

Title: SI Unit Fundamental Measurements

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Abstract:



The SI Measurement System and Fundamental Constants

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Perimeter Institute, May 8, 2018



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de recherches Canada

Canada





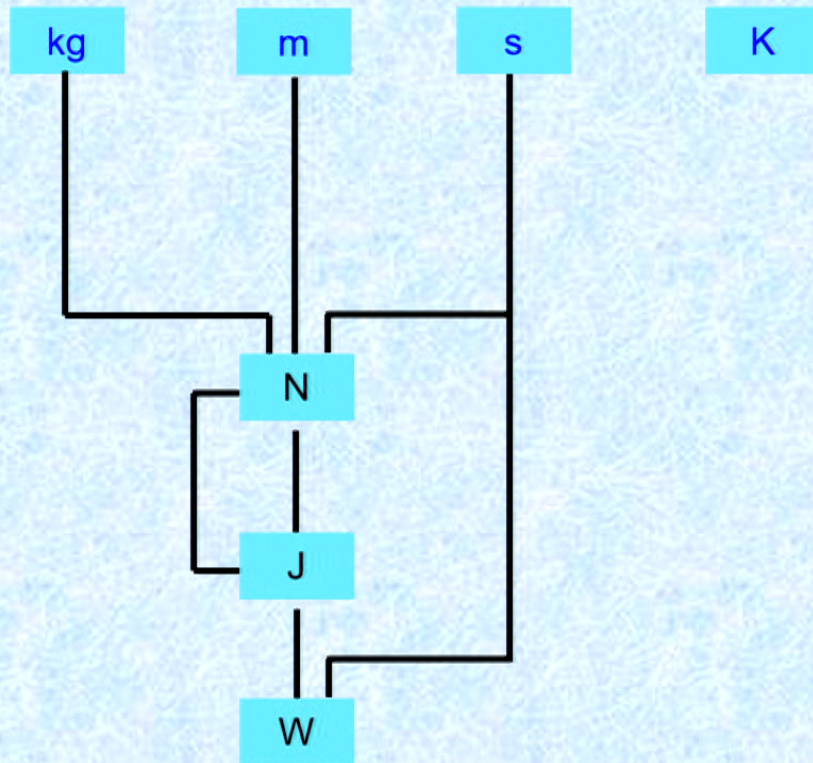
Outline

- The SI – Out with the old, In with the new
- Values of the Fundamental Constants - CODATA
- The Rydberg & fine structure constants
- Measuring h and e , XRCD and Kibble Balances
- Some other extreme measurements





The Original & Incomplete SI

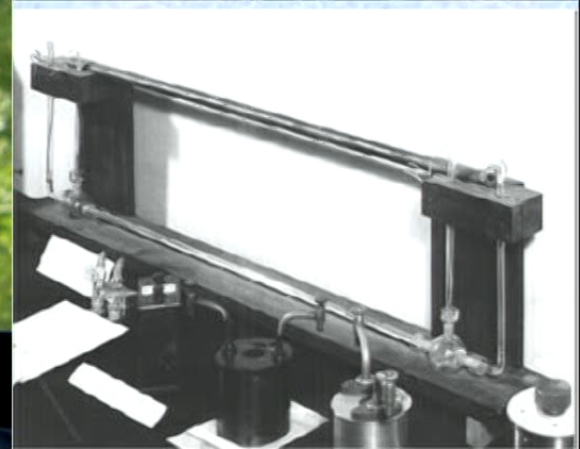
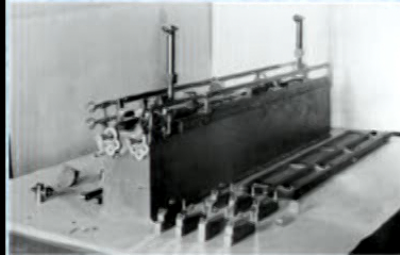


These four base units were fine but they did not address all measurements.

For example, electrical measurements were routinely performed outside the SI.



Physical Artifacts as Standards





But The SI Evolved ...

Changes and refinements in definitions of the second, the kelvin, the metre...

New base units: ampere, candela, mole

New SI units

*electrical, radiation and dose
photometry, radiometry*

*And the quiet introduction of **fundamental constants**.*



What is a fundamental constant?

- Fundamental physical properties - but what is fundamental?
- There are many 'constants', but most are inter-related.

Fundamental Constants are the basis of how we describe and model all of our observations of physical processes.

They frame

The BIG Picture!



What's a *BIG* Picture

Dynamic ranges in metrology, measurements and science.

Resistance

•Resistance Calibrations	$10^{-9} \Omega$ to $10^{16} \Omega$	→ 10^{25}
•Resistance Measurements	$10^{-27} \Omega$ to $10^{18} \Omega$	→ 10^{45}

Distance

•E. Cornell, asymmetry of electron & Diameter of earth orbit	$10^{-13} \text{ fm} = 10^{-28} \text{ m}$ $300 \times 10^6 \text{ km} = 3 \times 10^{11} \text{ m}$	→ 10^{40}
•Planck length (10^{-35} m) to solar system	$\sim 10^{12} \text{ m}$	→ 10^{47}

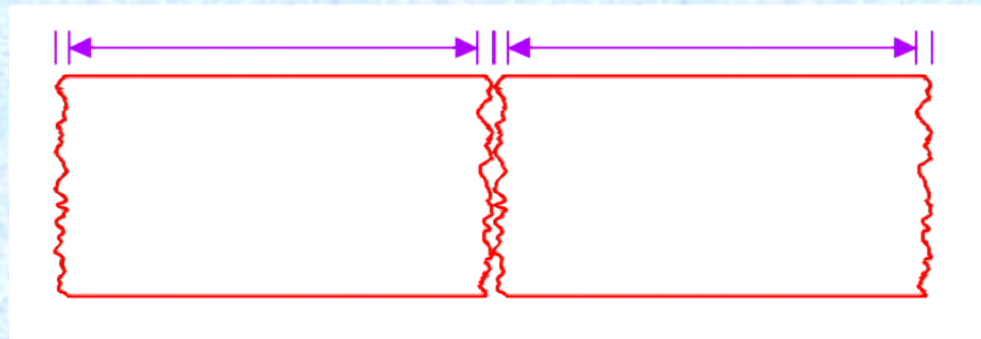
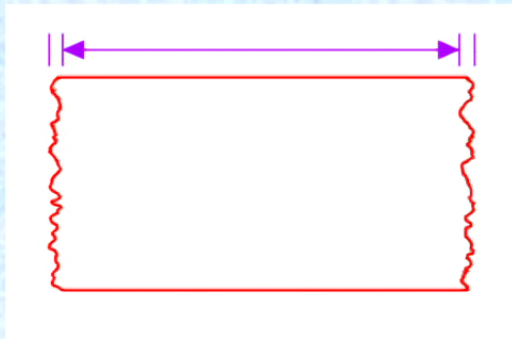
.....

Fundamental constants are essential to realize near perfect scaling over these enormous ranges. Physical artifacts are almost impossible to access over these ranges.



To See The BIG Picture

Consider the uncertainty of a physical meter stick. What happens as copies are combined or subdivided? The 'end effect' uncertainties accumulate!

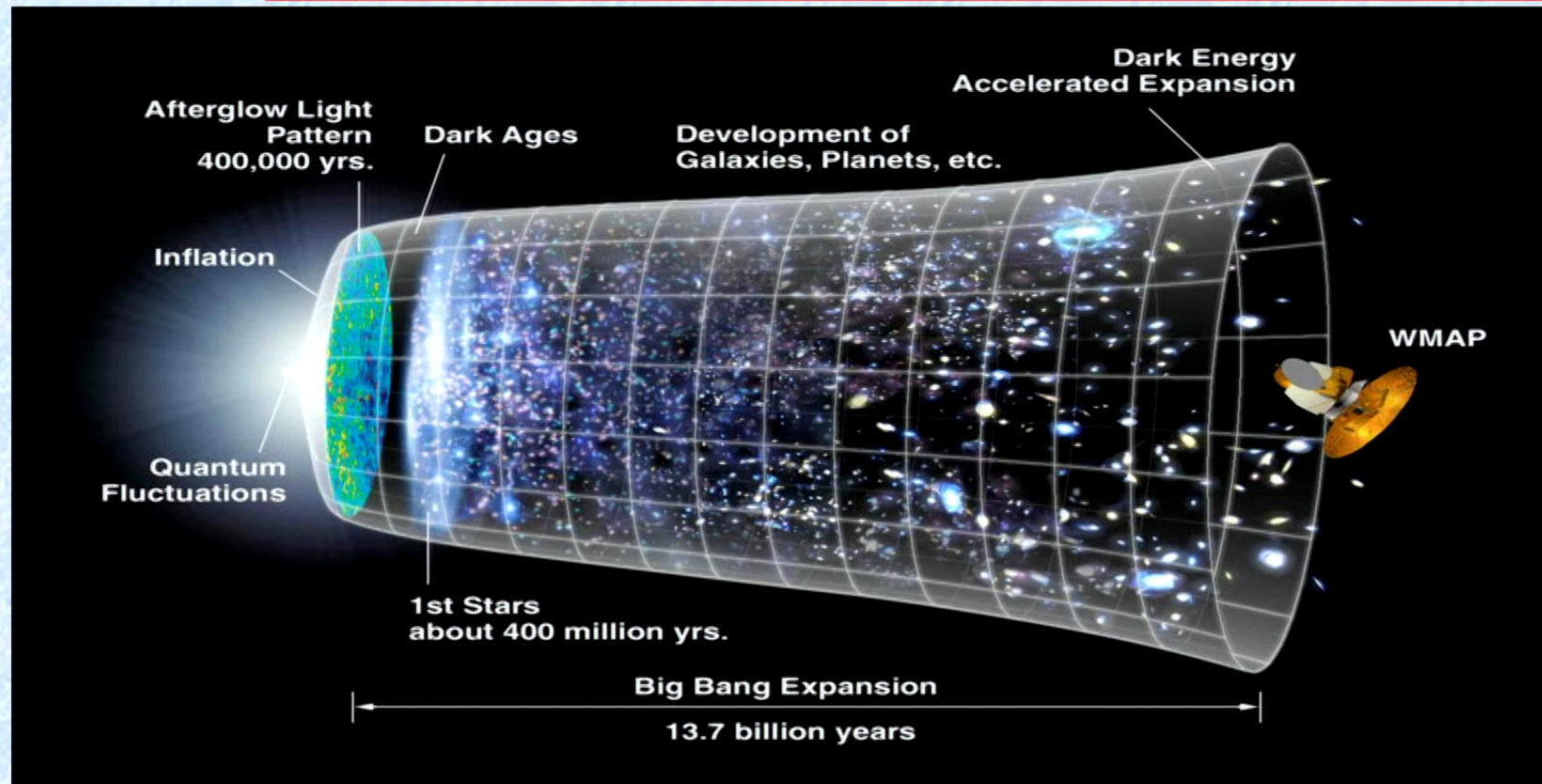


Coherent, quantum standards are essential to realize near perfect scaling over these enormous ranges with minimal loss of accuracy – no end effect uncertainty accumulation.

