Title: On the concepts of universality in physics and computer science

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Abstract: A central fact in computer science is that there are universal machines, that is machines that can run any other program. Recently, a somewhat similar notion of universality has been discovered in physics, by which some spin models can simulate all other models. In this work we shed light on the relation between the two concepts of universality

On the concepts of universality in physics and computer science

Work in progress: mostly thoughts, perspectives, …

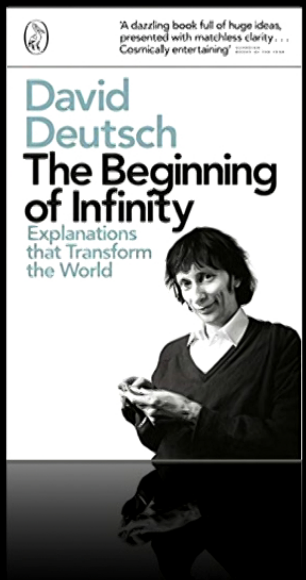
Gemma De las Cuevas (Innsbruck, Austria)

Joint work with Georg Moser

Algorithmic Information, Induction and Observers in Physics. PI, April 10, 2018

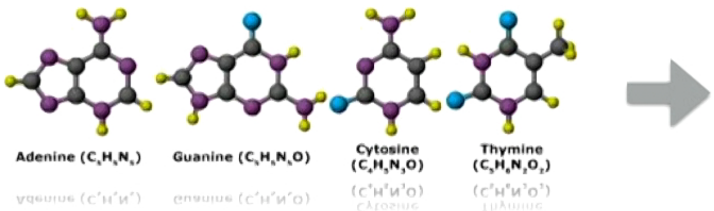
“Our universe is of astonishing simplicity: almost all physical observations can in principle be described by a few theories that have short mathematical descriptions.”

The concept of universality

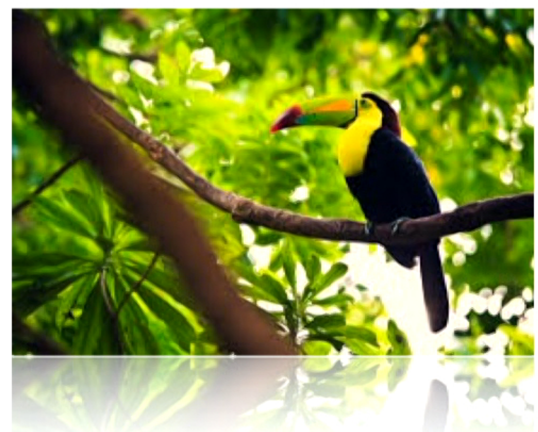


How “easy” it is to generate complexity

Genetic code



Biosphere



How “easy” it is to generate complexity

Alphabet

А Б В Г Д Е
Ё Ж З И Й К
Л М Н О П Р
С Т У Ф Х Ц
Ч Ш Щ Ъ Ы Ь
Э Ю Я

Language


БРАТЬЯ КАРАМАЗОВЫ

РОМАНЪ

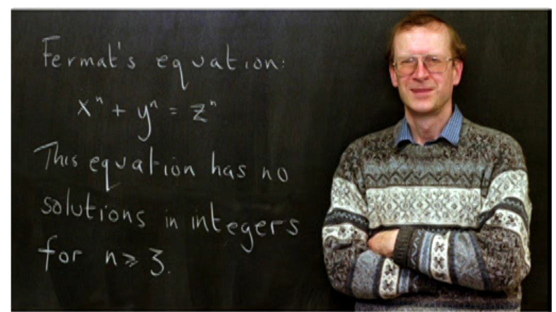
ОТЪ АВТОРА.

How “easy” it is to generate complexity

Axioms of the natural numbers

- 0 is a natural number
- every natural number has a successor
- 0 is not the successor of any natural number
- If the successor of x equals the successor of y, then x equals y
- the axiom of induction: if a statement is true of 0, and if the truth of that statement for a number implies its truth for the successor of that number, then the statement is true for every natural number.

Arithmetic



The jump to universality

When systems gradually become more complex,

they suddenly undergo a large change of functionality.

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They can suddenly can capture all possible complexity.

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They become *universal.*

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They become *universal.*

They are at the beginning of infinity.

Universality in writing

- Writing systems based on sounds: Universal

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capable of representing every possible word with a finite amount of symbols

Universality in writing

- Writing systems based on pictograms: Not universal

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Imagine you want to write TREASON but have no pictogram for it:

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Imagine you want to write TREASON but have no pictogram for it:

- You can invent a new pictogram.

Universality in writing

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Imagine you want to write TREASON but have no pictogram for it:

- You can invent a new pictogram.
- Or you can invent a rule:

Use the pictograms for **T**ent **R**ock **EA**gle **Z**ebra **N**ose

This rule triggers the jump to universality.

Universality in numeral systems

- Arabic numeral system: Universal

"2" can take infinitely many meanings: 2

20

2000

….

Universality in numeral systems

- Arabic numeral system: Universal

"2" can take infinitely many meanings: 2

20

2000

….

capable of representing every possible number with a finite amount of symbols (ten)

Universality in numeral systems

- Roman numeral system: Not universal

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  For numbers larger than a thousand, MMMMM….. Like tally marks

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  Imagine you want to write one million:

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Universality in numeral systems

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  For numbers larger than a thousand, MMMMM….. Like tally marks

  Imagine you want to write one million:

  - You could invent a new symbol
  - Or invent a new rule that triggers the jump to universality

Universality in acquisition of knowledge

- Critical mind: Universal

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"Problems can be solved" ➡ Open-ended generation of knowledge

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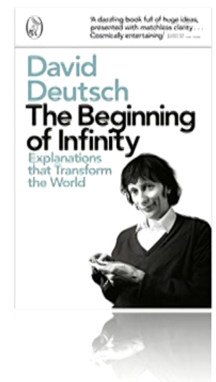

People become universal explainers.

