Title: Scattering Amplitudes, String Models and Gravitational Waves

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Abstract: The study of scattering amplitudes in field theory connects a wide range of problems, from the mathematics of string perturbation theory to computations related to gravitational waves. I will discuss a couple of topics that keep scattering amplitudes researchers busy, and that motivate an ongoing workshop at PI. First, I will give an overview of recent progress in describing interactions in particle theories in terms of "ambitwistor strings", a new type of field theory model inspired by string theory. The result is an elegant formalism for scattering amplitudes in certain field theories, based on the "scattering equations". This formalism brings a new light into the "double copy relation", discovered in string theory, that expresses perturbative gravity in terms of perturbative gauge theory. I will review the double copy for scattering amplitudes, and then I will discuss the recent application of this idea to classical solutions. One of the aims is to export to gravitational phenomenology the dramatic simplifications provided by the double copy for scattering amplitudes.

Scattering Amplitudes, String Models and Gravitational Waves

Ricardo Monteiro

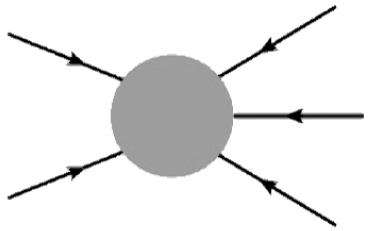
Queen Mary University of London

Colloquium, Perimeter Institute

18 April 2018

A peek into this week's workshop
"New Directions in Conventional and Ambitwistor String Theories"

Scattering Amplitudes



Calculable with Feynman diagrams:

- good: general, clear physical picture.
- bad: inefficient, symmetries obscured.

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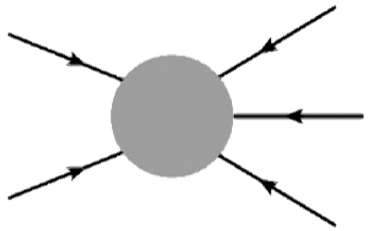

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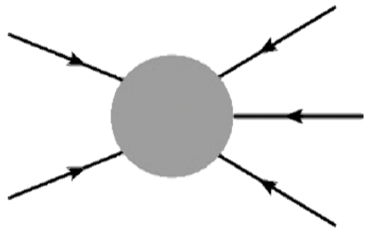

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Modern approaches:

- analyticity, kinematic variables (e.g. on-shell), symmetries.
- relations between theories (e.g. gravity vs gauge theory).
- new formulations of QFTs.

Why Strings?

QFT from low-energy string theory: alternative to Feynman expansion.

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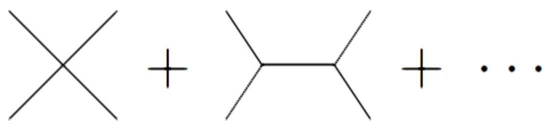
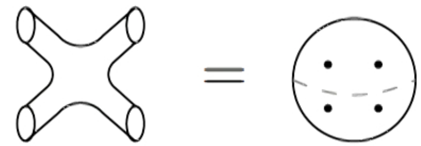


particle scattering
(many Feynman diagrams)



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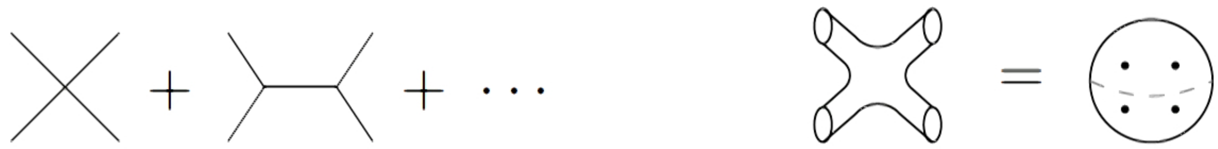

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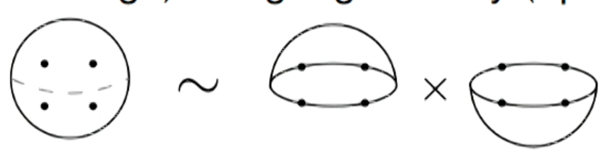
Insights: UV divergences, algebraic structure,
gravity (closed strings) v.s. gauge theory (open strings), . . .



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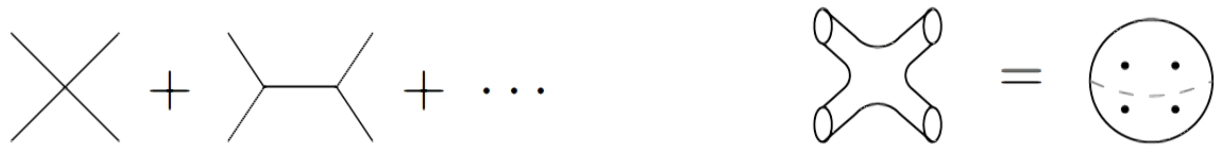

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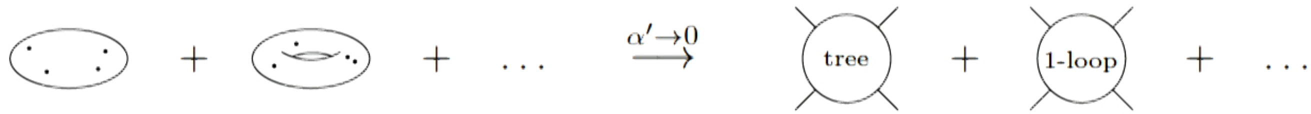
Hard: higher loop corrections (simpler at low energy), dropping susy.

World-sheet Models of (Massless) QFTs

String theory: field theory is $\alpha' \to 0$, massive modes decouple, $m_n^2 = c_n/\alpha'$.



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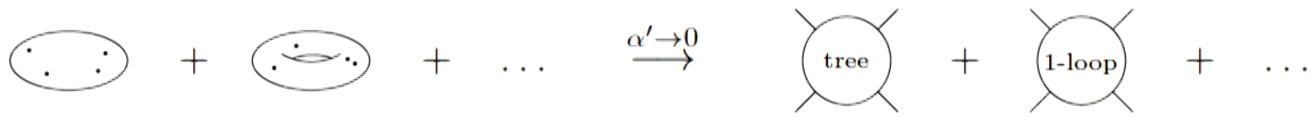


Target space is spacetime.

Is there truncated version just for QFT?