Examples - wavelengths of monochromatic radiation, flux quanta, single electron tunneling, anions, ...



The *BIG* Picture



Fundamental constants are key to the seeing the biggest pictures.



Some Obvious Fundamental Constants

Properties of particles

- mass of the electron
- mass of the proton
- charge of the electron...

Properties of quantized states

- flux quanta
- universal conductance

Properties of space

- speed of light c
- magnetic constant μ_0
- electric constant ϵ_0

Relationships between things

- gravitation constant - G
- between energies – h, k, R_∞
- between impedances - α

**But some are already fixed and most are inter-related.
And what are their values?**



ICSU & CODATA



Committee on Data for Science and Technology (CODATA) – was established in 1966 - is an interdisciplinary **Scientific Committee of the International Council for Science (ICSU)**, which works to improve the quality, reliability, management, and accessibility of data of importance to all fields of science and technology.

CODATA Task Group on Fundamental Constants - established in 1969 - “to periodically provide the scientific and technological communities with a self-consistent set of internationally **recommended values** of the basic constants and conversion factors of physics and chemistry based on all of the relevant data available at a given point in time.”

The Task Group sanctions the data selection and methodology of the adjustment of the recommended values of the constants.



Method of Least Squares

Setting the values of the fundamental constants.

The First Least-Squares Adjustment of the Fundamental Constants (LSA)

- Raymond T. Birge, 1929
- Cohen and Du Mond, 1965
- The 1973, 1986, 1998, 2002, 2006, 2010, 2014 CODATA Adjustment of the Fundamental Constants.

For example, the 2006 LSA had 150 input data values, 135 distinct types or observational equations, 79 adjusted constants or unknowns.

It is a variance weighted, generalized, multivariate least squares adjustment with accounting of covariances.

How to get the values

The NIST Reference on
Constants, Units, and Uncertainty

Information at the foundation of modern science and
technology from the [Physical Measurement Laboratory](#) of
NIST

URL: physics.nist.gov/constants

CODATA Internationally recommended 2014 values of the
Fundamental Physical Constants

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(e.g., electron mass, most misspellings okay)

Search by name

Search

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Find the [correlation coefficient](#) between any pair of constants

See also

[Article](#) on the 2014 adjustment of the values of the constants

[Wall Chart](#) and [Wallet Card](#) of the 2014 constants

[Background information](#) related to the constants

[Links](#) to selected scientific data

Previous Values [\(2010\)](#) [\(2006\)](#) [\(2002\)](#) [\(1998\)](#) [\(1986\)](#) [\(1973\)](#) [\(1969\)](#)

DEADLINE NOTICES (UPDATED)!

There will be an adjustment of the constants to provide the values for a [revision of the International System of Units \(SI\)](#) expected to take place in 2018. To be considered for use
adjustment, new results must be **accepted for publication by 1 July 2017**.

The 2018 CODATA adjustment of the fundamental constants will be based on the revised SI, which will significantly affect the uncertainties of many constants. For data to be con
in this adjustment, they must be **discussed in a publication preprint or a publication by 31 December 2018**.

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Proposed Changes to the SI

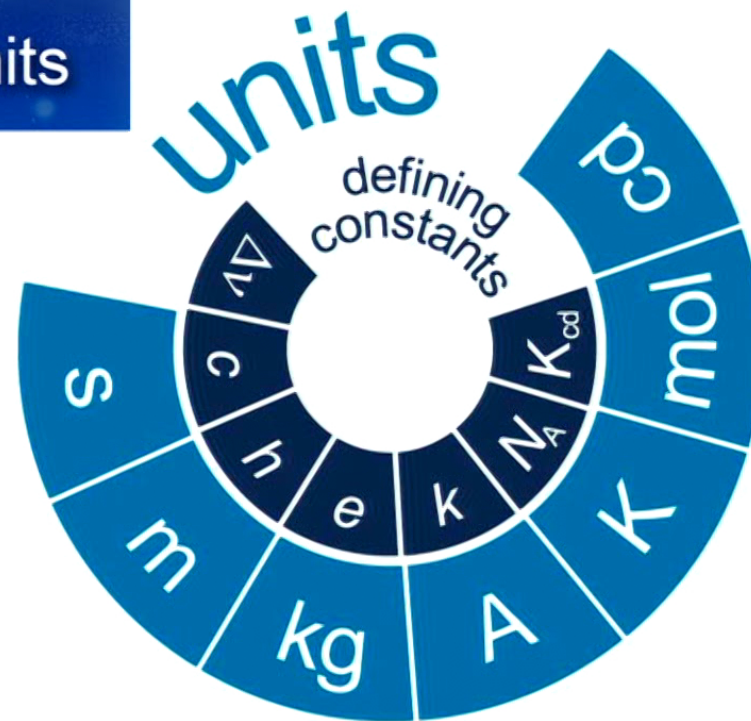
Modify the SI by exactly fixing the values of seven reference constants, five of which are **fundamental constants of physics**:

- h , Planck constant
- e , elementary charge
- k , Boltzmann constant
- N_A , Avogadro number
- c , speed of light
- $\Delta\nu$, the hyperfine splitting of ^{133}C
- K_{cd} , luminous efficacy of 540×10^{12} Hz radiation

and to make the SI units consistent with these values.

Revised International System of Units

SI International
System of Units

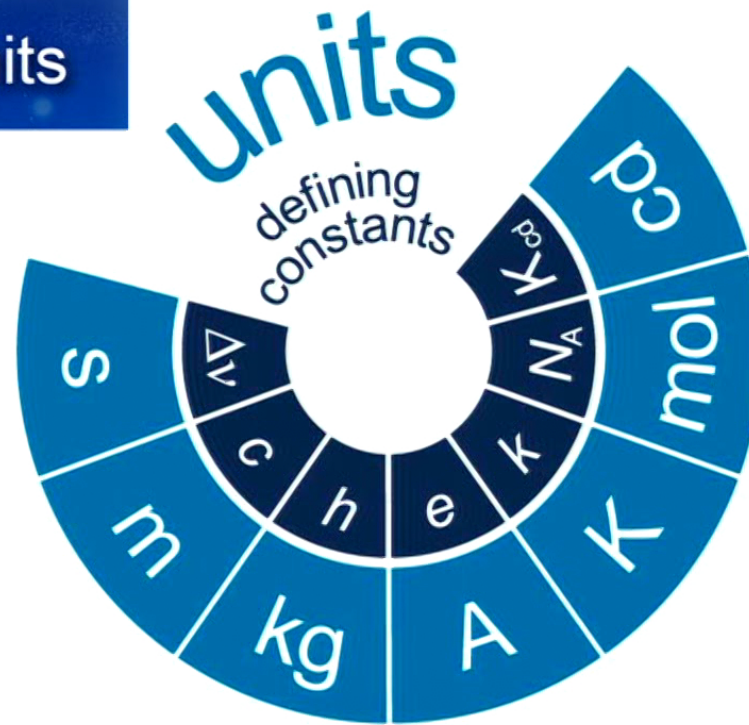


Accurate
Stable
Accessible
Versatile
Fundamental

A consistent and coherent set:
Based on our present understanding of nature

Revised International System of Units

SI International
System of Units



Accurate
Stable
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A consistent and coherent set:
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Changes in relative uncertainties $\times 10^{-9}$

Constant or Unit		Present SI (CODATA 2017)	Revised SI Defined h, e, k, N_A
Mass	$m(\kappa)$	exact	10
Planck constant	h	10	0
Avogadro	N_A	10	0
Elementary charge	e	5.0	0
Mass of electron	m_e	10	0.46
Flux quantum	$2e/h$	5.0	0
Mass of proton	m_p	10	0.62
dalton (amu)	u	10	0.62
Klitzing constant	h/e^2	0.23	0
Magnetic constant	μ_o	exact	0.23
Triple point of water	T_{TPW}	exact	370



A Hierarchy of Fundamental Constants

The speed of light	0 (fixed)	
The Rydberg	5.9×10^{-12}	
Relative atomic masses – (not a constant but important)		1.6×10^{-11}
The fine structure constant	2.3×10^{-10}	
The Planck Constant	1.2×10^{-8}	
The Avogadro Constant	1.2×10^{-8}	

But there are others

- The Boltzmann Constant, Gravitational Constant, the gas constant, ...



The speed of light, c

- The speed of light has not always been a fundamental constant. Even after 1905 many believed that it simply could not be constant.
- Now (since 1983) we take it for granted and use a fixed value with zero uncertainty.
 $c = 299\,792\,458 \text{ m / s}$
- Strangely, this was not the first exactly fixed fundamental constant. Yes, fundamental constants do evolve as our understanding evolves.



The Rydberg, R_∞



In the 1880s Rydberg described the wavelength of light radiated when an electron changes bound states in hydrogen. $1/\lambda = R_\infty(1/n_1^2 - 1/n_2^2)$ was originally applied only to hydrogen but now extends to other 'simple molecules'. Note: depends on $E = h\nu$

It involves classical electrodynamic forces, nuclear forces, effects of virtual particles (or states), ...

