Title: Non-Lorentzian geometry in gravity, string theory and holography

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Abstract: <p>I will present a brief introduction to non-Lorentzian geometries, an important example of such geometries being Newton-Cartan geometry and its torsionful generalization, which is the natural geometry to which non-relativistic field theories couple to. The talk will subsequently review how such geometries have in recent years appeared in gravity, string theory and holography. In particular, torsional Newton-Cartan geometry has been shown to appear as the boundary geometry for Lifshitz spacetimes. Furthermore, dynamical Newton-Cartan geometry is related to Horava-Lifsthiz gravity theories and appears in novel Chern-Simons theories of gravity in three dimensions. The latter can be obtained from a well-defined limit of the AdS3/CFT2 correspondence. Finally, I will briefly comment on how Newton-Cartan geometry appears in non-relativistic string theory.</p>

Non-Lorentzian Geometry in Gravity, String Theory and Holography

Perimeter Institute, Waterloo, Canada, May 29, 20189

Niels Obers (Niels Bohr Institute)

based on work:

1712.05784 (Hartong,Lei,NO,Oling), 1712.03980 (Grosvenor,Hartong,Keeler,NO),

1710.06885, 1710.04708, (de Boer,Hartong,NO,Sybesma,Vandoren)

1705.03535 (PRD) (Harmark,Hartong,NO)

1504.0746 (JHEP) (Hartong,NO), 1604.08054 (PRD) ((Hartong,Lei,NO)

1607.01753, 1607.01926 (Festuccia, Hansen, Hartong NO)

earlier work: 1409.1519 (PLB), 1409.1522 (PRD), 1502.00228 (JHEP) (Hartong,Kiritsis,NO)

1311.4794 (PRD) & 1311.6471 (JHEP) (Christensen,Hartong,NO,Rollier

Introduction

many branches of physics are not controlled by Lorentz symmetries, but by limits thereof or other effective symmetries:

- field theory (condensed matter/stat phys)
- gravity (approximations to GR/physics on null reductions/null hypersurfaces)
- holography (non-relativistic FT on boundary/non-relativistic bulk)
- string theory (limits)

non-Lorentzian geometry has in recent years seen revival and appearance in these contexts

In this review talk I will focus on a particular non-Lorentzian geometry:
Newton-Cartan (like) geometries
and appearances of this in holography, limit of AdS/CFT,
non-relativistic theories of gravity and string theory

Space-Time symmetries and Geometry

local symmetries of space and time ←→ geometry of space and time

Einstein: Lorentz ←→ Riemannian geometry

Einstein equivalence principle:
freely falling observers do not experience gravity
& laws of physics obey special relativity (local Lorentz sym.)

Cartan: Galilean ←→ Newton-Cartan geometry

- can be used to geometrize Newtonian gravity
& more general non-rel gravity: freely falling observers see Galilean laws of physics
- natural geometry on which non-rel FTs live (e.g. FQHE,: Son, 2013)

Non-Lorentzian geometry (general view)

very generally: take some symmetry algebra that includes space and time translations and spatial rotations (assume isotropic): "Aristotelian" symmetries
gauge the symmetry and turn space/time translations into local diffeomorphisms

Poincare	->	Lorentzian(pseudo-Riemannian) geometry	(relativistic)
Galilean/Bargmann	->	torsional Newton-Cartan geometry	(non-relativistic)
Carroll	->	Carrollian geometry	(ultra-relativistic)

crucial difference -> type of boosts		geometry
L: Lorentz	$t \to \gamma(t + \vec{v}\vec{x}/c^2) \quad , \quad \vec{x} \to \gamma(\vec{x} + \vec{v}t)$	$g_{\mu\nu}$
G/B: Galilean/Bargmann	$t \to t \quad , \quad \vec{x} \to \vec{x} + \vec{v}t$	$\tau_\mu \quad , \quad h^{\mu\nu} \quad , \quad m_\mu$
C: Carroll	$t \to t + \vec{v}\vec{x} \quad , \quad \vec{x} \to \vec{x}$	$v^\mu \quad , \quad h_{\mu\nu}$

Motivation

field theory

- use background field methods for systems with non-Lorentzian symmetries
  e.g. non-relativistic
  (compute EM tensor, Ward identities, anomalies)
- covariant formulation of hydrodynamics
- symmetry principle for construction of effective theories
- understand boundary geometry in non-AdS holography
  (different global spacetime symmetries on bdry)
- hydrodynamics (see talk Jelle Hartong on Friday June 1)

Motivation (cont'd)

