Title: On the New Large D Limit of Matrix Models and Phases of Matrix Quantum Mechanics and SYK Models

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Abstract: <p>In the first part of the talk, I will describe the new large N limit of tensor models, based on the â€œindexâ€• of graphs (in contrast to the standard large N expansion based on the â€œdegreeâ€•), and the associated new large D limit of matrix models. This new limit sheds an interesting light on the relation between disordered models Ã la SYK, tensor models and black holes. In the second part of the talk, I will apply these ideas to discuss the phase diagrams of some strongly coupled matrix quantum mechanics. The phase diagrams display many interesting features, including first and second order phase transitions and quantum critical points. Some of these phase transitions can be argued to provide a quantum mechanical description of the phenomenon of gravitational collapse. </p>

On the New Large D Limit of Matrix Models and Phases of Matrix Quantum Mechanics and SYK Models

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INTERNATIONAL SOLVAY INSTITUTES
Brussels

Quantum Gravity Seminar
Perimeter Institute for Theoretical Physics
Waterloo, May 3rd, 2018

Plan of the talk

1. Big picture

2. The new large N and large D limits of general matrix-tensor and tensor models (degree, index, melons and All That).

(F.F, Vincent Rivasseau and Guillaume Valette, arXiv:1709.07366, CMP; see also FF, arXiv:1701.01171; AFGLV, arXiv: 1710.07263)

3. Phase diagrams

(Tatsuo Azeyanagi, F.F. and Fidel Schaposnik, arXiv: 1707.03431, PRL; F.F. and Fidel Schaposnik, to appear)

Quantum Black Holes

(Unitarity puzzle, quasi-normal behaviour, chaos, horizon physics, BH interior…)

String Theory / D-Branes / Matrix Models /Holography / Matrix models

$$X^{a}_{\ b}$$

Quantum Black Holes

(Unitarity puzzle, quasi-normal behaviour, chaos, horizon physics, BH interior…)

String Theory / D-Branes / Matrix Models /Holography / Matrix models

$X^a_{\ b}$

↓

Quantum Black Holes

(Unitarity puzzle, quasi-normal behaviour, chaos, horizon physics, BH interior…)

↗

Fermionic disordered models

$J_{ijkl}\chi^i\chi^j\chi^k\chi^l$

String Theory / D-Branes / Matrix Models /Holography / Matrix models

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↓

Quantum Black Holes

(Unitarity puzzle, quasi-normal behaviour, chaos, horizon physics, BH interior…)

↗

Fermionic disordered models

$J_{ijkl}\chi^i\chi^j\chi^k\chi^l$

→

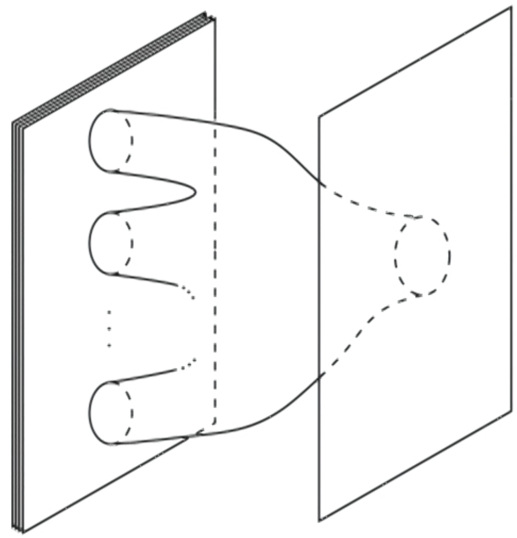
Melonic graphs

Tensor models

t_{abc}

The holographic description of black holes suggests quite generically that quantum black holes should be equivalent to large N, strongly coupled matrix quantum mechanical systems.

The typical set-up involves branes:



$$X^i_{\mu\, j}\,, \quad 1 \le \mu \le D = d - p - 1$$

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The index mu transforms in O(D) and is associated with the rotation transverse to the branes. Idea: take the large D limit and make the link with tensors (Ferrari 2017).

However, the large D limit cannot be vector-like, which is too trivial… Something new must happen here.

$$F_g = \sum_{\ell \ge 0} D^{1+g-\ell/2} F_{g,\ell}$$

$$F = \sum_{g \ge 0} N^{2-2g} F_g$$

There is also an interesting relationship with the large space-time dimension limit studied by Emparan et al.

$$L = ND\Big(\mathrm{tr}\big(\dot{X}^\dagger_\mu \dot{X}_\mu + m^2 X^\dagger_\mu X_\mu\big) - \sum_B t_B I_B(X)\Big)$$

Instead of taking the large D limit at fixed t_B, can we imagine a new enhanced scaling of the form

$$t_B = D^{g(B)} \lambda_B$$

Naively this seems impossible. If one enhances a coupling, diagrams containing a large number of the associated vertices will have an arbitrary high power of D. If the highest power of D is not bounded above, the large D limit cannot exist.

In other words, the 't Hooft's scalings is delicate. The typical situation is that, if the couplings are diminished, the limit becomes trivial; and if they are enhanced, the limit does not exist anymore.

But it turns out that a remarkable property holds: the powers of D and N in a diagram are not independent.

Intuition: the power of D is related to the number of O(D) loops in the diagram. The power of N is related to the genus of the surface on which the diagram can be drawn. But on a surface of a given genus, there is a constrain on the number of loops one can draw.

I will provide a nice proof tomorrow on the black board.

Result:

$$t_B = D^{g(B)} \lambda_B$$

Examples: $\mathrm{tr}\, X_\mu X_\mu X_\nu X_\nu$ genus zero: no enhancement

$\mathrm{tr}\, X_\mu X_\nu X_\mu X_\nu$ genus 1/2: enhancement

$\mathrm{tr}\, X_\mu X_\rho X_\mu X_\nu X_\rho X_\nu$ genus 1: enhancement

With this rule, one shows that the highest power of D of a planar diagram is D (the highest power of D at genus g is 1+g)

Standard large N approximation:

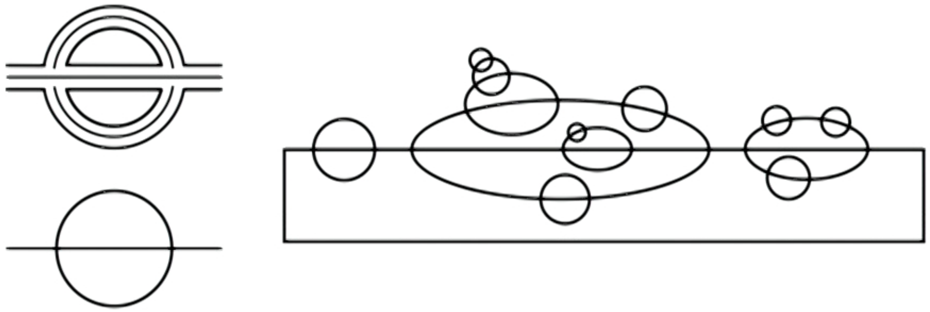
$$F = \sum_{g \geq 0} N^{2-2g} F_g$$

New large D approximation at fixed genus:

$$F_g = \sum_{\ell \geq 0} D^{1+g-\ell/2} F_{g,\ell}$$

The new large D and standard large N limits do not commute with each other.

The new large D limit yields a $1/\sqrt{D}$ expansion of Feynman diagram of fixed genus g.



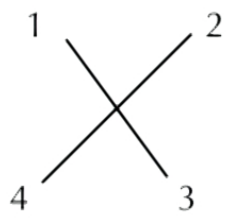
We are getting a new approximation for the sum over planar diagrams.

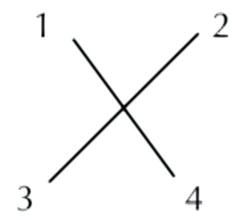
The BIG thing is that it seems that this new approximation, albeit quite simple, does capture correctly the qualitative physics associated with the full sum over planar diagrams!

Important graph theoretic ideas, I

A graph is just a set of vertices and edges relating the vertices. When you draw a graph on a piece of paper (surface), you actually provide more data:

1) you choose a particular cyclic ordering of the edges around the vertices.