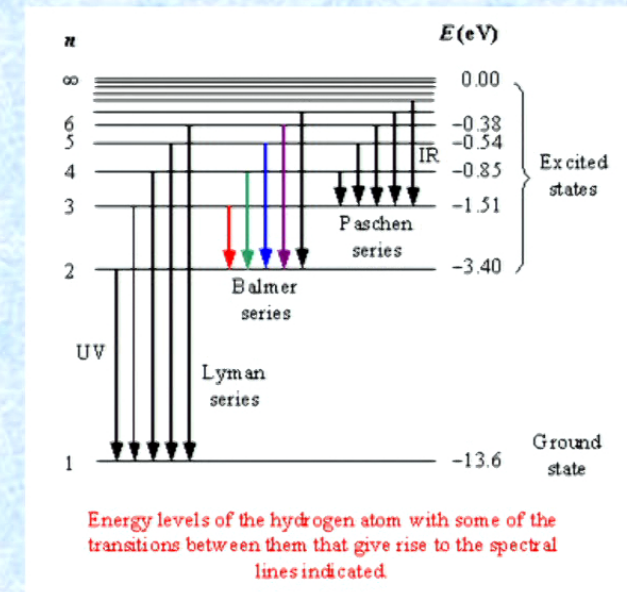
For different elements there is a different 'Rydberg'.

$R_M = R_\infty (1 + m_e/M)$, M is the mass of the protons

$$R_\infty = m_e e^4 / ((4\pi\epsilon_0)^2 h^3 4\pi c) = m_e e^4 / (8\epsilon_0^2 h^3 c) \\ = 10973731.568525(73) \text{ m}^{-1}$$

And

$$R_\infty = \alpha^2 m_e c / (4\pi h) = \alpha^2 e \lambda_e$$





Experimental Determinations of R_∞



Summary of measured transition frequencies considered for the determination of the Rydberg constant R_∞ (H is hydrogen and D is deuterium).

Authors, Laboratory,	Frequency interval(s)	Reported value (v/kHz)	Rel. stand uncert. u_r
(Fischer et al., 2004), MPQ	$\nu_H(1S_{1/2} - 2S_{1/2})$	2 466 061 413 187.074(34)	1.4×10^{-14}
(Weitz et al., 1995), MPQ	$\nu_H(2S_{1/2} - 4S_{1/2}) - 1/4 \nu_H(1S_{1/2} - 2S_{1/2})$	4 797 338(10)	2.1×10^{-6}
	$\nu_H(2S_{1/2} - 4S_{3/2}) - 1/4 \nu_H(1S_{1/2} - 2S_{1/2})$	6 490 144(24)	3.7×10^{-6}
	$\nu_D(2S_{1/2} - 4S_{1/2}) - 1/4 \nu_D(1S_{1/2} - 2S_{1/2})$	4 801 693(20)	4.2×10^{-6}
	$\nu_D(2S_{1/2} - 4D_{3/2}) - 1/4 \nu_D(1S_{1/2} - 2S_{1/2})$	6 494 841(41)	6.3×10^{-6}
(Huber et al., 1998), MPQ	$\nu_D(1S_{1/2} - 2S_{1/2}) - \nu_H(1S_{1/2} - 2S_{1/2})$	670 994 334.64(15)	2.2×10^{-10}
(de Beauvoir et al., 1997), LKB/SYRTE	$\nu_H(2S_{1/2} - 8S_{1/2})$	770 649 350 012.0(8.6)	1.1×10^{-11}
	$\nu_H(2S_{1/2} - 8D_{3/2})$	770 649 504 450.0(8.3)	1.1×10^{-11}
	$\nu_H(2S_{1/2} - 8D_{5/2})$	770 649 561 584.2(6.4)	8.3×10^{-12}
	$\nu_D(2S_{1/2} - 8S_{1/2})$	770 859 041 245.7(6.9)	8.9×10^{-12}
	$\nu_D(2S_{1/2} - 8D_{3/2})$	770 859 195 701.8(6.3)	8.2×10^{-12}
	$\nu_D(2S_{1/2} - 8D_{5/2})$	770 859 252 849.5(5.9)	7.7×10^{-12}
(Schwob et al., 1999, 2001), LKB/SYRTE	$\nu_H(2S_{1/2} - 12D_{3/2})$	799 191 710 472.7(9.4)	1.2×10^{-11}
	$\nu_H(2S_{1/2} - 12D_{5/2})$	799 191 727 403.7(7.0)	8.7×10^{-12}
	$\nu_D(2S_{1/2} - 12D_{3/2})$	799 409 168 038.0(8.6)	1.1×10^{-11}
	$\nu_D(2S_{1/2} - 12D_{5/2})$	799 409 184 966.8(6.8)	8.5×10^{-12}
(Bourzeix et al., 1996), LKB	$\nu_H(2S_{1/2} - 6S_{1/2}) - 1/4 \nu_H(1S_{1/2} - 3S_{1/2})$	4 197 604(21)	4.9×10^{-6}
	$\nu_H(2S_{1/2} - 6D_{3/2}) - 1/4 \nu_H(1S_{1/2} - 3S_{1/2})$	4 699 099(10)	2.2×10^{-6}
(Berkeland et al., 1995), Yale	$\nu_H(2S_{1/2} - 4P_{1/2}) - 1/4 \nu_H(1S_{1/2} - 2S_{1/2})$	4 664 269(15)	3.2×10^{-6}
	$\nu_H(2S_{1/2} - 4P_{3/2}) - 1/4 \nu_H(1S_{1/2} - 2S_{1/2})$	6 035 373(10)	1.7×10^{-6}
(Hagley and Pipkin, 1994), Harvard	$\nu_H(2S_{1/2} - 2P_{3/2})$	9 911 200(12)	1.2×10^{-6}
(Lundeen and Pipkin, 1986), Harvard	$\nu_H(2P_{1/2} - 2S_{1/2})$	1 057 845.0(9.0)	8.5×10^{-6}
(Newton et al., 1979), U. Sussex	$\nu_H(2P_{1/2} - 2S_{1/2})$	1 057 862(20)	1.9×10^{-5}



The Rydberg Theory (or theories!)

- Bohr model ca. 1915
- Sommerfeld fine structure correction 1922
- Evidence of QED effect since 1929
- Invention of QED and resolution of infinities 1948
- Lamb shift 1951
- and more....

It uses ALL of the classes of our theories!

The Rydberg is a measure of the binding of an electron to a proton.

Fig. 1. It is a relation between the $1s-2s$ transition frequency $\nu_H(1s-2s)$ and the Rydberg constant R_∞ . A correction for the difference between the center of gravity of the $1s$ and $2s$ hyperfine multiplets and their triplet component is not included. This figure is an example of a complicated relationship, and is not intended to be read.

$$\begin{aligned} \nu_H(1s-2s) - \frac{1}{4} R_\infty \left\{ 1 + \left[\frac{11}{48} (Z\alpha)^2 + \frac{65}{288} (Z\alpha)^4 + \frac{863}{14256} (Z\alpha)^6 + \dots \right] \right. \\ + \frac{m_e}{m_p} \left[-1 - \frac{19}{24} (Z\alpha)^2 - \frac{17}{64} (Z\alpha)^4 + \dots \right] \\ + \left(\frac{m_e}{m_p} \right)^2 \left[1 + \frac{43}{24} (Z\alpha)^2 + \dots \right] \\ + \left(\frac{m_e}{m_p} \right)^3 \left[-1 + \dots \right] \\ + \frac{(Z\alpha)^5}{\pi} \frac{m_e}{m_p} \left[-\frac{1}{9} \ln \frac{1}{(Z\alpha)^2} - \frac{2}{9} \ln R_\infty(2s) + \frac{64}{9} \ln R_\infty(1s) - \frac{112}{3} \ln 2 - \frac{905}{18} \right] \\ + \frac{(Z\alpha)^5}{\pi} \left(\frac{m_e}{m_p} \right)^2 \left[\frac{2}{3} \ln \frac{1}{(Z\alpha)^2} + \frac{8}{3} \ln R_\infty(2s) - \frac{64}{3} \ln R_\infty(1s) + 122 \ln 2 + \frac{989}{18} \right] + \dots \\ + \frac{m_e}{\pi} (Z\alpha)^4 \left[-\frac{28}{9} \ln \frac{1}{(Z\alpha)^2} - \frac{6}{9} \ln R_\infty(2s) + \frac{32}{9} \ln R_\infty(1s) - \frac{286}{15} \right] \\ + \frac{m_e}{\pi} (Z\alpha)^4 \frac{m_e}{m_p} \left[\frac{28}{3} \ln \frac{1}{(Z\alpha)^2} + \frac{6}{3} \ln R_\infty(2s) - \frac{32}{3} \ln R_\infty(1s) + \frac{14}{3} \right] \\ + \frac{m_e}{\pi} (Z\alpha)^4 \left(\frac{m_e}{m_p} \right)^2 \left[-\frac{16}{9} \ln \frac{1}{(Z\alpha)^2} - \frac{6}{9} \ln R_\infty(2s) + \frac{64}{9} \ln R_\infty(1s) - \frac{14}{3} \right] \\ + m_e (Z\alpha)^4 \left[\frac{16}{3} \ln 2 - \frac{2093}{225} \right] \\ + m_e (Z\alpha)^4 \frac{m_e}{m_p} \left[-1.4 \ln 2 + \frac{2093}{450} \right] \\ + \frac{m_e}{\pi} (Z\alpha)^4 \left[\frac{2}{3} \ln \frac{1}{(Z\alpha)^2} + \left(\frac{208}{9} \ln 2 + \frac{247}{95} \right) \ln \frac{1}{(Z\alpha)^2} - 71.628974 \right] + \dots \\ + \left(\frac{2}{\pi} \right)^2 (Z\alpha)^2 \left[-\frac{7}{2} \ln^2 \ln 2 - \frac{79\pi^2}{81} + \frac{15252}{1584} + \frac{23}{4} \zeta(3) \right] \\ + \left(\frac{2}{\pi} \right)^2 (Z\alpha)^2 \frac{m_e}{m_p} \left[\frac{31}{2} \ln^2 \ln 2 - \frac{77\pi^2}{27} - \frac{17203}{648} - \frac{63}{4} \zeta(3) \right] \\ + 70.2878 \left(\frac{2}{\pi} \right)^2 (Z\alpha)^2 \\ + \left(\frac{2}{\pi} \right)^2 (Z\alpha)^4 \left[\frac{56}{81} \ln^4 \frac{1}{(Z\alpha)^2} + \frac{1}{27} \ln^3 \frac{1}{(Z\alpha)^2} \right. \\ + \left(\frac{246337}{238320} - \frac{205\pi^2}{81} + \frac{1126}{135} - \frac{7\pi^2}{105} \ln 2 - \frac{248}{27} \ln^2 2 - 38.643332 \right) \ln \frac{1}{(Z\alpha)^2} \\ + 147(25) \left. \right] + \dots \\ + \left(\frac{2}{\pi} \right)^4 (Z\alpha)^4 \left[\frac{2480570611}{279360} + \frac{1761717\pi^2}{21600} - \frac{11137\pi^4}{9720} + \frac{7812}{27} \ln 2 - \frac{33030\pi^2}{135} \ln 2 \right. \\ + \frac{1877\pi^2}{405} \ln^2 2 + \frac{497}{81} \ln^3 2 + \frac{584697}{6912} \zeta(4) + \frac{887\pi^2}{216} \zeta(3) - \frac{195}{72} \zeta(5) \left. \right] + \dots \\ + \frac{m_e}{\pi^2} (Z\alpha)^3 \frac{m_e}{m_p} \left[\frac{2126}{81} - \frac{242\pi^2}{108} + \frac{14\pi^4}{3} \ln 2 - 14\zeta(3) - \frac{14}{3} \pi^2 \ln^2 \frac{1}{(Z\alpha)^2} \right] \\ - \frac{14}{9} (Z\alpha)^3 \left(\frac{m_e}{m_p} \right)^2 \\ + \dots \left. \right\} \end{aligned}$$



