

Title: Beyond isotropic & homogeneous loop quantum cosmology: theory and predictions

Speakers: Javier Olmedo

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Abstract: In this talk I will discuss several recent advances in loop quantum cosmology and its extension to inhomogeneous models. I will focus on spherically symmetric spacetimes and Gowdy cosmologies with local rotational symmetry in vacuum. I will discuss how to implement a quantum Hamiltonian evolution on these quantizations. Then, I will focus on how we can extract predictions from those quantum geometries, and finally analyze a concrete example: cosmological perturbations on Bianchi I spacetimes in LQC.

# Beyond isotropic & homogeneous loop quantum cosmology: theory and predictions

**Javier Olmedo**

**Penn State University**

Perimeter Institute, Waterloo, 06/07

# Motivation

- ◆ Singularity theorems in classical GR.
- ◆ What is the nature of the spacetime close to the high curvature regions?
- ◆ Can these theories resolve all singularities?
- ◆ What are the observables we can measure?
- ◆ Can simple models provide predictions so that we can falsify them or even the full theory?

## Motivation

- ◆ Symmetry reduced models allow us to realize concrete calculations (FRW, Bianchi, Schwarzschild, Kerr, ...).
- ◆ In quantum gravity, it is not obvious how to reduce the full quantum theory.
- ◆ The quantization of symmetry reduced models of GR can give us hints about the physics and mathematics of the full theory.
- ◆ For instance, we can study semiclassical sectors in agreement with GR and how quantum geometry can affect the predictions of the classical theory and its comparison with observations.

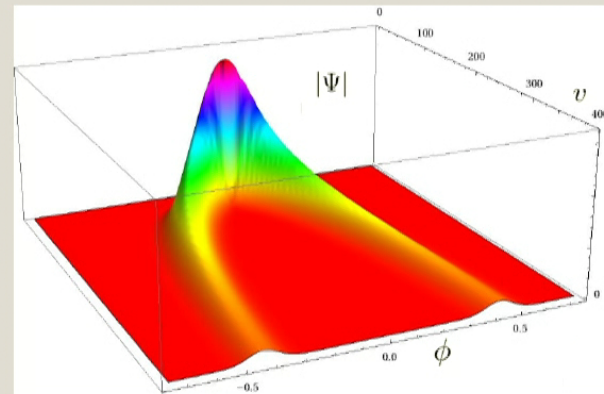
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# Homogeneous models in LQC

## ◆ Quantization of a FRW spacetime with a massless scalar field

- a) Quantum dynamics (improved scheme) ✓
- b) Singularity resolution (discrete quantum geometry) ✓
- c) Semiclassical sectors and effective dynamics ✓

$$H^2 = \frac{8\pi G}{3} \rho \left( 1 - \frac{\rho}{\rho_c} \right)$$



# Homogeneous models in LQC

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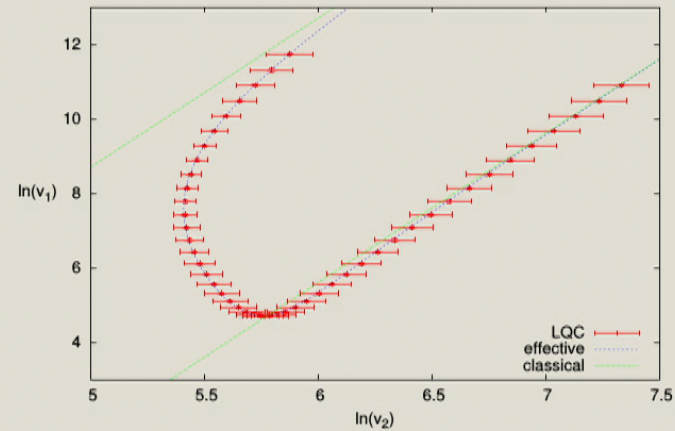
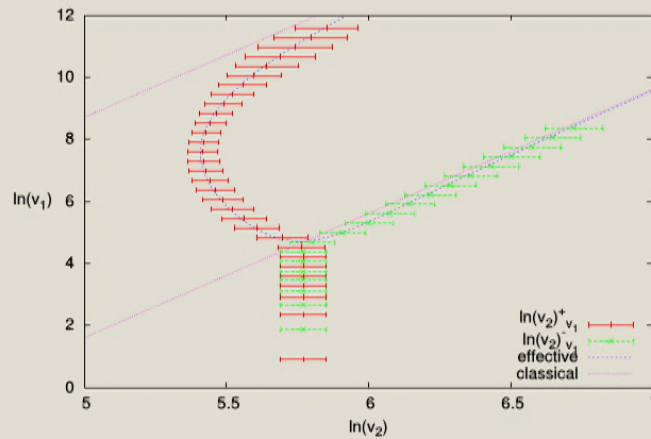
## ◆ Extensions:

- a) FRW with  $k = 1$  ✓
- b) FRW with  $\Lambda = \pm 1$  ✓
- c) Radiation dominated ✓
- d) Bianchi I ✓
- e) Bianchi II and IX
- f) Kantowski-Sachs.

# Homogeneous models in LQC

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- d) **Bianchi I** ✓
- e) Bianchi II and IX
- f) Kantowski-Sachs.



