Title: Aspects of approximate quantum error correction in holography

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Abstract: <p>Several equivalent formulations of quantum error correction condition will be introduced. Subtleties arise</p>

<p>when the error correction conditions hold only approximately. We will discuss an equivalent formulation that is </p>

<p>robust to the approximation error. One can leverage this tool to derive the existence of approximate quantum</p>

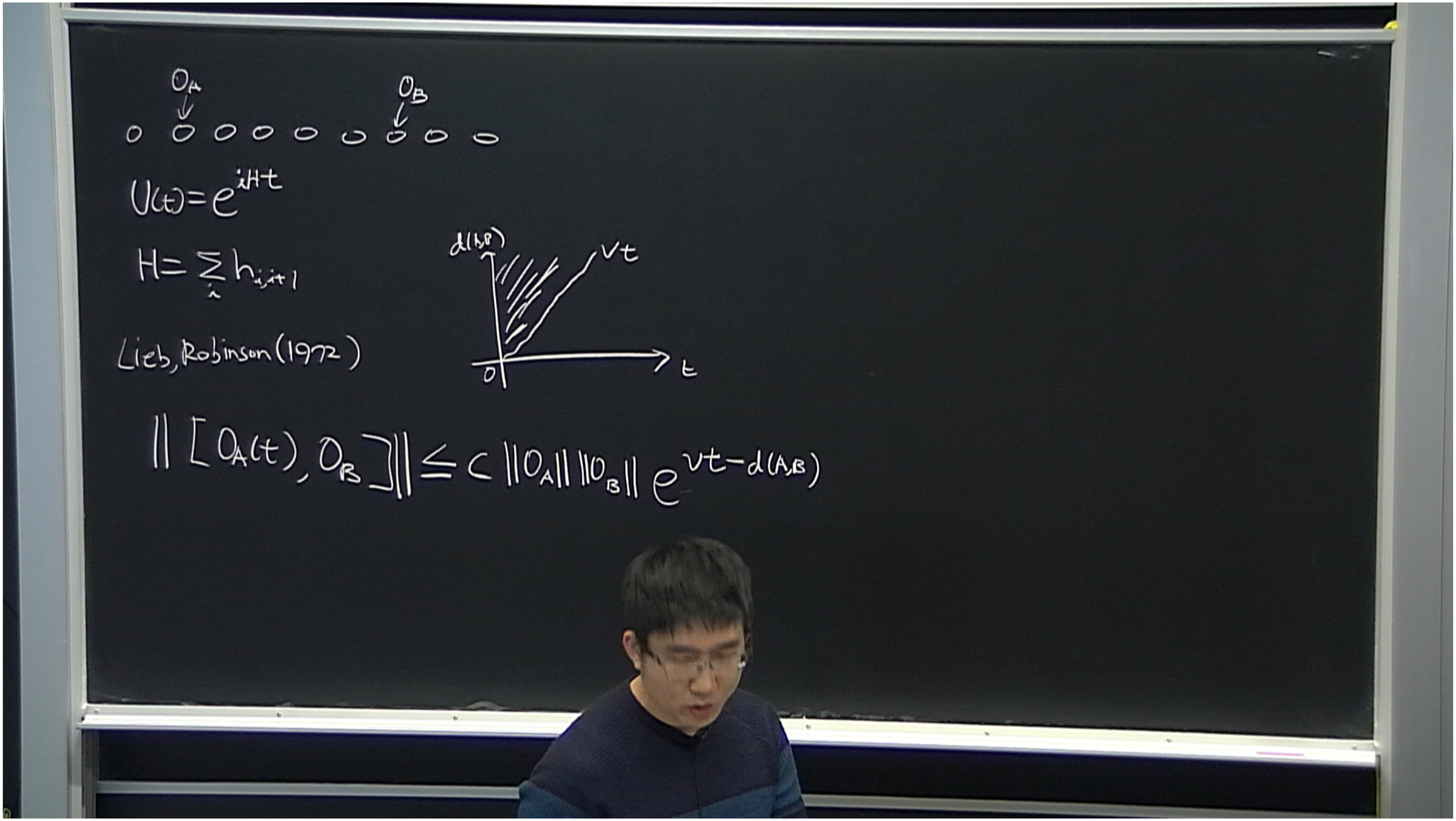
<p>error correcting code at low energy subspace of CFT that reproduces aspects of the holographic quantum error</p>

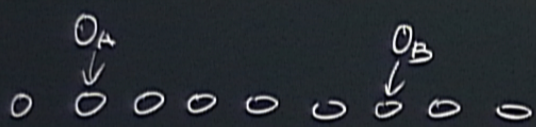
<p>correcting code. Using the same tool, we observe that two operators with greatly differing complexity approximately</p>