Relative Atomic Masses

Values of the relative atomic masses are often related to the arbitrarily defined atomic mass unit, $1 \text{ amu} = {}^{12}\text{C} / 12$. This is more correctly referred to as the atomic Relative standard mass $A_r(x)$.

Values of the relative atomic masses of the neutron and various atoms as given in the 2003 atomic mass evaluation together with the defined value for ${}^{12}\text{C}$.

We are not yet able to accurately calculate the mass of particular atoms. However, it is possible to accurately measure the mass of a single atom in comparison with another single atom.

Atom Relative	atomic Relative standard mass $A_r(X)$	uncertainty u_r
— n	1.008 664 915 74(56)	5.6×10^{-10}
— ${}^1\text{H}$	1.007 825 032 07(10)	1.0×10^{-10}
— ${}^2\text{H}$	2.014 101 777 85(36)	1.8×10^{-10}
— ${}^3\text{H}$	3.016 049 2777(25)	8.2×10^{-10}
— ${}^3\text{He}$	3.016 029 3191(26)	8.6×10^{-10}
— ${}^4\text{He}$	4.002 603 254 153(63)	1.6×10^{-11}
— ${}^{12}\text{C}$	12 (exact)	
— ${}^{16}\text{O}$	15.994 914 619 56(16)	1.0×10^{-11}
— ${}^{28}\text{Si}$	27.976 926 5325(19)	6.9×10^{-11}
— ${}^{29}\text{Si}$	28.976 494 700(22)	7.6×10^{-10}
— ${}^{30}\text{Si}$	29.973 770 171(32)	1.1×10^{-9}
— ${}^{36}\text{Ar}$	35.967 545 105(28)	7.8×10^{-10}
— ${}^{38}\text{Ar}$	37.962 732 39(36)	9.5×10^{-9}
— ${}^{40}\text{Ar}$	39.962 383 1225(29)	7.2×10^{-11}
— ${}^{87}\text{Rb}$	86.909 180 526(12)	1.4×10^{-10}
— ${}^{107}\text{Ag}$	106.905 0968(46)	4.3×10^{-8}
— ${}^{109}\text{Ag}$	108.904 7523(31)	2.9×10^{-8}
— ${}^{133}\text{Cs}$	132.905 451 932(24)	1.8×10^{-10}



Measuring Relative Atomic Masses

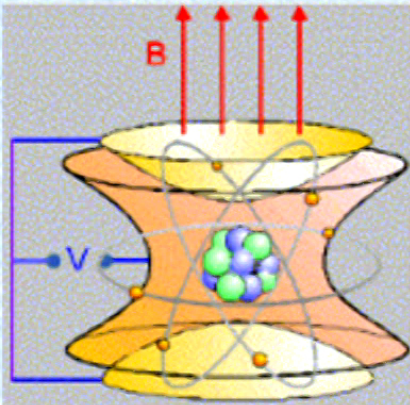
A charged single atom in a Penning trap is confined by an ac voltage between the vertical electrodes.

A magnetic field then causes three types of motion, axial, cyclotron and magnetron described by:

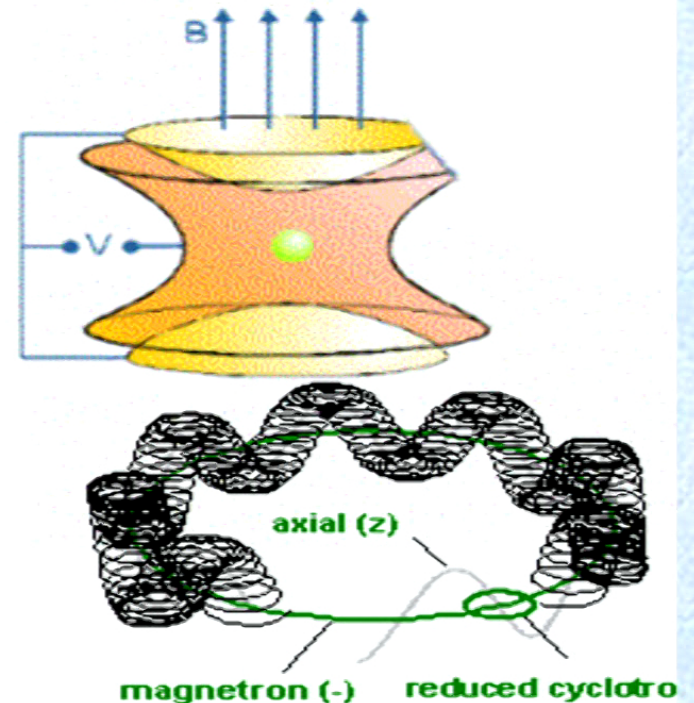
$$\omega_z = \sqrt{\frac{qV_0}{md^2}}$$

$$\omega_c = \frac{qB}{m}$$

$$\omega_m = \frac{V_0}{2d^2B} = \frac{\omega_z^2}{2\omega_c}$$



Relative masses can then be determined by frequency ratios.





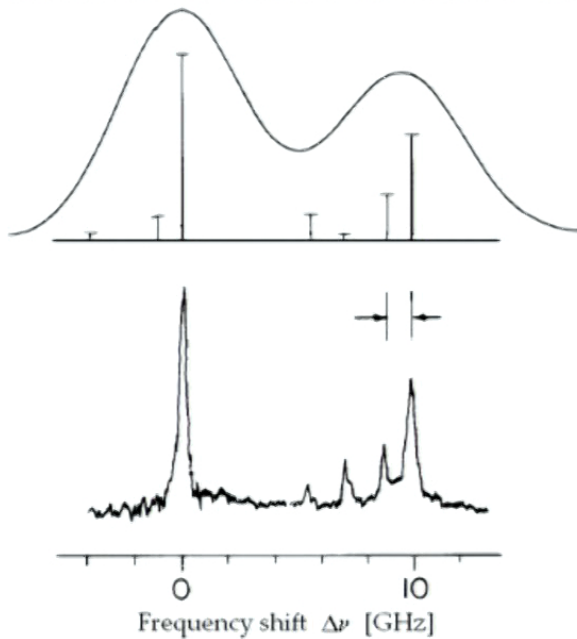
Fine Structure Constant

The simple Bohr model of hydrogen explained the coarse spectra of hydrogen. Sommerfeld explained the further splitting of this 'coarse structure' by considering elliptical electron orbits in a relativistic model.

The fine structure constant, α , is a measure of the strength of the interaction between an electron and photons.

α is also the ratio of the impedance of vacuum and the universal conductance and is related to several other constants.

Thus, α appears in all models that incorporate quantum and relativistic properties of charged particles. For this reason it is related to many other fundamental constants and often found in theory.



$$\alpha = \frac{e^2}{\hbar c 4\pi\epsilon_0} = 7.297\,352\,5376(50) \times 10^{-3} = \frac{1}{137.035\,999\,679(94)}$$



Measuring α

Anomalous magnetic moment of the electron or muonium.

Atomic recoil of various atoms.

Calculable capacitor, capacitance to resistance scaling and QHR.

d_{220} and h / neutron mass.

Gyromagnetic moment of the proton.

...

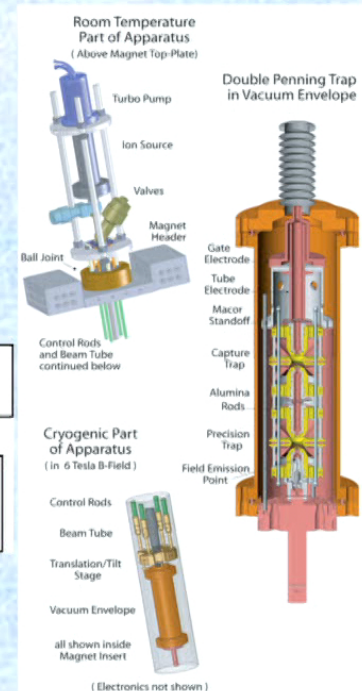
Very different measurements and diverse theories !