Universality in acquisition of knowledge

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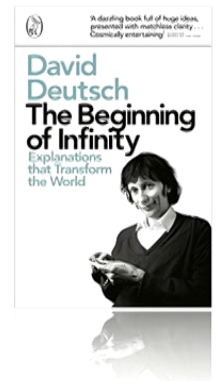

People become universal explainers.

- Knowledge given by authority (e.g. religion): Not universal.

The jump to universality

Some lessons

- It requires very little complexity of the underlying elements, but the correct "rules".

The jump to universality

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- It can only happen in digital systems

  since error correction is only possible in digital systems

  (in analogue systems every value can be correct)

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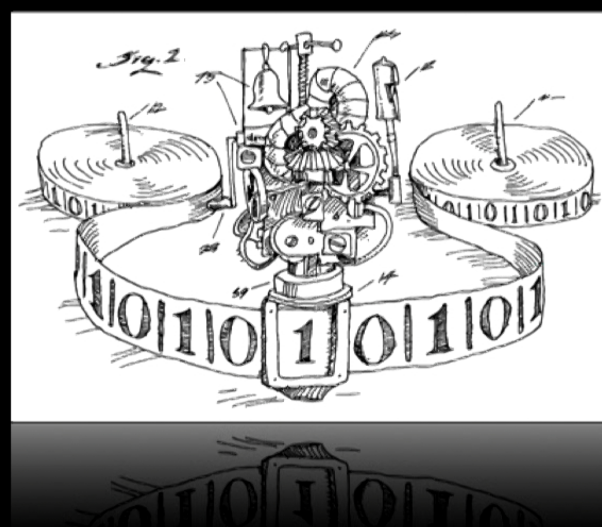
  (in analogue systems every value can be correct)

  The writing system a, b, c, …

  The numeral system 0, 1, 2 , …

  The genetic code A G T C

Universality in computer science



Universality in computer science

In the early days of computer science….

- Formalization of the notion of computation:

  Turing machines, Post systems,

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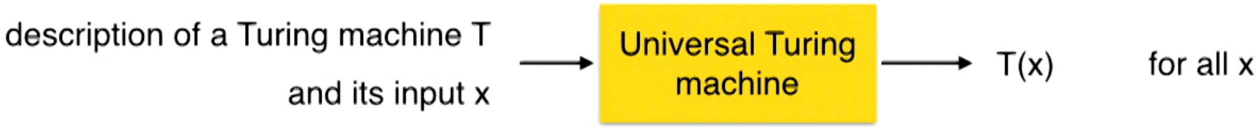

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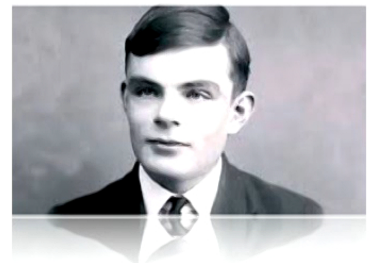

- Formalization of the notion of computation:

  Turing machines, Post systems,

  Recursive functions, lambda calculus, …

- Certain machines are "universal":

description of a Turing machine T and its input x → **Universal Turing machine** → T(x) for all x

it has finitely many rules, yet it can run any possible computation

Universality in computer science

A program can be interpreted as data, and thus fed to another program.

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Two concepts which were different (data and programs) are put at the same level

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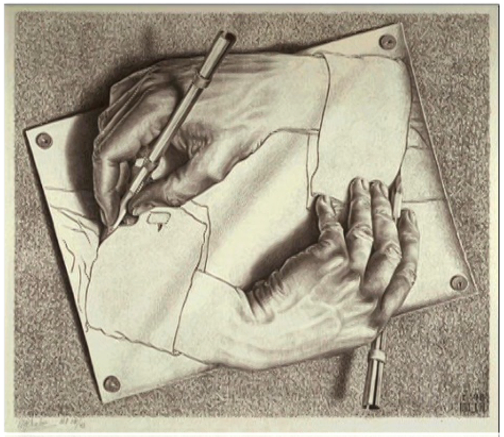
➡ self-reference



Undecidability

Self-reference and negation

“I am a liar”

Undecidability

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Cantor’s diagonal argument:

	1	2	3	4	5	….
1st set	0	1	1	1	1	….
2nd set	1	0	0	1	1	….
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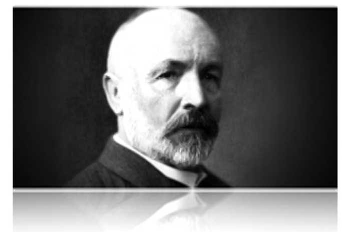
…						



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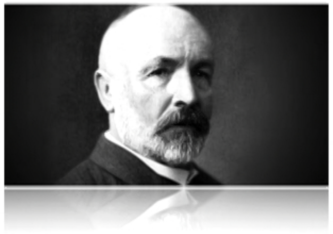

→ The natural numbers and its power set cannot be put in one-to-one correspondence.

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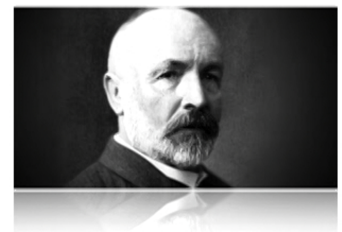

➡ The natural numbers and its power set cannot be put in one-to-one correspondence.

Gödel’s proof is also a diagonal argument.

Undecidability

Some lessons

- Crucial that the set is infinite

Undecidability

Some lessons

- Crucial that the set is infinite

- Against reductionism:

  this is a "fundamental truth" that appears at a higher level

  it happens whenever the system is capable of doing self-reference

Undecidability in maths

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- Almost every function is uncomputable:

  Set of Turing machines $\mathbb{N}$

  Set of functions $f : \mathbb{N} \to \mathbb{N}$

  Thus, most functions are uncomputable.

