Title: Progress in constructing an Asymptotically safe Standard Model

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Abstract: I outline a configuration in which the Standard Model can be embedded into an asymptotically safe gauge-Yukawa theory. The model can be though of as a minimal UV completion of the SM without gravity. I also discuss the remaining issues that need to be addressed for the scheme to be phenomenologically viable, and outline the different energy scales and possible signatures.

Towards an Asymptotically Safe SM

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w/ Francesco Sannino,
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Phys.Rev. D96 (2017) no.5, 056028; arXiv:1707.0663

Outline

- Motivation and recap of idea; the hierarchy versus triviality problem
- Asymptotically safe 4D QFTs
- Adding relevant operators
- Radiative symmetry breaking
- Towards an ASSM

Motivation: two problems to do with scalars

Hierarchy problem

Triviality problem

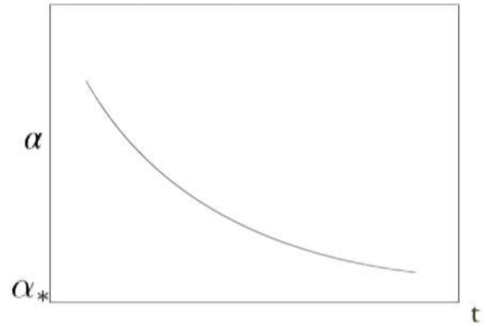
Hints from QCD

QCD is (unlike SUSY) a UV complete theory. Why?

1. ***There is (unlike string theory) no hierarchy problem:*** masses protected by chiral symmetry

2. ***There is (like string theory) no triviality problem:*** QCD is **asymptotically free**

$$\partial_t \alpha = -B\alpha^2$$



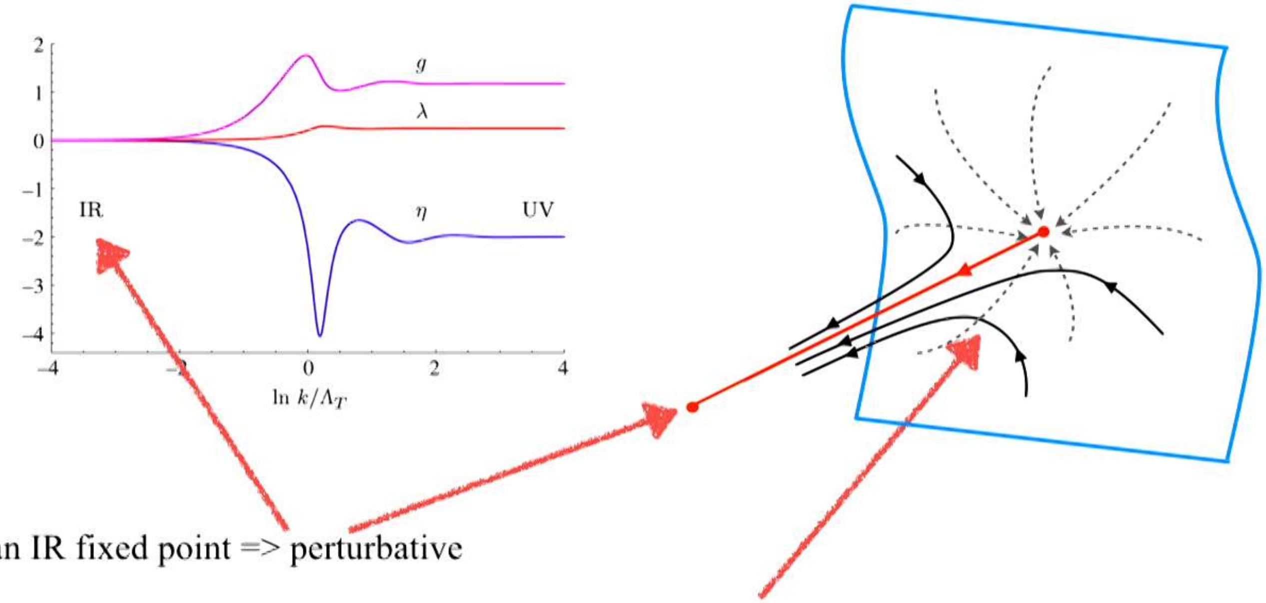
$$\alpha_* = 0$$

Note the philosophy of QCD: we do not mind running masses because they do not upset the Gaussian UV fixed point. We simply measure them and let them run. Or to put it another way: they are "relevant" operators that are effectively zero in the UV. They do not need to run to zero in the UV! (We also don't care too much about couplings blowing up in the IR.)

Gastmans et al '78
Weinberg '79
Peskin '80
Gawedski, Kupiainen '85
Kawai et al '90
de Calan et al '91
Reuter '96
Litim '03

The Basic idea

Weinberg used this as a basis for his proposal of UV complete theories



Gaussian IR fixed point => perturbative

Interacting UV fixed point => finite anomalous dimensions

In a field theory replace $1/\epsilon$ with $1/\gamma$ => divergences of marginal operators (which affect the fixed point) can be cured

Categorise the possible content of a theory as follows:

Irrelevant operators: would disrupt the UV fixed point - therefore asymptotically safe theories have to emanate precisely from UV fixed point where they are assumed zero (exactly renormalizable trajectory)

Marginal operators: can be involved in determining the UV fixed point where they become *exactly* marginal. Or can be marginally relevant (asymptotically free).

Relevant operators: become "irrelevant" in the UV but may determine the IR fixed point.

Dangerously irrelevant operators: grow in both the UV and IR (common in e.g. SUSY)

Harmless relevant operators: shrink in the UV and IR (not common/possible?)

Note relevant or marginally relevant operators still have "infinities" at the FP - just as quark masses, they still run at the FP just like any other relevant operator: but being relevant they do not affect the FP. (And by definition they become less important the higher you go in energy.)