Anomalous magnetic moment of the electron, a_e .

a penning trap experiment measuring mass ratios combined with QED theory

$$a_e(\text{QED}) = A_1 + A_2(m_e/m_\mu) + A_2(m_e/m_\tau) + A_3(m_e/m_\mu \cdot m_e/m_\tau). \quad (\text{B3})$$

$$A_i = A_i^{(2)} \left(\frac{\alpha}{\pi} \right) + A_i^{(4)} \left(\frac{\alpha}{\pi} \right)^2 + A_i^{(6)} \left(\frac{\alpha}{\pi} \right)^3 + A_i^{(8)} \left(\frac{\alpha}{\pi} \right)^4 + \dots \quad (\text{B4})$$



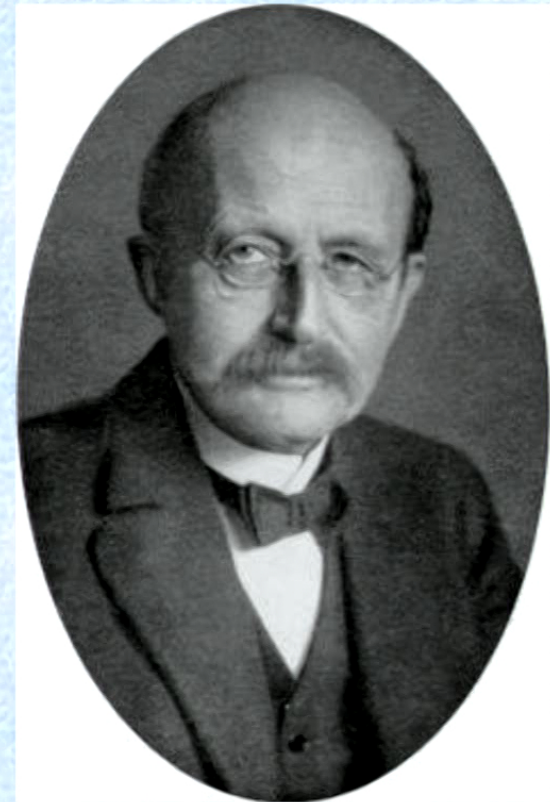


Planck Constant

$E = h\nu$ the energy of a photon

a link between heavy and
massless particles

Because of the accuracy and ease with which electromagnetic and optical frequencies can be measured, the Planck constant plays a critical role in physics and in the LSA of the Fundamental Constants.





h , e and N_A Equations

$$V = K_J f n = h/2e f n$$

$$R = R_K i = h/e^2 i$$

$$c^2 \epsilon_0 \mu_0 = 1$$

$$h K_J^2 R_K = 4$$

$$e^2 = 2 h \alpha / (\mu_0 c)$$

$$R_\infty = \alpha^2 m_e c / (4\pi h)$$

$$N_A = V_m(\text{Si}) / (a^3 / n)$$

$$V_m(\text{Si}) = 2^{0.5} M_u A_r(e) c \alpha^2 d_{220}^3 / h R_\infty$$

Josephson equation

Quantum Hall equation

electric and magnetic constants

Planck constant

elementary charge relationship

atomic mass relationship

Avogadro and Molar volume of Si

relationship between N_A and h

$$R_\infty = 10\,973\,731.568\,539(55) \text{ m}^{-1}$$

relative standard uncertainty 5.9×10^{-12}

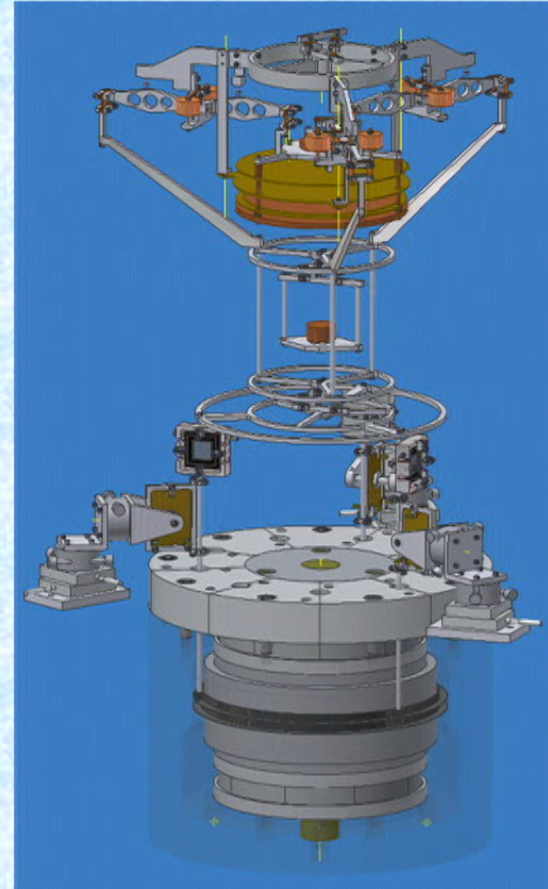
$$\alpha = 7.297\,352\,5698(24) \times 10^{-3}$$

relative standard uncertainty 2.3×10^{-10}

Measuring h and N_A

- Molar mass of silicon
(Avogadro and XRCD)
- Watt balances – with JVS and QHR
- Volt balances
- Gyromagnetic ratio of
the proton
- Faraday constant
- ...

The BIPM watt balance

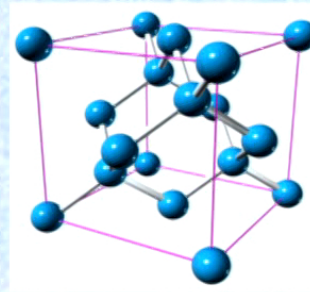




N_A and h by the XRCD method

h and N_A are related through the electron mass

$$N_A h = \frac{M(e) c \alpha^2}{2 R_\infty}$$



N_A is related to mass and lattice spacing

$$N_A = 8M/(\rho a^3)$$

Electron rest mass

Mass ratio

Number of Si atoms

Impurities, vacancies and surface layers

$$m_{\text{core}} = \frac{2hR_\infty}{c\alpha^2} \frac{\sum_i f_i A_r(^i\text{Si})}{A_r(\text{e})} \frac{8V_{\text{core}}}{a^3} - m_{\text{deficit}}$$

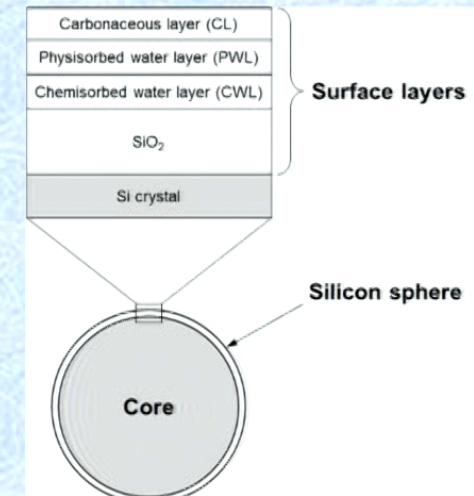
'weigh' the sphere

Isotope ratio and atomic weights

Volume and lattice spacing

A lot of chemical and surface analysis

$$m_{\text{sphere}} = m_{\text{core}} + m_{\text{SL}}$$





The XRCD in simple steps



The XRCD in simple steps

Grow a near perfect boule of ^{28}Si





The XRCD in simple steps

Grow a near perfect boule of ^{28}Si

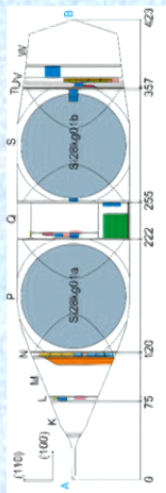
Measure:

Lattice spacing
Chemical Impurities
Voids & defects
Isotopic ratios

IAC



PTB

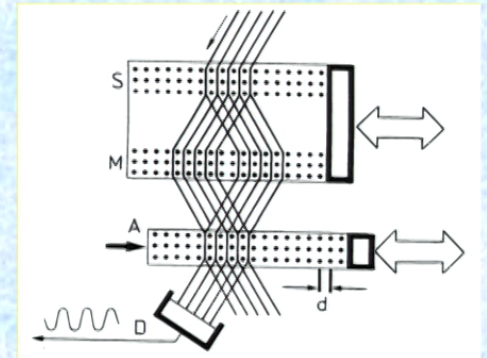
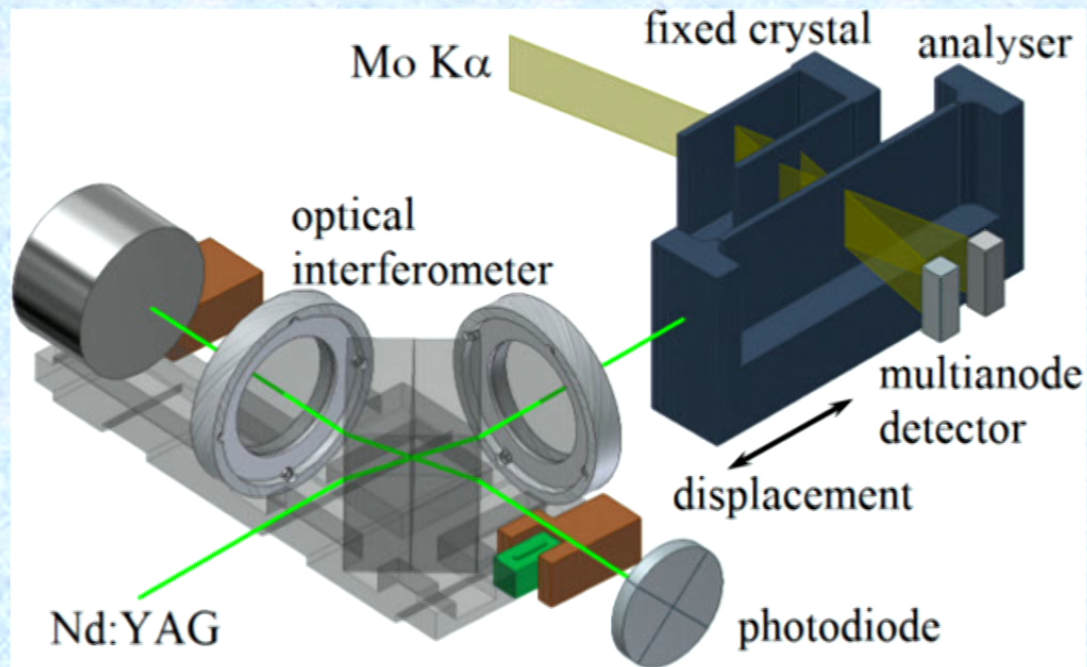
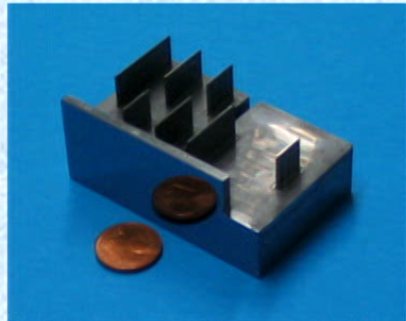