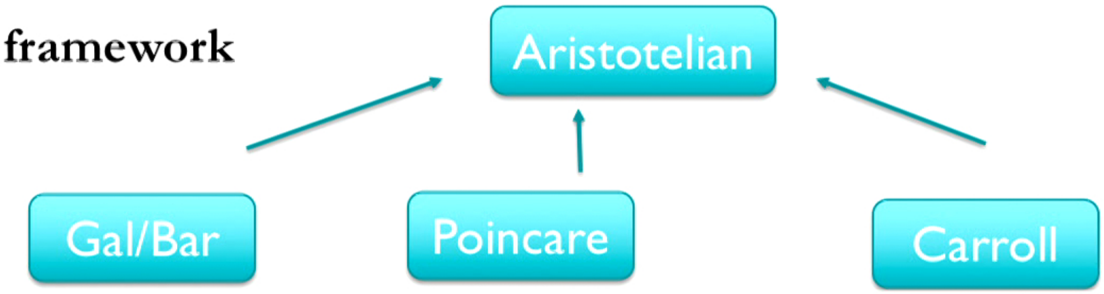
gravity/holography

- interesting in own right to find new theories of gravity
  (w. other local symmetries and still diff inv.)
- applications in holography as new bulk theories
  (relevant for non-AdS holography, but also limits of AdS/CFT)
- cosmology
- condensed matter
- toy models of quantum gravity, new insights into quantum behavior (BH ?)

string theory

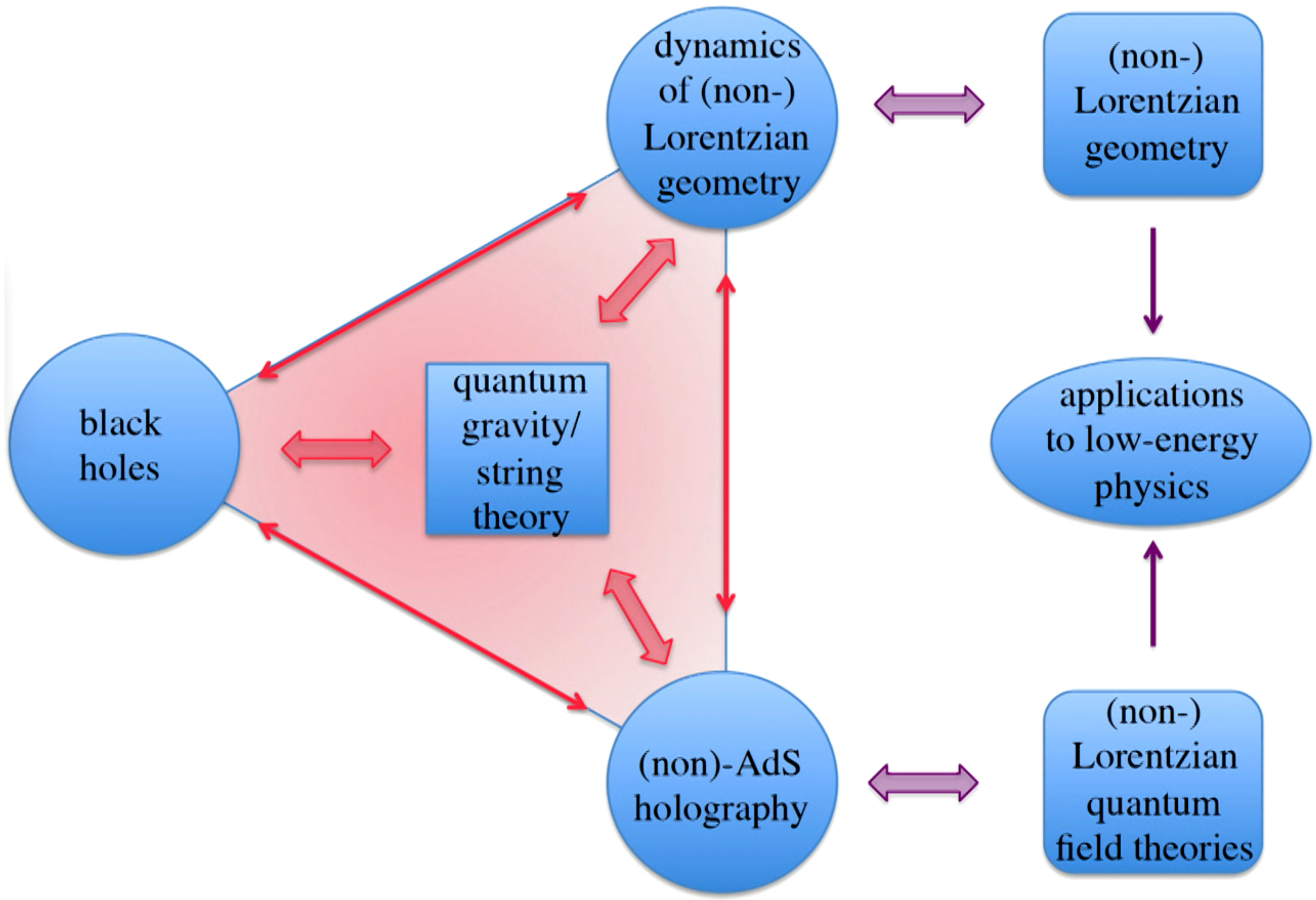
- tractable limits of string theory
- non-relativistic dispersion relations seen in limits of AdS/CFT
- new string theories ? (non-relativistic sigma models on a non-rel WS)
- non-Lorentzian target spaces and
  low energy effective actions as novel theories of gravity