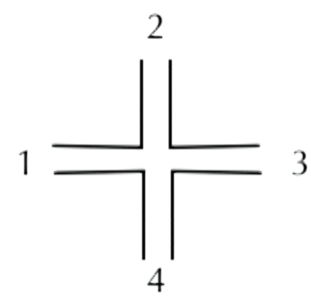


This additional information is irrelevant for ordinary field theories

$$\phi_1\phi_2\phi_3\phi_4 = \phi_1\phi_2\phi_4\phi_3$$

But it is relevant in the case of matrix models

$$\mathrm{tr}\, X_1X_2X_3X_4 \neq \mathrm{tr}\, X_1X_2X_4X_3$$



Important graph theoretic ideas, I

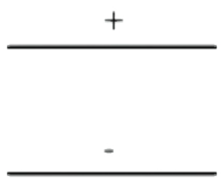
2) you choose a particular type (untwisted or twisted) for the edges

$\delta^a_d \delta^c_b$ Hermitian

$\delta_{ad}\delta_{bc} + \delta_{ac}\delta_{bd}$ Real symmetric

Important graph theoretic ideas, I

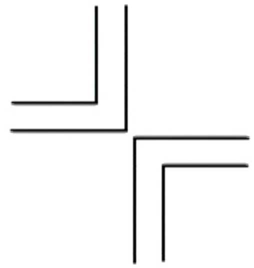
2) you choose a particular type (untwisted or twisted) for the edges



	$\delta^a_d \delta^c_b$	Hermitian
	$\delta_{ad}\delta_{bc} + \delta_{ac}\delta_{bd}$	Real symmetric

Important matrix model idea

$$S = N\Big(\operatorname{tr} X^2 + \sum_a N^{1-t_a}\tau_a I_a(X)\Big)$$



$$\operatorname{tr} X^2 \operatorname{tr} X^2$$

$$F = \sum_{h\geq 0} N^{2-2h} F_h$$

$$h = g + 1 - B + \sum_a (t_a - 1)$$

B is the number of connected components of the ribbon graph

$$g \geq 0$$

$$1 - B + \sum_a (t_a - 1) = \sum_a t_a - B - \sum_a 1 + 1 = \mathscr{L} \geq 0$$

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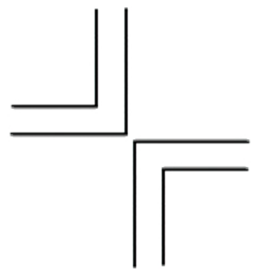

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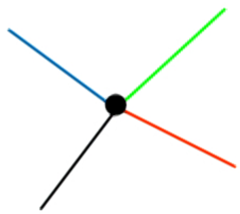
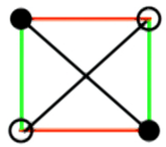
Important graph theoretic ideas, II

A d-bubble, or d-colored graph, is a regular graph with colored edges such that the d edges incident to any given vertex carry all the d possible colors.



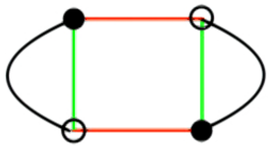
t_{abcd}

Interaction vertices for tensors of rank R are R-bubbles



Non-melonic
Non-bipartite

$\operatorname{tr} X_\mu X_\nu^\dagger X_\mu X_\nu^\dagger$

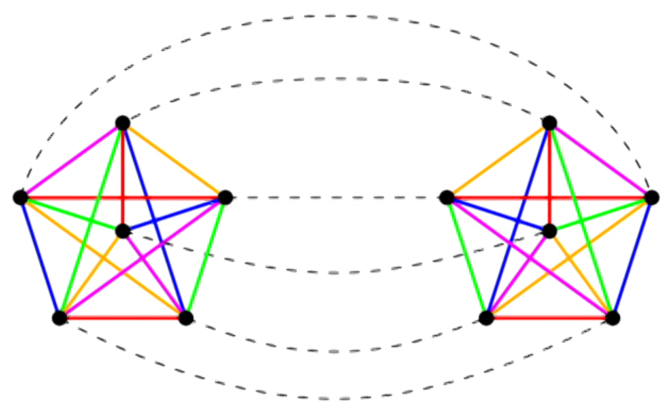


melonic
Bipartite

$\operatorname{tr} X_\mu X_\mu^\dagger X_\nu X_\nu^\dagger$

Important graph theoretic ideas, II

Feynman graphs for tensors of rank R are (R+1)-bubbles



Note that if you remove p colors from a d-bubble, you get a (d-p)-bubble.

Important graph theoretic ideas, II

There are many ways to draw a colored graph on a surface. Pick a cyclic ordering of the colors, pick a vertex type for each vertex and decide that the edges joining vertices of different types are untwisted. This defines a jacket.

There are now two distinct ways to naturally associate a positive number to any colored graph.

Important graph theoretic ideas, II

1) Consider all the possible ways to draw the graph on a surface (i.e. all the possible jackets) and sum of all the genera of these surfaces. This defines the degree (Gurau)

$$\deg \mathcal{B} = \frac{1}{2} \sum_{\text{cycles } \sigma \in S_d} g(\mathcal{B}; \sigma)$$

Graphs of degree zero are called melons; they are "superplanar".

2)Pick a particular color (0, propagators) and consider all the possible matrix model graphs embedded in the colored graph, by keeping the color 0 and any two other colors i and j ("matrix indices"). This graph is proportional to

$$N^{2-2h_{ij}} \qquad h_{ij} = g_{ij} + 1 - B_{ij} + \sum_a (t_{a,ij} - 1)$$

The sum of all the integers h associated to all the matrix model graphs one can build from the colored graph is the index (FRV)

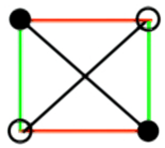
$$\mathrm{ind}_0\, \mathcal{B} = \frac{1}{2} \sum_{i<j} h_{ij}$$

Scalings

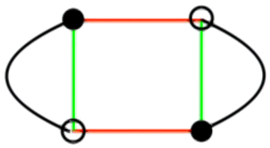
$$S = ND^r\Big(\mathrm{tr}\, X_{\mu_1\cdots\mu_r} X^T_{\mu_1\cdots\mu_r} + \sum_a N^{1-t(\mathcal{B}_a)}\tau_a I_{\mathcal{B}_a}(X)\Big)$$

Standard Bonzom-Gurau-Rivasseau scaling, suitably generalized to the non-bipartite and matrix/tensor case:

$$\tau_a = D^{t(\mathcal{B}_a)-c(\mathcal{B}_a)-\frac{2}{r!}\deg\mathcal{B}_a}\mu_a$$



$D^{-1}\,\mathrm{tr}\, X_\mu X_\nu^\dagger X_\mu X_\nu^\dagger$



$\mathrm{tr}\, X_\mu X_\mu^\dagger X_\nu X_\nu^\dagger$

Feynman graphs are proportional to $N^{2-h} D^{r+h-L}$

$$L = \frac{2}{(r+1)!}\deg\mathcal{B} + (r+2)\Big[\sum_a\big(c(\mathcal{B}_a)-1\big) - B + 1\Big]$$

D=N: the expansion is governed by the degree.

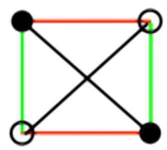
Only melonic interactions can contribute at leading order.

This limit is "vector model" like; not interesting.

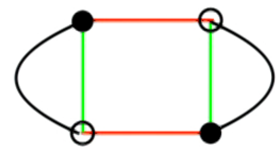
Scalings $S = N D^r \left(\mathrm{tr}\, X_{\mu_1\cdots\mu_r} X^T_{\mu_1\cdots\mu_r} + \sum_a N^{1-t(\mathcal{B}_a)} \tau_a I_{\mathcal{B}_a}(X) \right)$

$$\tau_a = D^{t(\mathcal{B}_a)-c(\mathcal{B}_a)-\frac{2}{r!}\deg\mathcal{B}_a}\mu_a$$

New scaling (FRV) $\boxed{\tau_a = D^{t(\mathcal{B}_a)-c(\mathcal{B}_a)+\frac{2}{(r+1)!}\deg\mathcal{B}_a}\lambda_a}$ Enhancement!



$\sqrt{D}\ \ \mathrm{tr}\, X_\mu X^\dagger_\nu X_\mu X^\dagger_\nu$



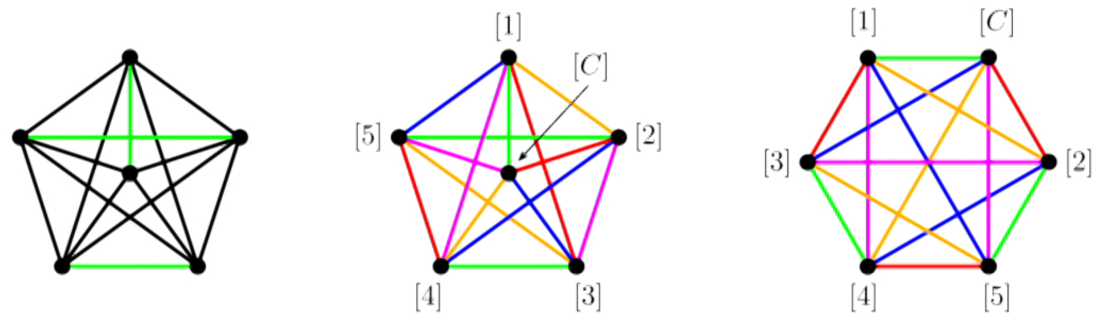
$\mathrm{tr}\, X_\mu X^\dagger_\mu X_\nu X^\dagger_\nu$

Feynman graphs are proportional to $N^{2-h} D^{r+h-\frac{\ell}{r+1}}$

$$\ell = 2\,\mathrm{ind}_0\,\mathcal{B} + (r+1)(r+2)\Big[\sum_a \big(c(\mathcal{B}_a)-1\big) - B + 1\Big]$$

The expansion is governed by the index. Many more graphs contribute, in particular non-melonic interactions can contribute at leading orders. This is the expansion we need for SYK-like physics.

