. Sannino [1707.06638]

- Realistic BSM models feature both Yukawa and quartic couplings and often semi-simple gauge groups.
- Explore a limit of QFTs

  Litim-Sannino (Large N_f, N_c) — D.F. Litim and F. Sannino [1406.2337]

$1/N_f$ counting

The goal is to expand the a gauge theory in $1/N_f$;

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{I=1}^{N_f} i\,\overline{\Psi}_I \gamma^\mu (\partial_\mu - i g\, A_\mu) \Psi^I$$

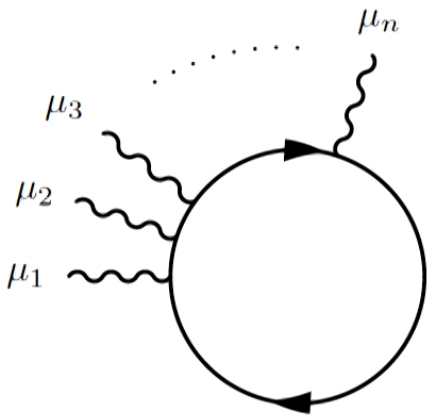
Introduce 't Hooft-like coupling

$$K = \frac{g^2 N_f}{4\pi^2}$$

$1/N_f \ll 1$ limit: $\quad K = \text{cst.} \quad g \sim \sqrt{1/N_f}$

What diagrams should we include?

Diagrams contain fermion loops with $n \geq 2$ gauge insertions;



$$\sim g^n N_f = \mathcal{O}\left(\frac{1}{N_f}\right)^{(n-2)/2}$$

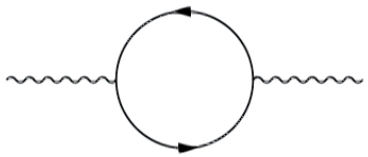
- Loops with $n = 2$ are "free"; internal gauge lines must be dressed as

$$= \mathcal{O}(1)$$

- Loops with $n \geq 3$ decreases the order of a diagram.

Large N_f beta function in QED

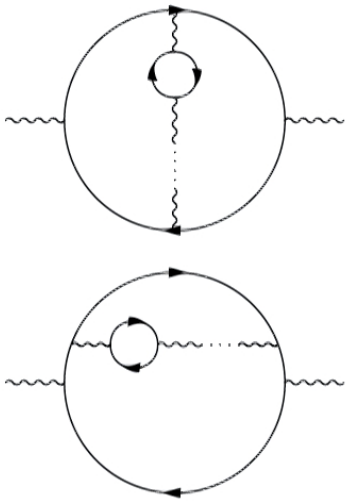
At LO in $1/N_f$ the beta function for the renormalized coupling, K, is



$$\beta_K = \frac{\mathrm{d}K}{\mathrm{d}t} = \frac{2K^2}{3} + \dots$$

At NLO there is an infinite number of diagrams:

A. Planques-Mestre and P. Pascual '84



$$\beta_K = \frac{2K^2}{3}\left[1 + \frac{1}{N_f}F_1(K)\right] + \dots$$

$$F_1(K) = \frac{3}{4}\int_0^K \mathrm{d}x\, \tilde{F}\left(0, \tfrac{2}{3}x\right)$$

$$\tilde{F}(0,x) = \frac{(1-x)(1-\frac{x}{3})(1+\frac{x}{2})\Gamma(4-x)}{3\Gamma(3-\frac{x}{2})\Gamma^2(2-\frac{x}{2})\Gamma(1+\frac{x}{2})}$$

F_1 $(\tilde{F})$ has a pole at $K = 15/2$ $(x = 5)$

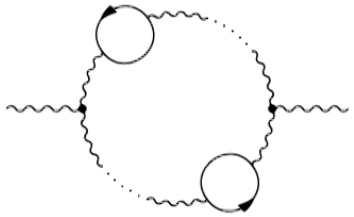
Non-Abelian gauge theories

In simple non-Abelian theories a new renormalized coupling is introduced

$$K = \frac{g^2 N_f \, S_2(R_\Psi)}{4\pi^2}$$

Self-interacting gluons give new contributions at NLO:

J.A. Gracey [hep-ph/9602214] B. Holdom [1006.2119]



$$\beta_K = \frac{2K^2}{3}\left[1 + \frac{1}{N_f}\frac{d(G)}{d(R_\Psi)} H_1(K)\right] + \dots$$

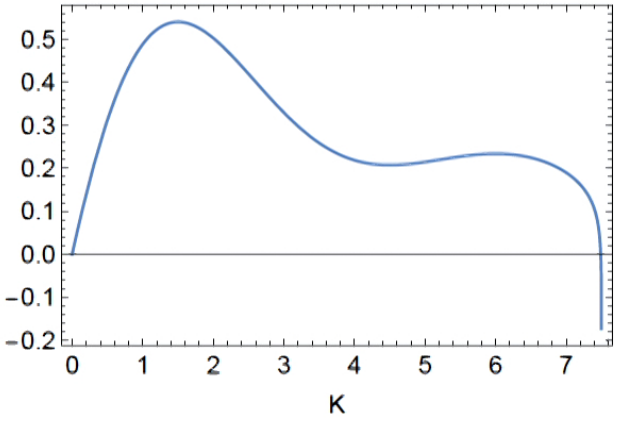
$$H_1(K) = -\frac{11 C_2(G)}{4 C_2(R_\Psi)} + \frac{3}{4}\int_0^K \mathrm{d}x \, \tilde{F}(0, \tfrac{2}{3}x)\tilde{G}(\tfrac{1}{3}x)$$

$$\tilde{G}(x) = 1 + \frac{C_2(G)}{C_2(R_\Psi)} \frac{20 - 43x + 32x^2 - 14x^3 + 4x^4}{4\,(2\,x - 1)\,(2\,x - 3)\,(1 - x^2)}$$

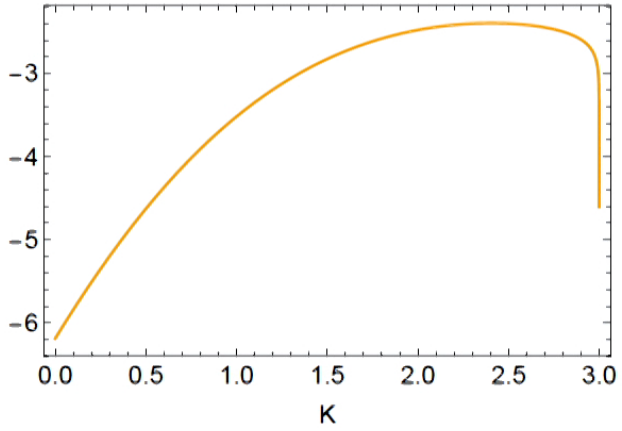
$H_1\,(\tilde{G})$ has a pole at $K = 3\;(x = 1)$

Zeros of the beta functions

Behavior of the NLO contributions ($N_c = 3$, $R_\Psi = \text{fund}$)



(a) Abelian, $F_1(K)$



(b) Non-Abelian, $H_1(K)$

$$\beta_K = \frac{2K^2}{3}\left[1 + \frac{1}{N_f}F_1(K)\right] \qquad \beta_K = \frac{2K^2}{3}\left[1 + \frac{1}{N_f}\frac{d(G)}{d(R_\Psi)}H_1(K)\right]$$

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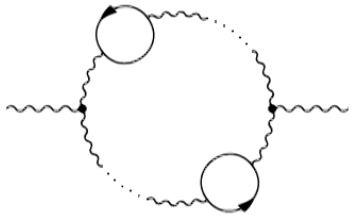

$$\beta_K = \frac{2K^2}{3}\left[1 + \frac{1}{N_f}\frac{d(G)}{d(R_\Psi)} H_1(K)\right] + \dots$$

$$H_1(K) = -\frac{11 C_2(G)}{4 C_2(R_\Psi)} + \frac{3}{4}\int_0^K \mathrm{d}x \, \tilde{F}(0, \tfrac{2}{3}x)\tilde{G}(\tfrac{1}{3}x)$$

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$H_1\ (\tilde{G})$ has a pole at $K = 3\ (x = 1)$

Asymptotic safety?