Classification framework



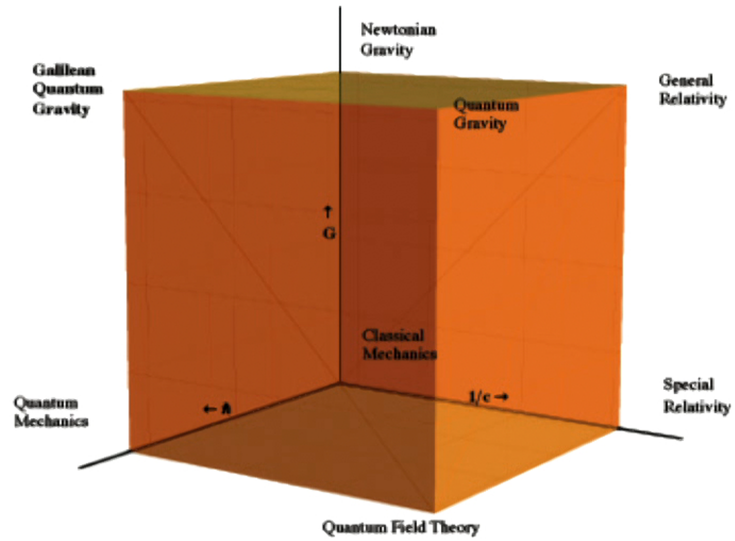
Symmetry	Galilean/Bargmann	Poincaré	Carroll
geometry	Newton–Cartan $\tau_\mu, h^{\mu\nu}, m_\mu$	pseudo-Riemannian. $g_{\mu\nu}$	Carrollian $v^\mu, h_{\mu\nu}$
causal structure	non-rel.	Minkowski	ultra-rel.
response	energy current momentum current symmetric stress	symmetric EM tensor	energy density momentum current symmetric stress
boost Ward-identity	momentum flux = mass flux	momentum current =energy current	energy flux = 0
scaling	$\forall z$	$z = 1$	$\forall z$
dynamical geometry	HL-gravity (w. $U(1)$) CS on non-rel. algebra + dyn. non-rel. sources	GR + dyn. rel. sources	ultra-relativistic gravity + dyn. ultra-rel. sources
holographic realization	EMD-model, ... HL/CS gravity	AdS/CFT	flat space

The Non-Lorentzian "Universe"



The 7th corner of the cube of physical theories

$(\hbar, G_N, 1/c)$



so far: no coherent approach to describe the corner "non-relativistic quantum gravity"

already interesting to find out what classical theory is

- new insights in the quantum theories of gravity
- approach relativistic quantum gravity by including 1/c corrections

Action principle for 7th corner: Hansen,Hartong,NO (to appear)

Outline

- intro to non-Lorentzian geometry,

in particular: Torsional Newton-Cartan (TNC) geometry (different perspectives)

- appearances of TNC geometry:..

- non-relativistic particles
- non-relativistic field theory:
- relation to Horava-Lifsthiz gravity
- non-AdS holography
- non-relativistic strings

- Chern-Simons theories of non-relativistic gravity and near-BPS limits of AdS/CFT

- Outlook

Newton-Cartan makes Galilean local

- NC geometry originally introduced by Cartan to geometrize Newtonian gravity

Eisenhart,Trautman,Dautcourt,Kuenzle,Duval,Burdet,Perrin,Gibbons,Horvathy,Nicolai,Julia…..