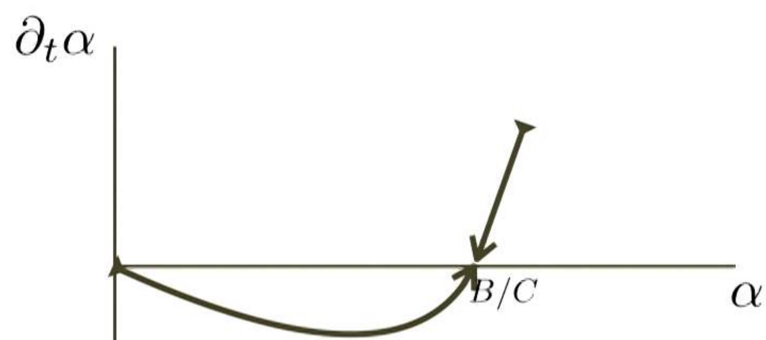
U.V. v. I.R. F.P.

Caswell-Banks-Zaks fixed point:

Take QCD with $SU(N_C)$ and N_F fermions but very large numbers of colours+flavours

$$\partial_t \alpha = -B\alpha^2 + C\alpha^3 \qquad B \propto \epsilon = \frac{N_F}{N_C} - \frac{11}{2}$$

Turns out C>0, B>0: theory has *stable* IR fixed point at α = B/C and *unstable* one in UV α = 0



Note perturbativity: $\implies B \ll C$

requires many fields (Veneziano limit) with $N_F \approx 11 N_C/2$

Familiar from Seiberg duality and weakly coupled $N_F \lesssim 3N_C$ $\mathcal{N}=1$ supersymmetry

Real situation requires several couplings to realise

Litim & Sannino '14

Need to add **scalars** and **Yukawa couplings:**

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\, F^{\mu\nu}F_{\mu\nu} + \mathrm{Tr}\left(\overline{Q}\, i\not{D}\, Q\right) + y\,\mathrm{Tr}\left(\overline{Q}\, H\, Q\right) + \mathrm{Tr}\,(\partial_\mu H^\dagger\, \partial^\mu H)$$
$$-u\,\mathrm{Tr}\,[(H^\dagger H)^2] - v\,(\mathrm{Tr}\,[H^\dagger H])^2\,,$$

H is an $N_F \times N_F$ scalar

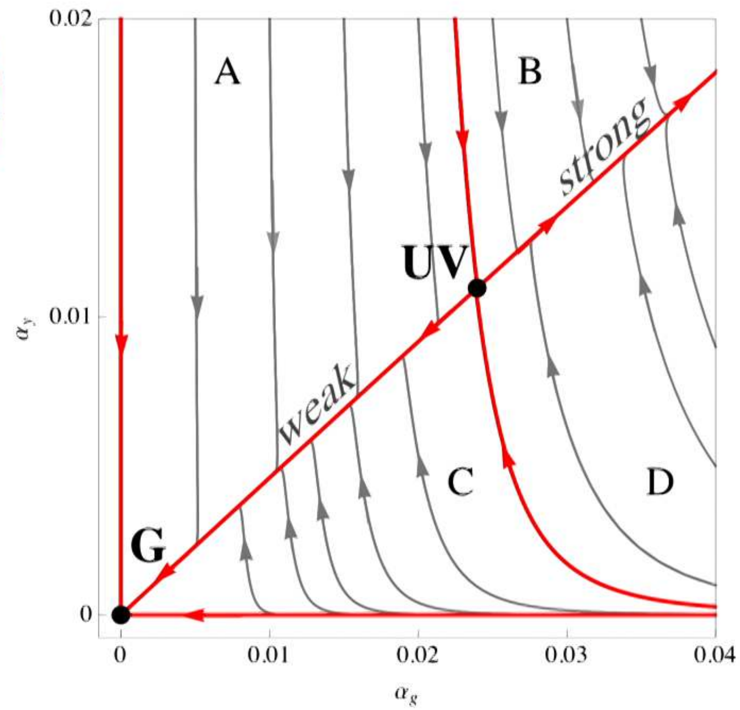
Initially have $U(N_F)_L \times U(N_F)_R$ flavour symmetry

Effect of Yukawa ….

$$\left(\alpha_g = \frac{g^2\, N_C}{(4\pi)^2}\,, \quad \alpha_y = \frac{y^2\, N_C}{(4\pi)^2} \right)$$

$$\beta_g = \alpha_g^2 \left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2}+\epsilon\right)^2 \alpha_y \right]$$

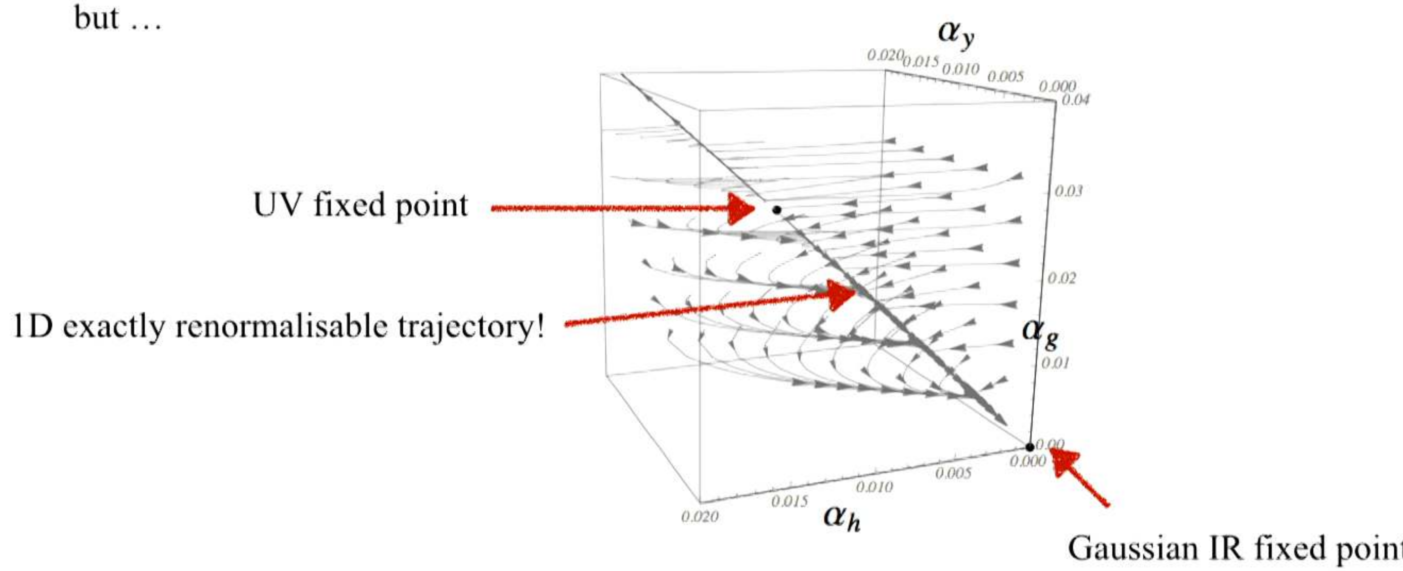
$$\beta_y = \alpha_y \left[(13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g \right]$$



