Title: Superradiance Beyond the Linear Regime

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Abstract:

Superradiance Beyond the Linear Regime

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Searching for New Particles with Black Hole Superradiance

Perimeter Institute, May 10 2018

Outline

- Motivation
  - why study non-linear aspects of superradiance?
  - why is it challenging?
- Methods
  - the BEST scheme to help reduce the computational cost for certain classes of problem
- Results
  - gravitational wave superradiance
  - Proca-field superradiance
- Conclusions

Black Hole Superradiance

- Well known that energy can be extracted from rotating black holes, in various ways
  - Penrose process, the Blandford-Znajek mechanism, scattering of bosonic fields (including gravitational waves) off the black hole
    - motivation for studying massive bosonic fields of particular astrophysical interest - *see the conference "Searching for New Particles with Black Hole Superradiance", Perimeter Institute, May 9-11 2018*
- Phenomenon not unique to black hole physics, but the explanation for why it must occur here connected to the laws of black mechanics, energy conditions and cosmic censorship
  - an excellent testbed then for exploring black hole physics and the dynamical strong-field regime of general relativity

Black Hole Superradiance

- Many aspects of superradiance well studied in the linear regime
  - this is justifiable when the spacetime changes slowly compared to the light-crossing time of the black hole, and/or when the metric perturbation away from Kerr is small
  - still much room for novel work here
- Open questions beyond linear approximations
  - (0^{th} order : sanity check on typical approach to incorporate back-reaction on the geometry using energy/angular momentum conservation)
  - evolution past the exponential growth phase (how the instability saturates), back-reaction on the geometry in scenarios where growth is rapid and/or a large fraction angular momentum extracted, for massive fields stability of "hair", if sufficiently long-lived dynamics in binary mergers, etc.

Computational Challenge

- Analytic methods unavailable to tackle full problem
- Can apply numerical methods then; few studies at present, dearth in part due to the computational challenge
  - one exception is the charged scalar/Reisner-Nordstrom case, which can be studied in spherical symmetry (see e.g. *Bosch, Green & Lehner, PRL 116 (2016); Baake & Rinne, PRD 94 (2016))*
  - the Kerr case requires at least axisymmetry, but the most interesting regimes have no symmetries
  - astrophysically interesting scenarios typically exhibit timescales not easily amenable to numerical treatment

Computational Challenge

- Here presenting results from a couple corners of parameter space where numerics are feasible today
  - large amplitude gravitational wave scattering off initially near-extremal Kerr : no symmetry assumptions, but short enough timescales O(100M)
  - growth to saturation of a complex Proca-field cloud from initially near-extremal Kerr, but restricting to a single azimuthal (m=1) mode and forcing axisymmetry in the geometry : achieve necessary $O(10^5 M)$ evolution
- Despite the above simplifications, still require a code “adapted” to the problem in some sense
  - a natural approach would be a spherical-polar based code. Time consuming writing a new Einstein code; also, spherical polar based coordinates not well adapted to eventual application to binaries
  - instead using an existing 3D Cartesian code; feasible for the single black hole case thanks to the Background Error Subtraction Technique (BEST)
    - a relatively simple addition that can be made to any code (finite difference, pseudo-spectral, Cartesian, spherical polar, Harmonic, BSSN, …) that can give an order of magnitude or more speedup in this kind of setting

The Background Error Subtraction Technique

Work with W. East, PRD 87 (2013)

- Original motivation was to study the tidal disruption of a main sequence star by a super-massive (O(10^7-10^8 $M_\odot$) black hole
  - disruption and subsequent accretion of material onto the black hole expected to produce electromagnetic emission on a timescale of order months after the event
  - several candidates observed, and with upcoming transient surveys, expect thousands over the coming decade
- With these systems, disruption occurs near the inner most stable orbit of the black hole, and hence the details of the disruption subject to strong relativistic effects, in particular black hole spin-induced orbital plane precession
  - In principle can then infer properties of the central supermassive BH by observation of the properties of the tidal flare

The Background Error Subtraction Technique

- The back-reaction of the star on the black hole metric is negligible, and following complete disruption the self-gravity of the stellar matter is unimportant in its dynamics

- Two main approaches have been employed to study this problem :

  - special relativistic hydrodynamics on a fixed background metric

    - correctly models orbital properties of unbound particle streams about a Kerr black hole

    - errors when the self-binding of the stellar matter is important

  - Newtonian hydrodynamics with a modified point-particle potential and event-horizon sink.

    - accurately models stellar self-gravity effects

    - not clear yet how well the existing proposed modified Newtonian potentials capture key properties of orbital dynamics about rapidly rotating black holes

BEST Algorithm

- Write the full solution $\mathbf{y}$ as (spatial dependence suppressed)

$$y(t) = Y(t) + \delta(t)$$

where $\mathbf{Y}$ is the known solution and δ the perturbation away from it.

- Let Δ be the single time step, discrete evolution operator

$$f(t = t_{n+1}) = \Delta\left[f(t = t_n)\right]$$

- Each time step, compute the solution error $\mathbf{E}$ of $\mathbf{Y}$ evolved with Δ

$$E_n = \Delta\left[Y(t = t_n)\right] - Y(t = t_{n+1})$$

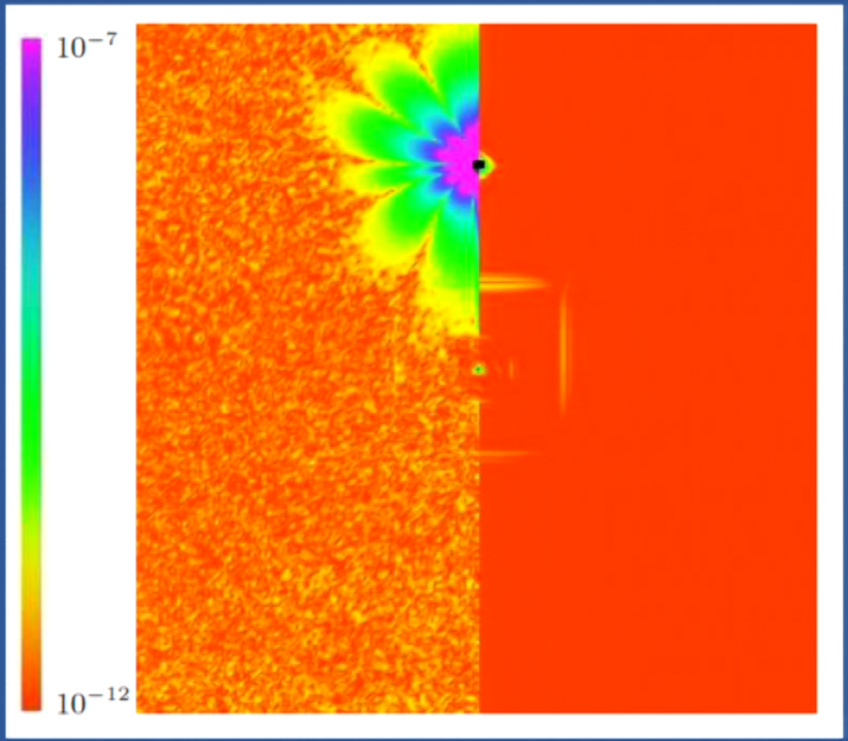
then evolve $\mathbf{y}$ with Δ and subtract $\mathbf{E}$

$$y(t = t_{n+1}) = \Delta\left[y(t = t_n)\right] - E_n$$

Comments

- It is simple to modify an existing code to use BEST, and the modification doesn't have to be adapted to any particular background, PDEs or solution method
  - each evolution step has to be performed twice, once with the full solution, once with the background, and then the correction step performed
    - certain situations can be optimized, e.g. if the background **Y** is static and the grid hierarchy is fixed (or infrequently changed)
  - memory needs to be allocated for a discrete exact solution
  - at worst the algorithm is twice as slow, though significant cost savings can be achieved if a large volume of the domain, where the exact solution dominates, can effectively be under-resolved
    - for the stellar disruption problem we achieved speed-ups of 40 times for axisymmetric (2D) head-on collision test problems, and over two orders of magnitude for the more relevant 3D case.

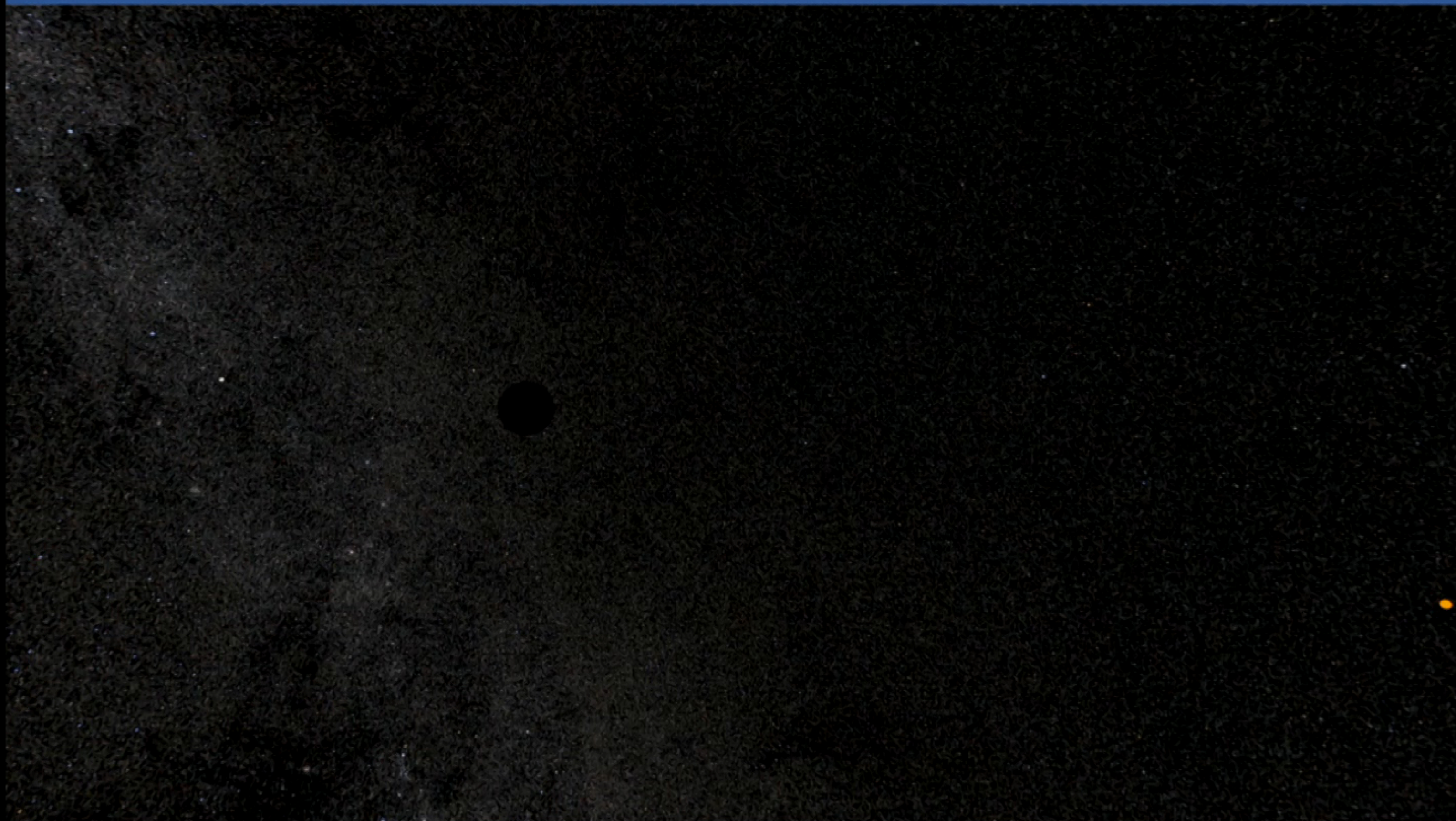
Example : head-on black hole/star encounter



- Truncation error in one of the metric components without (left) and with (right) BEST
- The star (center) has 6 additional levels of 2:1 refinement compared to the black hole, and here $m/M=10^{-6}$

Grazing collision

W. East, Astrophys.J. 795 (2014) 2



Animation by Ralf Kahler, KIPAC

Grazing collision

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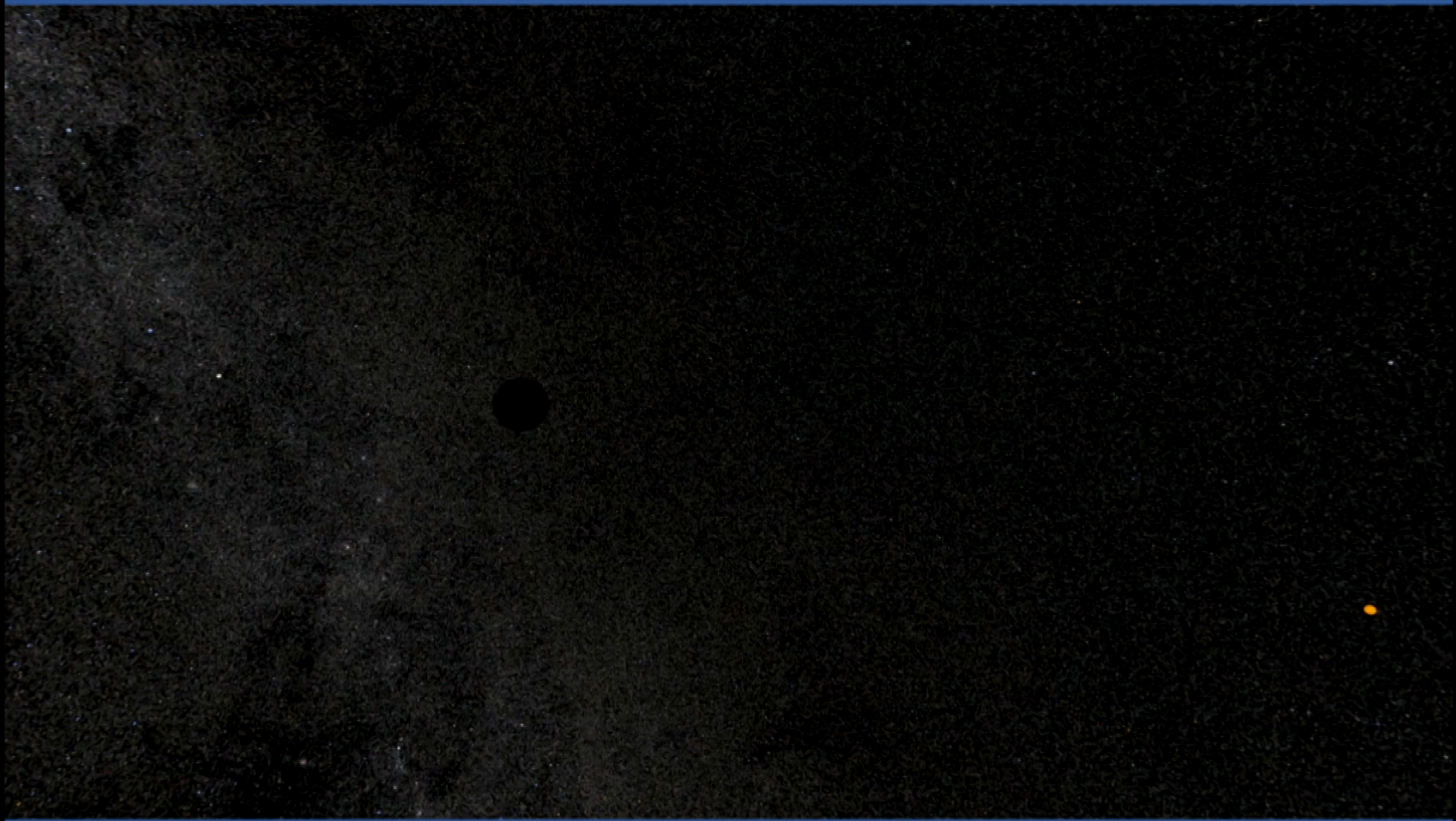