⇨ both Einstein's and Newton's theories of gravity admit geometrical formulations which are diffeomorphism invariant

- NC originally formulated in "metric" formulation
modern approach: vielbein formulation (shows underlying sym. principle better)

Andringa,Bergshoeff,Panda,de Roo

Riemannian geometry: tangent space is Poincare invariant

Newton-Cartan geometry: tangent space is Bargmann (central ext. Gal.) invariant

Galilean algebra $[J\,,P_a] = \epsilon_{ab}P_b\,, \quad [J\,,G_a] = \epsilon_{ab}G_b\,, \quad [H\,,G_a] = P_a$

Bargmann algebra $[P_a\,,G_b] = N\delta_{ab}$
(central extension N = mass generator)

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Different perspectives on TNC geometry

manifestations of Einstein's equivalence principle:

- gauging Poincare
- cosets (Minkowski = Poincare/Lorentz)
- Noether procedure in relativistic field theory

- Torsional Newton-Cartan geometry:
apply equivalence principle to any a Galilean boost structure

- gauging Bargmann
  Andringa,Bergshoeff,Panda,de Roo/Bergshoeff,Hartong,Rosseel/Hartong,NO
- cosets
  Grosvenor,Hartong,Keeler,NO
- Noether procedure in Galilean/Bargmann field theory
  Festuccia,Hansen,Hartong,NO

also:

- Null reductions — Julia,Nicolai
- Limits (c → infinity) — Bergshoeff,Rosseel,Zoje/van Bleeken

Gauging the Bargmann algebra

symmetry	generators	gauge field	parameters	curvatures
time translations	H	τ_μ	$\zeta(x^\nu)$	$R_{\mu\nu}(H)$
space translations	P_a	${e_\mu}^a$	$\zeta^a(x^\nu)$	${R_{\mu\nu}}^a(P)$
boosts	G_a	${\omega_\mu}^a$	$\lambda^a(x^\nu)$	${R_{\mu\nu}}^a(G)$
spatial rotations	J_{ab}	${\omega_\mu}^{ab}$	$\lambda^{ab}(x^\nu)$	${R_{\mu\nu}}^{ab}(J)$
central charge transf.	N	m_μ	$\sigma(x^\nu)$	$R_{\mu\nu}(N)$

impose curvature constraints:

$R_{\mu\nu}(H)$ $\quad R^a_{\mu\nu}(P)$ $\quad R_{\mu\nu}(N)$ $\quad$ (e.g. =0)

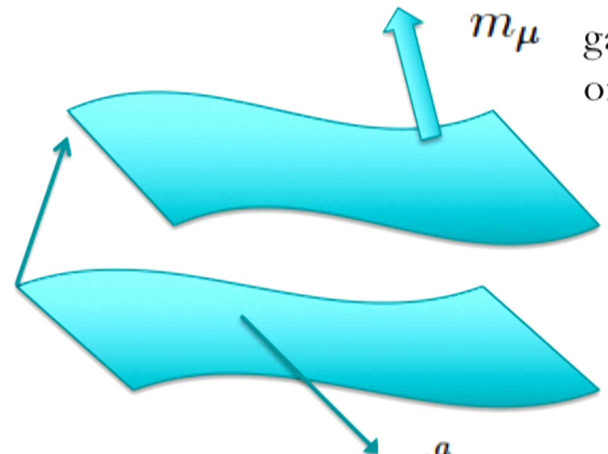
independent fields: τ_μ, e^a_μ, m_μ

= gauge fields of Hamiltonian, spatial translations and central charge

transformations:

$$\begin{aligned} \delta\tau_\mu &= \mathcal{L}_\xi \tau_\mu \\ \delta e^a_\mu &= \mathcal{L}_\xi e^a_\mu + \lambda^a \tau_\mu + {\lambda^a}_b e^b_\mu \\ \delta m_\mu &= \mathcal{L}_\xi m_\mu + \partial_\mu \sigma + \lambda_a e^a_\mu \end{aligned}$$

Newton-Cartan geometry



gauge field of central extension of Galilean algebra (Bargmann)

(inverse) spatial metric:

$$h^{\mu\nu} = e^\mu_a e^\nu_b \delta^{ab}$$

(degenerate: rank D-1)

NC geometry = no torsion $\longrightarrow$ $\tau_\mu = \partial_\mu t$ notion of absolute time

TTNC geometry = twistless torsion $\longrightarrow$ $\tau_\mu =$ HSO preferred foliation in equal time slices

