

Title: The shape of a more fundamental theory?

Date: May 31, 2018 02:30 PM

URL: <http://pirsa.org/18050064>

Abstract:

I suggest a minimal practical formal structure for a more fundamental theory than the Standard Model + GR and review a mechanism that produces such a structure. The proposed mechanism has possibilities of producing non-canonical phenomena in SU(2) and SU(3) gauge theories which might allow conditional predictions that can be tested.

The slides and other writings are posted on my web page

<http://www.physics.rutgers.edu/~friedan/#Perimeter>

During my visit to PI, I hope also to discuss informally a separate project in pure QFT, a scheme to construct a new kind of QFT of extended objects (also described on my web page).

The shape of a more fundamental theory?

Daniel Friedan

Rutgers University and the University of Iceland

Perimeter Institute, May 31, 2018

Abstract

I suggest a minimal practical formal structure for a more fundamental theory than the Standard Model + GR and review a mechanism that produces such a structure. The proposed mechanism has possibilities of producing non-canonical phenomena in $SU(2)$ and $SU(3)$ gauge theories which might allow conditional predictions that can be tested.

These slides and other writings are posted on my web page

<http://www.physics.rutgers.edu/~friedan/#Perimeter>

During my visit to PI, I hope also to discuss informally a separate project in pure QFT, a scheme to construct a new kind of QFT of extended objects (also described on my web page).

For the last 45 years, our most fundamental theory has been the Standard Model + General Relativity.

SM+GR is an *effective* QFT with UV cutoff $\sim (10^3\text{GeV})^{-1}$.

GR can be considered an effective QFT because quantum corrections in GR are completely negligible at this huge distance $(10^3\text{GeV})^{-1} = 10^{16}\ell_P$.

SM+GR describes almost all physics at distances $\gtrsim (10^3\text{GeV})^{-1}$.

Only dark matter, neutrino mixing, and some CP violation are unexplained.

I am interested in the possibility of formal fundamental physics:

1. hypothesize a “more fundamental” formal machinery that can “produce” the Standard Model + General Relativity
2. predict consequences beyond the SM+GR that can be checked experimentally

Prototypes:

- GR from Newtonian Gravity + Special Relativity
- Grand Unification from the SM

Grand Unification makes a *conditional* prediction that is testable. The RG acting on the space of grand unified theories *can* produce the SM. *If* the RG produces the SM, then it predicts proton decay (which unfortunately has not been seen).

In the 45 years since the SM was finished, none of the attempts at formal fundamental physics has worked.

One natural response is to give up, at least for now, perhaps hoping that experiment will eventually give more guidance.

Alternatively, it might be useful to reconsider the assumptions that have guided the enterprise over these 45 years.

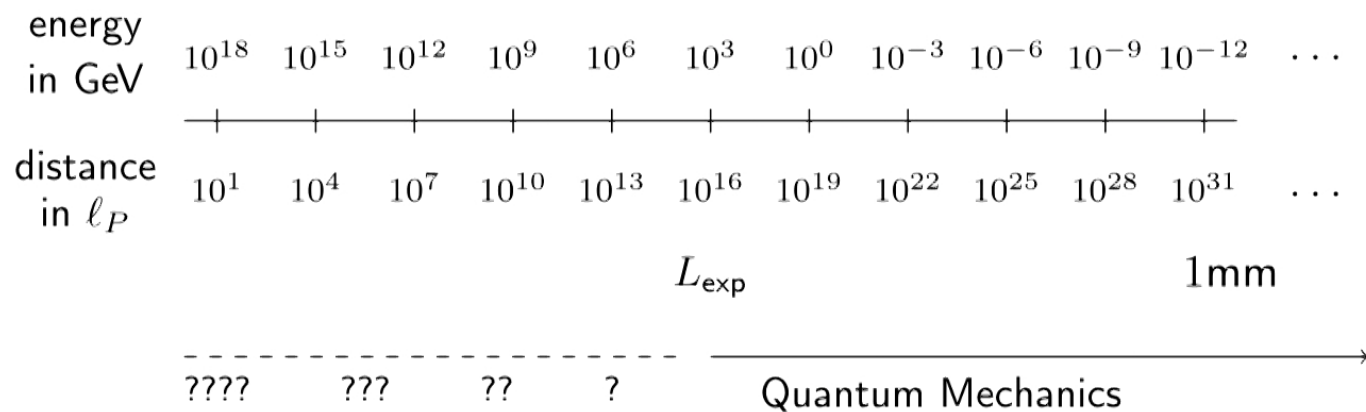
An analogy: hiking a mountain without a map. If after 45 years no measurable altitude has been gained, maybe it is time to backtrack and reconsider the previous choices of direction.