Martín-Benito, Mena Marugán, Pawłowski, Phys. Rev. D 80, 084038 (2009)

# Inhomogeneous models

- ◆ Spherically symmetric vacuum spacetimes (Bojowald, Swiderski, 2004-2005) and polarized Gowdy models (Banerjee, Date, 2007).
  - a) Symmetry reduction of the full theory in Ashtekar-Barbero variables.
  - b) Robust kinematical quantum description.
  - c) Well defined quantum Hamiltonian constraint (à la loop).
  
- ◆ Polarized Gowdy models – hybrid quantization (Martín-Benito, Mena-Marugán, Garay, 2008).
  - a) Partial gauge fixing.
  - b) Robust kinematical quantum description combining loop and Fock representation.
  - c) Well defined quantum constraints (à la hybrid).
  - d) Effective dynamics.

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## Inhomogeneous models

- ◆ Abelian constraint: Spherically symmetric spacetimes (Gambini, Pullin 2013; Gambini, O, Pullin 2013), (LRS) Gowdy cosmologies (Martín-de Blas, O, Pawłowski, 2015-2017) or 1+1 spacetimes like dilatonic black holes (Corichi, O, Rastgoo, 2016)

a) classical abelianization of the Hamiltonian constraint,

$$H_T = \int dx (NH + N^r H_r) = \int dx (\bar{N}H_{\text{new}} + \bar{N}^r H_r),$$

$$H \rightarrow H_{\text{new}} := \frac{(E^x)'}{E^\varphi} H - 2 \frac{\sqrt{E^x}}{E^\varphi} K_\varphi H_r \Rightarrow \{H_{\text{new}}(\bar{N}), H_{\text{new}}(\bar{M})\} = 0.$$

# Inhomogeneous models

◆ Abelian constraint: Spherically symmetric spacetimes (Gambini, Pullin 2013; Gambini, O, Pullin 2013), (LRS) Gowdy cosmologies (Martín-de Blas, O, Pawłowski, 2017) or 1+1 spacetimes like dilatonic black holes (Corichi, O, Rastgoo, 2016)

- a) classical abelianization of the Hamiltonian constraint,
- b) well-known kinematical Hilbert space,

$$|g, \vec{k}, \vec{\mu}\rangle = \text{---} \begin{array}{c} \dots \quad \mu_{j-1} \quad k_j \quad \mu_j \quad k_{j+1} \quad \mu_{j+1} \quad \dots \\ \bullet \quad \bullet \quad \bullet \end{array} \text{---}$$

$$\Psi_{g, \vec{k}, \vec{\mu}}(A_x, A_\varphi) = \prod_{e_j \in \mathcal{G}} \exp \left( ik_j \int_{e_j} dx A_x(x) \right) \prod_{v_j \in \mathcal{G}} \exp (i\mu_j A_\varphi(v_j)) .$$

$$\langle g', \vec{k}', \vec{\mu}' | g, \vec{k}, \vec{\mu} \rangle = \delta_{g', g} \delta_{\vec{k}', \vec{k}} \delta_{\vec{\mu}', \vec{\mu}} .$$

# Inhomogeneous models

◆ Abelian constraint: Spherically symmetric spacetimes (Gambini, Pullin 2013; Gambini, O, Pullin 2013), (LRS) Gowdy cosmologies (Martín-de Blas, O, Pawłowski, 2017) or 1+1 spacetimes like dilatonic black holes (Corichi, O, Rastgoo, 2016)

- a) classical abelianization of the hamiltonian constraint
- b) well-known kinematical Hilbert space
- c) Physical Hilbert space

$$\|\Psi_{phys}\|^2 = \left( \langle \Psi_{kin} | \eta_{diff(H_r)} \eta_{diff(H_{new})} \right) | \Psi_{kin} \rangle$$

and Dirac observables

$$\hat{O}(z) = \ell_{Pl}^2 \hat{k}_{Int(nz)},$$

and one physical global degree of freedom (either the mass  $\hat{M}$  or a densitized shear scalar  $\hat{h}$ ), are known.

## Black hole spacetimes

- ◆ The effective spacetime metric can be computed by means of parametrized (relational) observables. For instance, in the spatially flat gauge  $ds^2 = d\tilde{s}^2 + \langle \hat{E}^x(x) \rangle d^2\Omega$ , where

$$d\tilde{s}^2 = - \left\langle \left( 1 - \frac{\hat{r}_S}{\sqrt{\hat{E}^x(x)}} \right) \right\rangle dt^2 - \langle \eta \sqrt{\frac{\hat{r}_S}{[\hat{E}^x(x)]^{3/2}}} [\hat{E}^x(x)]' \rangle dt dx + \left\langle \left( \frac{[\hat{E}^x(x)]'}{4\hat{E}^x(x)} \right)^2 \right\rangle dx^2$$

- ◆ These geometries are discrete (piecewise constant  $x$ -functions).
- ◆ Close to where the singularity would be effective geometries are regular (singularity free).
- ◆ Interplay between the (fluctuating) discrete geometry and the (fluctuating) horizon (?).

