Title: Driven Quantum Dynamics: Will It Blend?

Date: Apr 25, 2018 04:00 PM

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Abstract: <p>Randomness is an essential tool in many disciplines of modern sciences, such as cryptography, black hole physics, random matrix theory, and Monte Carlo sampling. In quantum systems, random operations can be obtained via random circuits thanks to so-called q-designs and play a central role in condensed-matter physics and in the fast scrambling conjecture for black holes. Here, we consider a more physically motivated way of generating random evolutions by exploiting the many-body dynamics of a quantum system driven with stochastic external pulses. We combine techniques from quantum control, open quantum systems, and exactly solvable models (via the Bethe ansatz) to generate Haar-uniform random operations in driven many-body systems. We show that any fully controllable system converges to a unitary q-design in the long-time limit. Moreover, we study the convergence time of a driven spin chain by mapping its random evolution into a semigroup with an integrable Liouvillian and finding its gap. Remarkably, we find via Bethe-ansatz techniques that the gap is independent of q. We use mean-field techniques to argue that this property may be typical for other controllable systems, although we explicitly construct counterexamples via symmetry-breaking arguments to show that this is not always the case. Our findings open up new physical methods to transform classical randomness into quantum randomness, via a combination of quantum many-body dynamics and random driving.</p>

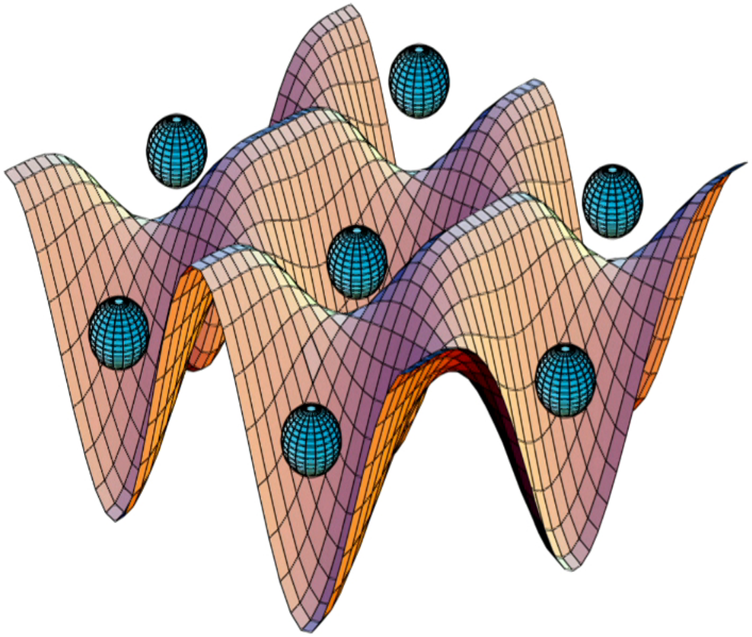
<p>Reference: L. Banchi, D. Burgarth, M. J. Kastoryano, Phys. Rev. X 7, 041015 (2017)</p>

Driven Quantum Dynamics: Will It Blend?

Leonardo Banchi (Imperial College, UK)

25 Apr 2018

Question



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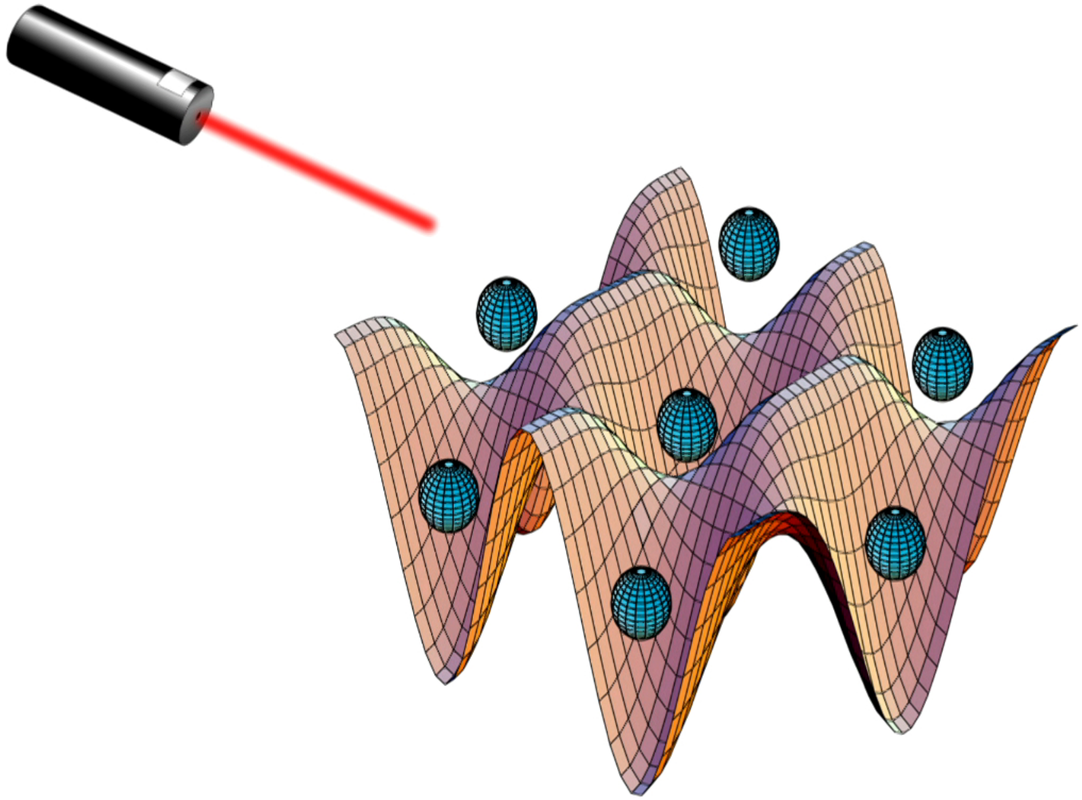

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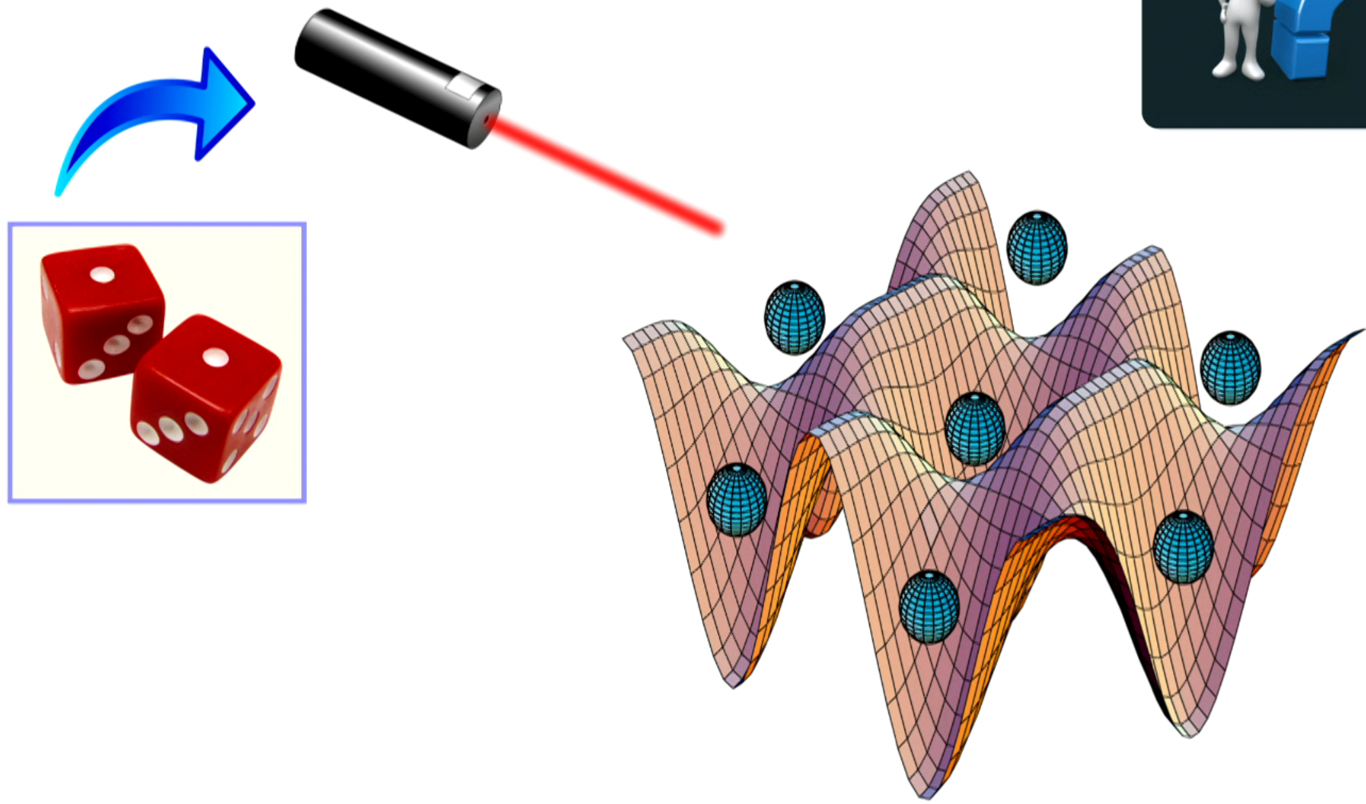

