Title: Efficient Preparation of Nontrivial Quantum States

Date: May 07, 2018 11:30 AM

URL: http://pirsa.org/18050040

Abstract:

Navigating the World of Quantum Matter

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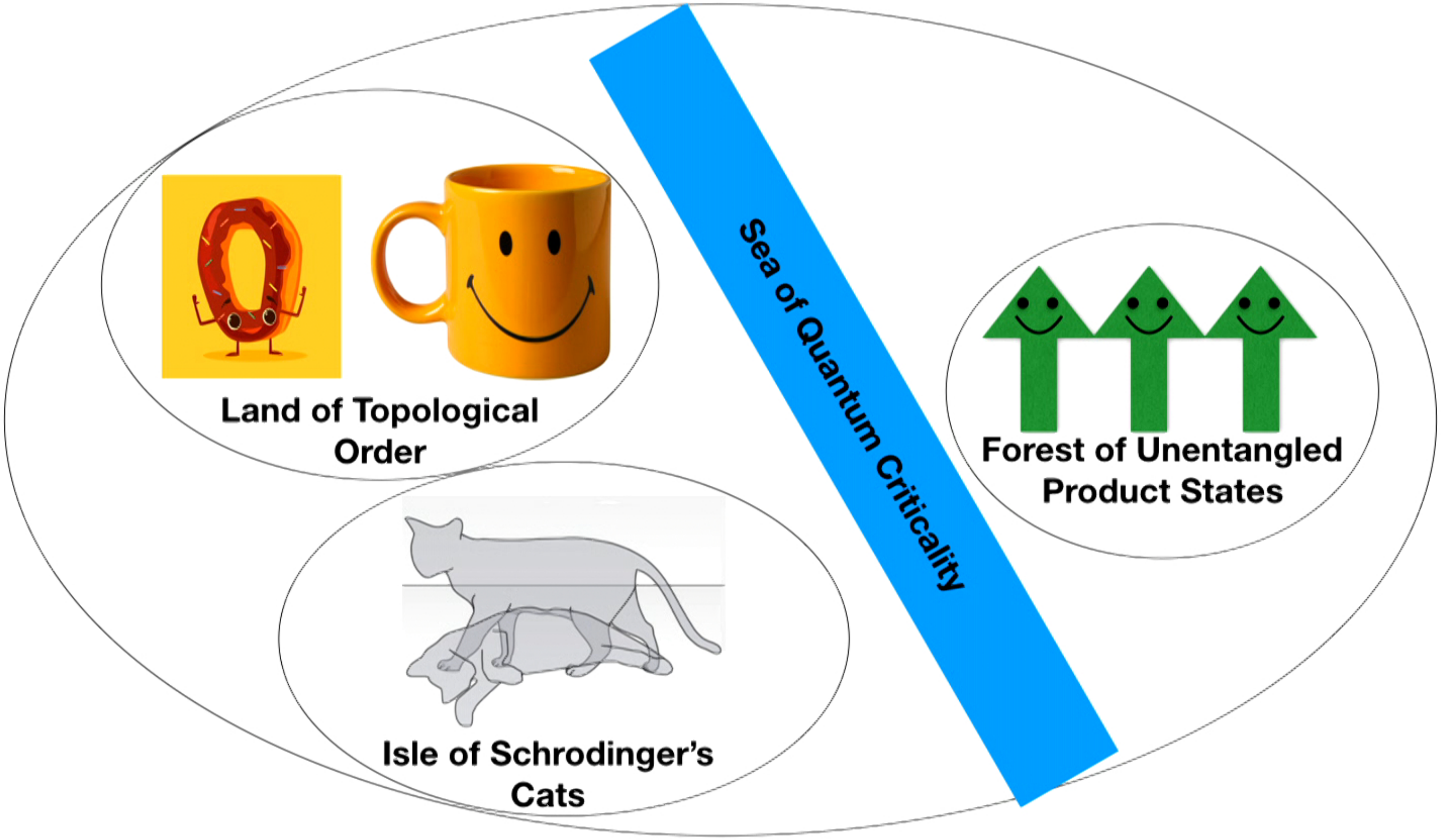

RESEARCH AREAS INCLUDE:

- Topological phases of matter
- Critical phases of matter and exotic quantum critical points
- State-of-the-art numerical and analytic approaches to the many-body problem
- Application of modern information and complexity theory to quantum many-body physics
- Quantum error correction, CFT, and bulk locality in holography and beyond
- Non-equilibrium phenomena
- Chaos, scrambling, and complexity in quantum matter and holography

Faculty:

- Yin-Chen He
- Tim Hsieh
- Beni Yoshida

Other faculty members connected to the initiative:

- Davide Gaiotto
- Daniel Gottesman
- Sung-Sik Lee
- Roger Melko
- Robert Myers
- Subir Sachdev (Maxwell Chair, Visiting)
- Guifre Vidal
- Jon Yard

Collaborators

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Harvard postdoc;
former PSI student

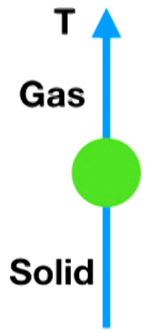
Cheryne Jonay

Current PSI student

WWH and TH, arXiv:1803.00026 (2018)

(Quantum) Phases of Matter

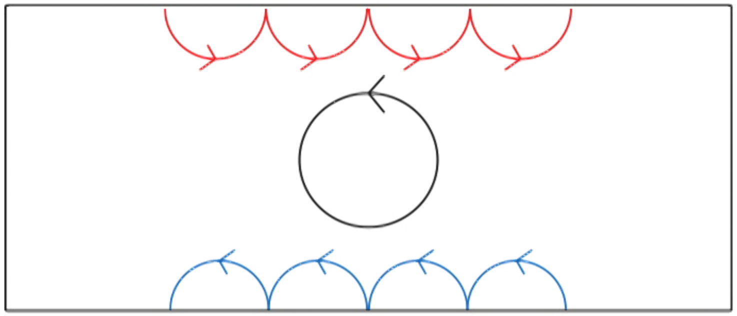
Classical: thermal fluctuations can drive a phase transition



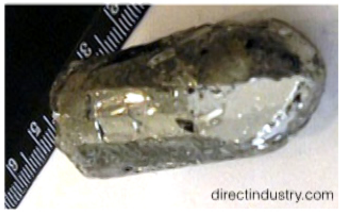
Quantum fluctuations can also drive transition