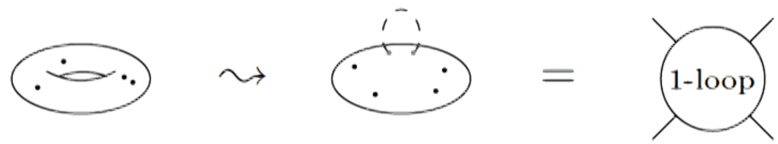
Ambitwistor strings: no α', only massless states.

Target space is space of (complex) null geodesics = *ambitwistor space*.

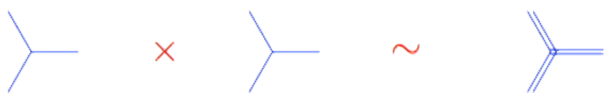
Formulas for amplitudes based on *scattering equations*.

Outline

Part I   Scattering Equations and Ambitwistor Strings



Part II   Double Copy from Gauge Theory to Gravity



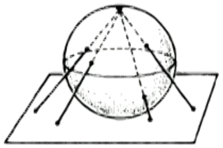
Scattering Equations

[Cachazo, He, Yuan '13 at Perimeter Institute]

Consider n massless particles, $k_i^2 = 0$, $i = 1, \ldots, n$, $\sum_{i=1}^{n} k_i = 0$.

$$E_i = \sum_{j \neq i} \frac{k_i \cdot k_j}{\sigma_i - \sigma_j} = 0, \quad \forall i$$

- kinematic invariants $s_{ij} = (k_i + k_j)^2 = 2\, k_i \cdot k_j \longrightarrow$ points $\sigma_i \in \mathbb{CP}^1$



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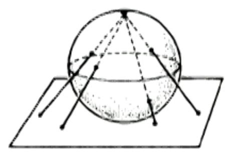

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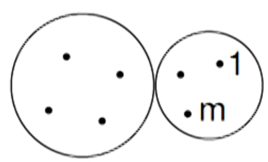

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Direct evaluation:

$$\mathcal{A} = \sum_{A=1}^{(n-3)!} \frac{\mathcal{I}}{J}\Big|_{\sigma_i=\sigma_i^{(A)}}$$

Scattering equations hard to solve, but no need for that!

[Dolan, Goddard; Cachazo, Gomez; Baadsgaard at al; Huang et al; Sogaard, Zhang; Cardona, Kalousios; Fu et al; . . .]

Examples: Yang-Mills theory and Gravity [Cachazo, He, Yuan '13]

Ingredients in Yang-Mills (gluon) $e^{ik\cdot x}\epsilon_\mu T^a$, Gravity (graviton, ...) $e^{ik\cdot x}\varepsilon_{\mu\nu}$:

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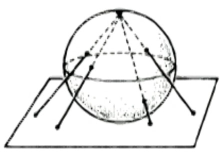

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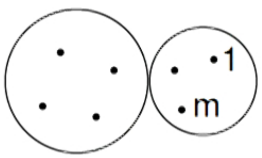

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Amplitudes are $\mathcal{A} = \int_{\mathfrak{M}_{0,n}} d\mu\ \mathcal{I}$. Clear n-particle structure in D dimensions!

Yang-Mills theory: $\mathcal{I}_{\mathrm{YM}} = \mathrm{Pf}\,'M(\epsilon_i) \times \mathcal{C}(a_i)$

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$\Rightarrow$ **Gravity $\sim$ YM^2** more later

Other Examples

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e.g. Gravity-Yang-Mills,
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[Cachazo, He, Yuan 14]

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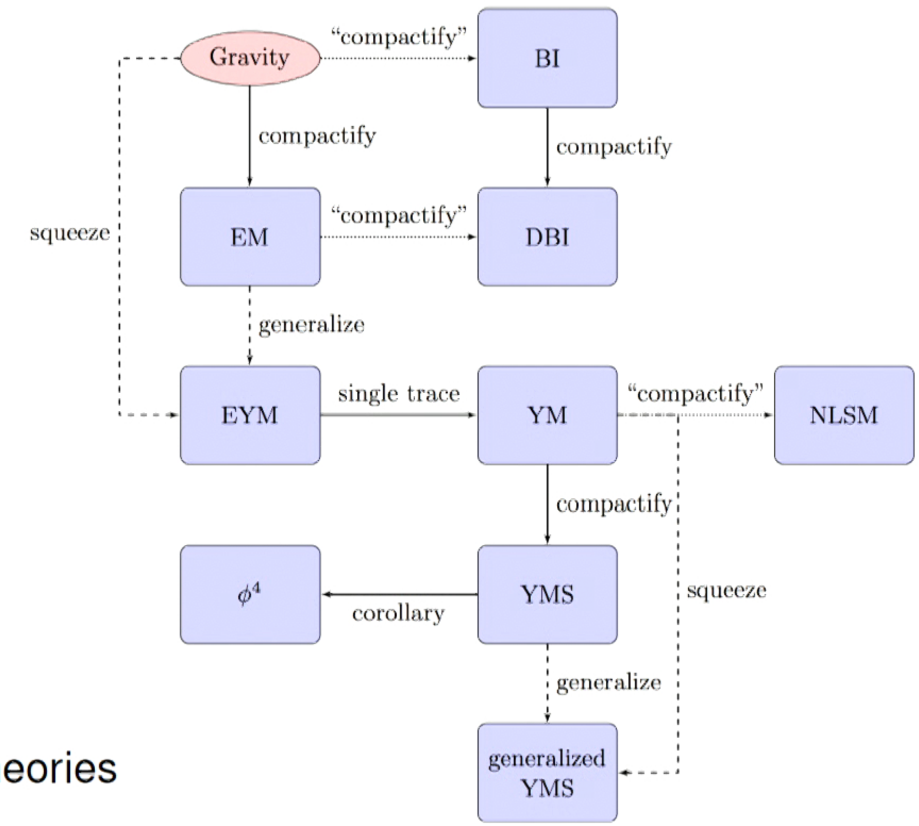

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Lessons:

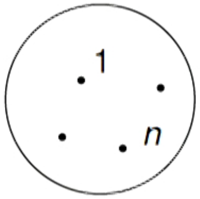
- relations between theories
- messy Feynman rules $\neq$ messy amplitudes

Ambitwistor strings

Geometry of Scattering Equations

$SL(2,\mathbb{C})$ invariant differential on $\mathbb{CP}^1$:

$$P_\mu(\sigma) = d\sigma \sum_{i=1}^{n} \frac{k_{i\,\mu}}{\sigma - \sigma_i}$$



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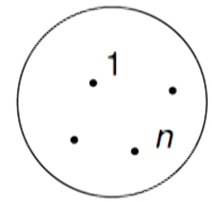

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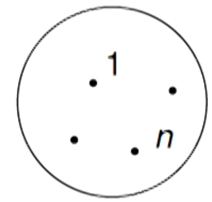

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Twistor string theory [Witten 03] $\longrightarrow$ CHY predecessor [Roiban, Spradlin, Volovich 04]
4D Yang-Mills theory.

