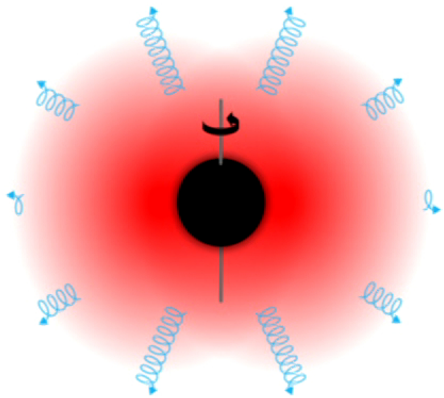
Title: Superradiant instabilities and rotating black holes

Date: May 09, 2018 11:30 AM

URL: http://pirsa.org/18050028

Abstract:

Superradiant instabilities on the Kerr spacetime



Sam Dolan

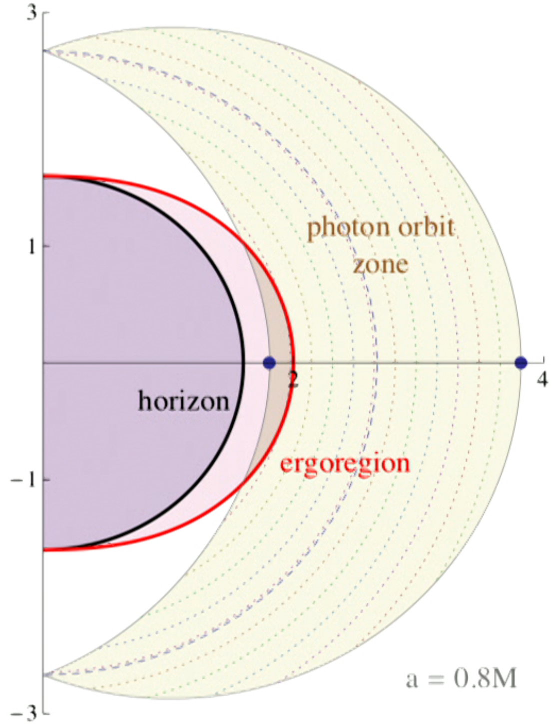
The University Of Sheffield.

@ Perimeter Institute
9th May 2018

1. The Kerr black hole

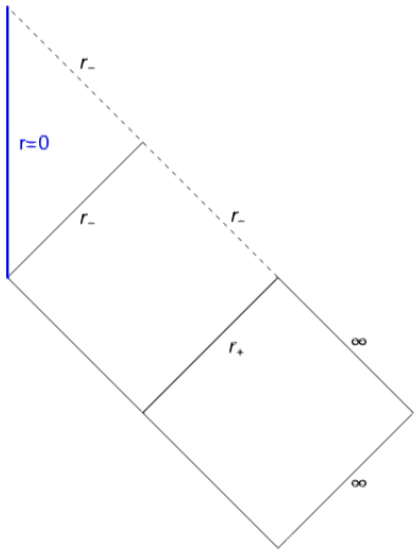
"The most shattering experience has been the realization that [Kerr's] solution of Einstein's equations of general relativity provides the absolutely exact representation of untold numbers of massive black holes that populate the universe." — *S. Chandrasekhar (1975).*

The Kerr spacetime



← Cross section of a Kerr BH

↓ Penrose-Carter diagram



Black hole mechanics

- **1973** **Four Laws of Black Hole Mechanics** (GR)
  1. First Law:

$$c^2 dM = \frac{\kappa}{8\pi}\frac{c^2}{G} dA + \Omega\, dJ$$

  2. Second Law: $dA \geq 0$

(NB. κ is surface gravity and Ω is angular frequency of horizon).

Bardeen, Carter & Hawking (1973)

"It can be seen that $\kappa/8\pi$ is analogous to temperature in the same way that A is analogous to entropy. It should however be emphasized that $\kappa/8\pi$ and A are distinct from the temperature and entropy of the black hole."

Entropy

- **1976 Hawking radiation** (GR + QFT)
- Black holes **radiate** like a black body with a temperature and entropy

$$T_H = \frac{\kappa}{2\pi}\frac{\hbar}{k_b c}, \qquad S = \frac{A}{4}\frac{k_b c^3}{G\hbar} \approx 10^{54}\left(\frac{M}{M_\odot}\right)^2 \mathrm{JK}^{-1}.$$

- The entropy in the Universe is **dominated** by black holes!

Object	**Entropy** (in JK^{-1})
The Sun	$\sim 10^{35}$
BH(Sol)	$\sim 10^{54}$
BH(Sag A*)	$\sim 10^{67}$.

- **GW150914**: merger of two black holes: $36 + 29 \to 62 + 3$. This created an entropy $\mathbf{1.7 \cdot 10^{22}}$ times of that in our Sun.

BH superradiance

- **Penrose process**: A process in which a black hole releases energy, angular momentum and/or charge by **increasing its horizon area**:
  - Particle-splitting in the ergoregion
  - Tidal heating
  - The Blandford-Znajek effect
  - Black hole superradiance
- **Rotational superradiance**: A stimulated or spontaneous radiation-enhancement mechanism by which energy & angular momentum is extracted from a system:
  - Zeldovich's conducting cylinder
  - Black hole superradiance
  - The Nottingham draining bathtub experiment [Torres et al., Nature Phys. **13**, 833 (2017)]

BH superradiance

- A rotating BH can be 'stimulated' to shed mass and angular momentum by a bosonic field.
- A perturbation ψ has a reflection coefficient $\mathcal{R} > 1$ iff

$$\omega\left(\omega - m\Omega\right) < 0$$

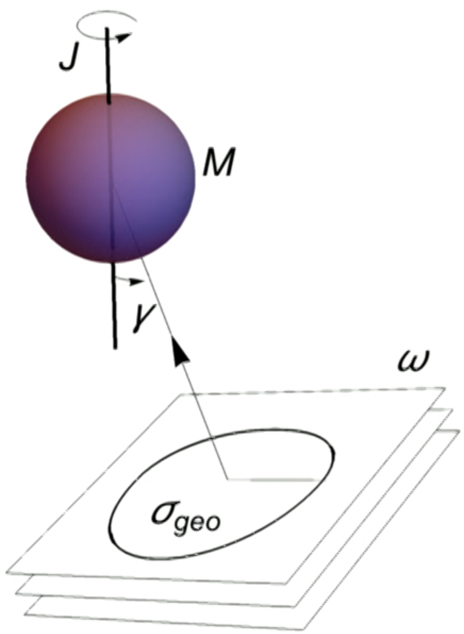
where $\psi \sim \exp(-i\omega t + im\phi)$ and $\Omega = a/(2Mr_+)$ is the angular frequency of event horizon.

- **Why?** **2nd law:** $dA \geq 0$
- Recall first law: $dM = \frac{\kappa}{8\pi}dA + \Omega dJ$

$$\Rightarrow \quad \frac{\kappa}{8\pi}\frac{dA}{dM} = 1 - \Omega\frac{dJ}{dM} = \frac{1}{\omega^2} \times \omega\left(\omega - m\Omega\right)$$

- Since dA is positive, dM must depend on sign of $\omega(\omega - m\Omega)$.