Si Lattice Spacing

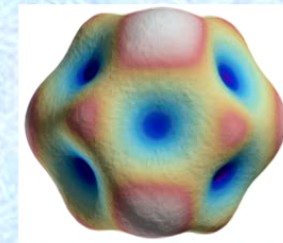
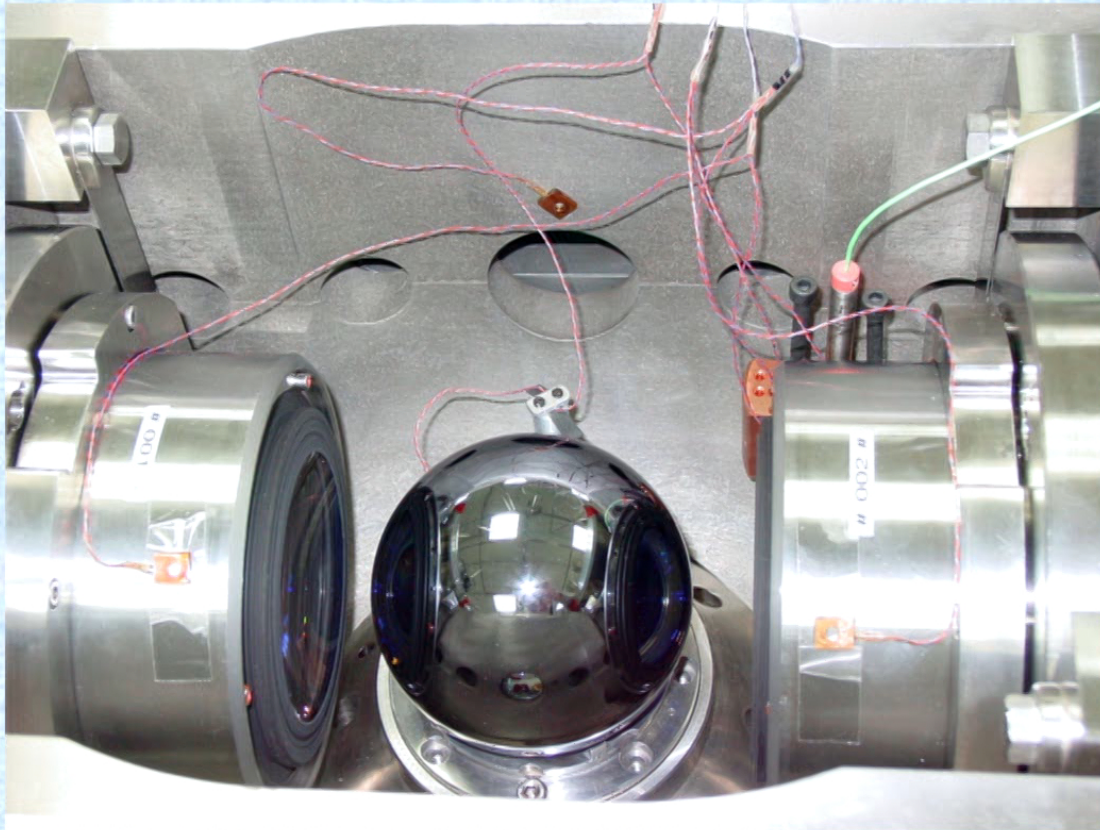
An X-ray Interferometry measures the Si lattice spacing. From a single crystal, grind out two thin walls and move them with respect to a third wall in an X-ray beam.

Relative standard uncertainty in a : 1.8×10^{-9}

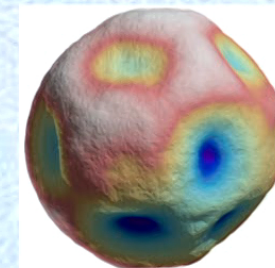


Volume Measurements

Fizeau interferometer at PTB



S5c



S8c

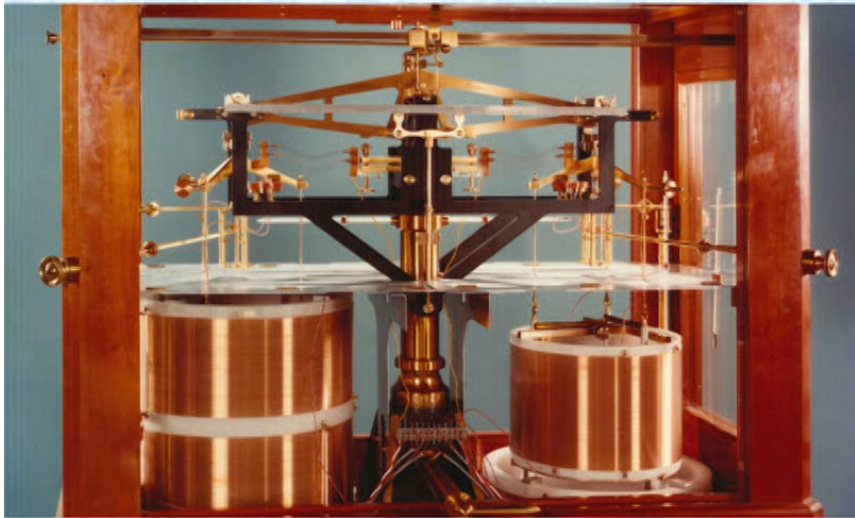


The Kibble Balance

What is the Kibble or watt balance, originally conceived by Bryan Kibble?

How is it related to Planck's constant and mass?

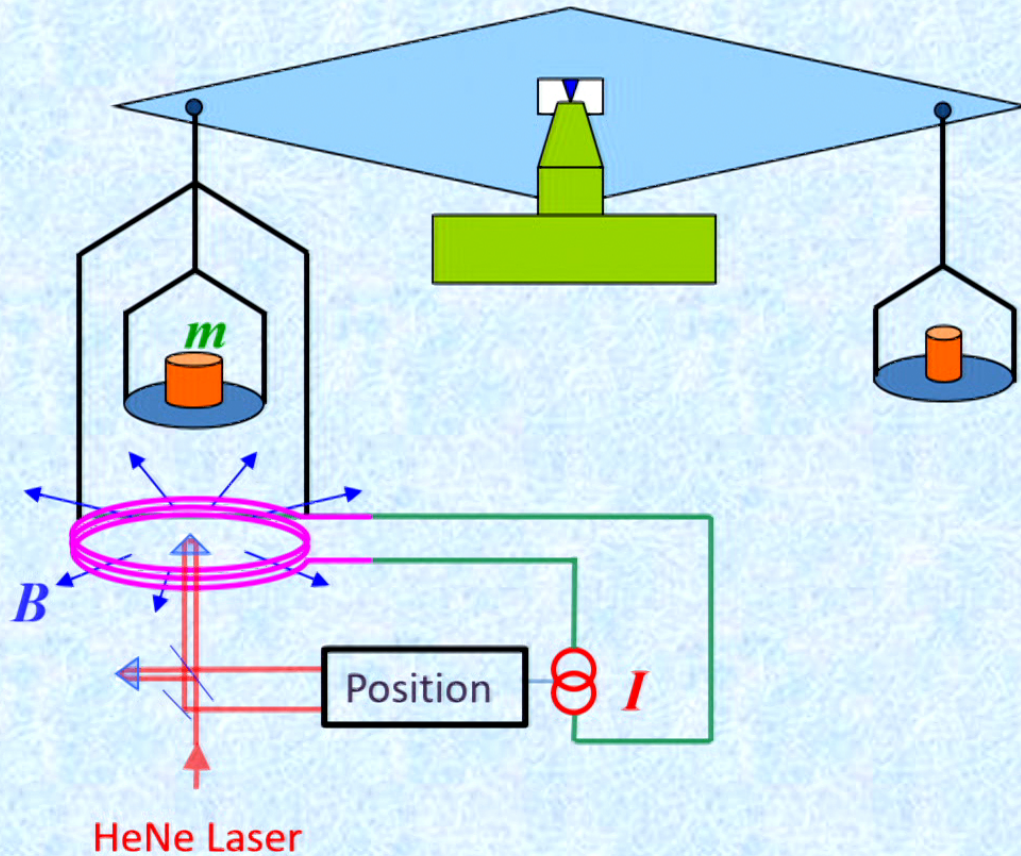
What is special about the NRC balance?



Bryan was trying to design a better ampere balance: determining a current from mass, length and time using the equivalence of electrical force and mechanical force.



Kibble Balance – weighing phase



Coil Position
Servoed to $z=0$

$$F = mg = BL I$$



Combining both phases

$$mg = BL I$$

weighing

$$V_c = v BL$$

moving

$$mgv = I V_c$$

The Kibble equation

Using the Josephson effect for voltage

$$V_c = n_1 f_1 \frac{h}{2e}$$

$$V_R = n_2 f_2 \frac{h}{2e}$$

& Ohm's law and the quantum Hall effect for resistance

$$I = \frac{V_R}{R}$$

$$R = n_3 \frac{h}{e^2}$$

so

$$m = \frac{1}{gv} \frac{V_c}{R} V_R = \frac{1}{gv} n_1 f_1 n_2 f_2 n_3 \frac{h}{4}$$

Mass is related to Planck's constant via gravity , velocity and frequency.