## Gowdy cosmologies with local rotational symmetry

- ◆ We also showed that it is possible to implement an improved dynamics scheme with  $E^x \rightarrow V = \sqrt{\mathcal{E}}E^x$ .
- ◆ On the physical Hilbert space, the quantum Hamiltonian evolution is defined as follows
  - ◆ Choice of phase space variable as time function  $T_j$  (on  $v_j$ ).
  - ◆ A family of unitary (norm preserving) transformations  $\hat{P}_{T_j}$  between  $\mathcal{H}_{\text{phy}}$  and  $\mathcal{H}_T$ .
  - ◆ Relevant operators (observables)  $\{\hat{O}(T)\}$  with suitable domains in  $\mathcal{H}_T$ .
- ◆ Finally, the evolution is defined via a family of operators  $|\psi_{T'}\rangle := \hat{U}_{T',T}|\psi_T\rangle = \hat{P}_{T'}(\hat{P}_T)^{-1}|\psi_T\rangle$ .
- ◆ As an example, we choose  $V(\theta)$  as time function (non monotonic). Evolution split on several charts (on each vertex)

(Martín-de Blas, O, Pawłowski - 2017).

## Gowdy cosmologies with local rotational symmetry

- ◆ The solutions to the constraint and inner physical product can be written as

$$\tilde{\Psi}(\vec{k}, h, \vec{v}) = \tilde{\Psi}(\vec{k}, h) e_{\vec{k}, h}(\vec{v}), \quad \langle \Phi | \Psi \rangle = \sum_{\vec{k} \in (\mathbb{Z}^*)^n} \int_0^{h_*(\vec{k})} dh \Phi^*(\vec{k}, h) \tilde{\Psi}(\vec{k}, h).$$

- ◆ We split the solutions as via  $\Psi^\pm(\nu) = \mathcal{F}^{-1} \theta(\pm b) [\mathcal{F} \Psi](b)$ . This defines the unitary maps  $\hat{U}_{\vec{v}', \vec{v}}^\pm = \hat{P}_{\vec{v}}^\pm (\hat{P}_{\vec{v}'}^\pm)^{-1}$  with

$$\tilde{\Psi}_{\vec{v}}^\pm(\vec{k}, h) = \hat{P}_{\vec{v}}^\pm \tilde{\Psi}(\vec{k}, h) = \frac{e_{\vec{k}, h}^\pm(\vec{v})}{|e_{\vec{k}, h}^\pm(\vec{v})|} \tilde{\Psi}(\vec{k}, h).$$

- ◆ The evolution on each chart, namely volume expanding and contracting, (in the Schrödinger picture)

$$|\Psi_{\vec{v}}^\pm\rangle = \sum_{\vec{k} \in (\mathbb{Z}^*)^n} \int_0^{h_*(\vec{k})} dh \tilde{\Psi}(\vec{k}, h) \frac{e_{\vec{k}, h}^\pm(\vec{v})}{|e_{\vec{k}, h}^\pm(\vec{v})|} |\vec{k}, h\rangle, \quad \langle \Psi_{\vec{v}}^\pm | \Psi_{\vec{v}}^\pm \rangle = 1, \quad \forall \vec{v}.$$

- ◆ Dirac observables  $\hat{O}_{\vec{v}'} = \hat{U}_{\vec{v}', \vec{v}}^\pm \hat{O}_{\vec{v}} (\hat{U}_{\vec{v}', \vec{v}}^\pm)^{-1}$

# Gowdy cosmologies with local rotational symmetry

- ◆ Merits:
  - ◆ Rigorous quantum Hamiltonian (local) evolution.
  - ◆ It is not necessary to rely on the classical theory.
- ◆ Limitations:
  - ◆ The time-dependent states are not solutions to the constraint (but are related with them via a bijection).
  - ◆ The observables have ambiguous physical meaning close to the turning points (there it is more convenient to switch the time function).
  - ◆ Not obvious application to phase space variables with multiple turning points.
- ◆ Prospects: Extension of this Hamiltonian evolution in the context of black hole spacetimes.

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# Quantum field theories on quantum geometries

- ◆ Test fields (no backreaction) on these quantum geometries (perturbations) experience a dressed effective geometry.

$$\int d^4x \mathcal{L}_\phi^{\text{class}} = \frac{1}{2} \int dt d^3x \sqrt{-g} \left[ -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m_\phi^2 \phi^2 \right] \rightarrow$$
$$\int d^4x \hat{\mathcal{L}}_\phi^{\text{dress}} = \frac{1}{2} \int dt d^3x \left[ -\langle \sqrt{-\hat{g}} \hat{g}^{\mu\nu} \rangle \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} - \langle \sqrt{-\hat{g}} \rangle m_\phi^2 \hat{\phi}^2 \right]$$

- ◆ The propagation of fields on these (inhomogeneous) quantum geometries is not well understood.
- ◆ Extraction of predictions: role of discrete geometry and quantum anisotropies on the CMB; Hawking radiation, spectrum of quasi-normal modes (O - 2018), ...



