Title: Constraining Asymptotic Safety using central charges

Date: Jun 06, 2018 03:45 PM

URL: http://pirsa.org/18060024

Abstract: I will present constraints from central charges and gradient flow relations on UV and IR interacting fixed points under perturbative control. It is possible to extend this methodology beyond perturbation theory for supersymmetric theories where the central charges are calculated to all orders. In this case, these constraints draw a complex map of possible RG flows, some of them compatible with Asymptotic Safety. Examples of such SUSY theories are discussed

Constraining Asymptotic Safety using central charges

Nicola Andrea Dondi

CP[3]-Origins, University of Southern Denmark

Asymptotic Safety in a Dark Universe

Perimeter Institute, 6th June 2018

CP[3] Origins

Cosmology & Particle Physics

Introduction

Constraints on perturbative fixed points

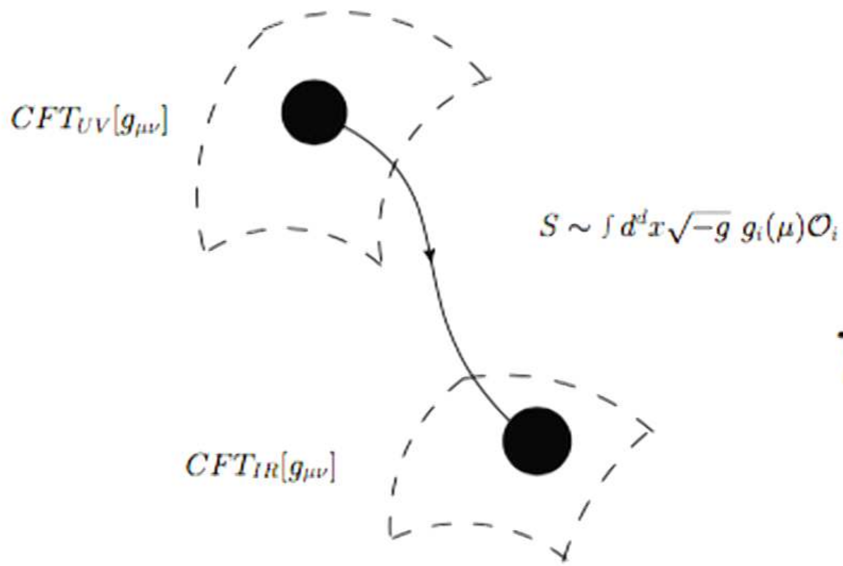
Extension to $\mathcal{N} = 1$ SUSY flows

Conclusion

Based on:

N.A.Dondi, V.Prochazka, F.Sannino [ArXiv:1712.05388]
B.Bajc, N.A.Dondi, F.Sannino [ArXiv:1709.07436]

RG Flows



$$T^{\mu}_{\mu} \supset \mathcal{A}\,\mathbb{I} + \beta_i \mathcal{O}^i$$

Weyl Anomaly: What?

- We can extend $CFT \to CFT[g_{\mu\nu}]$ such that the $CFT[g_{\mu\nu}]$ is $Diff \times Weyl$ invariant [ArXiv:1702.07079]
- A c-number anomaly is present:

$$\mathcal{A} = c\, W^2 - a\, E_4 \quad (d = 4);$$

where

$$E_4 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2,$$
$$W^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 2 R_{\mu\nu} R^{\mu\nu} + \frac{1}{3} R^2.$$

and E_4 is a topological term:

$$\chi(\mathcal{M}) = 32\pi^2 \int_{\mathcal{M}} d^4x\, E_4$$

- Quantum correction at every loop order

Weyl Anomaly: Why?

The anomaly coefficients satisfy constraints:

- c > 0 from positivity of 2-pt functions $\langle T_{\mu\nu} T_{\rho\sigma} \rangle$.
- $\frac{a}{c} \in \left[\frac{1}{3}, \frac{31}{18}\right] \xrightarrow[\mathcal{N}=1]{} \frac{a}{c} \in \left[\frac{1}{2}, \frac{3}{2}\right] \xrightarrow[\mathcal{N}=2]{} \frac{a}{c} \in \left[\frac{1}{2}, \frac{5}{4}\right] \xrightarrow[\mathcal{N}=4]{} \frac{a}{c} = 1$
- a > 0 from the above.