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- Many "easy" problems are undecidable:

  Take 5 3x3 matrices with integer entries: $M_1 = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$ ….. $M_5 = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$

  Is there a product of them which gives the 0 matrix? $M_1 M_2 M_2 M_1 M_5 \ldots$

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- By making reductions, these problems spread everywhere

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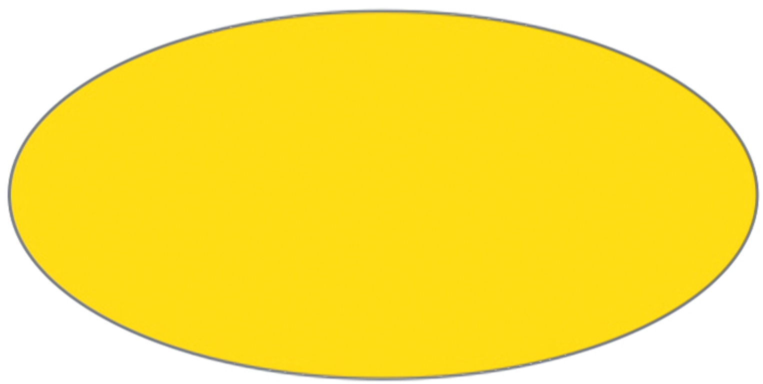
- A language is called recursively enumerable if it is recognized by a Turing machine.

Undecidability in computer science

- Consider the set of recursively enumerable languages:



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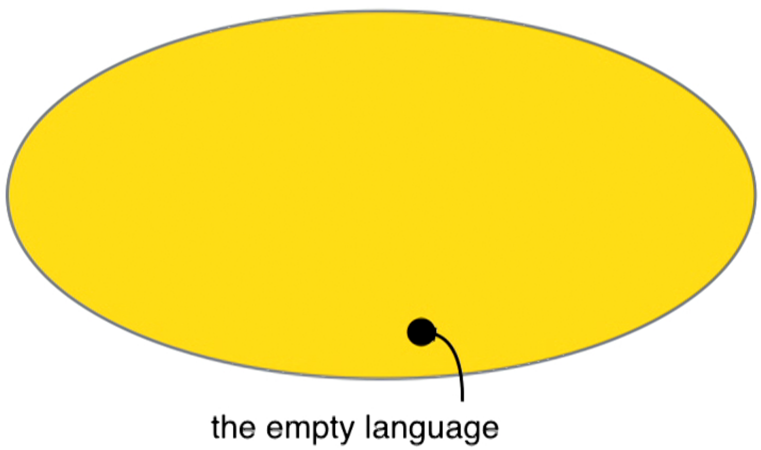

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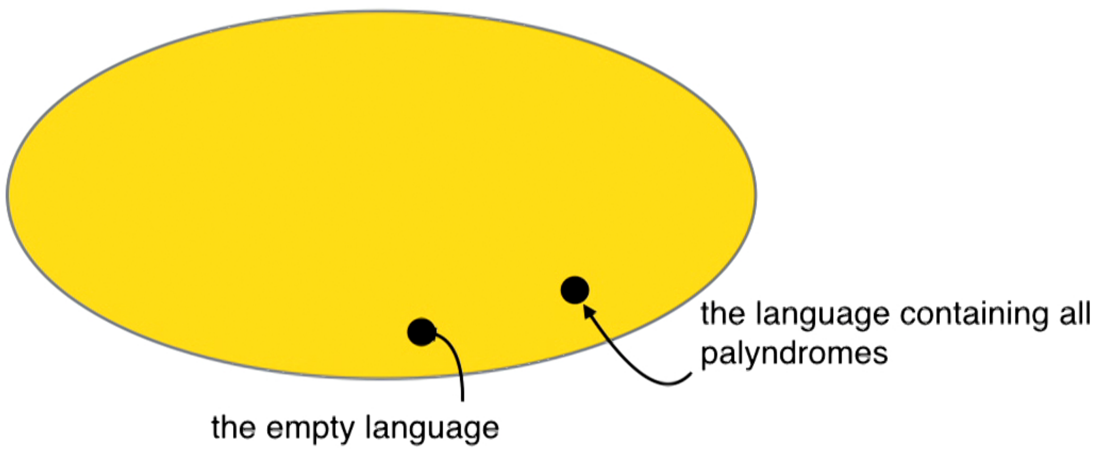

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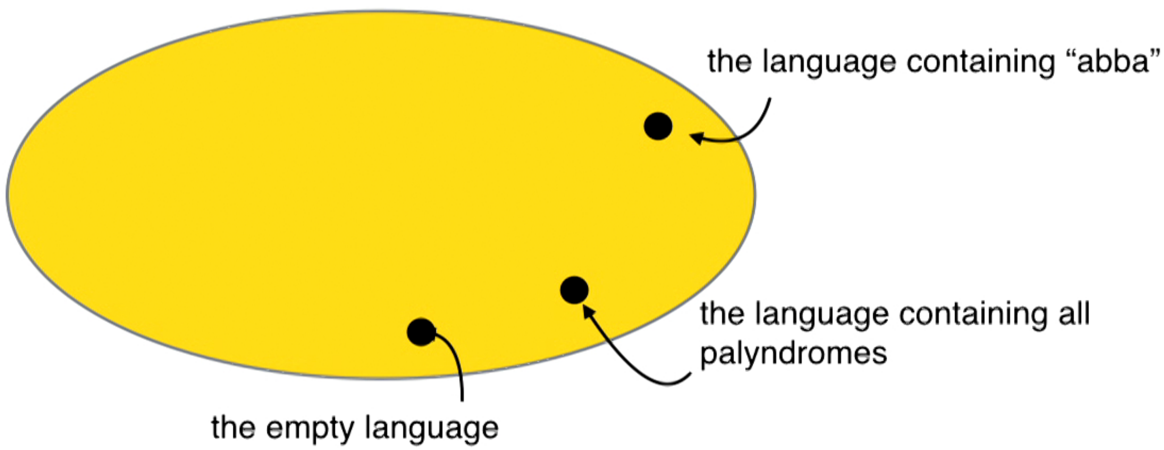