## QFT on quantum BHs

- ◆ For instance, the effective Hamiltonian of a test field (with parametrized observables in spatially flat gauge coordinates)

$$\hat{\mathcal{H}}_\psi^{\text{sph}} = \langle \psi | \hat{\mathcal{H}}^{\text{sph}} | \psi \rangle = \int dx \sum_{\ell m} \frac{1}{2} (\mathcal{P}_1 \hat{\pi}_{\ell m}^* \hat{\pi}_{\ell m} + \mathcal{P}_2 \partial_x \hat{\phi}_{\ell m}^* \partial_x \hat{\phi}_{\ell m} + (\mathcal{P}_3 m_\phi^2 + \mathcal{P}_4 \ell(\ell+1)) \hat{\phi}_{\ell m}^* \hat{\phi}_{\ell m}) + \hat{\pi}_{\ell m}^* \mathcal{P}_5 \partial_x \hat{\phi}_{\ell m},$$

$$\mathcal{P}_1(x) = \langle \psi | \left[ \frac{2}{(E^x)' \sqrt{E^x}} \right] | \psi \rangle, \quad \mathcal{P}_2(x) = \langle \psi | \left[ \frac{2\sqrt{E^x E^x}}{(E^x)'} \right] | \psi \rangle, \quad \mathcal{P}_3(x) = \langle \psi | \left[ \frac{(E^x)' \sqrt{E^x}}{2} \right] | \psi \rangle$$

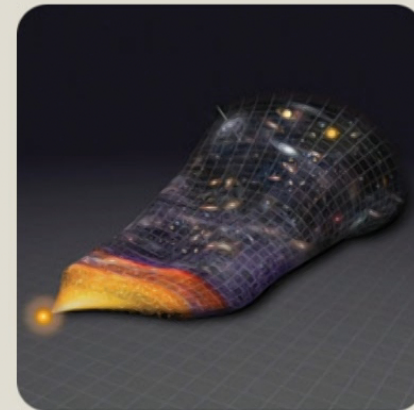
$$\mathcal{P}_4(x) = \langle \psi | \left[ \frac{(E^x)'}{2\sqrt{E^x}} \right] | \psi \rangle, \quad \mathcal{P}_5(x) = \langle \psi | -2\eta \sqrt{2\hat{M}} \left[ \frac{(E^x)^{1/4}}{(E^x)'} \right] | \psi \rangle.$$

$$\tilde{N}^2 = \mathcal{P}_4 \sqrt{\mathcal{P}_1 \mathcal{P}_2}, \quad \tilde{N}^x = \mathcal{P}_5, \quad \tilde{q}_{xx} = \frac{\mathcal{P}_4}{\sqrt{\mathcal{P}_1 \mathcal{P}_2}}, \quad \tilde{q}_{\theta\theta} = \sqrt{\frac{\mathcal{P}_2}{\mathcal{P}_1}} = \frac{\tilde{q}_{\varphi\varphi}}{\sin^2 \theta}, \quad \tilde{m}_\phi^2 = m_\phi^2 \sqrt{\frac{\mathcal{P}_1 \mathcal{P}_3}{\mathcal{P}_2 \mathcal{P}_4}}.$$

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## Example: Bianchi I & perturbations

- ◆ Homogeneity, isotropy and flatness are assumed in most of the present models of our Universe: inflation homogenizes and suppresses anisotropies and spatial curvature quickly.
- ◆ Together with quantum fluctuations of matter and geometry (quantum cosmological perturbations) we can provide predictions in very good agreement with observations.
- ◆ Upper bounds on anisotropies (shear) suggest that our Universe is isotropic at present.
- ◆ But this is not the end of the story  
...



## Example: Bianchi I & perturbations

- ◆ Cosmological perturbation theory on Bianchi I spacetimes has been studied in some detail (Pereira, Pitrou, Uzan, 2007-2008)
  - a) rigorous formulation in terms of gauge-invariant perturbations (Lagrangian formulation and SVT decomposition),
  - b) scalar and tensor perturbations are coupled dynamically if anisotropies are present,
  - c) although anisotropies do not induce dipoles, they “break” scale invariance, isotropy and introduce scalar-tensor and tensor-tensor cross-correlations.
  
- ◆ Questions not fully understood:
  - a) exact Fock quantization for the perturbations,
  - b) extension to the Planck regime where quantum gravity is relevant (LQC),
  - c) bounds on the anisotropies (shear) since perturbations can keep memory if they are large before (or close to) inflation.
  - d) Characterization of the cross-correlations.

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## Bianchi I spacetimes

- ◆ Let us consider a Bianchi I spacetime

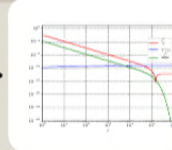
$$ds^2 = -N^2 d\tau^2 + a_1^2 dx_1^2 + a_2^2 dx_2^2 + a_3^2 dx_3^2.$$

- ◆ We will assume a scalar field  $\phi$  of mass  $m$  as matter content.
- ◆ The EOMs, in terms of  $a = (a_1 a_2 a_3)^{1/3}$  are

$$3H^2 = \kappa \left[ \frac{1}{2} \dot{\phi}^2 + V(\phi) \right] + \frac{1}{2} \tilde{\sigma}^2, \quad (\tilde{\sigma}_j^i)^{\cdot} = -3H \tilde{\sigma}_j^i,$$

$$\ddot{\phi} + 3H \dot{\phi} + V_\phi = 0, \quad \tilde{\sigma}_{ij} = \frac{1}{2} \dot{\gamma}_{ij},$$

with  $H = \frac{\dot{a}}{a}$ ,  $\tilde{\sigma}_j^i = \gamma^{ik} \tilde{\sigma}_{kj}$ ,  $\gamma_{ij} = \frac{h_{ij}}{a^2}$ .