F. Sannino and O. Antipin [1709.02354]

- Large N_f expansion seems to be valid for $N_f \gtrsim 10 N_c$ with $\Psi \in \boldsymbol{N_c}$
- Anomalous dimension as a check of the FP
  - Abelian: $\gamma_m \to \infty$ for $N_f \to \infty$
  - Non-Abelian: $\gamma_m \to 0$ for $N_f \to \infty$
- Series expansion of F_1 and H_1 seems to idicate worse convergence of the Abelian series

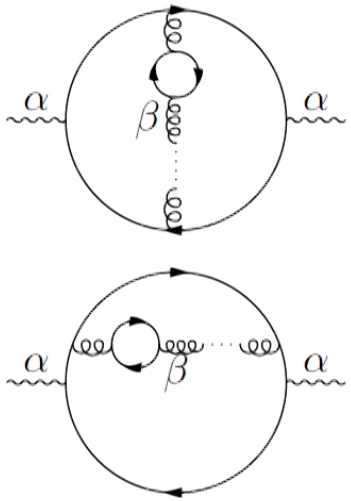
We probably should not trust the Abelian fixed point

Semi-simple extension

Consider a gauge group $G = \times_\alpha G_\alpha$ and fermions $\Psi_I \in N_f \times (\otimes_\alpha R^\alpha_\Psi)$

New effective flavor number

$$\mathcal{N} \equiv N_f \prod_\alpha d(R^\alpha_\Psi) \quad \text{and} \quad K_\alpha = \frac{g_\alpha^2 \mathcal{N} S_2(R^\alpha_\Psi)}{4\pi^2 d(R^\alpha_\Psi)}$$



New mixed diagrams are like the Abelian NLO diagrams, but

$$g^4 N_f \longrightarrow g_\alpha^2 g_\beta^2 \mathrm{Tr}\left[T^A_{R^\alpha_\Psi} T^B_{R^\alpha_\Psi} T^C_{R^\beta_\Psi} T^C_{R^\beta_\Psi}\right]$$
$$= \frac{d(R^\beta_\Psi)}{\mathcal{N}} K_\alpha K_\beta \delta^{AB}$$

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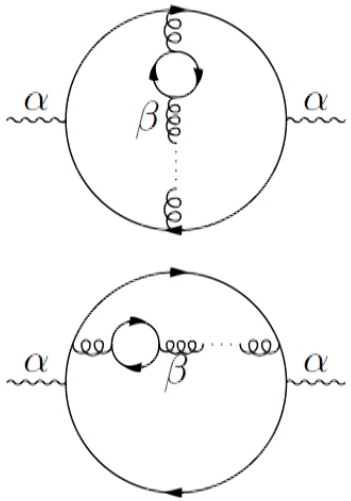
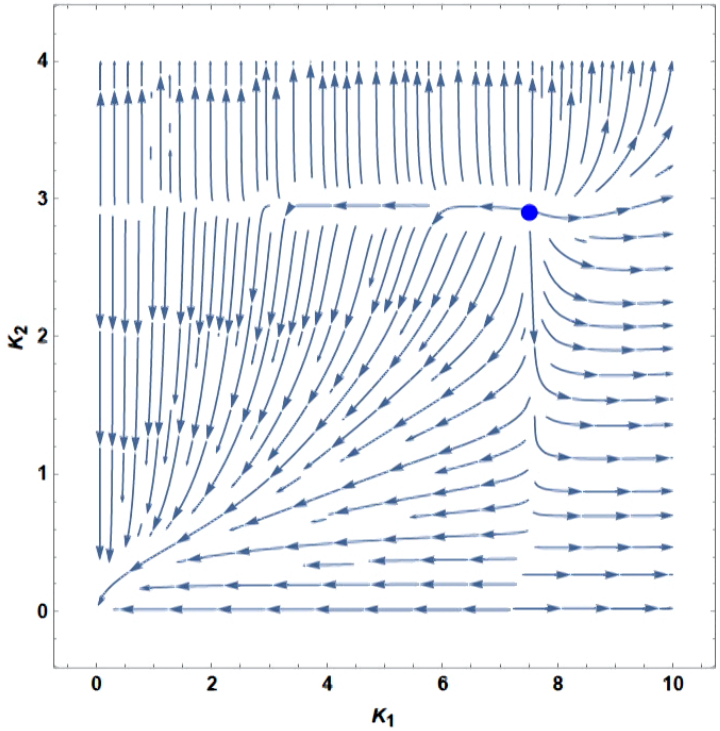