<p>commute in an appropriate code subspace. This leads to a notion of bulk locality in the entanglement shadow,</p>

<p>but the precise definition of complexity seems to play an important role in determining how well these operators commute</p>

<p>in the subspace.</p>

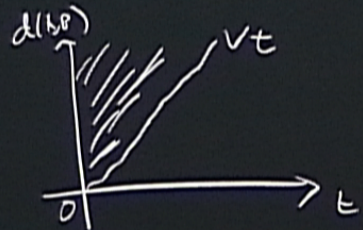

O_A
O_B
$U(t)=e^{iHt}$
$H=\sum_i h_{i,i+1}$
Lieb, Robinson (1972)
$d(A,B)$
vt
0
t
$\| [O_A(t), O_B] \| \leq c \|O_A\| \|O_B\| e^{vt-d(A,B)}$



$$U(t) = e^{iHt}$$

$$H = \sum_i h_{i,i+1}$$

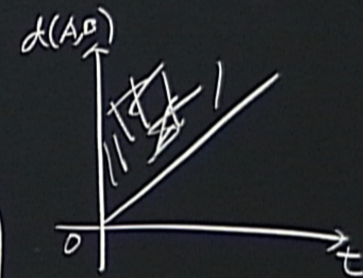
Lieb, Robinson (1972)



$$\| [O_A(t), O_B] \| \le c \|O_A\| \|O_B\| e^{vt - d(A,B)}$$

Ordinary, locally interacting QMS

$$\| P [O_A(t), O_B] P \| \le c \|O_A\| \|O_B\| f(t, d(A,B))$$



1) Define Subspace, O_B: logical operator

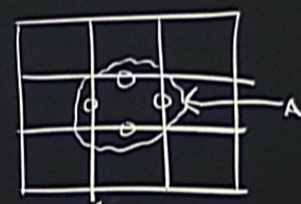
2) Understand EC against erasure

3) Choose $O_A(t)$ to be the "error"

4) Apply QEC condition

Correctability

$$R \circ \mathrm{Tr}_A[\rho] = \rho \quad \forall \rho$$

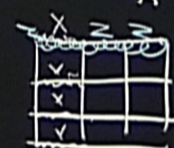


Local Indistinguishability

$$\rho^A = \omega^A$$

Cleanability

$$\forall U_L \;\; \exists U_{\bar{A}} \;\; \text{s.t.} \;\; U_{\bar{A}} \circ U = U \circ U_L$$



Robust Version

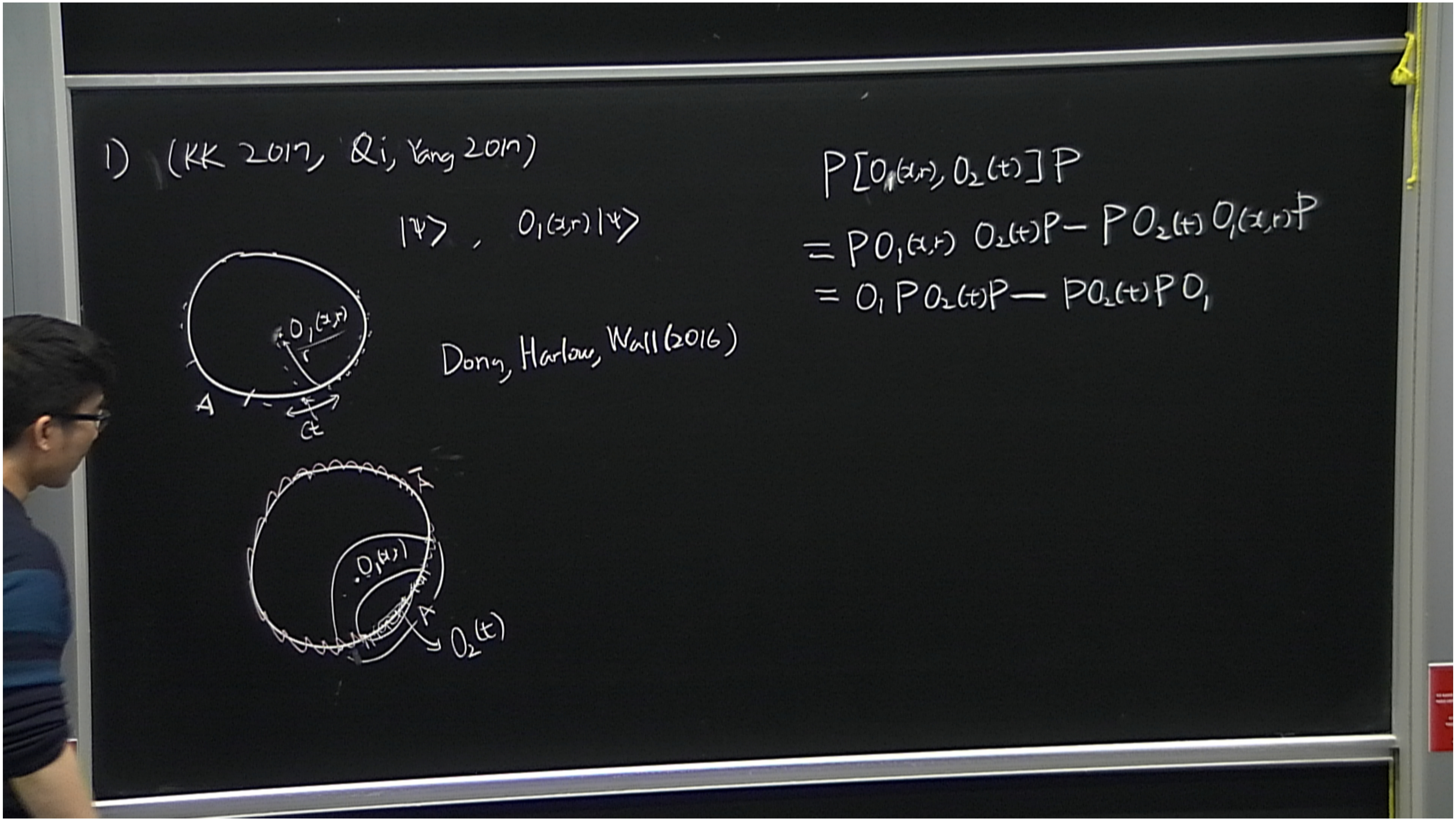
$$\|\mathcal{R}\circ \mathrm{Tr}_A - \mathcal{I}\|_\diamond \le \varepsilon \quad \longleftrightarrow \quad \|\rho^{AR} - \omega^{A}\otimes\rho^{R}\|_1 \le \varepsilon$$

