Title: The Measurement Postulates of Quantum Mechanics are Redundant

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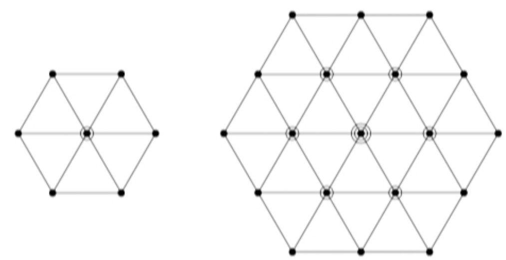
Abstract: <p>In order to think about the foundations of physics it is important to understand the logical relationships among the physical principles that sustain the building. As part of these axioms of physics there is the core hypothesis that, how the Universe is partitioned into systems and subsystems is a subjective choice of the observer that should not affect the predictions of physics. Other foundational principles are the Postulates of Quantum Mechanics. However, we prove that these are not independent from the â€œindependence of subsystem partitioningâ€• hypothesis described above. In particular, we prove that the mathematical structure of quantum measurements and the formula for assigning outcome probabilities are implied by the mentioned hypothesis and the rest of quantum postulates. </p>

The measurement postulates are redundant

Thomas Galley, Lluis Masanes, University College London



Postulates of QT

States

$$\psi \in \mathrm{P}\mathbb{C}^d$$

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Composite states

$$\mathbb{C}^d = \mathbb{C}^a \otimes \mathbb{C}^b$$

Outcome probability functions

$$\mathbf{f} \in \mathcal{F}_d = \{P(\text{outcome}|\psi) : \forall \text{ outcome}\}$$

$$\mathbf{f} : \mathrm{P}\mathbb{C}^d \to [0,1] \quad \text{continuous}$$

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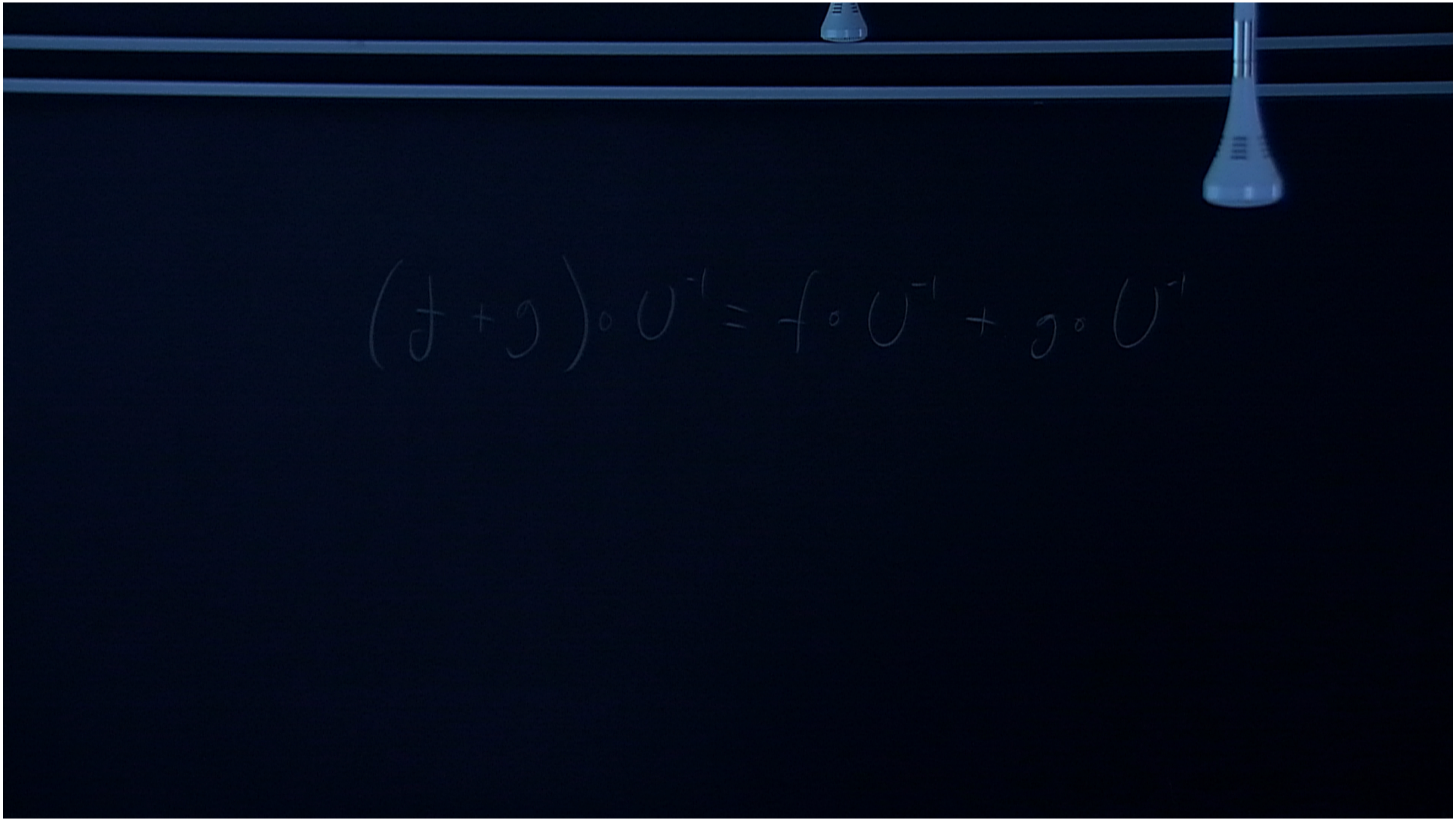
$$\mathbf{f} \mapsto \mathbf{f} \circ U^{-1} \quad \text{continuous and linear}$$


(f + g)∘U⁻¹ = f∘U⁻¹ + g∘U⁻¹

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$$V_d = \bigoplus_{\lambda} \mathbb{S}_\lambda \mathbb{C}^d$$