Four ’t Hooft-like couplings - flow could in principle be four dimensional

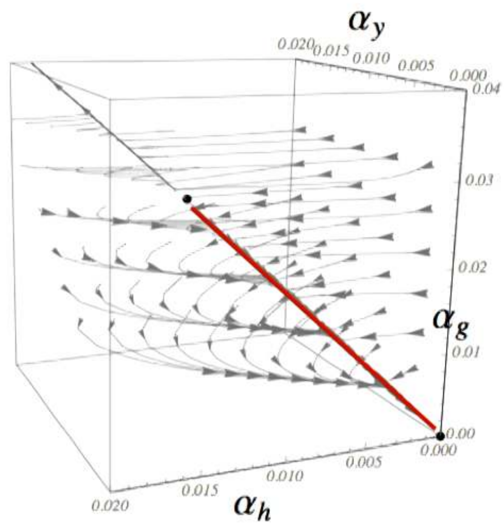
$$\alpha_g = \frac{g^2\, N_C}{(4\pi)^2}\,, \quad \alpha_y = \frac{y^2\, N_C}{(4\pi)^2}\,, \quad \alpha_h = \frac{u\, N_F}{(4\pi)^2}\,, \quad \alpha_v = \frac{v\, N_F^2}{(4\pi)^2}$$

but …



Along the critical-curve/exact-trajectory can parameterise the flow in terms of $\alpha_g(t)$

$$\alpha_y(t) = \frac{6}{13}\alpha_g(t) \ ,$$
$$\alpha_h(t) = 3\frac{\sqrt{23}-1}{26}\alpha_g(t) \ ,$$
$$\alpha_v(t) = \frac{3\sqrt{20+6\sqrt{23}}-6\sqrt{23}}{26}\alpha_g(t) \ ,$$



At the fixed point it is arbitrarily weakly coupled, $\alpha_g^* = \ 0.4561\,\epsilon$, where $\epsilon = \frac{N_F}{N_C} - \frac{11}{2}$

Solve Callan Symanzik eqn for them as usual =>

- **warm-up**; first restrict ourselves to the diagonal direction where mass-squared term looks like the following operator:

$$V \supset \frac{m_\phi^2}{4N_F} \left(\mathrm{Tr}(H + H^\dagger)\right)^2$$

$$\bar{\beta} = \frac{d\lambda^{(n)}(t)}{dt} = \frac{\partial \lambda_{eff}^{(n)}}{\partial t} + n\bar{\gamma}\lambda^{(n)}$$

Anomalous dimension of fields

t-dependence in one-loop calculation of V

Critique of the simplest example...

- **Purely** multiplicative: Hence the mass-squared has to be negative along the whole trajectory
- **We cheated:** in the sense that we ignored all the orthogonal directions!! These also get contributions at one-loop even though their masses were zero at tree-level

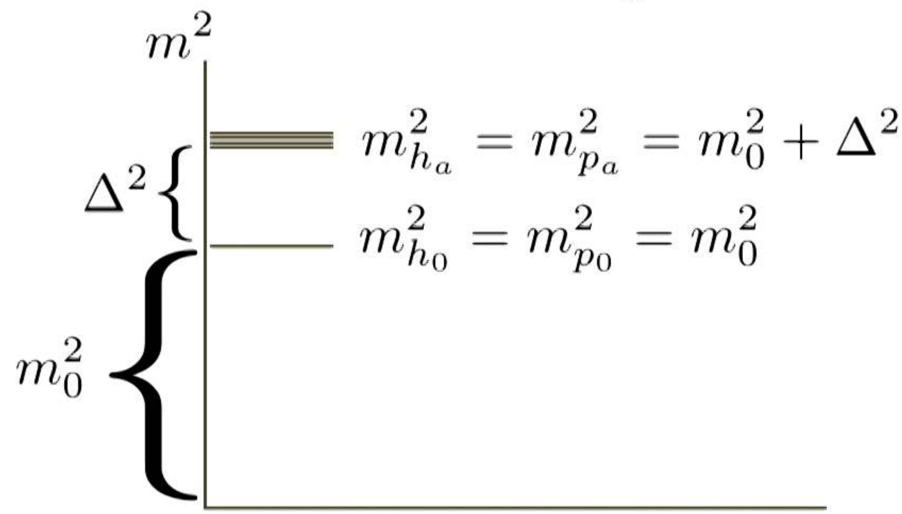
In order to address both these, organise the discussion in terms of the $U(N_F) \times U(N_F)$ **flavour symmetry that we break with the mass-squareds:**

$$H = \frac{(h_0 + ip_0)}{\sqrt{2N_F}} \mathbb{1}_{N_F \times N_F} + (h_a + ip_a)T_a$$

Non-trivial simple example...