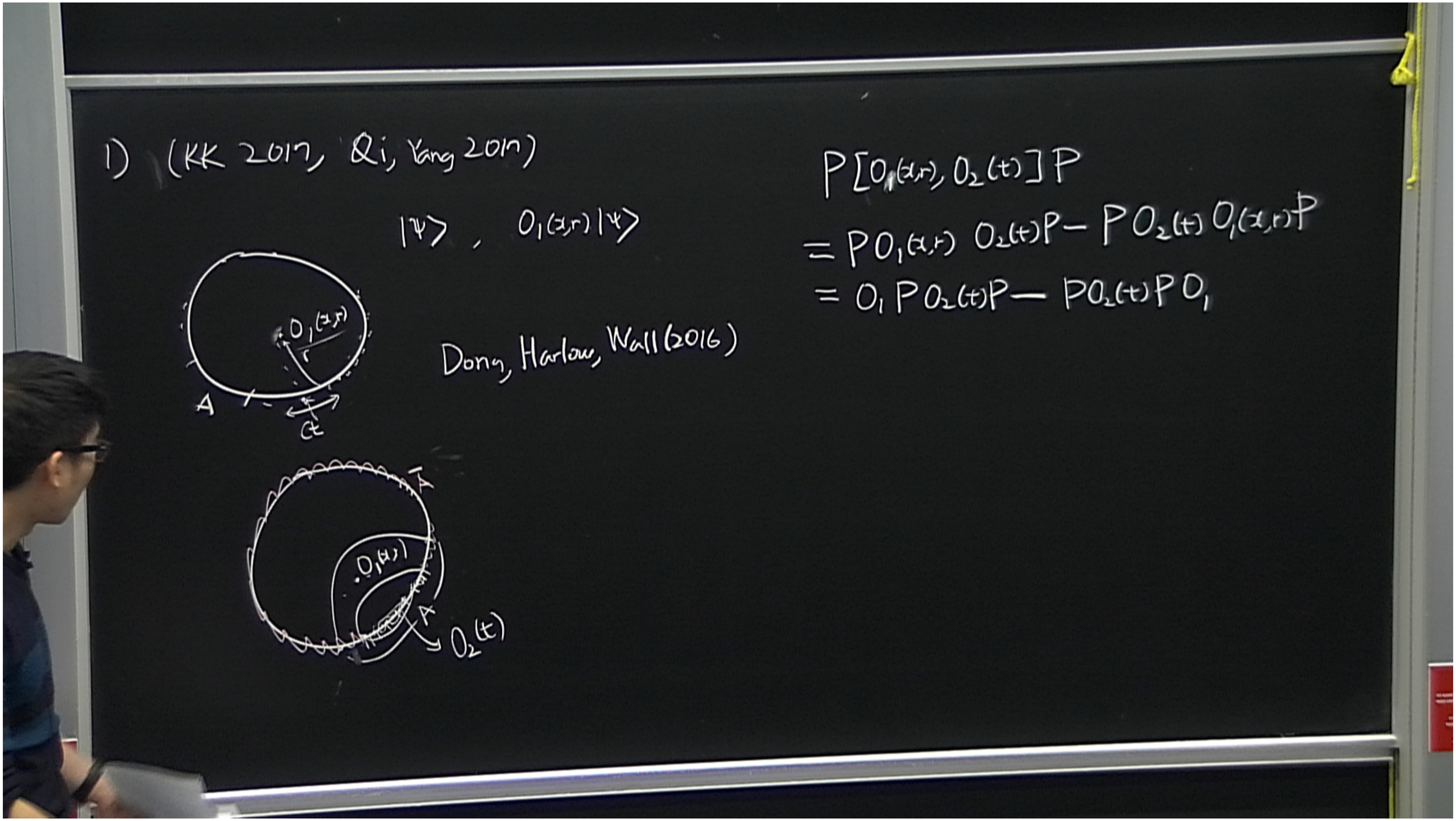
Unpublished

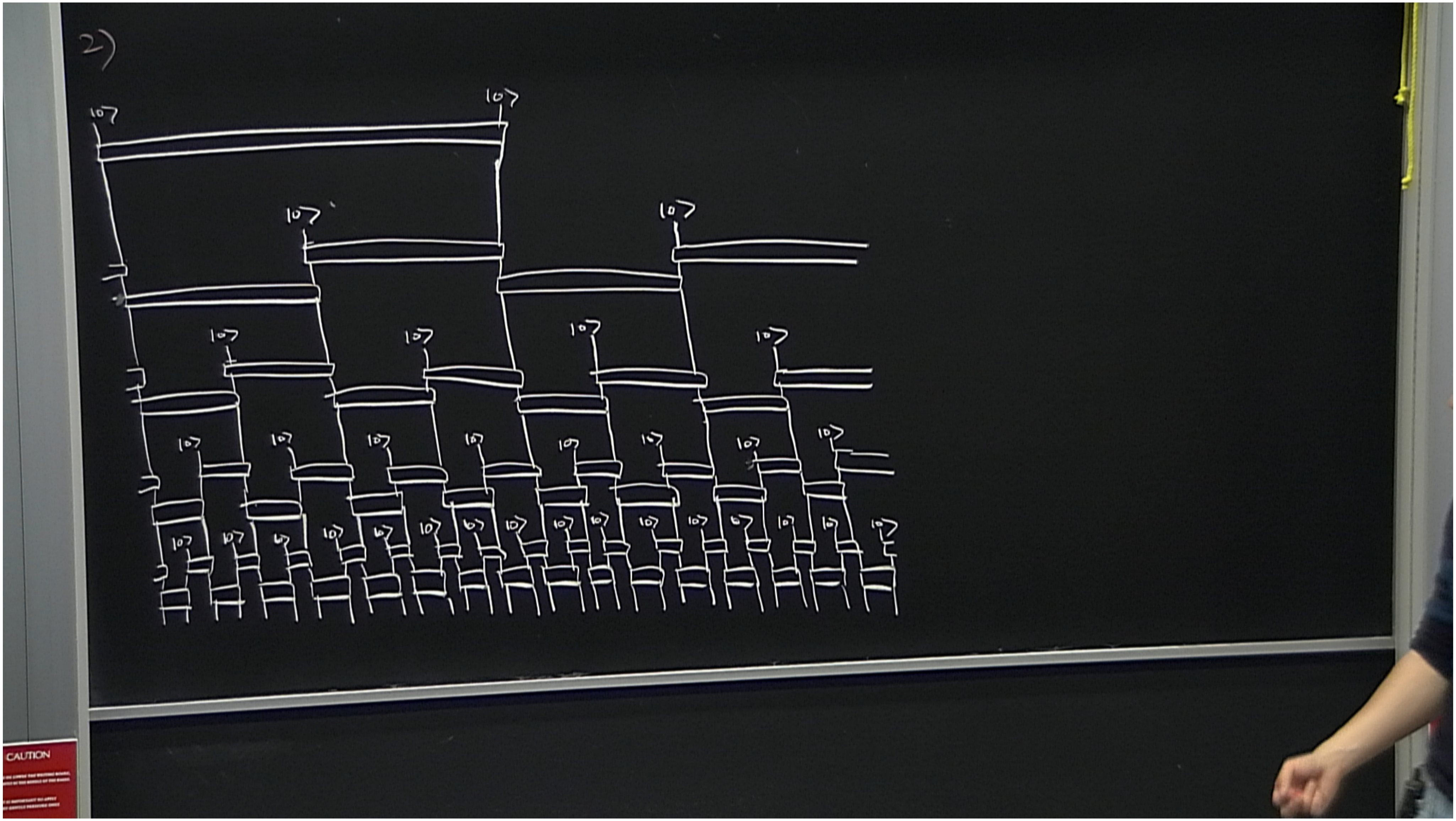
$$\sup_{U_L}\ \inf_{U_{\bar{A}}}\ \|U_{\bar{A}}\circ \mathcal{V} - \mathcal{V}\circ U_L\|_\diamond \le O(\varepsilon)$$