TNC geometry no condition on τ_μ

- in TTNC: torsion measured by $a_\mu = \mathcal{L}_{\hat{v}} \tau_\mu$
  geometry on spatial slices is Riemannian

Adding torsion to NC

Christensen,Hartong,Rollier,NO
Hartong,Kiritsis,NO/Hartong,NO
Bergshoeff,Hartong,Rosseel

- inverse vielbeins (v^μ, e^μ_a)

$$v^\mu \tau_\mu = -1\,, \qquad v^\mu e^a_\mu = 0\,, \qquad e^\mu_a \tau_\mu = 0\,, \qquad e^\mu_a e^b_\mu = \delta^b_a$$

can build Galilean boost-invariants

$$\hat{v}^\mu = v^\mu - h^{\mu\nu} M_\nu\,,$$
$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \tau_\mu M_\nu - \tau_\nu M_\mu\,,$$
$$\tilde{\Phi} = -v^\mu M_\mu + \tfrac{1}{2} h^{\mu\nu} M_\mu M_\nu\,,$$

-introduce Stueckelberg scalar chi:
(to deal with broken/unbroken N-sym)

$$M_\mu = m_\mu - \partial_\mu \chi.$$

affine connection of TNC (inert under G,J,N)

$$\Gamma^\rho_{\mu\nu} = -\hat{v}^\rho \partial_\mu \tau_\nu + \frac{1}{2} h^{\rho\sigma}\left(\partial_\mu \bar{h}_{\nu\sigma} + \partial_\nu \bar{h}_{\mu\sigma} - \partial_\sigma \bar{h}_{\mu\nu}\right)$$

with torsion $\Gamma^\rho_{[\mu\nu]} = -\frac{1}{2}\hat{v}^\rho(\partial_\mu \tau_\nu - \partial_\nu \tau_\mu)$

$$\nabla_\mu \tau_\nu = 0\,, \qquad \nabla_\mu h^{\nu\rho} = 0\,,$$

analogue of metric compatibility

connection obtained via Noether: [Festuccia,Hansen,Hartong,NO](1607)

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TNC from reduction and limiting procedures

- can also get TNC structures from null reduction [Julia,Nicolai]

$$ds^2 = \gamma_{AB}dx^A dx^B = 2\tau_\mu dx^\mu (du - m_\nu dx^\nu) + h_{\mu\nu}dx^\mu dx^\nu$$

$$\gamma^{uu} = 2\tilde{\Phi}\,, \qquad \gamma^{u\mu} = -\hat{v}^\mu\,, \qquad \gamma^{\mu\nu} = h^{\mu\nu}$$

- non-relativistic limits (two approaches)

- expand in 1/c

$$\begin{aligned} g_{\mu\nu} &= -c^2\tau_\mu\tau_\nu + h_{\mu\nu} - \tau_\mu m_\nu - \tau_\nu m_\mu + \mathcal{O}(c^{-1}) \\ g^{\mu\nu} &= h^{\mu\nu} + c^{-2}(-v^\mu v^\nu + \beta^{\mu\nu}) + \mathcal{O}(c^{-3}) \end{aligned}$$

van Bleeken(2017)

- add U(1) field to Poincare, combine with relativistic vielbeins and do limit (involves combining Hamiltonian with U(1)) Bergshoeff,Rosseel,Zojer(2015)

Up Next

- appearances of TNC geometry: non-relativistic …..

- particles
- field theory:
- HL gravity
- holography
- strings

Non-Relativistic particle from null reduction

non-relativistic geometries/field theories/gravity can also be approached by null-reduction

null-reduction of relativistic particle $$S = \int d\lambda \frac{1}{e} G_{\mathcal{M}\mathcal{N}} \dot{X}^{\mathcal{M}} \dot{X}^{\mathcal{N}}$$

target space with null Killing vector

$$ds^2 = G_{\mathcal{M}\mathcal{N}} dx^{\mathcal{M}} dx^{\mathcal{N}} = 2\tau_M dx^M \left(du - m_N dx^N\right) + h_{MN} dx^M dx^N$$

can solve EOM for e for $\dot{X}^u$.

can. conjugate momentum $$P_u = \frac{\partial L}{\partial \dot{X}^u} = \frac{2}{e} \tau_M \dot{X}^M$$

Legendre transform $$\tilde{L} = L - P_u \dot{X}^u$$

(does not contain X^u. & P_u indep. variable)

$$\tilde{S} = \int d\lambda \tilde{L} = \int d\lambda \left(-P_u m_N \dot{X}^N + \frac{P_u}{2} \frac{h_{\mathcal{M}\mathcal{N}} \dot{X}^M \dot{X}^N}{\tau_M \dot{X}^M} \right)$$

[Kuchar], [Bergshoeff et al]

for $P_u = m = \text{cst}$ action has TNC local target space symmetries

geodesic equation on flat NC space with m_t=Phi → Newton's law

Non-Lorentzian geometry in Field Theory

- in relativistic FT: very useful to couple to background (Riemannian) geometry
  -> compute EM tensors, study anomalies, Ward identities, etc.