These are "local" constraints.

A constraint on the RG flow: the a-theorem

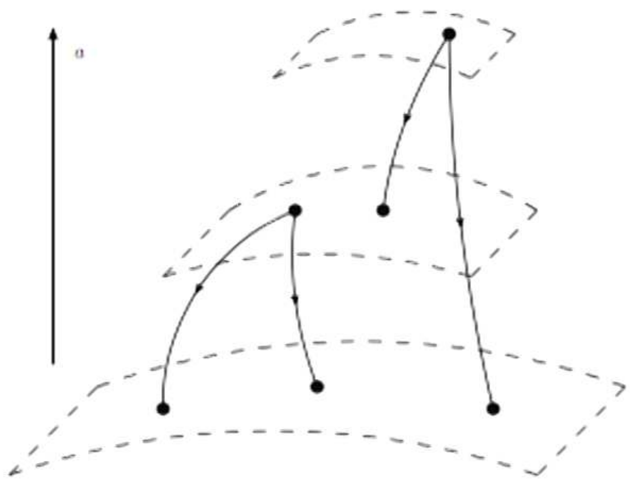
Consider a flow $CFT_{UV} \rightarrow CFT_{IR}$. It exists a quantity $a_{UV/IR}$ such that

- $a_{UV} > a_{IR}$ [ArXiv:1107.3987]
- Is possible to define a monotonic function of the energy scale $\tilde{a}(\mu)$ such that $\tilde{a}(0) = a_{IR}$, $\tilde{a}(\infty) = a_{UV}$ [ArXiv:1107.3987].
- The above $\tilde{a}$ satisfy a gradient flow equation

$$\partial_i \tilde{a} = -\chi_{ij}\beta^j + \partial_{[i,} w_{j]}\beta^j$$

χ_{ij} is a metric in coupling space
[Jack, Osborn Nucl.Phys. B343 (1990) 647-688].

Consequences of the a-theorem



- $d.o.f(UV) > d.o.f(IR)$
- Theory space is foliated
- Is there AS up there?

Weyl consistency conditions

An analogy in electromagnetism:

$$
\begin{aligned}
E_i(x) \quad &\rightarrow \quad \beta_i(g) = \chi_{ij}\beta^j \\
V(x) \quad &\rightarrow \quad \tilde{a}(g) \\
\nabla_i V = -E_i \quad &\rightarrow \quad \partial_i \tilde{a} = -\beta_i
\end{aligned}
$$

The last equation can be integrated if E_i is an irrotational field:

$$\nabla \times E = 0 \quad \rightarrow \quad \partial_i \beta_j - \partial_j \beta_i = 0$$

These are the *Weyl consistency condition*:
scheme-independent relations among β coefficients

Constraints on perturbative fixed points

We consider fixed point in very general Gauge-Yukawa theories:

$$\mathcal{L} = -\frac{1}{4g_a^2} F_{\mu\nu,a} F_a^{\mu\nu} + i\Psi_i^\dagger \bar{\sigma}^\mu D_\mu \Psi_i + \frac{1}{2} D_\mu \phi_A D^\mu \phi_A - \left(y_{ij}^A \Psi_i \Psi_j \phi_A + \text{h.c.} \right) - \frac{1}{4!} \lambda_{ABCD} \phi_A \phi_B \phi_C \phi_D \,,$$

$$c = \frac{1}{20(4\pi)^2} \left(2n_v + n_\psi + \frac{1}{6} n_\phi \right) + O(g^2, y^2)$$