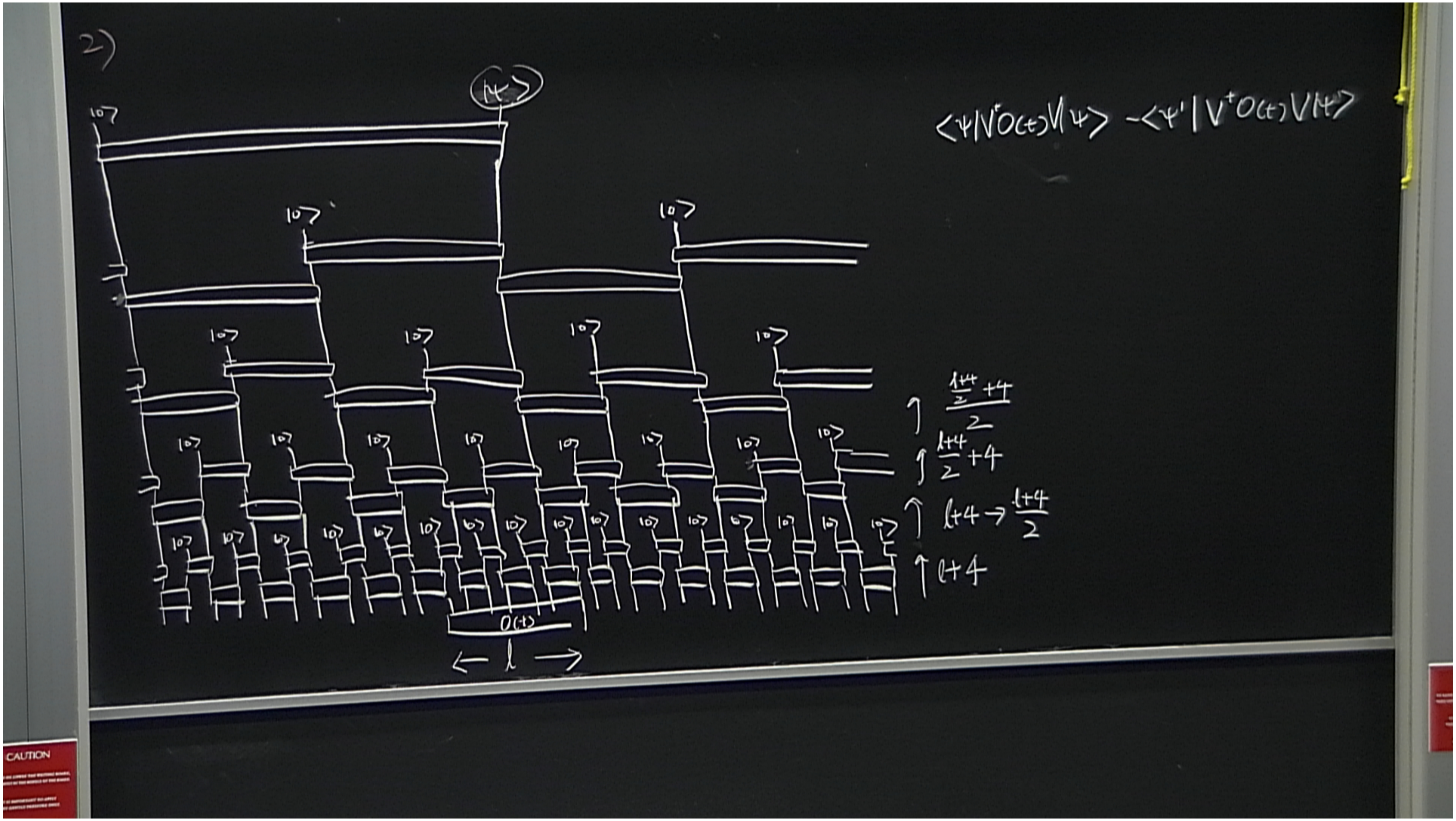
$$\|\Phi - \mathcal{E}\|_1 \le \varepsilon$$