(Later also 4D gravity. [Hodges, Cachazo, Geyer, Skinner, Mason 12])

Only tree level. [Berkovitz, Witten 04].

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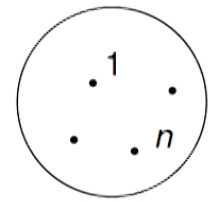

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Any D? Twistor $\rightsquigarrow$ Ambitwistor space [Mason, Skinner 13]

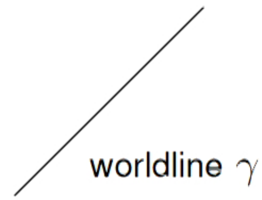
$=$ space of null geodesics of (complexified) spacetime.

Strings in Ambitwistor Space

[Mason, Skinner 13]

Worldline action for massless particle:

$$S_p = \int_\gamma p_\mu \, dx^\mu - \frac{1}{2} e \, p^2 \qquad d = d\lambda \, \partial_\lambda$$



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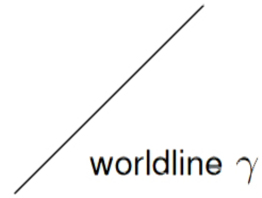

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Chiral complexification: “string”

$$S = \frac{1}{2\pi} \int_\Sigma P_\mu \, \bar\partial X^\mu - \frac{1}{2} e \, P^2 \qquad \bar\partial = d\bar\sigma \, \partial_{\bar\sigma}$$

- $P_\mu = d\sigma \, p_\mu(\sigma)$.
- e enforces $P^2 = 0$, same gauge freedom $(d\alpha \rightsquigarrow \bar\partial\alpha)$.

Ambitwistor space: (X^μ, P_ν) with $P^2 = 0, \; (X^\mu, P_\nu) \sim (X^\mu + \alpha P^\mu, P_\nu)$.

Quantisation of Ambitwistor String

[Mason, Skinner 13]

Technical but worth it

Action: $S = \frac{1}{2\pi}\int_\Sigma P_\mu\,\bar{\partial}\,X^\mu - \frac{1}{2}\,e\,P^2$ Amplitude: $\mathcal{A} = \left\langle \prod_{i=1}^{n} V_i \right\rangle$

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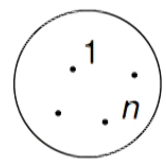
Riemann sphere $\mathbb{CP}^1 \Rightarrow P_\mu = d\sigma \sum_i \frac{k_{i\,\mu}}{\sigma - \sigma_i}$



$\Rightarrow \text{Res}_{\sigma_i} P^2 = E_i = 0$ are scattering equations $\Rightarrow \mathcal{A} = \int_{\mathfrak{M}_{0,n}} d\mu\;\mathcal{I}$

Quantisation of Ambitwistor String

[Mason, Skinner 13]

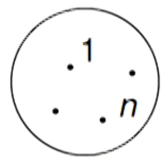
Technical but worth it

Action: $S = \frac{1}{2\pi}\int_\Sigma P_\mu\,\bar\partial X^\mu - \frac{1}{2}\,e\,P^2$ Amplitude: $\mathcal{A} = \left\langle \prod_{i=1}^{n} V_i \right\rangle$

Gauge fix $e = 0$: $V_i = \int d\sigma_i\,\delta(\mathrm{Res}_{\sigma_i} P^2)\,e^{ik_i\cdot X}\,\ldots$ $k_i^2 = 0$ massless

Integrate $X^\mu \Rightarrow P_\mu$ is meromorphic with simple poles at σ_i with residues $k_{i\,\mu}$.

Riemann sphere $\mathbb{CP}^1$ $\Rightarrow$ $P_\mu = d\sigma \sum_i \frac{k_{i\,\mu}}{\sigma - \sigma_i}$



$\Rightarrow$ $\mathrm{Res}_{\sigma_i} P^2 = E_i = 0$ are scattering equations $\Rightarrow$ $\mathcal{A} = \int_{\mathfrak{M}_{0,n}} d\mu\ \mathcal{I}$

Combine with world-sheet matter to reproduce various CHY formulas.

[also Ohmori 15; Casali, Geyer, Mason, RM, Roehrig 15]

E.g., add system of fermions $\rightsquigarrow$ Pfaffian.

Ambitwistor Strings vs. Ordinary Strings

Ambitwistor strings:

- chiral, massless states
- $\mathcal{A} = \int \left(\prod_i d\sigma_i \, \delta(E_i)\right) \; \mathcal{I}_L(\sigma_i) \, \mathcal{I}_R(\sigma_i)$

Ordinary (closed) strings:

- not chiral, infinite tower of massive states
- $\mathcal{A}_{\text{st}}(\alpha') = \int \left(\prod_i d\sigma_i d\bar{\sigma}_i\right) F(|\sigma_i - \sigma_j|) \; \mathcal{I}_L(\sigma_i) \, \mathcal{I}_R(\bar{\sigma}_i)$

Formulas: field theory limit is $\mathcal{A}_{\text{st}}(\alpha') \xrightarrow{\alpha' \to 0} \mathcal{A}$. Non-trivial!

[e.g. Bjerrum-Bohr, Damgaard, Tourkine, Vanhove 14, Mizera 17]

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Theories:

Surprise understanding from $\alpha' \to \infty$ with alternative quantisation.

[Siegel 15, +Huang, Yuan 16, Casali, Tourkine 16, +Herfray 17, Azevedo, Jusinskas 17, . . .]

Open problem "type II" superstring $\xrightarrow{\alpha' \to 0}$ "type II" ambitwistor string.