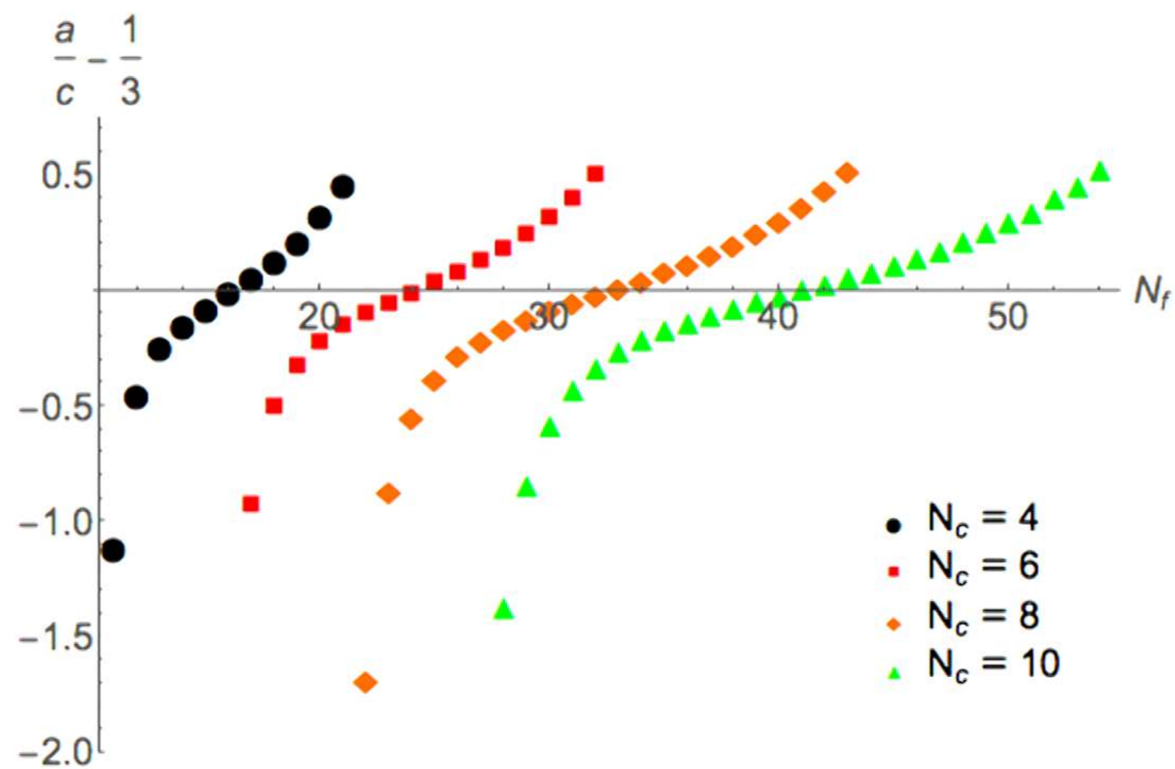
$$\tilde{a} = \frac{1}{360(4\pi)^2} \left(n_\phi + \frac{11}{2} n_\psi + 62 n_v \right) + O(g^2, y^2)$$

Example I

Fields	$[SU(N_c)]$	$SU_L(N_f)$	$SU_R(N_f)$	$U(N_s)$
ψ	$\square$	$\square$	1	1
$\tilde{\psi}$	$\overline{\square}$	1	$\square$	1
ϕ	$\square$	1	1	$\square$

$$\mathcal{L} = -v\,\mathrm{Tr}[\phi^\dagger \phi]^2 - u\,\mathrm{Tr}[(\phi^\dagger \phi)^2]$$

[arXiv:1706.06402]



a/c − 1/3
0.5
−0.5
−1.0
−1.5
−2.0
20
30
40
50
N_f
N_c = 4
N_c = 6
N_c = 8
N_c = 10

Example II

Fields	$[SU(N_c)]$	$SU(N_c - 4 + p)$	$SU(p)$
ψ	$\square$	1	$\square$
$\tilde{\psi}$	$\overline{\square}$	$\overline{\square}$	1
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1	1
M	1	$\square$	$\overline{\square}$
H	$\overline{\square}$	1	1

$$\mathcal{L}_H = y_H f_a \bar{\psi}_a A\, H + h.c.$$

$$\mathcal{L}_M = y_M[\delta_{ab} - f_a f_b]\, \bar{\psi}_a M_{bc} \psi_c + y_1 f_a f_b\, \bar{\psi}_a M_{bc} \psi_c + h.c.$$

[arXiv:1610.03130]

	$N_c = 5,\ p = 26$	$N_c = 6,\ p = 30$	$N_c = 8,\ p = 39$
α_g^*	1.41	0.0325	0.0481
α_H^*	6.12	0.151	0.241
α_M^*	0.652	0.0155	0.0233
α_1^*	0.312	0.00652	0.00801
θ_{UV}	-0.0428	-0.00585	-0.00602
a	-1311	14.7	21.6
c	710	47.5	126
a/c	-1.84	0.296	0.171
Δa	-1321	-0.537	-4.27