$$\|\Phi - \mathcal{E}\|_\diamond = \Omega(1)$$

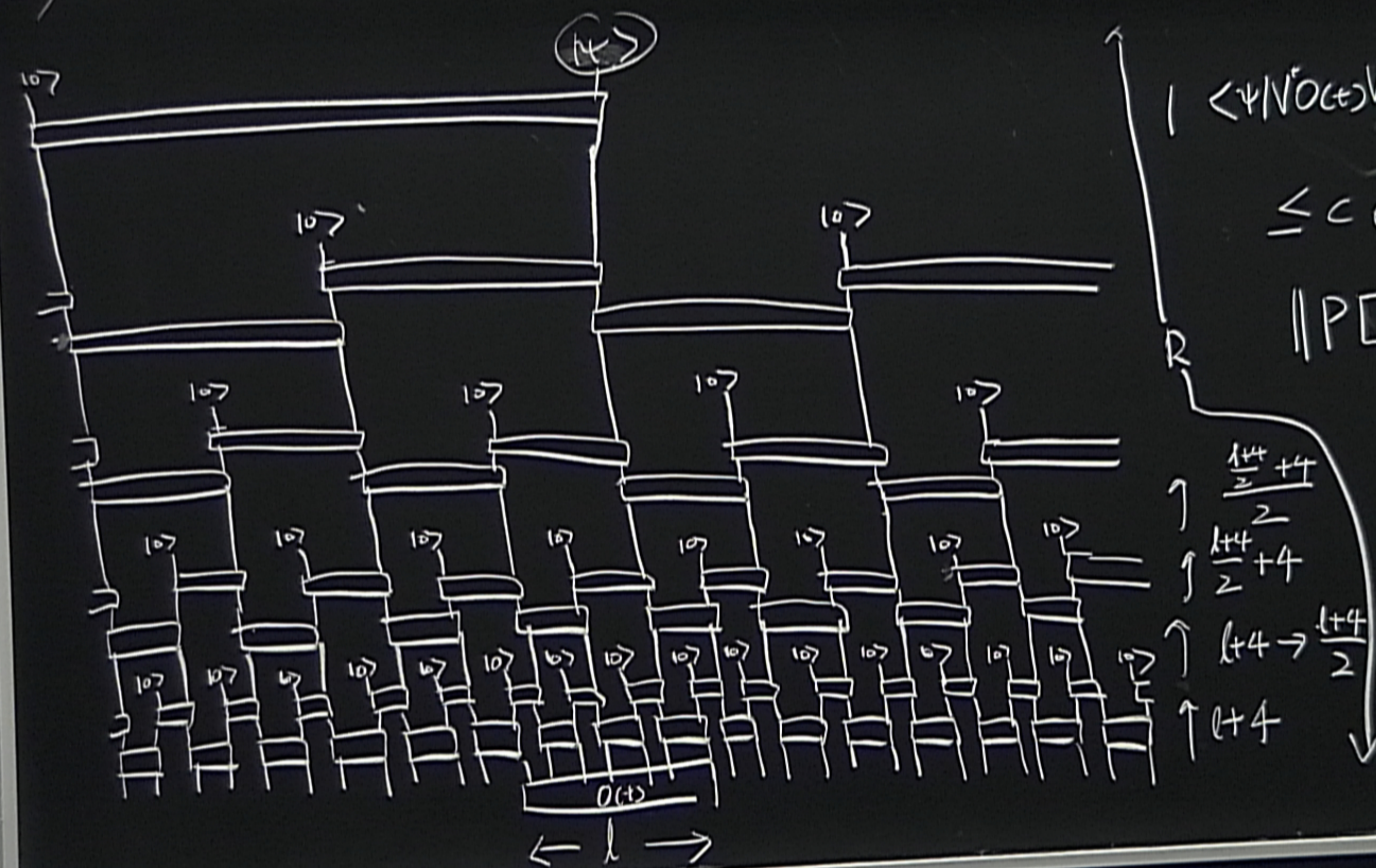

1) (KK 2017, Qi, Yang 2017)
|ψ⟩ , O1(x,r)|ψ⟩
O1(x,r)
r
A
ct
Dong, Harlow, Wall (2016)
A
O1(x,r)
A
O2(t)
P[O1(x,r), O2(t)] P
= P O1(x,r) O2(t) P − P O2(t) O1(x,r) P
= O1 P O2(t) P − P O2(t) P O1