Maximally single trace interactions are interactions that are single-trace with respect to all the possible matrix models you can build from the tensor models.

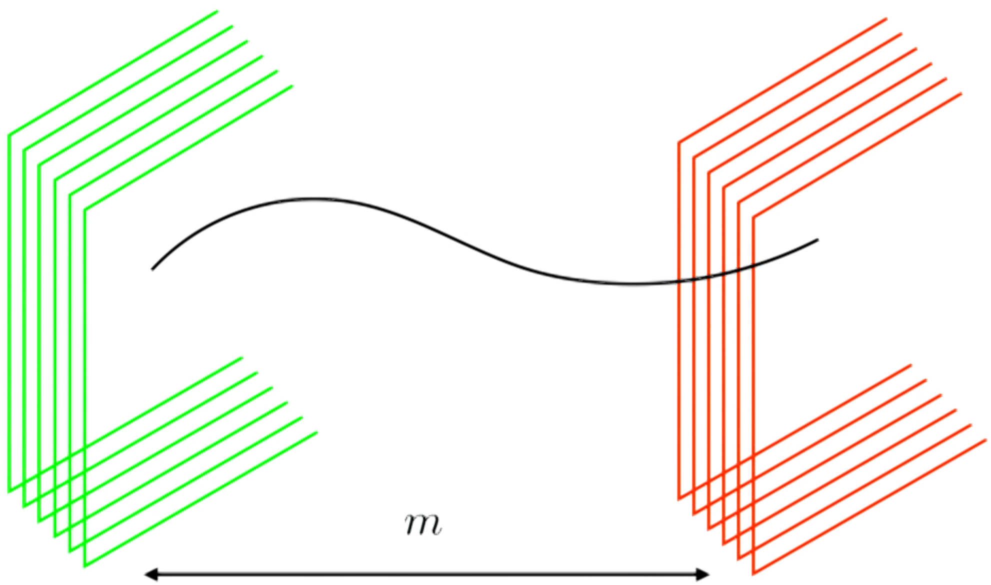


This is a new fascinating class of models which are extremely interesting to study. Deriving the form of the generalized melons (index zero graphs) for these cases in an outstanding open problem. We have solved it for the complete colored graph when R is a prime number (in which case it is MST).

A model:

$$H = ND\,\mathrm{tr}\Big(M\psi_\mu^\dagger\psi_\mu + \frac{1}{2}\Lambda\sqrt{D}\,\psi_\mu\psi_\nu^\dagger\psi_\mu\psi_\nu^\dagger\Big)$$

$$\left\{\psi^a_{\mu\,b}, (\psi^\dagger_\nu)^c{}_d\right\} = \frac{1}{ND}\delta_{\mu\nu}\delta^a_d\delta^c_b$$



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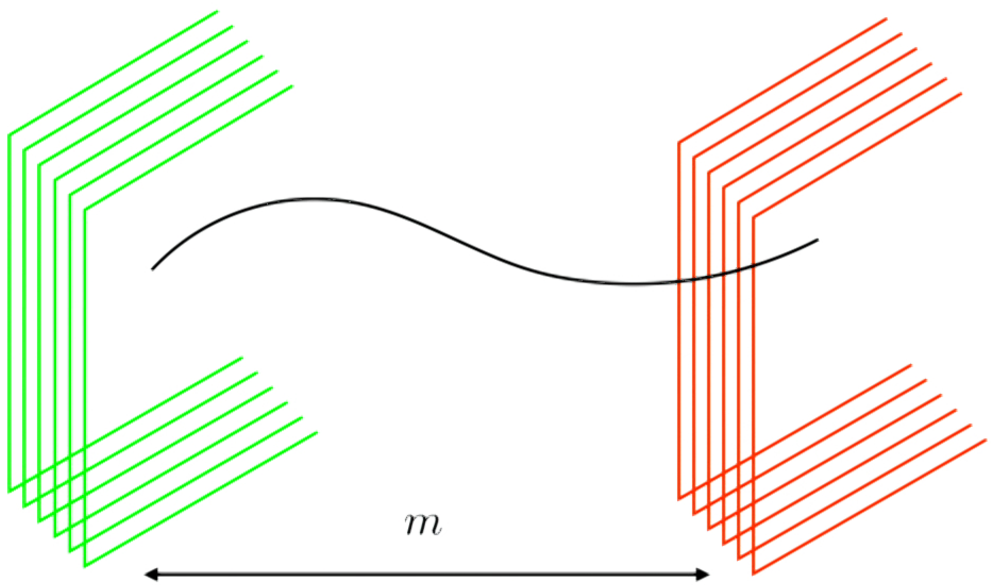

Parameters:

$$M\,, \quad \Lambda\,, \quad T = 1/\beta$$

$$m = M/\Lambda\,, \quad t = T/\Lambda$$

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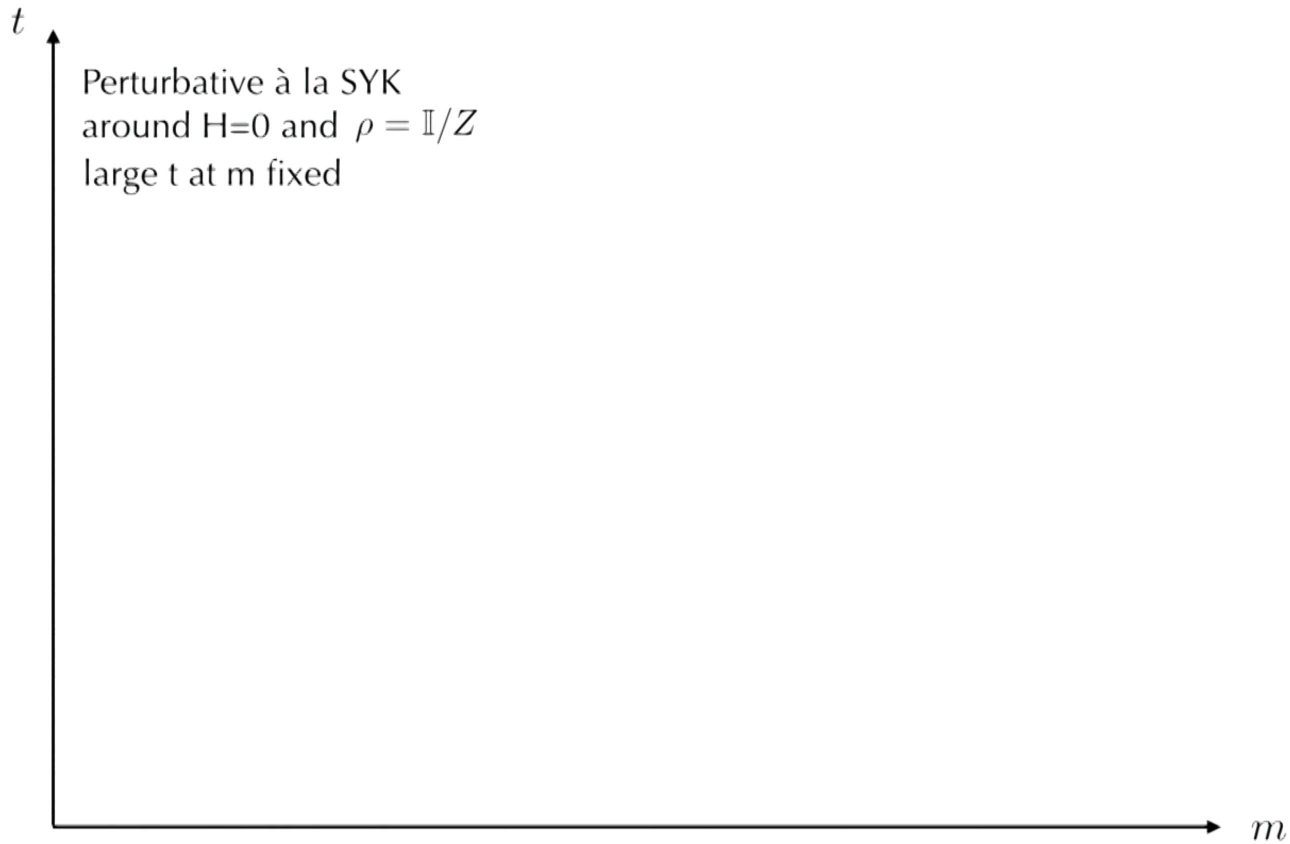