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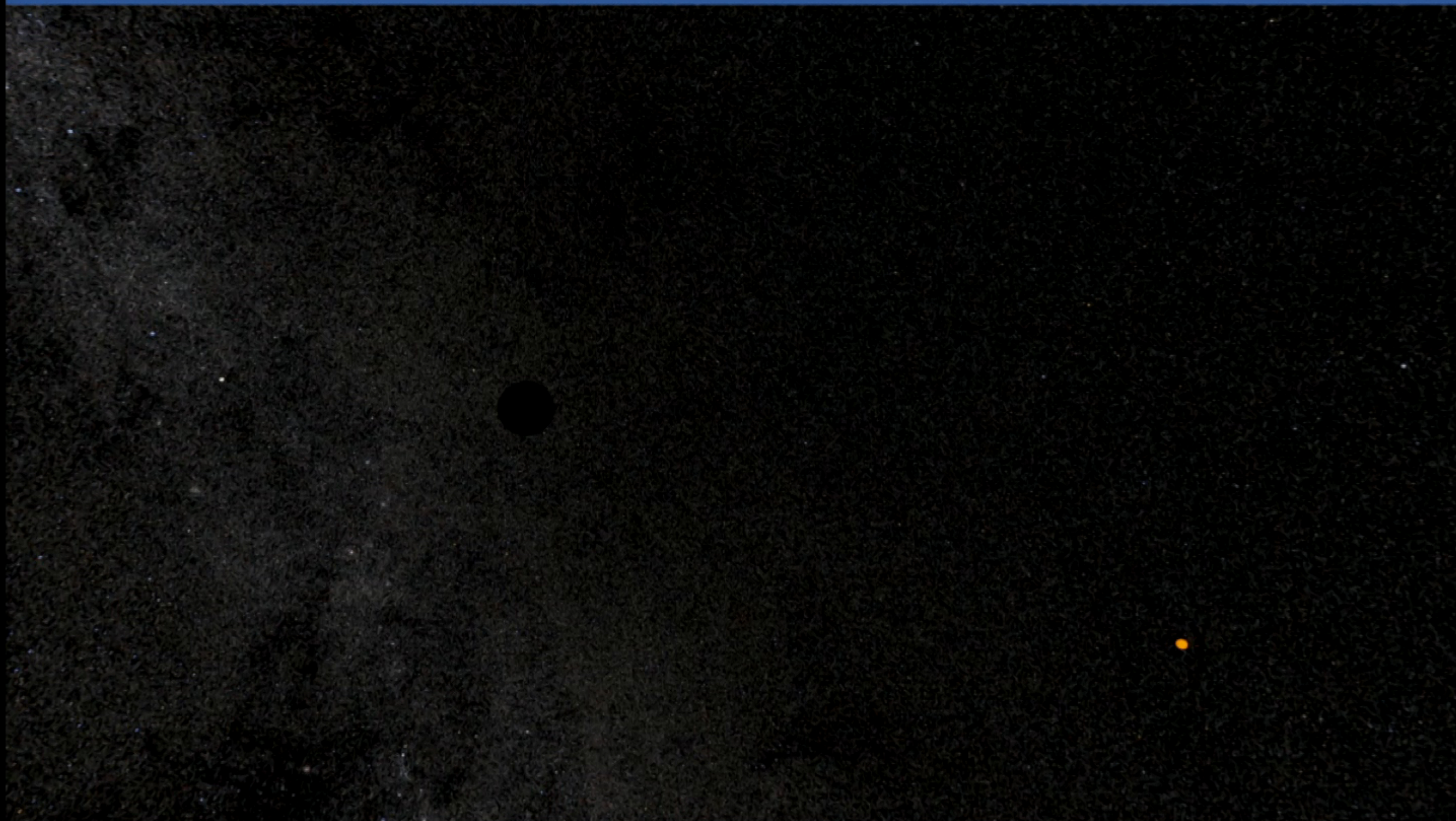

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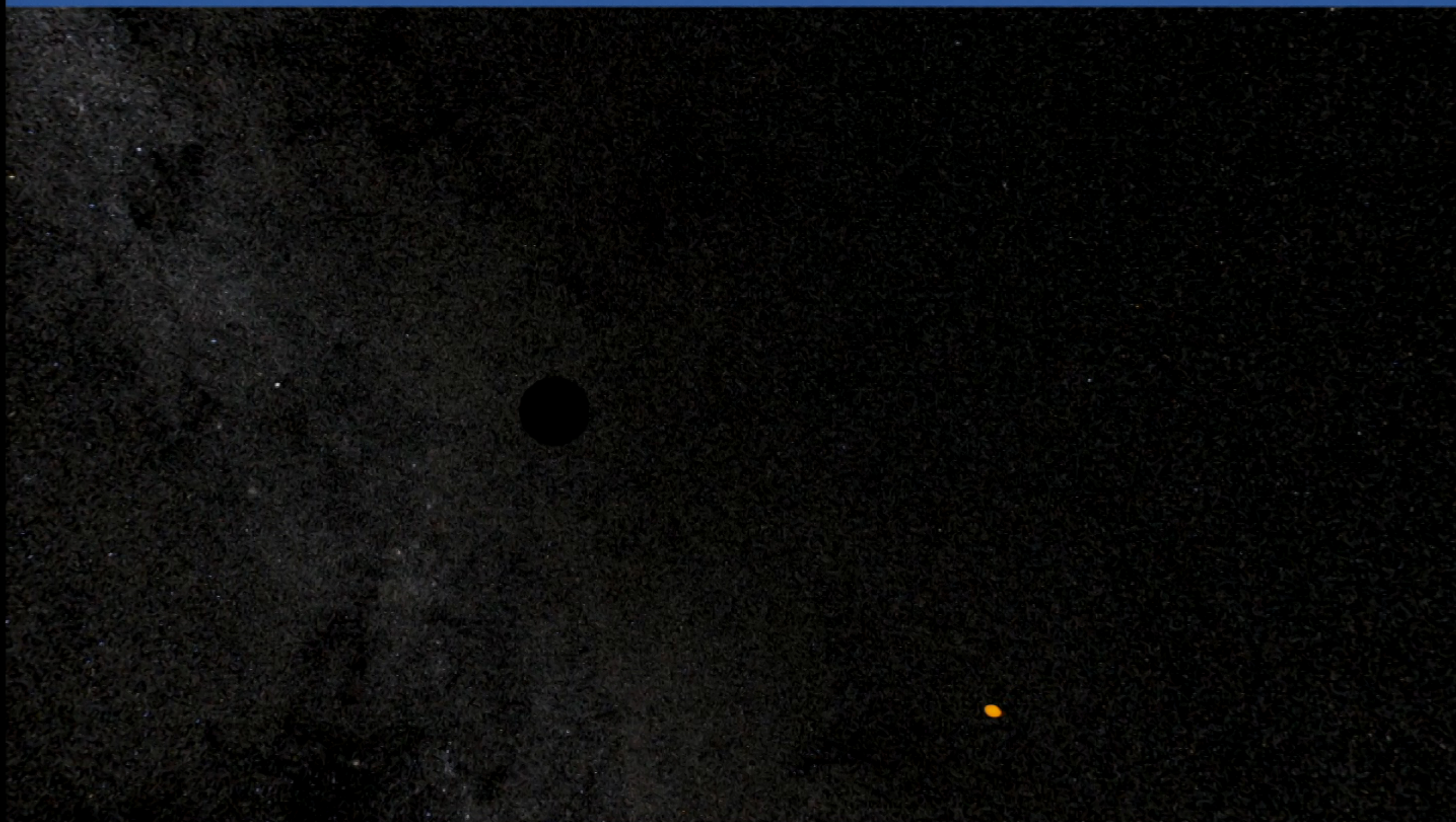

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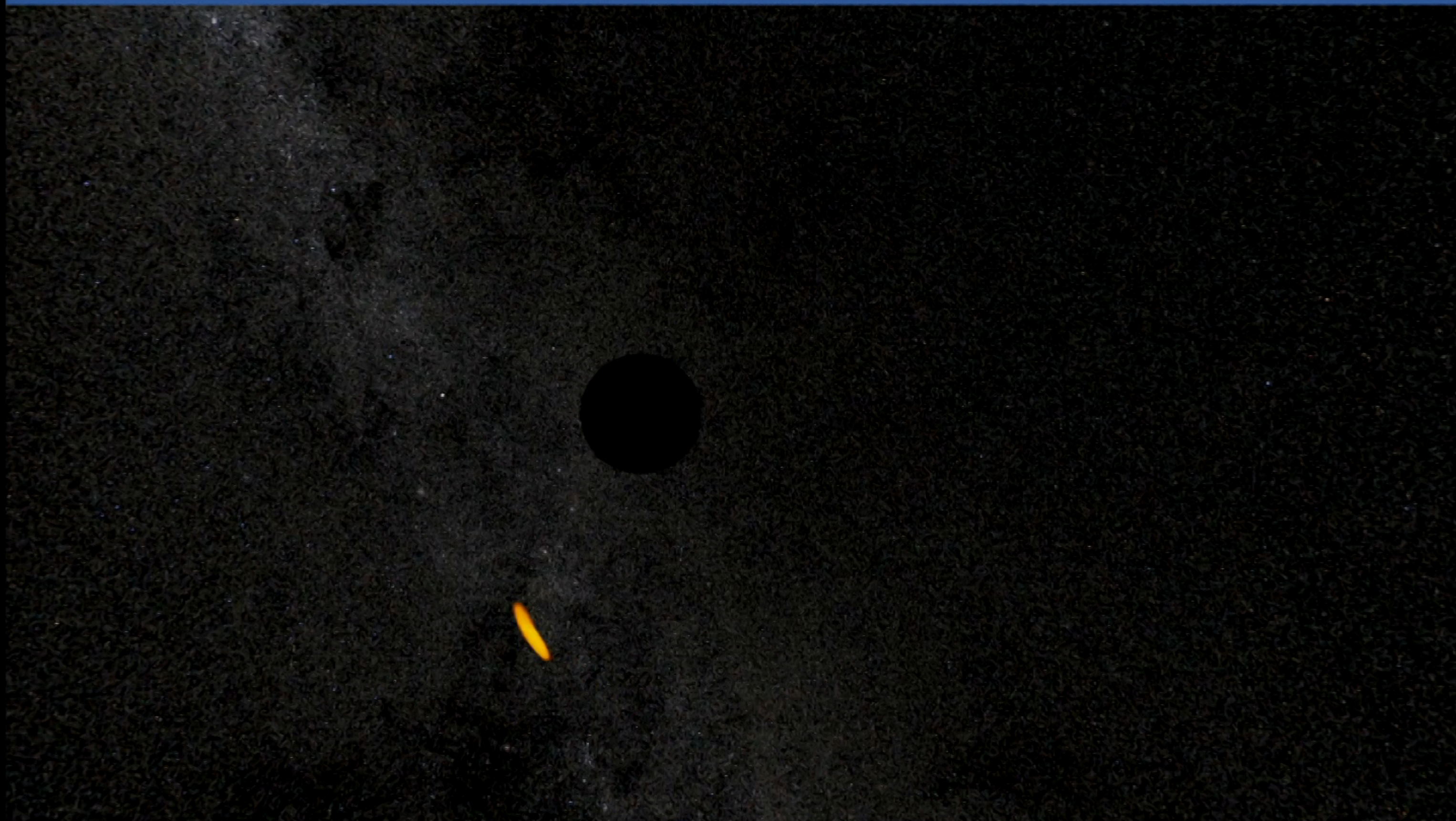


Animation by Ralf Kahler, KIPAC

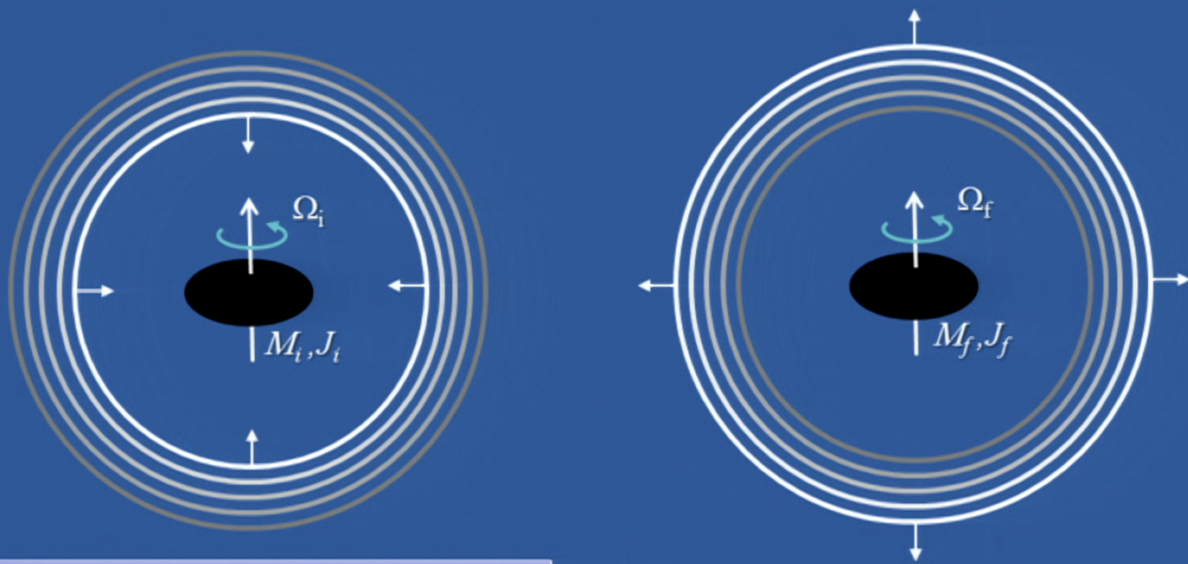
Kerr Black Holes

- One of the key outstanding problems pertaining to 4D asymptotically flat black holes in vacuum Einstein gravity can be summarized via the *final state conjecture (Penrose)*
  - *the final state of generic, non-singular vacuum initial data are a set unbound black holes, each settling down to a unique member of the Kerr family, plus radiation streaming to infinity.*
- "no hair" theorems consistent with this, but they do not incorporate dynamics
- Perturbative/weak-field results support this for large classes of initial data (quasi-normal modes, power-law tails, etc.)
- Numerical results in the non-linear regime are also consistent with this
  - though initial data explored is much less generic (mostly black hole mergers, and some work on gravitational collapse and interaction of gravitational waves with black holes)