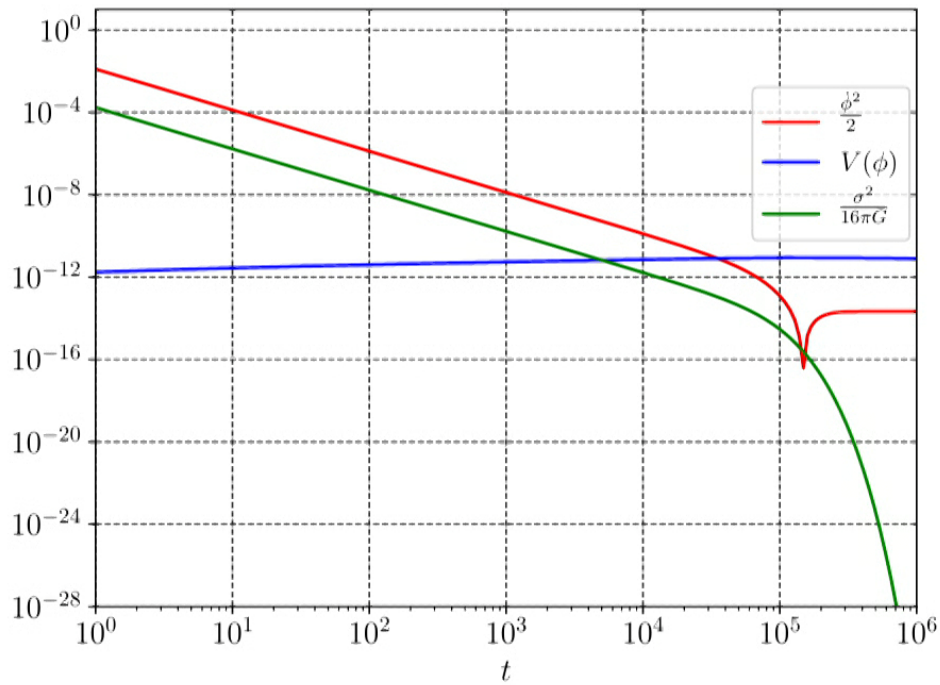


# Bianchi I spacetimes

◆ Let us consider a Bianchi I spacetime

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## Perturbations: 2nd order Hamiltonian

◆ The 2nd order Hamiltonian takes the form

$$H^{(2)} = \frac{1}{2} \int d^3 k \left\{ \pi_v \pi_v^* - \left[ \frac{z_s''}{z_s} - k^2 \right] v v^* - \sum_{\lambda=+, \times} \frac{1}{a^2} \left( 2\sqrt{\kappa} \bar{\pi}_\phi \sigma_\lambda \mathcal{F} \right)' (v^* \mu_\lambda + v \mu_\lambda^*) \right. \\ \left. + \sum_{\lambda=+, \times} \pi_{\mu_\lambda} \pi_{\mu_\lambda}^* - \left[ \frac{z_\lambda''}{z_\lambda} - k^2 \right] \mu_\lambda \mu_\lambda^* + \left[ 2\sigma_+ \sigma_\times - \left( 2a^2 \sigma_+ \sigma_\times \mathcal{F} \right)' \right] \mu_{1-\lambda} \mu_\lambda^* \right\},$$

$$\mathcal{F} = \frac{2\mathcal{H} + \sigma_{||}}{\frac{32\pi G}{3} a^2 \rho + \frac{2}{3} (2\sigma_{v_1}^2 + 2\sigma_{v_2}^2 + \sigma_+^2 + \sigma_\times^2)},$$

$$\frac{z_s''}{z_s} = \frac{a''}{a} - a^2 V_{\phi\phi} + \frac{1}{a^2} \left( 2\kappa \bar{\pi}_\phi^2 \mathcal{F} \right)',$$

$$\frac{z_\lambda''}{z_\lambda} = \frac{a''}{a} + 2\sigma_{(1-\lambda)}^2 + \frac{1}{a^2} \left( a^2 \sigma_{||} \right)' + \frac{1}{a^2} \left( 2a^2 \sigma_\lambda^2 \mathcal{F} \right)'.$$

## QFT for cosmological perturbations

- ◆ The Fourier modes  $\gamma = (\vec{q}, \vec{\pi})$ , with  $\vec{q} = (v, \mu_+, \mu_\times)$  and  $\vec{\pi}$  the conjugate momenta, can be written as a linear combination of the (orthonormal) basis of complex solutions  $\gamma^{(i)}$  and  $(\gamma^{(i)})^*$  with respect to the norm

$$\vec{q} = \sum_{i=1}^3 a_i \vec{q}^{(i)} + \text{c.c.},$$

$$\langle \gamma, \tilde{\gamma} \rangle = i\hbar \sum_{j=1}^3 \left( q_j^* \tilde{\pi}^j - (\tilde{\pi}^j)^* \tilde{q}_j \right).$$

Note: addends “j” preserved independently if fields uncoupled.