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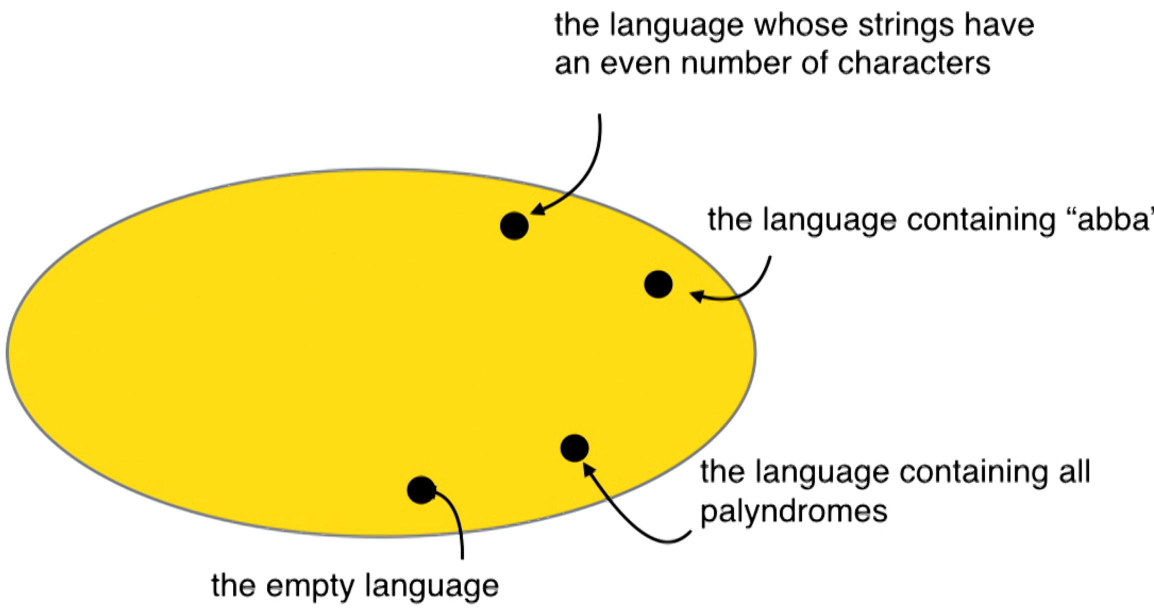

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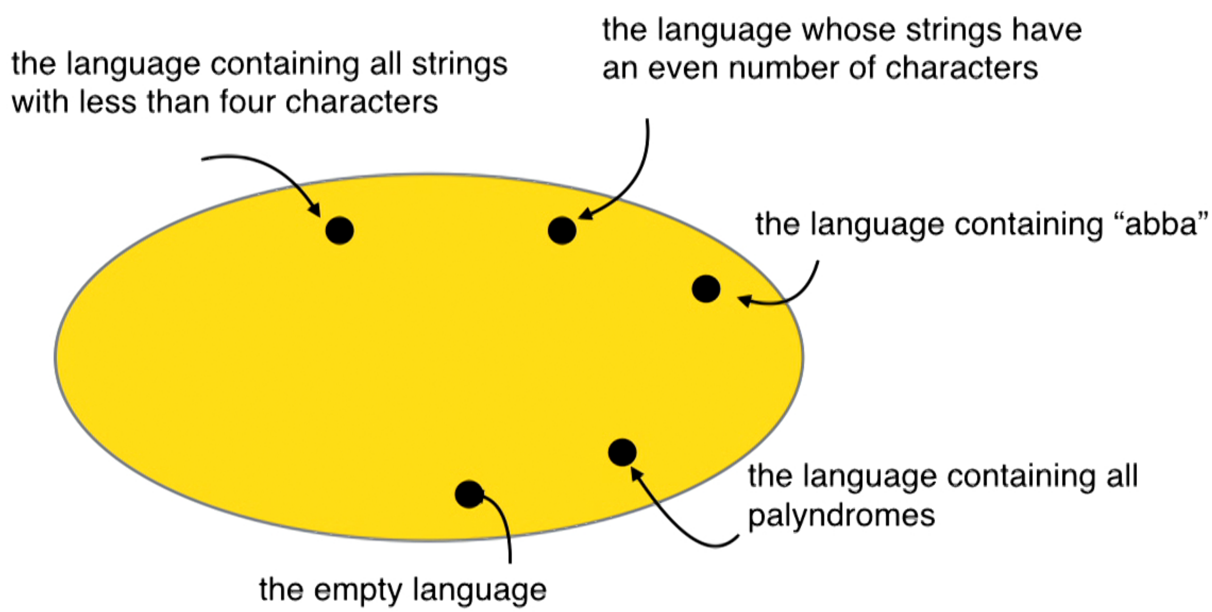