Example: Absorption by a Kerr black hole



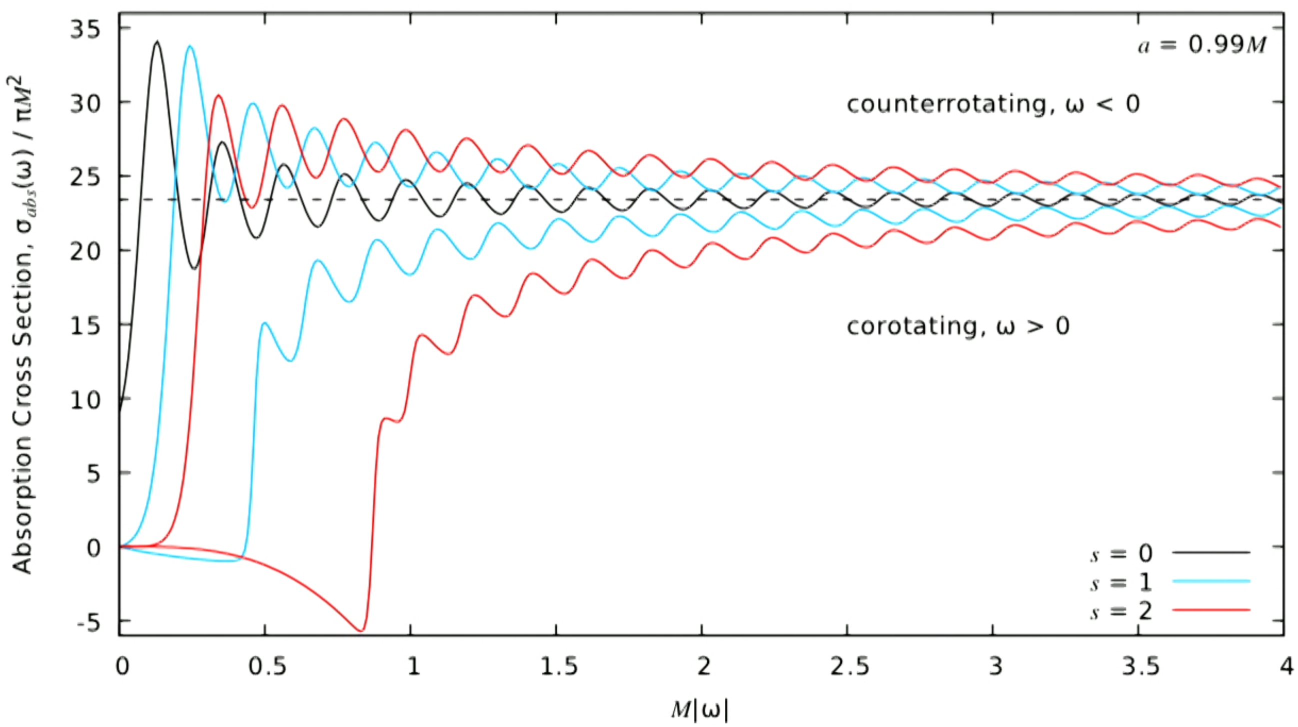
- Consider a monochromatic wave incident on a Kerr black hole in vacuum.
- Parameter: $M\omega = \frac{r_S}{\lambda\pi}$
- The **absorption cross section** σ_{abs} can be calculated from ODEs.

$$\sigma_{\text{abs}}(\omega) = \frac{4\pi^2}{\omega^2} \sum_{l=|\mathfrak{s}|}^{+\infty} \sum_{m=-l}^{+l} \left|S_{\mathfrak{s}lm\omega}(\gamma)\right|^2 \Gamma_{\mathfrak{s}lm\omega}.$$