1) (KK 2017, Qi, Yang 2017)
|ψ⟩ , O_1(x,r)|ψ⟩
O_1(x,r)
r
A
ct
Dong, Harlow, Wall (2016)
Ā
O_1(x,r)
A
O_2(t)
P[O_1(x,r), O_2(t)] P
= P O_1(x,r) O_2(t) P − P O_2(t) O_1(x,r) P
= O_1 P O_2(t) P − P O_2(t) P O_1


2)
|0⟩


2)
|0>
|ψ>
<ψ|V†O(t)V|ψ> − <ψ'|V†O(t)V|ψ>
l+4 → (l+4)/2
(l+4)/2 + 4
((l+4)/2 + 4)/2
O(t)
← l →
CAUTION

2)

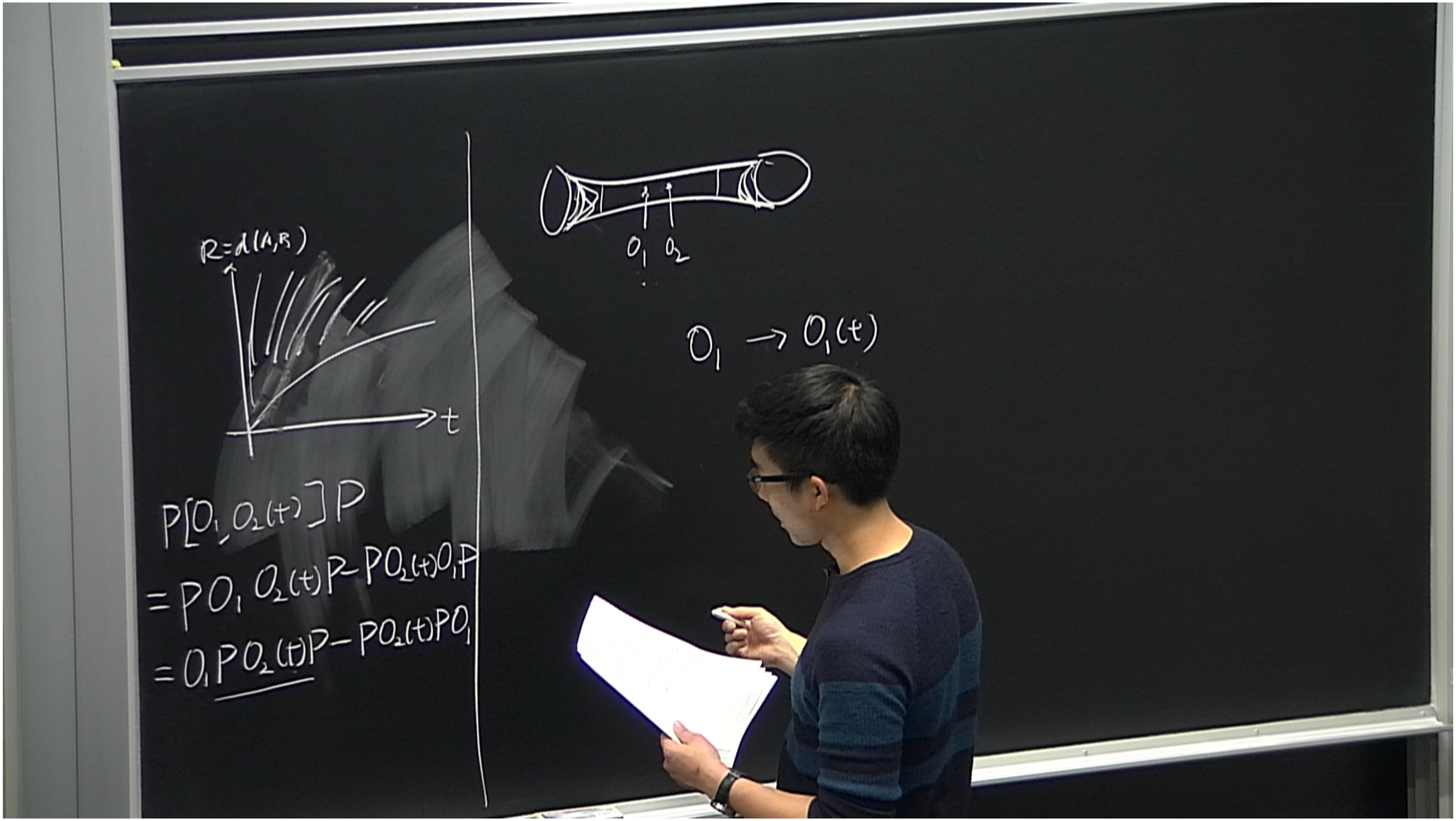


$$\left| \langle\psi|V^\dagger O(t) V|\psi\rangle - \langle\psi'|V^\dagger O(t) V|\psi'\rangle \right| \le c\, e^{-\alpha(R-\log vt)}$$