Seek to add a set of mass-squared operators whose flavour structure is closed under RG: simple example

$$V^{(2)}_{class} = m_0^2 \mathrm{Tr}(H^\dagger H) + 2\Delta^2 \sum_a \mathrm{Tr}(T_a H^\dagger)\mathrm{Tr}(T_a H)$$



Following the same procedure and after some work find the following answer in terms of two RG invariants (one for each independent bit of the flavour structure) (where υ=(1-1/NF^2)):

$$m_0^2 \;=\; \tilde{m}_*^2 \left(\frac{\alpha_g^*}{\alpha_g} - 1\right)^{-\frac{3f_{m_0}}{4\epsilon}} - \Delta_*^2\,\nu \left(\frac{\alpha_g^*}{\alpha_g} - 1\right)^{-\frac{3f_\Delta}{4\epsilon}},$$

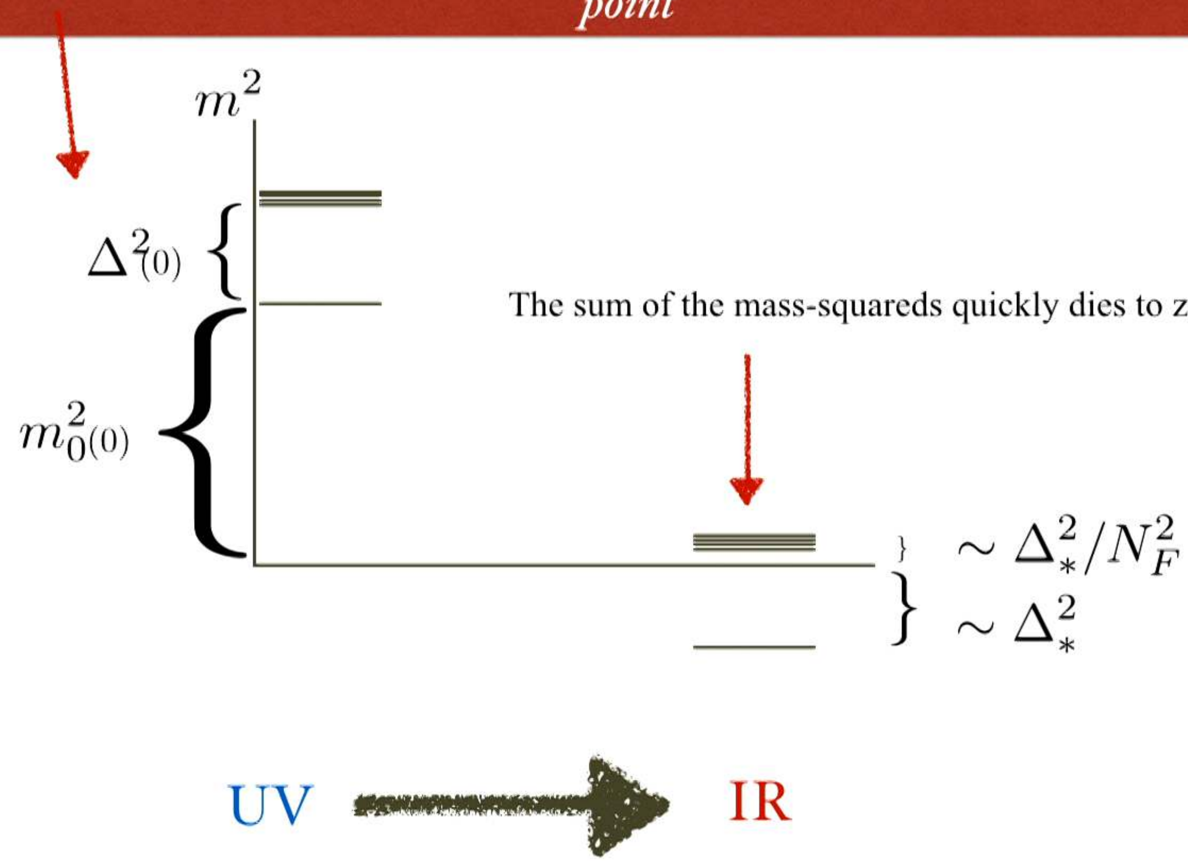
$$m_{a=1\ldots N_F^2-1}^2 \;=\; \tilde{m}_*^2 \left(\frac{\alpha_g^*}{\alpha_g} - 1\right)^{-\frac{3f_{m_0}}{4\epsilon}} + \Delta_*^2\,(1-\nu) \left(\frac{\alpha_g^*}{\alpha_g} - 1\right)^{-\frac{3f_\Delta}{4\epsilon}}$$

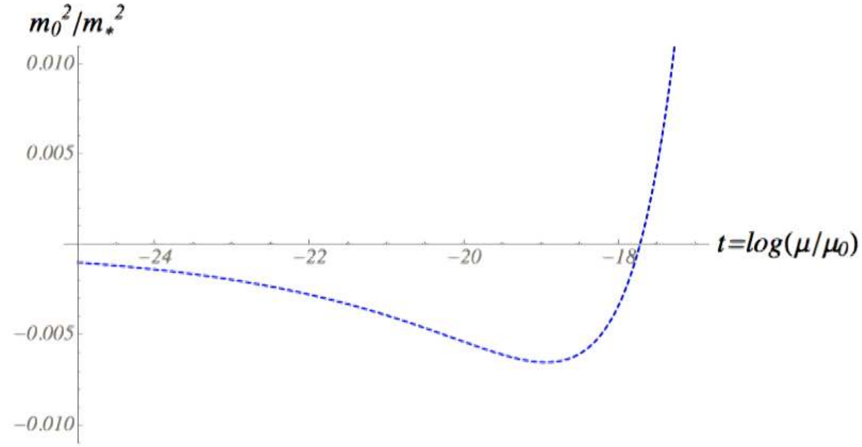
$$f_{m_0} > f_\Delta$$

Dies away quickly *in the IR*

Dies away slowly *in the IR*

Starting values get relatively closer in UV (note the masses are all shrinking in absolute terms in the IR) - full flavour symmetry restored precisely at fixed point





$$\alpha_{g,min} \xrightarrow{\epsilon \to 0} \frac{1}{2}\alpha_g^*$$

$$m_{0,min}^2 \sim -\tilde{m}_*^2$$

Note entire flow defined in terms of a set of flavour violating RG invariants.
Technically natural.