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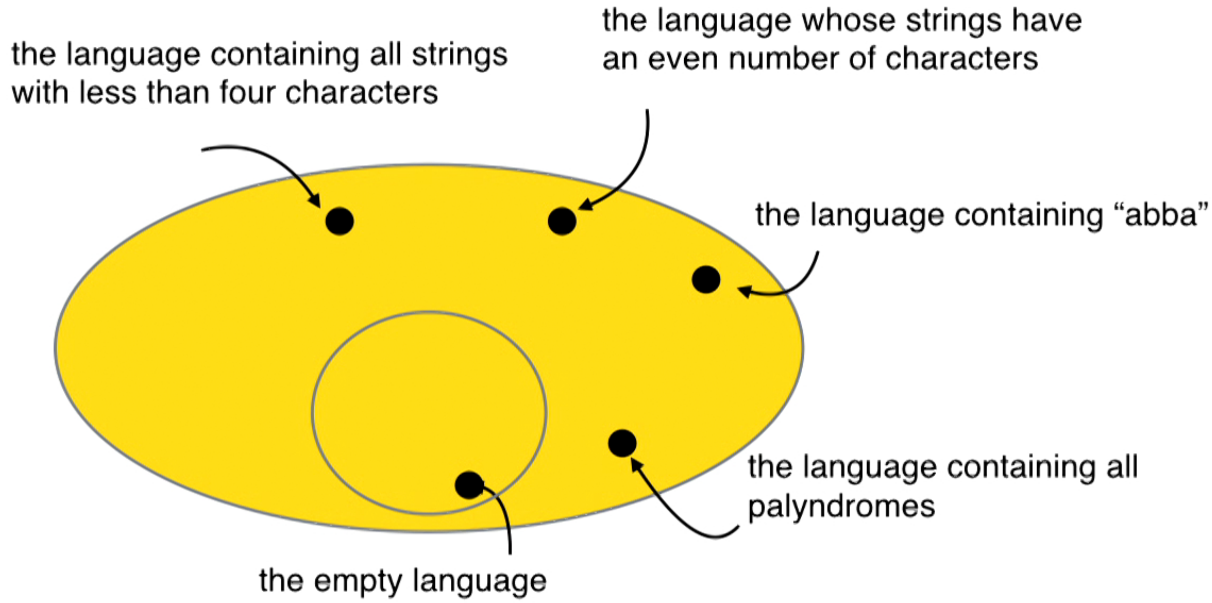

- A non-trivial property is a proper subset of this set.

Undecidability in computer science

- Rice's theorem: *Every* non-trivial property of the set of recursively enumerable languages is undecidable.

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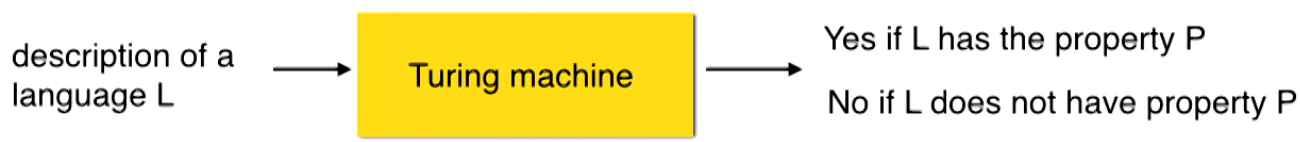

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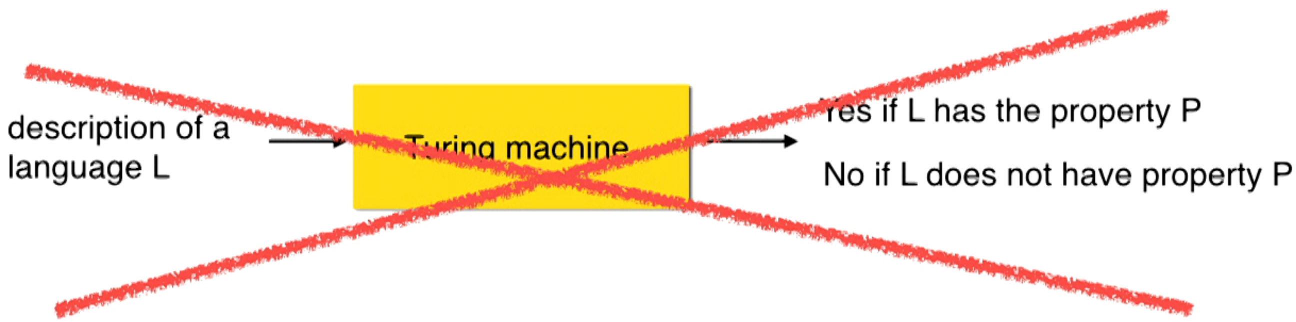

There is no Turing machine that does this job.

Undecidability in physics

- Largely unexplored

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  can we do some other mathematical abstraction of our physical problem
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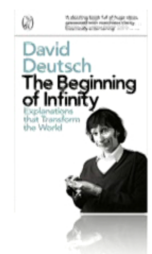
- Physics determines what is undecidable

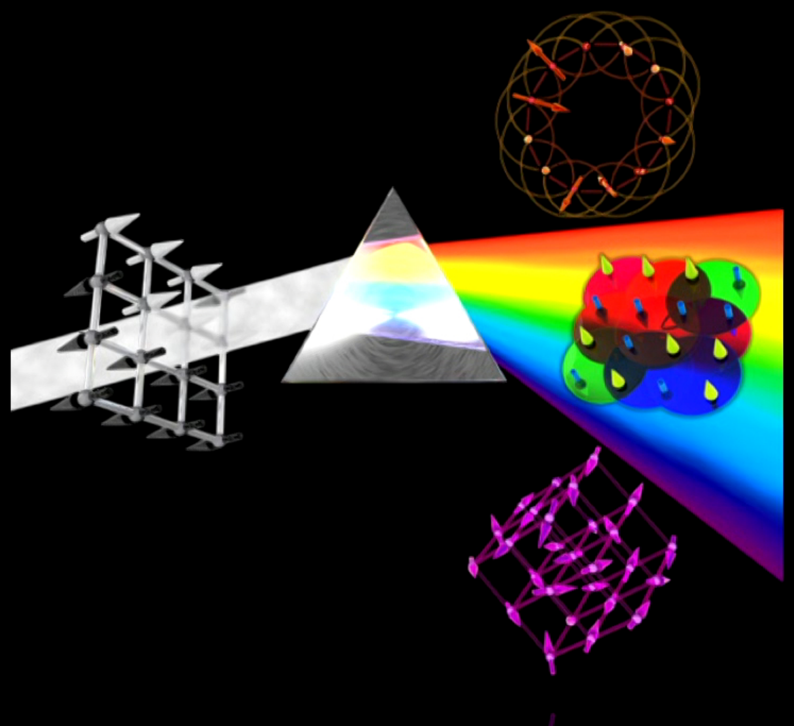
  All knowledge is generated by physical processes: e.g. a computation, a proof.

