Title: Normative probability in quantum mechanics

Date: Apr 11, 2018 10:00 AM

URL: http://pirsa.org/18040113

Abstract: In this talk I compare the normative concept of probability at the heart of QBism with the notion of probability implied by the use of Solomonoff induction in Markus Mueller's preprint arXiv:1712.01816.

Four tenets of QBism (see, e.g., Wikipedia)

1. All probabilities, including those equal to zero or one, are valuations that an agent ascribes to his or her degrees of belief in possible outcomes.

2. The Born rule is normative, not descriptive or prescriptive.

3. Quantum measurement outcomes are personal experiences for the agent gambling on them.

4. A measurement apparatus is conceptually an extension of the agent.

Tenet 1

1. All probabilities, including those equal to zero or one, are valuations that an agent ascribes to his or her degrees of belief in possible outcomes.

- Decision-theoretic, personalist probabilities
- No requirement that different agents assign the same probabilities
- QBism also explicitely rejects the key assumptions of Harrigan and Spekkens' ontological model framework, namely that there is an ontological variable λ that determines the outcome probabilities of a measurement.

Tenet 2

2. The Born rule is normative, not descriptive or prescriptive.

- It's not a law that constrains how nature behaves.
- It is a consistency requirement that guides an agent's decision making.

Tenet 3

3. Quantum measurement outcomes are personal experiences for the agent gambling on them.

- Wigner's friend

Tenet 4

4. A measurement apparatus is conceptually an extension of the agent.

- Agent $\neq$ Person
- An agent is an entity that is capable of using the Born rule normatively.

Three routes to QBism

Route 1 starts with de Finetti 1931

Route 2 starts with Einstein 1935

Route 3 starts with Wigner 1961

Route 3, starting with Wigner

Wigner 1961: Wigner's Friend

Caves, Fuchs, RS 2007: *Subjective probability and quantum certainty*: "facts for the agent"

Fuchs, Mermin, RS 2014: *An Introduction to QBism with an Application to the Locality of Quantum Mechanics*: "experience"

no-go theorems by Brukner 2015, Frauchiger and Renner 2016: strengthen the QBist case

Route 2, starting with Einstein

Einstein 1935 (letter to Schrödinger): assuming λ and locality $\Longrightarrow \psi$ is not a function of λ.

Bell 1961: assuming λ and locality and quantum theory $\Longrightarrow$ contradiction

QBism: rejects λ (and hence reject the EPR criterion of reality)

no-go theorems from PBR to Myrvold 2018: strengthen the QBist case

Lambda

The assumption of an ontological model:

For any measurement on a physical system, either the outcomes or their probabilities are determined by the system's real properties, λ. (Harrigan and Spekkens, 2007).

(Potentially misleading alternative labels for the same idea: "hidden variables", "realism".)

No-go theorems

Einstein 1927

Assuming λ (elements of physical reality) and locality (no spooky action at a distance) implies that ψ is not in one-to-one correspondence with λ.

Einstein 1935 (letter to Schrödinger, not EPR)

Assuming λ and locality implies ψ is not determined by λ.

PBR, Colbeck and Renner, Hardy, Gisin, . . .

Assuming λ plus further assumptions implies ψ is determined by λ.

Bell

Assuming λ and locality contradicts quantum mechanics.

Einstein to Schrödinger (1935, not EPR)

Consider the state $|\psi^{AB}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)$,

where $|0\rangle$ and $|1\rangle$ are the eigenstates of the spin Z operator.

Now, $\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) = \frac{1}{\sqrt{2}}(|+\rangle|+\rangle + |-\rangle|-\rangle)$,

where $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ are the eigenstates of the spin X operator.

Let $|\psi^{B}\rangle$ be the conditional state after a measurement on A:

- A measures Z.
- A measures X.

- $|\psi^{B}\rangle \in \{|0\rangle, |1\rangle\}$
- $|\psi^{B}\rangle \in \{|+\rangle, |-\rangle\}$

Einstein to Schrödinger (1935, not EPR)

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Einstein:

"[...] the real state of (AB) consists precisely of the real state of A and the real state of B, which two states have nothing to do with one another. The real state of B thus cannot depend upon the kind of measurement I carry out on A."

Implication, assuming locality (Caves,Fuchs,RS 2002):

$|\psi^B\rangle$ is not a function of "the real state at B", i.e., $|\psi^B\rangle$ is not a real property of the system at B.

Loophole-free Bell experiment

You have to give up either locality or λ.

QBism rejects λ.