- **To embed the SM - focus on breaking SU(Nc) to SU(3) colour with Ns new scalars …**

c.f. Pelaggi, Sannino Strumia Vigiani

	$SU(N_C)$	$SU(N_F)_L \supset SU(2)_L \times SU(n_g)$	$SU(N_F)_R \supset SU(2)_R \times SU(n_g)$	$SU(N_S)_R \supset SU(2)_R$
Q_a^i	$\square$	$\square \supset (\square, \square)$	1	1
$\tilde{Q}_i^a$	$\tilde{\square}$	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	1
H_i^j	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	$\square \supset (\square, \square)$	1
$\tilde{S}_{j=1..N_S}$	$\tilde{\square}$	1	1	$\tilde{\square}$

The new scalars give a similar UVFP …

Extension of Pati-Salam (XPS) - breaks to SU(3) if we choose $N_S = N_C - 2$

$$\frac{N_S}{N_C} \to 1\,; \quad \frac{N_F}{N_C} \to \frac{21}{4} + \epsilon$$

$$\tilde{S} = \left(\overbrace{\begin{matrix} \begin{pmatrix} \tilde{d}^c \\ \tilde{u}^c \end{pmatrix} & \begin{pmatrix} \tilde{e}^c \\ \tilde{\nu}^c \end{pmatrix} & \begin{pmatrix} \tilde{\phi}_{-\frac{1}{2}} \\ \tilde{\phi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\phi}_{-\frac{1}{2}} \\ \tilde{\phi}_{\frac{1}{2}} \end{pmatrix} \\ \tilde{T}_{-\frac{1}{6}} & \tilde{\phi}_{\frac{1}{2}} & \tilde{\phi}_0 & \cdots & \tilde{\phi}_0 \\ \vdots & \vdots & \vdots & & \vdots \\ \tilde{T}_{-\frac{1}{6}} & \tilde{\phi}_{\frac{1}{2}} & \tilde{\phi}_0 & \cdots & \tilde{\phi}_0 \end{matrix}}^{N_C} \right) \Bigg\} N_S = N_C - 2$$

- **To embed the SM - focus on breaking SU(Nc) to SU(3) colour with Ns new scalars …**

c.f. Pelaggi, Sannino Strumia Vigiani

Somewhat complementary to Bond, Litim; Bond, Hiller, Kowalska, Litim

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Q^i_a	$\square$	$\square \supset (\square, \square)$	1	1
$\tilde{Q}^a_i$	$\tilde{\square}$	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	1
H^j_i	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	$\square \supset (\square, \square)$	1
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$$\langle \tilde{S} \rangle = \tilde{V} \left(\overbrace{\begin{matrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & 1 & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{matrix}}^{N_C} \right) \Bigg\} N_S = N_C - 2$$

- **Explicit embedding looks like P-S**

$$Q = \left(\overbrace{\begin{array}{cccc} q_1 & \ell_1 & \cdots \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} \cdots \\ q_2 & \ell_2 & \cdots \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} \cdots \\ q_3 & \ell_3 & \cdots \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} \cdots \\ \vdots & \vdots & \ddots \end{array}}^{N_C} \right) \Bigg\} N_F \;\; ; \quad \tilde{Q} = \begin{pmatrix} \begin{pmatrix} u^c \\ d^c \end{pmatrix} & \begin{pmatrix} \nu^c_e \\ e^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \begin{pmatrix} s^c \\ c^c \end{pmatrix} & \begin{pmatrix} \nu^c_\mu \\ \mu^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \begin{pmatrix} b^c \\ t^c \end{pmatrix} & \begin{pmatrix} \nu^c_\tau \\ \tau^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

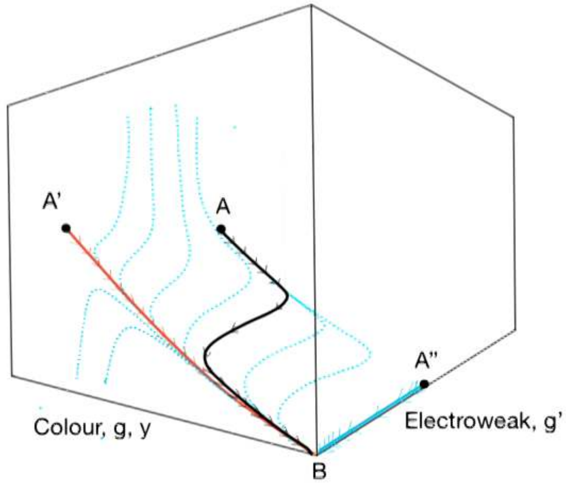
$$H = \begin{pmatrix} \begin{pmatrix} h^0_u & h^-_d \\ h^+_u & h^0_d \end{pmatrix}_{11} & \begin{pmatrix} h^0_u & h^-_d \\ h^+_u & h^0_d \end{pmatrix}_{12} & \begin{pmatrix} h^0_u & h^-_d \\ h^+_u & h^0_d \end{pmatrix}_{13} & \cdots \\ \begin{pmatrix} h^0_u & h^-_d \\ h^+_u & h^0_d \end{pmatrix}_{21} & \begin{pmatrix} h^0_u & h^-_d \\ h^+_u & h^0_d \end{pmatrix}_{22} & \begin{pmatrix} h^0_u & h^-_d \\ h^+_u & h^0_d \end{pmatrix}_{23} & \cdots \\ \begin{pmatrix} h^0_u & h^-_d \\ h^+_u & h^0_d \end{pmatrix}_{31} & \begin{pmatrix} h^0_u & h^-_d \\ h^+_u & h^0_d \end{pmatrix}_{32} & \begin{pmatrix} h^0_u & h^-_d \\ h^+_u & h^0_d \end{pmatrix}_{33} & \cdots \\ \vdots & \vdots & \vdots & H_0 \end{pmatrix}$$