Near-extremal Kerr Black Holes

- Other than the binary answer to whether the conjecture is true or not, much is unknown about details of the non-linear dynamics of spacetime approaching Kerr
- Near extremal Kerr black holes may be particularly interesting in this regard
  - extremal black holes have a class of zero-damped modes *[Detweiler],* and this can have consequences for the decay of near-extremal perturbations
    - early-time power-law instead of exponential decay *[Hod, Yang et al]*
    - modes may exhibit a parametric instability that could result in a turbulent-like cascade of energy *[Yang, Zimmerman and Lehner]*
    - likely related to the instability of scalar waves on extremal Kerr *[Aretakis]*
- *Superradiant scattering* of waves another interesting property of Kerr, and may be linked to all the above as the frequency of zero-damped modes becomes commensurate with the onset of superradiance in the extremal limit

Scattering a Wave off a Black Hole



$$Y_{-2}^{lm}(\theta,\varphi) f(r) \int A_{in}(\omega_{lm}) e^{i\omega_{lm}t} d\omega_{lm}$$

- Send in a pure spheroidal harmonic gravitational wave packet from infinity
- To leading order at the linear level get the same harmonic out, but with different amplitude spectrum $A_{out}(\omega_{lm})/A_{in}(\omega_{lm})=Z(\omega_{lm}, M,J)$
  - subleading/non-linear effects include mode coupling, and back scattering of the wave off the spacetime curvature leading to powerlaw tails

Superradiance

- Long known that for spinning black holes with rotational frequency Ω, if $\omega < m\Omega$ then $A_{out} > A_{in}$ — *superradiance*
  - "required" by the laws of black hole mechanics [subject to energy conditions/cosmic censor]
  - energy/angular momentum gain in the outgoing wave balanced by corresponding loss of the black hole's rotational energy/spin
- Until recently not explored beyond the quasi-stationary, linear level; open questions:
  - is cosmic censorship upheld?
  - how does the efficiency of black hole energy extraction compare to the quasi-stationary limit?
  - any unusual non-linear phenomena?

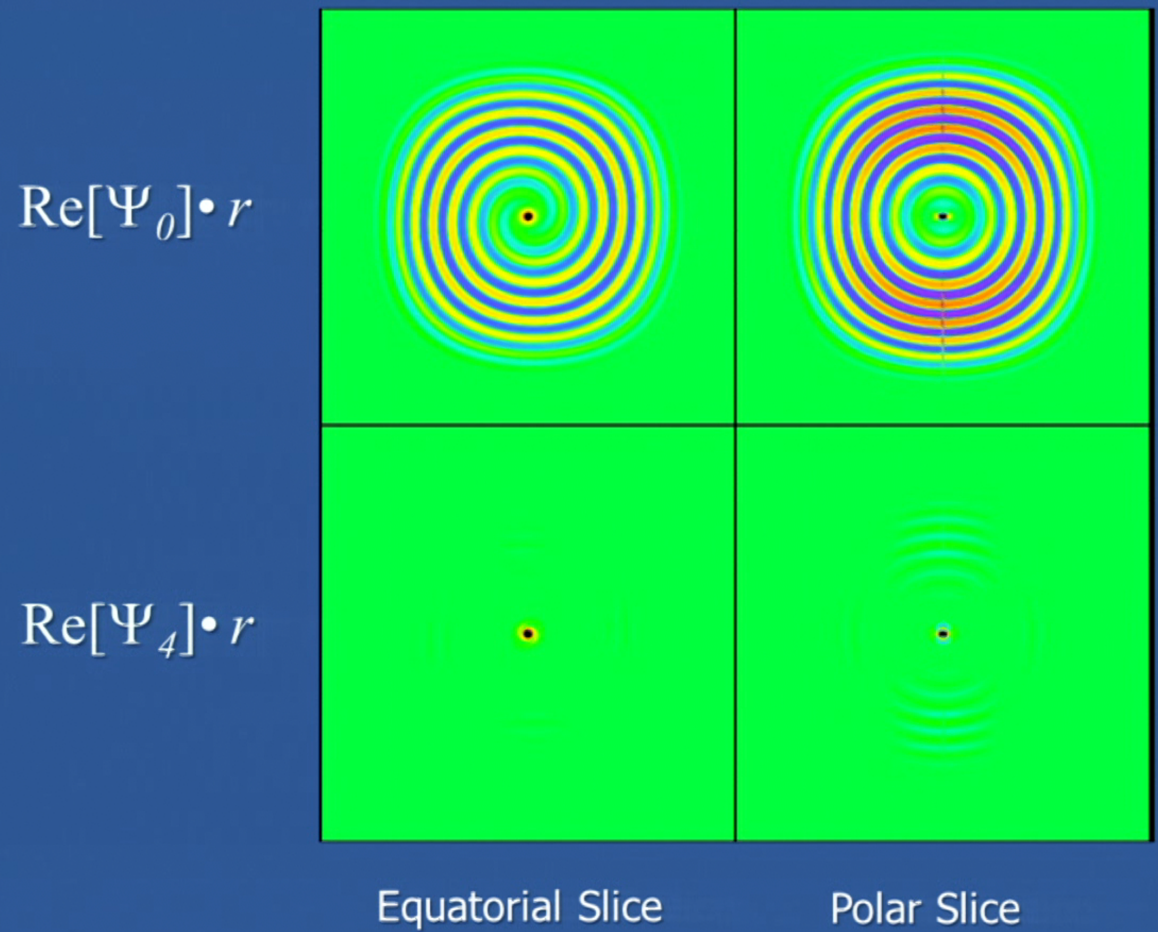
Setup

- Explore large amplitude superradiant scattering of gravitational wave packets off a Kerr black hole with dimensionless spin of $a=0.99$
- Superimpose an ingoing wave on a Kerr-Schild metric for the background solution to a Conformal Thin Sandwich constraint solve; evolve with 4th order, finite difference generalized harmonic code.
- For the ingoing wave:
  - $l=|m|=2$ spin-weight 2 spheroidal harmonic
  - radial profile is a sinusoid modulated by a Gaussian enveloped with characteristic width $r=10M$, centered at $r=40M$
  - three central frequencies:

    $\omega_0 M=0.75$, the linear estimate for maximum superradiance
    $\omega_0 M=0.87$, near the least-damped QNM frequency
    $\omega_0 M=1$, significantly above the superradiant and QNM frequencies

  - range of amplitudes; the maximum contributes roughly 10% to the total mass of the spacetime $(E\sim A^2)$