Outcome probability functions

$$U\psi = \psi$$

$$U \in \mathrm{SU}(d-1) \times \mathrm{U}(1) \subseteq \mathrm{SU}(d)$$

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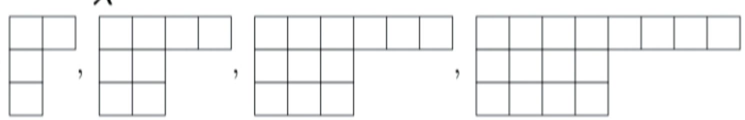
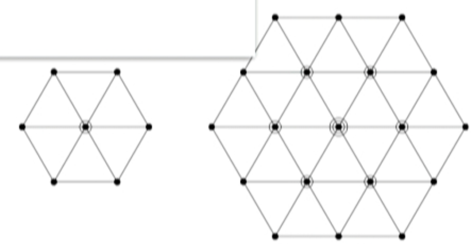
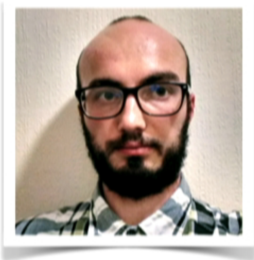
$\lambda =$ 



All alternatives to the Measurement Postulates for single systems



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Alternative Measurement Postulates in composite systems

Measurements

Composition

$$\star : \mathcal{F}_a \times \mathcal{F}_b \to \mathcal{F}_{ab} \qquad \text{bilinear}$$

$$(\mathbf{f} \star \mathbf{g})(\psi \otimes \phi) = \mathbf{f}(\psi)\mathbf{g}(\phi)$$

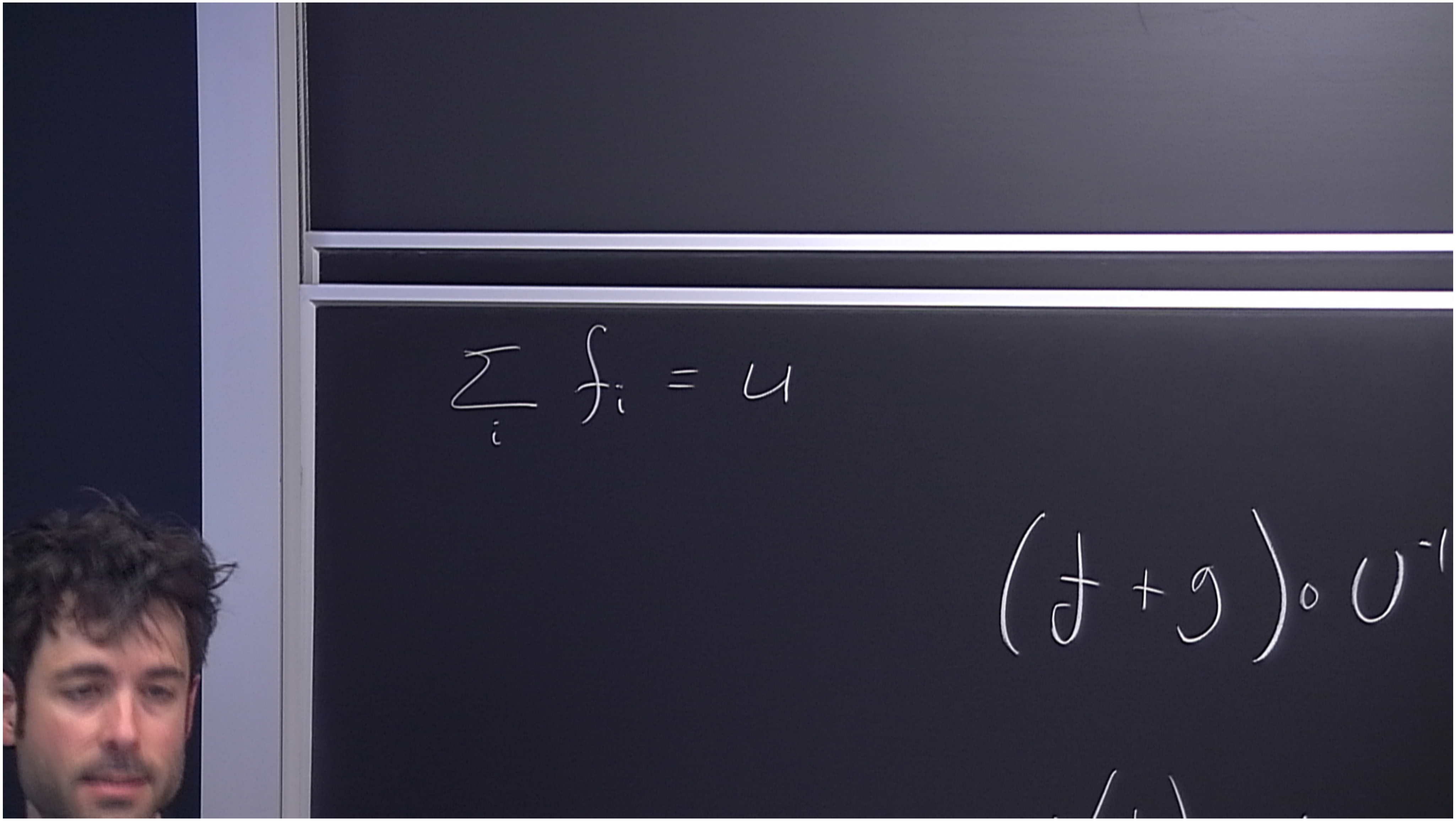
$$\mathbf{u}_{\mathsf{A}} \star \mathbf{u}_{\mathsf{B}} = \mathbf{u}_{\mathsf{AB}}$$

$$\forall\ \mathbf{f} \in \mathcal{F}_{ab}\ \exists\ \mathbf{g} \in \mathcal{F}_a : \mathbf{f}(\cdot \otimes \phi) = \mathbf{g}(\cdot)$$

$$(\mathbf{f} \star \mathbf{g}) \star \mathbf{h} = \mathbf{f} \star (\mathbf{g} \star \mathbf{h})$$

$$(f+g)\circ U^{-1} = f\circ U^{-1} + g\circ U^{-1}$$

$$u(f) = 1$$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Alternative Measurement Postulates in composite systems

$$V_{abc}|_{\mathrm{SU}(a)\otimes\mathrm{SU}(bc)} = \bigoplus_{(\lambda,\mu)} \mathbb{S}_\lambda\mathbb{C}^a \otimes \mathbb{S}_\mu\mathbb{C}^{bc}$$

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$\mathcal{F}_a$ $\mathcal{F}_{bc}$

$$\mathcal{F}_a \star \mathbf{u}_\mathsf{B} \star \mathbf{u}_\mathsf{C} \text{ is } \mathrm{SU}(bc) \text{ invariant } \Rightarrow \mathbf{u}_\mathsf{A} \star \mathbf{u}_\mathsf{B} \star \mathcal{F}_c$$

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Contradiction!