Question

A driven quantum system

$$H(t) = H_0 + g(t)V$$

Resulting unitary operation after a time T

$$U = \mathcal{T} e^{-i \int_0^T H(t)\, dt}$$

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What if $g(t)$ is random?

We show (under some conditions):

- After some time $\{U\}$ is fully random (Haar)
- Estimation of the blending time using the theory of open quantum systems and many-body techniques (Bethe Ansatz)

L. Banchi, D. Burgarth, M. J. Kastoryano, Phys. Rev. X 7, 041015 (2017)

Motivation

Very useful in quantum information processing

- Quantum encryption

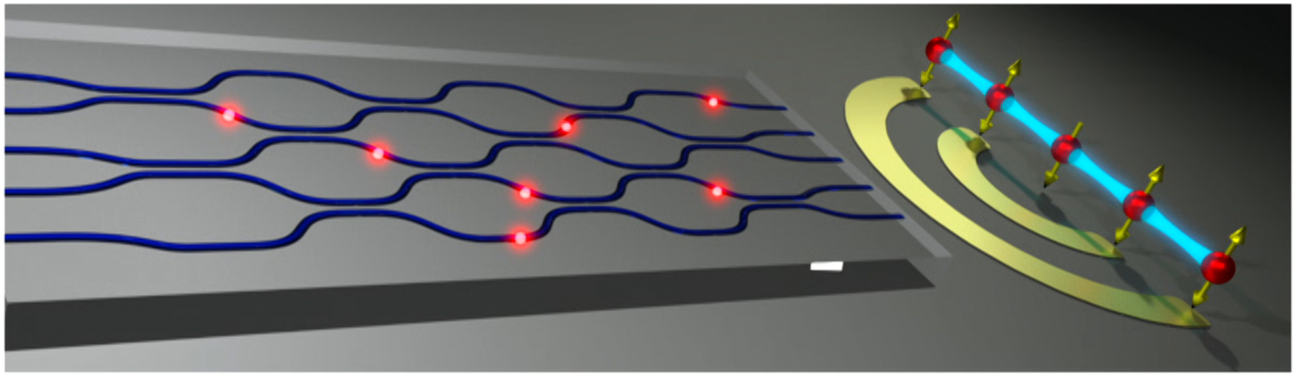
$$|\psi_1\rangle\,,\ |\psi_2\rangle\,,\ \ldots \qquad \longmapsto \qquad U_1\,|\psi_1\rangle\,,\ U_2\,|\psi_2\rangle\,,\ \ldots$$

- State tomography (2-design)
  - Optical tomography with $2N$-designs (N = number of photons)

Banchi, Kolthammer, Kim, (to appear)

- Noise estimation in open quantum systems
  (apply random unitaries such that the coherent part is averaged out)
- Generation of highly entangled states

Generation of highly entangled states



I. Pitsios, L. Banchi, A. S. Rab, M. Bentivegna, D. Caprara, A. Crespi, N. Spagnolo, S. Bose, P. Mataloni, R. Osellame, F. Sciarrino, Nature Communications 8: 1569 (2017)

Alternative: "Entanglement tsunami"

The generation of entanglement requires optimized dynamics, but it is typically achieved also with random operations... and in a fast way!

A. Nahum, J. Ruhman, S. Vijay, J. Haah / PRX 2017

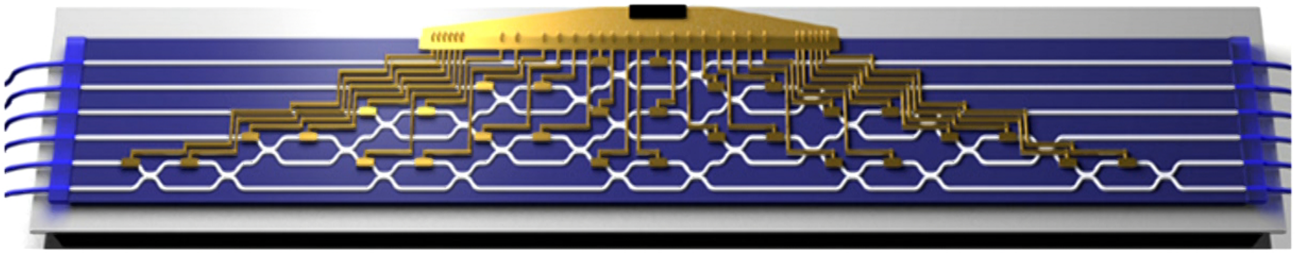
Estimation of the control time in quantum control problems

- If
$$U = \mathcal{T} e^{-i \int_0^T H(t)\, dt}$$
is Haar random, then all operations are achievable at time T.

- Minimal T provides an estimation of the control time

Other applications

credit: J. Carolan et al./Science 2015



Boson sampling experiments require random operations

Other applications

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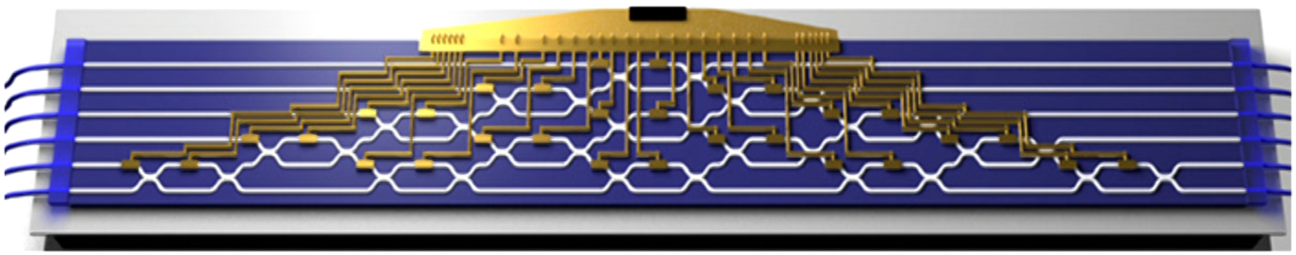
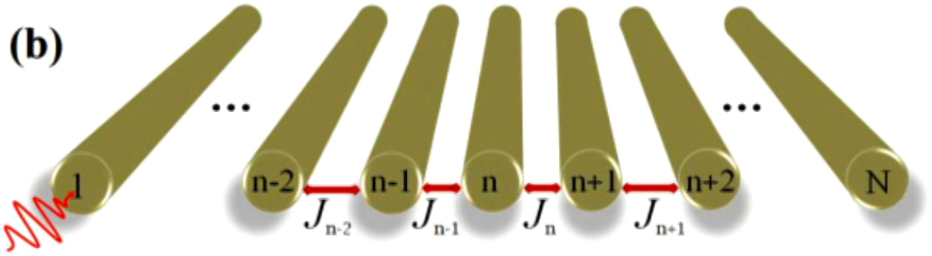