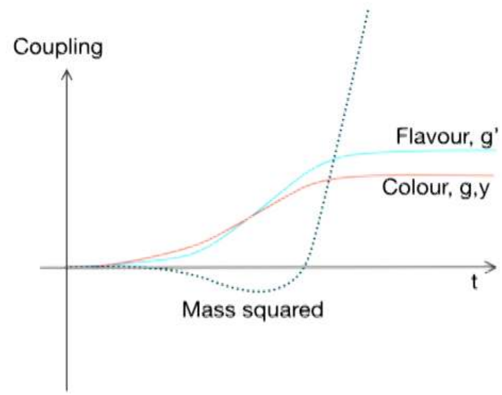
- Assignment implies 9 pairs of Higgses one for each Yukawa coupling

- **Extend Lagrangian …**

$$\mathcal{L}_{UVFP} = \mathcal{L}_{YM} + \mathcal{L}_{KE} + \frac{y}{\sqrt{2}}\mathrm{Tr}\left[\left(Q_L^\dagger H \cdot Q_R\right)\right] - u_1 \mathrm{Tr}\left[H^\dagger H\right]^2 - u_2 \mathrm{Tr}\left[H^\dagger H\, H^\dagger H\right]$$
$$- w_1\, \mathrm{Tr}[\tilde{Q}^\dagger \cdot \tilde{Q}]^2 - w_2\, \mathrm{Tr}[\tilde{Q}^\dagger \cdot \tilde{Q}\, \tilde{Q}^\dagger \cdot \tilde{Q}]\ ,$$

- **Want to do …**





- **A large number of extraneous states are already removed by the VEVs but what about the others?**

$$Q = \left(\overbrace{\begin{array}{ccccc} q_1 & \ell_1 & \cdots \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} \cdots \\ q_2 & \ell_2 & \cdots \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} \cdots \\ q_3 & \ell_3 & \cdots \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} \cdots \\ \vdots & \vdots & \ddots \end{array}}^{N_C}\right) \Bigg\} N_F \;\; ; \quad \tilde{Q} = \begin{pmatrix} \begin{pmatrix} u^c \\ d^c \end{pmatrix} & \begin{pmatrix} \nu_e^c \\ e^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \begin{pmatrix} s^c \\ c^c \end{pmatrix} & \begin{pmatrix} \nu_\mu^c \\ \mu^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \begin{pmatrix} b^c \\ t^c \end{pmatrix} & \begin{pmatrix} \nu_\tau^c \\ \tau^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$$H = \begin{pmatrix} \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{11} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{12} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{13} & \cdots \\ \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{21} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{22} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{23} & \cdots \\ \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{31} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{32} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{33} & \cdots \\ \vdots & \vdots & \vdots & H_0 \end{pmatrix} \qquad \tilde{S} = \left(\overbrace{\begin{array}{ccccc} \begin{pmatrix} \tilde{d}^c \\ \tilde{u}^c \end{pmatrix} & \begin{pmatrix} \tilde{e}^c \\ \tilde{\nu}^c \end{pmatrix} & \begin{pmatrix} \tilde{\phi}_{-\frac{1}{2}} \\ \tilde{\phi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\phi}_{-\frac{1}{2}} \\ \tilde{\phi}_{\frac{1}{2}} \end{pmatrix} \\ \tilde{T}_{-\frac{1}{6}} & \tilde{\phi}_{\frac{1}{2}} & \tilde{\phi}_0 & \cdots & \tilde{\phi}_0 \\ \vdots & \vdots & \vdots & & \vdots \\ \tilde{T}_{-\frac{1}{6}} & \tilde{\phi}_{\frac{1}{2}} & \tilde{\phi}_0 & \cdots & \tilde{\phi}_0 \end{array}}^{N_C}\right) \Bigg\} N_S = N_C - 2$$

- **Indeed a VEV for H0 removes all vector-like non-SU(2) charged fermions. But what about the extra SU(2)L and SU(2)R multiplets?**

- Chiral symmetry tells us we need to add $N_q = N_C - 4$ compensating fermions, along with Yukawa couplings to form Dirac masses leaving only the light PS states:

	$SU(N_C)$	$SU(N_F)_L \supset SU(2)_L \times SU(n_g)$	$SU(N_F)_R \supset SU(2)_R \times SU(n_g)$	$SU(N_S)_R \supset SU(2)_R$
Q^i_a	$\square$	$\square \supset (\square, \square)$	1	1
$\tilde{Q}^a_i$	$\tilde{\square}$	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	1
H^j_i	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	$\square \supset (\square, \square)$	1
$\tilde{S}_{j=1..N_S}$	$\tilde{\square}$	1	1	$\tilde{\square}$

		$SU(2)_L \times SU(n_g)$	$SU(2)_R \times SU(n_g)$	
$\tilde{q}_{j=1..N_q}$	1	$(\tilde{\square}, \tilde{\square})$	1	1
$q_{j=1..N_q}$	1	1	$(\square, \square)$	1

$$q = \left(\overbrace{\begin{array}{ccc} \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \end{array}}^{N_q = N_C - 4}\right)\Bigg\} 2 \times n_g = 6 \ ; \quad \tilde{q} = \left(\begin{array}{ccc} \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \end{array}\right)$$

Summary

- Considered perturbative asymptotically safe QFTs (gauge-Yukawa theories that require scalars)
- UV fixed points do not prefer *any* mass-squared - the entire flow of each relevant operator is described by a single RG-invariant (multiplicative renormalisation)
- Deviation from zero = breaking of scale invariance, c.f. non-zero quark masses = breaking of chiral symmetry
- Positive mass-squareds can be driven negative in the IR, akin to radiative symmetry breaking in MSSM => radiative symmetry breaking
- A minimal embedding of the SM within this set-up looks straightforward - work in progress
- Overall now has the "feel of" other RG systems with large numbers of degrees of freedom in the UV such as duality cascade. Gravity duals?