- background field methods for systems with non-Lorentzian symmetries require non-Lorentzian geometry

  -> FTs can have Lorentz/Galilean/Carrollian boost symmetries:
  background field methods for systems with:
  - non-relativistic (NR) symmetries require NC geometry (with torsion)
  - ultra-relativistic (UR) symmetries require Carrollian geometry

  -> there is full space-time diffeomorphism invariance when coupling to the right background fields

- Examples

  [Son, 2013], [Geracie, Son, Wu, Wu, 2014],…
  [Bergshoeff,Rosseel,Townsend,2018]

  * effective field theory for the FQHE uses NC geometry

  [Jensen,2014] [Kiritsis,Matsuo,2015]
  [Hartong,NO,Sanchioni,2016]
  [deBoer,Hartong,NO,Sybesma,Vandoren,2017]

  * non-relativistic (NR) hydrodynamics

Coupling FTs to TNC

[Hartong,Kiritsis,NO]
[Jensen]

- action functional $S(v^\mu, h^{\mu\nu}, m_\mu)$

EM tensor: $T^\mu{}_\nu$
mass current T^μ

$$\delta S = \int d^{d+1}x e(T_\mu dv^\mu + \frac{1}{2}T_{\mu\nu}\delta h^{\mu\nu} + J^\mu \delta m_\mu)$$

$$T^\mu{}_\nu = v^\mu T_\mu + h^{\mu\rho}T_{\rho\nu}$$

$T^t{}_t$, $T^t{}_i$, $T^i{}_t$, $T^i{}_j$ J^t , J^i

energy density, momentum flux, energy flux, spatial stress, mass density, mass flux

* important to have torsion in order to describe the most general energy current !

- from the various local symmetries:

particle number conservation (if extra local U(1))

mass flux= momentum flux (local Galilean boosts)

symmetric spatial stress (local rotations)

Diffeomorphism and scale Ward identities

- diffeos -> on-shell WI

$$\nabla_\nu T^\nu{}_\mu + \text{torsion terms} + \rho \nabla_\mu \tilde{\Phi} = 0$$

* conserved currents $\partial_\nu \left(e K^\mu T^\nu{}_\mu \right) = 0\,.$

for K a TNC Killing vector:

extra force term

- if theory has scale invariance:

can use TNC analogue of dilatation connection

$$z\mathcal{E} + \mathrm{Tr}\, T_{\text{spatial}} + 2(z-1))\rho\Phi = 0$$

z-deformed trace WI

Examples: Schrödinger model & Lifshitz model

- simplest toy model for coupling non-rel. scale-inv theory to TNC (z=2)

$$S = \int d^{d+1}x e \left(-i\phi^\star \hat{v}^\mu \partial_\mu \phi + i\phi \hat{v}^\mu \partial_\mu \phi^\star - h^{\mu\nu}\partial_\mu \phi \partial_\nu \phi^\star - 2\tilde{\Phi}\phi\phi^\star - V_0(\phi\phi^*)^{\frac{d+2}{d}} \right)$$

-> gives Schr. equation

has Bargmann invariance

- can also consider deformations preserving local scale inv: breaking Bargmann to Lifshitz

- other possibility: do not couple to $\tilde{\Phi}$ -> e.g. z=2 Lifshitz model

$$S = \int d^{d+1}x e \left[\frac{1}{2} (\hat{v}^\mu \partial_\mu \phi)^2 - \frac{\lambda}{2} (h^{\mu\nu} \nabla_\mu \partial_\nu \phi)^2 \right]$$

has Carrollian symmetry at leading derivatives, broken to Lifshitz by higher derivatives

Non-Lorentzian Geometry in Gravity

- interesting to make non-Lorentzian geometry dynamical
  -> "new" theories of gravity

have shown: dynamical Newton-Cartan (NC) = Horava-Lifshitz (HL) gravity

Hartong,NO (2015)

Horava (0812,0901);
Horava,Melby-Thompson (2010)

⇨ natural geometric framework with full diffeomorphism invariance

such theories of gravity interesting as
- other bulk theories of gravity in holographic setups
- effective theories (cond mat, cosmology)
- 3D case = Chern-Simons on non-rel algebras !