Theorem

$$\mathcal{F}_d = \{P(\text{outcome}|\psi) : \forall \text{ outcome}\}$$

$$\star : \mathcal{F}_a \times \mathcal{F}_b \to \mathcal{F}_{ab} \qquad \text{bilinear}$$

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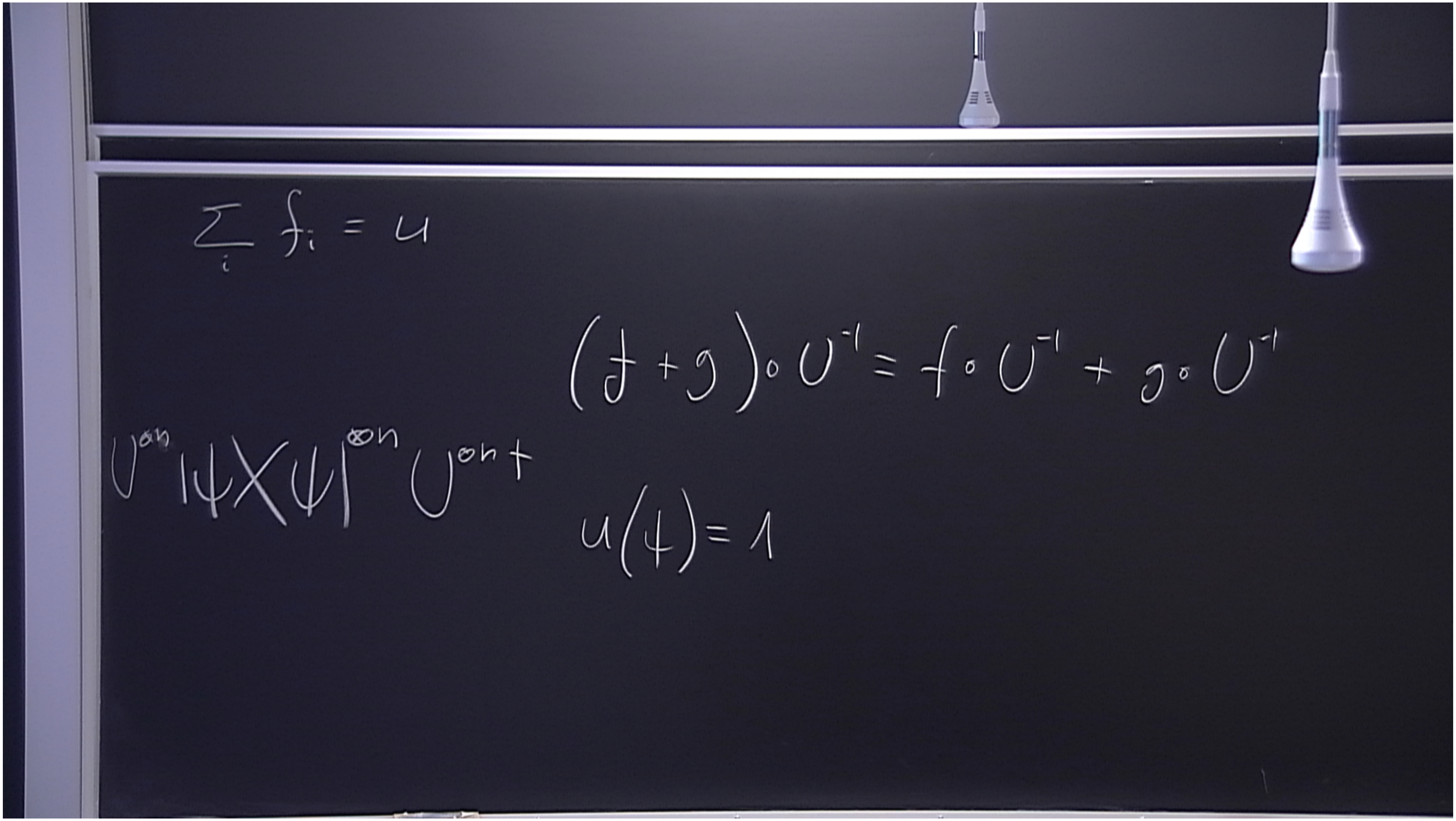
$\sum_i f_i = u$
$(f+g)\circ U^{-1} = f\circ U^{-1} + g\circ U^{-1}$
$U^{\otimes n}|\psi\rangle\langle\psi|^{\otimes n}U^{\otimes n\dagger}$
$u(\psi) = 1$

The Measurement Postulates of Quantum Theory are Redundant!

Is this a substitution of postulates?

- Associativity is weaker than the Born Rule.

Is this a substitution of postulates?

- Associativity is weaker than the Born Rule.
- Separate different categories of physical principles
  - What is a general probabilistic theory
  - Which general probabilistic theory

Relaxing continuity of OPFs

- We could assume finite dimension of $V_d = \bigoplus_{\lambda} \mathbb{S}_\lambda \mathbb{C}^d$
- This dimension is the number of parameters that are required to specify a mix state.

The Projection Postulate doesn't seem to be redundant.

Long tradition of derivations of the measurement postulates.

Relative-state interpretation

Everett, Hartle, DeWitt,
Graham, Geroch,

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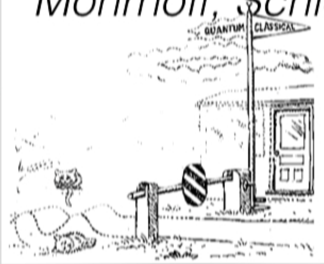

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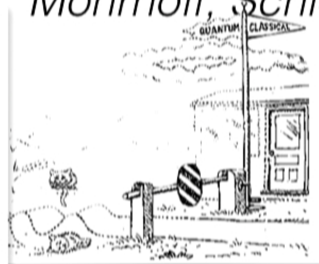

Critics

Zeh, Kent

Reward availability: All rewards are available to the agent at any macrostate: that is, the set of available acts always includes ones which give all of the agent's future selves the reward.

Branching availability: Given any set of positive real numbers $p_1, \ldots p_n$ summing to unity, an agent can always choose some act which has n different macrostates as possible outcomes, and gives weight p_i to the ith outcome.

Erasure: Given a pair of states $\psi \in E$ and $\varphi \in F$ in the same reward, there is an act $\widehat{U}$ available at E and an act $\widehat{V}$ available at F such that $\widehat{U}\psi = \widehat{V}\varphi$.

Problem continuity: For each event E, the set of acts available at E is an open subset of the set of unitary transformations from E to $\mathcal{H}$.

