

Title: Phases of Gravitational Collapse in AdS

Date: May 23, 2018 11:00 AM

URL: <http://pirsa.org/18050063>

Abstract: <p>Generically, a small amount of matter introduced to anti-de Sitter spacetime leads to formation of a black hole; however, the high degree of symmetry of AdS means that some initial distributions of matter (possibly also technically generic) oscillate indefinitely. Whether a given initial profile leads to a horizon at arbitrarily small amplitudes is of great interest for a number of reasons, not least because horizon formation corresponds holographically to thermalization in CFT. We will present an overview of approaches the question and show a phase diagram of the stability behavior of AdS in the presence of a scalar field, including an analysis of nonperturbative phases at the border of perturbative stability and instability.</p>

A Process Theoretic Reconstruction of Quantum Theory

John H. Selby, Carlo Maria Scandolo & Bob Coecke
arXiv:1802.00367

What is a reconstruction?

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Set of postulates $\{P_i\}$ along with a proof that

$$P_0 \wedge \cdots \wedge P_n \iff \text{Quantum Theory}$$

Why reconstruct quantum theory?

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 - ▶ c.f. special relativity

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 - ▶ QT: axioms are too abstract to be counter-intuitive

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- ▶ Restarted by Lucien Hardy 2001 "*Quantum theory from five reasonable axioms*" (arXiv:quant-ph/0101012).

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- ▶ Conceptually consistent
 - ▶ operational
 - ▶ informatic
 - ▶ logic-based
 - ▶ ...

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but still to be...

- ▶ Mathematically precise
- ▶ Simple and as easy to use as standard postulates

What we do differently?

What we do differently?

We work in the framework of

Process Theories

Introduction

Process theories

The reconstruction

Conclusion

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Process theories

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Conclusion

The development of process theories

Linear algebra \rightarrow Category theory \rightarrow Diagrams \rightarrow Process theories

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e.g. entanglement swapping

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$$\frac{1}{2} \langle \Psi^+ |_{A,C} \langle \Psi^+ |_{C,B}$$
 where $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$$\rho_{A,C,B} = |\Psi^+\rangle\langle\Psi^+|_{A,C} = |\Psi^+\rangle\langle\Psi^+|_{C,B} = \frac{1}{4} \left[\begin{array}{l} |000\rangle\langle 000| + |000\rangle\langle 011| \\ + |001\rangle\langle 000| + |001\rangle\langle 011| \\ + |011\rangle\langle 000| + |011\rangle\langle 011| \\ + |010\rangle\langle 000| + |010\rangle\langle 011| \\ + |100\rangle\langle 000| + |100\rangle\langle 011| \\ + |110\rangle\langle 000| + |110\rangle\langle 011| \\ + \text{some other terms} \end{array} \right]$$

$$\text{tr}_{C,B} \left(|\Psi^+\rangle\langle\Psi^+|_{C,C} \rho_{A,C,B} \right)$$

$$= \text{tr}_{C,B} \left(\frac{1}{2} \left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11| \right) \rho_{A,C,B} \right)$$

$$= \frac{1}{2} \langle 00 | \rho_{A,C,B} | 00 \rangle + \frac{1}{2} \langle 00 | \rho_{A,C,B} | 11 \rangle + \frac{1}{2} \langle 11 | \rho_{A,C,B} | 00 \rangle + \frac{1}{2} \langle 11 | \rho_{A,C,B} | 11 \rangle$$

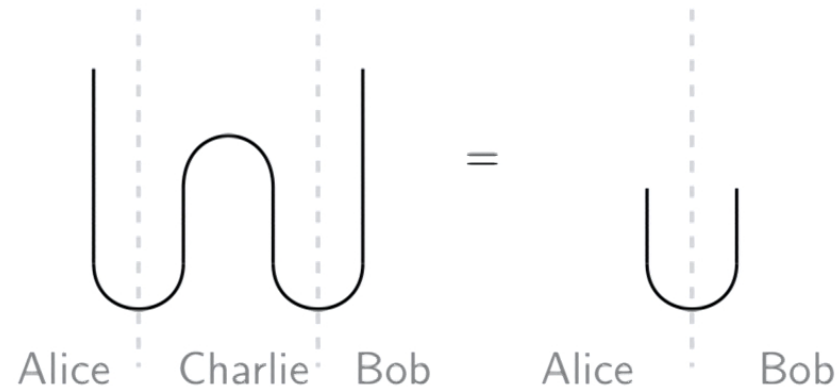
$$= \frac{1}{2} [|00\rangle\langle 00| + |00\rangle\langle 11| + |00\rangle\langle 11| + |00\rangle\langle 00| + |00\rangle\langle 11| + \dots]$$
 probably

$$= \frac{1}{2} [|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|]$$

$$= \mathbb{1} |\Psi^+\rangle\langle\Psi^+|_{A,B}$$

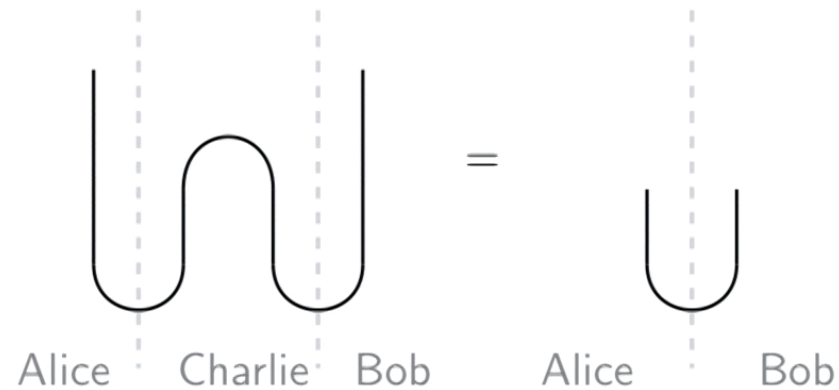
The development of process theories

Linear algebra \rightarrow Category theory \rightarrow Diagrams \rightarrow Process theories
e.g. entanglement swapping