- Low-frequency co-rotating modes undergo **superradiance**

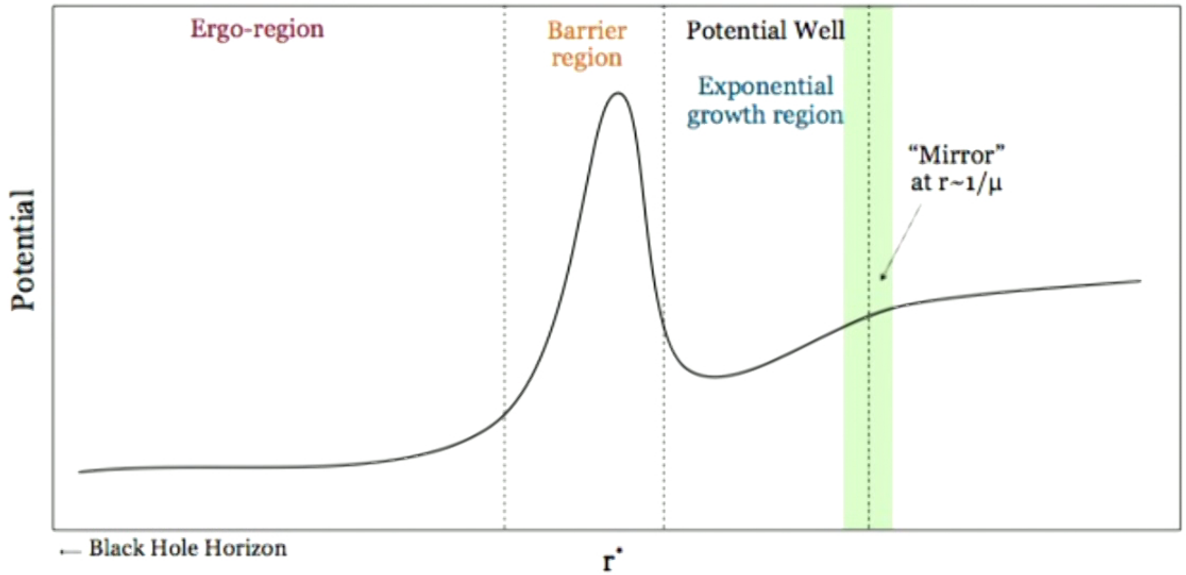
⇒ σ_{abs} can be **negative**

Abs. cross sections ($\gamma = 0$) for EM & GW waves



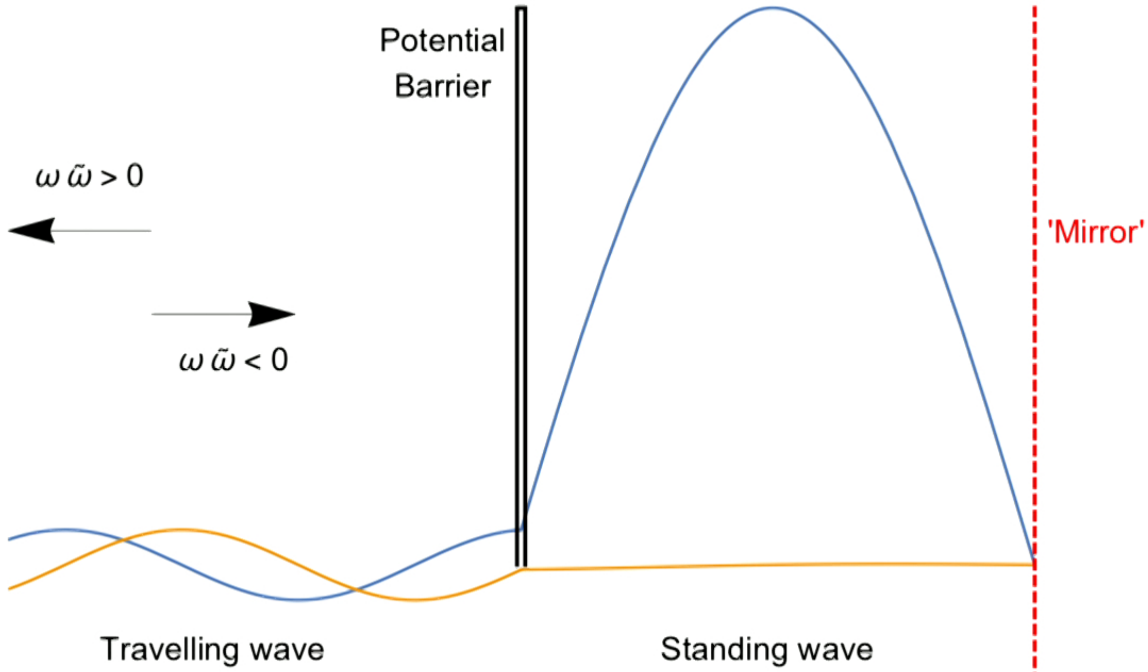
Superradiant instabilities

The effective potential



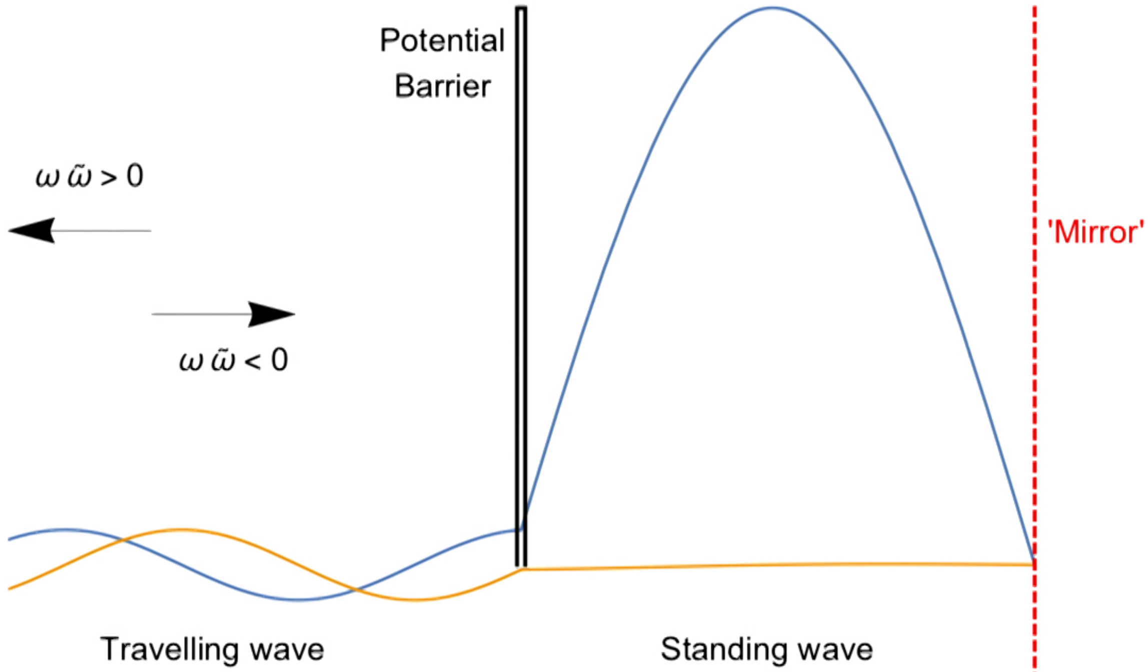
Arvanitaki, Dimopoulos, Dubovsky *et al.*,
"String Axiverse", Phys. Rev. D **81**, 123530 (2010) arXiv:0905.4720

Toy model



$$\tilde{\omega} = \omega - m\Omega, \qquad \Omega = a/2Mr_+$$

Toy model



$$\widetilde{\omega} = \omega - m\Omega, \qquad \Omega = a/2Mr_+$$

Bound states of scalar field

- A scalar field Φ satisfying $\Box\Phi - \mu^2\Phi = 0$ which is regular on $\mathcal{H}^+$ and as $r \to \infty$ has a **discrete spectrum** of complex frequencies

$$\omega = \hat{\omega} + i\nu$$

labelled by azimuthal m and total l ang. mom., and overtone $\hat{n}$.

- In limit $\alpha \equiv M\mu \ll l$, there is a **hydrogenic spectrum** with fine structure corrections:

$$\frac{\hat{\omega}}{\mu c^2} \approx 1 - \frac{\alpha^2}{2n^2} - \frac{\alpha^4}{8n^4} + \frac{(2l - 3n + 1)\alpha^4}{n^4(l + 1/2)} + \frac{2am/M\alpha^5}{n^3 l(l + 1/2)(l + 1)} + \ldots$$

where $n = \hat{n} + l + 1$.

- The fine and hyperfine structure terms were recently found by Baumann, Chia & Porto, arXiv:1804.03208.
- For Schwarzschild BH, all states decay $\nu < 0$.
- For Kerr BH, states satisfying the superradiant condition, $0 < \hat{\omega} < m\Omega$ will **grow**, $\nu > 0$. The co-rotating dipole mode $l = m = 1$ is dominant

Bound states of scalar field

- The bound state spectrum (ω/μ) is determined by two dimensionless parameters

$$0 \leq a_* = \frac{J}{M^2} < 1, \qquad M\mu \equiv \frac{GM\mu}{\hbar c} \sim \frac{\text{horizon radius}}{\text{Compton wavelength}}.$$

- For superradiance, we need $\mu \lesssim m\Omega$, and $\Omega \leq 1/2M$.
- The instability is significant for $M\mu \sim 1$, but exponentially-suppressed for large $M\mu$.
- For a pion π^0 + astrophysical BH, $M\mu \sim 10^{18}$ (!)
- The instability is only significant for primordial black holes . . . or **ultra-light bosonic fields** such as axions.

Growth of bound states: Key results

- Zouros & Eardley (1979):

$$M\nu \sim 10^{-7} e^{-1.84 M\mu}, \qquad M\mu \gg 1.$$

- Detweiler (1980):

$$M\nu \sim -\frac{1}{12}(M\mu)^9 \left(\mu - \Omega\right) r_+, \qquad M\mu \ll 1,\ l = 1$$

Growth of bound states: Key results

- Zouros & Eardley (1979):

$$M\nu \sim 10^{-7} e^{-1.84 M\mu}, \qquad M\mu \gg 1.$$

- Detweiler (1980):

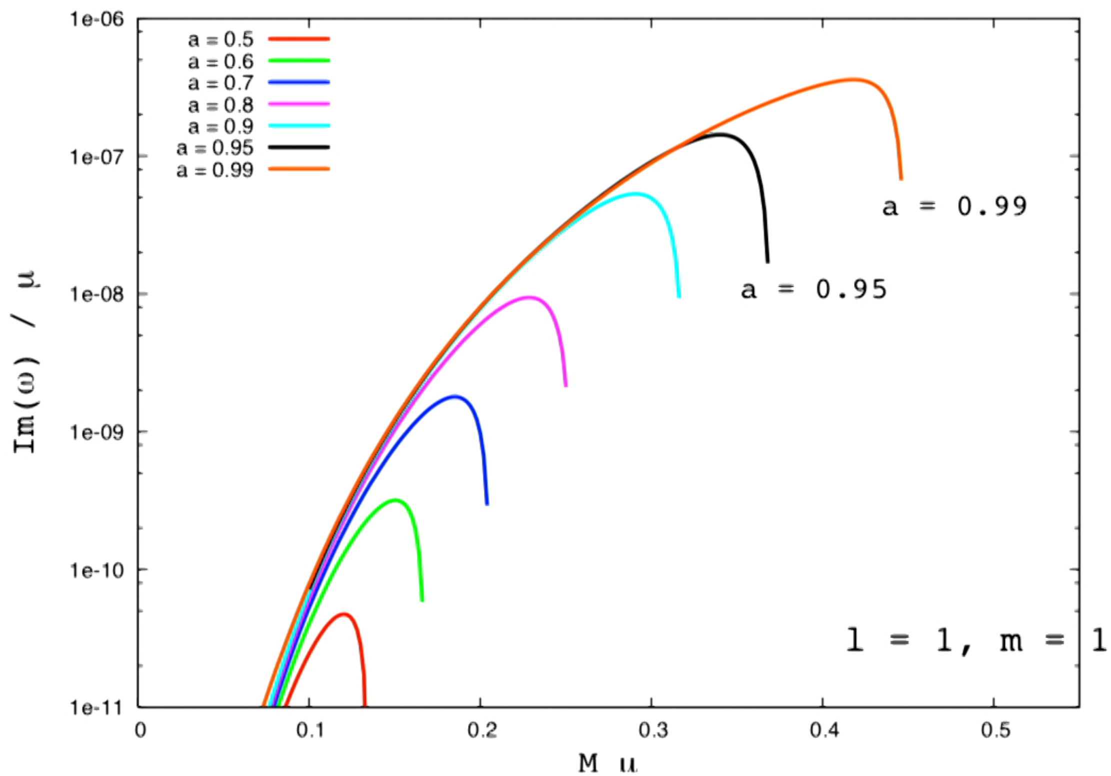
$$M\nu \sim -\frac{1}{12}(M\mu)^9 (\mu - \Omega)\, r_+, \qquad M\mu \ll 1, \; l = 1$$