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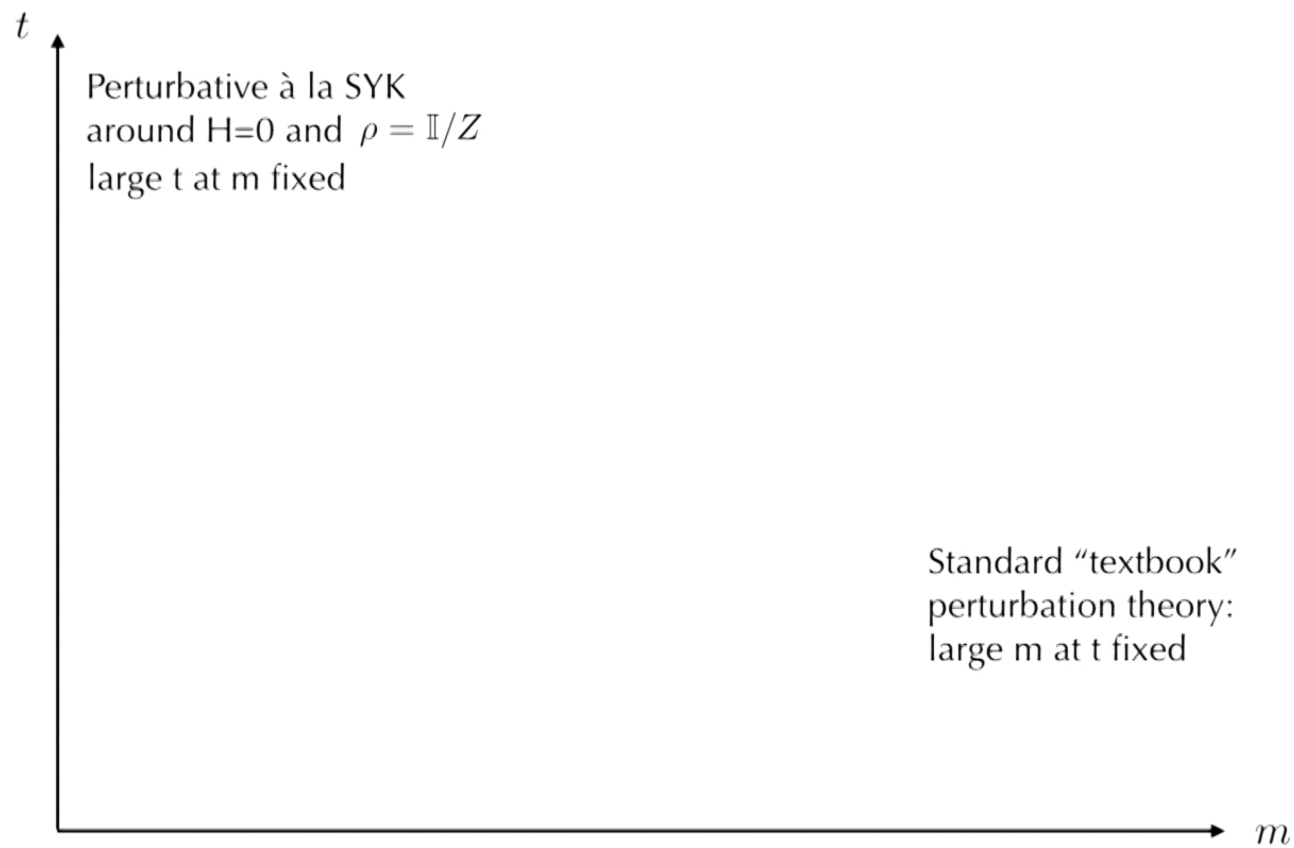

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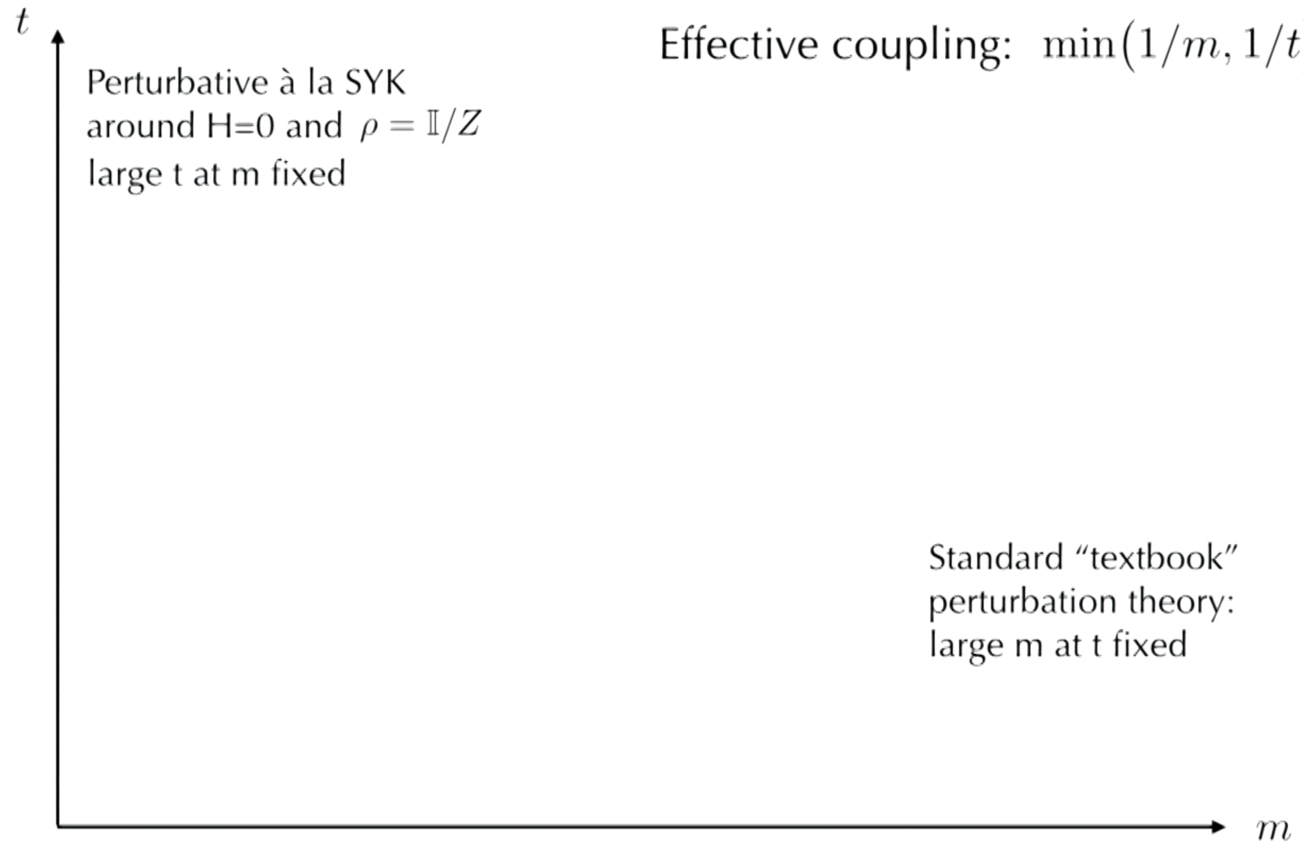
Effective coupling: $\min\big(1/m, 1/t\big)$