Griffin,Horava,Melby-Thompson (2012)
Janiszweski,Karch (2012)

[Bergshoeff,Rosseel],[Hartong,Lei,NO]

Non-relativistic FT and coupling to NC/HL gravity

have tools/building blocks to covariantly couple HL/NC gravity to non-relativistic field theories:

Jensen (1408)
Hartong,Kiritsis,NO (1409)

- massless/massive spin (0),1/2,1,3/2 for no-torsion
  using non-relativistic contraction: [Bergshoeff,Rosseel,Zojer]1512
- spin ½ for generic NC using non-rel limit: [Fuini,Karch,Uhlemann]1510
- GED (Galilean electrodynamics) coupled to TNC

[Festuccia,Hansen,Hartong,NO]1607

methods to construct non-trivial (interacting) non-rel field theories

- from scratch (use symmetries)
- take non-rel. (c-> infinity) limits
- perform null reduction of D+1 dim relativistic theories

Non-relativistic Stings on Newton-Cartan geometry

perform similar
as for rel particle by
null reducing Polyakov action

$$S = -\frac{T}{2}\int d^2\sigma \sqrt{-\gamma}\gamma^{\alpha\beta} g_{\alpha\beta}$$

$$\tilde{S} = \int d^2\sigma \left(-P_u^\alpha m_\alpha + \frac{1}{2} P_u^\gamma \tau_\gamma \left(v^\alpha v^\beta - e^\alpha e^\beta \right) h_{\alpha\beta} \right)$$

- stringy counterpart of non-rel. particle action coupling to TNC
- has all local TNC symmetries for P_u = constant
- for $P_u^\sigma = 0,\ P_u^\tau = P = \text{cst}$ and use static gauge $\tau = t,$

action on flat NC background becomes standard non-rel string which has 1+1 dimensional world volume Poincare sym.

$$S = T\int dt dx \left(\frac{1}{2}\left(\partial_t \vec{Y}\right)^2 - \frac{1}{2}\left(\partial_x \vec{Y}\right)^2 \right)$$

More on non-relativistic strings

punchlines:

- new non-relativistic limit of string theory involving non-Lorentzian world-sheet and target space geometries:
  -> Newton-Cartan geometry and generalizations ,

- specific non-rel. WS limit connects to previously studied limits of AdS/CFT

- Landau-Lifshitz model (arising from Krusczenski limit/Spin matrix theory) is a non-rel. ST with NC-like target space geometry

- target space dynamics ?: some type of non-relativistic gravity

Chern-Simons theories on non-relativistic algebras

3D Einstein gravity = CS gauge theory
-> insights into classical and quantum properties of theory
holographic dualities with 2D CFTs
black holes

- can 3D non-relativistic gravity theories be reformulated as CS ?
- can we get this from a limiting procedure of AdS3 gravity ?

yes:

→ CS on (alternate real form of) Newton-Hooke: provides 3D CS theory that can be considered the non-relativistic analogue of AdS3/CFT_2

on CFT side: zooming into near-BPS bound Hartong,Lei,NO,Olling(2017)

Note: CS on extended Bargmann first considered by: Papageorgiou,Schroers (0907) and also (& SUSY extension) considered in Hartong,Lei,NO(2016) and relation to 3D HL, & Bergshoeff,Rosseel (2016) from non-relatvistic limit of SO(1,2) CS with two U(1) fields added.

Chern-Simons on non-relativ. algebras need extension

CS Lagrangian $\mathcal{L} = \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$

need invariant bilinear form -> non-trivial requirement for non. rel algebras (non-semi simple Lie algebras)

Galilean algebra $[J\,, P_a] = \epsilon_{ab} P_b\,, \quad [J\,, G_a] = \epsilon_{ab} G_b\,, \quad [H\,, G_a] = P_a$

Bargmann algebra
(central extension = mass generator) $[P_a\,, G_b] = N \delta_{ab}$

2+1 dim special:
can further extend with a central term $[G_a, G_b] = S\epsilon_{ab},$

number of generators: 8

Newton-Hooke (cosmo cst.): $[G_a\,, G_b] = S\epsilon_{ab}\,, \quad [H\,, P_a] = -\Lambda_c G_a\,,$
$[P_a\,, P_b] = \Lambda_c S \epsilon_{ab}\,.$

Chern-Simons on non-relativ. algebras need extension

CS Lagrangian $\mathcal{L} = \text{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)$

need invariant bilinear form -> non-trivial requirement for non. rel algebras (non-semi simple Lie algebras)