It might be especially useful to question the truisms and mathematical idealizations that have governed the enterprise.

A truism: that Quantum Gravity is needed

On the contrary,

- It is implausible that any proposed Quantum Gravity can be checked experimentally, since the smallest distance presently accessible to experiment is $L_{\text{exp}} \sim (10^3 \text{ GeV})^{-1} = 10^{16} \ell_P$.
- Assuming that Quantum Mechanics is valid over 16 orders of magnitude in distance below where there is evidence is a presumptuous extrapolation.



A leading edge high energy scattering experiment has a certain size L and probes our ignorance at distances $< L$.

at distances $> L$: QFT at distances $< L$: ?

Preparation of initial scattering states and detection of final scattering states are described by the QFT at scale $> L$.

The scattering amplitudes between such QFT states capture the physics at distances $< L$.

The minimal formalism for distances $> L$ is an effective QFT.

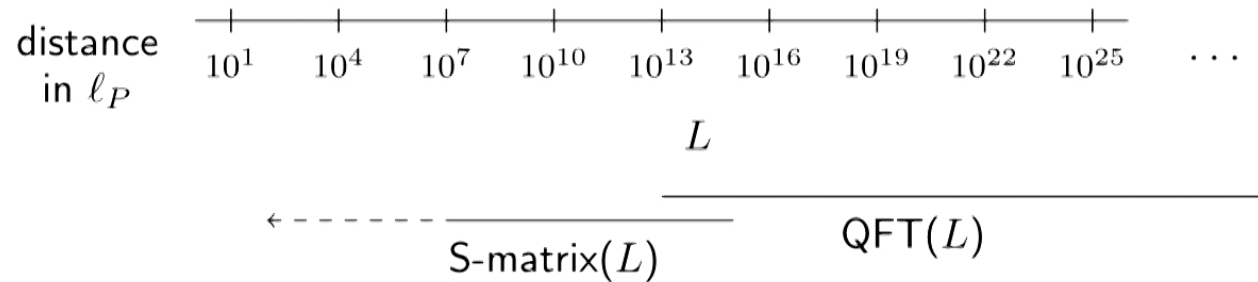
- An *effective QFT* is a QFT with UV cutoff L .

The minimal formalism for distances $< L$ is an effective S-matrix.

- An *effective S-matrix* is an S-matrix with IR cutoff L .

L is a sliding distance scale. What we mean by “short distance physics” is relative. Progress pushes L smaller and smaller.

A minimal practical formalism for fundamental physics:



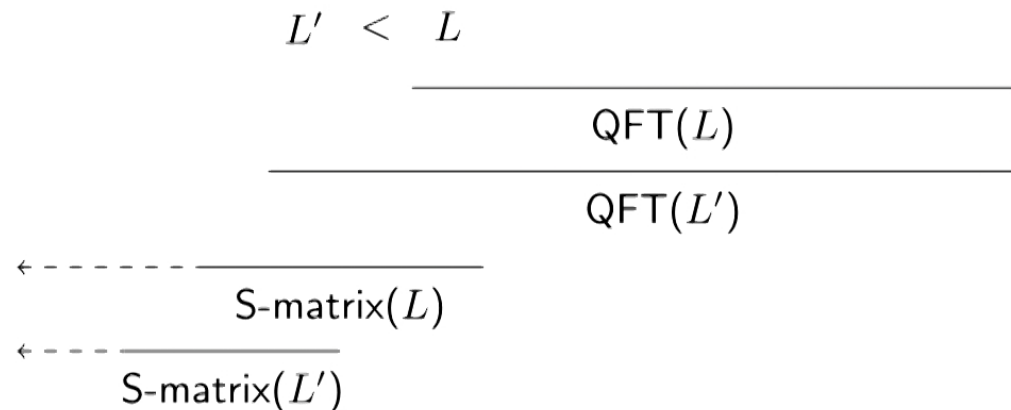
For every $L \gg 1$

- a QFT(L) = an effective QFT with UV cutoff L describing physics at distances $> L$, and
- an S-matrix(L) = an effective S-matrix with IR cutoff L describing physics at distances $< L$,

satisfying the following consistency conditions:

For $L' < L$ (both $\gg 1$)

- (1) $\text{QFT}(L') \supset \text{QFT}(L)$ via the QFT RG acting from L' up to L .
- (2) $\text{S-matrix}(L) \supset \text{S-matrix}(L')$ via the “S-matrix RG” which acts by using the scattering states at the larger scale L to make the scattering states at the smaller scale L' .
- (3) $\text{S-matrix}(L)$ agrees with the S-matrix derived from $\text{QFT}(L')$ where both apply — between L' and L .