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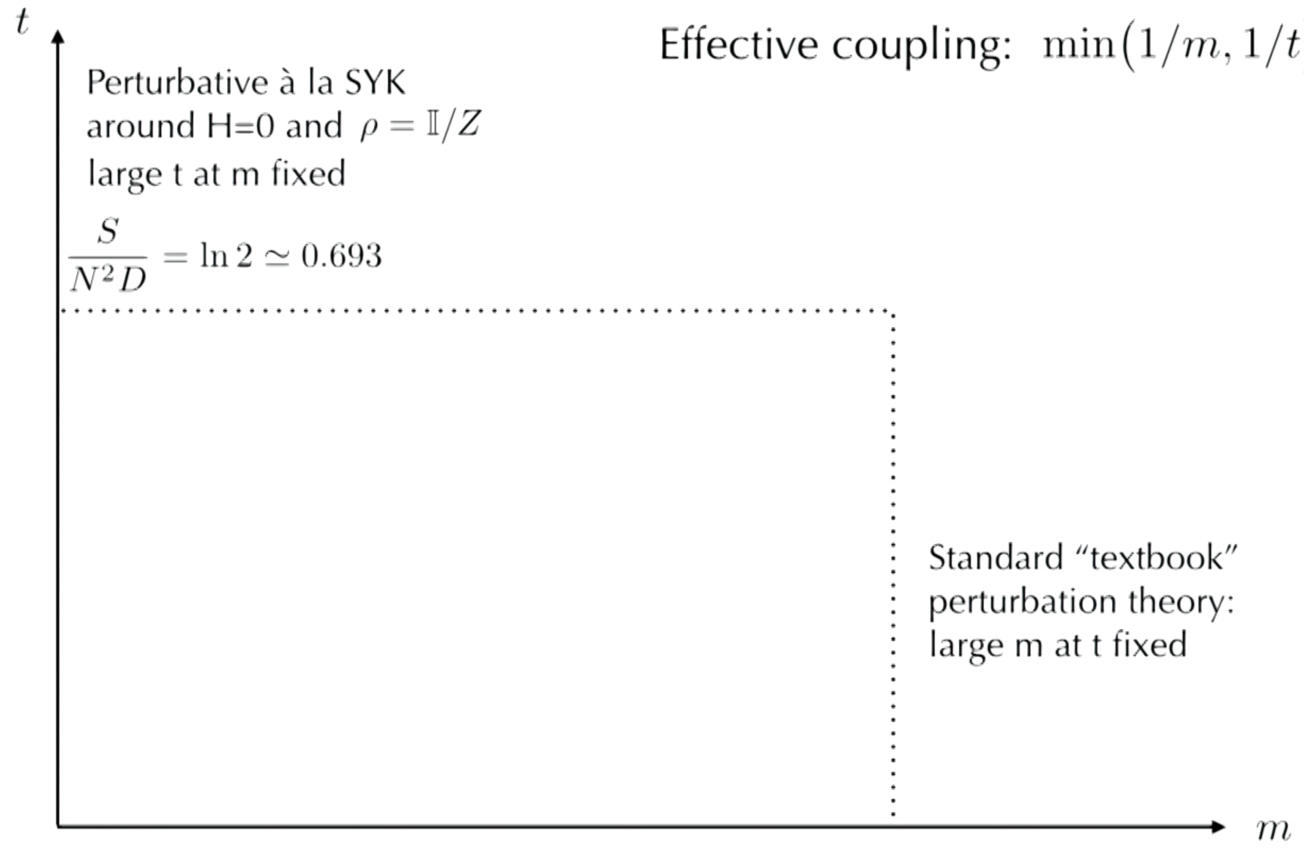

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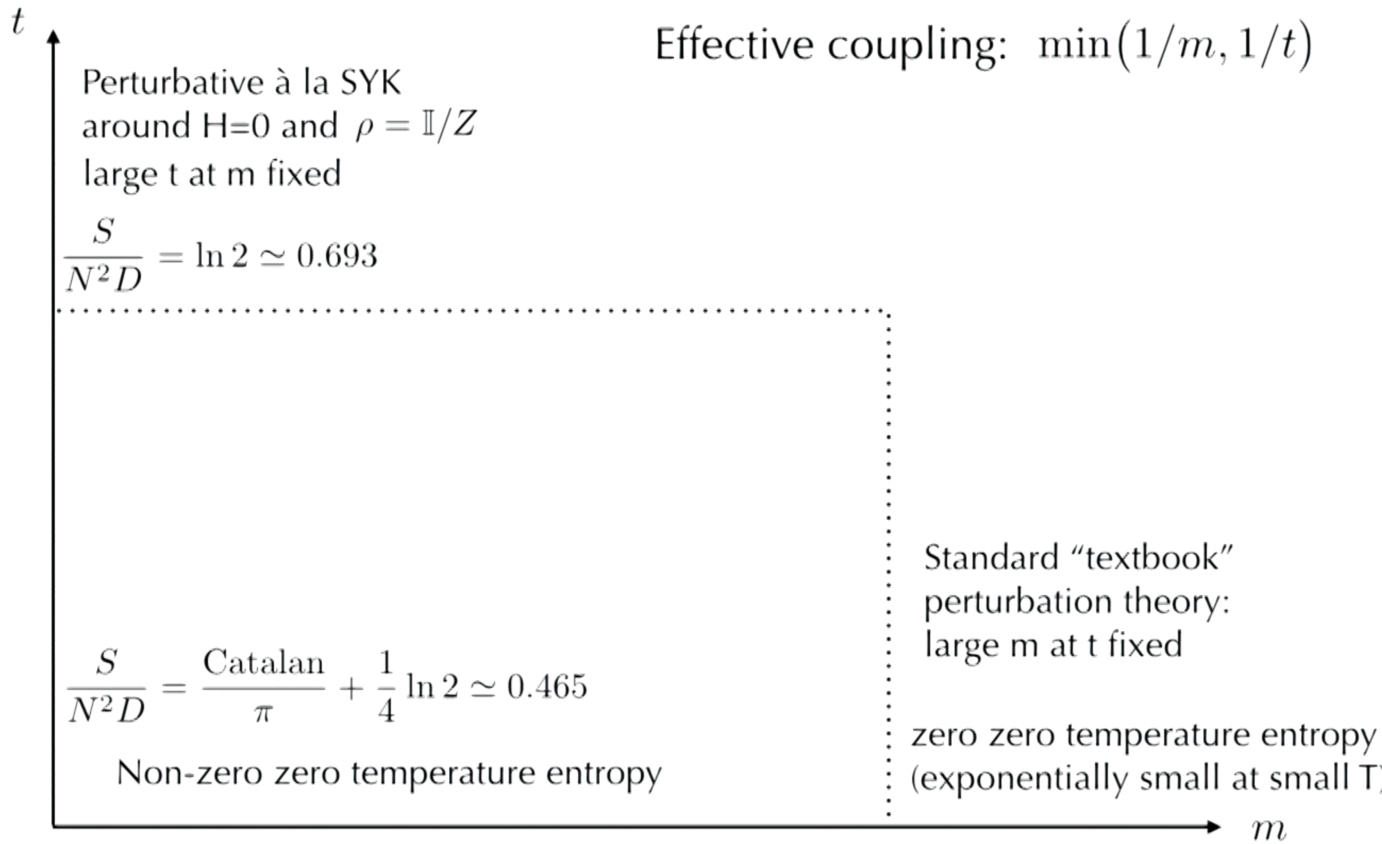
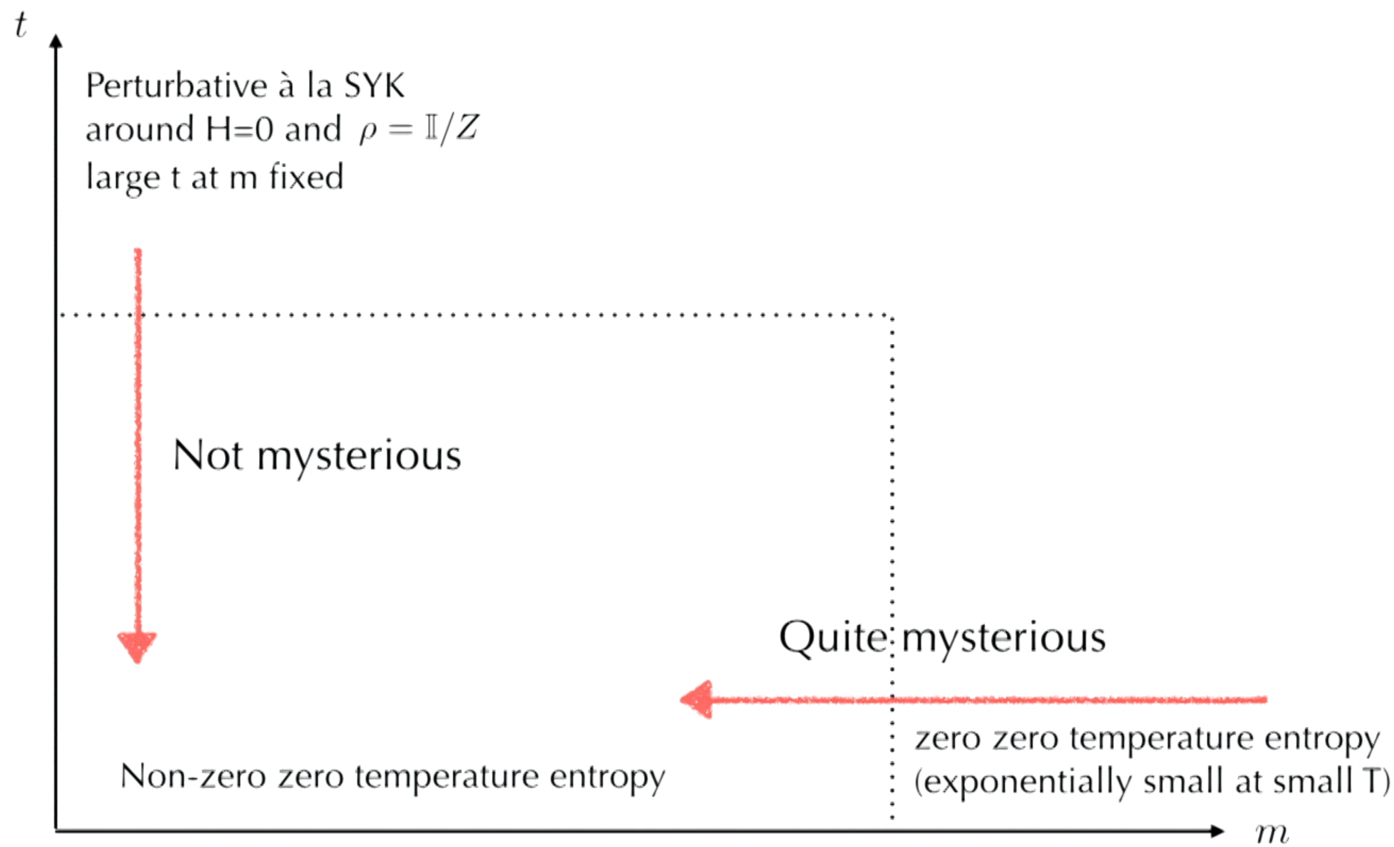
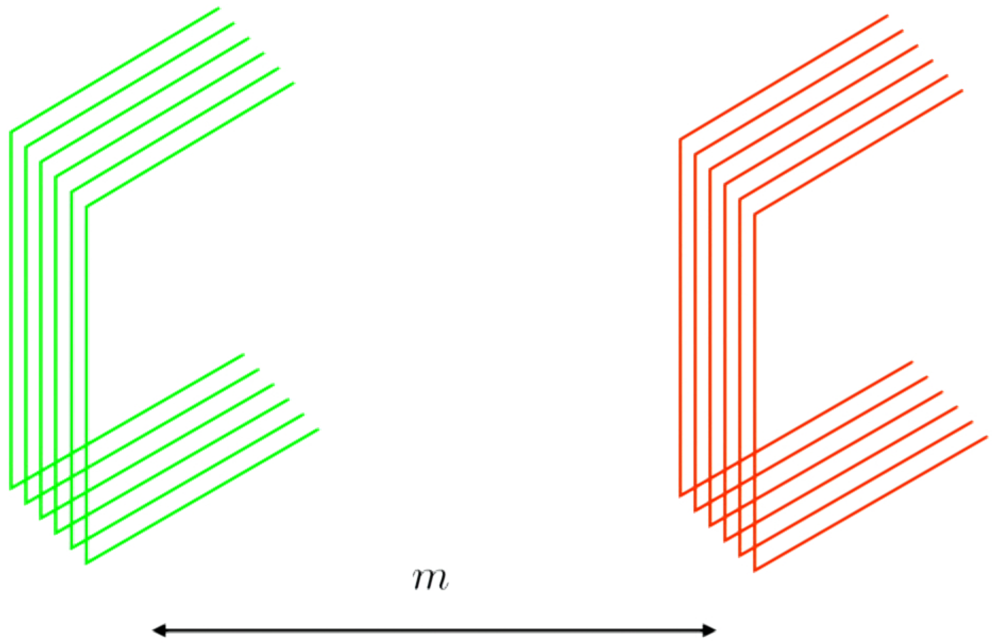
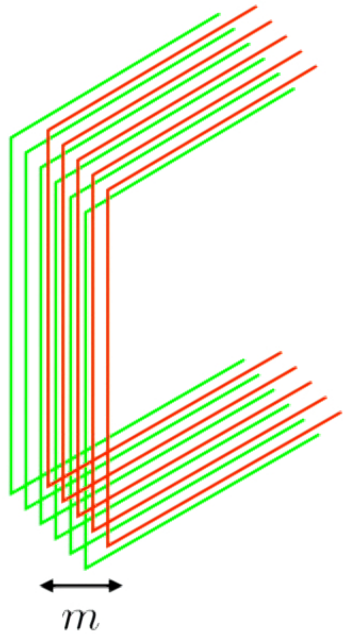