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(1) The probability for a particular outcome, *i.e.*, for the occurence of a specific value of a measured physical quantity, is identified with the probability for the eigenstate of the measured observable with eigenvalue corresponding to the measured value—an assumption that would follow from the *eigenvalue–eigenstate link*.

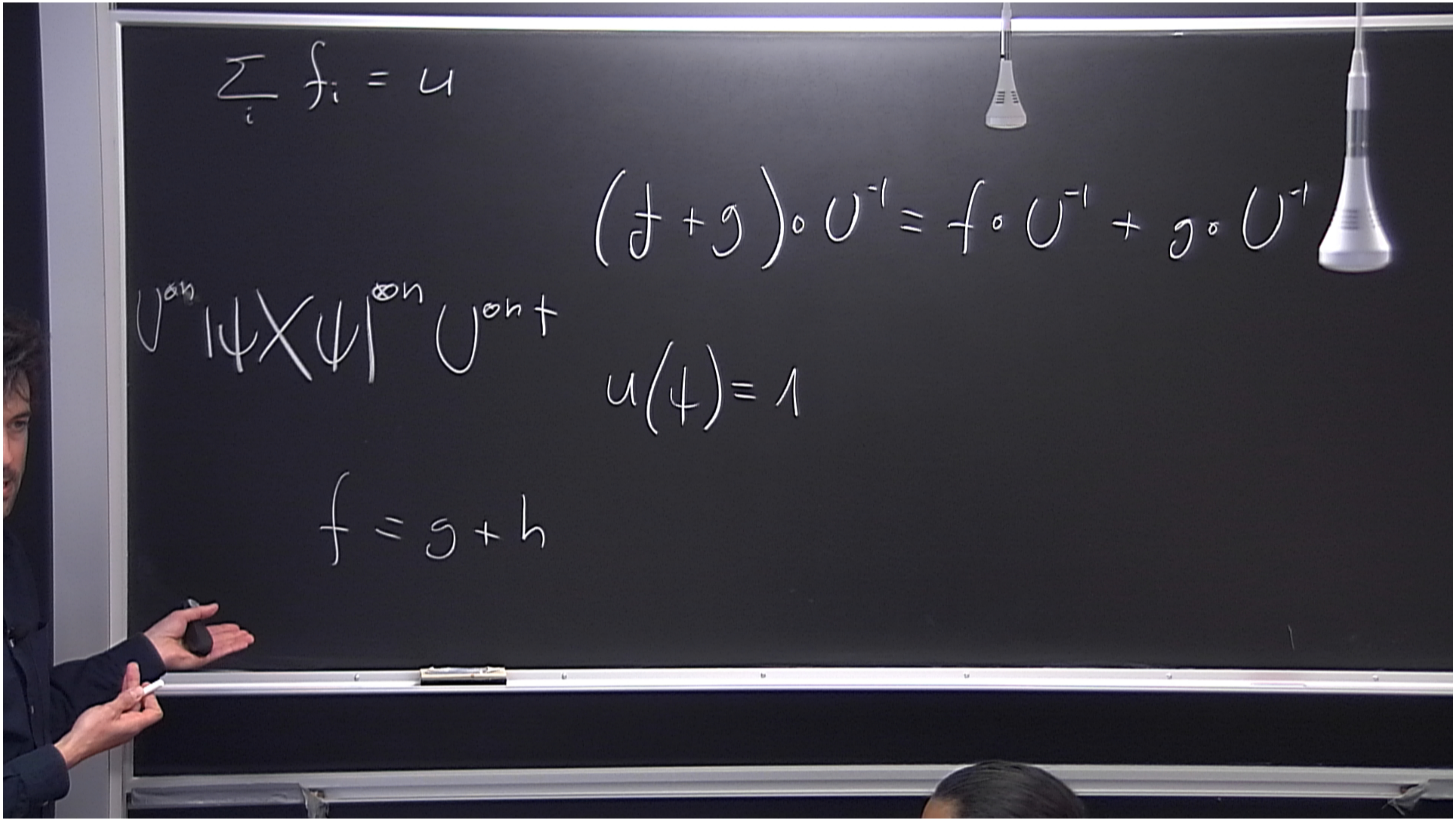
(2) Probabilities of a system $\mathcal{S}$ entangled with another system $\mathcal{E}$ are a function of the *local* properties of $\mathcal{S}$ only, which are exclusively determined by the state vector of the *composite* system $\mathcal{SE}$.

(3) For a composite state in the Schmidt form $|\psi_{\mathcal{SE}}\rangle = \sum_k \lambda_k |s_k\rangle|e_k\rangle$, the probability for $|s_k\rangle$ is *equal* to the probability for $|e_k\rangle$.

(4) Probabilities associated with a system $\mathcal{S}$ entangled with another system $\mathcal{E}$ remain *unchanged* when certain transformations (namely, Zurek's "envariant transformations") are applied that only act on $\mathcal{E}$ (and similarly for $\mathcal{S}$ and $\mathcal{E}$ interchanged).

Is this useful for the Many-Worlds Interpretation?


Thank you!