- Numerical results for intermediate regime $M\mu \sim 1$ found by Furuhashi *et al.* (2004), Cardoso *et al.* (2005), Dolan (2007) and others.
- Minimum e-folding time $\tau_{\text{min}} = 1/\nu_{\text{max}}$,

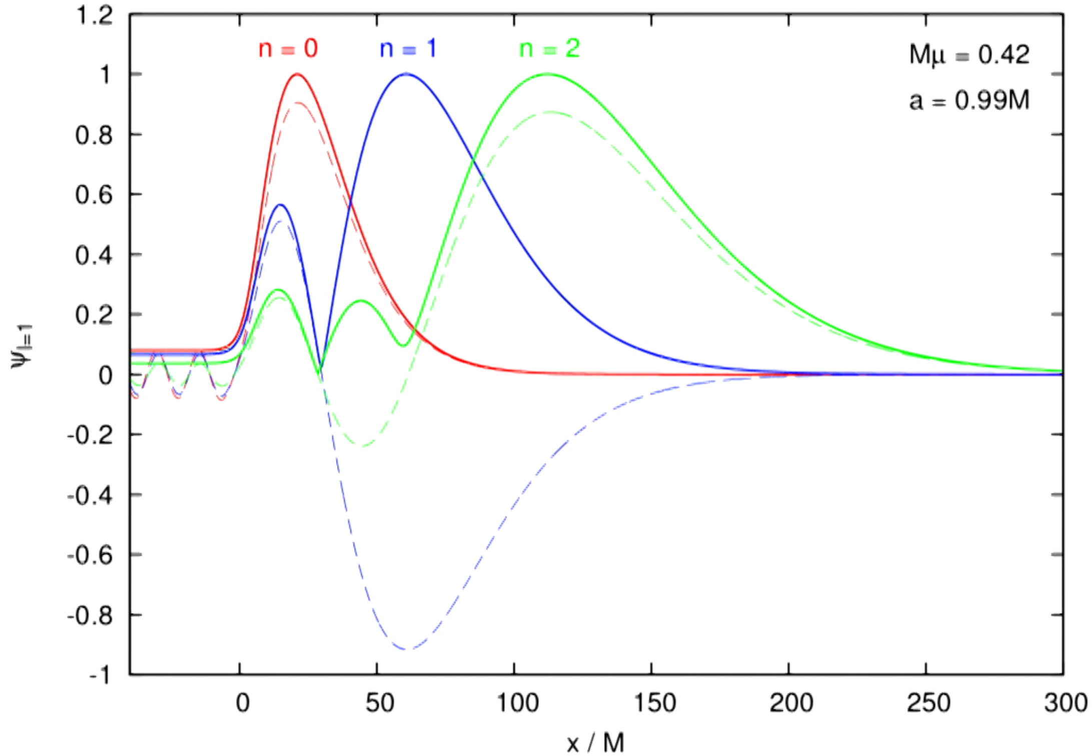
$$\tau_{\text{min}} \approx 5.81 \times 10^6 \, GM/c^3 \approx 29 \text{ sec} \times \left(\frac{M}{M_\odot}\right)$$

for $a \approx 0.997M$ and $M\mu \approx 0.45$.

Unstable Bound States: $M\nu > 0$



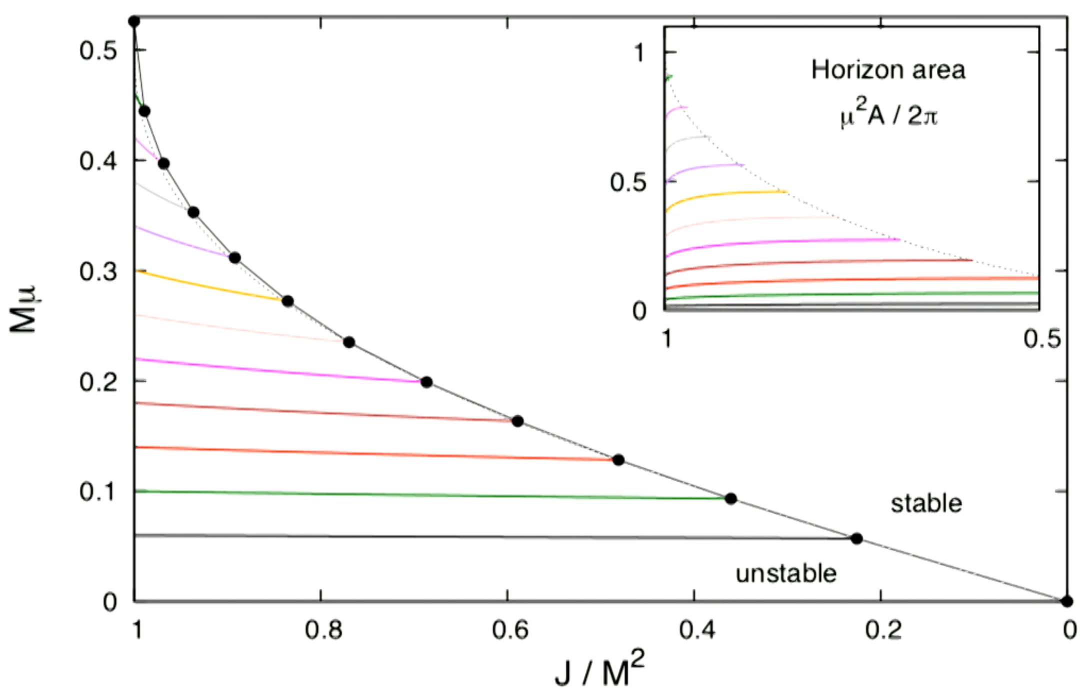
What is the profile of the bound states?



How do BH parameters evolve?

(in the linear regime)

Evolution of black hole under superradiant instability



The Dirac field

Bound states of the Dirac field

- The Dirac field also has a bound state spectrum [**Lasenby** et al., Phys. Rev. D 72, 105014 (2005)].
- It is also hydrogenic with fine & hyperfine structure [Dolan & Dempsey, CQG, 32 (2015) 184001]

$$\frac{\hat{\omega}}{\mu c^2} \approx 1 - \frac{\alpha^2}{2n^2} + \frac{\alpha^4}{n^4}\left(\frac{15}{8} - \frac{3n}{2j+1} - \frac{3n}{2l+1}\right) + \frac{\beta_{jln} am/M\alpha^5}{n^5} + \dots$$

cf. the scalar field spectrum:

$$\frac{\hat{\omega}}{\mu c^2} \approx 1 - \frac{\alpha^2}{2n^2} - \frac{\alpha^4}{n^4}\left(\frac{1}{8} + \frac{(2l-3n+1)}{(l+1/2)}\right) + \frac{2am/M\alpha^5}{n^3 l(l+1/2)(l+1)} + \dots$$

- But **all modes decay**, so there is no instability
- There is no (classical) superradiance for fermionic fields.