Topological Quantum Phases

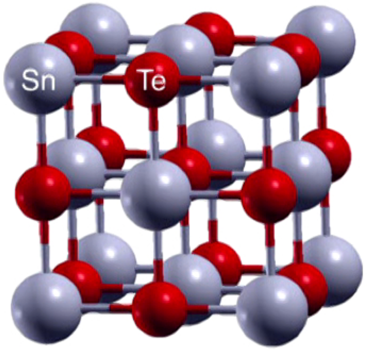
The original:
Integer quantum Hall



Recent offspring:
Topological materials

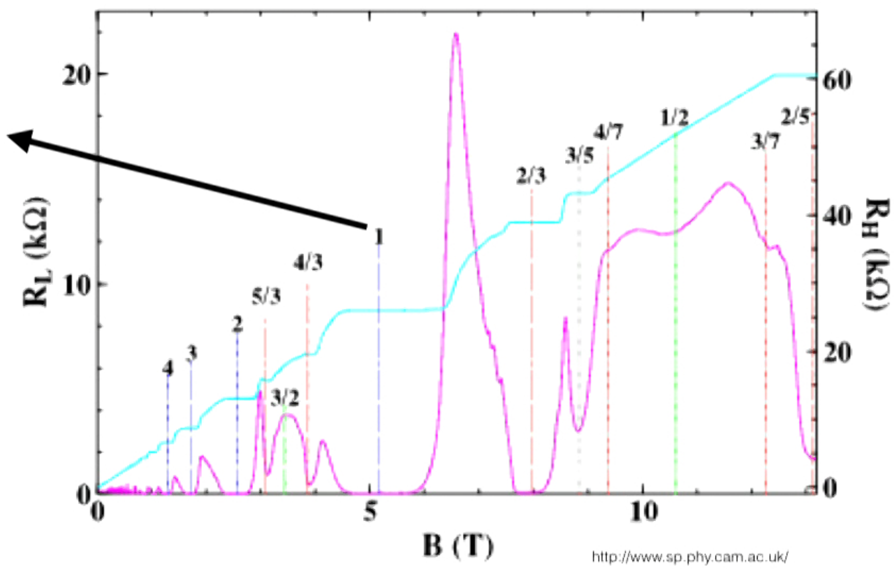


Bi_2Se_3



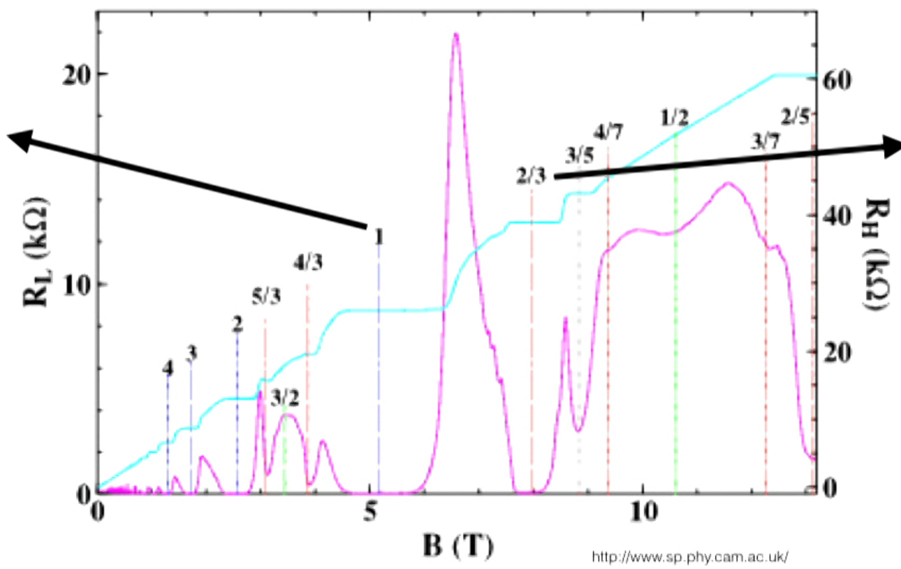
Fractional ones are rare

Integer
Quantum
Hall
Variety



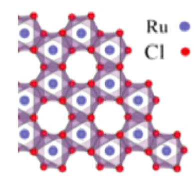
Fractional ones are rare

Integer Quantum Hall Variety



Fractional Quantum Hall Variety

Herbertsmithite



$RuCl_3$

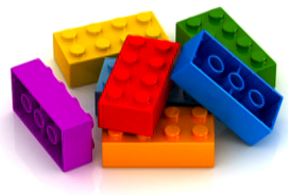
Challenging to realize in materials

Topological bootstrap:

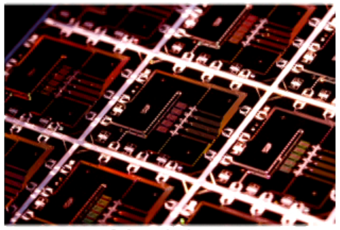
TH, Y-M. Lu, and A. Ludwig, Sci. Adv. 3 (10), e1700729

Synthetic Quantum Systems

- Highly tunable system of qubits with precise characterization and control



- Superconducting circuits



e.g. Martinis group
UCSB/Google

- Trapped ions

e.g. Monroe group
U. Maryland/IonQ

Quantum Computation/ Simulation

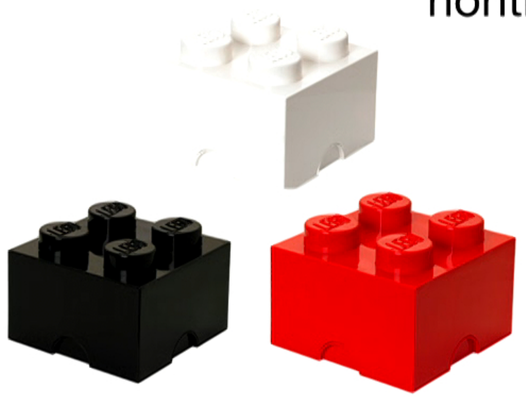
- Near term: 50-100 qubits, potentially useful for optimization problems
- Eventually: quantum algorithms like factoring
- Quantum simulation:
  - probe intractable quantum systems