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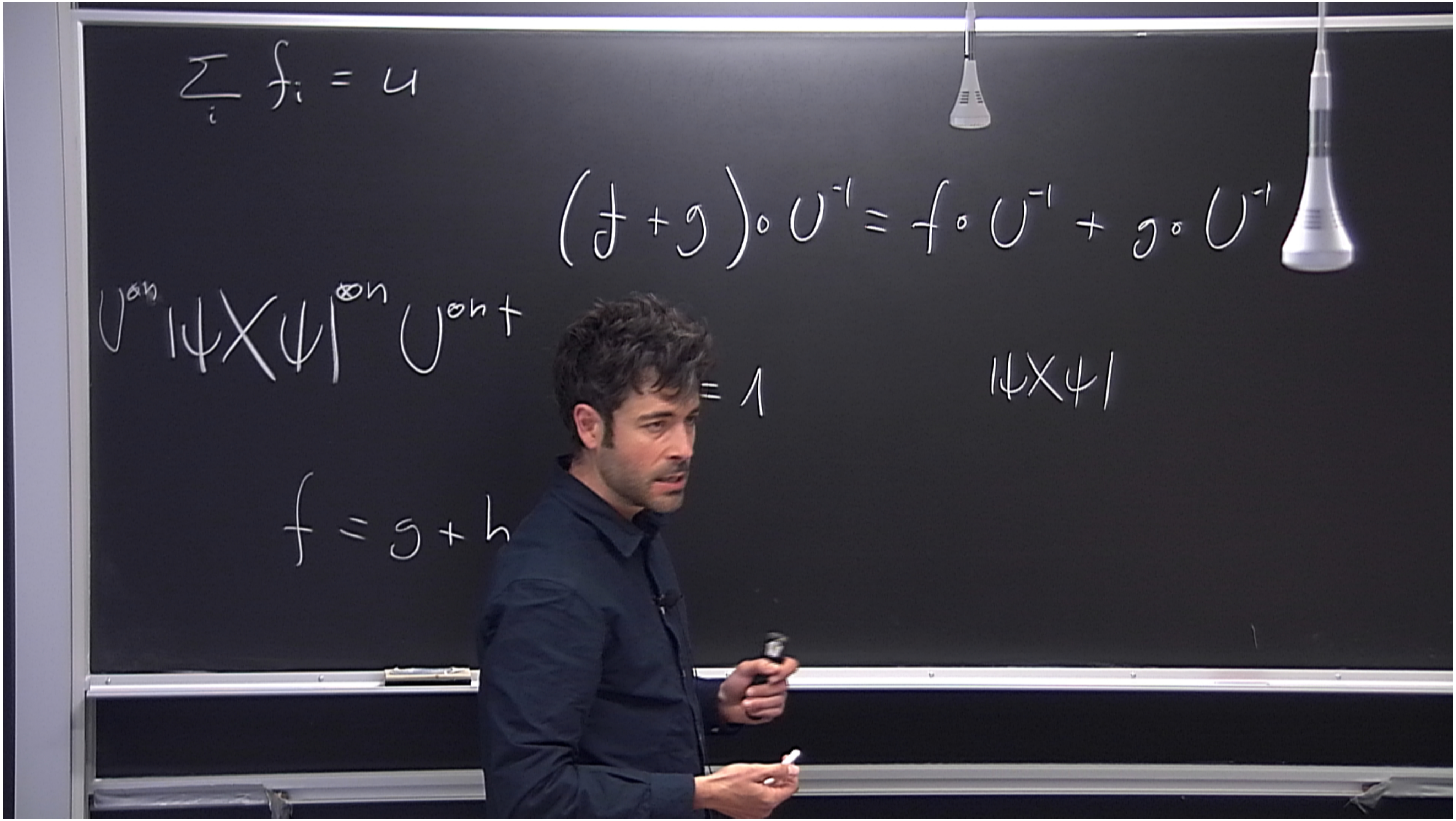
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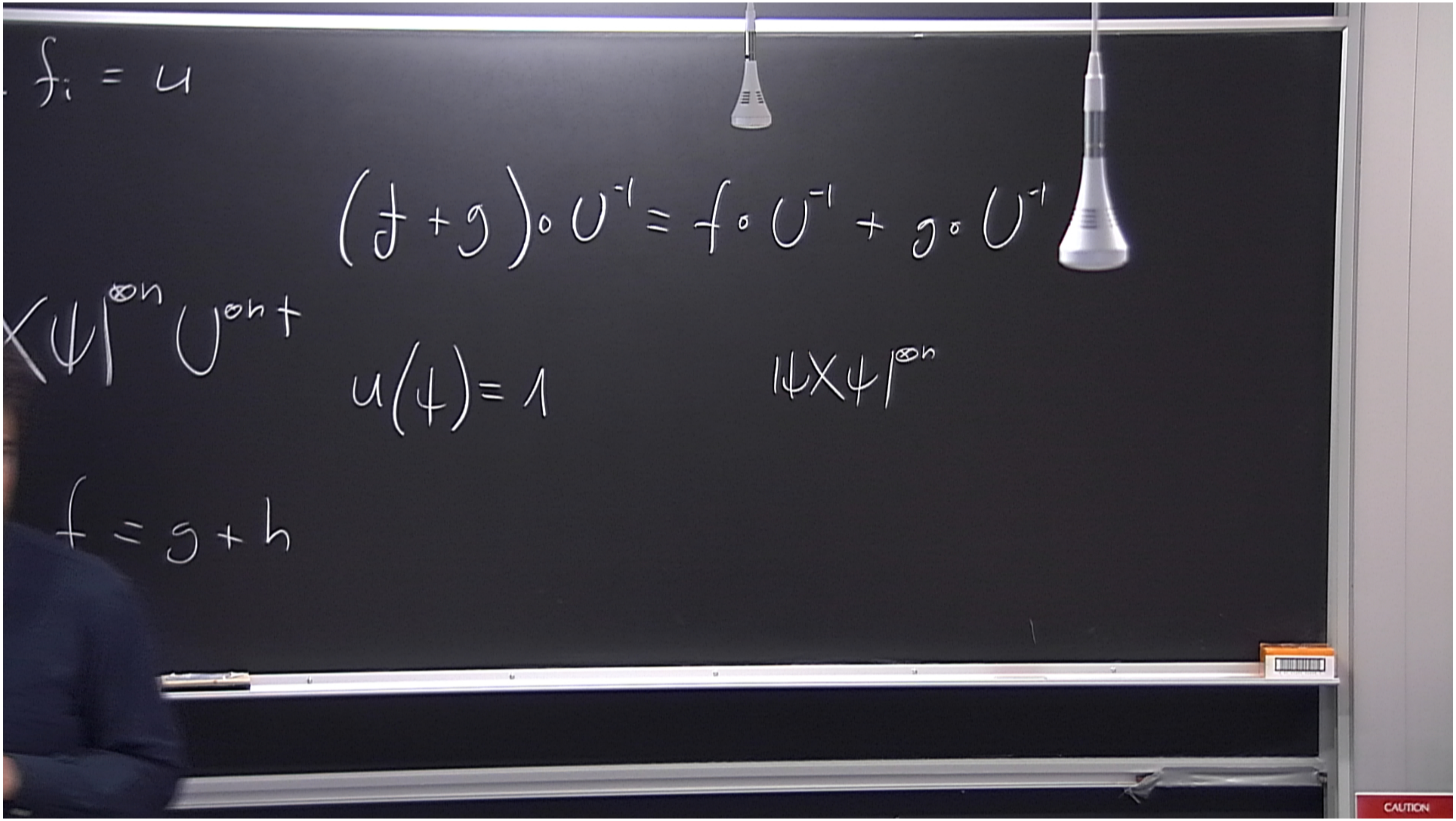