The development of process theories

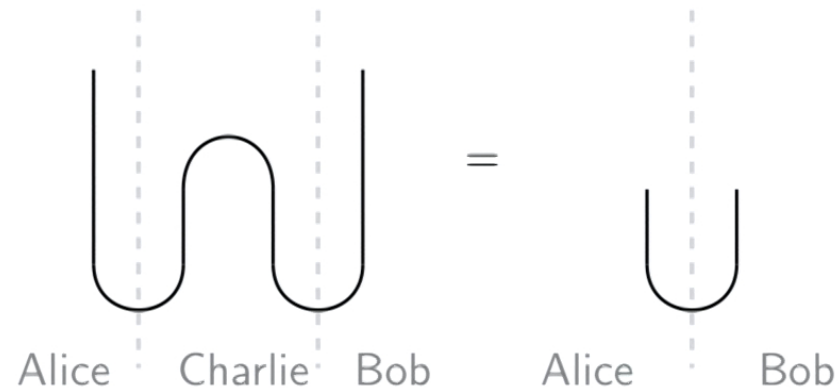
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- ▶ Conceptually appealing
- ▶ Convenient notation
- ▶ Automation

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- ▶ Conceptually appealing
- ▶ Convenient notation
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- ▶ Wide applicability

Conceptual shift

Physics is about...

...*states* and their *evolution*.

Conceptual shift

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~vs.~

...*processes* and their *composition*.

Processes

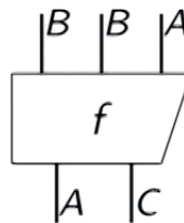
A process theory is defined by a collection of *processes*, e.g.

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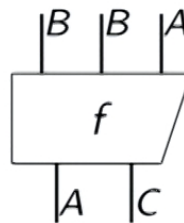
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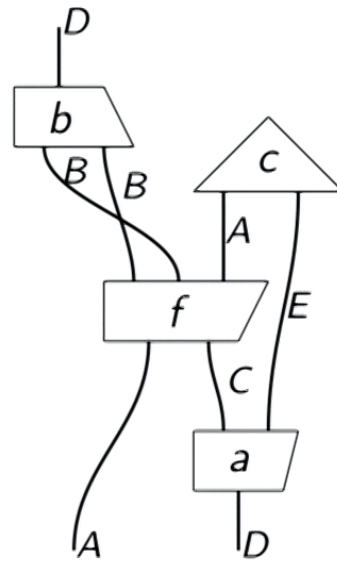


These could represent...

- ▶ Physical processes
- ▶ Mathematical representations of physical processes
- ▶ Operational description of a piece of lab equipment
- ▶ Chemical processes
- ▶ Words
- ▶ Cognitive processes
- ▶ Data processing
- ▶ Abstract mathematical transformations

Composition

These can be *composed* to form *diagrams*, e.g.

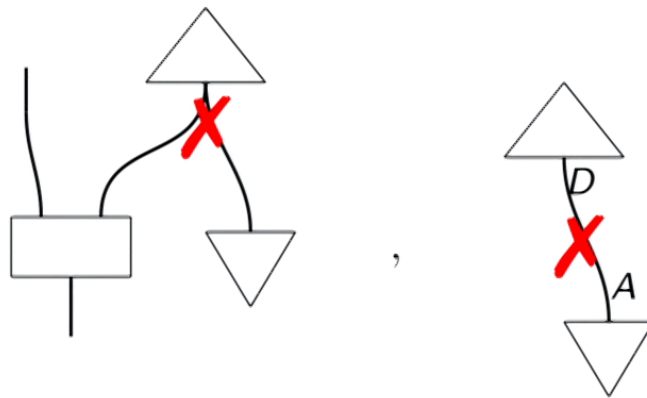


Constraints

This composition isn't completely free...