Absence of Dirac superradiance

- The Dirac current $J^a \equiv \overline{\Psi}\gamma^\mu \Psi$ is conserved: $\nabla_a J^a = 0$.
- The radial component of the current takes the form

$$J^r = \frac{1}{\Sigma}\left(|R_1(r)|^2 - |R_2(r)|^2\right)\left(|S_1(\theta)|^2 + |S_2(\theta)|^2\right)$$

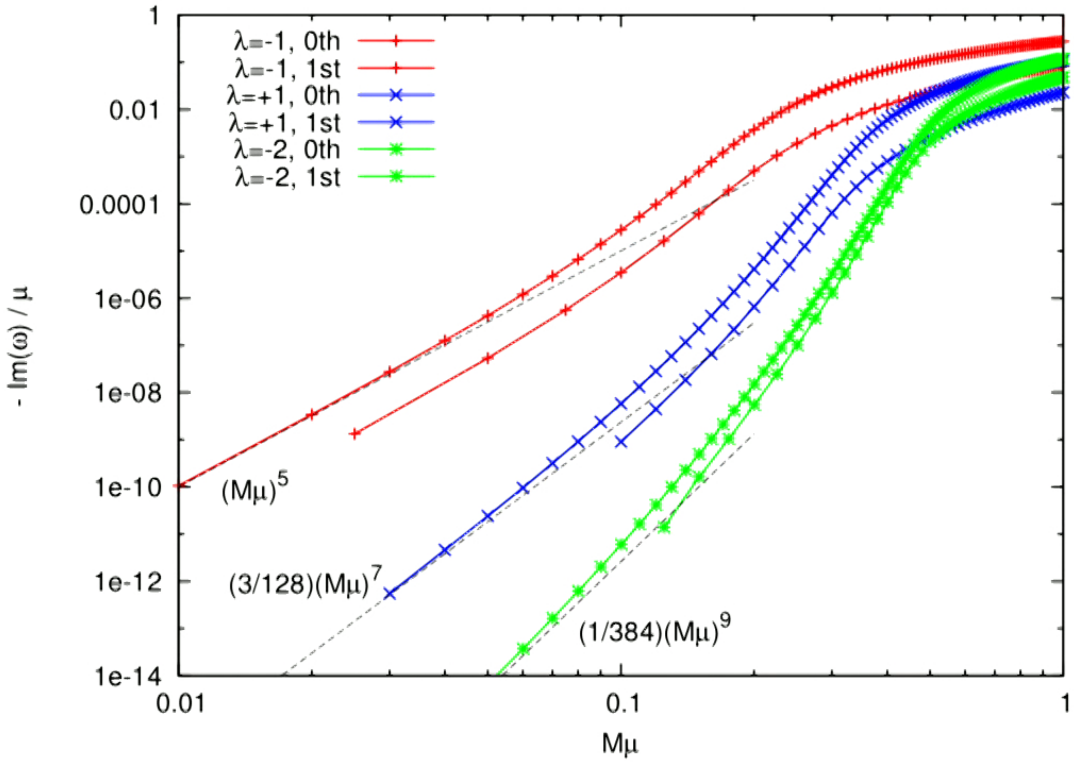
and the spinor that is regular on the future horizon $\mathcal{H}^+$ has the form

$$\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} \sim \begin{pmatrix} \beta\sqrt{\Delta} \\ 1 \end{pmatrix} \exp(-i\widetilde{\omega} r_*), \qquad r \to r_+$$

implying that $J^r < 0$ on r_+. Hence there is a flux **into** the black hole.

- **Q**. But what happened to the zeroth law $dA > 0$?
- **A**. The Dirac field **violates** the **weak energy principle** which asserts that $-T_{ab}t^a t^b \geq 0$ for any timelike vector t^a.

Decay times for Dirac fields on Schwarzschild



Power-law scaling

Instabilities: Proca field (Massive vector boson)

e.g. ‘The string photiverse’:
spin-1 non-trivial gauge field configurations

Bound states of the Proca field

- The Proca field states have three spin polarizations: $S = +1$, 0 and -1.
- Under spatial inversion, $S = +1$ and $S = -1$ are even-parity, and $S = 0$ is odd-parity.
- In 2012, Joao Rosa & I looked at the Schwarzschild case $a = 0$, finding:
  - The odd-parity $S = 0$ mode satisfies a 2nd-order radial equation

$$\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - f(r)\left[\frac{l(l+1)}{r^2} + \mu^2\right]\right) R(r) = 0$$

  - The even-parity $S = \pm 1$ modes satisfy a **pair** of coupled 2nd-order ODEs.
  - The decay rate scales as

$$\mathrm{Im}(\omega/\mu) \propto (M\mu)^{4l+2S+5}$$

Bound states of the Proca field

There has been interest in calculating the growth rates for the Proca field (massive vector boson) for several years. Highlights include:

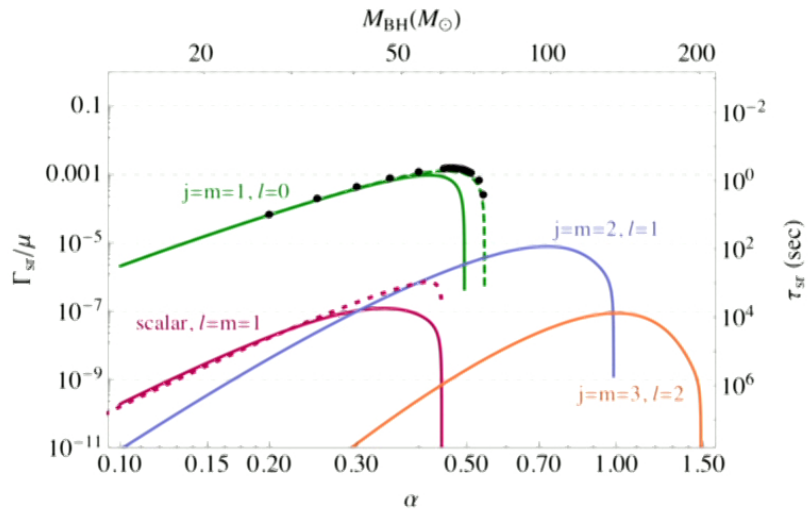
- *"Superradiant instabilities in astrophysical systems"*, Witek, Cardoso, Ishibashi & Sperhake, Phys. Rev. D **87**, 043513 (2013).
- *"Black-Hole Bombs and Photon-Mass Bounds"*, Pani, Cardoso, Gualtieri, Berti & Ishibashi Phys. Rev. Lett. **109**, 131102 (2012).
- *"Superradiant Instability and Back-reaction of Massive Vector Fields around Kerr Black Holes"*, East & F. Pretorius, Phys. Rev. Lett. **119**, 041101 (2017).
- *"A modern approach to superradiance"*, Endlich & Penco, JHEP 2017: 52 (2017).
- *"Black Hole Superradiance Signatures of Ultralight Vectors"*, Baryakhtar, Lasenby & Teo, Phys. Rev. D **96**, 035019 (2017).
- *"Massive Vector Fields in Kerr-NUT-(A)dS Spacetimes: Separability and Quasinormal Modes"*, Frolov, Krtous, Kubiznak & Santos, arXiv:1804.00030.