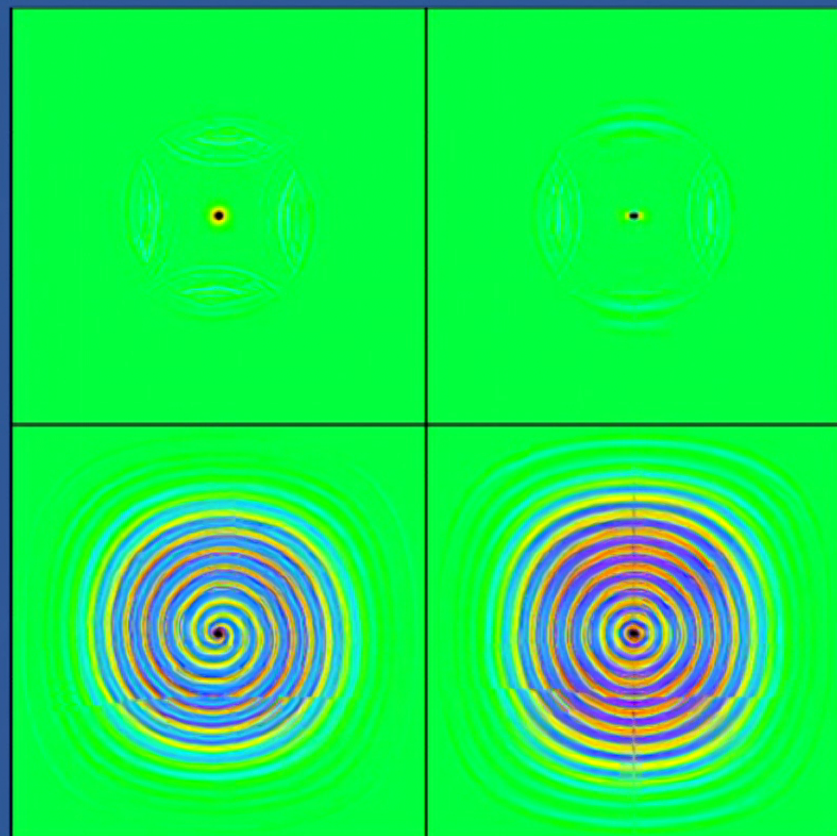
Sample Evolution



Sample Evolution

$\mathrm{Re}[\Psi_0]\bullet r$

$\mathrm{Re}[\Psi_4]\bullet r$

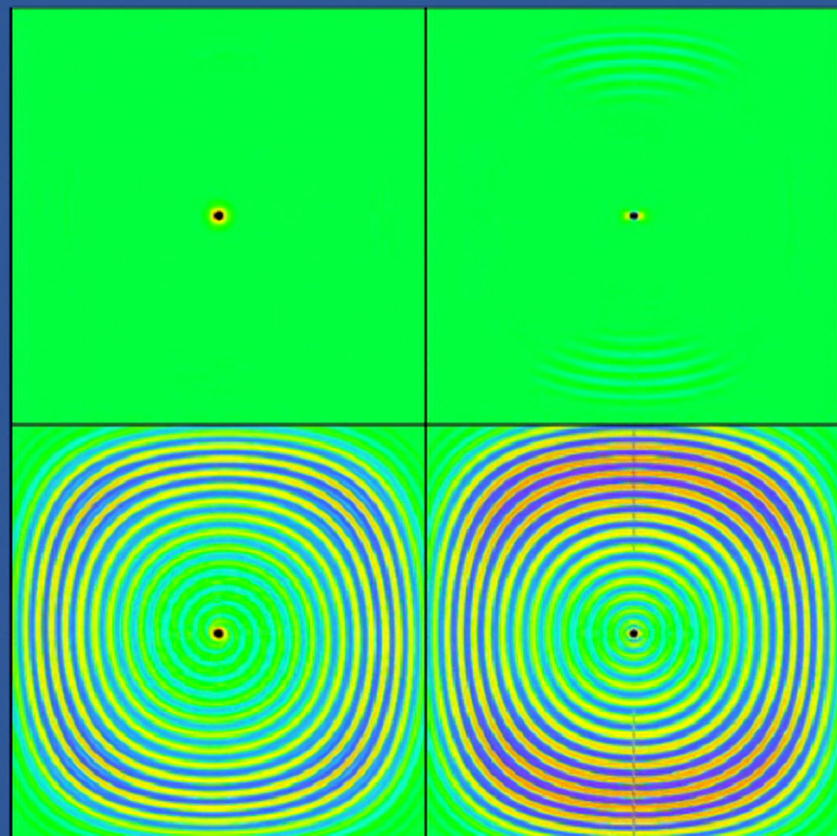


Equatorial Slice   Polar Slice

Sample Evolution

$\mathrm{Re}[\Psi_0] \bullet r$

$\mathrm{Re}[\Psi_4] \bullet r$



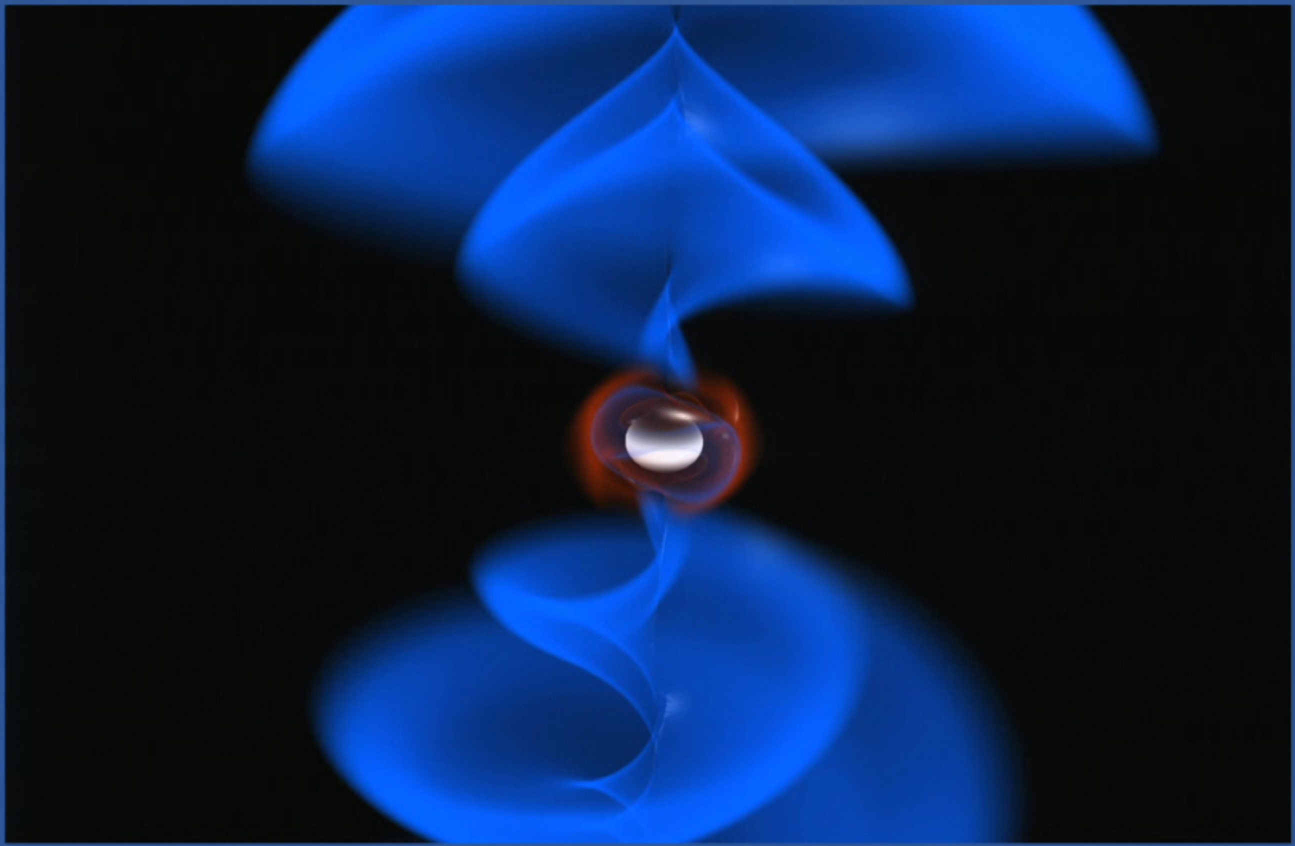
Equatorial Slice   Polar Slice

Sample Evolution

Blue :
$Re[\Psi_0] \bullet r$

Red :
$Re[\Psi_4] \bullet r$

Apparent horizon grey-scale shaded by scalar curvature



Animation by Ralf Kahler, KIPAC

Sample Evolution

Blue :
$\mathrm{Re}[\Psi_0] \cdot r$

Red :
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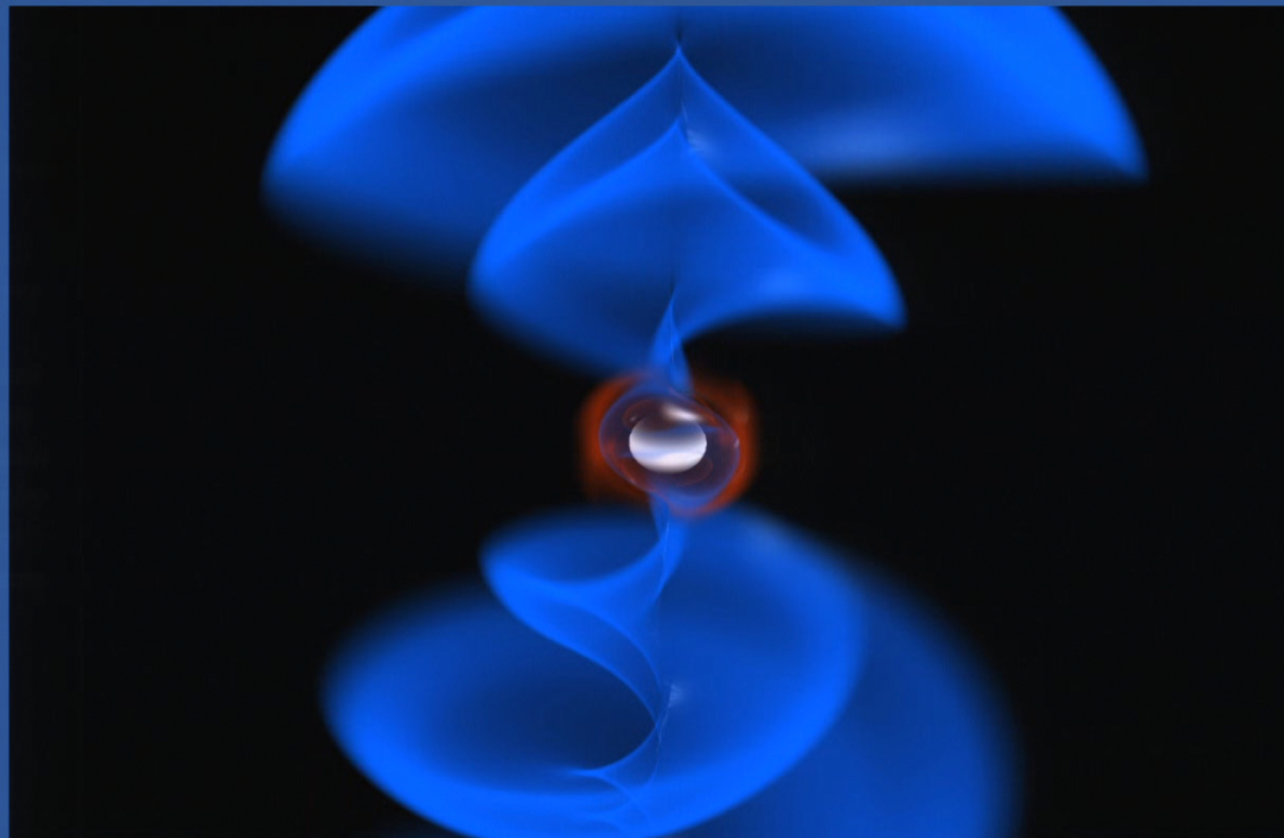

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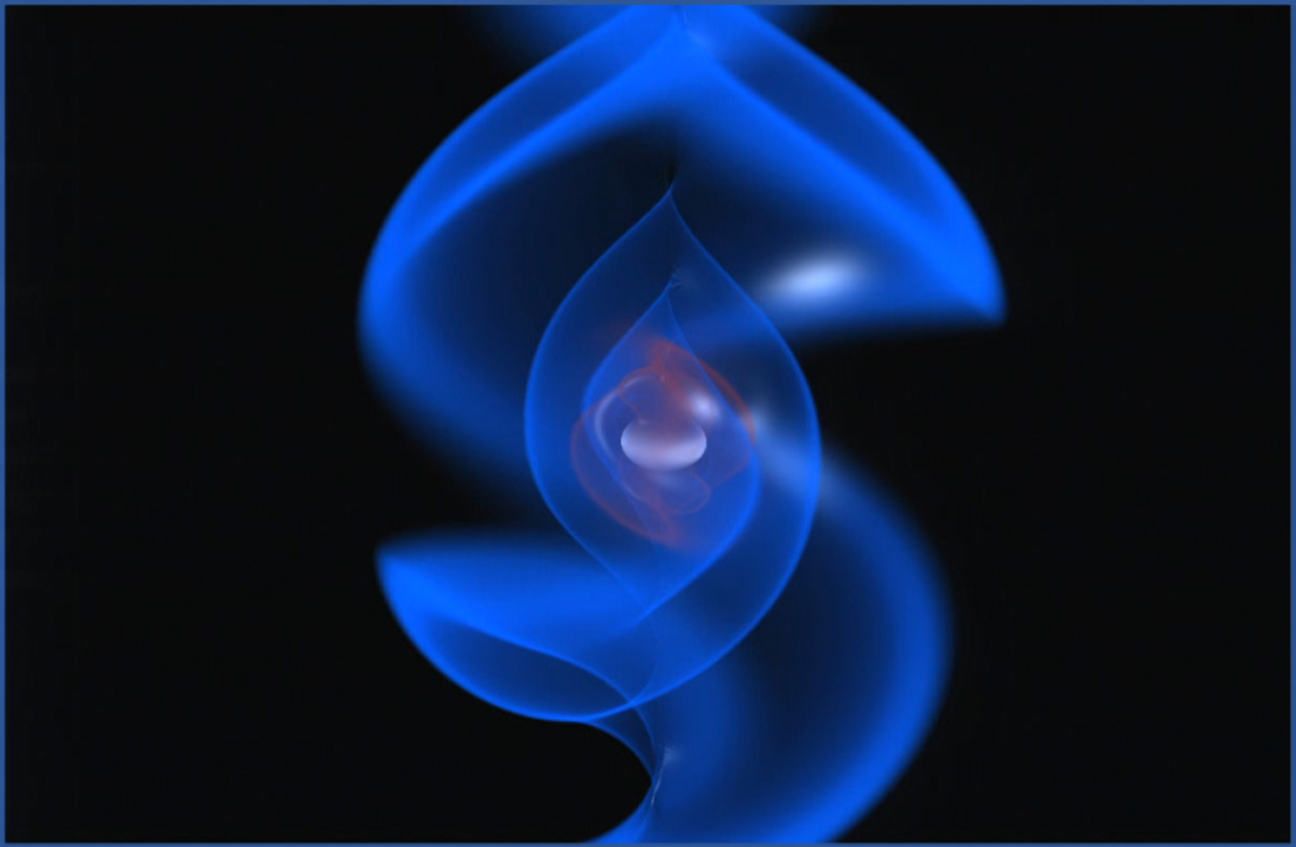

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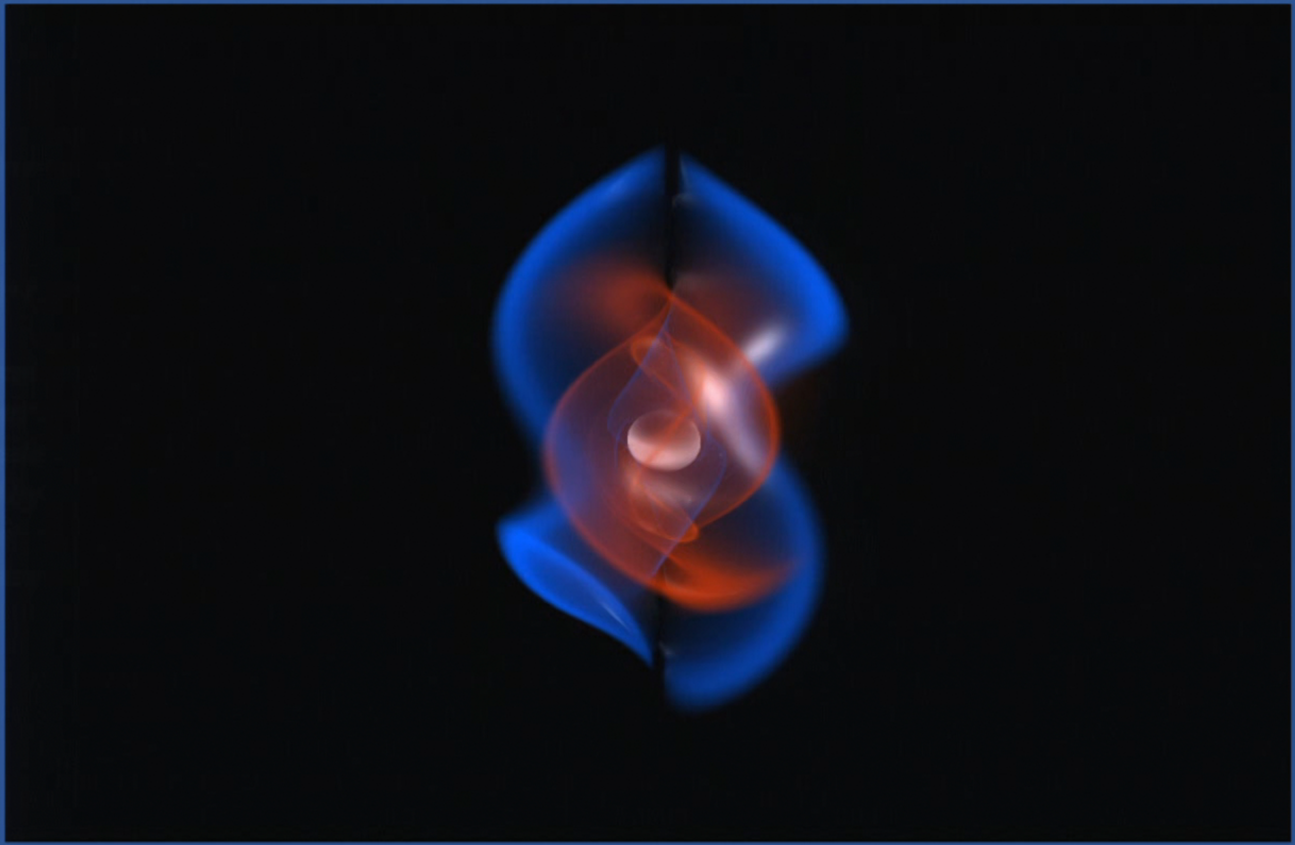

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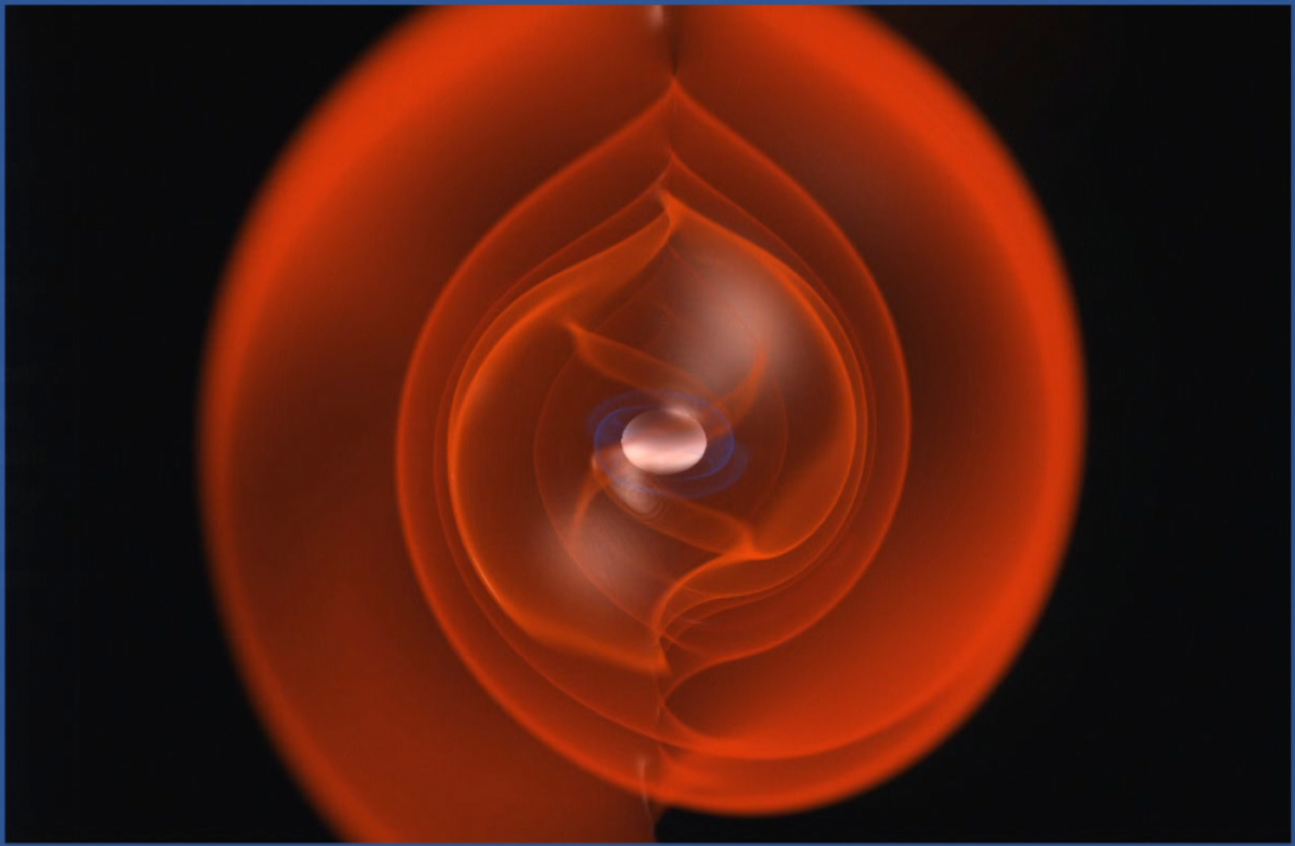