Ambitwistor Strings at Higher Genus

Loop corrections? [Adamo, Casali, Skinner 13]

Example: one loop needs torus  τ $(\sigma \sim \sigma+1 \sim \sigma+\tau)$

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- sphere: $P_\mu = d\sigma \sum_i \frac{k_{i\,\mu}}{\sigma - \sigma_i}$
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- genus g: $P_\mu = \sum_{I=1}^{g} \ell_\mu^{(I)} \omega_I + \sum_i k_{i\,\mu} \varpi_{i,*}$

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- genus g: $P_\mu = \sum_{I=1}^{g} \ell_\mu^{(I)} \, \omega_I + \sum_i k_{i\,\mu} \, \varpi_{i,*}$

Scattering equations still $P^2 = 0$.

Higher-genus surfaces hard (Jacobi θ functions)...

From the Torus to the Nodal Sphere

[Geyer, Mason, RM, Tourkine 15]

Torus scattering equations localise both σ_i and τ, but too hard to solve.

Loop integrand should be easier, like tree level.

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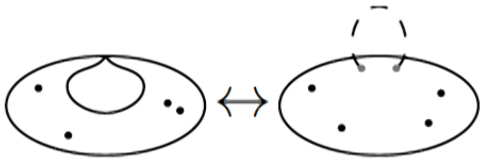

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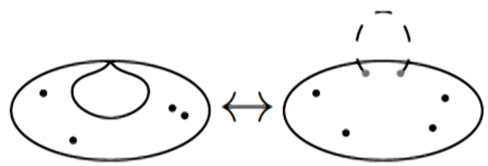
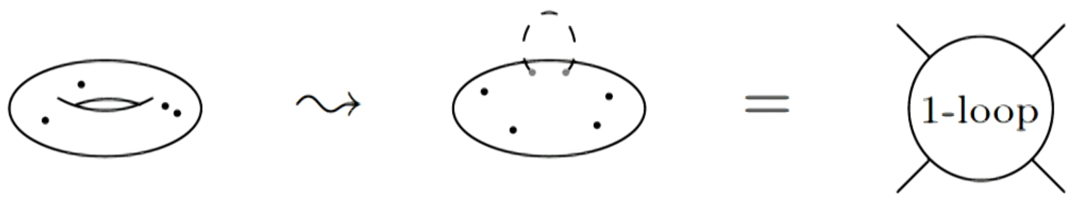


How to get there? Residue theorem on τ integration: localises on $\tau = i\infty$.

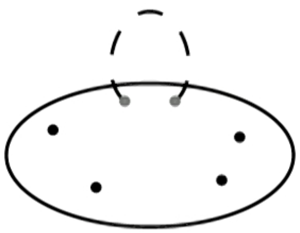


Different approach: use elliptic parametrisation. [Cardona, Gomez 16]

New One-Loop Formula

[Geyer, Mason, RM, Tourkine 15]

Final result: $\mathcal{A}^{(1)} = \int d^D\ell \, \frac{1}{\ell^2} \int_{\mathfrak{M}_{0,n+2}} d\mu^{(1)} \; \mathcal{I}^{(1)}$



Like tree level, but now for one-loop integrand!

- two new "particles": loop momentum insertions $\pm\ell$
- $P_\mu = d\sigma \left(\frac{\ell_\mu}{\sigma - \sigma_{+\ell}} + \frac{-\ell_\mu}{\sigma - \sigma_{-\ell}} + \sum_i \frac{k_{i\,\mu}}{\sigma - \sigma_i} \right)$
- $\mathcal{I}^{(1)} = \lim_{\tau \to i\infty} \mathcal{I}^{\text{torus}}$
- one-loop scattering equations depend on ℓ_μ

New Formalism

CHY-type expression for loop integrand:

$$\mathcal{A}^{(1)} = \int d^D\ell \, \frac{1}{\ell^2} \int_{\mathfrak{M}_{0,n+2}} d\mu^{(1)} \; \mathcal{I}^{(1)}$$

Previous derivation applied to “type II supergravity in 10D”.

More general?

- Higher genus is very restrictive (modular invariance).
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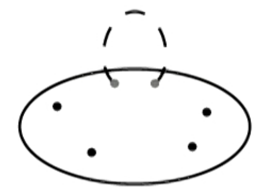

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More general?

- Higher genus is very restrictive (modular invariance).
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Proposal: generic loop expansion is nodal expansion on sphere.

Look directly for $\mathcal{I}^{(1)}$ in nodal sphere, in any D.

New CHY-type formulas

[Geyer, Mason, RM, Tourkine 15]

Yang-Mills: $\mathcal{I}^{(1)}_{\text{YM}} = \mathcal{I}^{(1)}_{\text{kin}}(\epsilon)\, \mathcal{I}^{(1)}_{\text{colour}}$ Gravity: $\mathcal{I}^{(1)}_{\text{Grav}} = \mathcal{I}^{(1)}_{\text{kin}}(\epsilon)\, \mathcal{I}^{(1)}_{\text{kin}}(\tilde{\epsilon})$

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- Susy: $\mathcal{I}^{(1)}_{\text{kin}}(\epsilon) = \text{vector} + \text{fermion in the loop}$
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New formula: $\mathcal{A}^{(1)} = \int d^D\ell \, \frac{1}{\ell^2}\, \mathfrak{I}(\ell)\,, \qquad \mathfrak{I}(\ell) = \int_{\mathfrak{M}_{0,n+2}} d\mu^{(1)}\ \mathcal{I}^{(1)}$

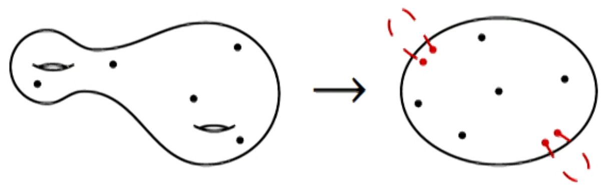
Explicit integrand $\mathfrak{I}(\ell)$? No need to solve the scattering equations.

New type of propagator structure.