Bound states of the Proca field

- Baryakhtar, Lasenby & Teo (2017) found an analytic approximation for the growth rate:

$$\mathrm{Im}(\omega) \sim (M\mu)^{2j+2l+5}(m\Omega - \omega)$$

- East (2017) obtained numerical data for the growth rate from time-domain simulations. This is Fig. 2 from BLT ↓



Separability of the Proca field

- Frolov, Krtous, Kubiznak & Santos have shown something remarkable: the equations governing the even-parity ($S = \pm 1$) modes of the Proca field are separable!
- With the ansatz $A^a = B^{ab}\nabla_b \Psi$ for the vector field, and a multiplicative separability ansatz for Ψ, FKKS find that

$$\frac{d}{dr}\left[\Delta \frac{dR}{dr}\right] + \left(\frac{K_r^2}{\Delta} + \frac{2-q_r}{q_r}\frac{\sigma}{\nu} - \frac{q_r \mu^2}{\nu^2}\right) R(r) = 0$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta \frac{dS}{d\theta}\right] - \left[\frac{K_\theta^2}{\sin^2\theta} + \frac{2-q_\theta}{q_\theta}\frac{\sigma}{\nu} - \frac{q_\theta \mu^2}{\nu^2}\right] S(\theta) = 0$$

where

$$K_r = am - (a^2 + r^2)\omega, \quad K_\theta = m - a\omega \sin^2\theta,$$

$$q_r = 1 + \nu^2 r^2, \quad q_\theta = 1 - \nu^2 a^2 \cos^2\theta, \quad \sigma = \omega + a\nu^2(m - a\omega).$$

- Here ν is the separation constant (impose regularity on $S(\theta)$ at poles).
- In the limit $a \to 0$, $S = Y_{lm}(\theta)$ and $\omega/\nu - \mu^2/\nu^2 = -l(l+1)$.

The 'Killing tower'

- A **tower** of 'Killing objects' are generated from the primary tensor h_{ab}
- The Hodge dual of the primary tensor is the **Killing-Yano tensor** $f = {}^\star h$, whose derivative is totally antisymmetric:

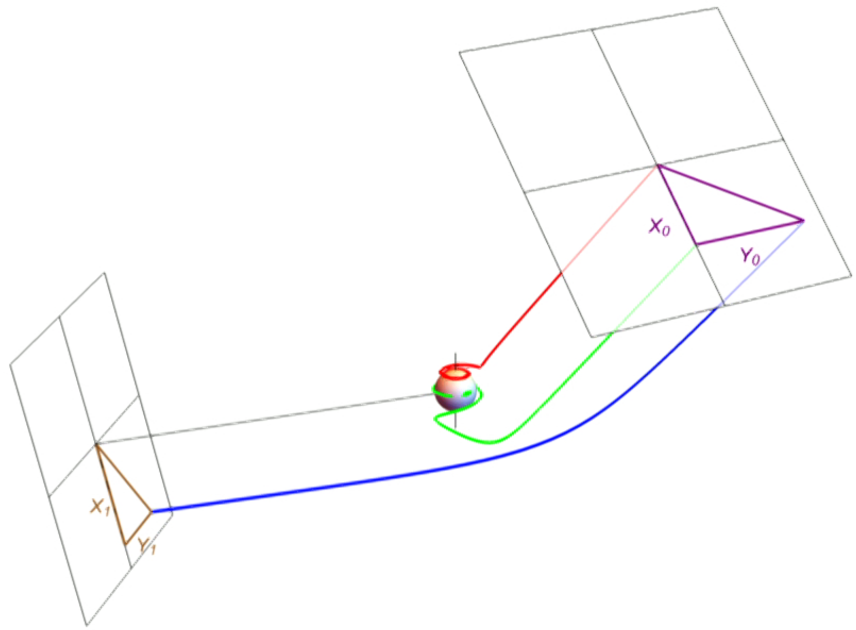
$$\nabla_a f_{bc} = \nabla_{[a} f_{bc]} = \frac{1}{2} \varepsilon_{abcd} \xi^d_{(t)}$$

- The two-forms f and h generate the **Killing tensor** K_{ab} and the **conformal Killing tensor** Q_{ab}:

$$\begin{aligned} K_{ab} &\equiv f_a{}^c f_{bc} \quad \Rightarrow \quad \nabla_{(a} K_{bc)} = 0 \\ Q_{ab} &\equiv h_a{}^c h_{bc} \quad \Rightarrow \quad \nabla_{(a} Q_{bc)} = g_{(ab} h_{c)d} \xi^d_{(t)} \end{aligned}$$

- The second Killing vector is $\xi^a_{(\psi)} = -K^a{}_b \xi^b_{(t)}$.
- See Frolov, Krtouš and Kubizník, Living Reviews in Relativity. **20**:6 (2017).

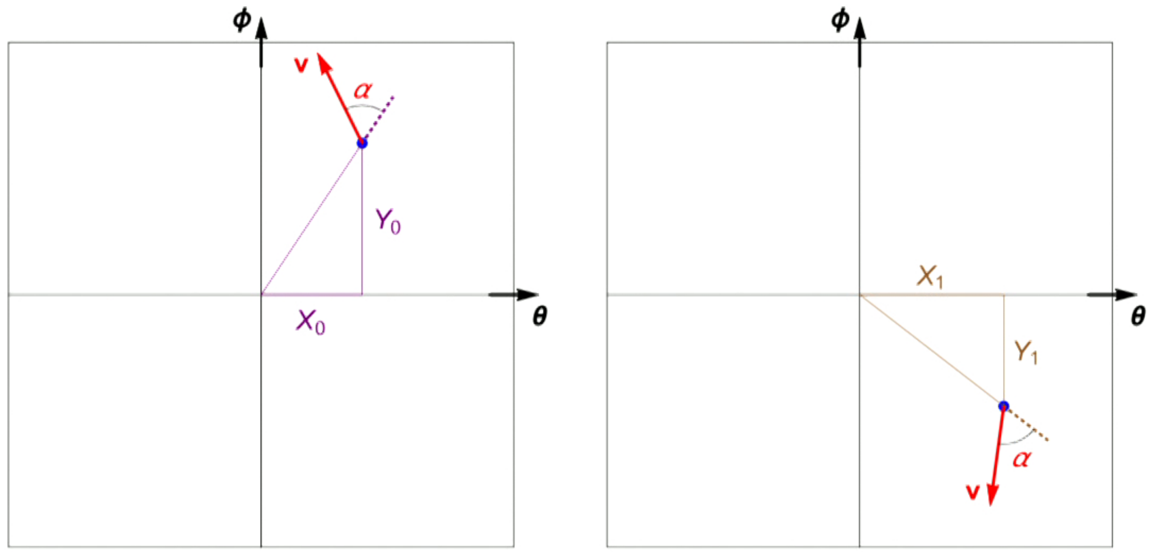
Conserved quantities for rays



$$X_0^2 + Y_0^2 = X_1^2 + Y_1^2 = \mathcal{K}/E^2$$ Carter constant

$$Y_0 \sin \gamma_0 = -Y_1 \sin \gamma_1 = L_z/E$$ Azimuthal ang. mom.

Rays and conserved quantities



← Before After →

$$\hat{Z} = \frac{i}{\sqrt{K}} \left(f_{ab} + i h_{ab}\right) v^a k^b = \exp\left(i\alpha\right).$$

$\hat{Z}$ is constant along the ray. The phase α is the precession angle for v^a.