  Thus, its scope and limitations are conditioned by the laws of nature.

  E.g. only a finite amount of information can be processed in a finite amount of time.



Universality in physics



GDLC and T. S. Cubitt, Science 351, 228 (2016)

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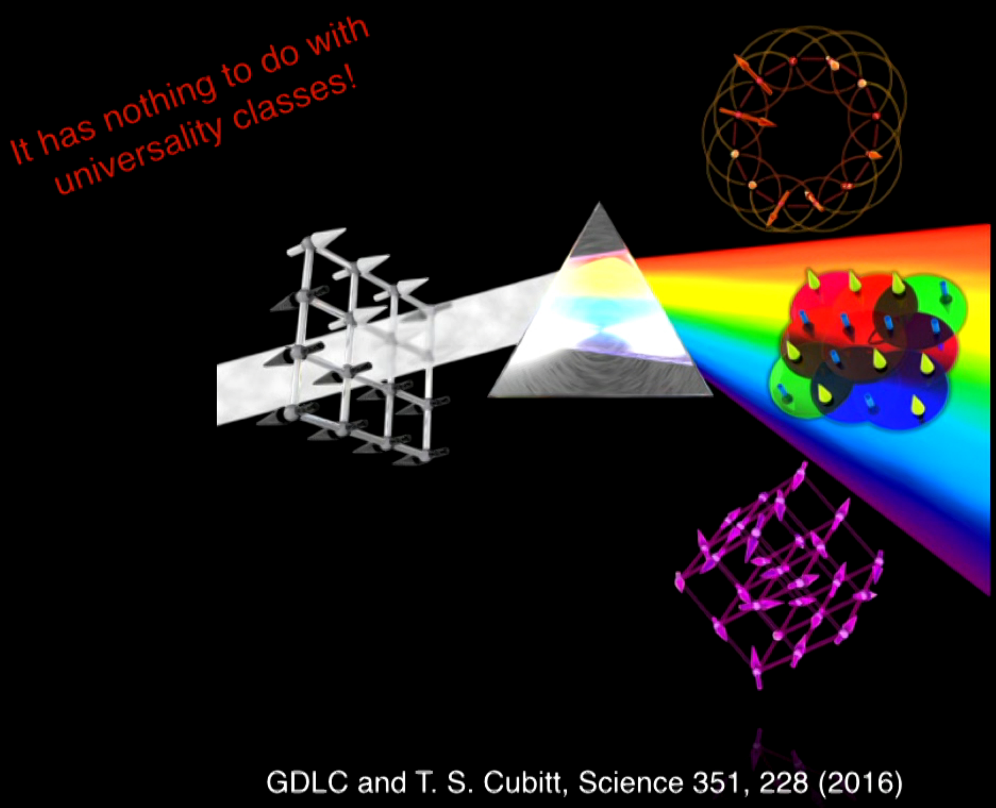

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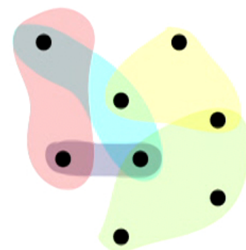
Spin models

A spin model:
- a set of classical degrees of freedom (the spins)
- a cost function (the hamiltonian)



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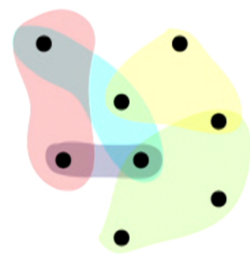

- Tool to model complexity “bottom up”

Spin models

- The 1D Ising model with coupling *J*:

Spin configuration	$\mapsto$	Energy	for any *n*
$s_1, \ldots, s_n$		$J \sum_{i=1}^{n-1} s_i s_{i+1}$	

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$$J \sum_{i=1}^{n-1} s_i s_{i+1}$$

x = Spin configuration → The 1D Ising model with coupling J → Energy of input x

for all x

Spin models

- The 2D Ising model with fields:

Spin configuration and coupling strengths	$\mapsto$	Energy	for any *n*
$s_1, \ldots, s_n \# h_1, \ldots, h_n \# J_{1,2}, \ldots, J_{n-1,n}$		$\sum_{(i,j)} J_{i,j} s_i s_j + \sum_i h_i s_i$	

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$$s_1, \ldots, s_n \# h_1, \ldots, h_n \# J_{1,2}, \ldots, J_{n-1,n} \qquad \sum_{(i,j)} J_{i,j} s_i s_j + \sum_i h_i s_i$$

x = Spin configuration and coupling strengths → **The 2D Ising model with fields** → Energy of input x

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Spin models

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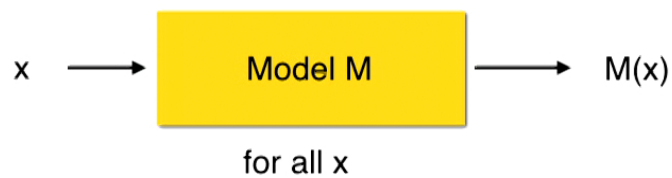