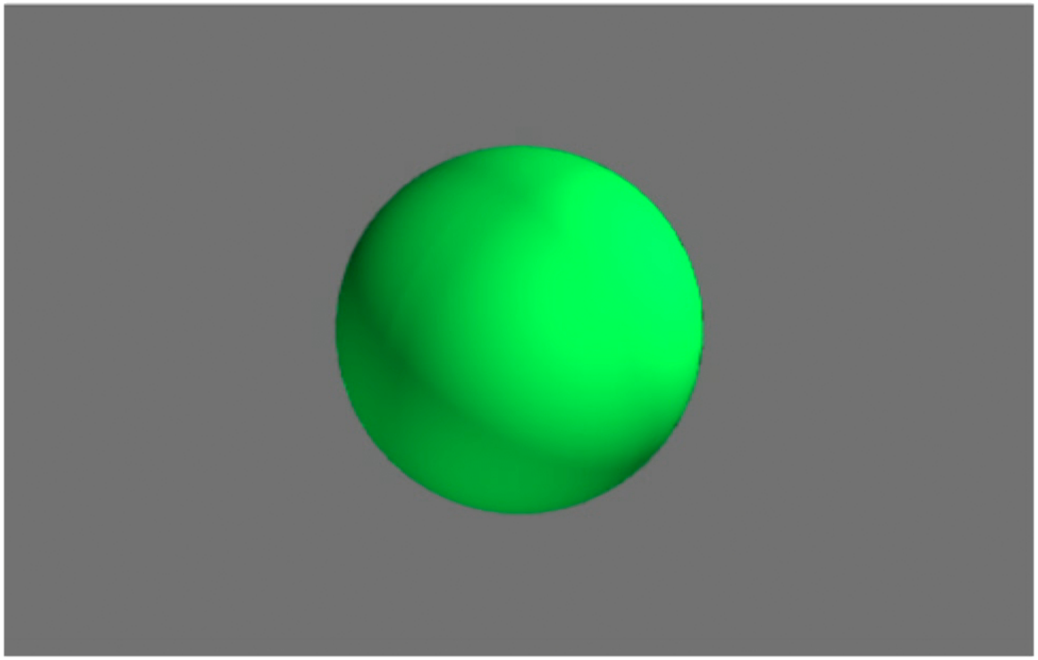
Boson sampling experiments require random operations



credit: Perez-Leija et al./PRA 2013

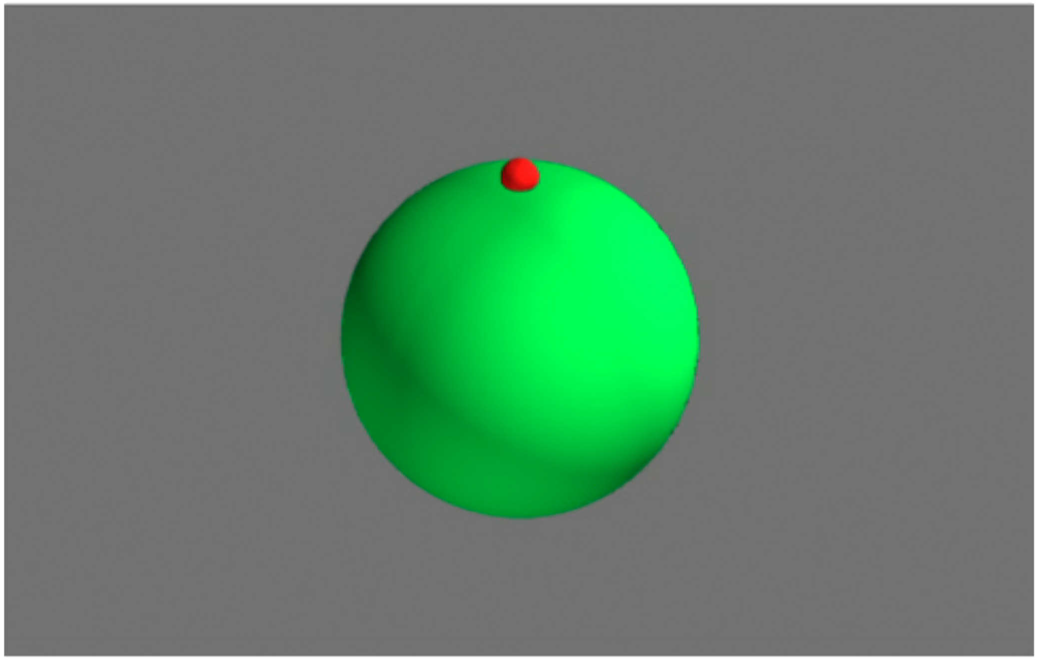
How?

Geometric picture



$$U = \mathcal{T} e^{-i \int_0^T H(t)\, dt}$$

Geometric picture



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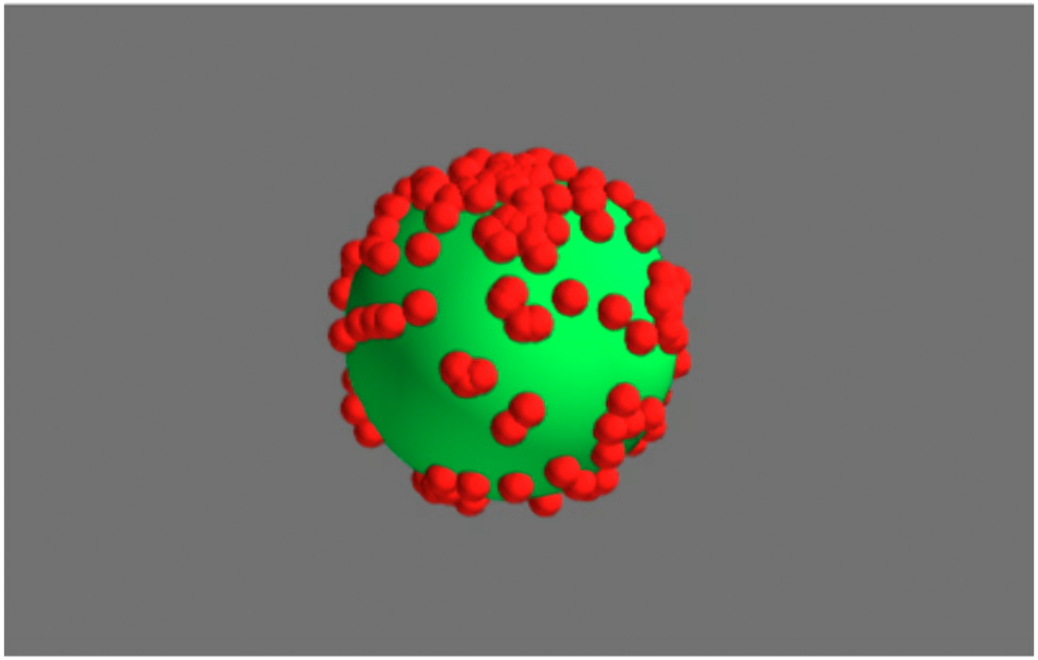