Quantum Computation/ Simulation

- Near term: 50-100 qubits, potentially useful for optimization problems
- Eventually: quantum algorithms like factoring
- Quantum simulation:
  - probe intractable quantum systems
  - realization of nontrivial states of matter

Main Goal

Protocols for transforming trivial (unentangled) product state into nontrivial quantum state



Source: Pinterest

Achieve:

- Cat state
- Quantum critical point
- Topological order

Main Tool

Quantum approximate optimization algorithm (QAOA)

Farhi, Goldstone, Gutmann (2014)

Original goal: find good solution to a classical optimization problem

Find string of bits satisfying as many constraints on bits as possible

In physics terms: find configuration of spins which minimizes energy, e.g. $S_1^z S_2^z$

QAOA: not adiabatic

Quantum adiabatic algorithm:

Simple Hamiltonian	Target/cost Hamiltonian
H_X	H_t
$\|+\rangle$	$\|\psi_t\rangle$

QAOA: not adiabatic

Quantum adiabatic algorithm:

Simple Hamiltonian — Interpolate → Target/cost Hamiltonian

$H_X \longrightarrow H_t$

$|+\rangle \longrightarrow |\psi_t\rangle$

If energy gap closes, need to go very slowly

Variational approach (QAOA)

$$|\psi\rangle = e^{-i\beta_p H_X} e^{-i\gamma_p H_t} \cdots e^{-i\beta_1 H_X} e^{-i\gamma_1 H_t} |+\rangle$$

Choose evolution times to minimize energy $\langle\psi|H_t|\psi\rangle$

QAOA as preparation scheme for *quantum* state

Quantum target Hamiltonian H_t

$$|\psi(\vec{\gamma}, \vec{\beta})\rangle_p = e^{-i\beta_p H_X} e^{-i\gamma_p H_I} \cdots e^{-i\beta_1 H_X} e^{-i\gamma_1 H_I} |+\rangle$$

Depth p is number of operations allowed

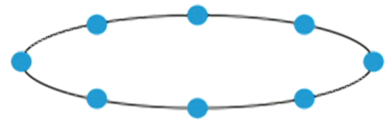
Minimize energy cost

$$F_p(\vec{\gamma}, \vec{\beta}) = {}_p\langle\psi(\vec{\gamma}, \vec{\beta})|H_t|\psi(\vec{\gamma}, \vec{\beta})\rangle_p$$

Wecker, Hastings, Troyer (2015)

This talk: how well can QAOA prepare non-trivial states, which are separated from product states by a phase transition?

Transverse Field Ising Model



$$H_{\text{TFIM}} = -\sum_{i=1}^{L} Z_i Z_{i+1} - g \sum_{i=1}^{L} X_i$$

$$|+++\ldots\rangle$$

$$X|+\rangle = |+\rangle$$

Warmup: GHZ (Cat) State

Target: $|\psi_t\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow ...\rangle + |\downarrow\downarrow\downarrow ...\rangle)$