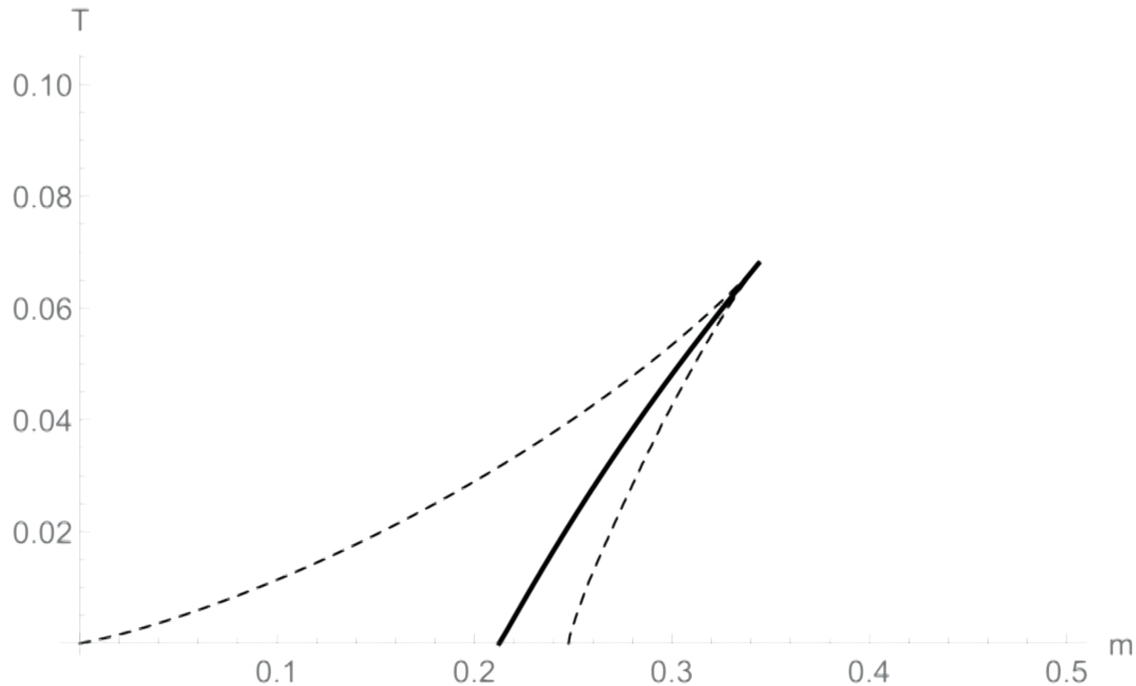


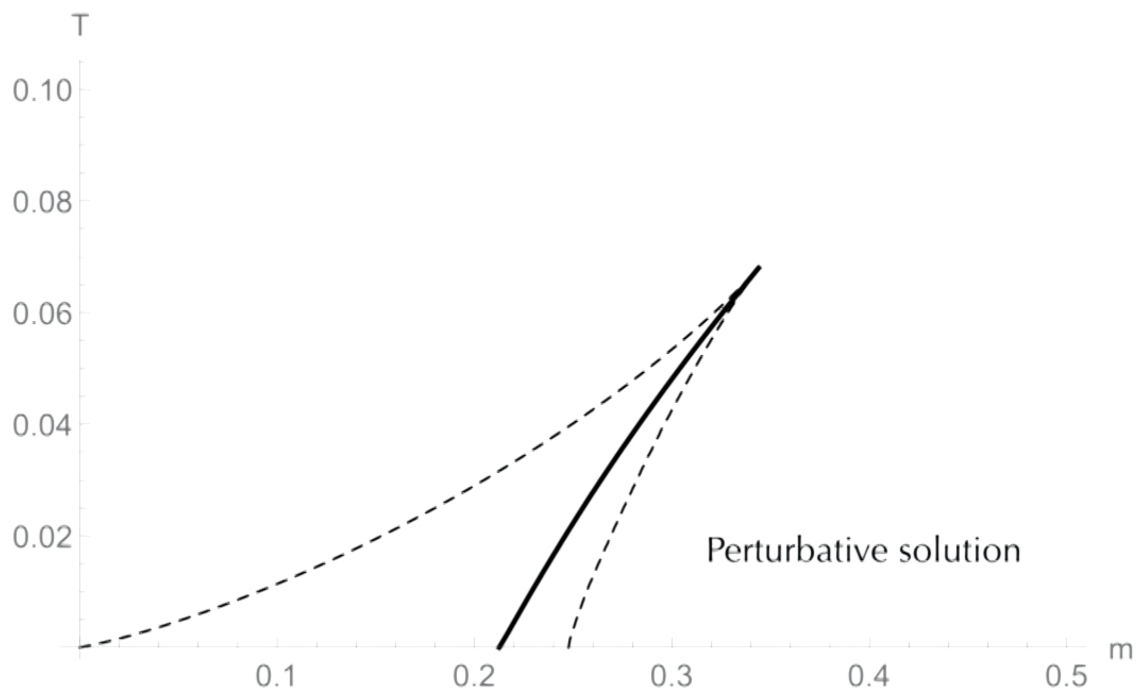
t
Perturbative à la SYK
around H=0 and $\rho = \mathbb{I}/Z$
large t at m fixed
Not mysterious
Quite mysterious
zero zero temperature entropy
(exponentially small at small T)
Non-zero zero temperature entropy
m