Route 1, starting with de Finetti

de Finetti 1931: *Probabilismo*. "probability does not exist"

Savage 1954: *The Foundations of Statistics*. Probability from decision theory

Caves, Fuchs, RS 2002: Quantum probabilities as Bayesian probabilities

Spekkens 2004: *In defense of the epistemic view of quantum states: a toy theory*, gives compelling arguments for an epistemic view of quantum states, even if from an ontological perspective

Fuchs, RS 2013: *Quantum Bayesian coherence*, personalist probability in QBism fully spelled out

Decision-theoretic personalist probability

My probability of an event E is p if I regard \$$p$ as the fair price of a lottery ticket that pays \$1 if E occurs.

Consistency (or Dutch-book coherence)

My probabilities (i.e., valuations) do not satisfy the rules of probability theory
$\Longrightarrow$ Sure Loss

Example: $p = 1.5$ ("\$1.50 is a fair price for a ticket that pays \$1 in the best case.")

Probability theory as a tool to detect inconsistency

3 coins are tossed.

Example 1: $p(HHH) = 0.5$ OK

Example 2: $p(\text{at least } 2H) + p(\text{at least } 2T) = 0.9$ not OK

- Probability theory provides relations between probabilities.
- It does not tell you what the correct probabilities are.

Normative versus descriptive rules

Example (Allais):

1. Given a choice between 1a or 1b, which do you choose?
2. Given a choice between 2a or 2b, which do you choose?

	1a	**1b**	**2a**	**2b**
payoff	p	p	p	p
0		0.01	0.89	0.9
$\$5 \times 10^6$	1	0.89	0.11	
$\$10 \times 10^6$		0.10		0.1

"Normative": How should you act?

"Descriptive": How do irrational agents act?

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Frequencies and repeated trials

Frequencies are data—probabilities are degrees of belief

Frequencies can be assigned probabilities. Probabilities can be refined on the basis of measured frequencies.

Exchangeability characterises repeated trials

For N trials,

$\rho^{(N)}$ exchangeable $\Longrightarrow \rho^{(N)} = \int \rho \otimes \rho \otimes \ldots \otimes \rho \, d\rho$

(this is the quantum de Finetti theorem)

Quantum Baysianism?

Bayes rule $p_{\text{Wed}}(AB) = p_{\text{Wed}}(A|B)p_{\text{Wed}}(B)$ follows from consistency (no sure loss)

Assume you observe B on Thursday morning:

$p_{\text{Thu}}(A) = p_{\text{Wed}}(A|B)$ requires additional assumptions

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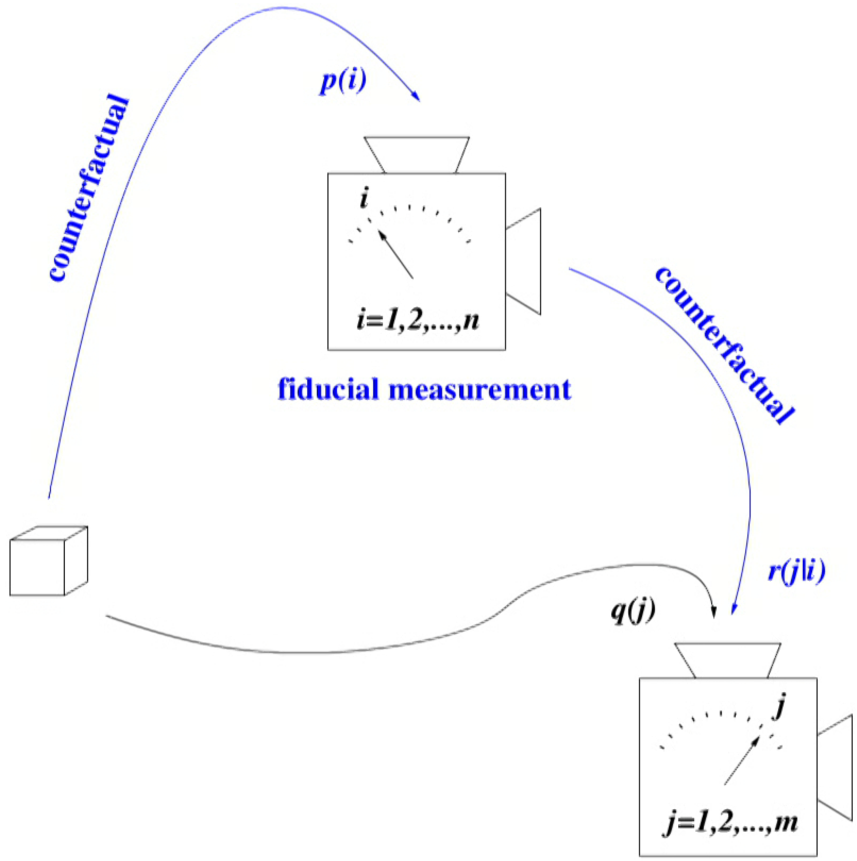
Bayesian updating is not fundamental!