$$H_t = -\sum_{i=1}^{L} Z_i Z_{i+1}$$

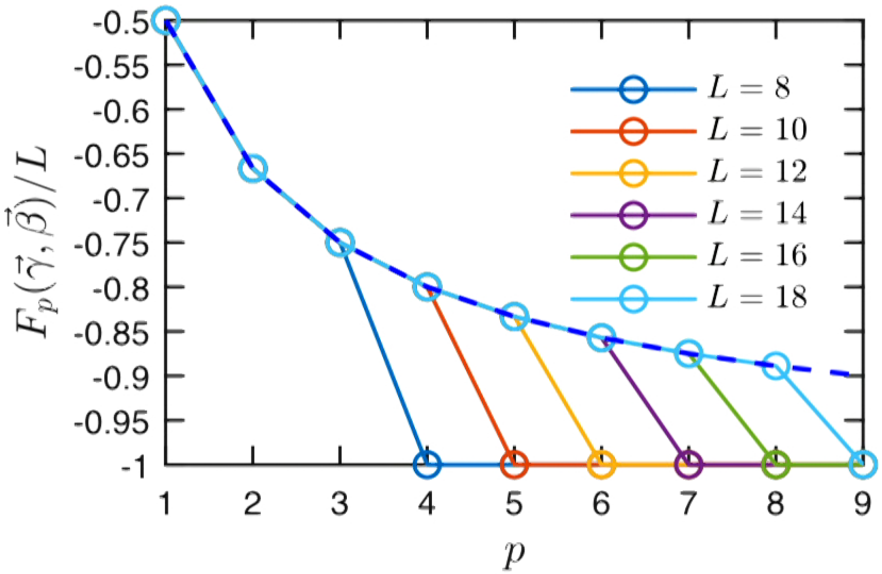
$$H_X = -\sum_i X_i$$

$$H_I = -\sum_{i=1}^{L} Z_i Z_{i+1}$$

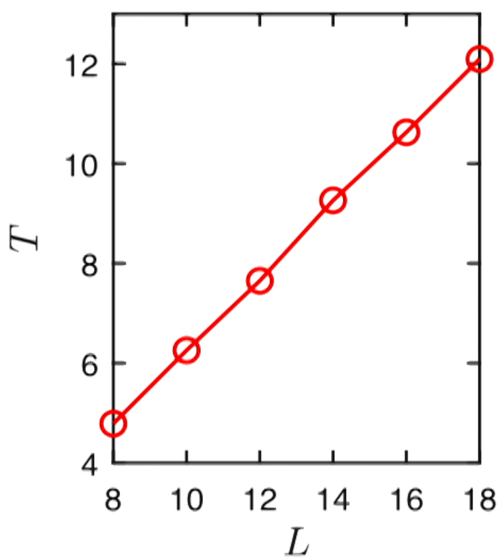
$$|\psi(\vec{\gamma},\vec{\beta})\rangle_p = e^{-i\beta_p H_X} e^{-i\gamma_p H_I} \cdots e^{-i\beta_1 H_X} e^{-i\gamma_1 H_I} |+\rangle$$

Cost function to be minimized: $F_p(\vec{\gamma},\vec{\beta}) = {}_p\langle\psi(\vec{\gamma},\vec{\beta})|H_t|\psi(\vec{\gamma},\vec{\beta})\rangle_p$