SUSY Weyl anomalies

- An *R-symmetry* is an automorphism of the SUSY algebra
- In $\mathcal{N} = 1$ we can have a $U(1)_R$ that commutes with bosonic generators and acts on the fermionic ones as

$$[Q_\alpha, R] = Q_\alpha, \quad [\bar{Q}_\alpha, R] = -\bar{Q}_{\dot{\alpha}}$$

- Is always defined up to a non-R symmetry:

$$R \sim R + \sum_i s_i F^i$$

where F_i are conserved charges associated to global $U(1)$ symmetries.

- since $[D, R] = 0$ they can be simultaneously diagonalised, and their eigenvalues are proportional:

$$R(\mathcal{O}) = \frac{2}{3}\Delta(\mathcal{O}) = \frac{2}{3}\left[1 + \frac{1}{2}\gamma(\mathcal{O})\right]$$

  Every gauge invariant primary has to satisfy $R(\mathcal{O}) \geq \frac{2}{3}$

- $\partial_\mu j^\mu_R$ is part of a supermultiplet together with T^μ_μ
- R is not anomalous $\implies$ SCFT

$$\mathcal{L} \supset \int d^2\theta \frac{1}{2g^2} \mathrm{Tr}\mathcal{W}^\alpha \mathcal{W}_\alpha + \int d^2\theta h W[\Phi_i]$$

The running of the couplings is determined by

$$\beta_g = -\frac{3g^3}{(4\pi)^3} f(g^2)\left[T(\mathcal{G}) + \sum_i T(r_i)(R_i - 1)\right]$$

$$\beta_h = \frac{3}{2} h\left[R_W - 2\right]$$

Central charges:

$$a = 2d(\mathcal{G}) + \sum_i d(r_i)[3(R_i-1)^3 - (R_i - 1)]$$

$$c = 4d(\mathcal{G}) + \sum_i d(r_i)[9(R_i-1)^3 - 5(R_i - 1)]$$

[arXiv:hep-th/9708042, arXiv:hep-th/9711035]

Finding a fixed point becomes an algebraic problem, solve n_β constraints in R_i, $i = 1..n$ unknown.

- $n_\beta > n \to$ only gaussian
- $n_\beta = n \to$ interacting FP candidate
- $n_\beta < n \to$ a-maximisation [arXiv:hep-th/0304128]

Next: Apply constraints on the putative FP!

SQCD + 2 Adjoints

Fields	$[SU(N_c)]$	$SU_L(N_f)$	$SU_R(N_f)$	$U_V(1)$
W_α	Adj	1	1	0
Q	$\square$	$\overline{\square}$	1	1
$\widetilde{Q}$	$\overline{\square}$	1	$\square$	-1
X	Adj	1	1	0
Y	Adj	1	1	0

[ArXiv:hep-ph/0011382]

General superpotential:

$$W \sim h_1 \,\mathrm{Tr}[(\tilde{Q}Q)^n X^{k_1} Y^{k_2}] + h_2 \,\mathrm{Tr}[(\tilde{Q}Q)^m X^{l_1} Y^{l_2}]\,.$$