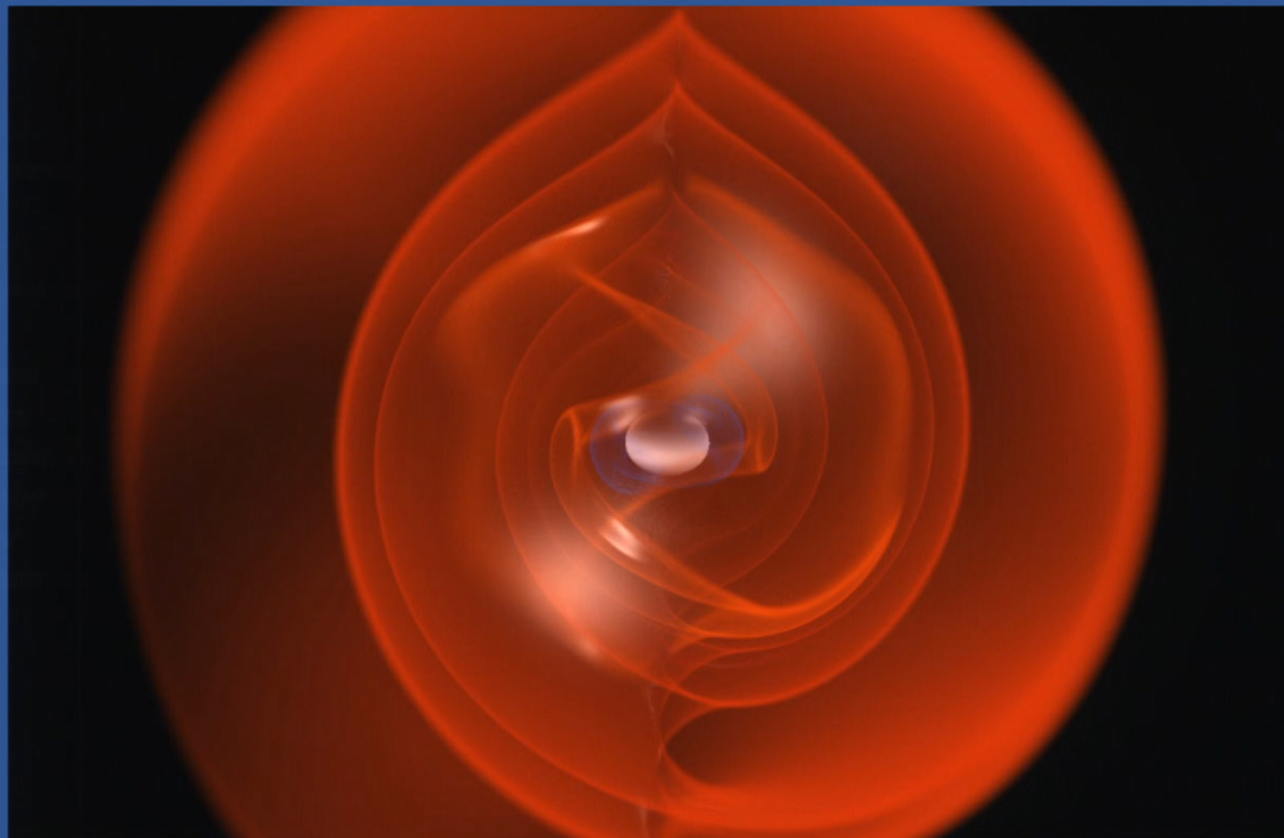
Animation by Ralf Kahler, KIPAC

Sample Evolution

Blue :
$\mathrm{Re}[\Psi_0]\bullet r$

Red :
$\mathrm{Re}[\Psi_4]\bullet r$

Apparent horizon grey-scale shaded by scalar curvature



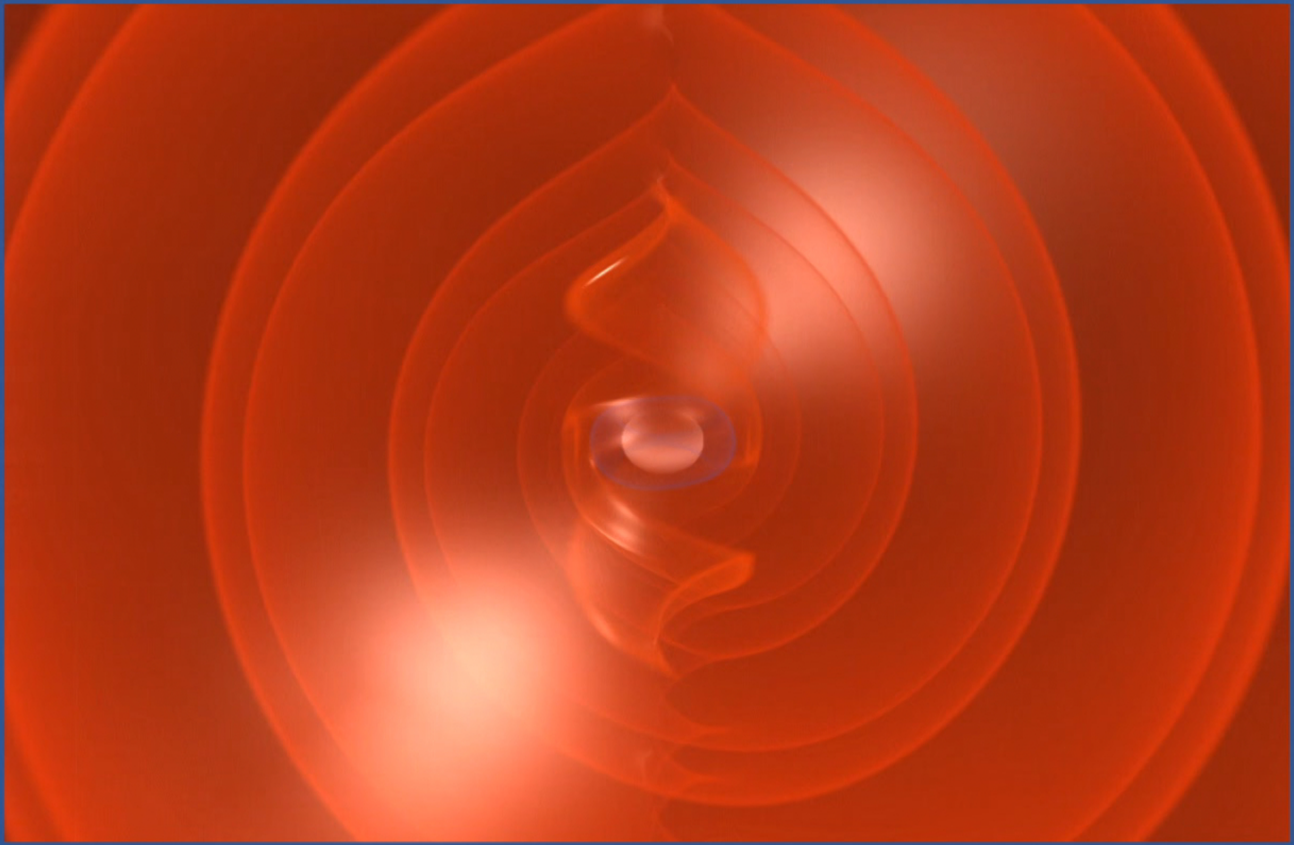
Animation by Ralf Kahler, KIPAC

Sample Evolution

Blue :
Re[Ψ_0]• r

Red :
Re[Ψ_4]• r

Apparent horizon grey-scale shaded by scalar curvature



Animation by Ralf Kahler, KIPAC

Sample Evolution

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Red :
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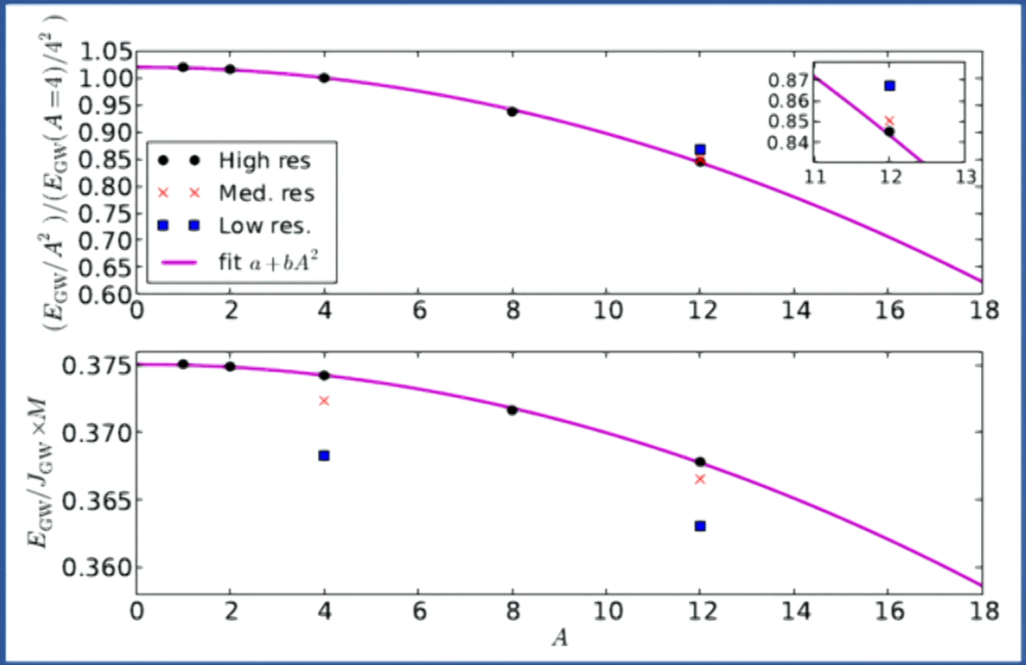
Apparent horizon grey-scale shaded by scalar curvature

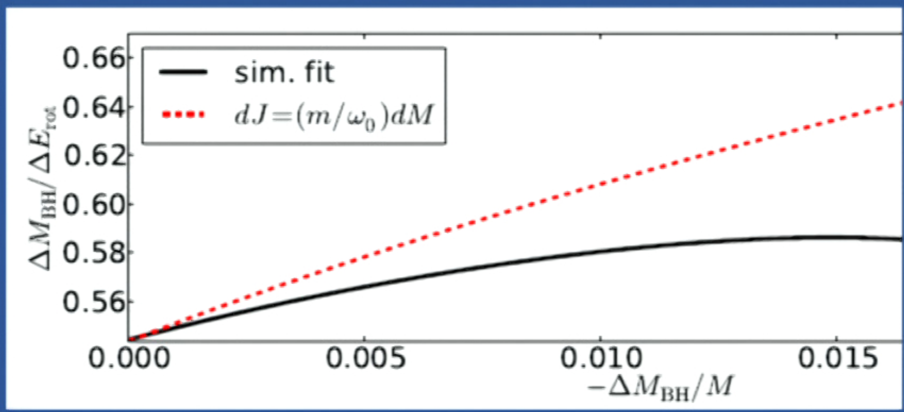
Animation by Ralf Kahler, KIPAC

$\omega_0 M=0.75$ case – Superradiant amplification



- $A=0$ limit consistent with linear theory, giving ~ 40% enhancement in energy of the outgoing wave
- Can understand part of the drop with A as the interaction immediately beings to decreases Ω, hence lowers the superradiant efficiency for the "late" part of the incoming wave

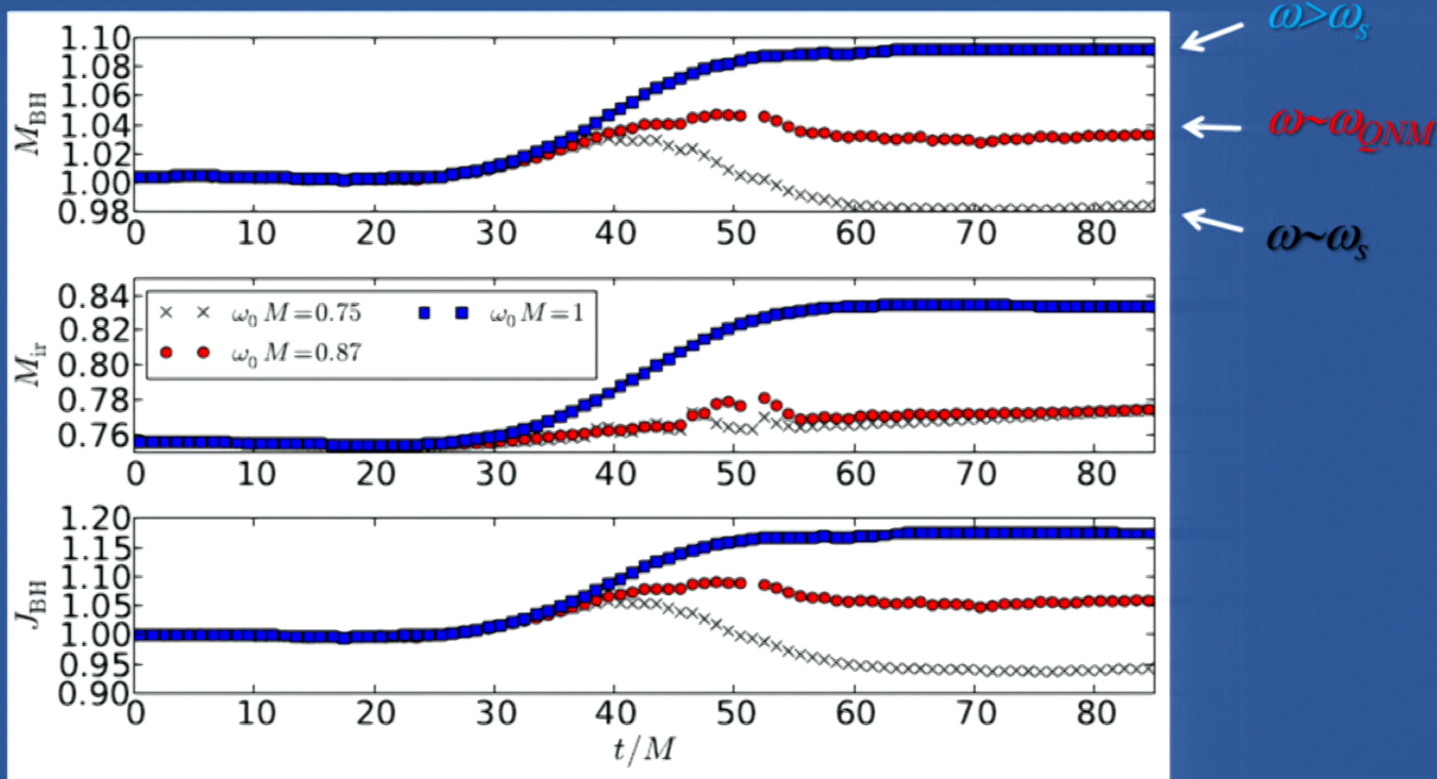
$\omega_0 M=0.75$ case – Rotational energy extraction efficiency