For short distance physics, there is only S -matrix(L). We send things in and measure what comes out.

There is no presumption of a quantum mechanical model of short distance physics, no assumption there there is Quantum Mechanics all the way down.

Having an S -matrix does not guarantee an underlying quantum mechanical hamiltonian. Given a hamiltonian, an S -matrix can be derived, but not vice versa.

I came to these ideas during the period 1977 – 2002 in the process of formulating a mechanism that *produces* such a formal structure.

The mechanism is a certain mathematically natural 2d nonlinear model (2d-NLM), called the λ -*model*, whose target manifold is the space of classical space-time fields which describe the classical string backgrounds = the space of 2d coupling constants of the string worldsheet.

At every $L \gg 1$, the λ -model produces a *quantum* string background consisting of

- an effective QFT(L) in the form of a functional measure on the manifold of space-time fields with UV cutoff L
- an effective 2d-QFT of the string worldsheet which gives an effective string S-matrix(L) with IR cutoff L .

I suggest exploring the λ -model because

1. It produces consistent realizations of the formal structure described above: a QFT(L) and an S-matrix(L) for $L \gg 1$.
2. The 2d mechanism that produces QFT(L) does not necessarily correspond to canonical quantization.

There are concrete possibilities of non-perturbative 2d effects in the λ -model — winding modes and 2d instantons — that will produce non-canonical degrees of freedom and non-canonical interactions in QFT(L).

These possibilities arise when the space-time fields include SU(2) and SU(3) gauge fields.

So there are concrete possibilities of testable conditional predictions of the form: if QFT(L) contains the SM, then it predicts such and such non-canonical degrees of freedom and such and such non-canonical interactions.

The 2d-RG as a mechanism for space-time physics (1977–79)

In the general renormalizable 2d-NLM

$$\int \mathcal{D}X e^{-\int d^2z g_{\mu\nu}(X) \partial X^\mu \bar{\partial} X^\nu} \quad X(z, \bar{z}) \in M$$

the coupling constants are given by a Riemannian metric $g_{\mu\nu}(X)$ on a manifold M .

The 2d-RG

$$\Lambda \frac{\partial}{\partial \Lambda} g_{\mu\nu}(X) = -R_{\mu\nu}(X) + O(R^2)$$

drives the 2d-NLM to a solution of $R_{\mu\nu} = 0$

This was extremely exciting (at least for me).

The 2d-RG is a *mechanism* that *produces* solutions of a GR-like space-time field equation $R_{\mu\nu} = 0$.

It suggested the possibility of actually answering the question

Where does space-time field theory come from?

or even

Where do the laws of physics come from?

with a quite unexpected mechanism: the 2d-RG.

By the late 1970s it had become clear that there are too many effective QFTs. A mechanism was needed that would *produce* effective QFTs more selectively than the QFT RG.

The 2d-RG seemed promising in that it was a mechanism that at least produced *classical* field theory.

The 2d-RG incorporated into string theory (1981–85)

- The 2d-RG fixed point equation $\beta = 0$ as consistency condition for the string S-matrix recipe (2d scale invariance)
- A string background as a 2d-NLM of the worldsheet with degrees of freedom $X^\mu(z, \bar{z})$ etc. such that the 2d coupling constants are the space-time metric $g_{\mu\nu}(X)$ plus non-abelian gauge fields, scalar fields, fermion fields, etc.
- The $\beta = 0$ equation of this 2d-NLM (generalizing $R_{\mu\nu} = 0$) as a semi-realistic classical field equation which includes GR and potentially the SM
- The string S-matrix at low momentum agrees with the S-matrix of the perturbative canonical quantization of the classical field equation $\beta = 0$.

Questions (1987)

1. $\beta = 0$ is a only consistency condition for the string recipe.
How does the 2d-RG act in string theory as a *mechanism*?
2. Where does *quantum* field theory come from?
What produces a functional integral over space-time fields?
3. What is the *quantum* string background, which should be given by a quantum state of a QFT?
(as opposed to the classical string backgrounds given by classical fields solving $R_{\mu\nu} = 0$.)
4. What can produce an *effective* string S-matrix with IR cutoff in an *effective* quantum background described by an effective QFT with UV cutoff?