New values of h , e , k , N_A

In the Revised SI these will be the new values of h , e , k and N_A .
They will have no uncertainty!

h , Planck constant = $6.626\,070\,15 \times 10^{-34} \text{ Js}$

e , Elementary charge = $1.602\,176\,634 \times 10^{-19} \text{ C}$

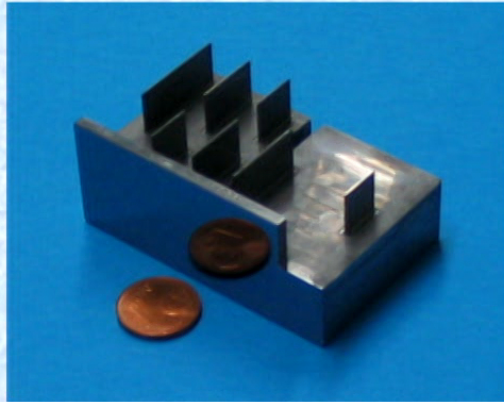
k , Boltzmann constant = $1.380\,649 \times 10^{-23} \text{ JK}^{-1}$

N_A , Avogadro number = $6.022\,140\,76 \times 10^{23} \text{ mol}^{-1}$

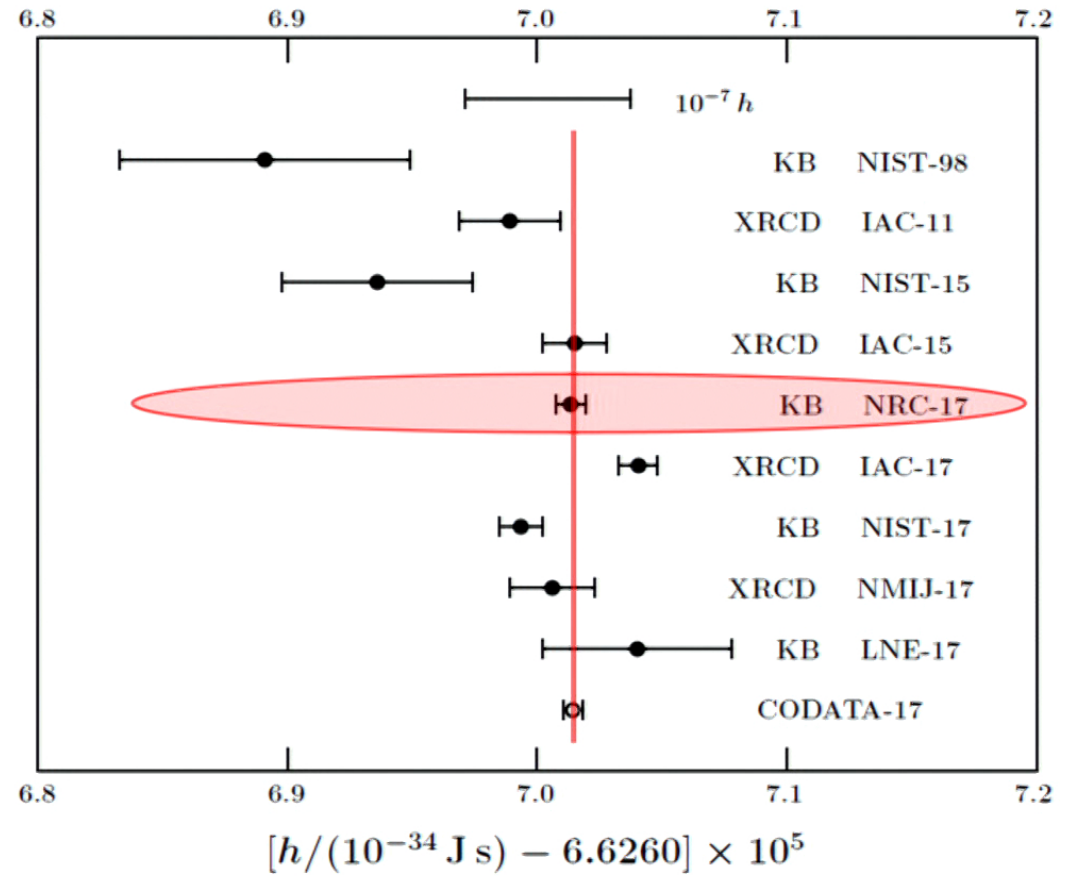
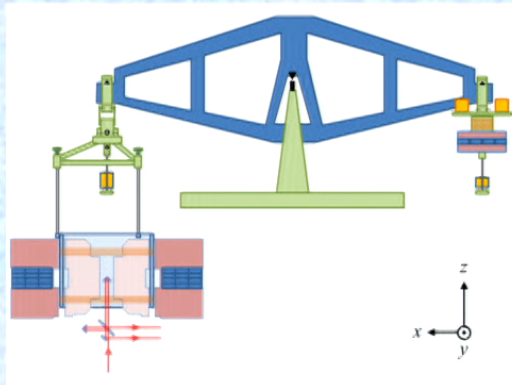


Planck Constant - 2017 LSA

XRCD



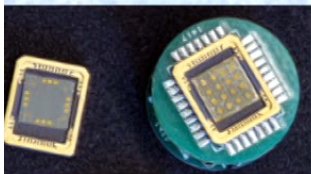
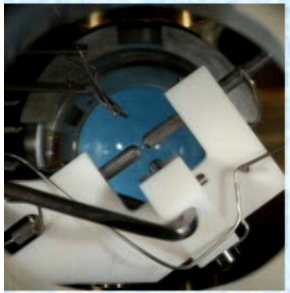
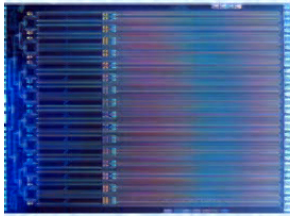
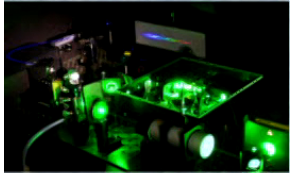
KB





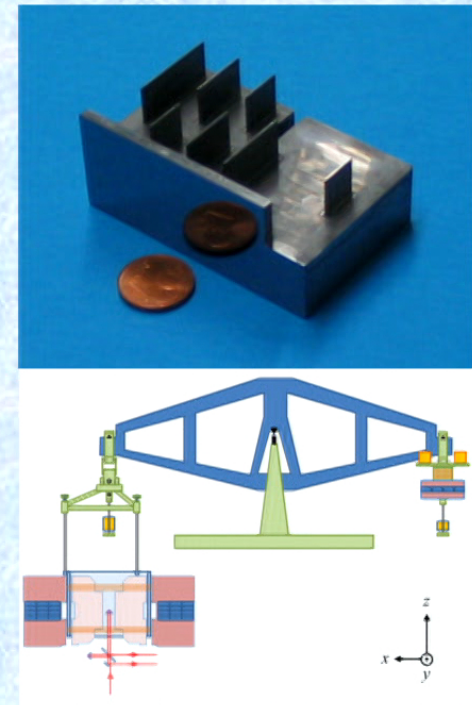
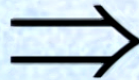
Other Extreme Measurements

Quantity	System	Relative Uncertainty
Frequency -	<i>Combs</i>	10^{-22}
Voltage -	<i>JVS</i>	10^{-11} to 10^{-19}
Time -	<i>Ion traps</i>	10^{-17}
Resistance -	<i>QHR</i>	10^{-11}
Current -	<i>CCCs</i>	10^{-11}
Mass -	<i>Vacuum balance</i>	10^{-10}
Temperature -		10^{-7}





The Evolution of the SI





More on Measurements.....

NRC Metrology Research Centre:

- **Canada's National Metrology Institute**
 - Accurate, traceable measurements
 - New measurement methods for emerging technologies

Moving from artifact-based calibrations towards direct measurement of fundamental constants and primary scales:

- Development of *inherent standards* by applying quantum-based technologies to metrology



BIPM Workshop: The Quantum Revolution in Metrology, France, September 2017



Topics:

- Advances in quantum electrical standards
- Single-photon measurements, entangled photon sources
- Highly entangled systems for metrology: entangled optical clocks
- Quantum standards for mass, force, pressure, vacuum, temperature, acoustics and vibration
- Emerging ideas in quantum metrology

137 Participants from 27 countries; 40 National Metrology Institutes, Universities, and Research Labs



Quantum Standards

Existing and evolving quantum standards:

- *Electrical units* (Josephson Voltage, ohm: Quantum Hall Effect devices) >25 years
- *Atomic clocks* (time and frequency) >60 years

New and forthcoming quantum standards:

- *Radiometry*: single photon measurements, single and entangled photon sources
- *Mass, pressure, vacuum, temperature, acoustics and vibration*

Future quantum standards/technologies include:

- Atomic scale magnetometry, entangled clocks
- Improved sensitivity and resolution due to quantum entanglement



Few Photon Metrology at NRC

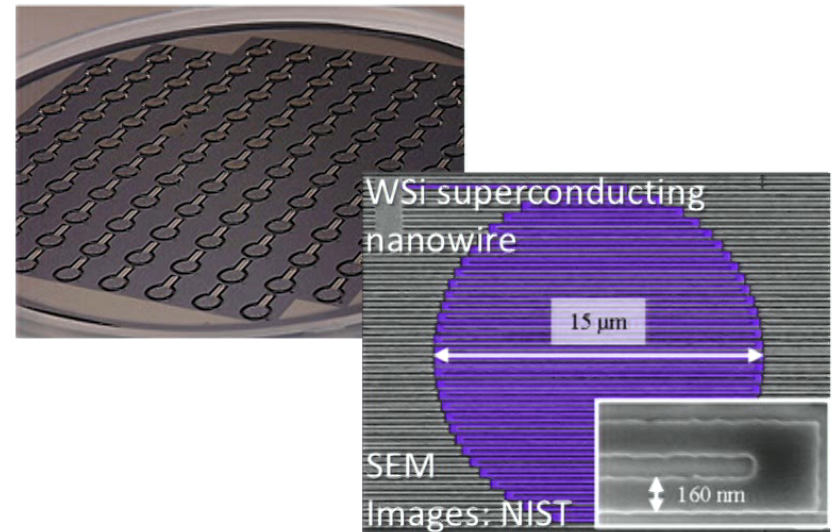
Project Goals:

- Single-photon detector efficiency measurements
 - Free space and fibre-coupled detectors
- Construction superconducting nanowire single-photon detector system (NIST)
- Characterize single-photon sources (NRC)



Looking forward:

- Multi-particle quantum state measurements



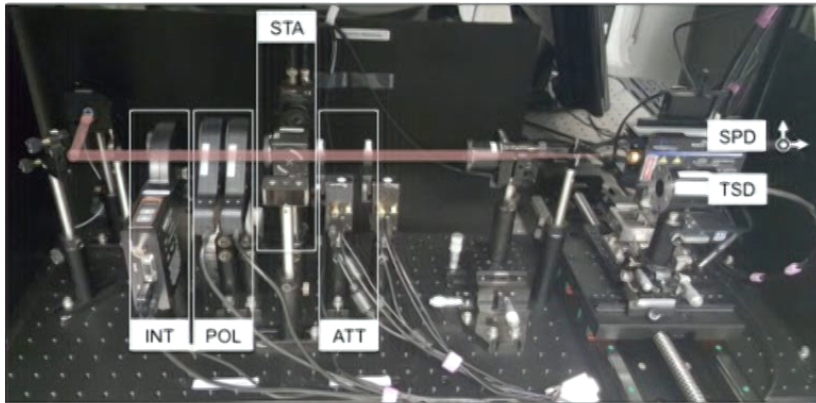


Single photon detector efficiency

Single-photon detector calibration capabilities → Free space Si SPADs

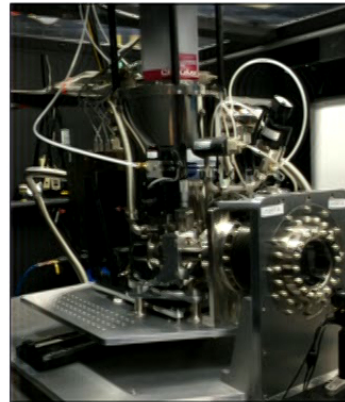
$$\eta = \frac{\# \text{ of detected photons per second}}{\# \text{ of incident photons per second}}$$