Separability

Field		Massless ($\mu = 0$)	Massive ($\mu \neq 0$)
Scalar $\Box\Phi = \mu^2\Phi$	$s = 0$	✓ Carter '68	✓ Brill et al '72
Spinor $i\not{D}_a\psi = \mu\psi$	$s = \frac{1}{2}$	✓ Unruh '73	✓ Chandrasekhar '76
Electromagnetic $dF = 0 = \delta F$	$s = 1$	✓ Teukolsky '72	—
Proca $\Box A^a = \mu^2 A^a$, $\nabla_a A^a = 0$	$s = 1$	—	Frolov, Krtouš, Kubizňák & Santos **2018** even-parity ✓, odd-parity ??
Gravitational	$s = 2$	✓ Teukolsky '72	??

Separability of the Proca field

- Frolov, Krtous, Kubiznak & Santos have shown something remarkable: the equations governing the even-parity ($S = \pm 1$) modes of the Proca field are separable!
- With the ansatz $A^a = B^{ab}\nabla_b \Psi$ for the vector field, and a multiplicative separability ansatz for Ψ, FKKS find that

$$\frac{d}{dr}\left[\Delta \frac{dR}{dr}\right] + \left(\frac{K_r^2}{\Delta} + \frac{2-q_r}{q_r}\frac{\sigma}{\nu} - \frac{q_r \mu^2}{\nu^2}\right) R(r) = 0$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left[\sin\theta \frac{dS}{d\theta}\right] - \left[\frac{K_\theta^2}{\sin^2\theta} + \frac{2-q_\theta}{q_\theta}\frac{\sigma}{\nu} - \frac{q_\theta \mu^2}{\nu^2}\right] S(\theta) = 0$$

where

$$K_r = am - (a^2 + r^2)\omega, \quad K_\theta = m - a\omega \sin^2\theta,$$
$$q_r = 1 + \nu^2 r^2, \quad q_\theta = 1 - \nu^2 a^2 \cos^2\theta, \quad \sigma = \omega + a\nu^2(m - a\omega).$$

- Here ν is the separation constant (impose regularity on $S(\theta)$ at poles).
- In the limit $a \to 0$, $S = Y_{lm}(\theta)$ and $\omega/\nu - \mu^2/\nu^2 = -l(l+1)$.

Growth rates: Proca field

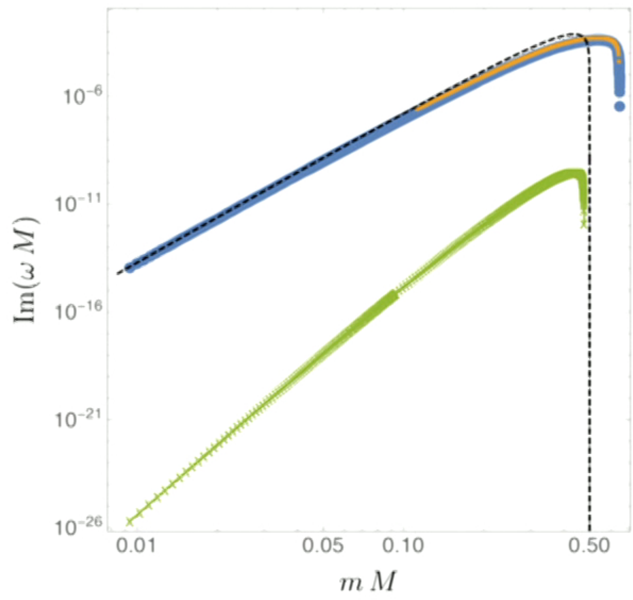
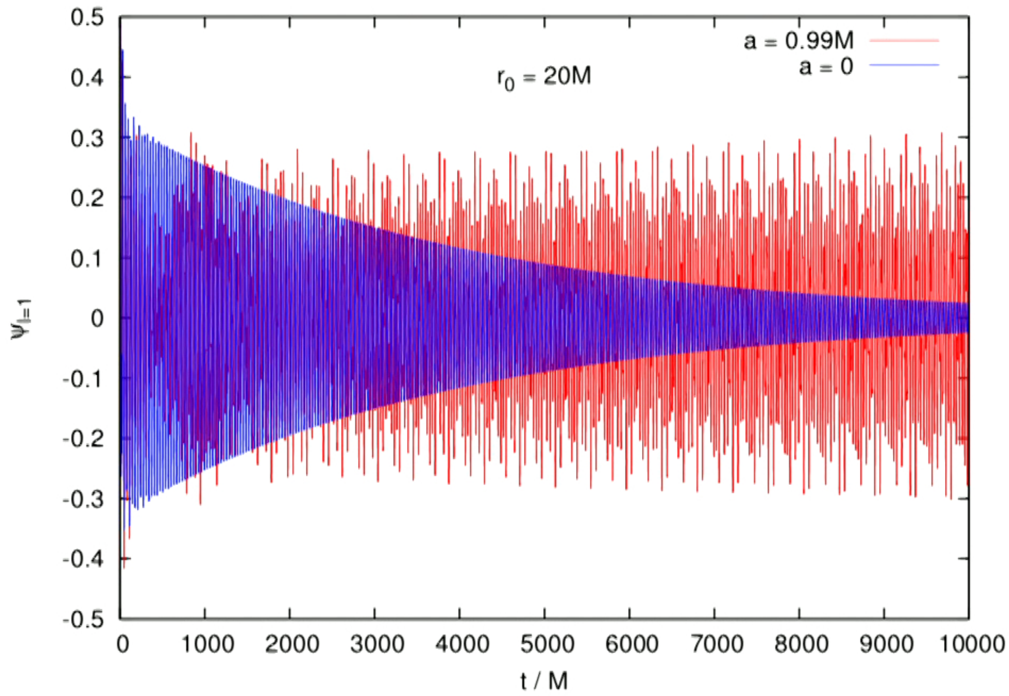


Fig. 1 from FKKS (arXiv:1804.00030) showing the growth rate for the even-parity $l = m = 1$ modes with $S = -1$ (blue) and $S = +1$ (green).

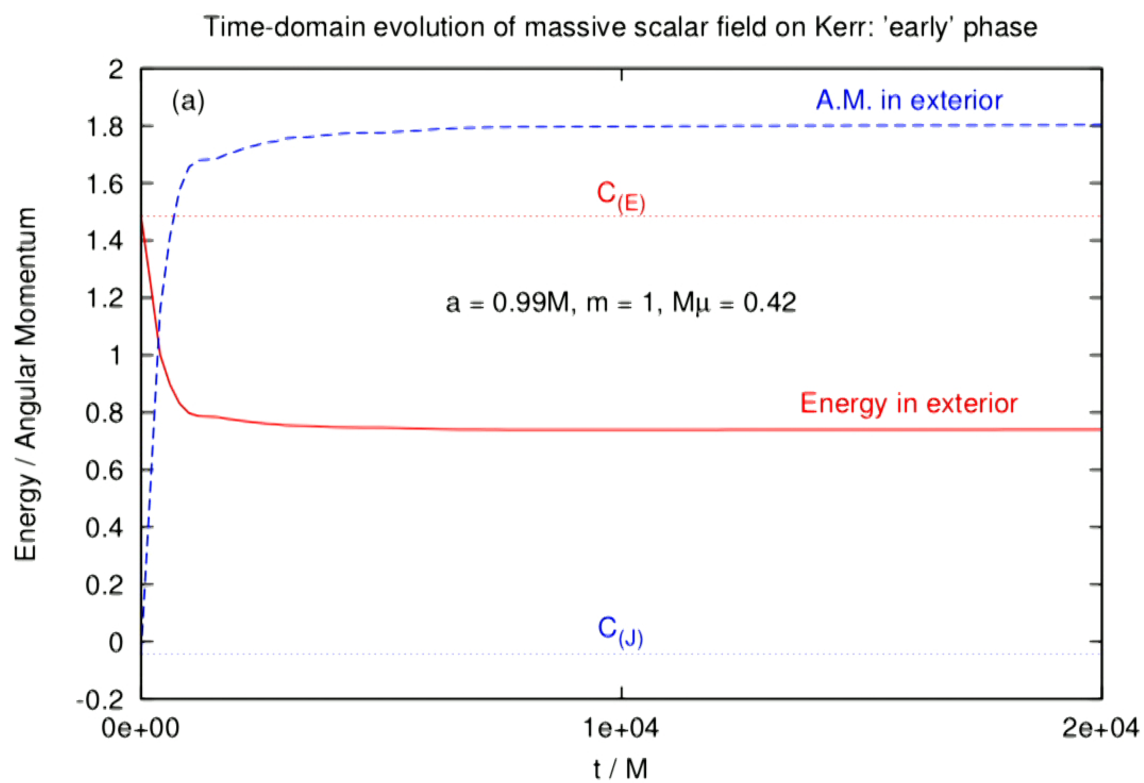
Scalar field evolution with 'mirror'