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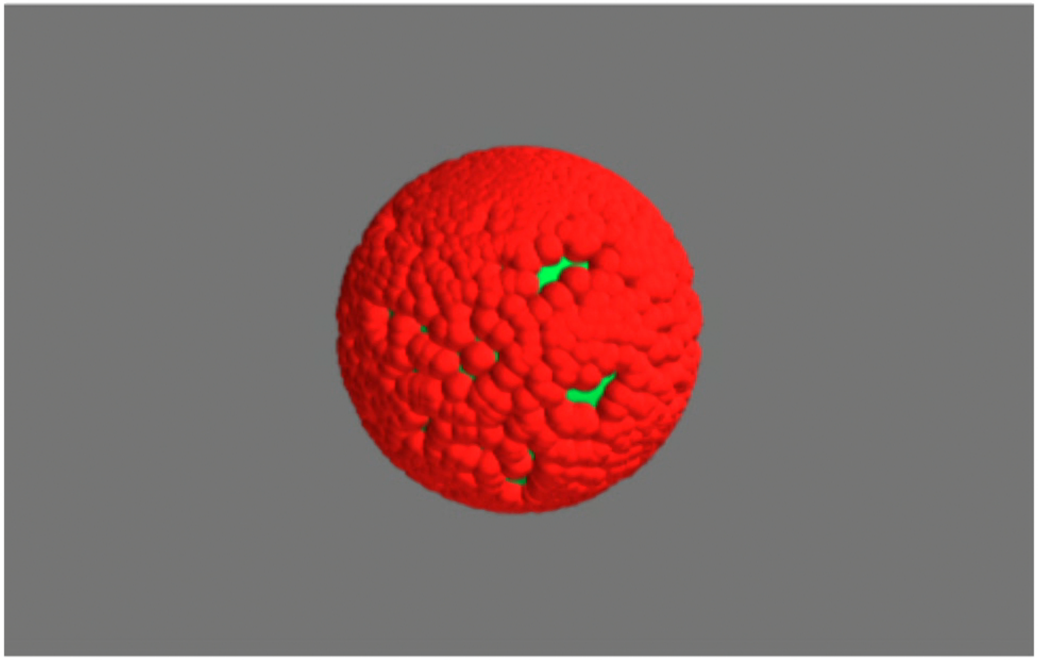

$$U = \mathcal{T} e^{-i \int_0^T H(t)\, dt} \qquad H(t) = H_0 + g(t) V$$

How “random” is the evolution?

Comparing probability distributions on unitaries as a physical process

$$\left\| \mathbb{E}_U \left[U \rho U^\dagger \right] - \int U \rho U^\dagger \, \mu_{\mathrm{Haar}}(dU) \right\| < \epsilon$$

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Comparing probability distributions on unitaries as a physical process

$$\left\| \mathbb{E}_U \left[U^{\otimes q} \rho\, U^{\otimes q \dagger} \right] - \int U^{\otimes q} \rho\, U^{\otimes q \dagger} \mu_{\mathrm{Haar}}(dU) \right\|_? < \epsilon$$

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q-design

$$\left\| \mathbb{E}_U \left[U^{\otimes q} \rho \, U^{\otimes q \dagger} \right] - \int U^{\otimes q} \rho \, U^{\otimes q \dagger} \mu_{\mathrm{Haar}}(dU) \right\|_{\diamond} < \epsilon$$

No single (global) measurement can distinguish between the two processes with probability larger than ϵ

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Using vectorisation

$$X = \sum_{ij} X_{ij} \left| i \right\rangle \left\langle j \right| \quad \mapsto \quad \left| X \right\rangle\rangle = \sum_{ij} X_{ij} \left| ij \right\rangle$$

$|AX\rangle\rangle = A \otimes 1 |X\rangle\rangle$ and $|XA\rangle\rangle = 1 \otimes A^T |X\rangle\rangle$. With $U^{\otimes q,q} = U^{\otimes q} \otimes (U^{\otimes q})^*$

Expanders (weaker)

$$e(\mu_U, q) = \left\| \mathbb{E}_U \left[U^{\otimes q,q} \right] - \int U^{\otimes q,q} \mu_{\mathrm{Haar}}(dU) \right\|_\infty < \epsilon$$

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Operational way of measuring if two probability distributions are “close enough”

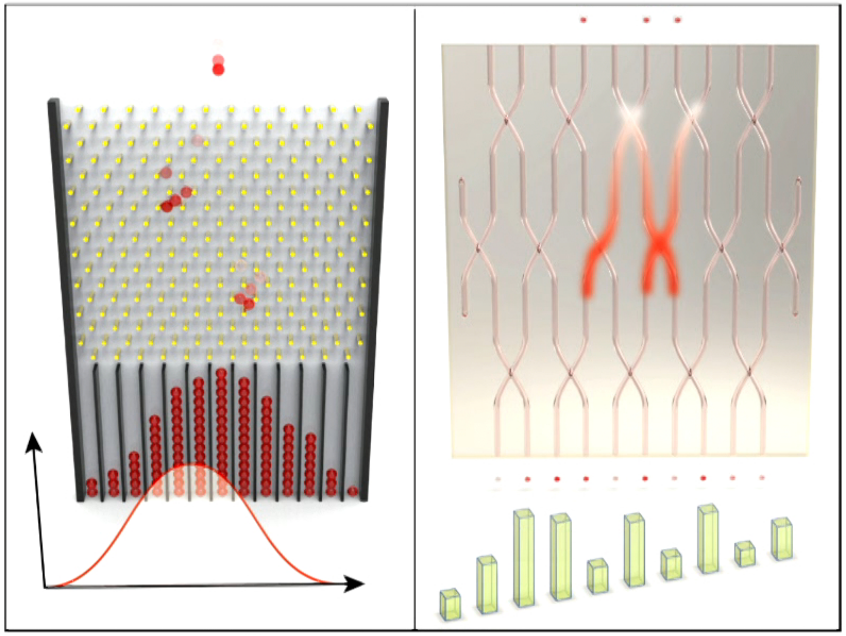
$$e(\mu_U, q) \leq 2q \, \mathcal{W}(\mu_U, \mu_{\text{Haar}}) \qquad \text{Brandao et al. (2016)}$$

Physical picture

In boson sampling experiments the photon distribution follows

$$|\mathrm{per}(\tilde{U})|^2 = \sum_{\sigma,\sigma'} \prod_{i,j=1}^{q} \tilde{U}_{i,\sigma(i)} \tilde{U}^*_{j,\sigma'(j)} = \mathrm{Tr}\left[U^{\otimes q,q}\mathcal{K}_{\mathrm{b.s.}}\right]$$

q is the number of injected photons – "quantum Plinko machine"

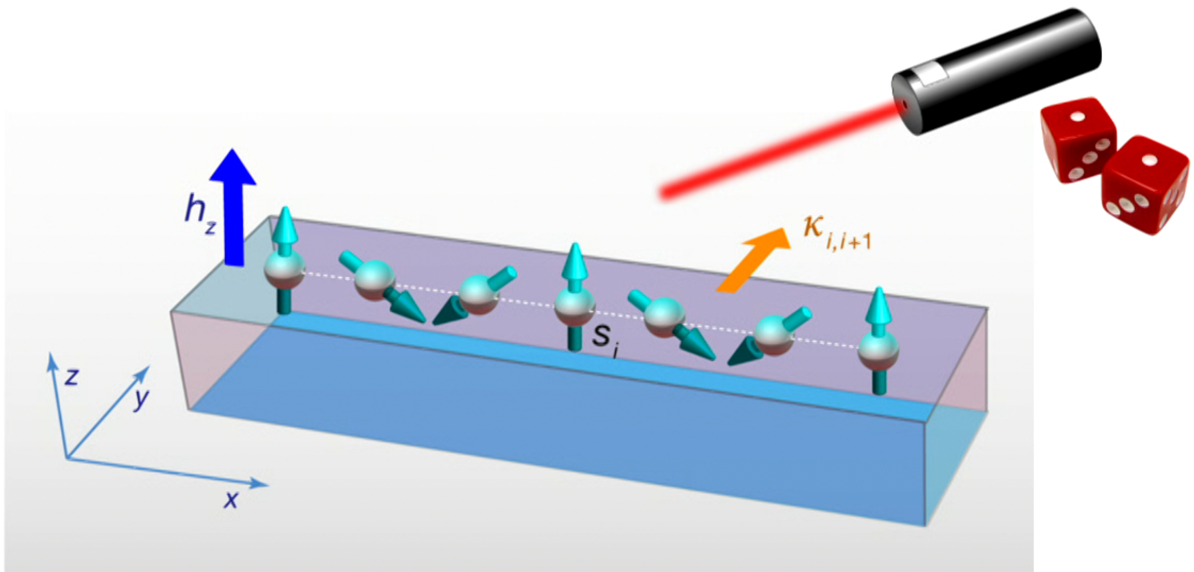


Physical picture

Convergence of spin correlations in randomly driven XY spin chain

$$\langle S_i^\alpha(t) S_{i+q}^\beta(t) \rangle = \mathrm{Tr}\left[U^{\otimes q,q} \mathcal{K}_{\mathrm{XY}} \right] \qquad \text{for } \alpha, \beta \in \{x, y\}$$