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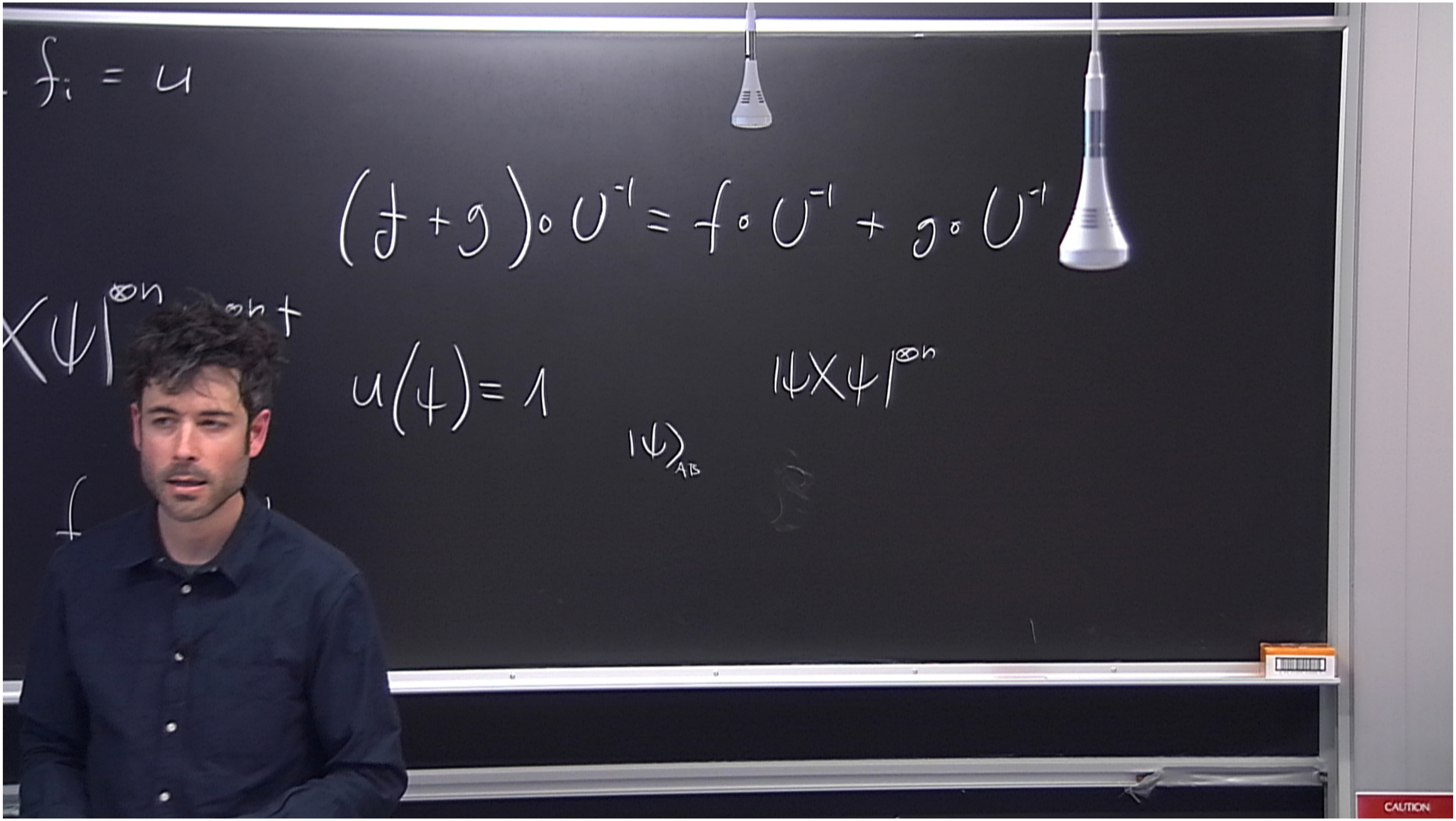
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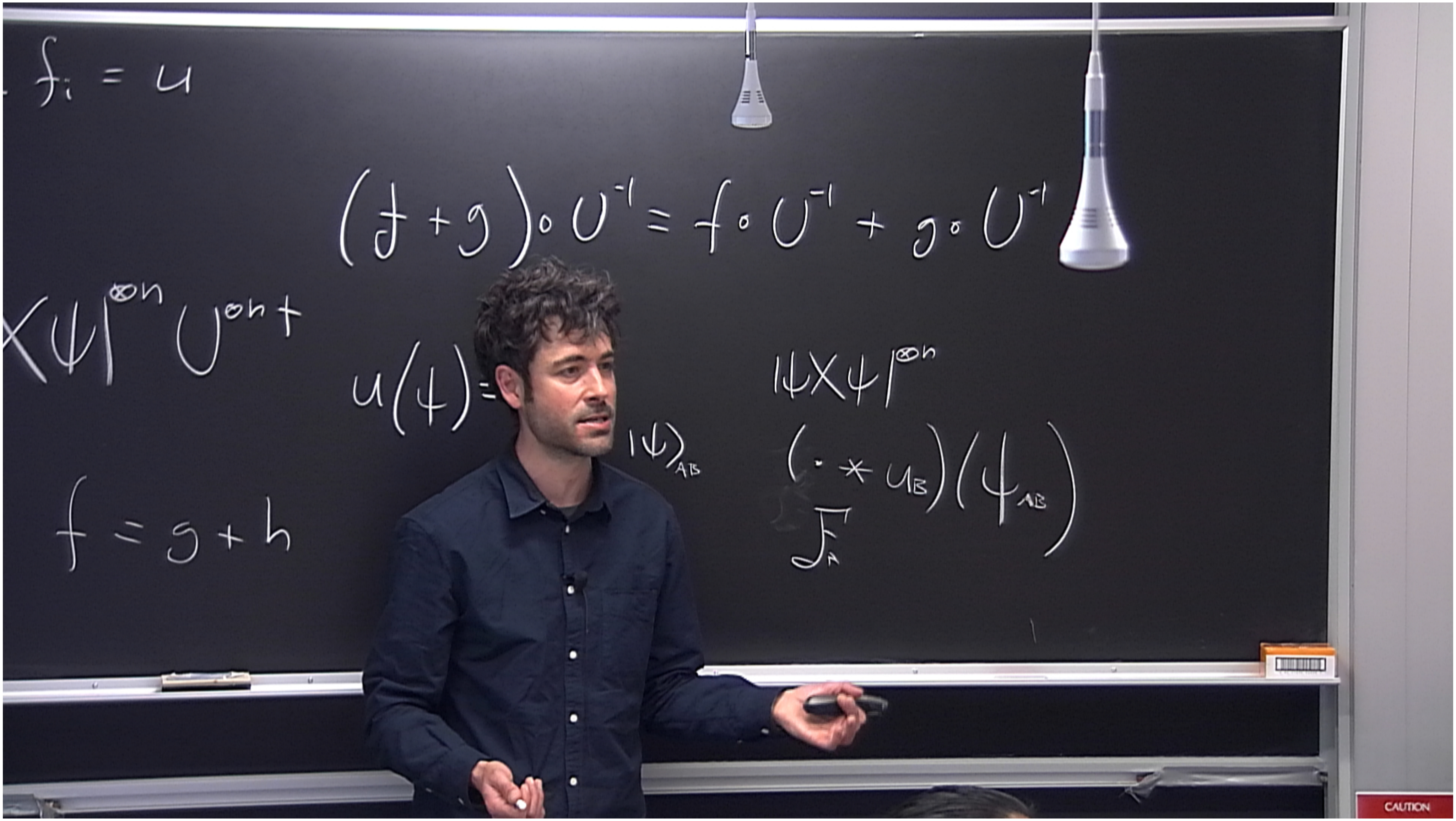