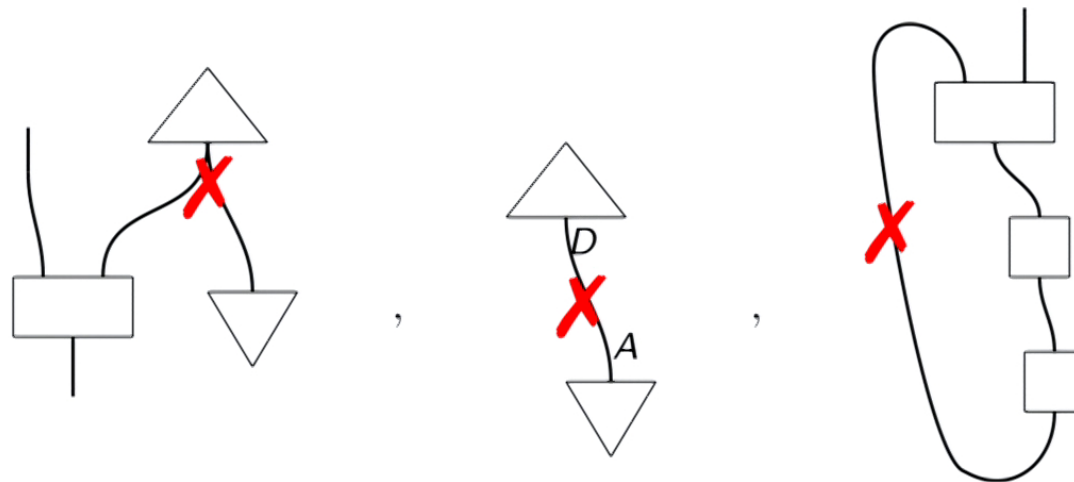
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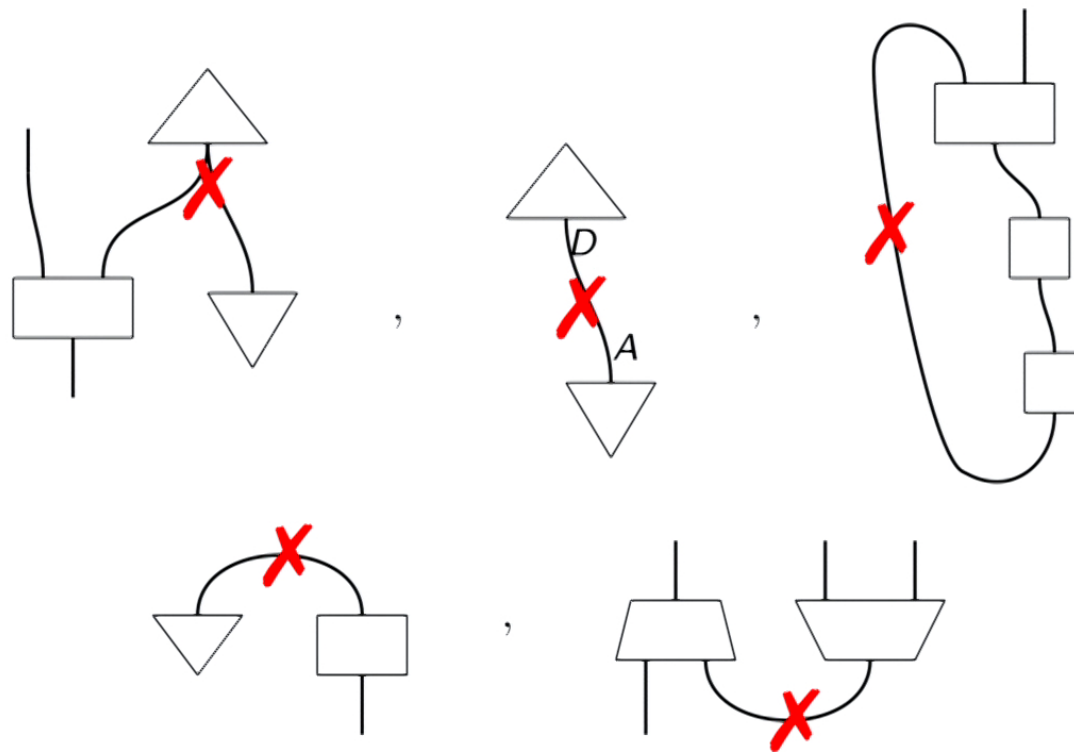
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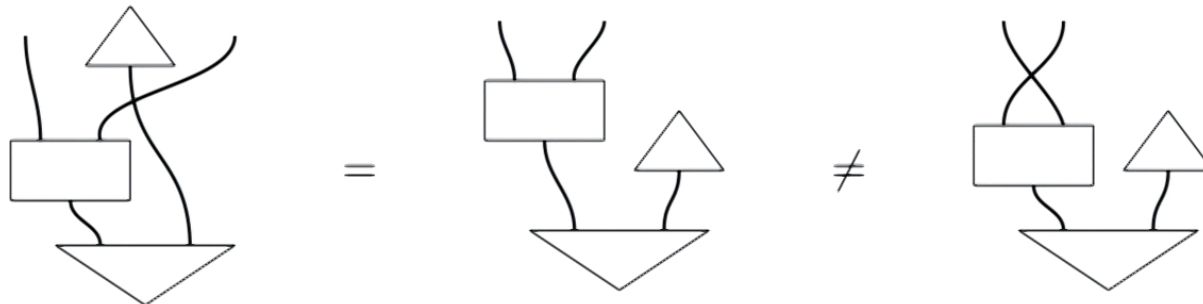
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Diagrammatic rule

Only *connectivity* matters, e.g.



Special processes

Special processes



Symmetries and dualities

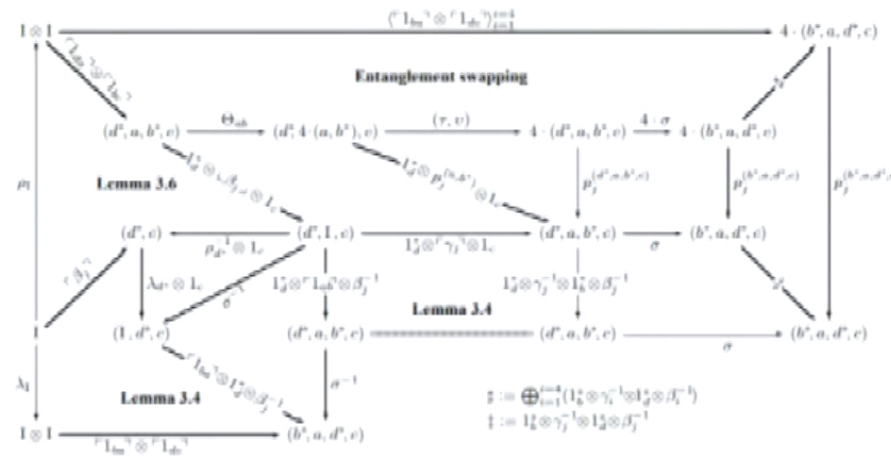
Choi-Jamiołkowski isomorphism a.k.a. bending wires

Is this an isomorphism?