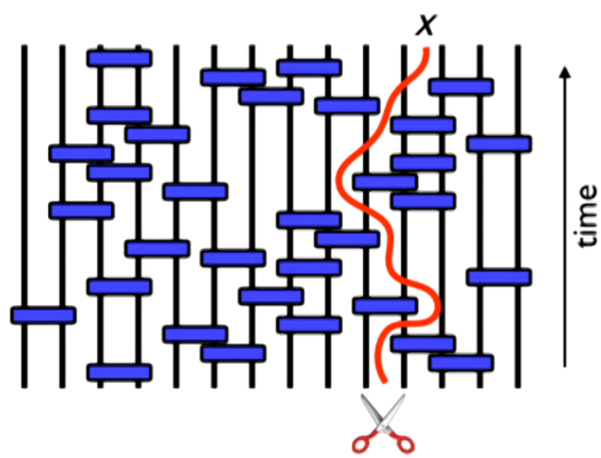
q is the distance between spins



Physical picture

Renyi entropies are studied in information scrambling

$$S_q = \frac{1}{1-q}\,\mathrm{Tr}[\rho^q] \equiv \frac{1}{1-q}\,\mathrm{Tr}[(\rho\otimes\rho\otimes\cdots\otimes\rho)\,\mathcal{P}]$$



If $\rho = \mathrm{Tr}_{\mathrm{ancilla}}[U\ |\psi\rangle\langle\psi|\ U^\dagger]$ then $\mathbb{E}[\mathrm{Tr}(\rho^q)] = \mathrm{Tr}\left[U^{\otimes q,q}\mathcal{K}_{\mathrm{Renyi}}\right]$

Results

Random pulse

Stochastic driving of a quantum system

$$\hat{H}(t) = H + g(t)V$$

e.g. with random amplitudes and phases

$$g(t) = \sum_{k=1}^{K} A_k \cos(\omega_k t + \varphi_k)$$

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The average is over the random amplitudes, phases etc.

$$\mathbb{E}_U \left[U^{\otimes q} \rho U^{\otimes q \dagger} \right] = \mathbb{E} \left[\left(\mathcal{T} \mathrm{e}^{-\mathrm{i} \int_0^T \hat{H}(s)\, ds} \right)^{\otimes q} \rho \left(\mathcal{T} \mathrm{e}^{\mathrm{i} \int_0^T \hat{H}(s)\, ds} \right)^{\otimes q} \right]$$

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Simplification when $g(t)$ is

- Gaussian (central limit theorem)
- Harmonic $\mathbb{E}[g(t+s)g(t)] = c(s)$
- Short correlations $Tc(Ts) \simeq \frac{\sigma}{2}\delta(s)$

Simplifications

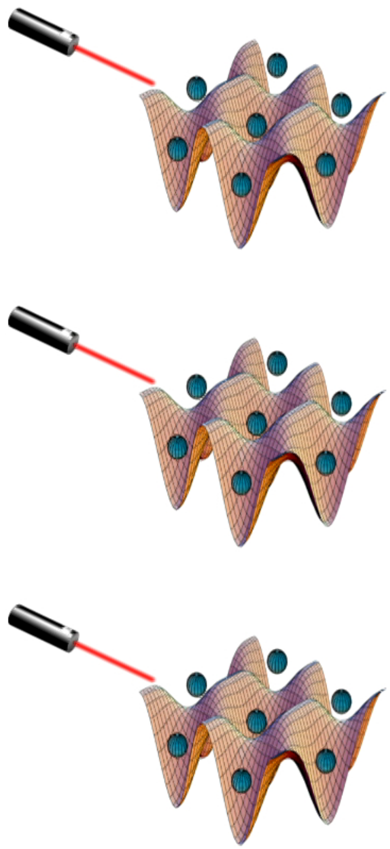
$$\mathbb{E}_U\left[U^{\otimes q}\rho U^{\otimes q\dagger}\right] = \mathbb{E}\left[\left(\mathcal{T}e^{-\mathrm{i}\int_0^T \hat{H}(s)\,ds}\right)^{\otimes q}\rho\left(\mathcal{T}e^{\mathrm{i}\int_0^T \hat{H}(s)\,ds}\right)^{\otimes q}\right]$$
$$\simeq e^{-T\mathcal{L}^q}\rho\ ,$$

where

$$\mathcal{L}^q\rho = -\mathrm{i}\left[\mathrm{H}^{\oplus\mathrm{q}},\rho\right] - \frac{\sigma}{2}\left[\mathrm{V}^{\oplus\mathrm{q}},\left[\mathrm{V}^{\oplus\mathrm{q}},\rho\right]\right]$$

$X^{\oplus q}{=}X\oplus X\oplus\ldots$, where $\oplus$ is the Kronecker sum $X\oplus Y{=}X\otimes 1+1\otimes Y$

Cartoon picture



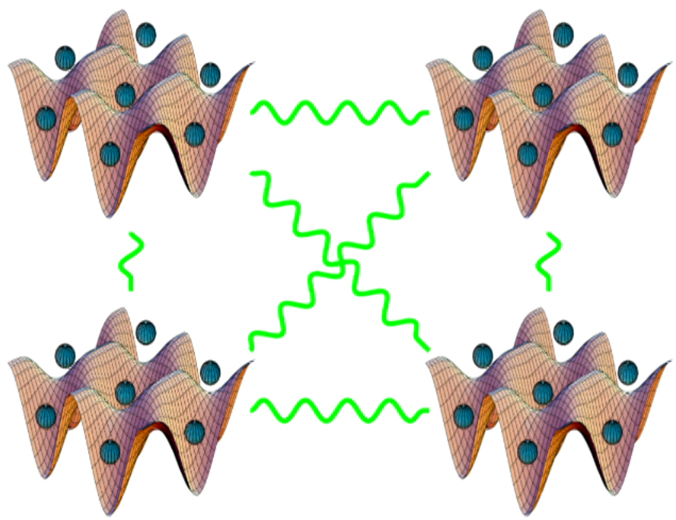
Each "replica" is initially

decoupled from the others

Cartoon picture

After the average over the random pulses these copies are interacting (*à la replica trick*), but in a dissipative way

$$\mathcal{L}^q\rho = -\mathrm{i}\left[\mathrm{H}^{\oplus \mathrm{q}}, \rho\right] - \frac{\sigma}{2}\left[\mathrm{V}^{\oplus \mathrm{q}}, \left[\mathrm{V}^{\oplus \mathrm{q}}, \rho\right]\right]$$



shown for $q = 4$

Controllability implies uniform blending

First central result

If $H + g(t)V$ is **fully-controllable**, then

$$\lim_{t\to\infty} e^{t\mathcal{L}^q}\rho = \int_{\mathrm{Haar}} U^{\otimes q}\rho U^{\otimes q\,\dagger}\, dU$$