- ◆ Quantum fields given by

$$\hat{\vec{q}} = \sum_{i=1}^3 \hat{a}_i \vec{q}^{(i)} + \hat{a}_j^\dagger (\vec{q}^{(i)})^*, \quad [\hat{a}_i(\mathbf{k}), \hat{a}_j^\dagger(\mathbf{k}')] = \delta_{ij} \delta^{(3)}(\mathbf{k} + \mathbf{k}')$$

# Bianchi I spacetimes in LQC

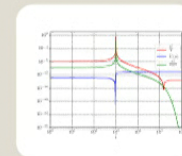
- ◆ In LQC the effective dynamics is determined by (Ashtekar, Wilson-Ewing, Mena-Marugán, Martín-Benito, ...)

$$\mathcal{H}_{\text{eff}} = \frac{1}{8 \pi G \gamma^2 v} \left[ - \left( \frac{\sin(\bar{\mu}_1 c_1)}{\bar{\mu}_1} \frac{\sin(\bar{\mu}_2 c_2)}{\bar{\mu}_2} p_1 p_2 + \text{cyclic terms} \right) + \bar{\pi}_\phi^2 + v^2 V(\phi) \right],$$

where  $v^2 = p_1 p_2 p_3$ ,  $\bar{\mu}_1 = \lambda \sqrt{\frac{|p_1|}{|p_2 p_3|}}$  (and similarly for  $\bar{\mu}_2$  and  $\bar{\mu}_3$ ), and  $\lambda^2 = 4 \sqrt{3} \pi \gamma \ell_{\text{Pl}}^2$ .

- ◆ The energy density, mean Hubble parameter and shear are bounded above (Gupt, Singh, 2012-2013)

$$\rho_{\text{max}} = 0.41 \rho_{\text{Pl}}, \quad H_{\text{max}} = \frac{8.34}{\ell_{\text{Pl}}}, \quad \sigma_{\text{max}}^2 = \frac{11.57}{\ell_{\text{Pl}}^2}.$$





# Bianchi I spacetimes in LQC

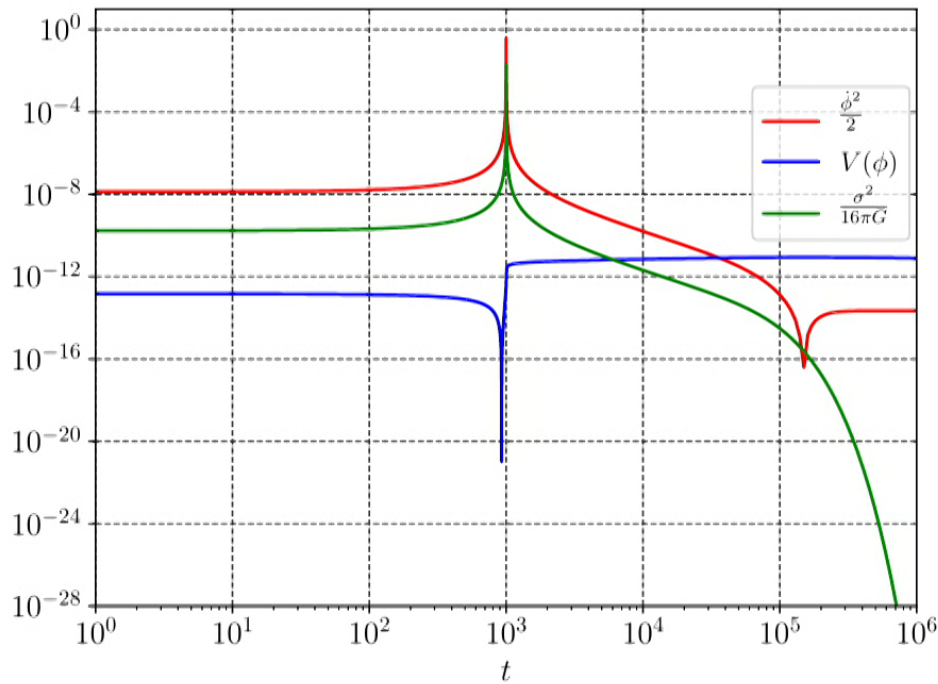
◆ In [Mena]