The development of process theories

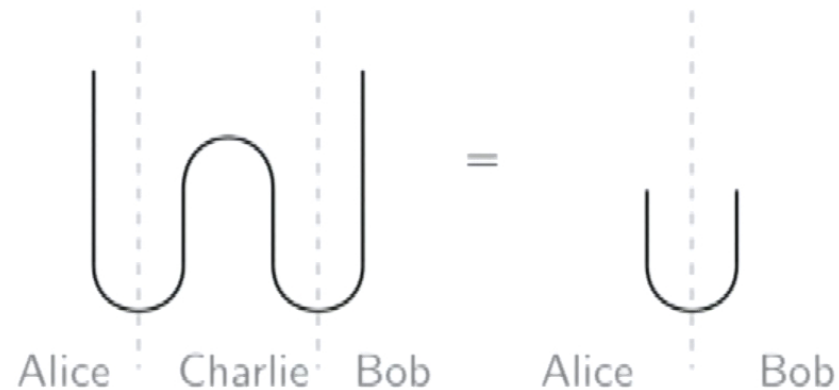
Linear algebra → Category theory → Diagrams → Process theories
e.g. entanglement swapping

Proof of Theorem 9.4 (entanglement swapping). The top trapezoid is the statement of the Theorem. We have a diagram of the form below for each $j \in \{1, 2, 3, 4\}$. To simplify the notation of the types we set (a', b, c', d) for $Q_a^c \otimes Q_b \otimes Q_c^d \otimes Q_d$ etc. We ignore the scalars – which cancel out against each other – in this proof.



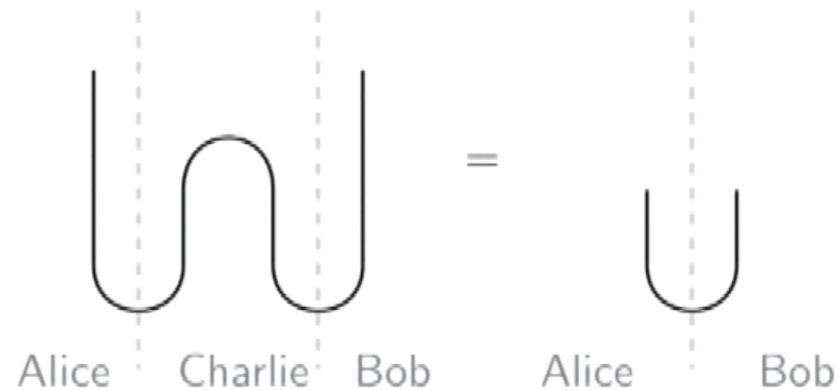
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...*processes* and their *composition*.

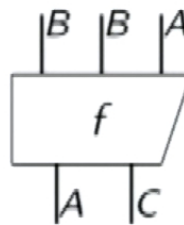
- ▶ novel framework for exploring potential physical theories

Processes

A process theory is defined by a collection of *processes*, e.g.

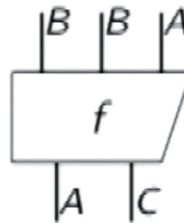
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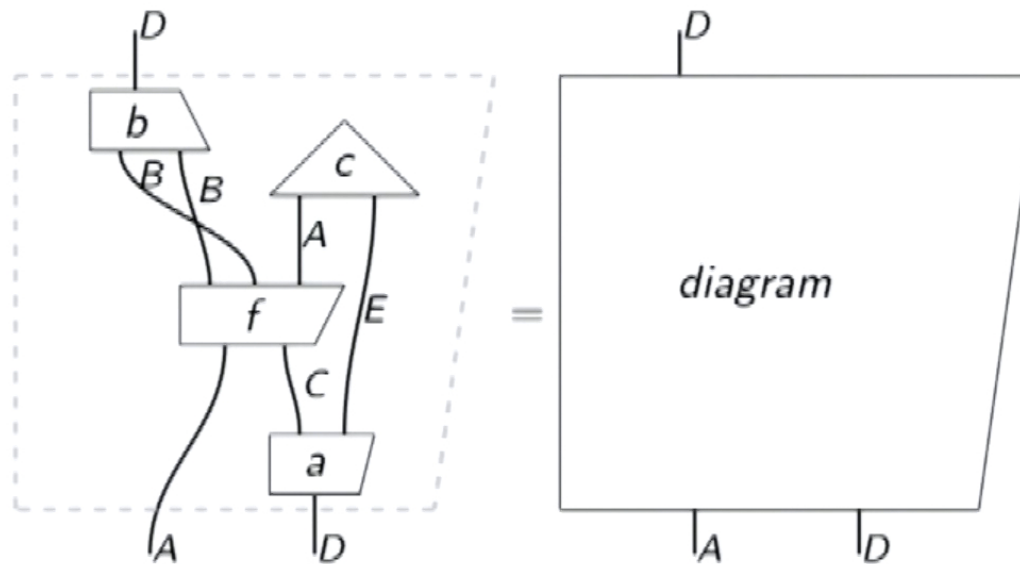