Recap of the idea

- ***The SM is "classically" scale invariant*** - tree level Lagrangian has no mass
- Coleman Weinberg mechanism leads to spontaneous breaking at a scale because the scale invariance is anomalous. (Huge amount of interest since 2012)
- Compute effective potential and renormalize it

$$V_{eff} = \frac{\lambda}{4!}|\phi|^4 + \frac{3g^4}{64\pi^2}|\phi|^4\left(\log\frac{|\phi|}{\mu} - \frac{25}{6}\right) \qquad \frac{\partial^2 V}{\partial\phi^2}|_{\phi=0} = 0 \quad \frac{\partial^4 V}{\partial\phi^4}|_{\phi=\mu} = \lambda$$

We imposed by hand no generation of mass terms!

Minimization leads to dimensional transmutation

$$\langle\phi\rangle = \mu e^{\frac{11}{6} - \frac{4\pi^2\lambda}{9g^4}}$$

- ***Heuristically unlikely to work from a UV fixed point:*** CW is all about ***IR*** scale invariance where $\phi=0$ - which is why it is a strange starting point for solving the problems of large UV thresholds.
- Proof (already shown numerically by Litim, Mojaza, Sannino but can do it analytically): for example choose the real trace direction …

$$H = \frac{\phi}{\sqrt{2N_F}} \mathbb{1}_{N_F \times N_F} \implies V_{class}^{(4)} = \frac{4\pi^2}{N_F^2} (\alpha_h + \alpha_v)\, \phi^4$$

- Effectively $$\lambda = 32\pi^2 \frac{3}{N_F^2} (\alpha_h + \alpha_v)$$
- Also define $$\kappa = 32\pi^2 \frac{1}{N_F^2} (3\alpha_h + \alpha_v)$$

Corrections all of order $\alpha\lambda$, so no perturbative minimum without a mass-squared for ϕ

$$V = \frac{\lambda}{4!}\phi^4 + \frac{m_\phi^2}{2}\phi^2 + \frac{1}{64\pi^2}\left(m_\phi^2 + \frac{\lambda}{2}\phi^2\right)^2 \left(\log \frac{m_\phi^2 + \frac{\lambda}{2}\phi^2}{\mu^2} - \frac{3}{2}\right)$$
$$- \frac{(4\pi)^2}{4N_F N_C}\alpha_y^2\phi^4 \left(\log \frac{(4\pi)^2\alpha_y\phi^2}{\sqrt{N_F N_C}\mu^2} - \frac{3}{2}\right)$$
$$+ \frac{(N_F^2 - 1)}{64\pi^2}\left(\frac{\kappa}{2}\phi^2\right)^2 \left(\log \frac{\frac{\kappa}{2}\phi^2}{\mu^2} - \frac{3}{2}\right) + \frac{N_F^2}{64\pi^2}\left(\frac{\lambda}{6}\phi^2\right)^2 \left(\log \frac{\frac{\lambda}{6}\phi^2}{\mu^2} - \frac{3}{2}\right)$$

- ***Heuristically unlikely to work from a UV fixed point:*** CW is all about ***IR*** scale invariance where $\phi=0$ - which is why it is a strange starting point for solving the problems of large UV thresholds.
- Proof (already shown numerically by Litim, Mojaza, Sannino but can do it analytically): for example choose the real trace direction …

$$H = \frac{\phi}{\sqrt{2N_F}} \mathbb{1}_{N_F\times N_F} \quad \Longrightarrow \quad V^{(4)}_{class} = \frac{4\pi^2}{N_F^2}(\alpha_h + \alpha_v)\,\phi^4$$

- Chiral symmetry tells us we need to add $N_q = N_C - 4$ compensating fermions, along with Yukawa couplings to form Dirac masses leaving only the light PS states:

	$SU(N_C)$	$SU(N_F)_L \supset SU(2)_L \times SU(n_g)$	$SU(N_F)_R \supset SU(2)_R \times SU(n_g)$	$SU(N_S)_R \supset SU(2)_R$
Q^i_a	$\square$	$\square \supset (\square, \square)$	1	1
$\tilde{Q}^a_i$	$\tilde{\square}$	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	1
H^j_i	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	$\square \supset (\square, \square)$	1
$\tilde{S}_{j=1..N_S}$	$\tilde{\square}$	1	1	$\tilde{\square}$

		$SU(2)_L \times SU(n_g)$	$SU(2)_R \times SU(n_g)$	
$\tilde{q}_{j=1..N_q}$	1	$(\tilde{\square}, \tilde{\square})$	1	1
$q_{j=1..N_q}$	1	1	$(\square, \square)$	1

$$q = \left(\overbrace{\begin{matrix} \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \end{matrix}}^{N_q = N_C - 4} \right) \Bigg\} 2 \times n_g = 6 \; ; \quad \tilde{q} = \begin{pmatrix} \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \end{pmatrix}$$