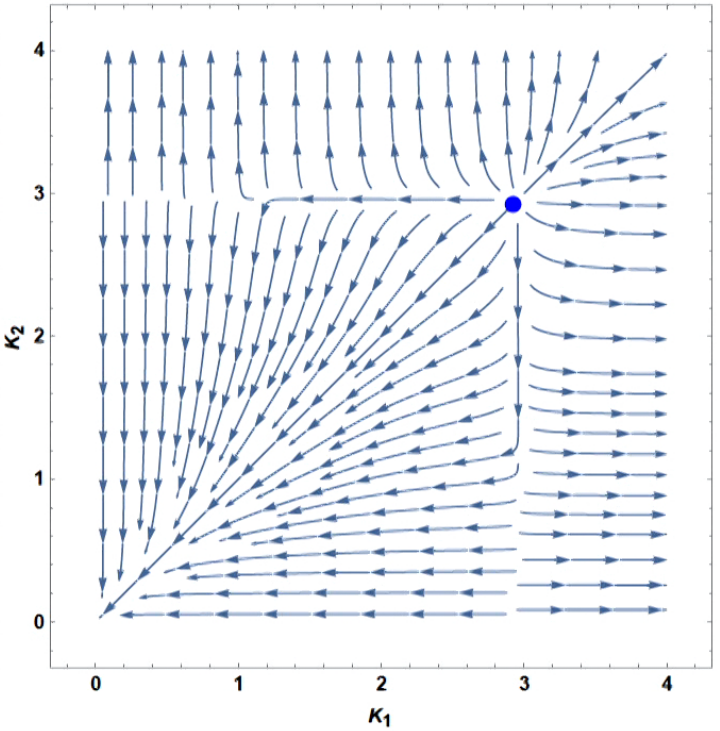
The full NLO beta function

$$\beta_{K_\alpha} = \frac{2K_\alpha^2}{3}\Big[1 + \frac{d(G_\alpha)}{\mathcal{N}} H_1^{(\alpha)}(K_\alpha) + \frac{1}{\mathcal{N}} \sum_{\beta \neq \alpha} d(G_\beta) F_1(K_\beta)\Big]$$

Semi-simple phase diagram



(c) An Abelian and a non-Abelian group



(d) Two non-Abelian groups

Generic gauge-Yukawa theory

Fields	$SO(1,3)^+$	$SU(N_f)$	$\times_\alpha G_\alpha$
Ψ	$(\frac{1}{2},0)\oplus(0,\frac{1}{2})$	N_f	$\otimes_\alpha R^\alpha_\Psi$
χ	$(\frac{1}{2},0)$	1	$\otimes_\alpha R^\alpha_\chi$
ϕ	$(0,0)$	1	$\otimes_\alpha R^\alpha_\phi$