QBism should really be Quantum Bettabilitarianism (following Oliver Wendell Holmes)

The Born as a normative consistency requirement



Born rule

$q(j) = \mathrm{tr}(\rho E_j)$

quantum state

$\rho \longleftrightarrow p(i)$

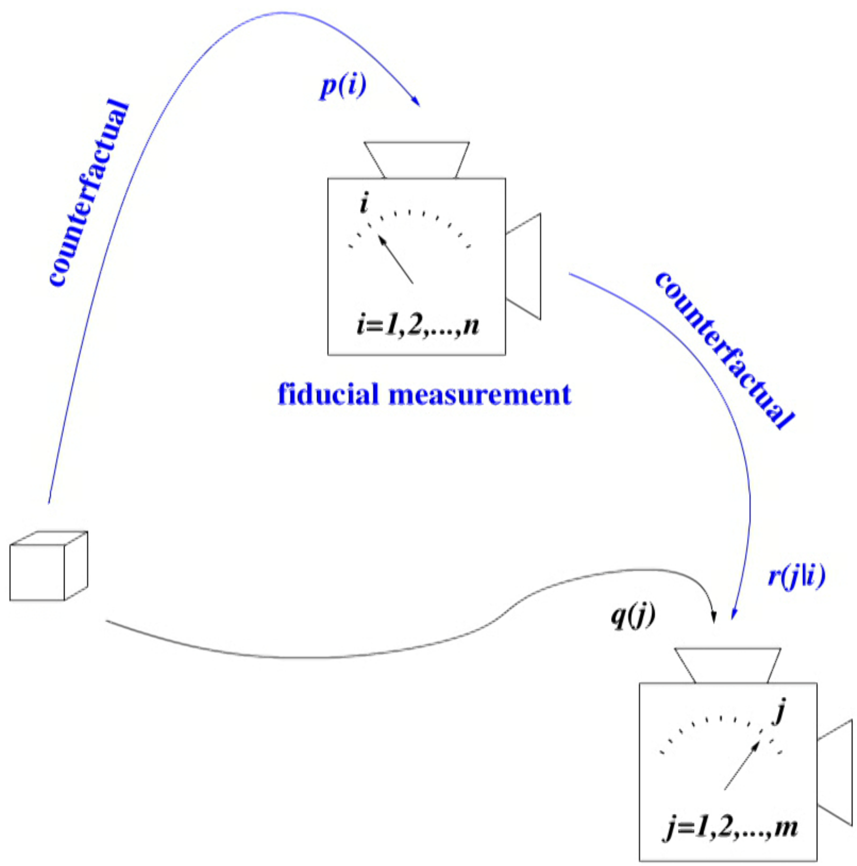
POVM

$\{E_j\} \longleftrightarrow r(j|i)$

Born rule, rewritten

$q(j) = f\big(p(i), r(j|i)\big)$

The Born as a normative consistency requirement



Born rule, rewritten

$$q(j) = f\left(p(i), r(j|i)\right)$$

Markus Müller's postulates

1. Every observer is described by an observer graph A; the complete list of all observations that the observer successively experiences corresponds to the sequence of binary strings in an A-history.

2. After having experienced an A-history $x = (x_1, \ldots, x_n)$, the observer will subsequently experience one of the strings from the set of possible next strings, $A(x_n)$, at random. The probability of any $y \in A(x_n)$ is given by the conditional algorithmic probability $P(y|x; A)$.

(An *observer graph* is a (computable, rooted) directed graph over the finite binary strings. An *A-history* is a path through the observer graph A, starting at its root.)

Key prediction of Markus' theory

"With high probability, observers see simple probabilistic 'laws of nature', and find themselves to be part of a larger dynamical system that they may call 'the universe'."

So: observers are fundamental and participate in the creation of the world.

And yet the theory is incompatible with QBism.

Probabilities

Markus: A fundamental stochastic process governed by conditional algorithmic probabilities $P(y|x;A)$.

QBism: A set of normative consistency criteria, where Bayesian updating is not fundamental. A world teeming with novelty that can not be captured by a stochastic process.

What is an agent?

Markus: Every [agent] is described by an observer graph A.

QBism's tenet 4: A measurement apparatus is conceptually an extension of the agent.

Hence, in QBism, the boundary between agent and world is fluid.

Participatory realism

In Markus' theory, observers are fundamental and the world emerges.

In QBism, both agents and the world are fundamental. QBism's very starting point is an agent's action on the external world.

QBism is participatory realism.