These questions led in a direction that departed from the mainstream of string theory, which was then assuming some mathematical idealizations as truisms:

1. The string S-matrix as an asymptotic, idealized S-matrix without IR cutoff — a “theory of everything”.
2. The string backgrounds as the backgrounds for such asymptotic string S-matrices: the solutions of $\beta = 0$, i.e., the Calabi-Yau manifolds ($R_{\mu\nu} = 0$) and generalizations.
3. The assumption that the low momentum physics of string theory *is* the (supersymmetric) QFT that happens to have the same low momentum scattering amplitudes as the asymptotic string S-matrix.

The λ -model (1988-2002)

Consider a 2d-NLM of the string worldsheet, with

$$\begin{aligned}\lambda^i &= \text{the 2d coupling constants,} \\ \phi_i(z, \bar{z}) &= \text{the corresponding 2d scaling fields,} \\ |\phi_i\rangle &= \text{the corresponding states on the circle.}\end{aligned}$$

The index i labels the modes of the space-time fields, e.g.,

$$\phi_i(z, \bar{z}) = e^{ip_\mu(i)X^\mu} h_{\mu\nu}(i) \partial X^\mu \bar{\partial} X^\nu \quad i \leftrightarrow p_\mu(i), h_{\mu\nu}(i)$$

Inserting the perturbation

$$e^{\int d^2z \lambda^i \phi_i(z, \bar{z})}$$

makes $\{\lambda^i\}$ a system of local coordinates on the space of 2d-QFTs.

The 2d scaling-dimensions and the 2d β -function are

$$\dim(\phi_i) = 2 + \delta(i) \quad \dim(\lambda^i) = -\delta(i) \quad \dim(\lambda^i \phi_i) = 2$$

$$\beta^i(\lambda) = -\delta(i)\lambda^i + O(\lambda^2)$$

where

$$\delta(i) = p(i)^2$$

The marginal couplings

$$\dim(\lambda^i) = -\delta(i) = 0$$

parametrize the $\beta = 0$ submanifold of 2d-QFTs.

The 2d-RG drives the worldsheet towards the $\beta = 0$ submanifold.

$(ds)^2 = \mu^2 |dz|^2$ is the 2d metric. $\Lambda^{-1} \ll \mu^{-1}$ is a 2d UV cutoff.

The cutoff string propagator (the cutoff 2d-cylinder) is

$$\int_0^{\ln(\Lambda/\mu)} d\tau \left(\sum_i |\phi_i\rangle e^{-\tau\delta(i)} \langle\phi_i| \right) = \sum_i |\phi_i\rangle \frac{1 - e^{-L^2\delta(i)}}{\delta(i)} \langle\phi_i|$$

where

$$e^{-L^2\delta(i)} = (\Lambda/\mu)^{-\delta(i)} \quad L^2 = \ln(\Lambda/\mu)$$

The only propagating modes are those satisfying

$$\delta(i) > L^{-2} \quad \text{which is} \quad p(i)^2 > L^{-2}$$

So the 2d UV cutoff Λ^{-1} is an IR cutoff L on the string S-matrix.

An effective 2d-QFT with 2d UV cutoff Λ^{-1} gives an effective string S-matrix(L) with $L^2 = \ln(\Lambda/\mu)$.

What are the effective 2d coupling constants at 2d scale Λ^{-1} ?

The perturbations at 2d scale Λ^{-1} are parametrized by microscopic coupling constants $\lambda^i(\Lambda)$.

Their effects are suppressed by the 2d-RG running from Λ^{-1} to μ^{-1}

$$\lambda^i(\mu) = (\Lambda/\mu)^{-\delta(i)} \lambda^i(\Lambda) = e^{-L^2\delta(i)} \lambda^i(\Lambda) \quad \dim(\lambda^i) = -\delta(i)$$

If $L^2\delta(i) > 1$ then $\lambda^i(\Lambda)$ is effectively irrelevant.

The effectively marginal couplings at 2d scale Λ^{-1} are those that satisfy

$$\delta(i) < L^{-2} \quad \text{which is} \quad p(i)^2 < L^{-2}$$

So there is a UV cutoff L on the modes of the space-time fields that describe the effective 2d-QFT.

Now let the λ^i vary on the worldsheet, becoming sources $\lambda^i(z, \bar{z})$.
(Wait a bit for the rationale.)