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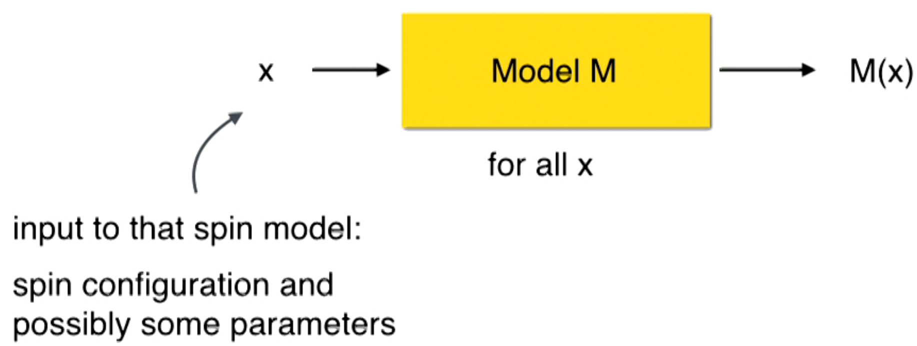

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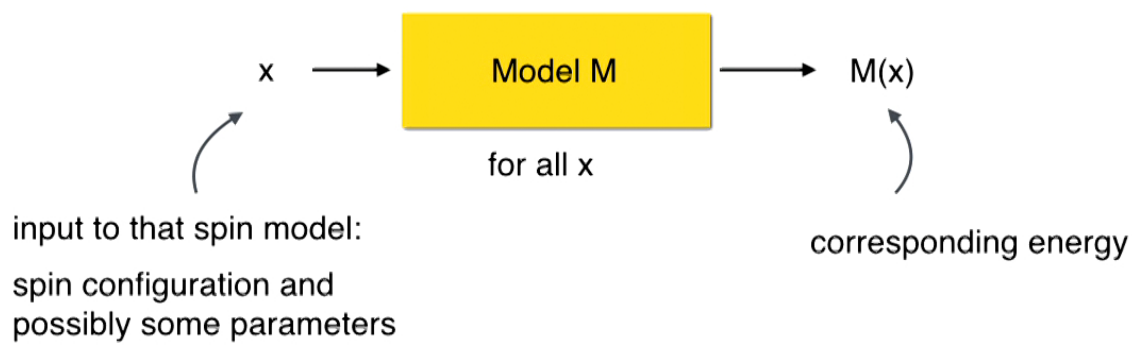

Spin models

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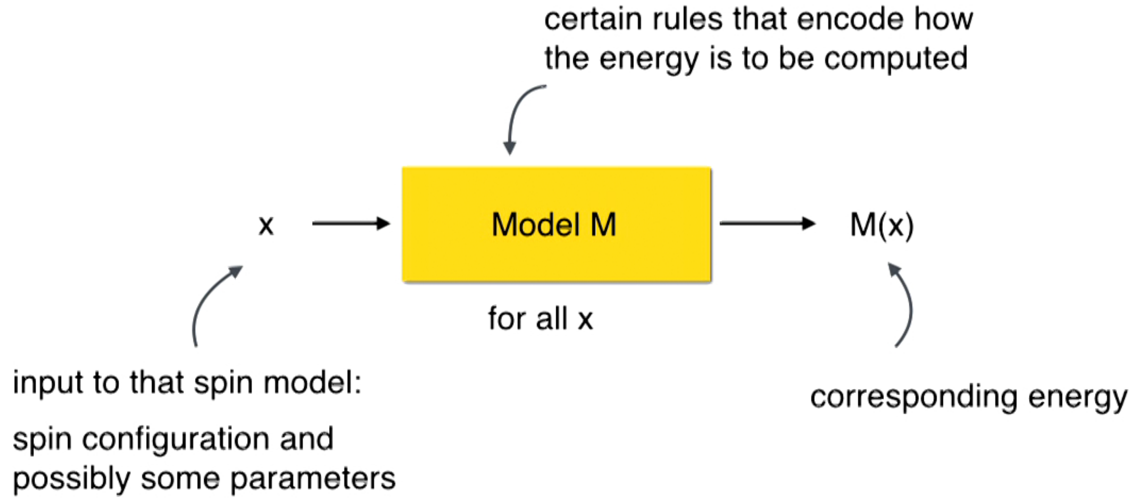

Simulation of spin models

- A spin model M *simulates* a spin model M' if there is a description such that

Description of (model M' and its input x) → Model M → Energy of model M' on input x

for all x

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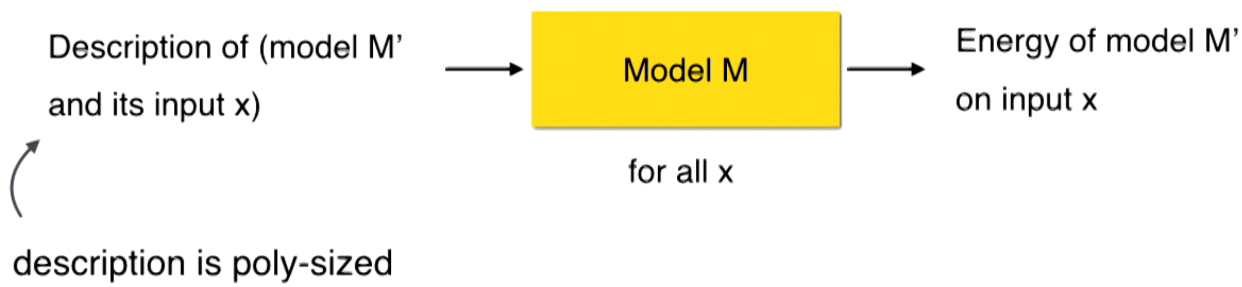

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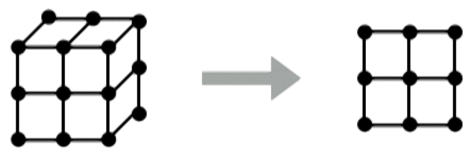
Put all couplings outside a layer to 0



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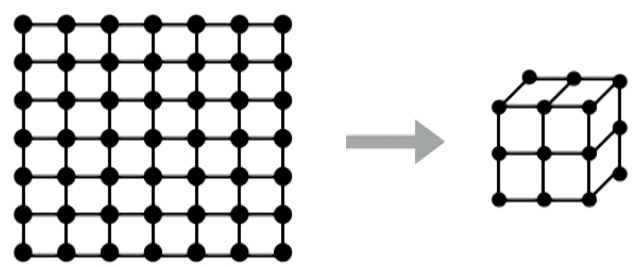
Project 3D lattice to 2D, use auxiliary spins, and coupling strengths…..