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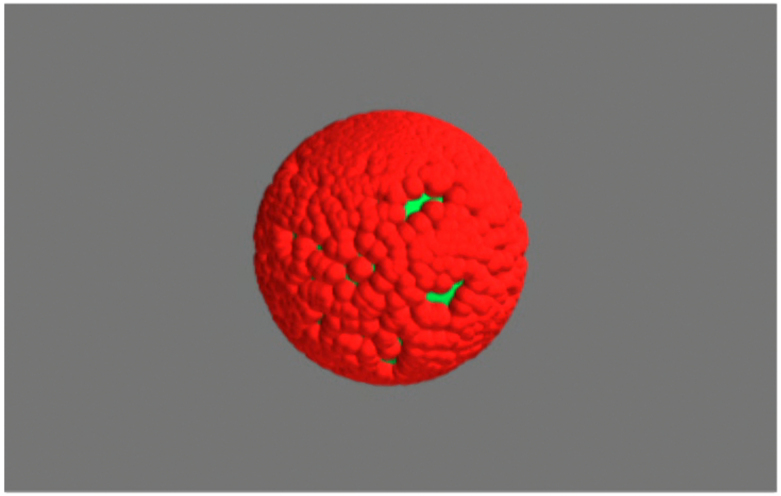

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Proof idea:

fully controllable means, H, V, $[H, V]$, $[H, [H, V]]$, $\ldots$ generate the full Lie algebra SU(d)

Schur Weyl duality: $(\mathbb{C}^d)^{\otimes q} = \otimes_\lambda \mathcal{P}^\lambda \otimes U^\lambda$
$\mathcal{P}^\lambda$ irreducible representation of the symmetric group S_q
$\mathcal{U}^\lambda$ irreducible representation of SU(d)

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Convergence time

$$\left\| e^{t\mathcal{L}^q} - \lim_{t\to\infty} e^{t\mathcal{L}^q} \right\|_\eta \lesssim A_\eta \, e^{-\lambda^* t}$$

λ^* is the Liouvillean gap, the eigenvalue of $-\mathcal{L}^q$ with minimal non-zero real part

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How do we estimate λ^*?

Complicated problem:

- $\mathcal{L}^q$ is formed by q interacting copies of the original Hilbert space
- Huge Hilbert space
- Restriction to low q is not enough

Many body theory

Let's write the Liouvillean using "vectorised" notation

$$\mathcal{L}_q = -i(H^{\otimes q} \otimes 1 - 1 \otimes H^{\otimes q}) - \frac{\sigma}{2}(V^{\otimes q} \otimes 1 - 1 \otimes V^{\otimes q})^2$$

Second quantized notation: for any operator H

$$H^{\otimes q} = \sum_{ij,u} H_{ij} a^{\dagger}_{iu} a_{ju}$$

q now is the total number of "virtual" particles

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Hubbard-like model (non Hermitean)

$$\mathcal{L}_q = -i \sum_{\alpha\beta u} H_{\alpha\beta}(a^{\dagger}_{\alpha u\uparrow} a_{\beta u\uparrow} - a^{\dagger}_{\beta u\downarrow} a_{\alpha u\downarrow})$$
$$- \frac{\sigma}{2} \sum_{\alpha\beta uv} V_{\alpha\alpha} V_{\beta\beta}(n_{\alpha u\uparrow} - n_{\alpha u\downarrow})(n_{\beta v\uparrow} - n_{\beta v\downarrow})$$

where $n_x = a^{\dagger}_x a_x$ and V is diagonal.

Mean field predictions

Mean field solution

The gap λ^* is independent on q

- Confirmed in "typical" numerical simulations
- Powerful result: the convergence time the same for all the moments[1]
- Validity because "everything interacts with everything"

[1] For certain choices of the norm...

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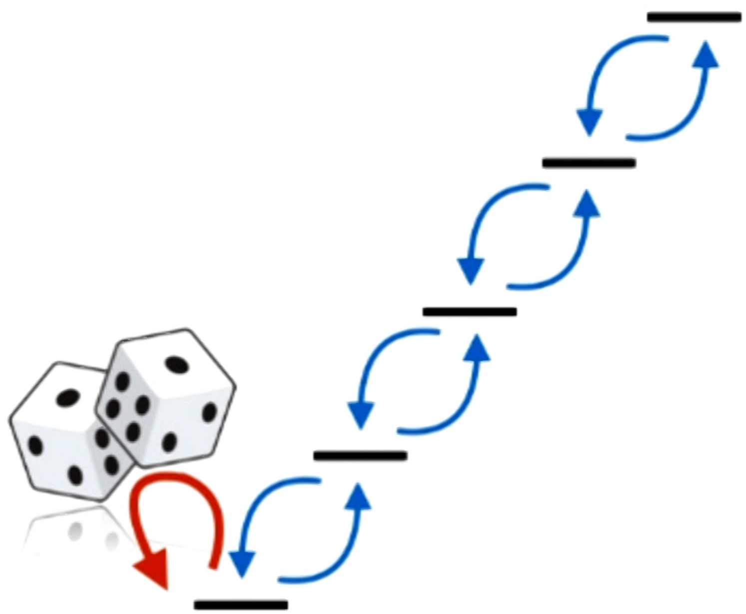
But...

we found (uncommon) **counterexamples**

- (replica?) symmetry breaking in tensor powers

[1] For certain choices of the norm...

Exactly solvable model: symmetric case



$$\mathcal{L}_q = -\mathrm{i}\sum_{\alpha}(\mathrm{a}^\dagger_{\alpha\uparrow}\mathrm{a}_{\alpha+1,\uparrow} - \mathrm{a}^\dagger_{\alpha\downarrow}\mathrm{a}_{\alpha+1,\downarrow} + \mathrm{h.c.}) - \frac{\sigma}{2}(\mathrm{n}^\uparrow_1 - \mathrm{n}^\downarrow_1)(\mathrm{n}^\uparrow_1 - \mathrm{n}^\downarrow_1)$$

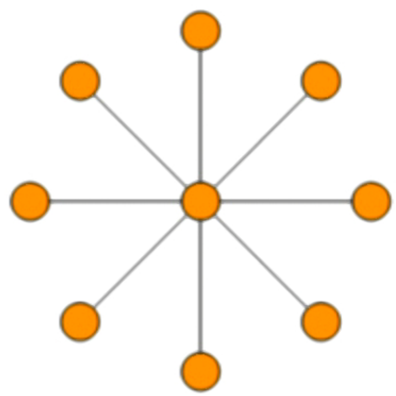
Exactly solvable model: symmetric case

Strong driving limit $\sigma \gg 1$

$$K_i^+ = \tilde{a}_{i\uparrow}^\dagger \tilde{a}_{i\downarrow}^\dagger$$
$$K_i^- = (K_i^+)^\dagger$$
$$K_i^z = (\tilde{n}_{i\uparrow} + \tilde{n}_{i\downarrow} + 1)/2$$

SU(1,1) Richardson-Gaudin model

$$\hat{\mathcal{L}}_q = \frac{2}{\sigma} - \frac{8}{\sigma} \sum_{k=1}^{L-1} g_k K_0 \cdot K_k$$



Exactly solvable model: symmetric case

Spectrum from Bethe Ansatz

$$\lambda = -\frac{2}{\sigma}\left(\sum_k g_k n_k + 4\sum_\alpha \frac{1}{\omega_\alpha}\right)$$

n_k is the number of unpaired particles in mode k

$$\sum_k \frac{n_k + 1}{\omega_\alpha - 2g_k^{-1}} + \frac{1}{\omega_k} + 2\sum_{\beta\neq\alpha}\frac{1}{\omega_\alpha - \omega_\beta} = 0$$

Solutions related to the roots of Heine-Stieltjes polynomials, so $2g_{k+1}^{-1} < \omega_\alpha < 2g_k^{-1}$

The gap is made with unpaired particles

$$\text{gap} \equiv \lambda^* = \frac{8}{\sigma L}\sin^2\left(\frac{\pi}{L}\right) = \mathcal{O}(L^{-3})$$