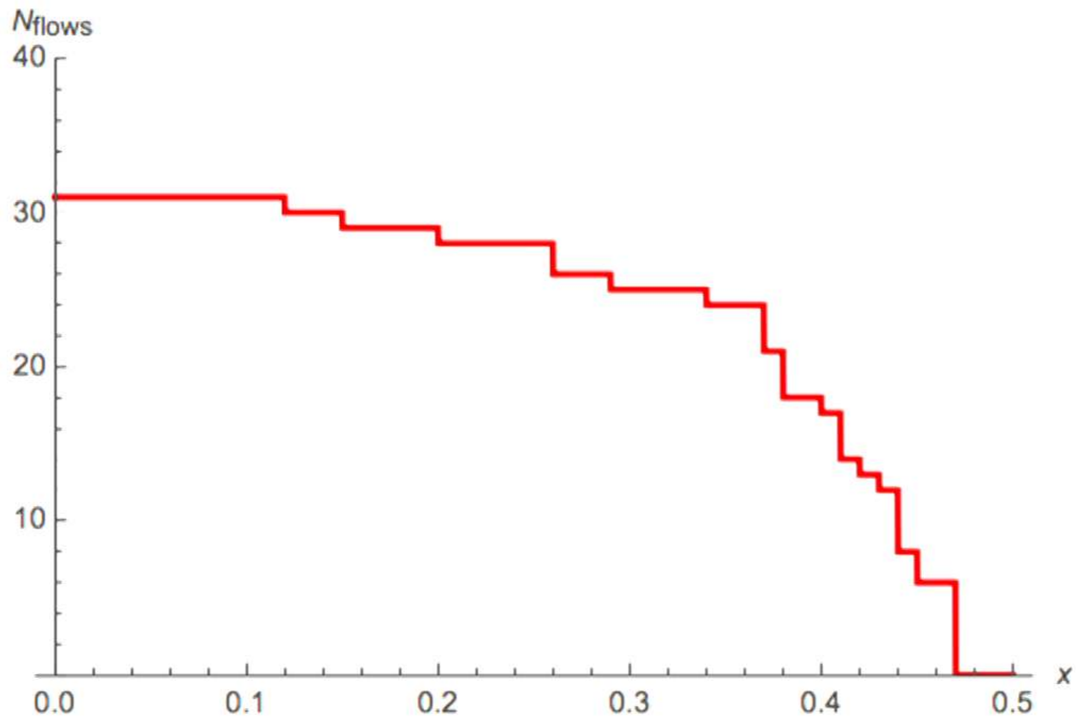
β function constraints:

$$\begin{aligned} &2nR_Q + k_1 R_X + k_2 R_Y = 2\,, \\ &2mR_Q + l_1 R_X + l_2 R_Y = 2\,, \qquad\qquad (x = N_c/N_f) \\ &x(R_X + R_Y - 1) + (R_Q - 1) = 0\,. \end{aligned}$$

No violation of unitarity bounds:

$$n = 0 \wedge 2 \leq k_1 + k_2 \leq 6 \quad \vee \quad k_1 = k_2 = 0 \wedge 1 \leq n \leq 3.$$

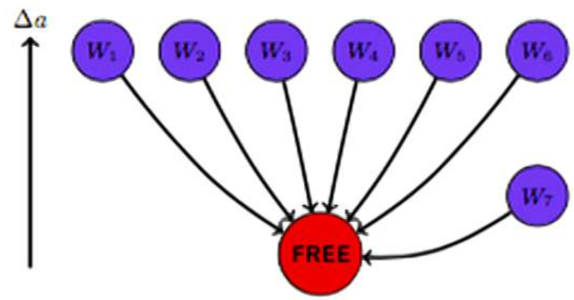
and similar for m, l_1, l_2



CW starts at $x = \frac{1}{3 - n_{Adj}}$

At $x = 0.46$ we have the following possibilities:

$$W_1 \sim \mathrm{Tr}[X^6] + \mathrm{Tr}[\tilde{Q}QX^4] ,$$
$$W_2 \sim \mathrm{Tr}[X^6] + \mathrm{Tr}[(\tilde{Q}Q)^2X^2] ,$$
$$W_3 \sim \mathrm{Tr}[X^6] + \mathrm{Tr}[(\tilde{Q}Q)^3] ,$$
$$W_4 \sim \mathrm{Tr}[\tilde{Q}QX^4] + \mathrm{Tr}[(\tilde{Q}Q)^2X^2] ,$$
$$W_5 \sim \mathrm{Tr}[\tilde{Q}QX^4] + \mathrm{Tr}[(\tilde{Q}Q)^3] ,$$
$$W_6 \sim \mathrm{Tr}[(\tilde{Q}Q)^2X^2] + \mathrm{Tr}[(\tilde{Q}Q)^3] ,$$
$$W_7 \sim \mathrm{Tr}[X^5] + \mathrm{Tr}[(\tilde{Q}Q)^3] .$$



Conclusions

- Within perturbation theory, these can be regarded as consistency checks.
- In SUSY, they determine the flow structure non-perturbatively
- These constraints are necessary but not sufficient for the existence of a FP

Can we implement SUSY breaking so to retain a UV fixed point and flow to SM?