- Multiple filters are used to attenuate the input of a nano-joule laser beam to a level measurable by the single photon detector

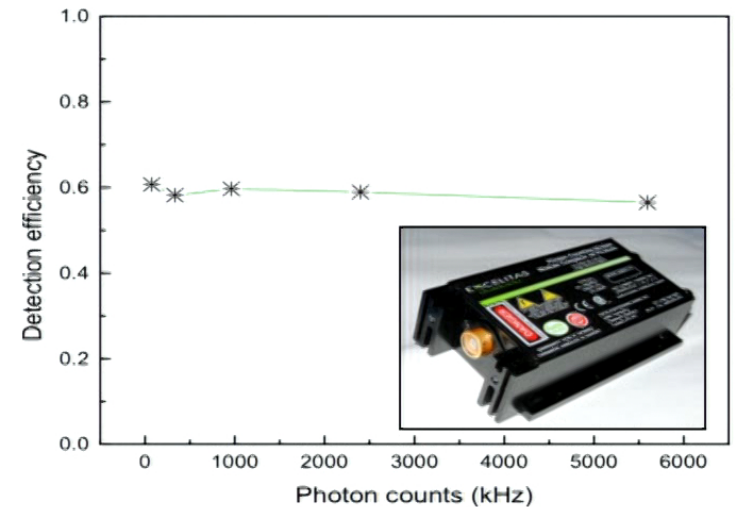


Efficiency measurements by substitution method

SI
→



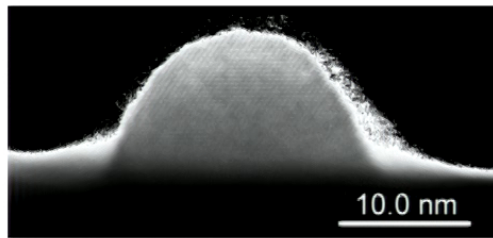
NRC cryogenic radiometer



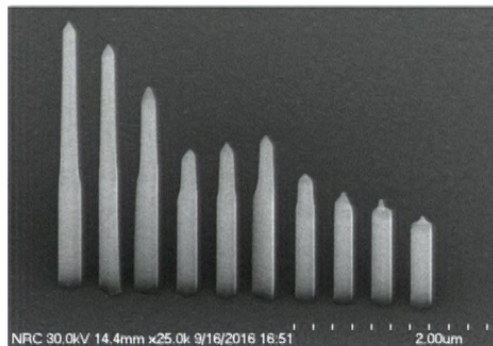


NRC single photon source

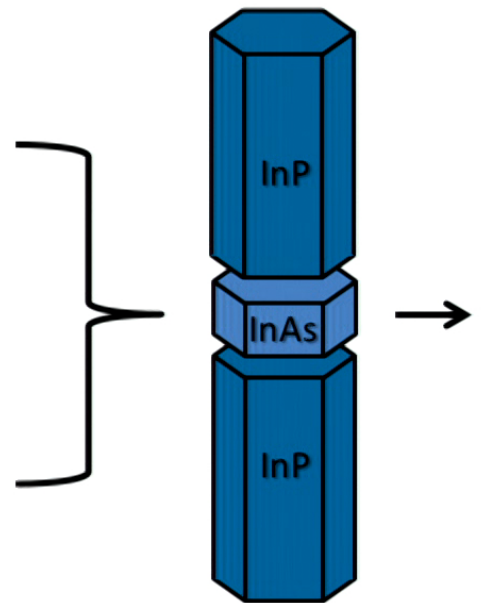
- Dan Dalacu and Robin Williams at NRC



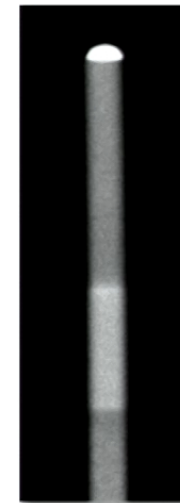
Self-assembled quantum dot



Site-controlled nanowires



Structure of quantum dot
embedded in nanowire



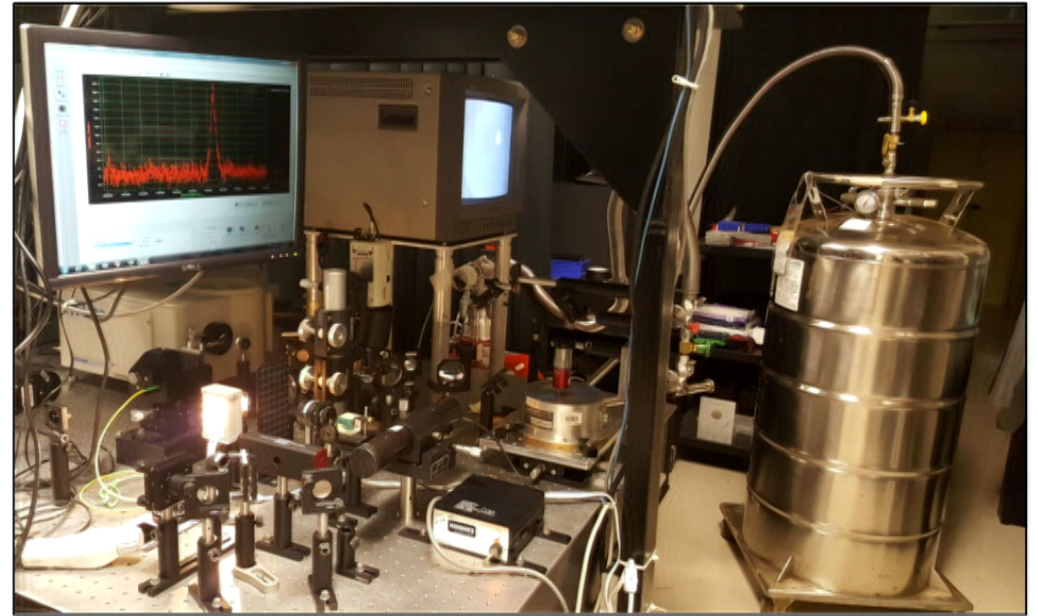
InAs
quantum
dots within
InP nanowire



Single photon source efficiency

Single photons on demand

- Presently no standard way to characterize source efficiency
- Efficiency ϵ : probability of collecting a photon
- Brightness β : number of photons collected per pump excitation pulse

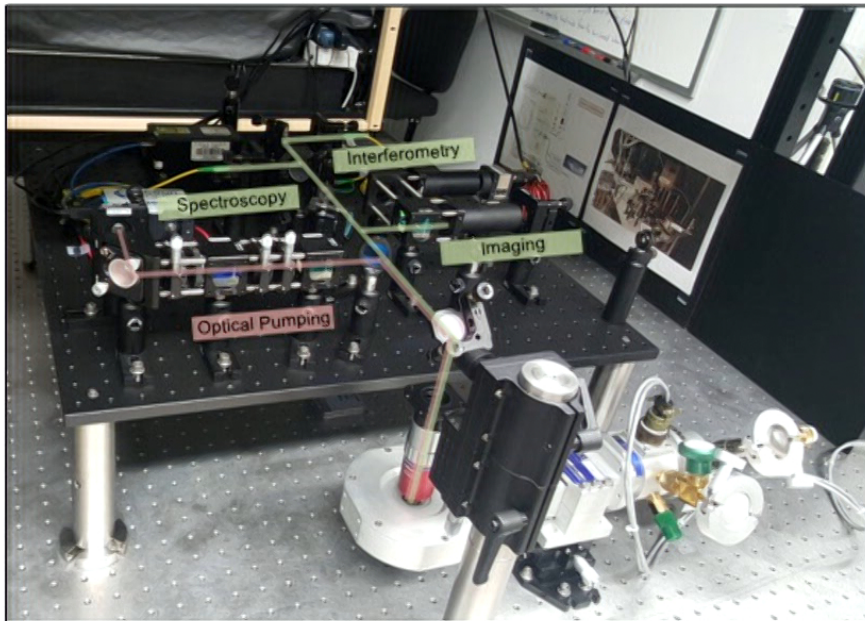


Single photon source characterization apparatus



Few photon measurements

NRC quantum dot nanowire photon sources → photons for multiparticle entangled states
Higher-order entangled states - double or triple quantum dots in semiconductor nanowire



New single photon source measurement apparatus

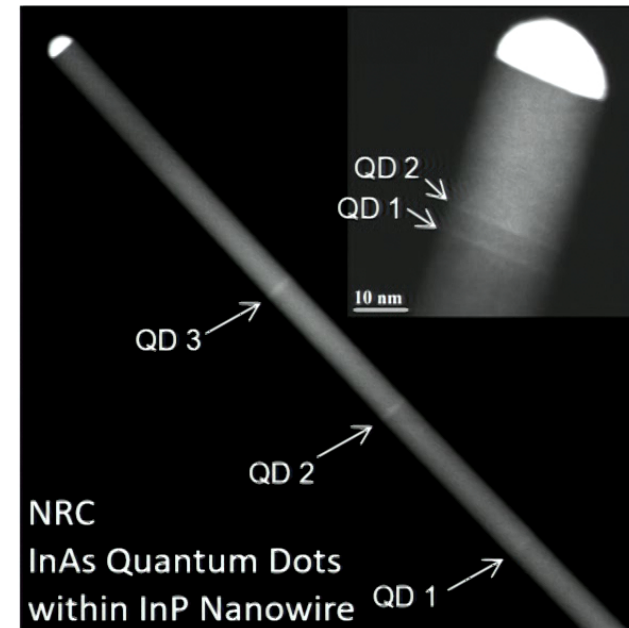


Image: D. Dalacu

Thank you!