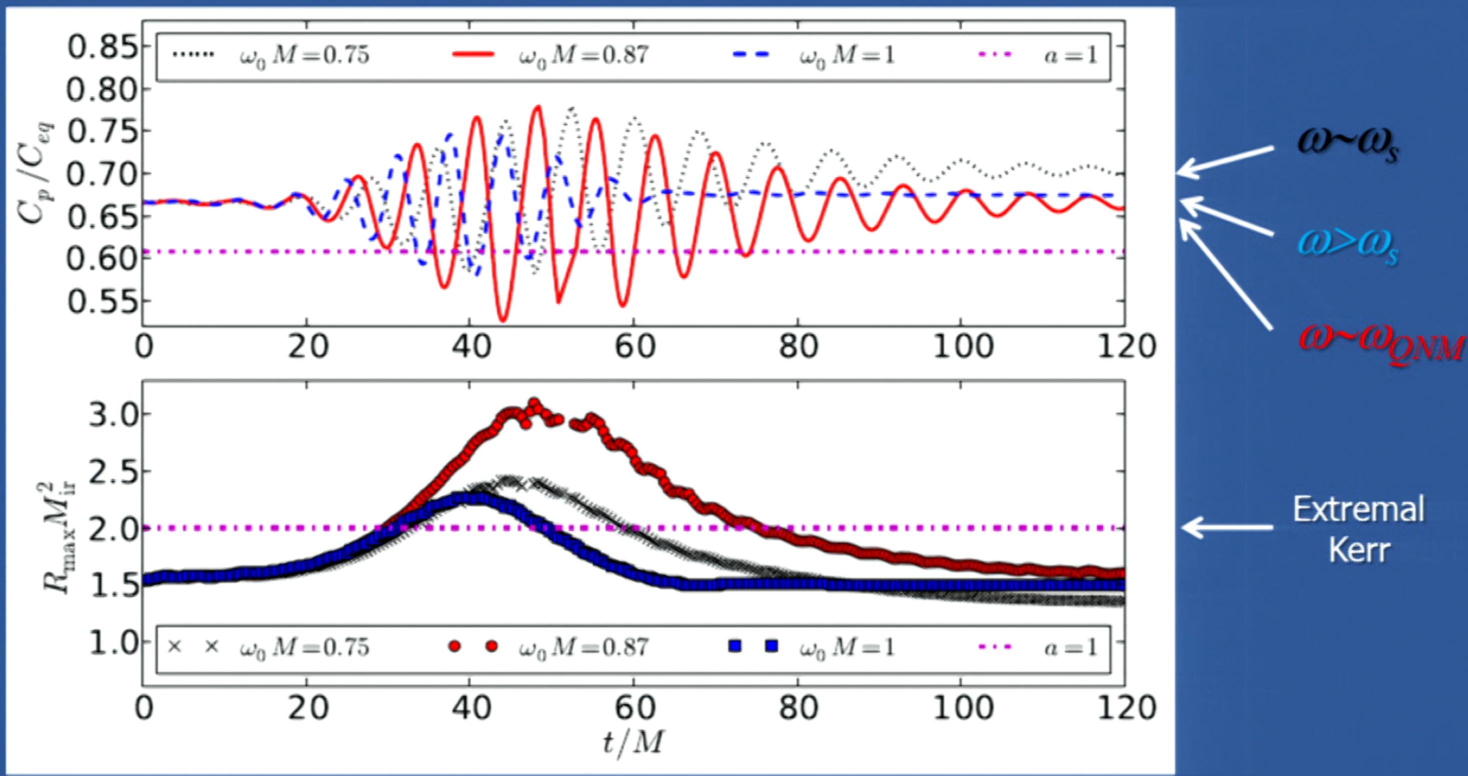
Quasi-stationary interaction from linear theory

Rapid O(10M) interaction from simulations

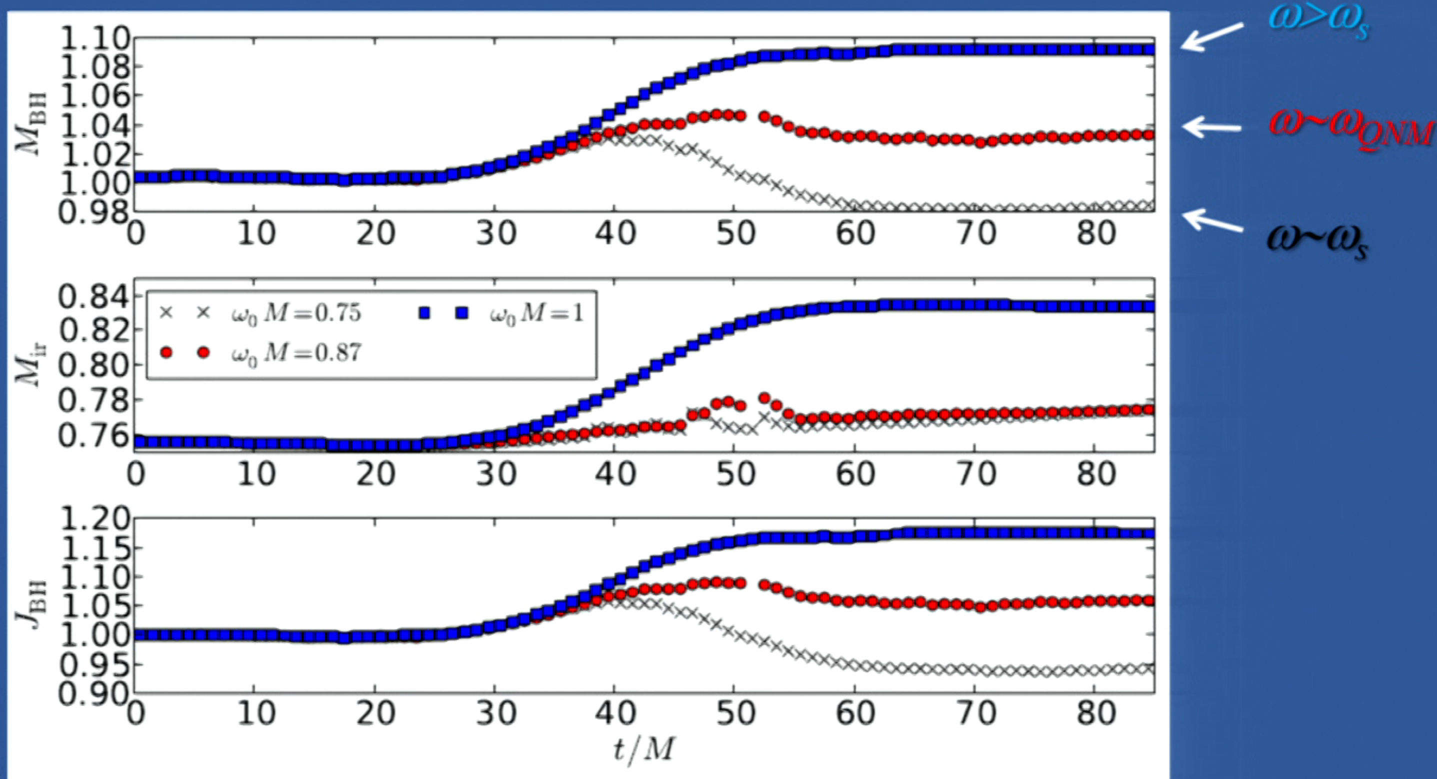
Dynamical Horizon Properties



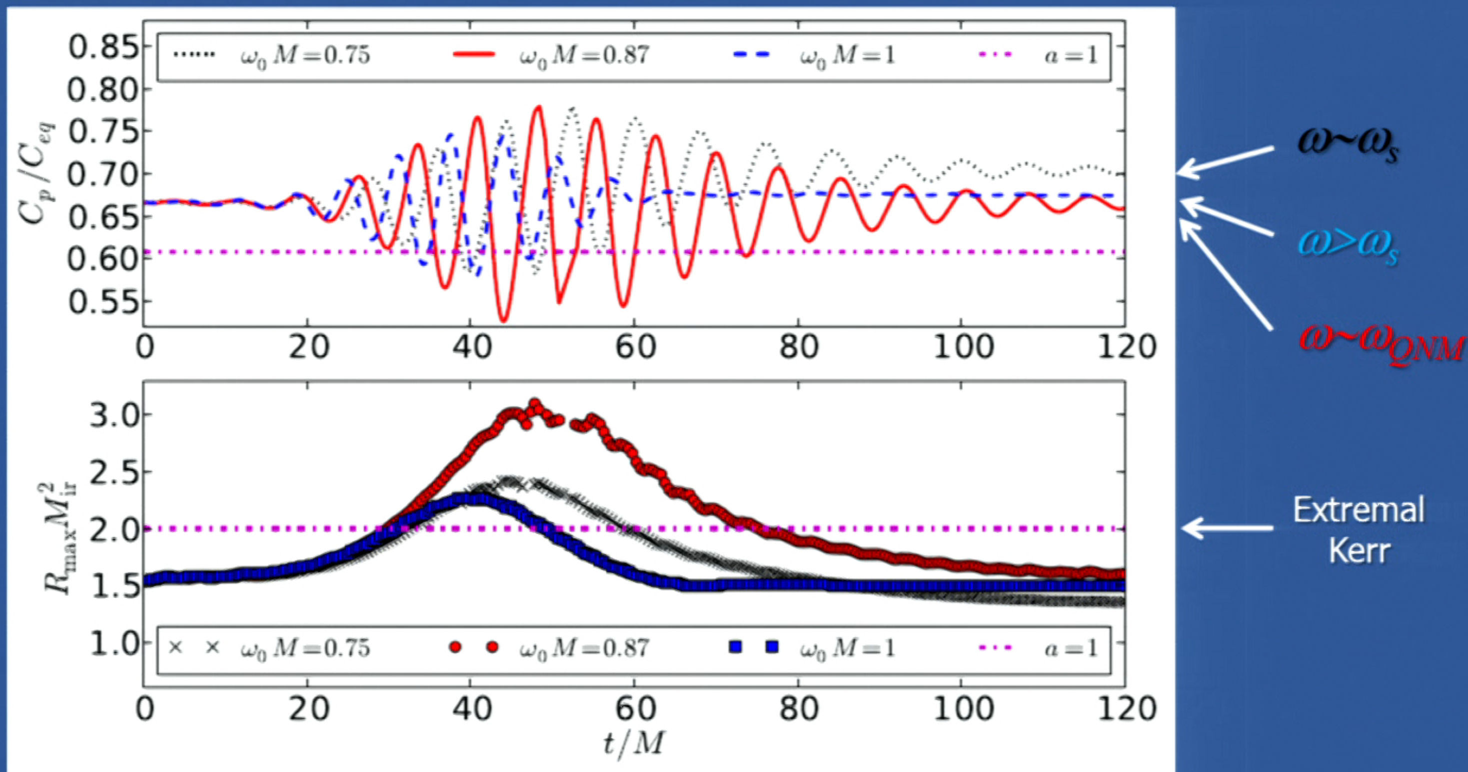
Dynamical Horizon Properties



Dynamical Horizon Properties



Dynamical Horizon Properties



Massive-particle Superradiance

- Same basic physics compared to gravitational wave superradiance:
  - if an incident field mode with angular velocity ω/m interacts with a black hole with angular velocity Ω, if $\omega/m<\Omega$ any energy/angular momentum absorbed by the black hole must be negative (as measured at ∞) to satisfy the second law of black hole mechanics $\rightarrow$ the reflected field gains energy relative to the incident
- The difference here is (for appropriate parameters) the field's self interaction acts as a confining potential
  - modes satisfying the superradiant condition are then trapped by the potential, and can undergo repeated "scattering" interactions with the black hole
  - at the linear level, this leads to exponential growth of the field

Questions for a non-linear study

- How efficient is the energy extraction? How does the instability saturate?
  - typical growth rates are slow relative to local dynamical timescales (black hole light-crossing time, field oscillation), so might expect slow transfer of energy and that evolution is always linear relative to some "hairy" black hole state *[Arvanitaki et al. PRD 81 (2009); Brito, Cardoso & Pani, CQG 32 (2015), ...]*
  - gravitational wave superradiant results then also imply this process is highly efficient
  - on the other hand, studies of analogous charged superradiance suggest non-linear phase of growth might be more explosive and "overshoot" *[Bosch, Green & Lehner, PRL 116 (2016)]*
  - our results consistent with the first two bullet points above, albeit with the restriction we have not yet addressed issues related to multiple modes, higher order harmonics, etc.

Proca-field Simulations

- As a first step toward addressing some of these questions, we look at a pair of massive vector fields, with single $m=1$ azimuthal mode, and "tune" the phases of the fields to achieve an axisymmetric stress-energy tensor, hence spacetime
  - massive vector fields have faster growth rates than scalar fields, and the m=1 mode, when unstable, is the fastest growing mode
  - simplest to achieve the tuning using a single complex field; but regardless, this is an *initial condition we impose*, and has nothing to do with a complex field
  - further tune the parameters of the scenario to minimize growth time by starting with a near-extremal ($a=0.99$) Kerr black hole, and Proca field mass parameter $\underline{u}$ $(=uM) = 0.25$ .. 0.5

The above choices are not physically motivated, rather they are to reduce the computational cost to make simulations of the relevant instability time-scales $O(10^5$ M) feasible

Proca-field Simulations

- For a Proca field X^a, the stress-energy tensor and equations of motion are

$$T_{ab} = \frac{1}{2}(F_{ac}\bar{F}_{bd} + \bar{F}_{ac}F_{bd})g^{cd} - \frac{1}{4}g_{ab}F_{cd}\bar{F}^{cd} + \frac{\mu^2}{2}(X_a\bar{X}_b + \bar{X}_aX_b - g_{ab}X_c\bar{X}^c),$$