Good for obtaining loop integrands from trees! [Baadsgaard et al 15, CHY 15, Skinner, Roehrig 17] [He, Schlotterer 16, + Zhang 17, Geyer, RM 17]

Higher loops

Two loops:



- done: 4-pts in SUGRA and SYM [Geyer, Mason, RM, Tourkine 16] [genus 2: Adamo, Casali 15]
- soon: n-pts in SUGRA and SYM [Geyer, RM]
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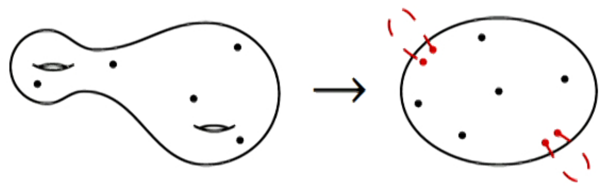

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- in progress: n-pts in non-SUSY gravity and YM

All genus plausibility:

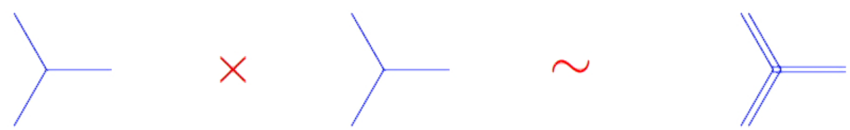
genus g	$\dim_{\mathbb{C}}(\mathfrak{M}_{g,n}) = n + 3g - 3$
degenerations	$-g$
g-nodal sphere	$\dim_{\mathbb{C}}(\mathfrak{M}_{0,n+2g}) = (n + 2g) - 3$

More amazing things

- Curved background: consistency $\Rightarrow$ Einstein eqns. [Adamo, Casali, Skinner 14]
- Scattering on plane wave backgrounds. [Adamo, Casali, Mason, Nekovar 17]
- Clear soft limits ($k_n \to 0$). [Cachazo, Strominger 14; Casali 14] [Schwab, Volovich 14] [Adamo, Casali, Skinner 15; Geyer, Lipstein, Mason 14; CHY 15; . . .]
- 4D ambitwistor strings: $P^2 = 0 \rightsquigarrow P_\mu = \sigma_\mu^{\alpha\dot{\alpha}}\, \lambda_\alpha\, \tilde{\lambda}_{\dot{\alpha}}$ [Geyer, Lipstein, Mason 14]
- Pure spinor ambitwistor string. [Berkovits 13; Gomez, Yuan 13]
- Ambitwistor string field theory. [Reid-Edwards, Riccombeni 17]
- Form factors and correlation functions. [He, Zang, Liu 16; Brandhuber et al 16] [Adamo, RM, Paulos 17; Bork, Onishchenko 17]
- New algebraic / geometric interpretations. [Mizera 17; Cruz, Kniss, Weinzierl 17] [Arkani-Hamed, Bai, He, Yan 17; Frost 18; He, Yan, Zhang 18]

Part II

Double Copy from Gauge Theory to Gravity



Motivation

Feynman rules for GR: expand Einstein-Hilbert Lagrangian [DeWitt '66]

$$\frac{\delta^3 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}} \longrightarrow$$

$$\begin{aligned}
&\mathrm{Sym}\Big[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})\\
&-\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda})+\tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+\tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho})+P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu})\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad-P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})\Big],
\end{aligned}$$

$$\frac{\delta^4 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}\delta\varphi_{\iota'''\kappa'''}} \longrightarrow$$

$$\begin{aligned}
&\mathrm{Sym}\Big[-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})+\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})\\
&+\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{4}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{2}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})\\
&+\tfrac{1}{4}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{24}(p^\sigma p^\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{12}(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\iota\kappa})+\tfrac{1}{2}P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})\\
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\end{aligned}$$

$+$ infinite number of higher-point vertices...

Gravity $\sim$ YM ^{2}

Free states

- Polarisation: $\boxed{\varepsilon^{\mu\nu} = \epsilon^{\mu}\,\tilde{\epsilon}^{\nu}}$ (graviton $h_{\mu\nu}$ + dilaton Φ + B-field $B_{\mu\nu}$)
- Degrees of freedom match: $(D-2)^2$.

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- "Factorisation" of ϵ_{μ}, $\tilde{\epsilon}_{\nu}$ preserved by interactions!
- **Double copy** $\boxed{\mathcal{A}_{\text{grav}}(\varepsilon_i^{\mu\nu}) \sim (\text{prop})^{-1}\ \mathcal{A}_{\text{YM}}(\epsilon_i^{\mu}) \times \mathcal{A}_{\text{YM}}(\tilde{\epsilon}_i^{\mu})\ \big|_{\text{colour stripped}}}$
- Well established at tree level, e.g. $\mathcal{I}_{\text{grav}}(\varepsilon_i^{\mu\nu}) = \text{Pf}\,'M(\epsilon_i^{\mu}) \times \text{Pf}\,'M(\tilde{\epsilon}_i^{\mu})$
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Classical solutions

- Suggests correspondence between classical theories.
- Map between solutions?

KLT and BCJ double copies [Kawai, Lewellen, Tye '86] [Bern, Carrasco, Johansson '08]

String theory

Vertex operators: $V_{\text{closed}}(\varepsilon^{\mu\nu} = \epsilon^{\mu}\tilde{\epsilon}^{\nu}) \sim V_{\text{open}}(\epsilon^{\mu})\,\bar{V}_{\text{open}}(\tilde{\epsilon}^{\nu})$

Scattering amplitudes:

KLT relations

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Field theory $(\alpha' \to 0)$

Alternative diagrammatic version: BCJ colour-kinematics duality

E.g. $n = 4$: $\quad \mathcal{A}_{\text{YM}}(\epsilon) = \dfrac{n_s(\epsilon)\,c_s}{s} + \dfrac{n_t(\epsilon)\,c_t}{t} + \dfrac{n_u(\epsilon)\,c_u}{u}\,, \qquad c_s = f^{a_1 a_2 b} f^{b a_3 a_4}$

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Jacobi identity $\boxed{c_u = c_s - c_t}$. Also true that $\boxed{n_u = n_s - n_t}$.

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Any n: kinematic Jacobi's not automatic, but always possible at tree level!