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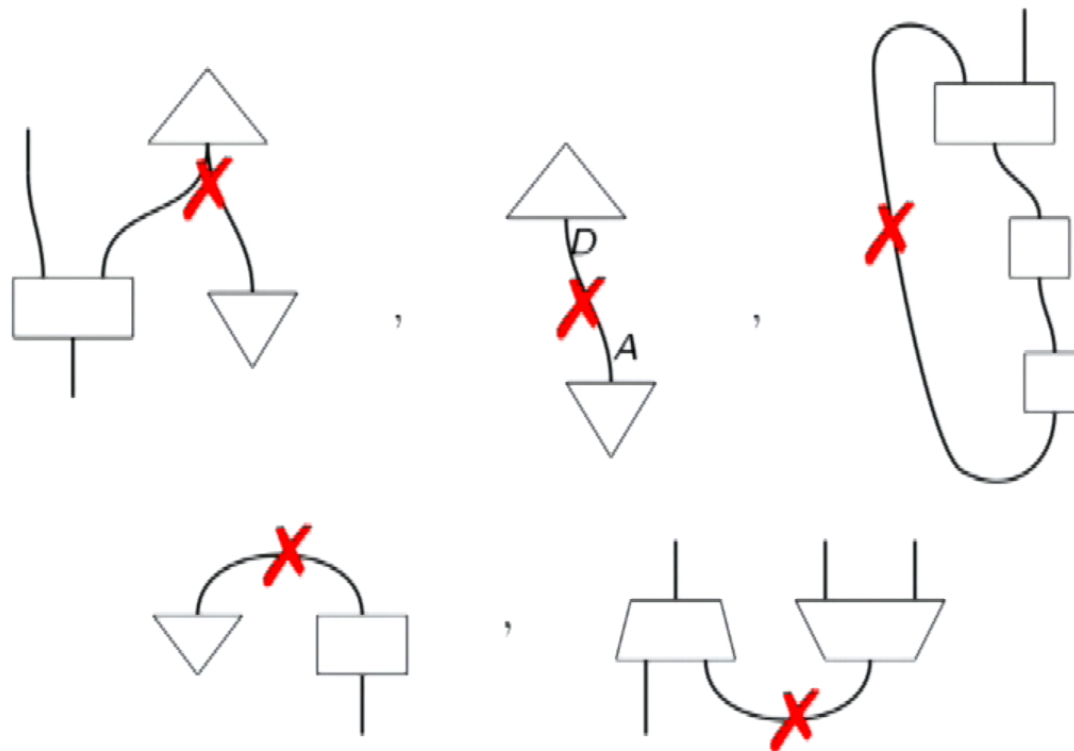
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Such a diagram *must* correspond to another process in the theory.

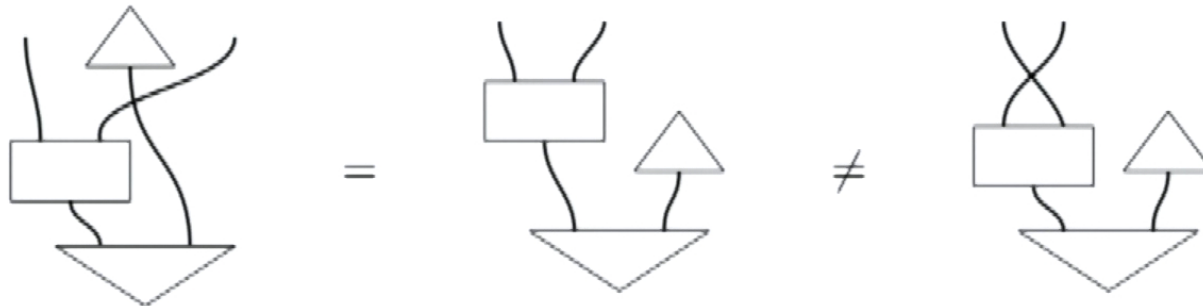
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Only *connectivity* matters, e.g.



Special processes



Examples from quantum foundations

Theory	Processes
QT	CP maps between $\mathcal{B}(\mathcal{H})$
ProbCT	Stochastic maps between sets
C*-Alg	CP maps between C*-algebras
PossCT	Relations between sets
Spek	Subtheory of PossCT

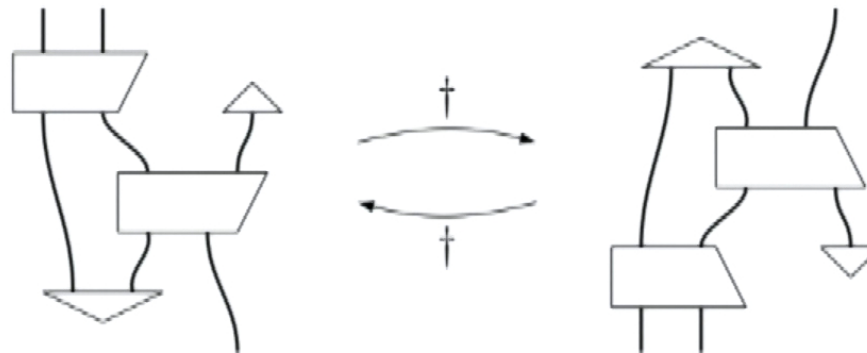
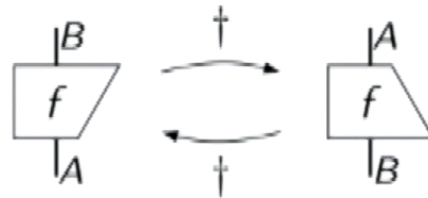
All of these process theories have something in common...

Symmetries and dualities

with an intuitive diagrammatic representation!