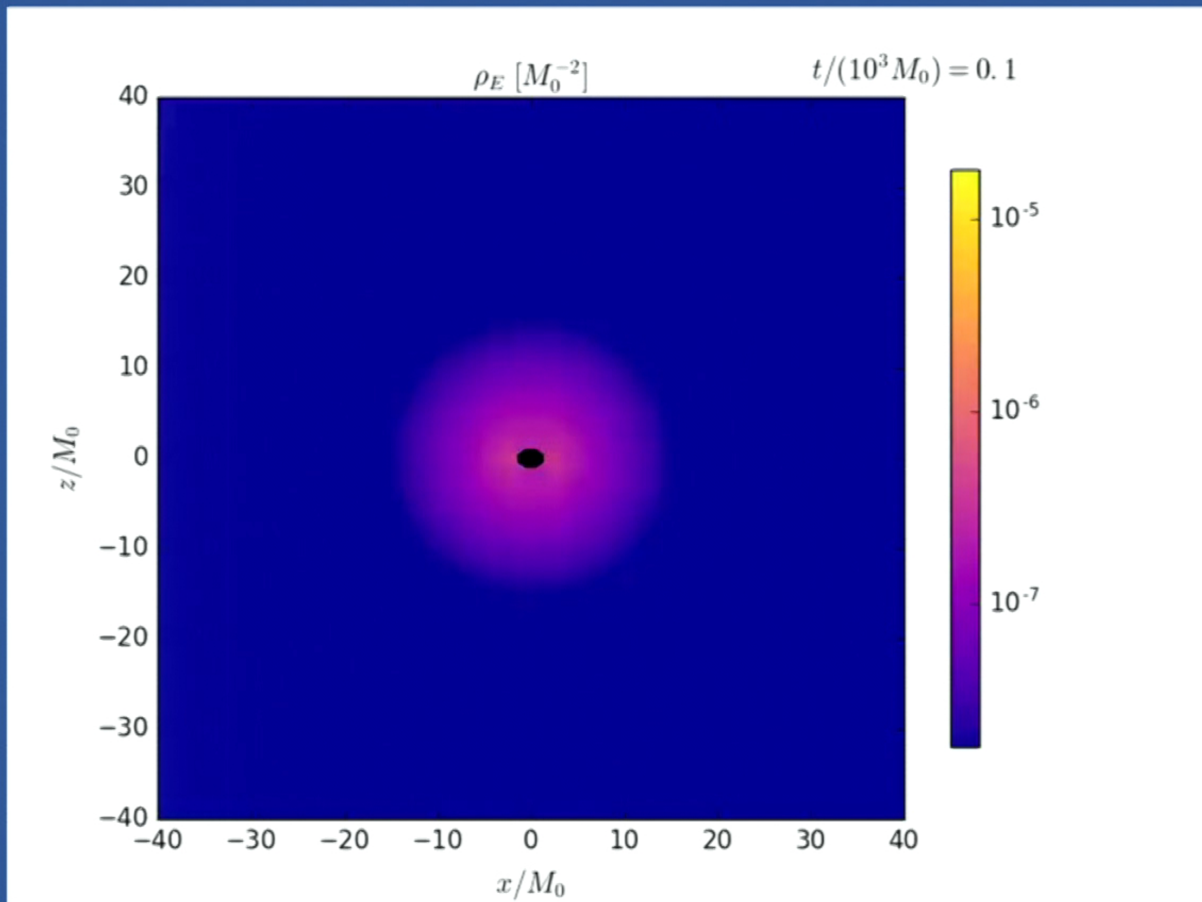
$$F_{ab} = \nabla_a X_b - \nabla_b X_a \qquad \nabla_a F^{ab} = \mu^2 X^b$$

- To allow for an axisymmetric spacetime with azimuthal Killing vector $(\partial/\partial\Phi)^a$, impose the following condition on X^a (here $m=1$):

$$\mathcal{L}_\phi X_a = imX_a$$

- Evolve the Einstein-Proca system with 4th order finite difference methods, generalization harmonic evolution with constraint damping, BEST around Kerr, modified "Cartoon" method to implement symmetry in a 3D Cartesian code, AMR, start with a sub-truncation-error seed field (i.e. not solving constraint equations) …

Sample evolution : $a=0.99$, $\underline{u}=0.3$



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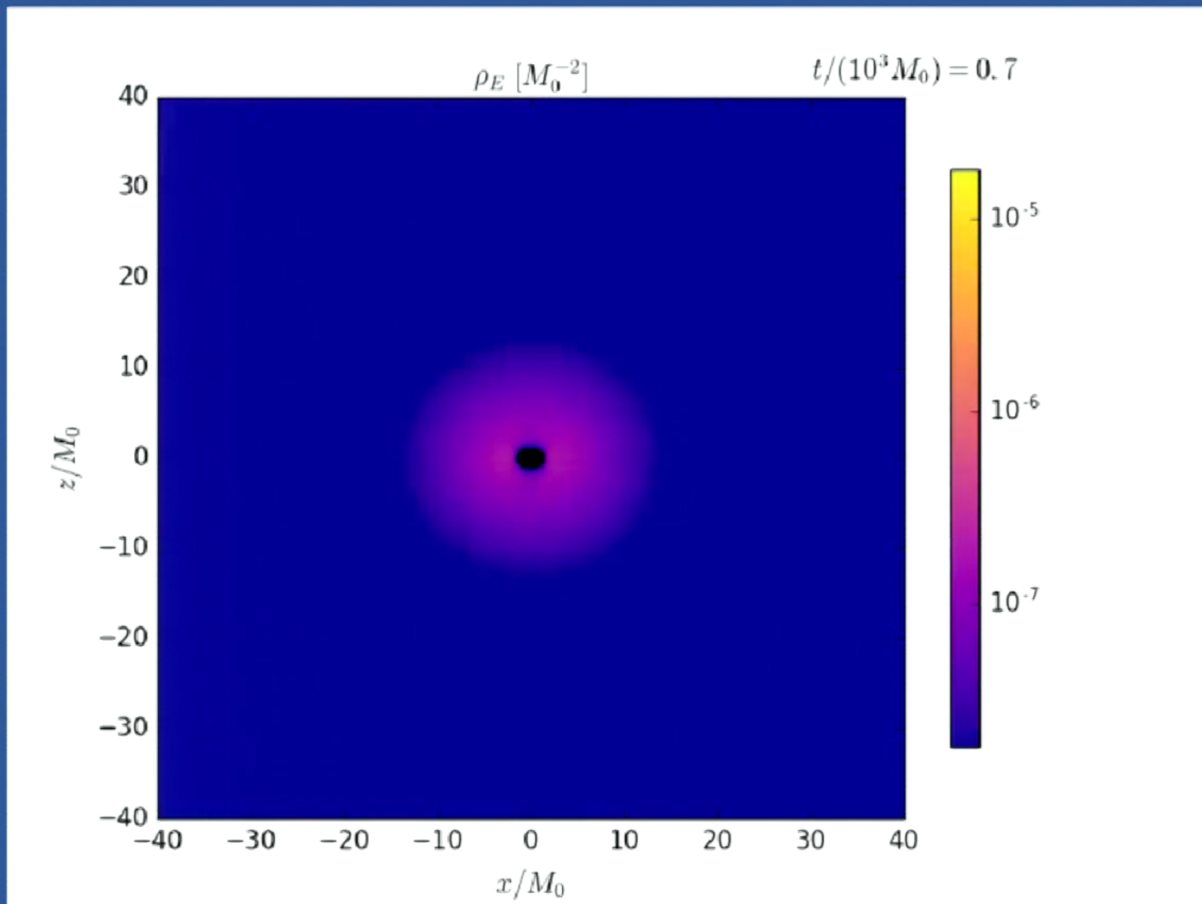
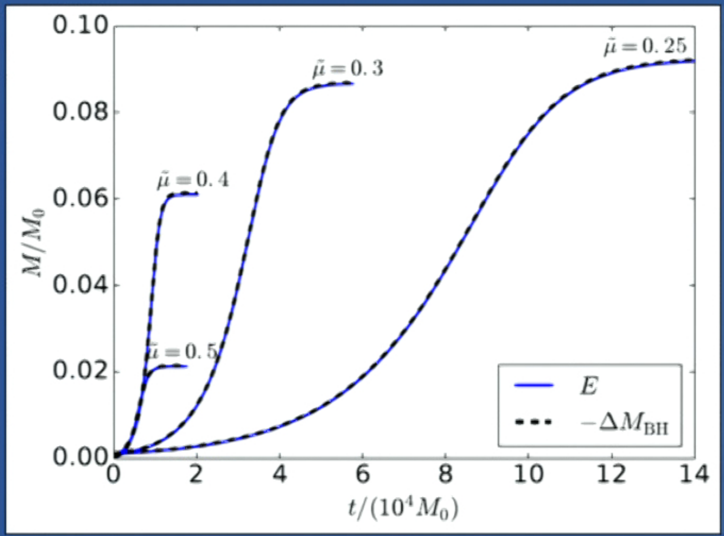
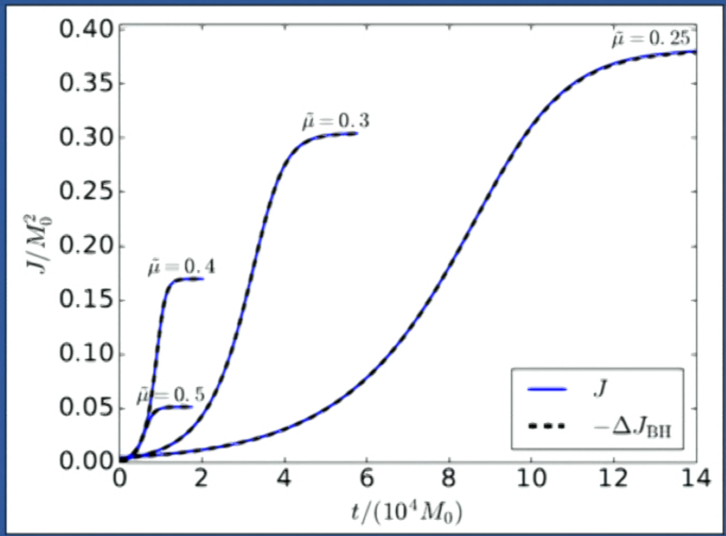
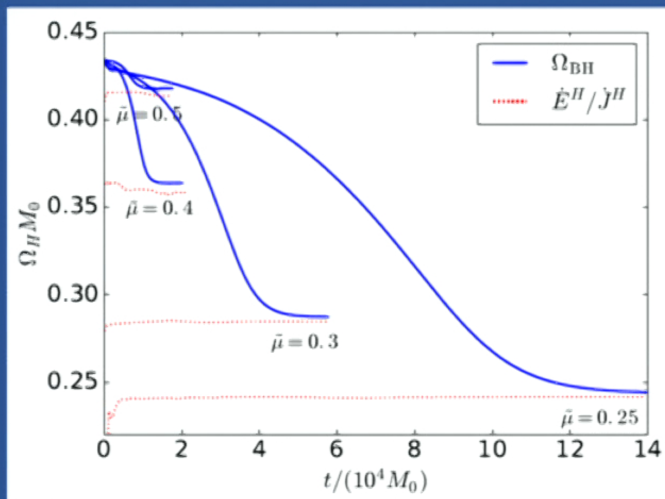


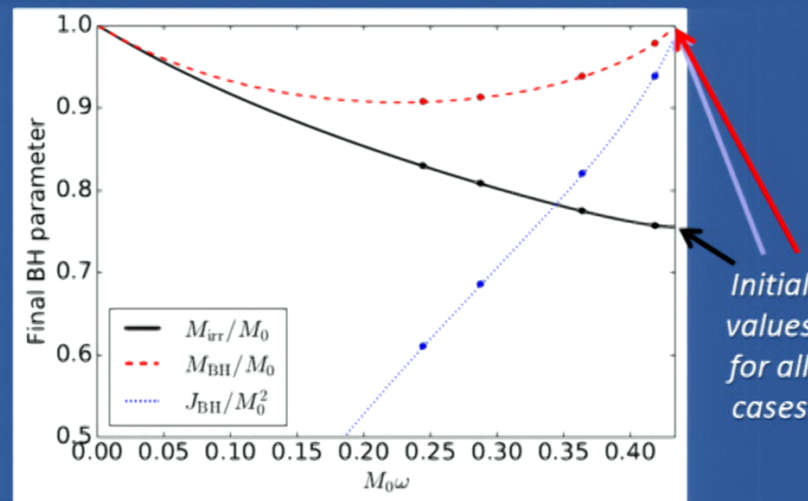
Mass and angular momentum transfer





Saturation of the instability





- horizon angular velocity and horizon fluxes
  - at the linear level, the field fluxes across the horizon satisfy $\dot{E}/\dot{J} = \omega/m$
- curves are estimates of the final black hole parameters assuming a constant rate of $\dot{E}/\dot{J} = \omega/m$ across the horizon until the superradiance condition terminates; dots are results of numerical evolution
- seems likely that the "end state" (before $m>1$ modes become significant) are the "hairy" black hole solutions found by *Herdeiro, Radu & Runarsson [QCG 33, 2016]*

Gravitational Wave Emission

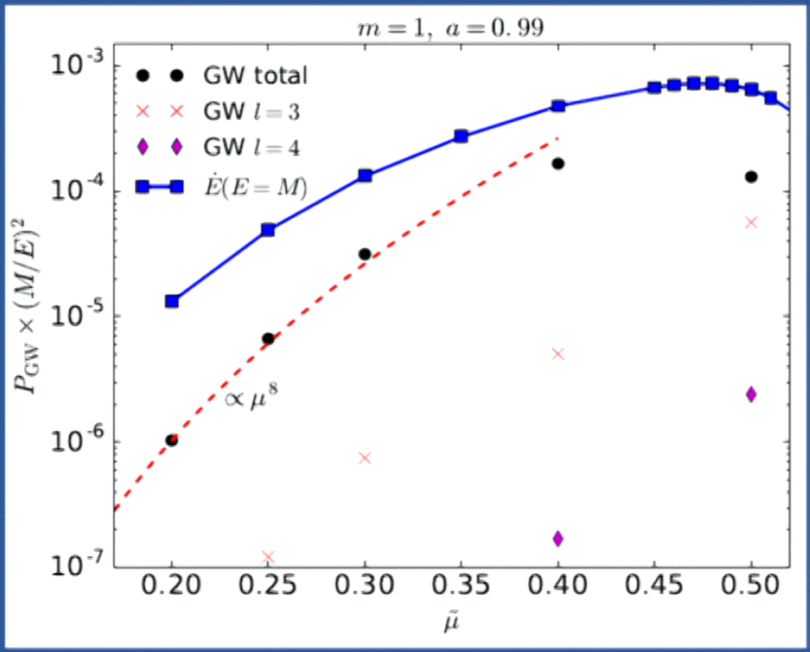
Work by W. East, PRD 96 (2017)

- Due to the axisymmetric ansatz, negligible gravitational wave emission (zero angular momentum-carrying waves)

- Estimate the GW emission from a single (real) Proca field the taking late time profile of one component of the complex field, transferring it to a 3D code, and measuring the emission from that:

- E is the total energy in the cloud; black dots show GW power, blue dots $m=1$ superradiant growth prior to saturation, normalized so that the ratio for a given scenario is $(P_{GW}/\dot{E})(E/M)$

- Example : For the GW150914 event remnant ($a\sim0.7$, $M=62M_o$), the $m=1$ mode for $\underline{u}=0.18$ ($4x10^{-13}$ eV) would grow from a single particle to saturation ($E/M=0.018$) in ~ 7 hours; over the next ~11 hours the cloud would emit 180Hz GWs at ~10^{50} ergs/s



Gravitational Wave Emission

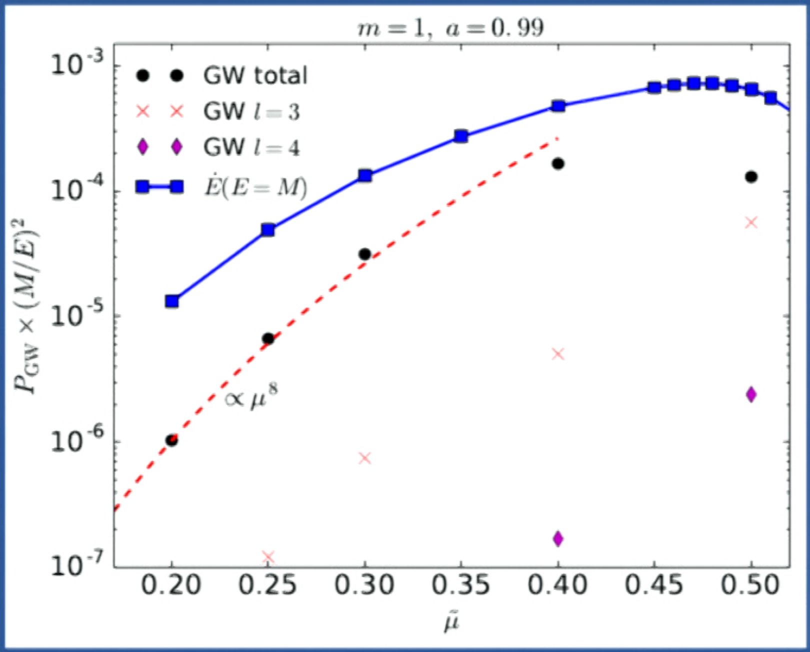
Work by W. East, PRD 96 (2017)