Double copy table (via KLT, BCJ or CHY)

Examples:

$$(\mathcal{N}=4\ \text{SYM}) \times (\mathcal{N}=4\ \text{SYM}) \sim (\mathcal{N}=8\ \text{SUGRA})$$
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Many more examples:

$$(\text{YM}) \times (\text{YM + scalar}) \sim (\text{Gravity-Yang-Mills})$$
$$(\text{YM}) \times (\text{NLSM}) \sim (\text{Born-Infeld})$$
$$(\text{SYM}) \times (\text{Z-theory}) \sim (\text{open superstring})$$

[Chiodaroli, Gunaydin, Johansson, Roiban; Kampf, Novotny, Trnka; Broedel, Dixon; Du, Feng, Fu; Cachazo, He, Yuan]
[Carrasco, Mafra, Schlotterer; Azevedo, Chiodaroli, Johansson, Schlotterer; Bargheer, He, McLoughlin; Huang, Johansson, Lee]
[. . .]

Loop-level BCJ conjecture

[Bern, Carrasco, Johansson '10]

$$\mathcal{A}_{\text{YM}} = \sum_{\alpha \in \text{cubic}} \int d^{LD}\ell \, \frac{n_\alpha c_\alpha}{D_\alpha} \quad \xrightarrow[n_\alpha \pm n_\beta \pm n_\gamma = 0]{c_\alpha \pm c_\beta \pm c_\gamma = 0} \quad \mathcal{A}_{\text{grav}} = \sum_{\alpha \in \text{cubic}} \int d^{LD}\ell \, \frac{n_\alpha \tilde{n}_\alpha}{D_\alpha}$$

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UV surprises in 4D SUGRA: finite for $\mathcal{N} > 4$?

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Ambitwistor strings provide first principles approach:

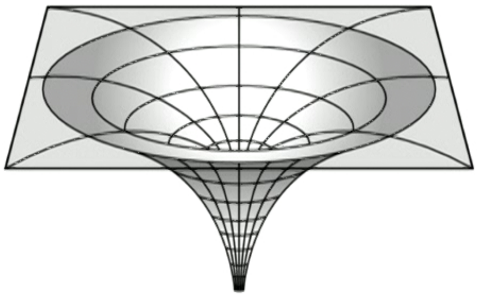
[He, Schlotterer 16, + Zhang 17; Geyer, RM 17]

- one loop follows from tree level.
- different from BCJ conjecture (propagators). Important guidance!

Double copy for classical solutions

Double copy for black holes?

Question: is there a double copy for classical solutions?



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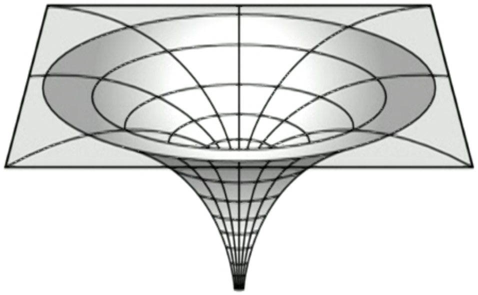

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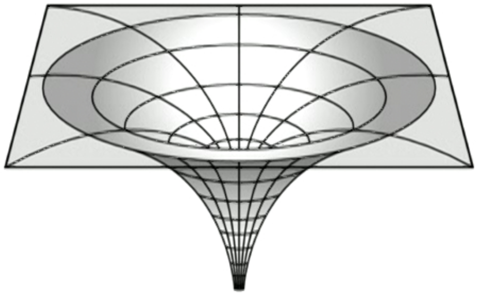


Still...

- Should work in perturbation theory.

  Examples: Schwarzchild [Duff 73; Neill, Rothstein 13], shockwave [Saotome, Akhoury '12].

- Inspiration: re-write perturbative gravity in KLT-ish way. [Bern, Grant 99]

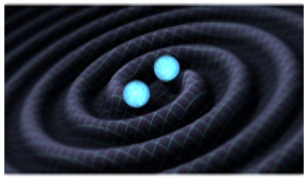
Classical double copy

Map of exact solutions (Kerr-Schild symmetry). [RM, O'Connell, White, Luna, Nicholson 14-16]

Map of linearised solutions. [Anastasiou, Borsten, Duff, Hughes, Nagy, Marrani, Zoccali 14-17] [Cardoso, Nagy, Nampuri, Inverso 16-18]

World-line EFT: radiation from point charges (**GWs in gravity**).

[Goldberger, Ridgway, Prabhu, Thompson, Li 16-17]



BCJ-ish perturbation theory for gravity.

[Luna, RM, Nicholson, Ochirov, O'Connell, Westerberger, White 16]

New forms of GR action. [Cheung, Remmen 16-17]

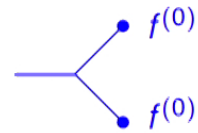
Alternative approach: look at on-shell quantities (amplitudes).

[Bjerrum-Bohr, Donoghue, Vanhove, Holstein, Plante, El-Menoufi 13-17] [Cachazo, Guevara 17; Guevara 17]

Double copy on plane wave backgrounds. [Adamo, Casali, Mason, Nekovar 17]

New rules for perturbative gravity

Example: $f^{(0)}$ is linearised solution, $f^{(1)}$ is first non-linear correction.

$f^{(1)} =$ 

Gauge theory field A^a_μ

$$A^{(1)a\mu}(-p_1) = \frac{i}{2p_1^2}\int d^Dp_2 d^Dp_3 \delta^D(p_1+p_2+p_3)\, \boxed{f^{abc}\; V^{\mu\beta\gamma}}\; A^{(0)b}_\beta(p_2) A^{(0)c}_\gamma(p_3)$$

YM vertex $\quad V(p_1,p_2,p_3)^{\mu\beta\gamma} = (p_1-p_2)^\gamma \eta^{\mu\beta} + (p_2-p_3)^\mu \eta^{\beta\gamma} + (p_3-p_1)^\beta \eta^{\gamma\mu}$

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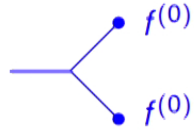
$f^{(1)} =$ 

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Great simplification: index factorisation, c.f. $\sim$100 terms in GR 3-pt vertex!

The higher the order, the bigger the payoff!

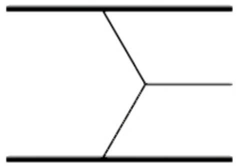
Pure Einstein gravity

Double copy: $(\text{YM})^2 \sim$ Einstein $h_{\mu\nu}$ + dilaton Φ + B-field $B_{\mu\nu}$

Pheno applications: only want pure gravity!

E.g. GWs from inelastic black hole scattering.

How to project out the dilaton?