Preparation of GHZ State



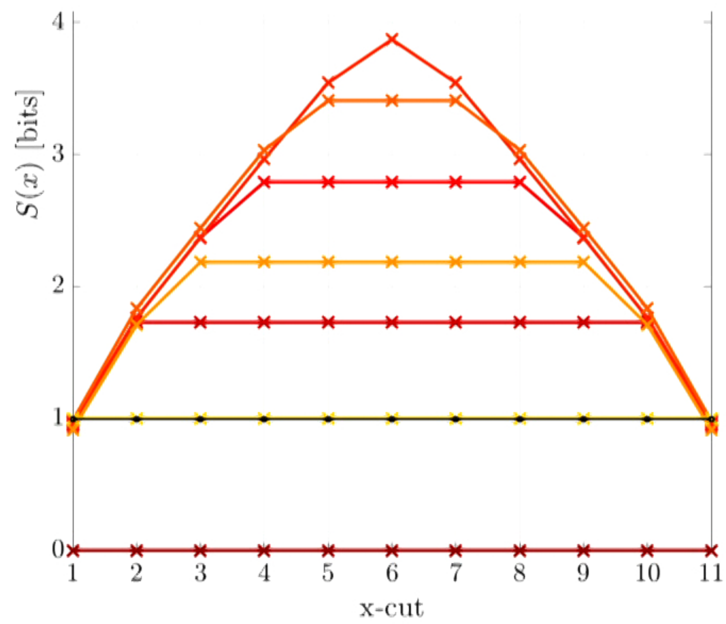
Perfect fidelity achieved at p = L/2



Minimum time to prepare GHZ

QAOA explores big Hilbert Space

L = 12, optimal GHZ preparation sequence



Entanglement growth during evolution of state

Quantum Critical State

Target: ground state of

$$H_t = -\sum_{i=1}^{L} Z_i Z_{i+1} - \sum_{i=1}^{L} X_i$$

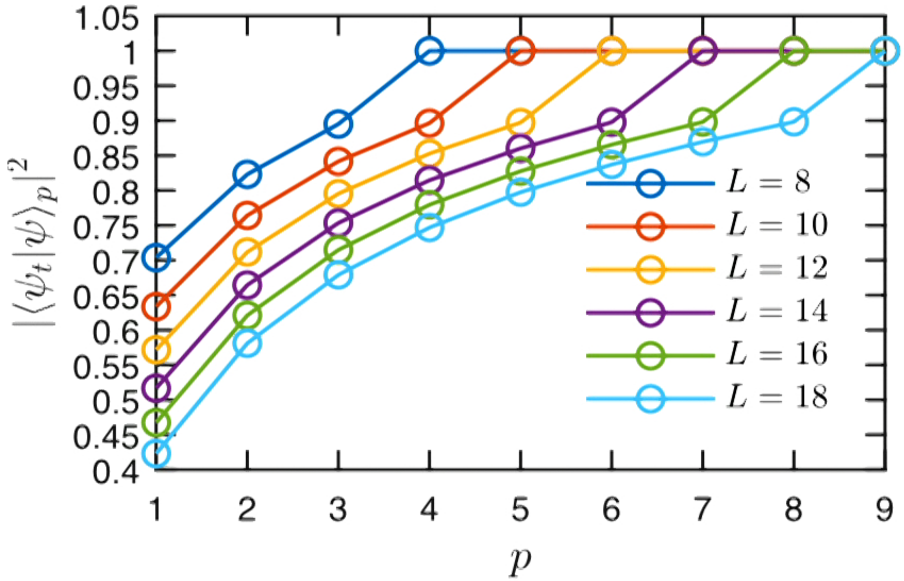
$$H_X = -\sum_i X_i$$

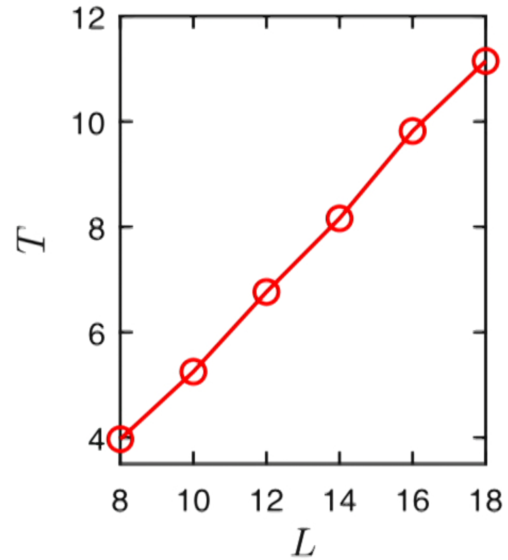
$$H_I = -\sum_{i=1}^{L} Z_i Z_{i+1}$$

$$|\psi(\vec{\gamma}, \vec{\beta})\rangle_p = e^{-i\beta_p H_X} e^{-i\gamma_p H_I} \cdots e^{-i\beta_1 H_X} e^{-i\gamma_1 H_I} |+\rangle$$

Cost function to be minimized: $F_p(\vec{\gamma}, \vec{\beta}) = {}_p\langle\psi(\vec{\gamma}, \vec{\beta})|H_t|\psi(\vec{\gamma}, \vec{\beta})\rangle_p$

Quantum Critical State Preparation





Perfect fidelity achieved at p = L/2

Minimum preparation time

Concrete Protocol

$$|\psi(\vec{\gamma},\vec{\beta})\rangle_p = e^{-i\beta_p H_X} e^{-i\gamma_p H_I} \cdots e^{-i\beta_1 H_X} e^{-i\gamma_1 H_I} |+\rangle$$

$L = 10, T = 5.250$:

$$(0.2473, 0.6977, 0.4888, 0.6783, 0.5559, 0.6567, 0.5558, 0.6029, 0.4598, 0.3068)$$

$L = 12, T = 6.7651$:

$$(0.2809, 0.6131, 0.6633, 0.4537, 0.8653, 0.4663, 0.6970, 0.6829, 0.4569, 0.7990, 0.3565, 0.4304)$$

$L = 14, T = 8.1604$:

$$(0.3090, 0.5710, 0.6923, 0.5648, 0.5391, 0.9684, 0.3979, 0.6852, 0.8235, 0.4474, 0.6930, 0.6465, 0.4120, 0.4104)$$