L is the length. The gap is independent on q

Exactly solvable model: other cases

Fermionic representation

$S_j^- = \tilde{a}_{j\uparrow}\tilde{a}_{j\downarrow}$
$S_j^+ = (S_j^-)^\dagger$
$S_j^z = (\tilde{a}_{j\uparrow}^\dagger\tilde{a}_{j\uparrow} + \tilde{a}_{j\downarrow}^\dagger\tilde{a}_{j\downarrow} - 1)/2$

SU(2) Gaudin model

$$\hat{\mathcal{L}}_q = -\frac{2}{\sigma} + \frac{8}{\sigma}\sum_{k=1}^{L-1} g_k\, S_0 \cdot S_k$$

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SU(2) Gaudin model

$$\hat{\mathcal{L}}_q = -\frac{2}{\sigma} + \frac{8}{\sigma}\sum_{k=1}^{L-1} g_k\, S_0 \cdot S_k$$

Generic case

$$X^{(j)}_{(x,\uparrow),(y,\uparrow)} = \frac{\tilde{a}_{jx\uparrow}^\dagger \tilde{a}_{jy\uparrow} - \tilde{a}_{jy\uparrow}\tilde{a}_{jx\uparrow}^\dagger}{2} ,$$
$$X^{(j)}_{(x,\downarrow),(y,\downarrow)} = \frac{\tilde{a}_{jx\downarrow}\tilde{a}_{jy\downarrow}^\dagger - \tilde{a}_{jy\downarrow}^\dagger \tilde{a}_{jx\downarrow}}{2} ,$$
$$X^{(j)}_{(x,\uparrow),(y,\downarrow)} = \tilde{a}_{jx\uparrow}^\dagger W \tilde{a}_{jy\downarrow}^\dagger ,$$
$$X^{(j)}_{(x,\downarrow),(y,\uparrow)} = \tilde{a}_{jx\downarrow} W \tilde{a}_{jy\uparrow} .$$

SU(2q) Gaudin model

$$\hat{\mathcal{L}}_q = -\frac{2q}{\sigma} + \frac{4}{\sigma}\sum_{k=1}^{L-1} g_k \sum_{\alpha\beta} X^{(0)}_{\alpha\beta} X^{(k)}_{\beta\alpha} ,$$

Exactly solvable model: generic cases

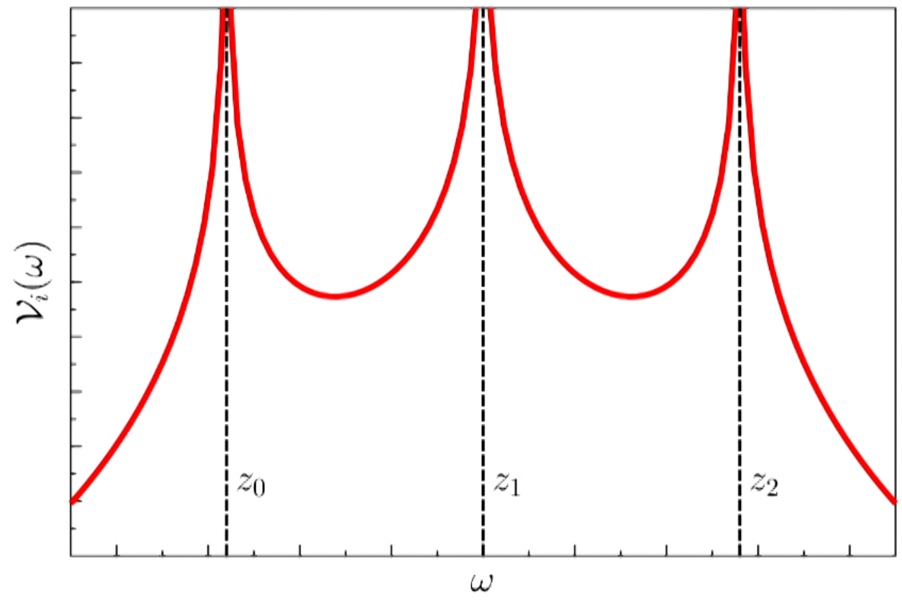
Spectrum from Bethe Ansatz

$$\lambda = -\frac{2}{\sigma}\left[\sum_{k=1}^{L-1} g_k \left(n_{\downarrow k} + n_{\uparrow k}\right) + 4\sum_{\alpha}\frac{1}{\omega_{q,\alpha}}\right]$$

n_k is the number of unpaired particles in mode k

$$\sum_{\beta}\frac{2}{\omega_{j,\beta}-\omega_{j,\alpha}} = \sum_{k=0}^{L-1}\frac{\mu_j^k}{z_k-\omega_{j,\alpha}} + \sum_{\beta,\pm}\frac{1}{\omega_{j\pm 1,\beta}-\omega_{j,\alpha}}$$

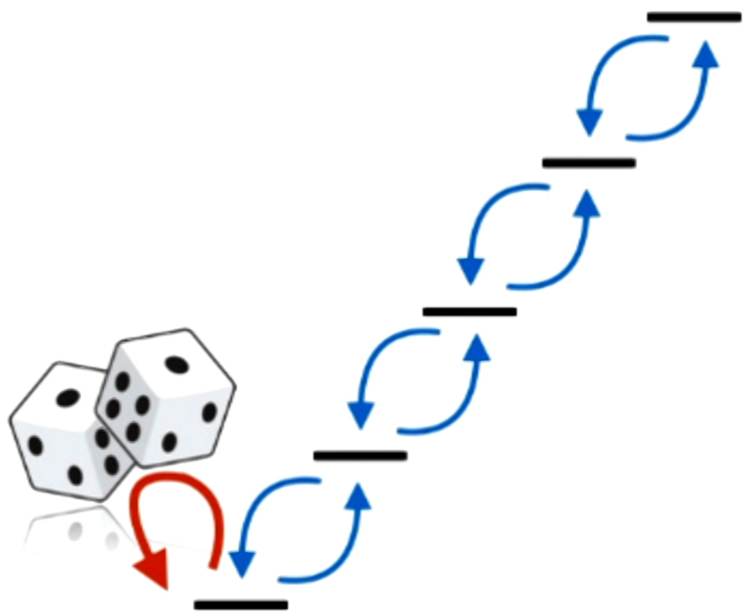
Exactly solvable model: generic cases



Final result

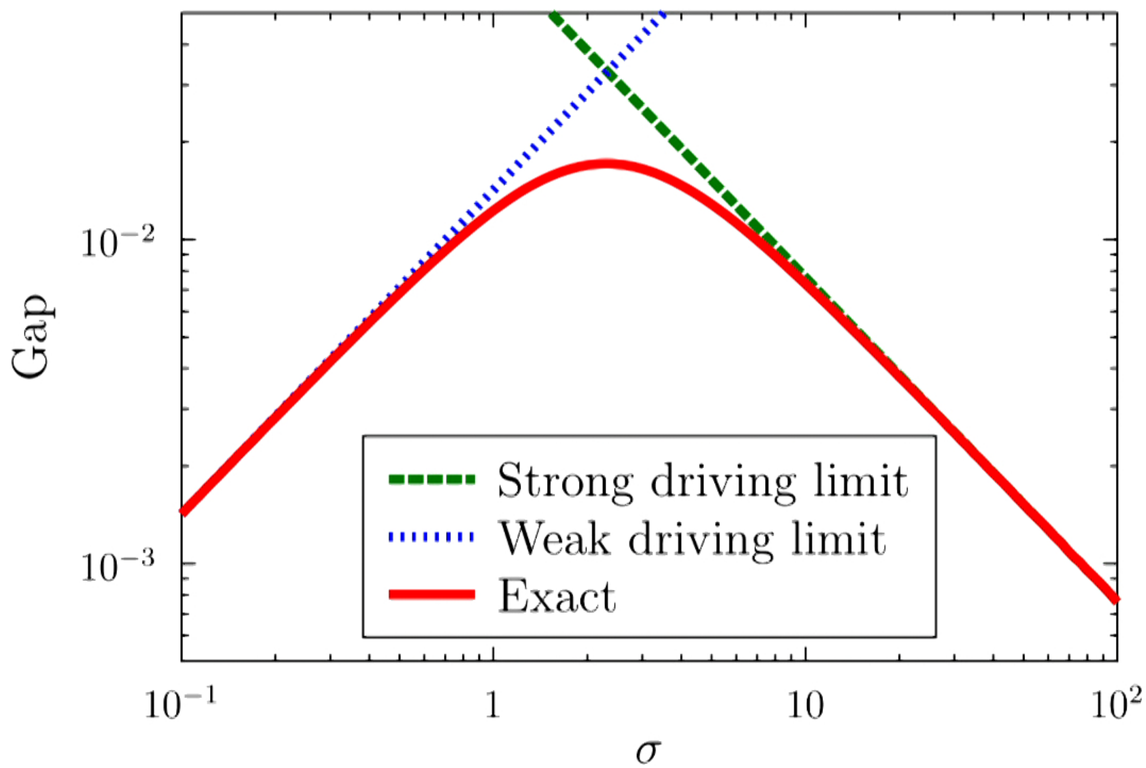
- Gap independent on q
- Mean field analysis is rigorous