Symmetries and dualities

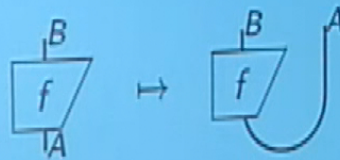
The adjoint a.k.a. the dagger



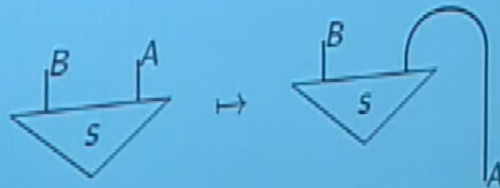
Symmetries and dualities

Choi-Jamiołkowski isomorphism a.k.a. bending wires

From maps to bipartite states:



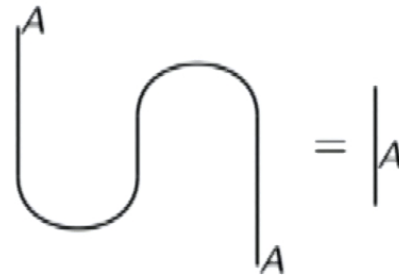
From bipartite states to maps:



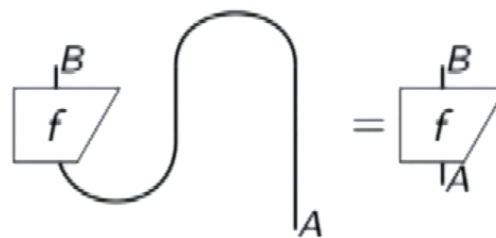
Symmetries and dualities

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Is this an isomorphism? We just need that...



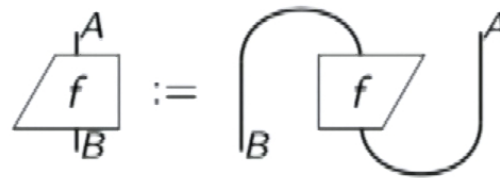
such that, for example,



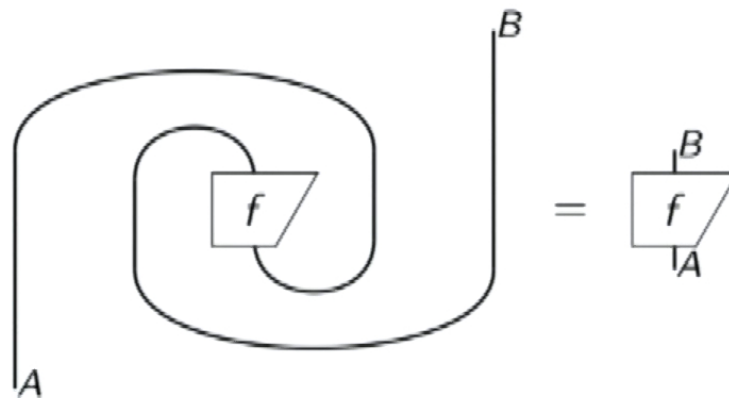
Symmetries and dualities

Transpose from Choi

We can define a transpose as...



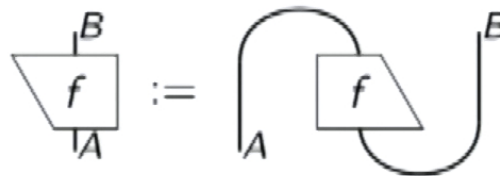
The transpose is idempotent...



Symmetries and dualities

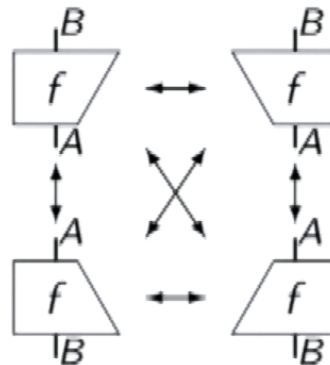
Conjugation from transpose and dagger

We can define generalised conjugation as...

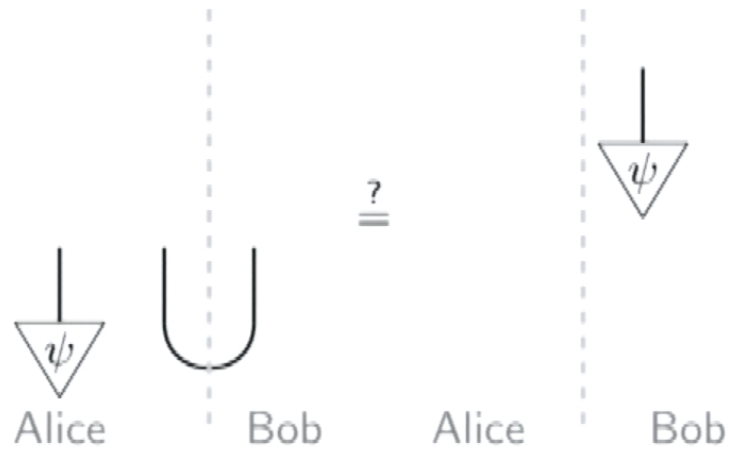


i.e. as the composition of transpose and dagger.

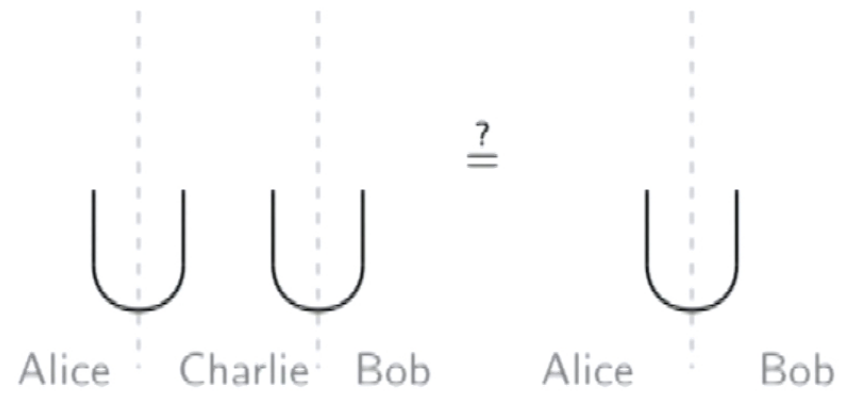
Hence, our example process theories have $Z_2 \times Z_2$ symmetry:



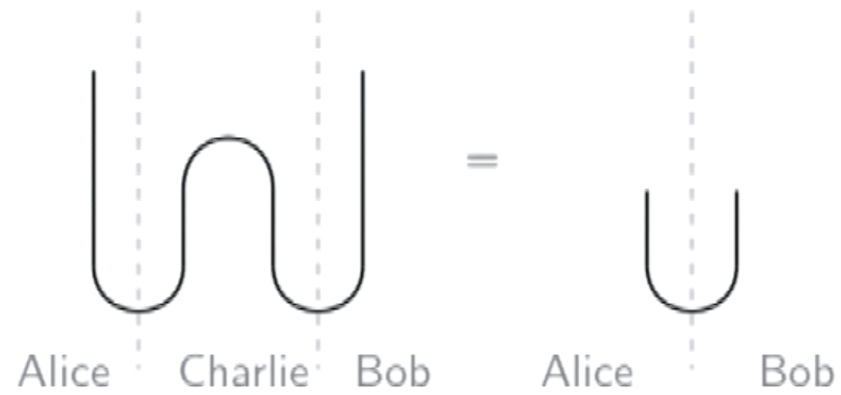
Calculation 1: Teleportation



Calculation 2: Entanglement sharing

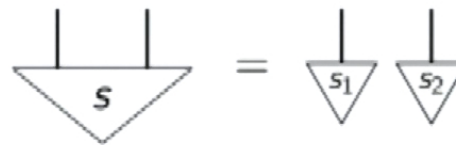


Calculation 2: Entanglement sharing



Calculation 3: Existence of correlated states

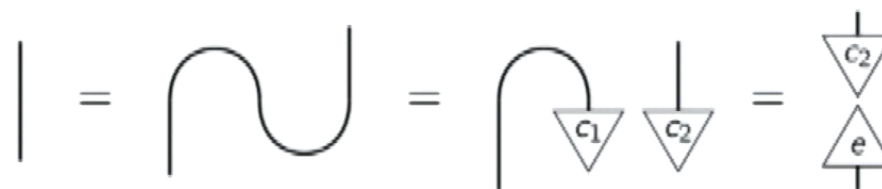
Assume that there are no correlated states



then, in particular, the cup separates:

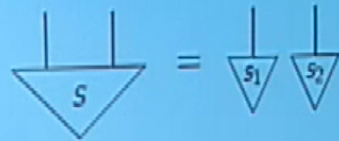


so we have:

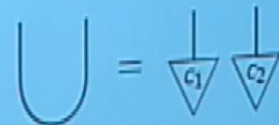


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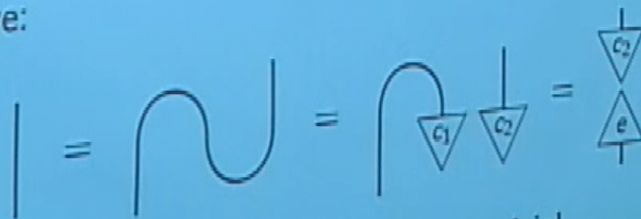
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so we have:



That is, all wires separate and the theory is trivial.

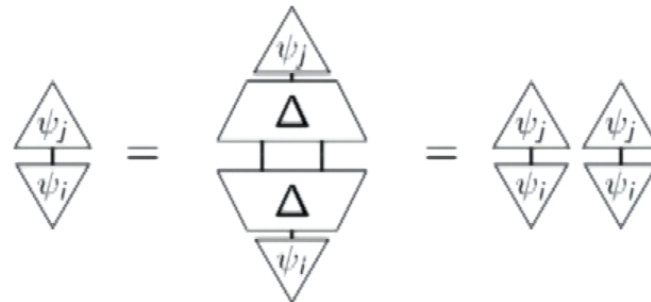


Calculation 4: Orthogonality of clonable states

Suppose we have an isometry Δ which clones the states ψ_i , i.e.



Then...



Hence...

$$\langle \psi_j | \psi_i \rangle \in \{0, 1\}$$

Summary so far

- ▶ Basic introduction to process theories
- ▶ Examples of process theories
- ▶ Diagrammatic representation of important structures
- ▶ Important results with simple diagrammatic proofs
- ▶ Captured some of the important structure of quantum theory

This poses the question:

what else do we need to add to recover all of quantum theory?

Introduction

Process theories

The reconstruction

Conclusion

Postulate 1:

The theory is a process theory.

Postulate 2:

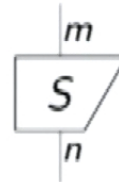
The theory has a probabilistic classical interface.

Postulate 2a:

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Postulate 2a:

- ▶ Classical probability theory is a subtheory,
- ▶ labeled by thin grey lines,
- ▶ such that

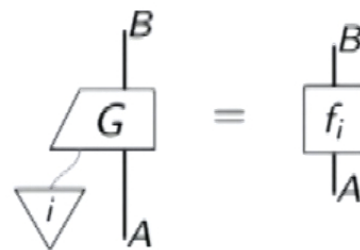


is a stochastic map from an n to an m -level classical system.