Galilean algebra $[J\,, P_a] = \epsilon_{ab} P_b\,, \quad [J\,, G_a] = \epsilon_{ab} G_b\,, \quad [H\,, G_a] = P_a$

Bargmann algebra
(central extension = mass generator) $[P_a\,, G_b] = N \delta_{ab}$

2+1 dim special:
can further extend with a central term $[G_a, G_b] = S\epsilon_{ab},$

number of generators: 8

Newton-Hooke (cosmo cst.):

$$[G_a\,, G_b] = S\epsilon_{ab}\,, \quad [H\,, P_a] = -\Lambda_c G_a\,,$$
$$[P_a\,, P_b] = \Lambda_c S \epsilon_{ab}\,.$$

Zooming into near-BPS in AdS3/CFT2

any d-dimensional CFT with U(1) flavor symmetry, BPS bound and free coupling constant admits limit in which one zooms in to spectrum close to lightest charged state of theory on $\mathbb{R} \times S^d$

- turn on chemical potential for charge: $D - Q$ has non-negative spectrum
  - zoom into the 1-loop corrections of the dilatation operator

→ symmetry algebra is Inonu-Wigner contraction of $so(2, d+1) \oplus u(1)$ leading to algebra with scale but no conformal generators

- for 2D CFT: rotation is abelian and useful to add 2nd U(1) and considering contraction of $so(2,2) \oplus u(1) \oplus u(1)$

i.e. two copies of $sl(2,\mathbb{R}) \oplus u(1)$.

after limit: two copies of P_2^c, two-dimensional centrally extended Poincare

admits infinite dimensional extension: left and right moving warped Virasoro algebra

Bulk and boundary interpretations of algebra

bulk

SO(2,2) AdS3 isometry

↓

3D Newton-Hooke
(alternative real form)

→ gives rise to CS action that describes:

pseudo Newton-Cartan geometry

↑

simple emergence of holographic direction
(NC time is the radial direction)

boundary

SL(2,R) xU(1) x SL(2,R) x U(1)

↓ phase space maps !

$P_2^c \oplus P_2^c$

$$[\mathcal{P}_a, \mathcal{K}_b] = -2i\mathcal{N}\eta_{ab} - 2i\mathcal{S}\epsilon_{ab}\,,$$
$$[\mathcal{D}, \mathcal{P}_a] = i\mathcal{P}_a\,, \quad [\mathcal{D}, \mathcal{K}_a] = -i\mathcal{K}_a\,,$$
$$[\mathcal{M}, \mathcal{P}_a] = i\epsilon_a{}^b\mathcal{P}_b\,, \quad [\mathcal{M}, \mathcal{K}_a] = i\epsilon_a{}^b\mathcal{K}_b\,.$$

new algebra: scaling non-conformal alg.

enhances to warped Virasoro

$$[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n} + 2\pi\gamma_1 m(m^2-1)\delta_{m+n},$$
$$[\mathcal{L}_m, \mathcal{N}_n] = -n\mathcal{N}_{m+n} - 4\pi i\gamma_2 m(m+1)\delta_{m+n}.$$

- holography with non-relativistic bulk gravity dual and new type of CFT on bdry

Main features

- describes near-BPS sector of CFT (sym algebra is warped Virasoro)
- phase space of limiting theory continuously connected to parent theory
- vacuum is the homogeneous (non-rel) coset $(\mathbf{P}_2^c \times \mathbf{P}_2^c)/(\mathbf{P}_2^c \times U(1))$

→ pseudo Newton-Cartan geometry in some sense mimimal setup of holography (only need to reconstruct foliation structure)

bulk action in 2nd order form: $\mathcal{L} = e\left(h^{\mu\rho}h^{\nu\sigma}K_{\mu\nu}K_{\rho\sigma} - (h^{\mu\nu}K_{\mu\nu})^2 - \tilde{\Phi}\mathcal{R}\right)$ + cosmo constant

new: non-relativistic bulk gravity dual to a relativistic FT

- relation to spin chains: $E = \alpha^{-2}D = \mathcal{N} + \frac{g}{2}\mathcal{D}$, $J = -\alpha^{-2}Q_1 = \mathcal{N} - \frac{g}{2}\mathcal{D}$ $\qquad g = \alpha^{-2}$

$\mathcal{N}$ = length of spin chain

$\mathcal{D}$ = one-loop dilation operator

limit $g \to 0$ $\qquad (E-J)/g$ fixed