$\mathcal{H}_{\text{eff}}$

with

$\lambda^2 =$

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boundary



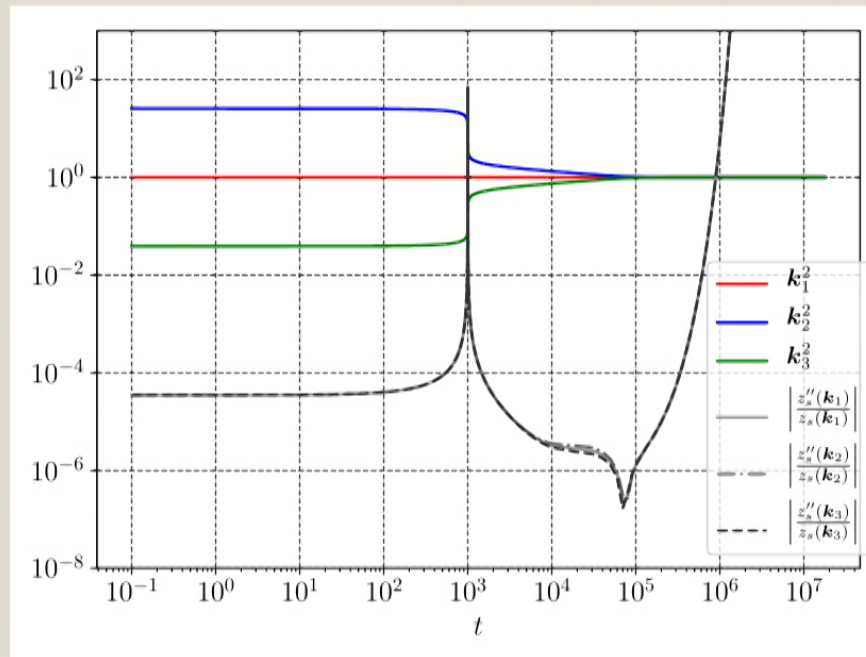
Wilson-Ewing,

$+ v^2 V(\phi)$

and  $\bar{\mu}_3$ ), and

shear are

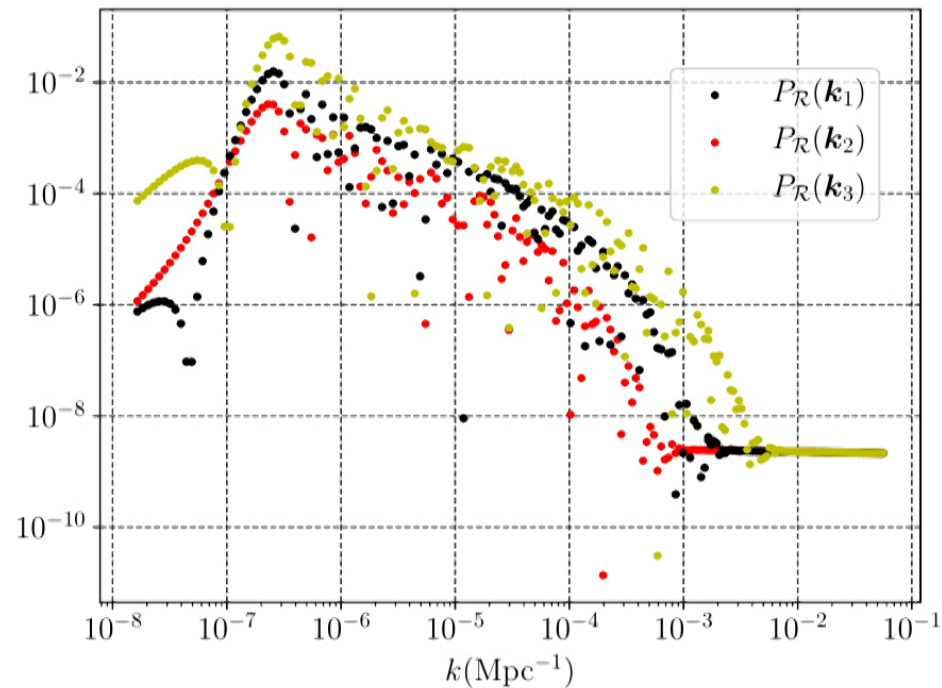
# Adiabatic conditions



Before the bounce the spacetime is (approximately) isotropic and there is a well defined adiabatic regime where we assume the 0th order adiabatic vacuum state.

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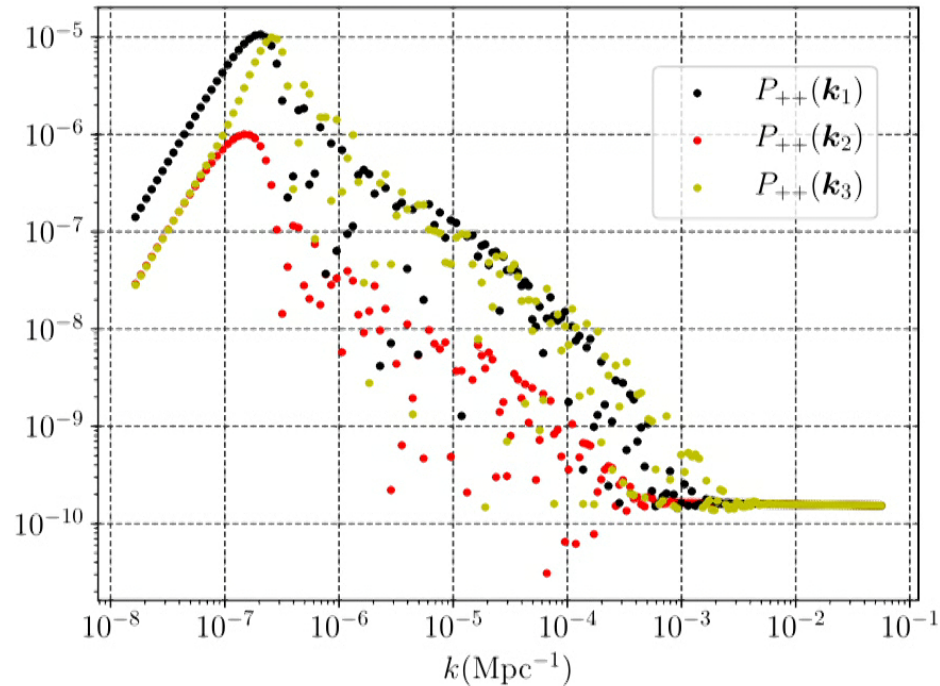
## Scalar power spectra



The isotropy of the scalar and tensor power spectra at the end of inflation is broken for large wavelengths.