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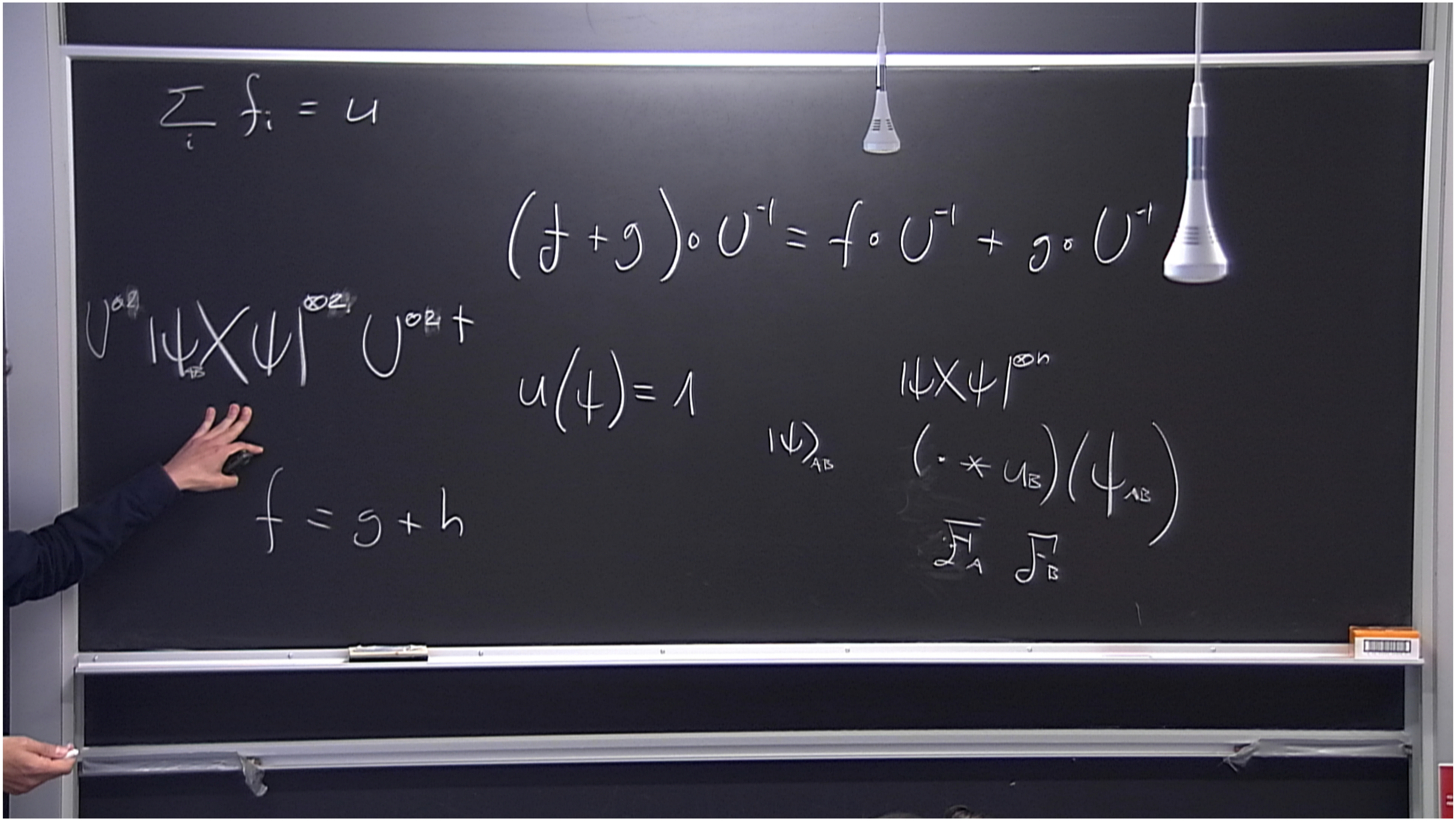
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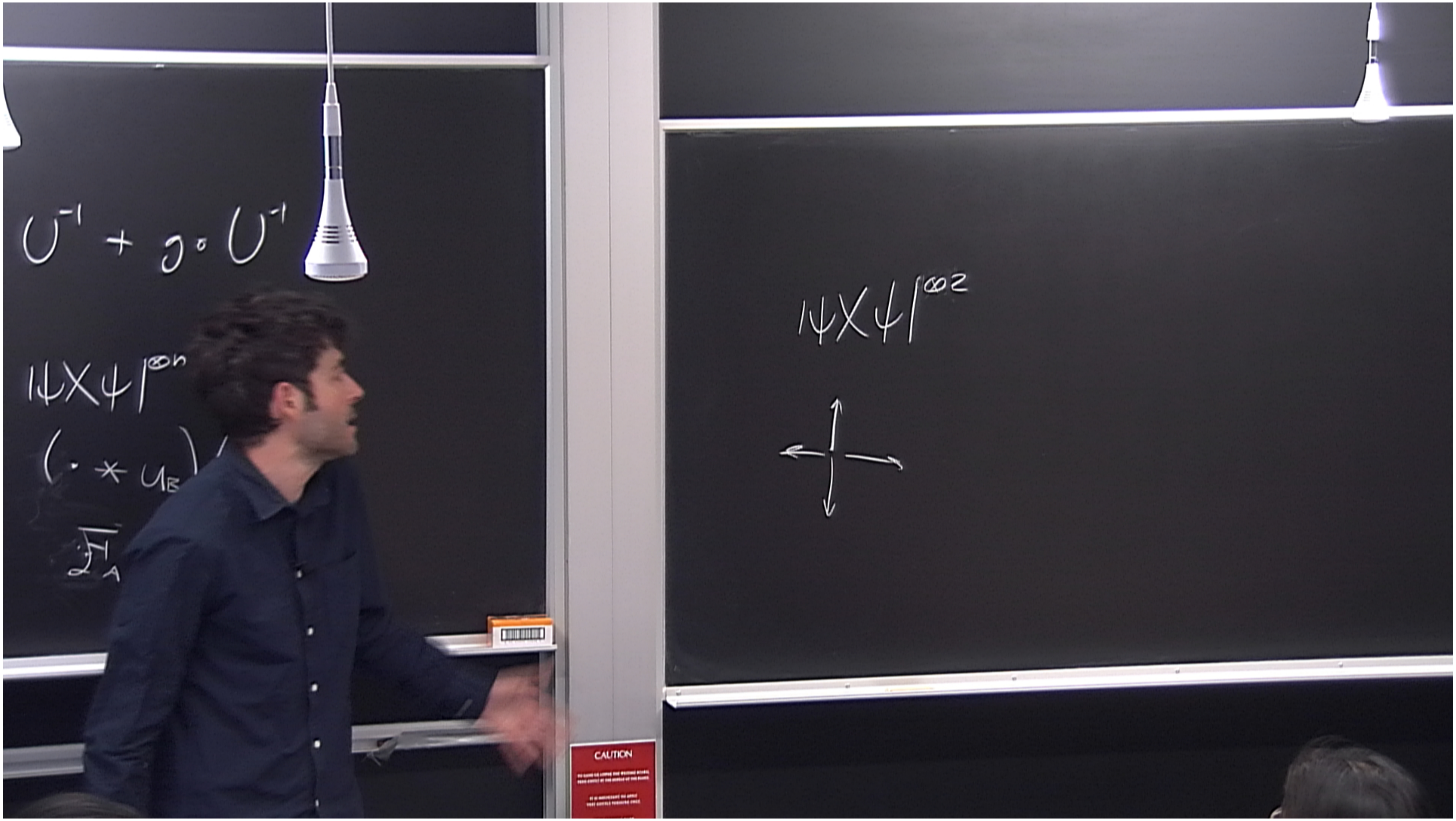