Exactly solvable model: symmetric case



$$\mathcal{L}_q = -\mathrm{i}\sum_{\alpha}(\mathrm{a}^{\dagger}_{\alpha\uparrow}\mathrm{a}_{\alpha+1,\uparrow} - \mathrm{a}^{\dagger}_{\alpha\downarrow}\mathrm{a}_{\alpha+1,\downarrow} + \mathrm{h.c.}) - \frac{\sigma}{2}(\mathrm{n}^{\uparrow}_{1} - \mathrm{n}^{\downarrow}_{1})(\mathrm{n}^{\uparrow}_{1} - \mathrm{n}^{\downarrow}_{1})$$

Numerical check



Finally?

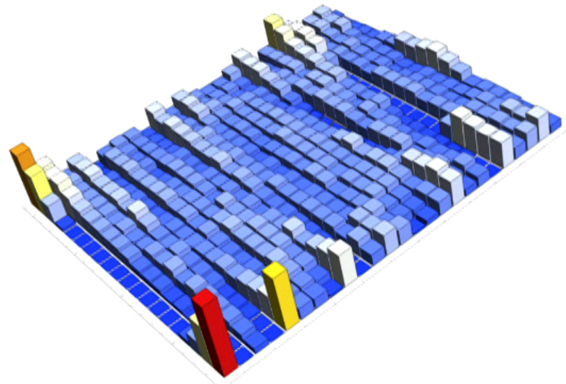
Putting things together

- We need full controllability
- All the moments (typically) converge at the same time
- Open quantum system theory to estimate this time

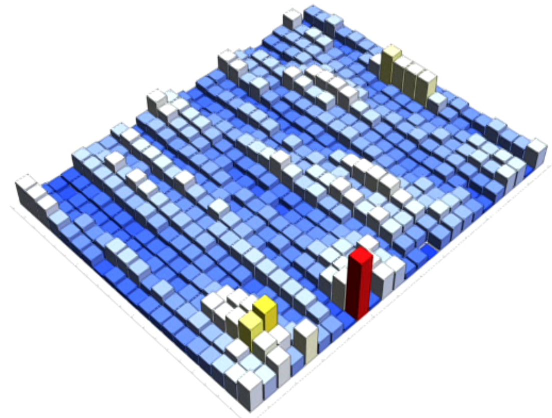
Numerical tests with stochastic pulse

$$g(t) = \sum_{k=1}^{K} A_k \cos(\omega_k t + \varphi_k)$$

Non controllable case



Controllable, but short time

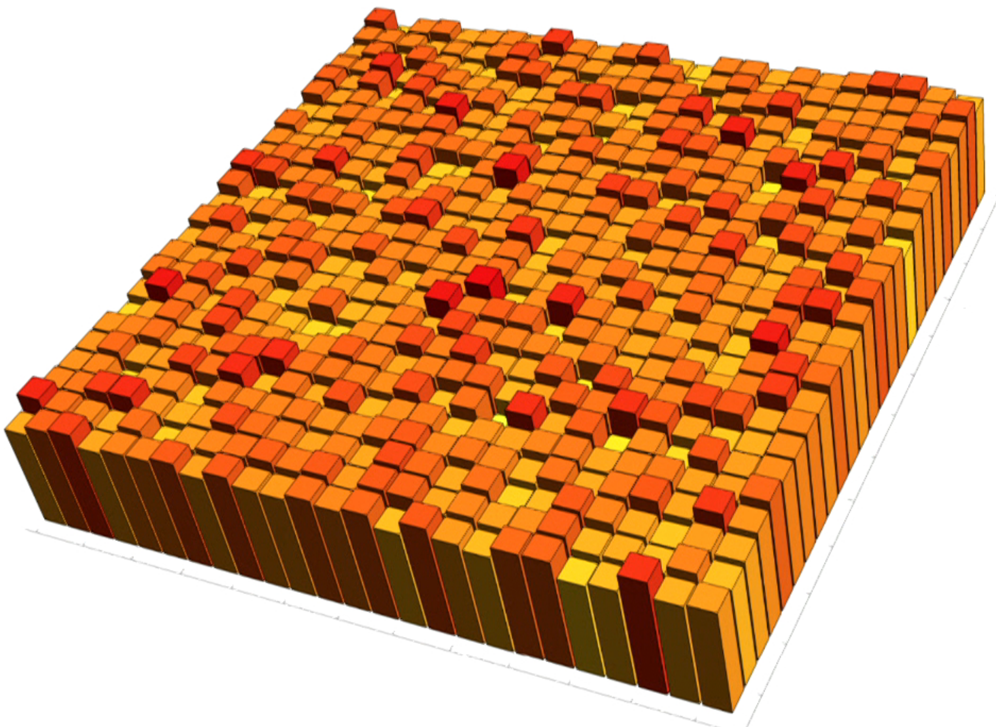


Angle decomposition

$$dU(\varphi_1, \ldots, \varphi_{L^2}) = \prod_{j=1}^{L^2} d\varphi_j \ ,$$

Numerical tests with stochastic pulse

Fully controllable case, after the blending time



Conclusions

Answered this question

When, and how rapidly, a quantum system subject to dynamical noise produces a fully-random (i.e. Haar-uniform) distribution of unitary evolutions?

Different tools from

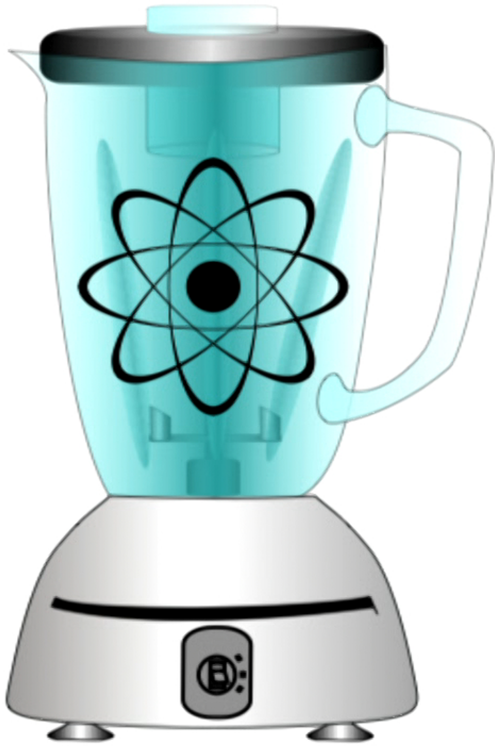
- quantum information (quantum control, q-design)
- open quantum systems (dynamical semigroup, "low energy" Liouvilleans)
- condensed matter physics (Bethe ansatz, mean field in replica space)

Explicit applications:

- Boson sampling experiments
- Estimation of the control time
- Entanglement generation in many-body settings

L. Banchi, D. Burgarth, M. J. Kastoryano, Phys. Rev. X 7, 041015 (2017)

Questions?



Yes, it blends!