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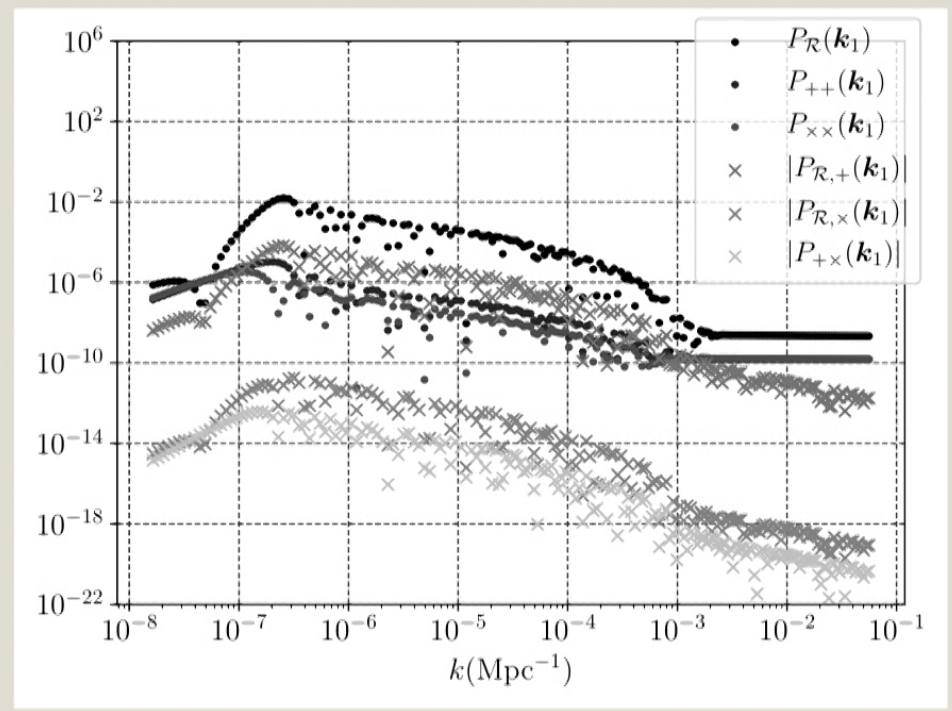
## Tensor power spectra



The isotropy of the scalar and tensor power spectra at the end of inflation is broken for large wavelengths.

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# Correlation functions



Cross-correlations can be large for small wavenumbers. For large wavenumbers they behave as  $P_{AB} = \frac{P_{AB}^{(o)}(\hat{k})}{k} \cos(\phi_{AB}(\mathbf{k}))$ , with  $(A, B) = \{\mathcal{R}, +, \times\}$  and where  $P_{AB}^{(o)}(\hat{k})$  grow with  $\sigma^2(t_B)$ .

## Upper bounds in the shear

- ◆ Given a fixed number of  $e$ -folds, it is possible to find upper bounds for the shear. For  $k = k'$ , let us define the estimator

$$\Delta\mathcal{P}_{\mathcal{R}}(k, k', \delta k) = 2 \left| \frac{\bar{\mathcal{P}}_{\mathcal{R}}(k, \delta k) - \bar{\mathcal{P}}_{\mathcal{R}}(k', \delta k)}{\bar{\mathcal{P}}_{\mathcal{R}}(k, \delta k) + \bar{\mathcal{P}}_{\mathcal{R}}(k', \delta k)} \right|.$$

Then, for  $\phi_B = 1.083$  (i.e.  $k_{LQC}$  at  $\ell \simeq 20$ ), and for  $k = k' = 2.1 \cdot 10^{-3} (\text{Mpc})^{-1}$  (or equivalently  $\ell \simeq 30$ ), we have

$\sigma^2(t_B)$	$\Delta\mathcal{P}_{\mathcal{R}}(k_1, k_2, \delta k)$	$\Delta\mathcal{P}_{\mathcal{R}}(k_1, k_3, \delta k)$	$\Delta\mathcal{P}_{\mathcal{R}}(k_2, k_3, \delta k)$
0.1	0.038	0.31	0.36
0.2	0.024	0.71	0.73
0.3	0.038	1.0	1.1
0.4	0.033	1.25	1.27
1.0	0.020	1.84	1.85

## Summary

- ◆ Loop quantum gravity techniques applied to symmetry reduced models successfully deals with several fundamental questions: singularity resolution, semiclassical geometries, effective description, etc.
- ◆ Full quantization of inhomogeneous cosmological and black hole models is available.
- ◆ Their quantum dynamics, semiclassical sectors and effective geometries need to be further explored.
- ◆ Test quantum fields on these quantum geometries deserve additional attention (dressed effective metric): the fundamental discretization and fluctuations of these quantum geometries can modify our understanding of several well-known phenomena of QFTs on classical curved spacetimes. Further research in this direction seems very promising.

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