Make the $\lambda^i(z, \bar{z})$ fluctuate at 2d distances $< \Lambda^{-1}$, governed by the 2d-NLM

$$\int \mathcal{D}\lambda \ e^{-\int d^2z \ g_{\text{str}}^{-2} G_{ij}(\lambda) \partial\lambda^i \bar{\partial}\lambda^j} \ e^{\int d^2z \ \lambda^i(z, \bar{z}) \phi_i(z, \bar{z})}$$

where

- $G_{ij}(\lambda)$ = the natural metric on the manifold of 2d-QFTs
- g_{str} = the string coupling constant
- $\lambda^i(z, \bar{z}) \in \mathcal{M}$ = the target space
 - = the manifold of classical string backgrounds
 - = the manifold of worldsheet 2d-QFTs
 - = the manifold of classical space-time fields.

This 2d-NLM is the λ -model.

on the 2d distance scale:



$$L^2 = \ln(\Lambda/\mu) \gg 1$$

The λ -fluctuations at 2d distances $< \Lambda^{-1}$ produce an effective 2d-QFT with UV cutoff Λ^{-1} .

This effective 2d-QFT in turn gives an effective string S-matrix(L) with IR cutoff L .

The λ -model is designed precisely to implement the "S-matrix RG".

The λ -fluctuations are designed precisely to replicate the froth of small handles.

Integrating out the λ -fluctuations at 2d scales from Λ'^{-1} to Λ^{-1}

$$\Lambda'^{-1} < \Lambda^{-1} \quad L' > L$$

takes the effective S-matrix(L') with the larger IR cutoff L' to the effective S-matrix(L) with the smaller IR cutoff L

$$\text{S-matrix}(L') \supset \text{S-matrix}(L)$$

by, in effect, integrating out the froth of small handles at 2d scales between Λ'^{-1} and Λ^{-1} , thereby integrating out the string modes with $p(i)^2$ from L'^{-2} up to L^{-2} .

(The basic calculation is shown in the Appendix.)

The λ -model (like any 2d-NLM) is specified by two pieces of data

- the metric $g_{\text{str}}^{-2} G_{ij}(\lambda)$ on the target manifold
- a measure $d\lambda \rho(\lambda)$ on the target manifold \mathcal{M} which gives the functional volume element

$$\int \mathcal{D}\lambda = \prod_{(z, \bar{z})} \int_{\mathcal{M}} d\lambda(z, \bar{z}) \rho(\lambda(z, \bar{z}))$$

$d\lambda \rho(\lambda)$ is called the *a priori* measure.

At 2d scale Λ^{-1} , a point (z, \bar{z}) represents a 2d block $\Lambda^{-1} \times \Lambda^{-1}$. The measure $d\lambda \rho(\lambda)$ summarizes the fluctuations inside a block.

$d\lambda \rho(\lambda)$ evolves under the 2d-RG, diffusing in the target manifold \mathcal{M} due to the λ -fluctuations. At the same time, it is driven by the beta-function $\beta^i(\lambda)$ because the λ^i are not exactly marginal, they flow with the 2d scale Λ^{-1} towards the $\beta = 0$ submanifold.

$d\lambda \rho(\lambda)$ evolves by the driven diffusion equation

$$\Lambda \frac{\partial}{\partial \Lambda} \rho(\lambda) = \nabla_i (g_{\text{str}}^2 G^{ij} \partial_j + \beta^i) \rho(\lambda)$$

(taking $d\lambda$ to be the metric volume element on \mathcal{M}).

$d\lambda \rho(\lambda)$ at scale Λ^{-1} is produced by integrating out the λ -fluctuations from 2d distance ~ 0 up to Λ^{-1} , driving it to the equilibrium measure

$$d\lambda \rho(\lambda) \rightarrow d\lambda e^{-\frac{1}{g_{\text{str}}^2} S(\lambda)} \quad \text{where} \quad \beta^i = G^{ij} \partial_j S$$

Recall that the λ^i are the spacetime field modes with UV cutoff L .

So $d\lambda \rho(\lambda)$ is the functional integral of an effective QFT(L) with classical action $\frac{1}{g_{\text{str}}^2} S(\lambda)$.

Thus the λ -model *produces* a QFT(L) at every $L \gg 1$.

The consistency conditions are satisfied:

$S\text{-matrix}(L) \supset S\text{-matrix}(L')$ for $L > L'$ by design of the λ -model.

$QFT(L') \supset QFT(L)$ via the QFT RG because of the 2d RG — the decoupling of irrelevant operators.