The most general Lagrangian with real scalars and Weyl fermions

$$\mathcal{L} = -\tfrac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} + \sum_{i=1}^{N_f}\overline{\Psi}_i i\gamma^\mu D_\mu \Psi^i + i\bar{\chi}_i\bar{\sigma}^\mu(D_\mu\chi)^i + \tfrac{1}{2}(D_\mu\phi)^a(D_\mu\phi)^a$$
$$-\tfrac{1}{2}\left(y_{aij}\phi^a\chi^i\chi^j + \text{h.c.}\right) - \tfrac{1}{24}\lambda_{abcd}\phi^a\phi^b\phi^c\phi^d$$

Perturbative counting

$$\lambda \sim y^2 \sim g^2 \sim \frac{1}{\mathcal{N}}$$

ensures that higher order contributions in regular loop counting can be consistently ignored.

Large N_f machinery: the bubble chain

In the Landau gauge $\xi = 0$,

$$D^{(n)}_{\mu\nu}(p) = \underbrace{\quad \text{ } \quad}_{n} = \frac{-i}{p^2}\Delta_{\mu\nu}(p)\,\Pi_0^n(p^2)$$

The single bubble contribution is ($d = 4 - \epsilon$ dimensions)

$$\Pi_0(p^2) = -2K_0\frac{\Gamma^2(2-\frac{\epsilon}{2})\Gamma(\frac{\epsilon}{2})}{\Gamma(4-\epsilon)}\left(-\frac{4\pi\mu^2}{p^2}\right)^{\epsilon/2}$$

The dimension-dependent contribution from the bubble shows up in every beta-function as $\Gamma_0^{-1}(\frac{\epsilon}{2}K) \quad \rightarrow$ pole at $K = 15/2$

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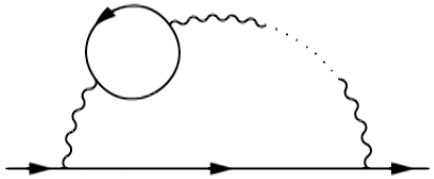
Fermion self-energy



n-bubble contribution to Weyl-spinor self-energy

$$-i\Sigma_\chi^{(n)}(p) = (i\tilde{g}_0)^2 C_2(R_\chi)\mu^\epsilon \int \frac{\mathrm{d}^d k}{(2\pi)^d}\bar{\sigma}^\mu \frac{i\sigma\cdot(p-k)}{(p-k)^2}\bar{\sigma}^\nu D^{(n)}_{\nu\mu}(k)$$

$Z_\chi^{(1)}$ is given by the simple pole of

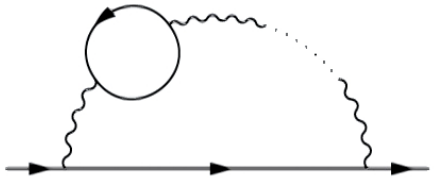
$$\frac{\mathrm{d}\Sigma_\chi}{\mathrm{d}\bar{\sigma}\cdot p} = -\frac{9}{16\mathcal{N}}\frac{d(R_\Psi)C_2(R_\chi)}{S_2(R_\Psi)}\sum_{n=1}^{\infty}\left(-\frac{2K_0}{3}\right)^n \frac{1}{n\epsilon^n}H_\psi(n,\epsilon)$$

$H_\psi(n,\epsilon)$ is a regular function in $n\epsilon$ and ϵ. A. Planques-Mestre and P. Pascual '84

Resummation required to extract the $1/\epsilon$ pole

$$K_0 = Z_K^{-1}K = K\left[1 - \frac{2K}{3\epsilon} + O\left(\frac{1}{\mathcal{N}}\right)\right]$$

Self-energies

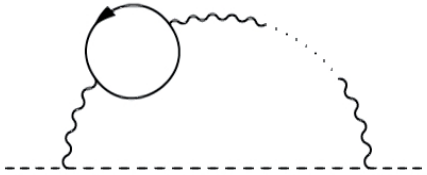


$$Z_\chi^{(1)} = \frac{d(R_\Psi)}{4\mathcal{N}\, S_2(R_\Psi)} C_2(R_\chi) \int_0^K \mathrm{d}x\, x H_0(\tfrac{2}{3}x)$$