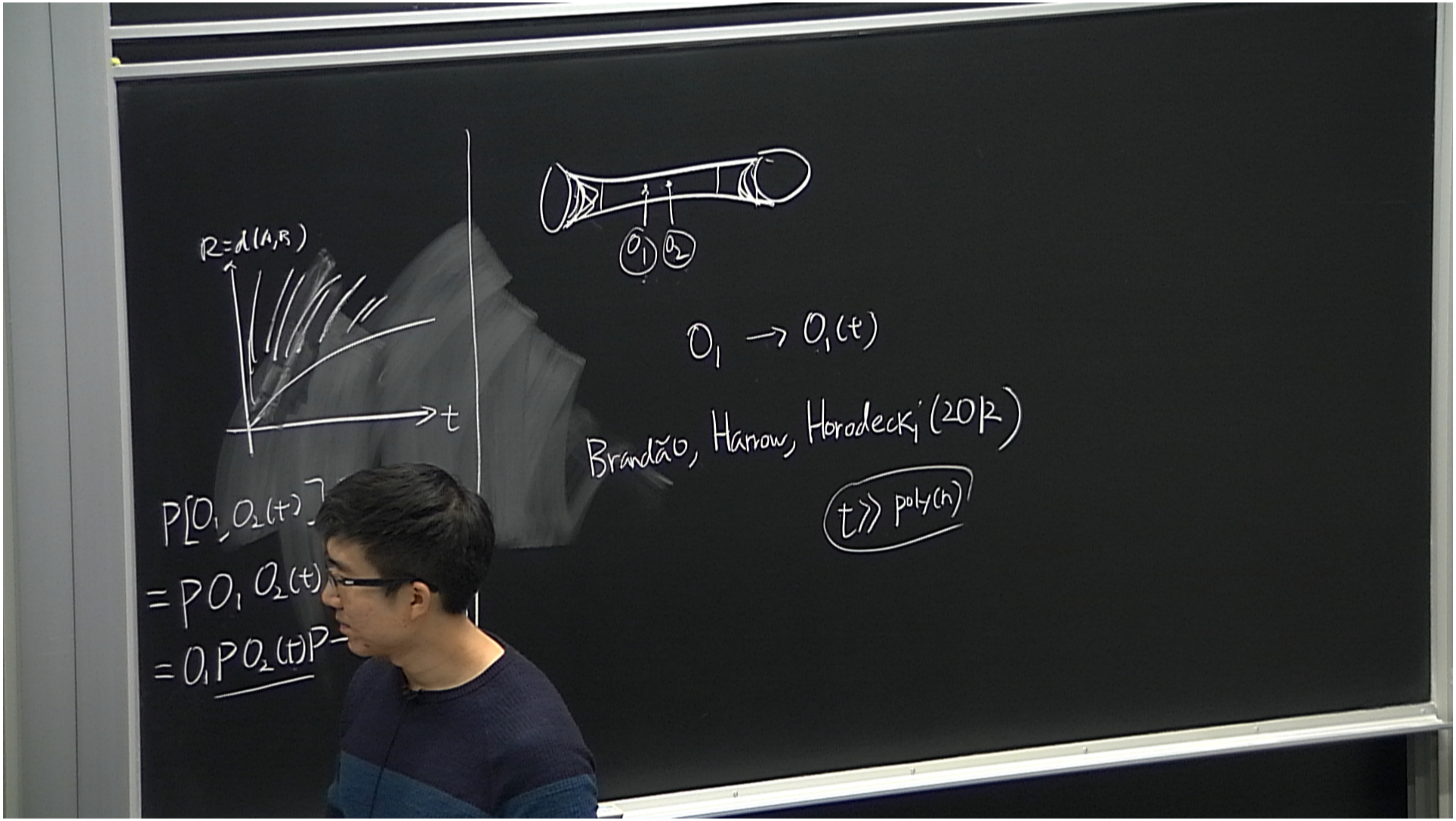
$$\|P[O_1, O_2(t)]P\| \le c\, e^{-\alpha(R-\log vt)} = c(vt)^\alpha e^{-\alpha R}$$

$$O(t) = e^{-iHt}\, O\, e^{iHt}$$

$$U^\dagger H U$$



R=d(A,B)
t
O1 O2
O1 → O1(t)
P[O1, O2(t)]P
=PO1 O2(t)P − PO2(t)O1P
=O1PO2(t)P − PO2(t)PO1


R=d(A,B)
t
O1
O2
O1 → O1(t)
Brandão, Harrow, Horodecki (2012)
t ≫ poly(n)
= P O1 O2(t)
= O1 P O2(t) P