$S\text{-matrix}(L)$ and $QFT(L)$ agree on amplitudes at scale $\sim L$ because the scattering amplitudes of $S\text{-matrix}(L)$ near the IR cutoff L are given by the 2d correlation functions near the 2d UV cutoff Λ^{-1} , which are determined by the *a priori* measure $d\lambda \rho(\lambda)$ which is $QFT(L)$.

The λ -model is a nonperturbative 2d-NLM, with possibilities of nonperturbative semi-classical effects:

- winding modes associated to π_1 of the target manifold \mathcal{M}
- 2d instantons associated to $\pi_2(\mathcal{M})$

where $\mathcal{M} =$ the manifold of space-time fields

π_k (the manifold of $SU(N)$ gauge fields in \mathbb{R}^4) = $\pi_{k+3}(SU(N))$

So there are winding modes ($k = 1$) when $\pi_4(SU(N)) \neq 0$

$$\pi_4(SU(2)) = \mathbb{Z}_2$$

and there are 2d instantons ($k = 2$) when $\pi_5(SU(N)) \neq 0$

$$\pi_5(SU(2)) = \mathbb{Z}_2 \quad \pi_5(SU(3)) = \mathbb{Z}$$

The winding modes for $SU(2)$ and the 2d-instantons for $SU(2)$ and $SU(3)$ offer possibilities of conditional predictions.

If the λ -model produces SM+GR then it also produces

- *non-canonical degrees of freedom from the \mathbb{Z}_2 winding mode in the manifold of $SU(2)$ gauge fields on \mathbb{R}^4*
- *non-canonical interactions from the 2d instanton in the manifold of $SU(2)$ gauge fields and the 2d instantons in the manifold of $SU(3)$ gauge fields on \mathbb{R}^4*

\mathbb{Z}_2 winding mode in the manifold of $SU(2)$ gauge fields on \mathbb{R}^4

Let $A(x_+, x_-, u)$ be a zero-size instanton at x_+ and a zero-size anti-instanton at x_- with relative orientation $u \in SU(2)/\mathbb{Z}_2$.

$$\mathcal{L}(x) = \frac{1}{8\pi} \text{tr} (F_{\mu\nu}(x) F^{\mu\nu}(x)) = \delta^4(x - x_+) + \delta^4(x - x_-)$$

$$\mathcal{L}^{\text{top}}(x) = \frac{1}{8\pi} \text{tr} (F_{\mu\nu}(x) * F^{\mu\nu}(x)) = \delta^4(x - x_+) - \delta^4(x - x_-)$$

The winding mode representing the nontrivial element in $\pi_1 = \mathbb{Z}_2$ is the nontrivial closed geodesic loop in $SU(2)/\mathbb{Z}_2$

$$\theta \mapsto A(x_+, x_-, u(\theta)) \quad u(0) = 1 \quad u(2\pi) = -1$$

This loop in \mathcal{M} has zero length, so the winding mode will be a 2d field of scaling dimension = 0 + quantum corrections, so it has a chance of participating in the *a priori* measure which is the space-time QFT.

The \mathbb{Z}_2 winding mode is *bi-local* in space-time, depending on the two space-time points x_+ , x_- .

The 2d instantons for $SU(2)$ and $SU(3)$ gauge fields are nontrivial 2-spheres in slightly more complicated configurations of zero-sized instantons and anti-instantons.

To do:

A huge amount of foundational technical work is still to be done.

More urgent is to find out if the λ -model can in fact make conditional predictions of observable non-canonical effects in $SU(2)$ and $SU(3)$ gauge theory in 4 dimensions.

1. Figure out how to calculate semi-classical corrections to the *a priori* measure of a 2d-NLM coming from winding modes and 2d instantons.
2. Calculate the corrections to the canonical SM
 - from the bi-local winding mode in the manifold of $SU(2)$ gauge fields.
 - from the multi-local 2d instantons in the manifolds of $SU(2)$ and $SU(3)$ gauge fields

Especially tantalizing is the top-down construction of $\text{QFT}(L)$.

The λ -model operates from 2d distance ~ 0 up to Λ^{-1} .

Recall that $L^2 = \ln(\Lambda/\mu)$.

So the λ -model builds $\text{QFT}(L)$ from space-time distance $\sim \infty$
down to L .

Unnaturalness could be natural in $\text{QFT}(L)$.

Appendix

- The λ -model as S-matrix RG (the basic calculation)
- Some philosophy
- Some motivations