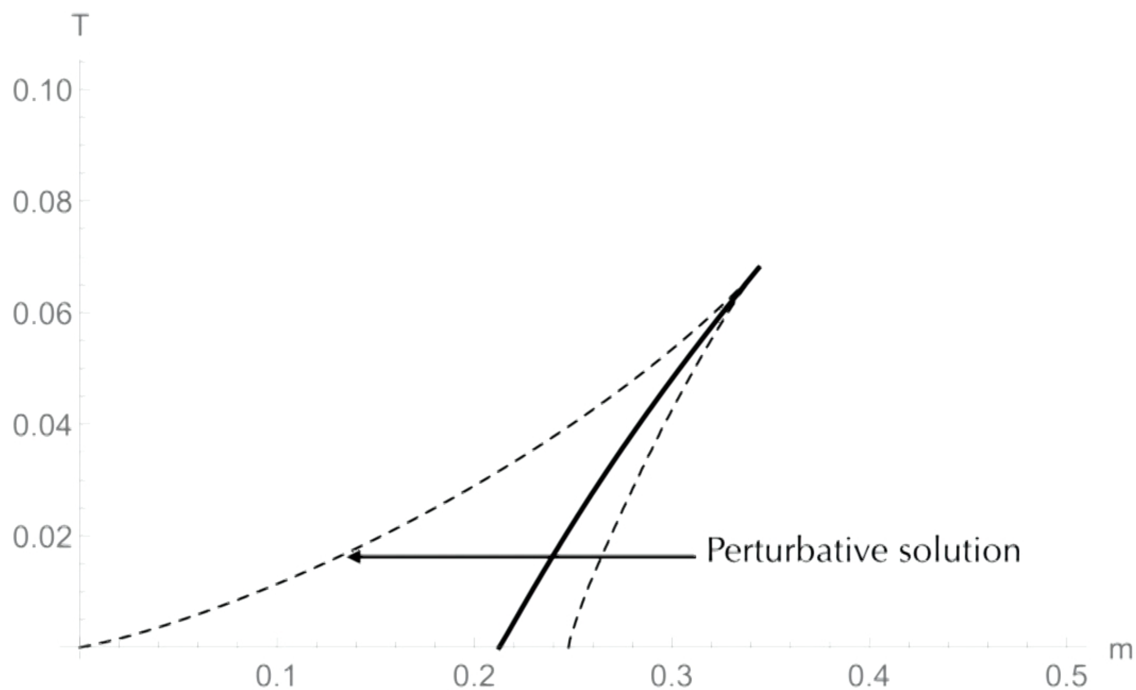

m



We may expect that, below some critical distance, the stack of branes gravitationally collapse to a black hole…


T
0.10
0.08
0.06
0.04
0.02
0.1
0.2
0.3
0.4
0.5
m


T
0.10
0.08
0.06
0.04
0.02
Perturbative solution
0.1
0.2
0.3
0.4
0.5
m


T
0.10
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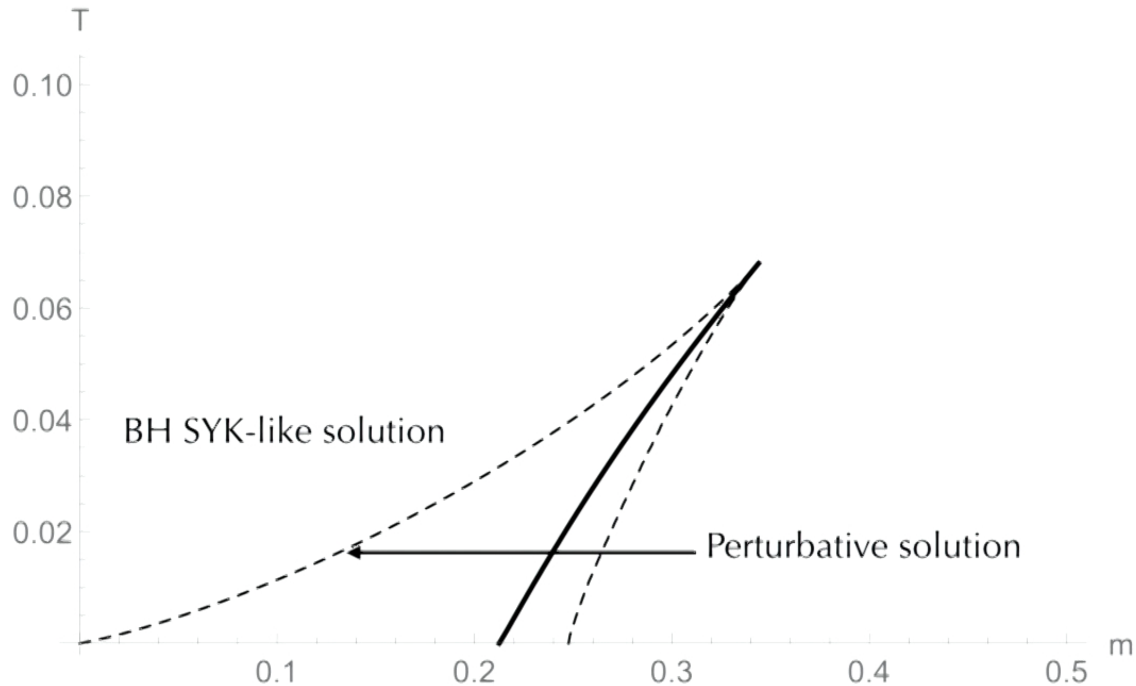
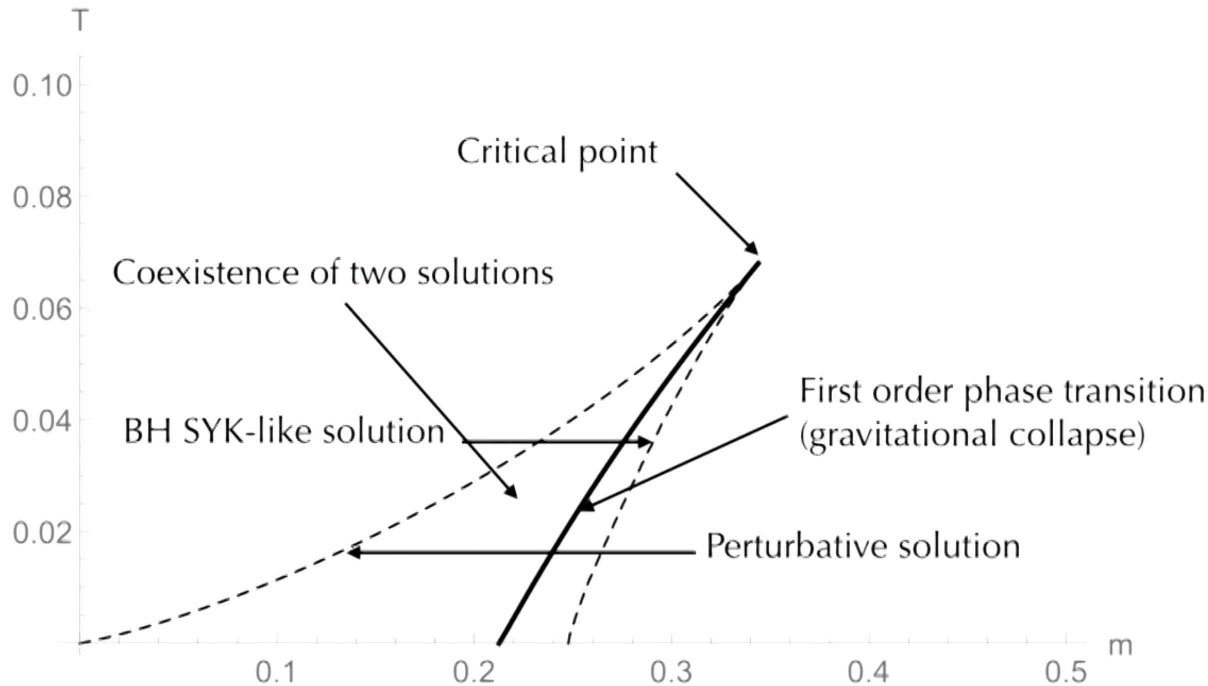
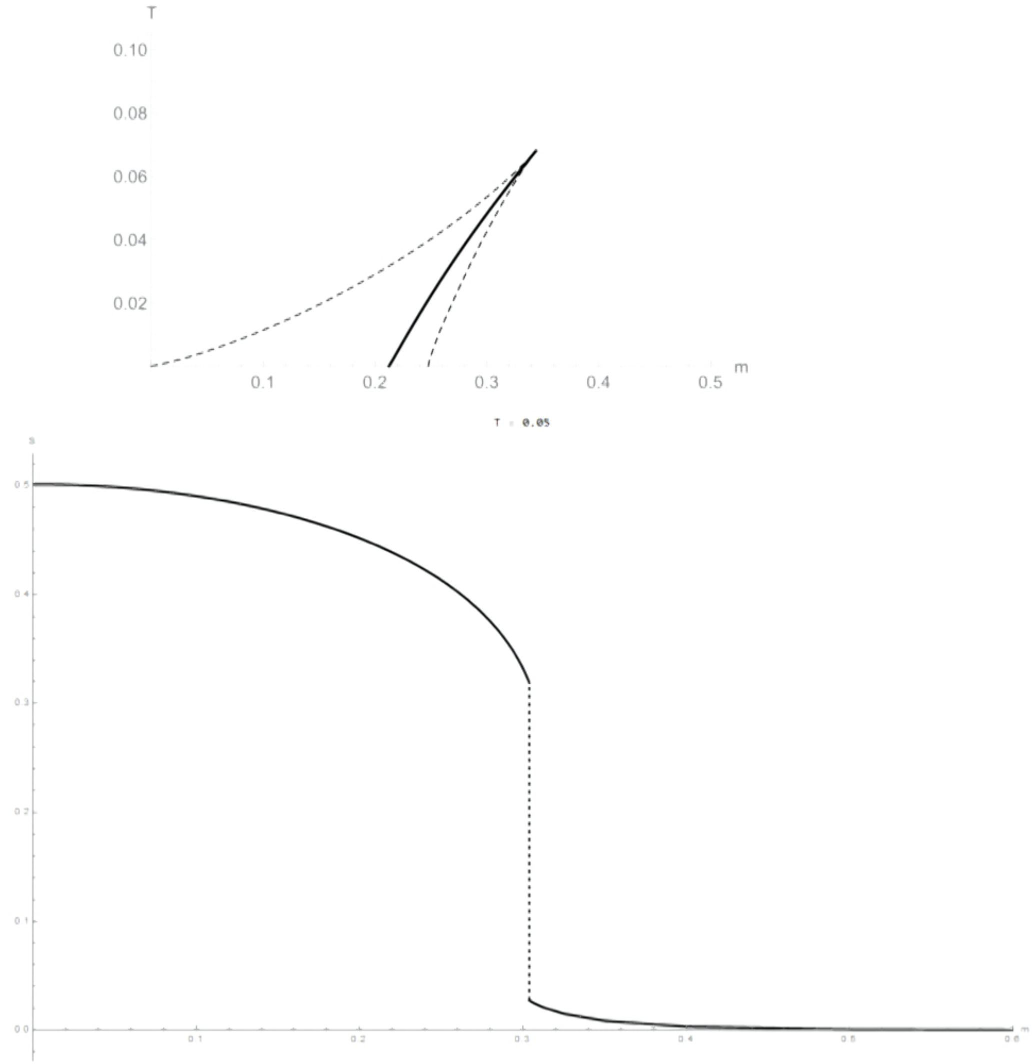
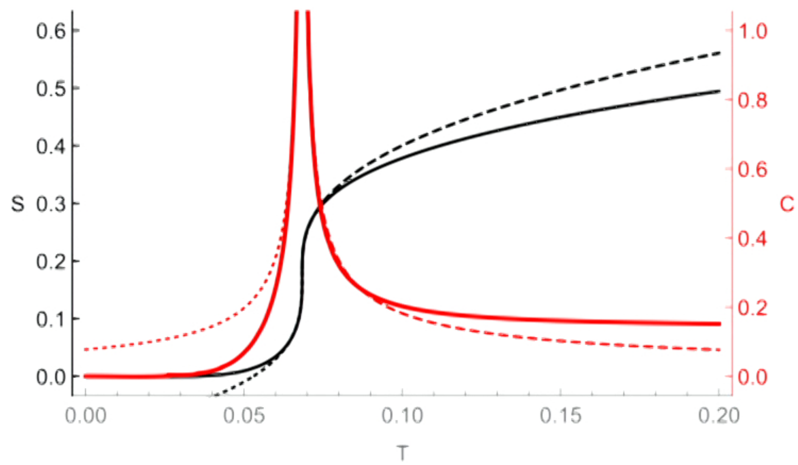
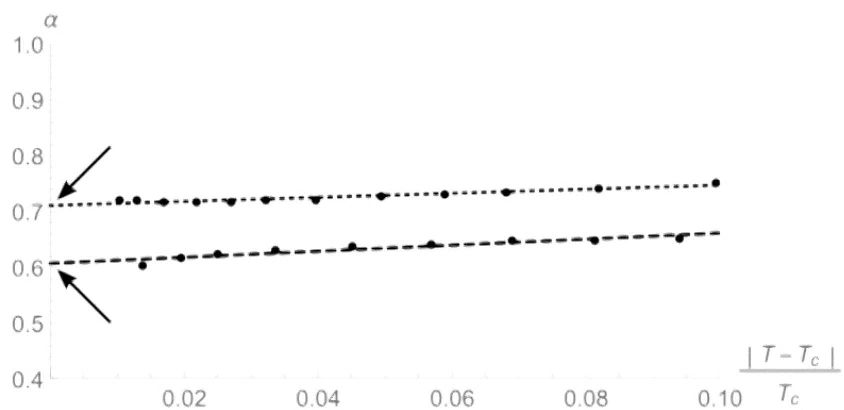
0.02
Perturbative solution
0.1
0.2
0.3
0.4
0.5
m