The field as a function of time, at $r = 10M$



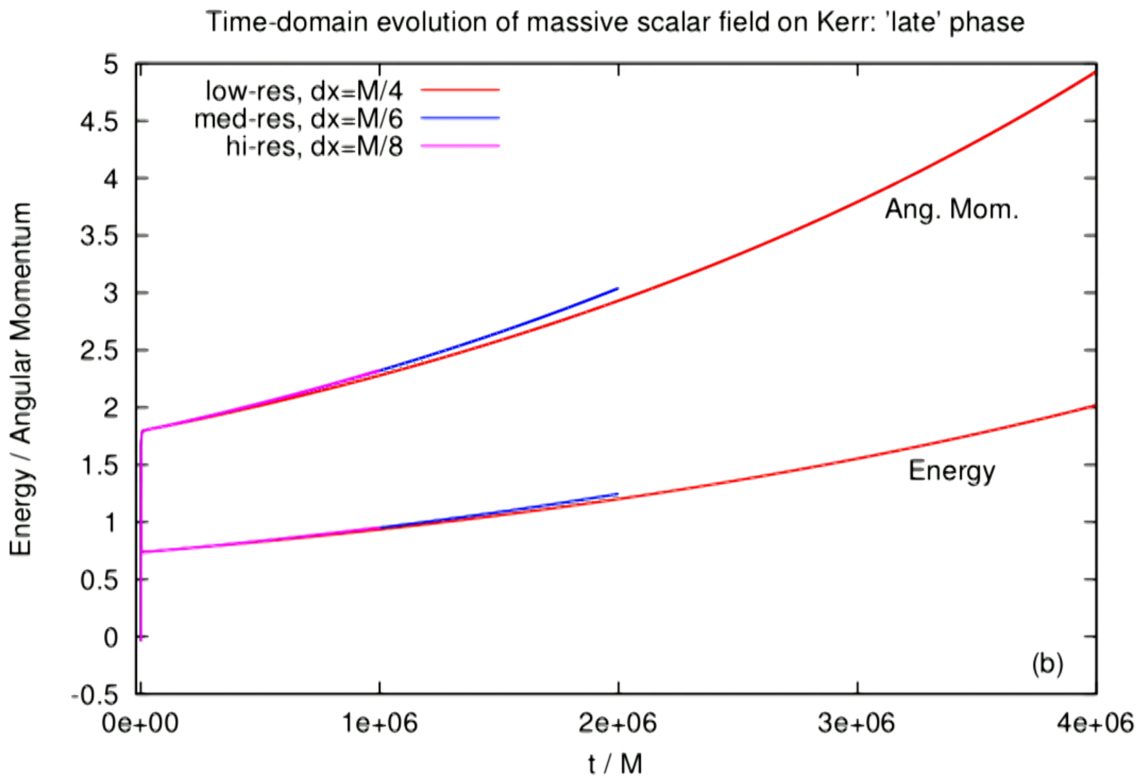
Evolution of massive scalar field

'Early' times: $t \lesssim 10^4 M$



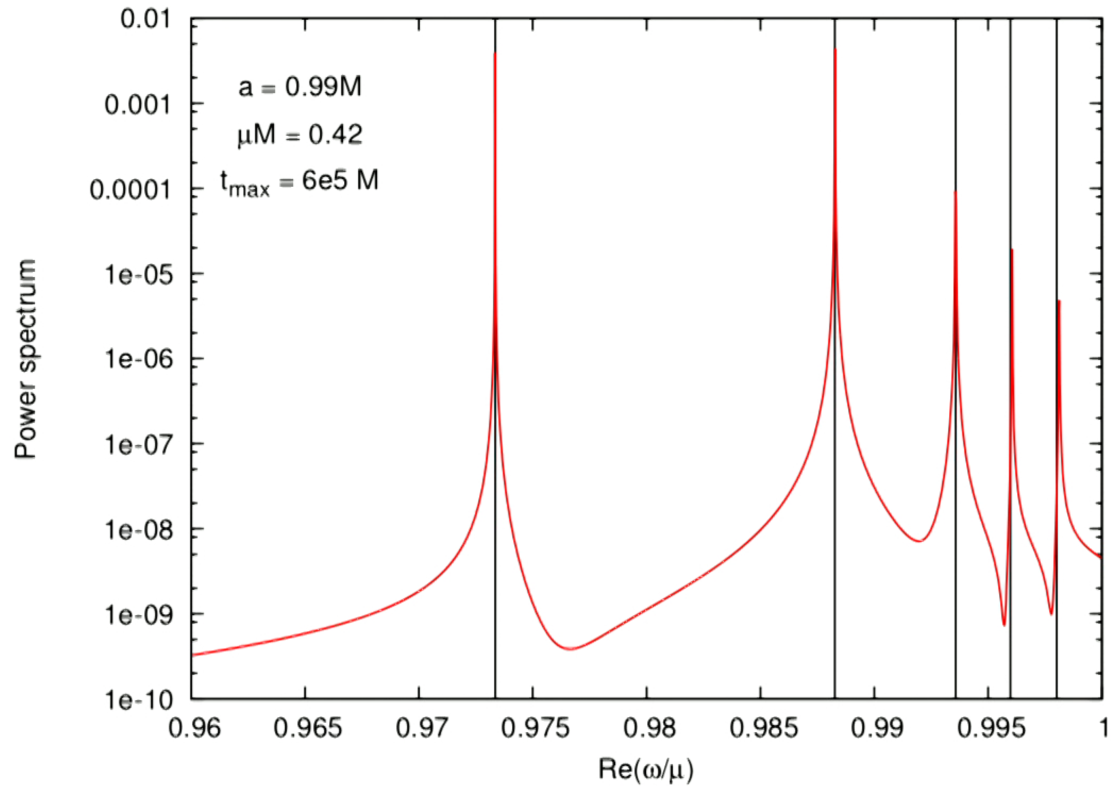
Evolution of massive scalar field

'Late' times: $t \lesssim 4 \times 10^6 M$



Evolution of massive scalar field

Fourier analysis: recovering the bound state spectrum



Final thoughts

- Superradiant instabilities are now well characterised in the linearized regime.
- The Proca instability is up to 10^4 faster than the scalar instability. (Massive $s = 2$ particles would be faster still).
- New: the Proca field separates on the Kerr spacetime!
- Superradiant instabilities can generate 'hairy' black holes [H&R; Pretorius & East], gravitational wave sirens, axion annihilations or explosive phenomena (Bosenovas).
- Using BH surveys & gravitational-wave detectors to search for fundamental ultra-light bosons is a viable prospect, in an era in which next-generation particle accelerators may be prohibitively expensive.