The new function $H_0(x) = H_\psi(0,x)/x$ and is given by

$$H_0(x) = \frac{(1-\frac{x}{3})\Gamma(4-x)}{3\Gamma(3-\frac{x}{2})\Gamma^2(2-\frac{x}{2})\Gamma(1+\frac{x}{2})} = 1 - \frac{5}{12}x - \frac{35}{144}x^2 + \dots$$

The scalar self-energy is computed in a similar manner



$$Z_\phi^{(1)} = \frac{3\, d(R_\Psi)}{2\mathcal{N}\, S_2(R_\Psi)} C_2(R_\phi) \int_0^K \mathrm{d}x\, H_0(\tfrac{2}{3}x)$$

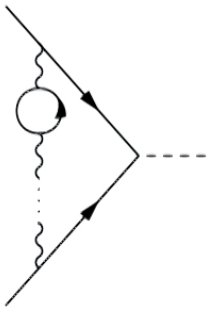
Yukawa beta function

The Yukawa beta function begins at order

$$\mathcal{O}(y^3) = \mathcal{O}(yg^2K^n) = \mathcal{O}\left(\frac{y}{\mathcal{N}}\right)$$

Only one kind of diagrams contribute to the vertex correction in the Landau gauge

K. Kowalska and E.M. Sesslo [1712.06859]



$$\delta y^{(1)}_{aij} = -\frac{3d(R_\Psi)}{2\mathcal{N}} \frac{y_{akj}\, C_2(R_\chi)^k{}_i + y_{aik}\, C_2(R_\chi)^k{}_j - y_{bij}\, C_2(R_\phi)_{ab}}{2S_2(R_\Psi)} \times \int_0^K \mathrm{d}x \, (1 - \tfrac{1}{6}x) H_0(\tfrac{2}{3}x)$$

Yukawa beta function

The Yukawa beta function at LO in $1/\mathcal{N}$ including regular 1-loop contributions

$$\beta_{y,aij} = \frac{1}{32\pi^2}\left(y_b y_b^\dagger y_a + y_a y_b^\dagger y_b\right)_{ij} + \frac{1}{32\pi^2}\mathrm{Tr}\left[y_a y_b^\dagger + y_a^\dagger y_b\right] y_{bij}$$
$$+\frac{1}{8\pi^2}(y_b y_a^\dagger y_b)_{ij} - \sum_\alpha \frac{d(R_\Psi^\alpha)}{8\mathcal{N}} \frac{y_{bij}\, C_2(R_\phi^\alpha)_{ab}}{S_2(R_\Psi^\alpha)} K_\alpha^2\, H_0(\tfrac{2}{3}K_\alpha)$$
$$-\sum_\alpha \frac{3d(R_\Psi^\alpha)}{4\mathcal{N}} \frac{y_{akj}\, C_2(R_\chi^\alpha)^k{}_i + y_{aik}\, C_2(R_\chi^\alpha)^k{}_j}{S_2(R_\Psi^\alpha)} K_\alpha\, H_0(\tfrac{2}{3}K_\alpha)$$

Abelian: $K_\alpha \to 15/2$ in the UV forces y free

Non-Abelian: $K_\alpha \to 3$ in the UV opens an interacting FP

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Non-Abelian: $K_\alpha \to 3$ in the UV opens an interacting FP