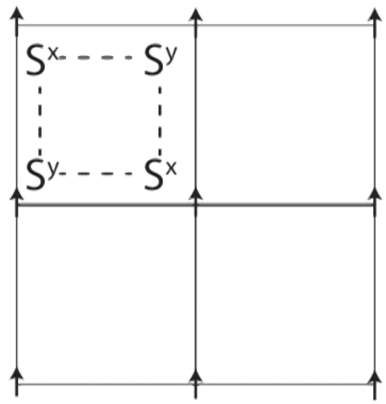
$L = 16, T = 9.8198$:

$$(0.3790, 0.5622, 0.5638, 0.7101, 0.9046, 0.3210, 0.6738, 0.8377, 0.8616, 0.4004, 0.5624, 0.9450, 0.5224, 0.6466, 0.4119, 0.5172)$$

$L = 18, T = 11.1485$:

$$(0.3830, 0.4931, 0.7099, 0.7010, 0.5330, 0.6523, 0.6887, 1.0405, 0.3083, 0.6215, 0.9607, 0.5977, 0.6209, 0.5597, 0.7850, 0.5851, 0.4132, 0.4948)$$

Topologically Ordered State



$$H_t = -\sum_{i=1}^{L}\sum_{j=1}^{L} \sigma^x_{i,j+1}\sigma^y_{i+1,j+1}\sigma^x_{i+1,j}\sigma^y_{i,j}$$

Kitaev toric code / Wen plaquette model

Exhibit fractionalization: excitations can be neither bosons not fermions

“Surface code” is promising for fault-tolerant quantum computer

Toric Code via QAOA

$$H_t = -\sum_{i=1}^{L}\sum_{j=1}^{L} \sigma^x_{i,j+1}\sigma^y_{i+1,j+1}\sigma^x_{i+1,j}\sigma^y_{i,j}$$

$$H_X = -\sum_i X_i$$

$$H_I = -\sum_{i=1}^{L}\sum_{j=1}^{L} \sigma^x_{i,j+1}\sigma^y_{i+1,j+1}\sigma^x_{i+1,j}\sigma^y_{i,j}$$

An operator duality maps each diagonal into a TFIM chain

Diagonals can be prepared in parallel

Use optimal angles from GHZ prep

Perfect fidelity at depth p = L / 2

Shortcuts via Long Range Interactions

Previously: achieved perfect fidelity, but with O(L) circuit depth

Low-depth quantum circuits are highly desirable

Long range (decaying with power law) interactions are natural in trapped ion systems

$$H = -\sum_{i<j}^{L} Z_i Z_j \left(\frac{1}{|i-j|^{\alpha}} \right) - g \sum_{i=1}^{L} X_i \qquad 0 < \alpha < 3$$