- **To embed the SM - focus on breaking SU(Nc) to SU(3) colour with Ns new scalars …**

c.f. Pelaggi, Sannino Strumia Vigiani

Somewhat complementary to Bond, Litim; Bond, Hiller, Kowalska, Litim

	$SU(N_C)$	$SU(N_F)_L \supset SU(2)_L \times SU(n_g)$	$SU(N_F)_R \supset SU(2)_R \times SU(n_g)$	$SU(N_S)_R \supset SU(2)_R$
Q_a^i	$\square$	$\square \supset (\square, \square)$	1	1
$\tilde{Q}_i^a$	$\tilde{\square}$	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	1
H_i^j	1	$\tilde{\square} \supset (\tilde{\square}, \tilde{\square})$	$\square \supset (\square, \square)$	1
$\tilde{S}_{j=1..N_S}$	$\tilde{\square}$	1	1	$\tilde{\square}$

The new scalars allow a similar UVFP …

Extension of Pati-Salam (XPS) - breaks to SU(3) if we choose $N_S = N_C - 2$

$$\frac{N_S}{N_C} \to 1\,; \quad \frac{N_F}{N_C} \to \frac{21}{4} + \epsilon$$

$$\langle \tilde{S} \rangle = \tilde{V} \left(\overbrace{\begin{matrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & 1 & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{matrix}}^{N_C} \right) \Bigg\} N_S = N_C - 2$$

- Chiral symmetry tells us we need to add $N_q = N_C - 4$ compensating fermions, along with Yukawa couplings to form Dirac masses leaving only the light PS states:

	$SU(N_C)$	$SU(N_F)_L \supset SU(2)_L \times SU(n_g)$	$SU(N_F)_R \supset SU(2)_R \times SU(n_g)$	$SU(N_S)_R \supset SU(2)_R$
Q^i_a	$\Box$	$\Box \supset (\Box, \Box)$	1	1
$\tilde{Q}^a_i$	$\tilde{\Box}$	1	$\tilde{\Box} \supset (\tilde{\Box}, \tilde{\Box})$	1
H^j_i	1	$\tilde{\Box} \supset (\tilde{\Box}, \tilde{\Box})$	$\Box \supset (\Box, \Box)$	1
$\tilde{S}_{j=1..N_S}$	$\tilde{\Box}$	1	1	$\tilde{\Box}$

		$SU(2)_L \times SU(n_g)$	$SU(2)_R \times SU(n_g)$	
$\tilde{q}_{j=1..N_q}$	1	$(\tilde{\Box}, \tilde{\Box})$	1	1
$q_{j=1..N_q}$	1	1	$(\Box, \Box)$	1

$$q = \left(\overbrace{\begin{matrix} \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \psi_{\frac{1}{2}} \\ \psi_{-\frac{1}{2}} \end{pmatrix} \end{matrix}}^{N_q = N_C - 4} \right) \Bigg\} 2 \times n_g = 6 \;\; ; \quad \tilde{q} = \left(\begin{matrix} \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \\ \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\psi}_{-\frac{1}{2}} \\ \tilde{\psi}_{\frac{1}{2}} \end{pmatrix} \end{matrix} \right)$$

- **A large number of extraneous states are already removed by the VEVs but what about the others?**

$$Q = \left(\overbrace{\begin{array}{cccc} q_1 & \ell_1 & \cdots \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} & \cdots \\ q_2 & \ell_2 & \cdots \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} & \cdots \\ q_3 & \ell_3 & \cdots \begin{pmatrix} \Psi_{\frac{1}{2}} \\ \Psi_{-\frac{1}{2}} \end{pmatrix} & \cdots \\ \vdots & \vdots & \ddots & \end{array}}^{N_C}\right)\Bigg\} N_F \ ; \quad \tilde{Q} = \begin{pmatrix} \begin{pmatrix} u^c \\ d^c \end{pmatrix} & \begin{pmatrix} \nu_e^c \\ e^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \begin{pmatrix} s^c \\ c^c \end{pmatrix} & \begin{pmatrix} \nu_\mu^c \\ \mu^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \begin{pmatrix} b^c \\ t^c \end{pmatrix} & \begin{pmatrix} \nu_\tau^c \\ \tau^c \end{pmatrix} & \cdots \begin{pmatrix} \tilde{\Psi}_{-\frac{1}{2}} \\ \tilde{\Psi}_{\frac{1}{2}} \end{pmatrix} \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$$H = \begin{pmatrix} \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{11} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{12} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{13} & \cdots \\ \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{21} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{22} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{23} & \cdots \\ \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{31} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{32} & \begin{pmatrix} h_u^0 & h_d^- \\ h_u^+ & h_d^0 \end{pmatrix}_{33} & \cdots \\ \vdots & \vdots & \vdots & H_0 \end{pmatrix} \qquad \tilde{S} = \left(\overbrace{\begin{array}{ccccc} \begin{pmatrix} \tilde{d}^c \\ \tilde{u}^c \end{pmatrix} & \begin{pmatrix} \tilde{e}^c \\ \tilde{\nu}^c \end{pmatrix} & \begin{pmatrix} \tilde{\phi}_{-\frac{1}{2}} \\ \tilde{\phi}_{\frac{1}{2}} \end{pmatrix} & \cdots & \begin{pmatrix} \tilde{\phi}_{-\frac{1}{2}} \\ \tilde{\phi}_{\frac{1}{2}} \end{pmatrix} \\ \tilde{T}_{-\frac{1}{6}} & \tilde{\phi}_{\frac{1}{2}} & \tilde{\phi}_0 & \cdots & \tilde{\phi}_0 \\ \vdots & \vdots & \vdots & & \vdots \\ \tilde{T}_{-\frac{1}{6}} & \tilde{\phi}_{\frac{1}{2}} & \tilde{\phi}_0 & \cdots & \tilde{\phi}_0 \end{array}}^{N_C}\right)\Bigg\} N_S = N_C - 2$$

- **Indeed a VEV for H0 removes all vector-like non-SU(2) charged fermions. But what about the extra SU(2)L and SU(2)R multiplets?**