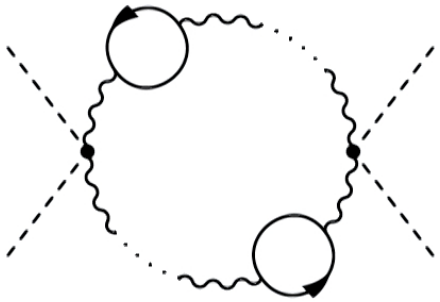
Quartic beta function

The quartic beta function begins at

$$\mathcal{O}(\lambda)^2 = \mathcal{O}(y)^4 = \mathcal{O}(\lambda y^2) = \mathcal{O}(\lambda g^2 K^n) = \mathcal{O}(g^4 K^n) = \mathcal{O}\left(\frac{1}{\mathcal{N}}\right)^2$$

Leading single gauge contribution to the Quartic beta function

G.M. Pelaggi *et. al.* [1708.00437]



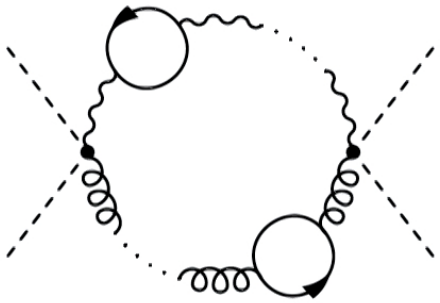
The divergence occurs at $p = 0$, where the loop integral is insensitive to the location of the bubbles;

$$\operatorname{div}\Lambda^{(n)} = \tfrac{1}{2} n K_{\alpha,0}^{n+2}\, L(n,\epsilon)$$

$$\delta\lambda^{(1)}_{abcd} = \frac{24\pi^2 d^2(R^\alpha_\Psi)}{\mathcal{N}^2 S_2^2(R^\alpha_\Psi)} A^\alpha_{abcd}\, K^2_\alpha (1 - \tfrac{1}{6}K_\alpha) H_0(\tfrac{2}{3}K_\alpha)$$

Quartic beta function

The leading mixed gauge contribution to the quartic beta function differs from the simple gauge case



The divergence takes the form

$$\begin{aligned}\operatorname{div}\Lambda^{(n)} &= K_{\alpha,0}K_{\beta,0}\sum_{m=0}^{n} K_{\alpha,0}^{n-m}K_{\beta,0}^{m}\,L(n,\epsilon)\\ &= \frac{K_{\alpha,0}K_{\beta,0}}{K_{\alpha,0}-K_{\beta,0}}\left(K_{\alpha,0}^{n+1}-K_{\beta,0}^{n+1}\right)L(n,\epsilon).\end{aligned}$$

$$\delta\lambda^{(1)}_{abcd} = B^{\alpha,\beta}_{abcd}\frac{48\pi^2}{\mathcal{N}^2}\frac{d(R^{\alpha}_{\Psi})\,d(R^{\beta}_{\Psi})}{S_2(R^{\alpha}_{\Psi})\,S_2(R^{\beta}_{\Psi})}\frac{K_\alpha K_\beta}{K_\alpha-K_\beta}\int_{K_\beta}^{K_\alpha}\mathrm{d}x\,(1-\tfrac{1}{6}x)H_0(\tfrac{2}{3}x)$$

Quartic beta function

The quartic beta function at LO in $1/\mathcal{N}$

$$\beta_{\lambda,abcd} = \frac{1}{16\pi^2}\underbrace{L_{abcd}}_{3\times\lambda^2} - \frac{1}{4\pi^2}\underbrace{H_{abcd}}_{6\times y^4} + \frac{1}{32\pi^2}\left[\lambda_{abce}\mathrm{Tr}(y_e y_d^\dagger + y_e^\dagger y_d) \;+\; 3 \text{ perm.}\right]$$

$$-\sum_\alpha \frac{3\,d(R_\psi^\alpha)}{4\mathcal{N}\,S_2(R_\psi^\alpha)}\left[\lambda_{abce}\,C_2(R_\phi^\alpha)_{ed} \;+\; 3 \text{ perm.}\right] K_\alpha\, H_0(\tfrac{2}{3}K_\alpha)$$