Quantum Criticality via Long-Range Interactions

$$H_t = -\sum_{i=1}^{L} Z_i Z_{i+1} - \sum_{i=1}^{L} X_i$$

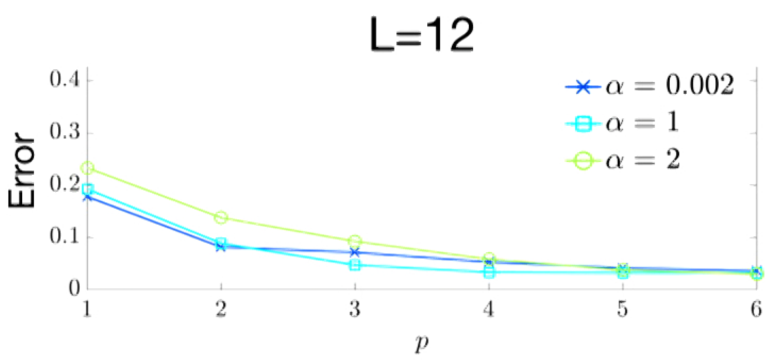
$$H_X = -\sum_i X_i$$

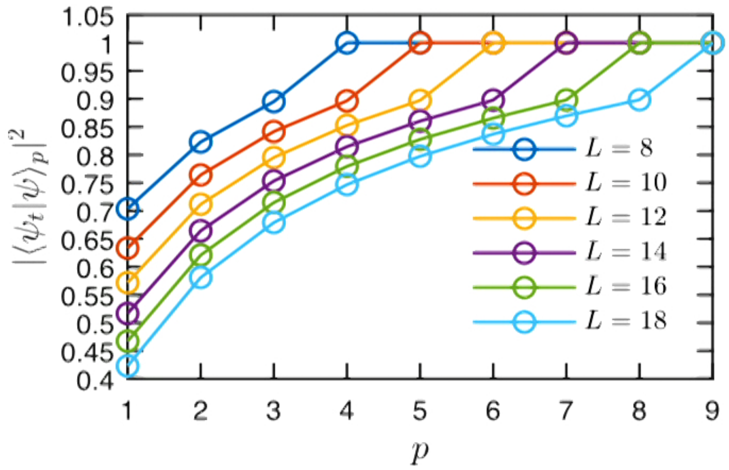
$$H_I = -\sum_{i<j}^{L} Z_i Z_j \left(\frac{1}{|i-j|^{\alpha}} \right)$$

$$|\psi(\vec{\gamma}, \vec{\beta})\rangle_p = e^{-i\beta_p H_X} e^{-i\gamma_p H_I} \cdots e^{-i\beta_1 H_X} e^{-i\gamma_1 H_I} |+\rangle$$

Cost function to be minimized: $F_p(\vec{\gamma}, \vec{\beta}) = {}_p\langle\psi(\vec{\gamma}, \vec{\beta})|H_t|\psi(\vec{\gamma}, \vec{\beta})\rangle_p$

Quantum Criticality via Long-Range Interactions





Compare with short-range QAOA

Power-law interactions "compatible" with critical state?

Non-trivial quantum states via QAOA

- Explicit protocols for preparing GHZ, quantum critical, and topologically ordered states **with perfect fidelity** and O(L) circuit depth
- This talk: emphasis on concrete protocols
- several conceptual aspects
    - circuit complexity of quantum states
    - variational wavefunctions for numerics

To Do

- Experiments: carry out protocols! (with feedback)
- Theory:
  - Develop protocols for other interesting states
  - Understand better how QAOA approach works
  - Find a better acronym

Thanks!

WWH and TH, arXiv:1803.00026 (2018)

GORDON AND BETTY MOORE FOUNDATION

PERIMETER INSTITUTE

Effect of Errors

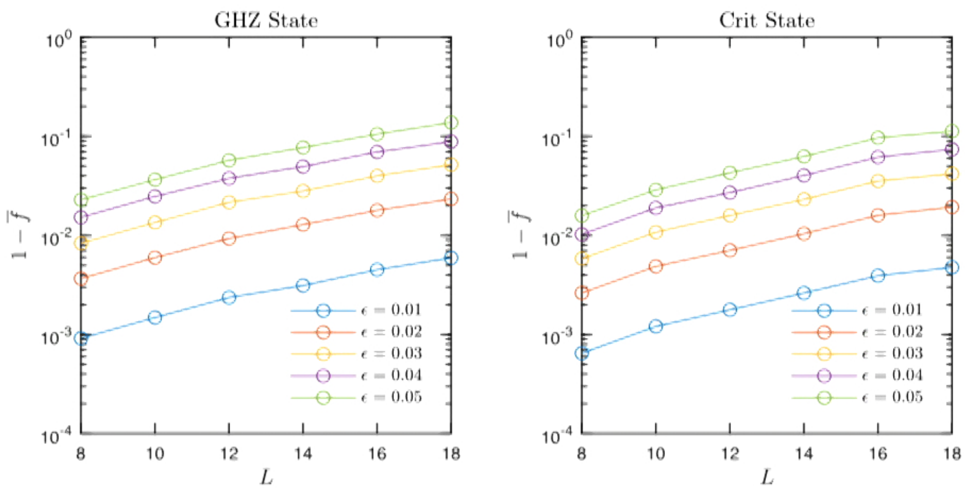


Figure 8. Effect of errors of strength ϵ on the QAOA preparation of GHZ and critical states for system size L. Plotted is the infidelity averaged over 1000 error realizations (denoted by the overline).