Import techniques from amplitudes:

- projector (gauge choice) [Luna, RM, Nicholson, Ochirov, O'Connell, Westerberger, White 16] [Bern, Cheung, Chi, Dixon, Nohle, Edison 15]
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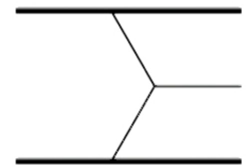

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Competitive for post-Minkowskian or post-Newtonian calculations?

Advanced state of the art, but amplitudes techniques may hope to help:

Double copy + On-shell techniques + Brute force QCD methods

Conclusion

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- Ambitwistor strings describe perturbative QFTs.
- Formulas for YM and gravity, SUSY/no-SUSY.

$$\boxed{\text{loop expansion} = \text{nodal expansion}}$$

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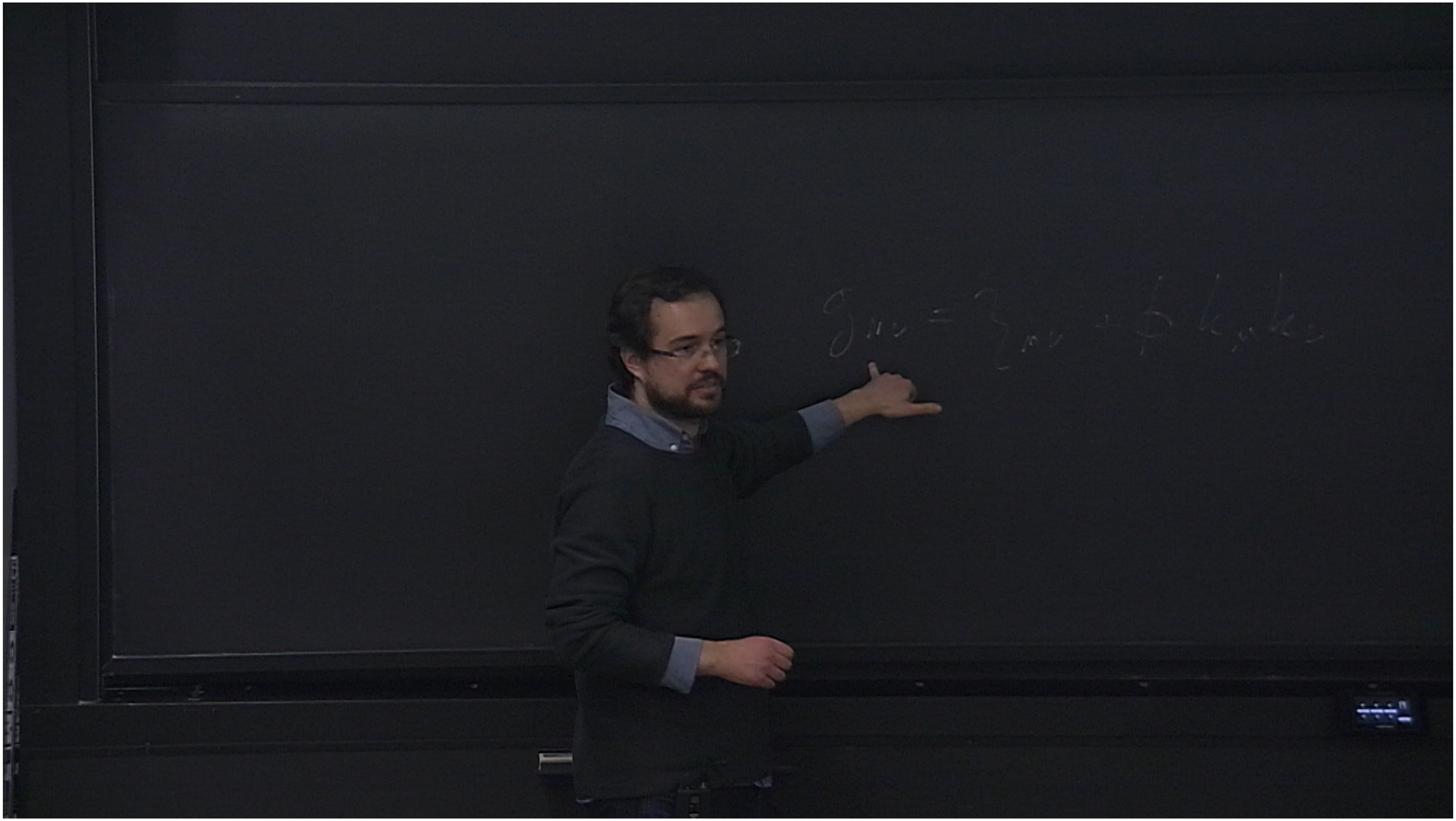
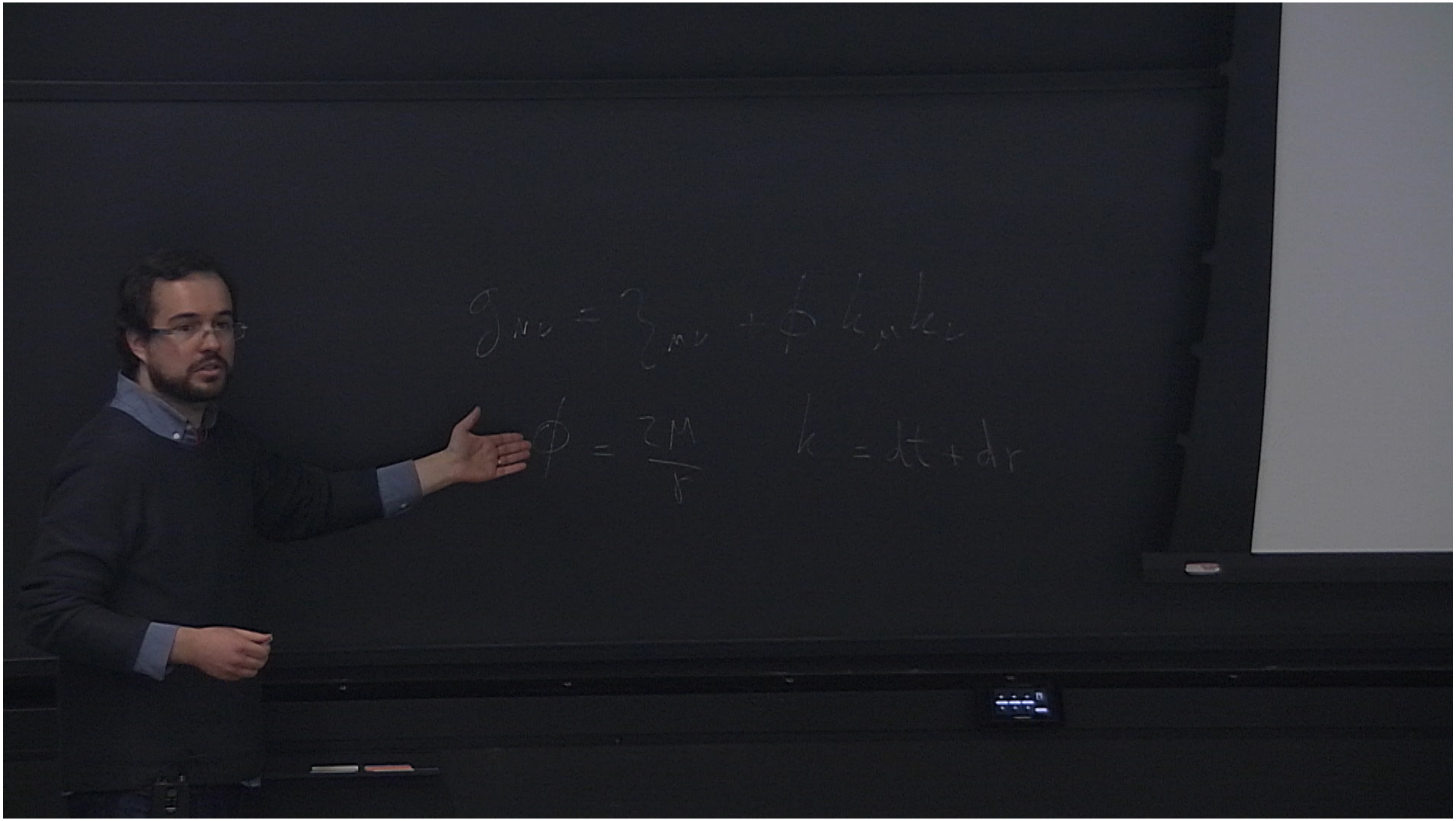
Many open questions