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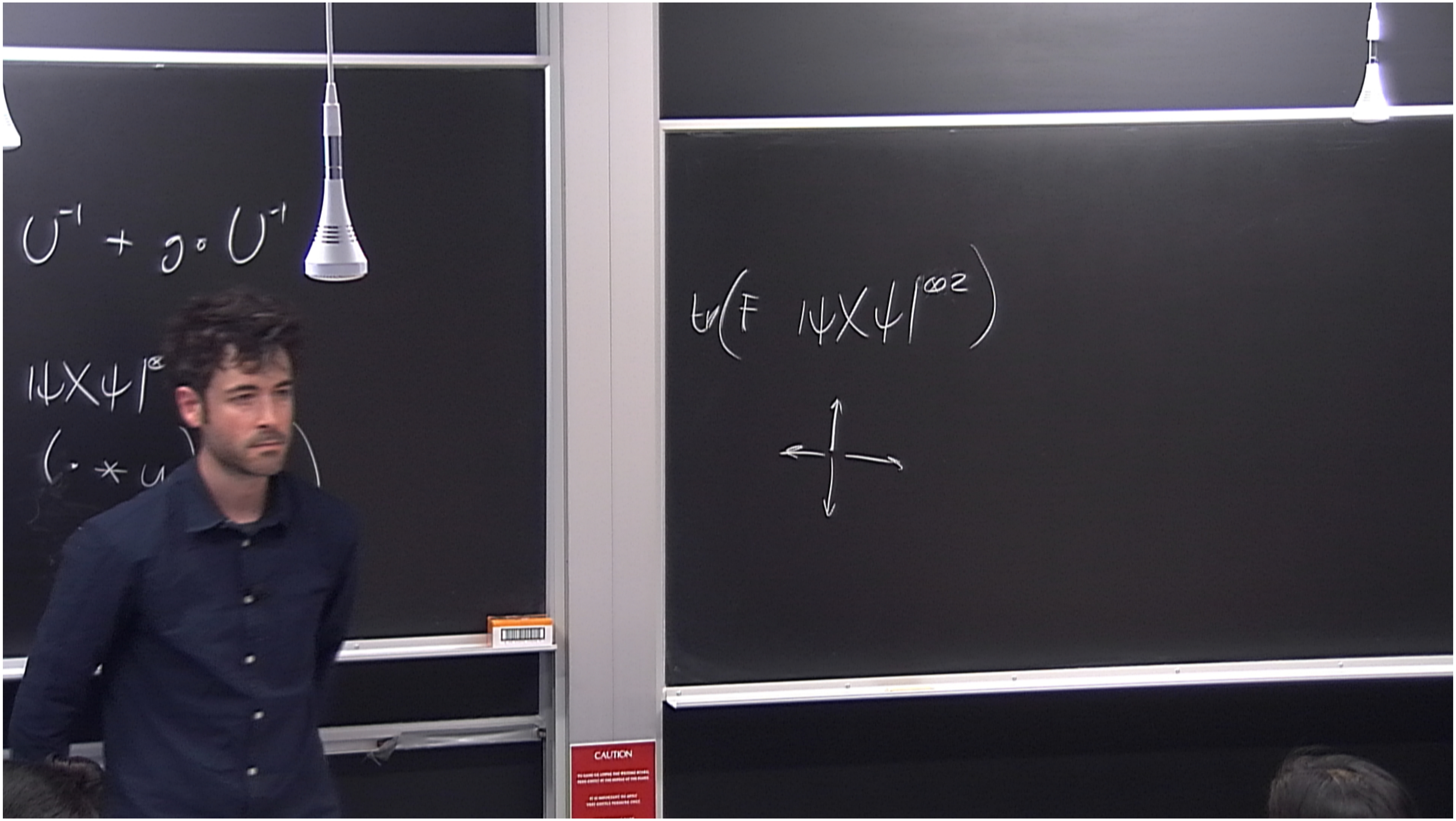
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