- Due to the axisymmetric ansatz, negligible gravitational wave emission (zero angular momentum-carrying waves)

- Estimate the GW emission from a single (real) Proca field the taking late time profile of one component of the complex field, transferring it to a 3D code, and measuring the emission from that:

- E is the total energy in the cloud; black dots show GW power, blue dots $m=1$ superradiant growth prior to saturation, normalized so that the ratio for a given scenario is $(P_{GW}/\dot{E})(E/M)$

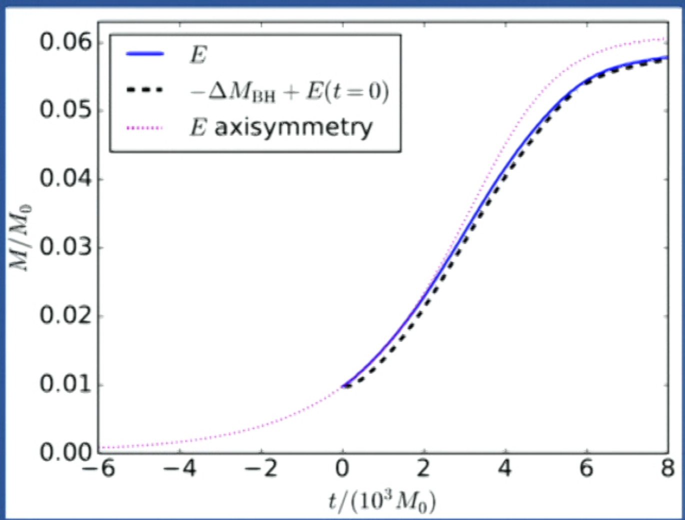
- Example : For the GW150914 event remnant ($a{\sim}0.7$, $M=62M_0$), the $m=1$ mode for $\underline{u}=0.18$ ($4x10^{-13}$ eV) would grow from a single particle to saturation ($E/M=0.018$) in ~ 7 hours; over the next ~11 hours the cloud would emit 180Hz GWs at ~10^{50} ergs/s

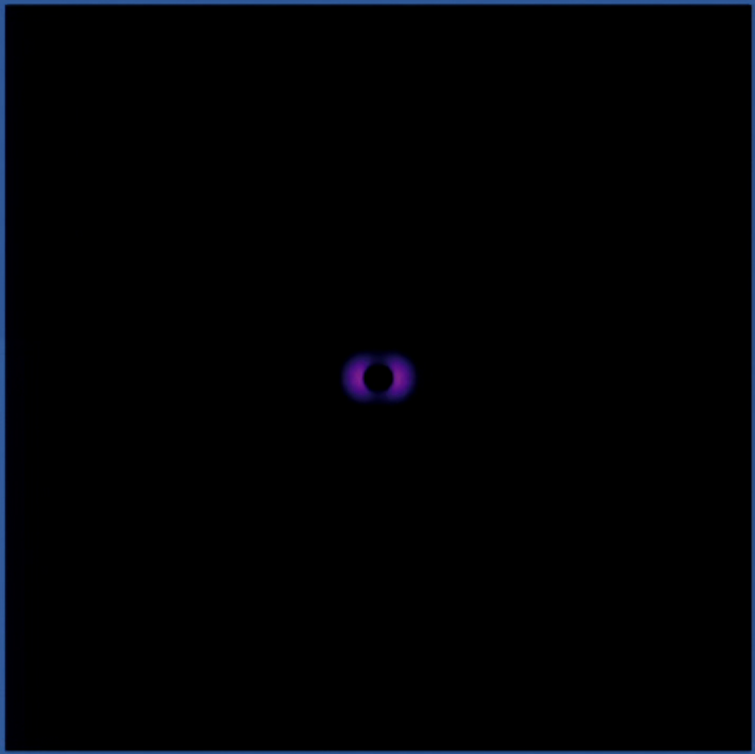


Late time evolution and GW emission in 3D; *a=0.99*, *u=0.4*

W. East, to be published

- As on previous page, an axisymmetric code used to evolve initial linear phase; then transfer real part to a 3D code for the remainder of the evolution, and calculate corresponding GW emission



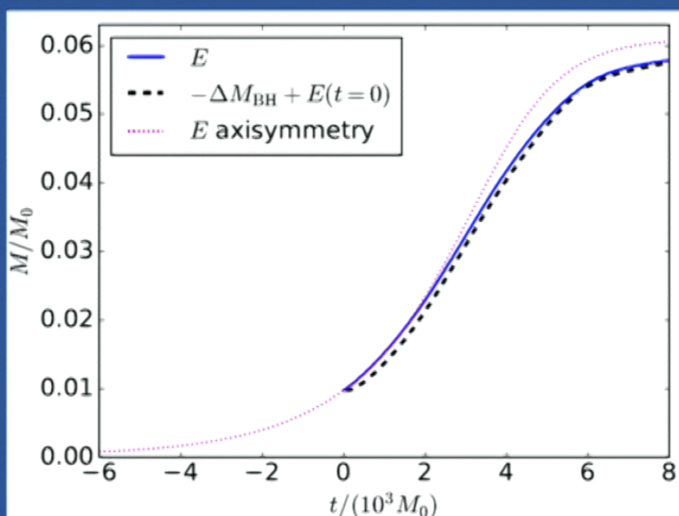


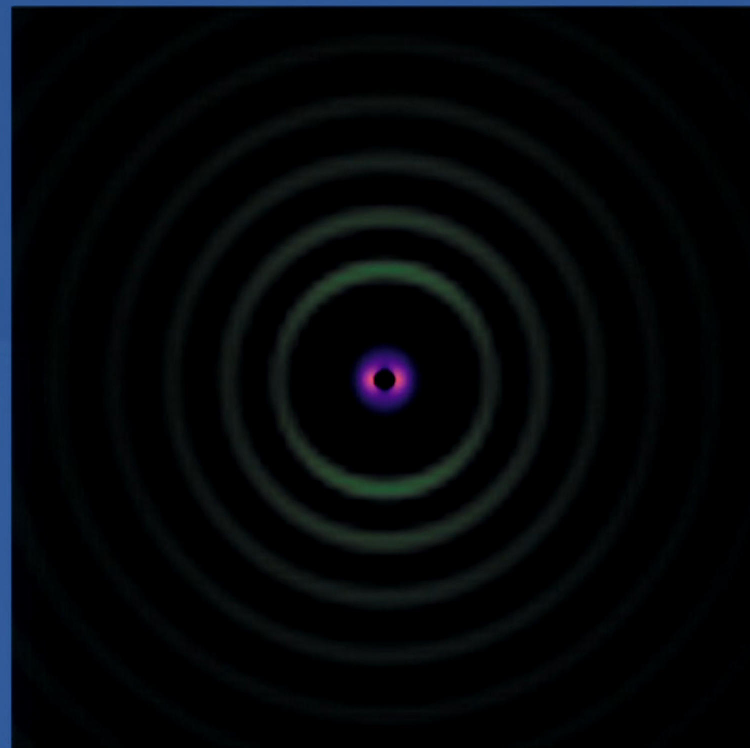
Proca field (inner region) plus gravitational wave emission (outer region)

Late time evolution and GW emission in 3D; *a=0.99*, *u=0.4*

W. East, to be published

- As on previous page, an axisymmetric code used to evolve initial linear phase; then transfer real part to a 3D code for the remainder of the evolution, and calculate corresponding GW emission





Proca field (inner region) plus gravitational wave emission (outer region)

Late time evolution and GW emission in 3D; a=0.99, $\underline{u}$=0.4

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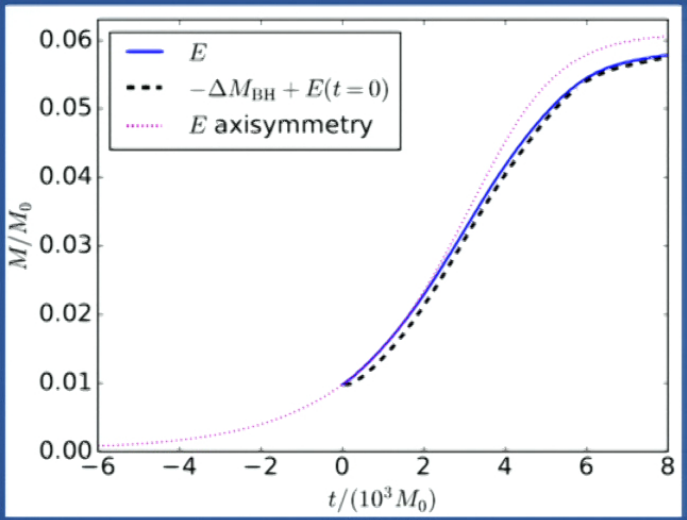

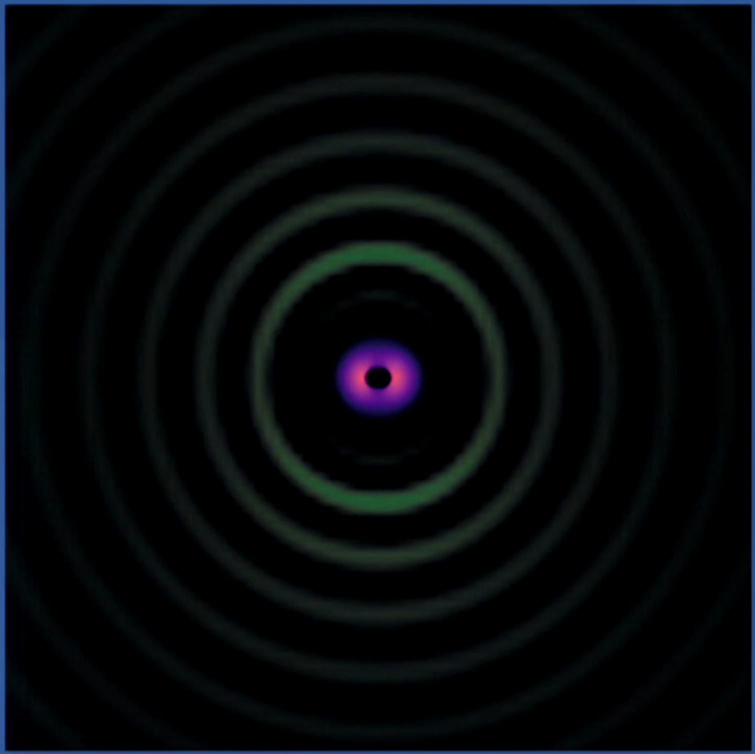

Proca field (inner region) plus gravitational wave emission (outer region)

Late time evolution and GW emission in 3D; a=0.99, $\underline{u}$=0.4

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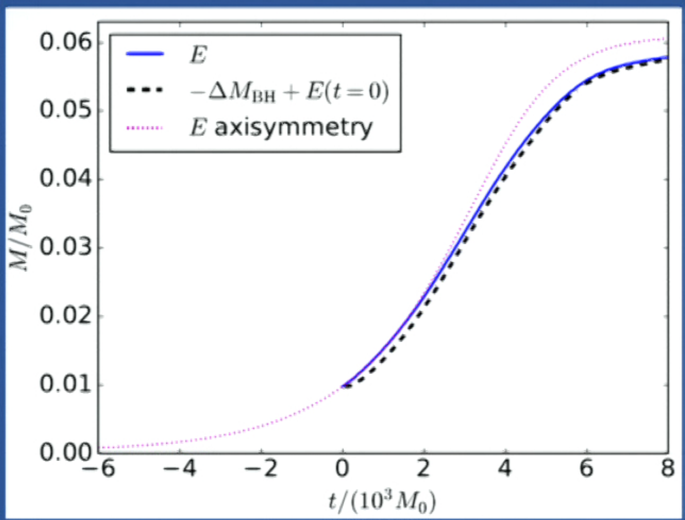

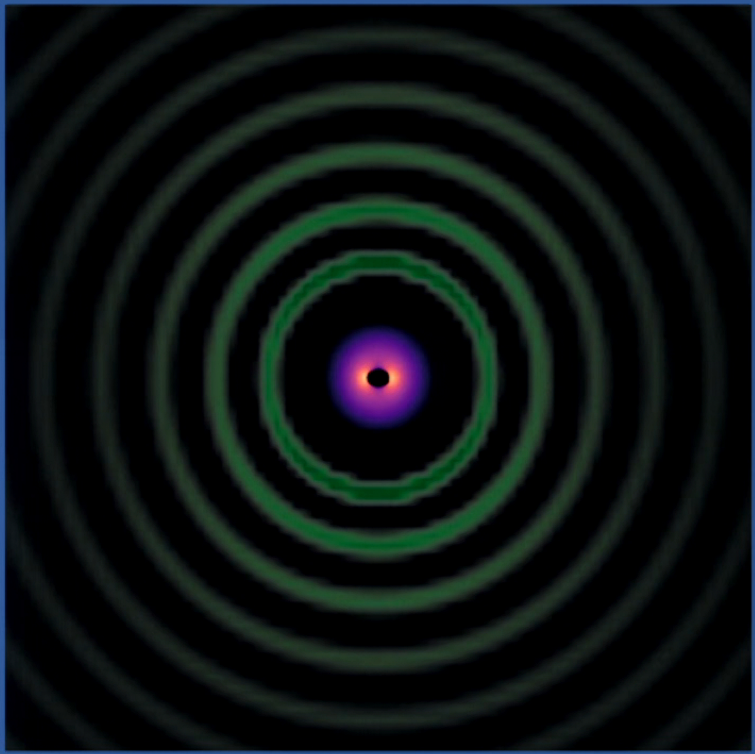

Proca field (inner region) plus gravitational wave emission (outer region)

Conclusions

- So far results are somewhat unsurprising
  - for rapid GW superradiance
    - less efficient than quasi-stationary process at extracting rotational energy; no violation of cosmic censorship; see mode-coupling exciting higher l,m modes (primarily the 4,4 here)
    - however, large horizon oscillations approaching the zero-damped limit with relatively small changes in horizon properties suggests this is still an interesting area to explore further
  - for slower Proca field superradiance
    - results justify a "quasi-adiabatic" approach
    - however, limited range of initial data (single m mode), relatively short timescales (cannot address stability of cloud), only simple form of potential studied
- Beyond single black holes, most interesting for future non-linear studies may be hairy black hole mergers *[c.f. Baumann, Chia & Porto arXiv:1804.03208]*