- All-loop story for ambitwistor strings and double copy?
- Ambitwistor strings: Insights on string theory? Efficient loop integration?
- Classical double copy: Tool for exact solutions? Application to GWs?
- Beyond perturbation theory?

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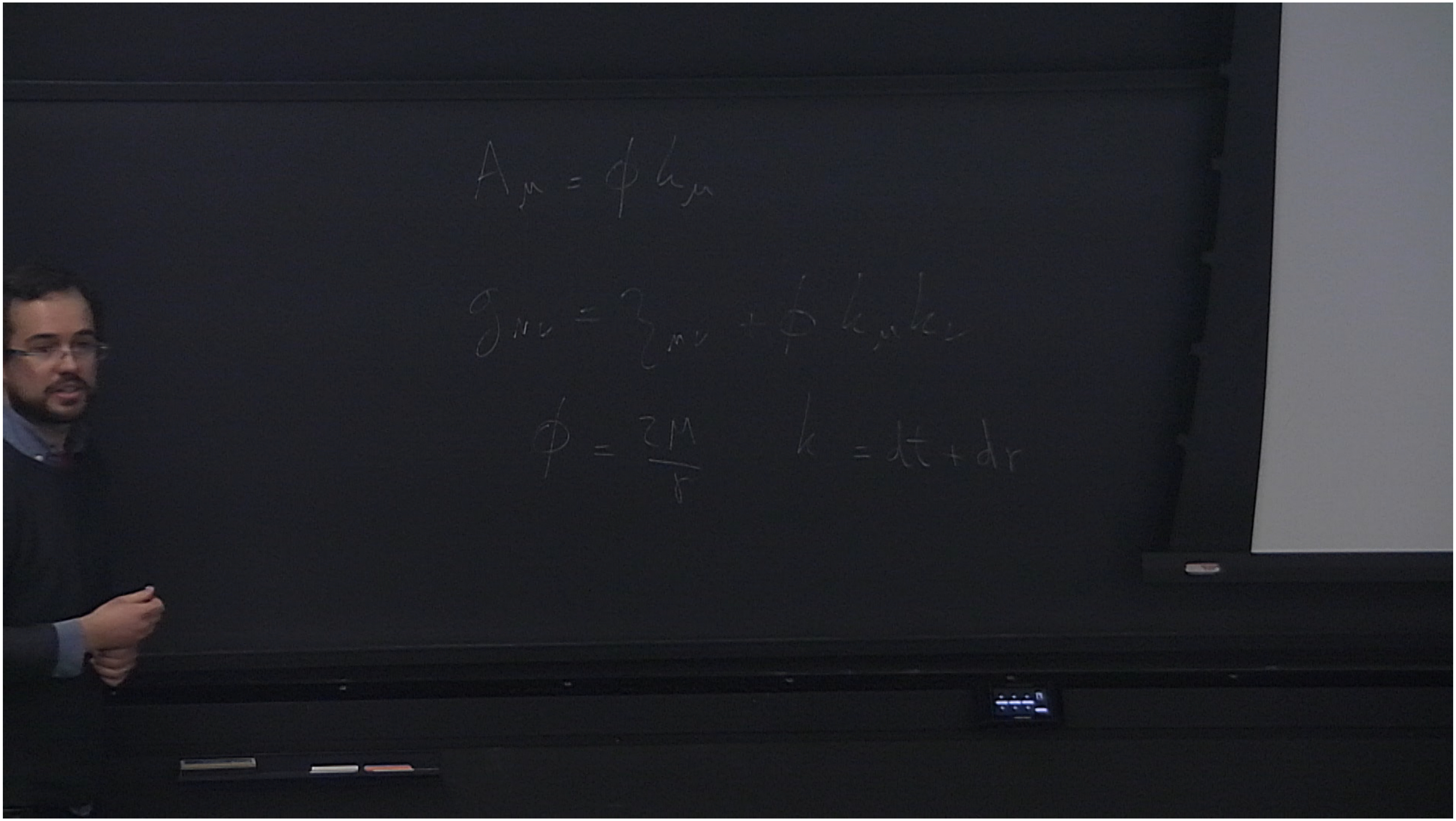
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