T
0.10
0.08
0.06
0.04
0.02
BH SYK-like solution
Perturbative solution
0.1
0.2
0.3
0.4
0.5
m


T
0.10
0.08
0.06
0.04
0.02
Critical point
Coexistence of two solutions
BH SYK-like solution
First order phase transition
(gravitational collapse)
Perturbative solution
0.1
0.2
0.3
0.4
0.5
m

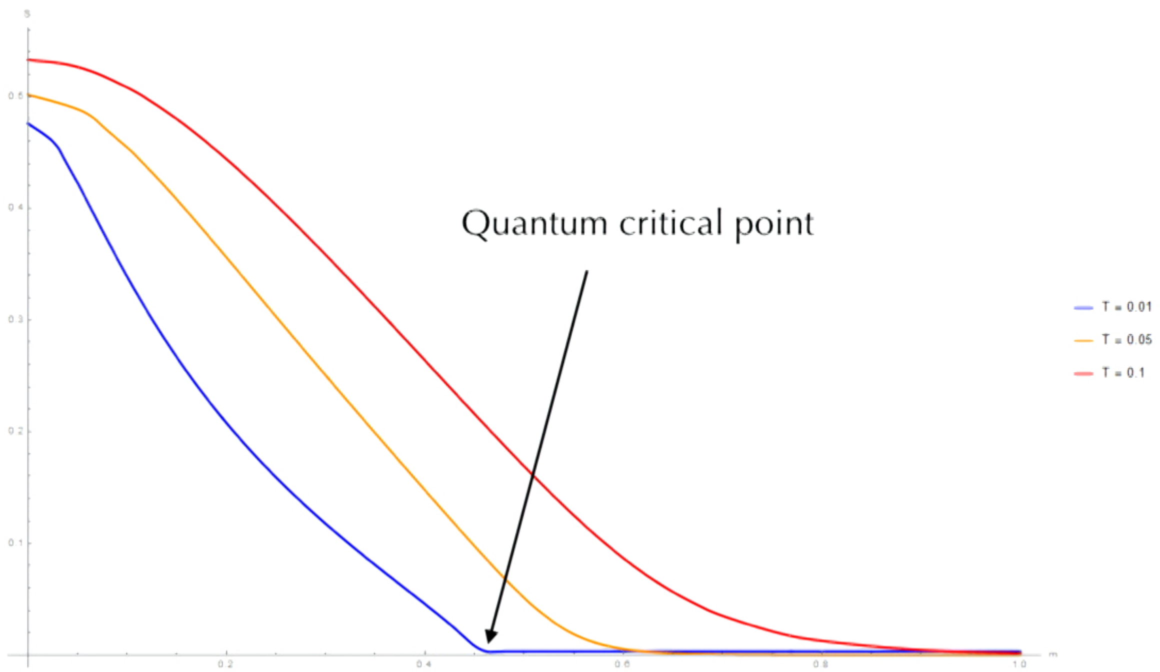

T
0.10
0.08
0.06
0.04
0.02
0.1
0.2
0.3
0.4
0.5
m
T = 0.05


S
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.00
0.05
0.10
0.15
0.20
T
C
1.0
0.8
0.6
0.4
0.2
0.0


α
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.02
0.04
0.06
0.08
0.10
| T − T_c | / T_c

Another more "realistic" model:

$$H = ND\,\mathrm{tr}\Big(M\psi^\dagger_\mu\psi_\mu + \frac{1}{2}\Lambda\sqrt{D}\big(\psi_\mu\psi_\nu\psi_\mu\psi^\dagger_\nu + \mathrm{H.c.}\big)\Big)$$



Understanding what is going on is particularly important in the case of the bosonic systems, because then the "SYK-like" perturbation theory does not exist at all.

Coupling constant: 1/m