$$+\frac{24\pi^2}{\mathcal{N}^2}\sum_\alpha A^\alpha_{abcd}\frac{d^2(R_\psi^\alpha)}{S_2^2(R_\psi^\alpha)}\left[(K_\alpha^2 - \tfrac{1}{3}K_\alpha^3)\,H_0(\tfrac{2}{3}K_\alpha) + \tfrac{2}{3}(K_\alpha^3 - \tfrac{1}{6}K_\alpha^4)\,H_0'(\tfrac{2}{3}K_\alpha)\right]$$

$$+\frac{48\pi^2}{\mathcal{N}^2}\sum_{\alpha<\beta} B^{\alpha,\beta}_{abcd}\frac{d(R_\psi^\alpha)\,d(R_\psi^\beta)}{S_2(R_\psi^\alpha)\,S_2(R_\psi^\beta)}\frac{K_\alpha K_\beta}{K_\alpha - K_\beta}$$

$$\times\left[(K_\alpha - \tfrac{1}{6}K_\alpha^2)\,H_0(\tfrac{2}{3}K_\alpha) - (K_\beta - \tfrac{1}{6}K_\beta^2)\,H_0(\tfrac{2}{3}K_\beta)\right]$$

Quartic beta function

The quartic beta function at LO in $1/\mathcal{N}$

$$\beta_{\lambda,abcd} = \frac{1}{16\pi^2}\underbrace{L_{abcd}}_{3\times\lambda^2} - \frac{1}{4\pi^2}\underbrace{H_{abcd}}_{6\times y^4} + \frac{1}{32\pi^2}\left[\lambda_{abce}\mathrm{Tr}(y_e y_d^\dagger + y_e^\dagger y_d) \;+\; 3\text{ perm.}\right]$$

$$-\sum_\alpha \frac{3\,d(R_\psi^\alpha)}{4\mathcal{N}\,S_2(R_\psi^\alpha)}\left[\lambda_{abce}\,C_2(R_\phi^\alpha)_{ed} \;+\; 3\text{ perm.}\right] K_\alpha\, H_0(\tfrac{2}{3}K_\alpha)$$

$$+\frac{24\pi^2}{\mathcal{N}^2}\sum_\alpha A^\alpha_{abcd}\frac{d^2(R_\psi^\alpha)}{S_2^2(R_\psi^\alpha)}\left[(K_\alpha^2 - \tfrac{1}{3}K_\alpha^3)\,H_0(\tfrac{2}{3}K_\alpha) + \tfrac{2}{3}(K_\alpha^3 - \tfrac{1}{6}K_\alpha^4)\,H_0'(\tfrac{2}{3}K_\alpha)\right]$$

$$+\frac{48\pi^2}{\mathcal{N}^2}\sum_{\alpha<\beta} B^{\alpha,\beta}_{abcd}\frac{d(R_\psi^\alpha)\,d(R_\psi^\beta)}{S_2(R_\psi^\alpha)\,S_2(R_\psi^\beta)}\frac{K_\alpha K_\beta}{K_\alpha - K_\beta}$$

$$\times\left[(K_\alpha - \tfrac{1}{6}K_\alpha^2)\,H_0(\tfrac{2}{3}K_\alpha) - (K_\beta - \tfrac{1}{6}K_\beta^2)\,H_0(\tfrac{2}{3}K_\beta)\right]$$

Abelian: $K_\alpha \to 15/2$ in the UV forces $\lambda \sim -\exp(\mathcal{N})$

Non-Abelian: $K_\alpha \to 3$ in the UV opens an interacting FP

take e.g. $y \to 0$

Summary

- The zeros of the gauge beta functions persists when generalized to semi-simple gauge theories
- The leading $1/N_f$ gauge contribution share the same structure as evident from the appearance of $H_0(\frac{2}{3}K_\alpha)$
- If the non-Abelian gauge couplings reach FPs, then the Yukawa and quartic coupling can become safe

Other than lattice computations, what would it take to convince you that the non-Abelianx FP is real?

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