Postulate 2b:

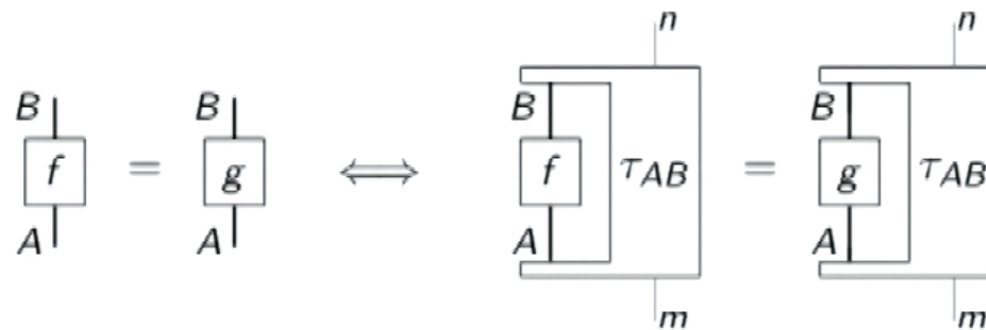
- ▶ Classical control, i.e. how we control the world,
- ▶

For all $\left\{ \begin{array}{c} |B \\ \boxed{f_i} \\ |A \end{array} \right\}_{i=1}^n$ there exists $\begin{array}{c} |B \\ \text{▱} G \\ |A \end{array}$ such that



Postulate 2c:

- ▶ Tomography, i.e. how we learn about the world,
- ▶ For all pairs of systems (A, B) there exists a 'test' τ_{AB} such that



Consequences of the classical interface

- ▶ Processes $A \rightarrow B$ live in convex sets
- ▶ Processes act linearly
- ▶ Unique discarding effect, denoted

$$\overline{\top}_A$$

- ▶ Essentially we have “causal generalised probabilistic theories”¹

¹Up to minor technicalities.

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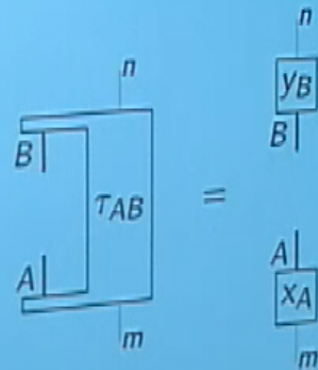
$$\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ | \\ A \end{array}$$

- ▶ Essentially we have “causal generalised probabilistic theories”¹
- ▶ We've reached the starting point of most other reconstructions

¹Up to minor technicalities.

Postulate 2c':

- ▶ Tomography can be performed *locally*,
- ▶ i.e. we can always choose τ_{AB} such that

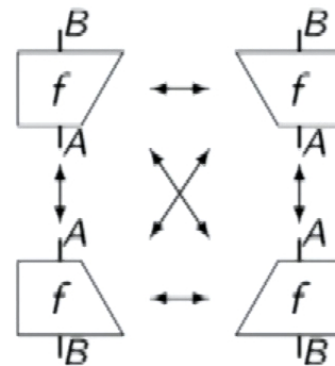


Postulate 3:

The theory has the fundamental diagrammatic symmetries.

Postulate 3:

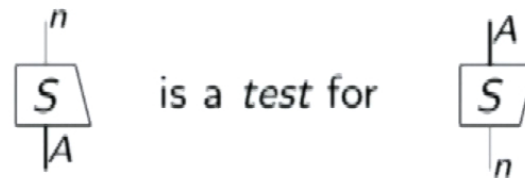
- ▶ a dagger,
- ▶ bending wires,
- ▶



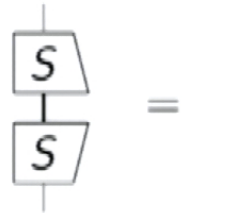
however, we need something more to ensure that the \dagger corresponds to the 'right' dagger, namely...

Postulate 3':

- ▶ *Sharpening the dagger,*



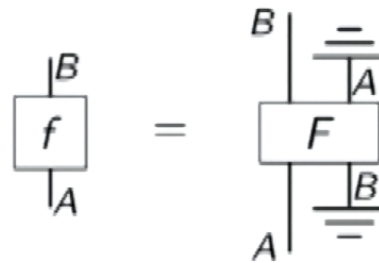
that is, if the states are 'testable' then



and if n is 'maximal' then the test is a 'complete' measurement.

Postulate 4:

- ▶ All processes have dagger-symmetric purifications:



Postulate 4:

- ▶ All processes have dagger-symmetric purifications:

$$\begin{array}{c} |B \\ \square \\ f \\ \square \\ |A \end{array} = \begin{array}{c} |B \\ \square \\ F \\ \square \\ |A \end{array} \begin{array}{c} \overline{\overline{|A}} \\ \overline{\overline{|B}} \end{array}$$

- ▶ which moreover are essentially unique.

Consequences of the postulates

- ▶ Processes $A \rightarrow B$ live in convex cones which are
 - ▶ homogeneous
 - ▶ strongly self dual
 - ▶ i.e. systems correspond to Euclidean Jordan Algebras
- ▶ the only EJAs that compose 'correctly' are C^* -algebras, therefore...

Processes are CP-maps between finite dimensional C^* -algebras
a.k.a.

the process-theoretic description of quantum theory!

Summary

Given

- ▶ the process theory framework,
- ▶ a finite local probabilistic classical interface,
- ▶ diagrammatic symmetries where, in particular,
- ▶ the dagger is sharp, and
- ▶ that processes have dagger-symmetric purifications,

we have reconstructed the process theoretic description of quantum theory.

Future work

- ▶ Can we go beyond finite dimensional quantum theory?
 - ▶ e.g. following the use of non-standard analysis of Stefano Gogioso & Fabrizio Genovese (arXiv:1703.09594)
- ▶ Can we avoid invoking a classical interface?
 - ▶ e.g. following the categorical reconstruction of Sean Tull (arXiv:1804.02265)