Universal spin models

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  these spin models are machines that only need to compute energies…

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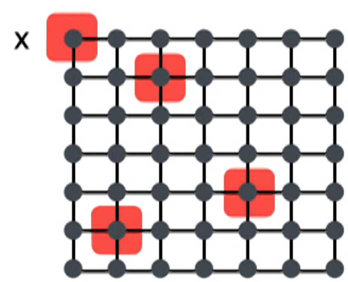
spin configuration and coupling strengths of 2D Ising model



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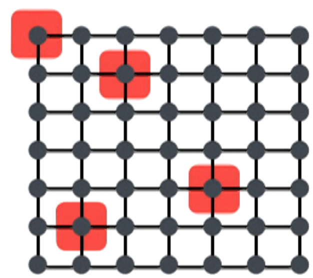

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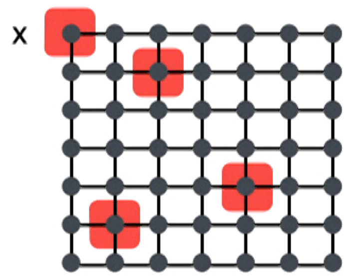

- Apparently simple model. Enough to reach this form of universality.
- 2D Ising model without fields is not universal….

Universal spin models

- Full characterization of universality in physics:

  A spin model is universal if and only if

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- I will argue that our notion of universality is weaker than that in computer science.

Connections between the two notions of universality

Work in progress

with Georg Moser

Machines and languages

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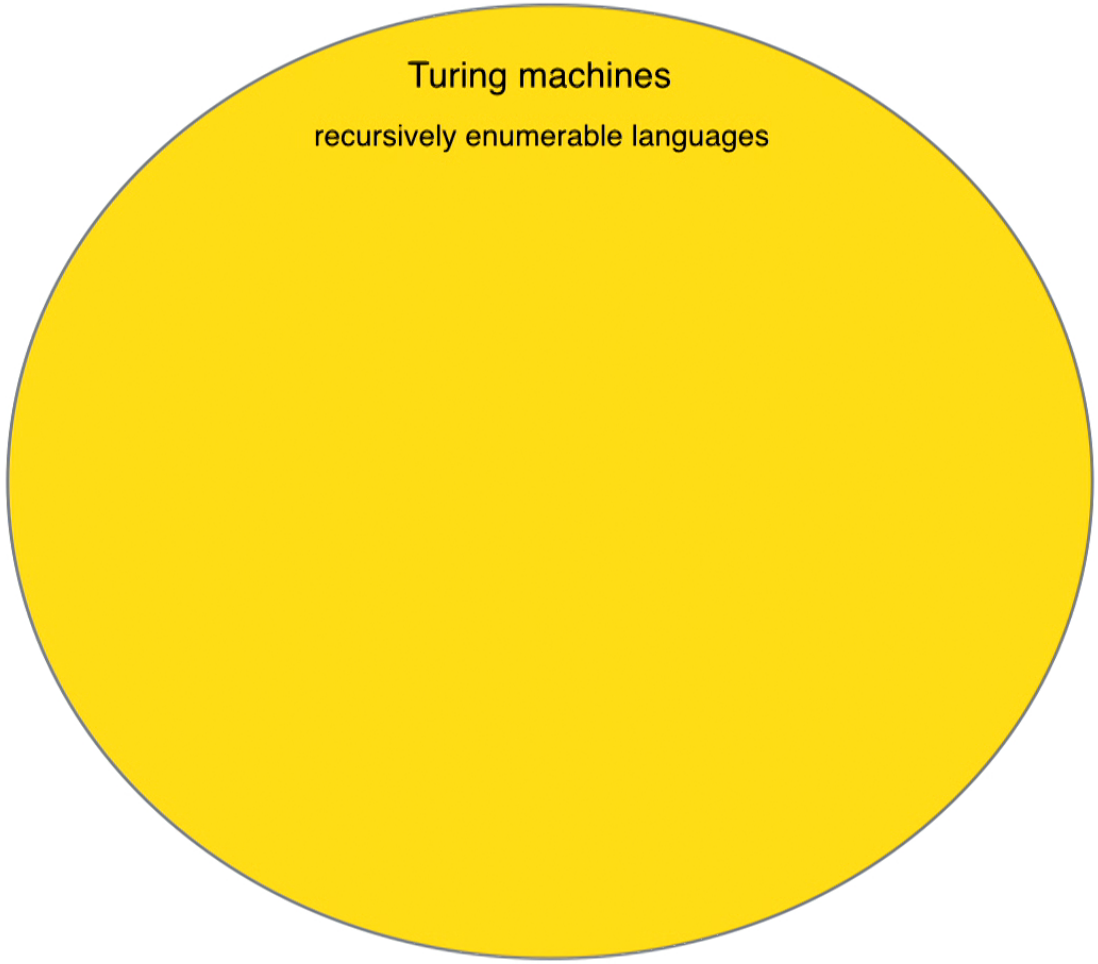
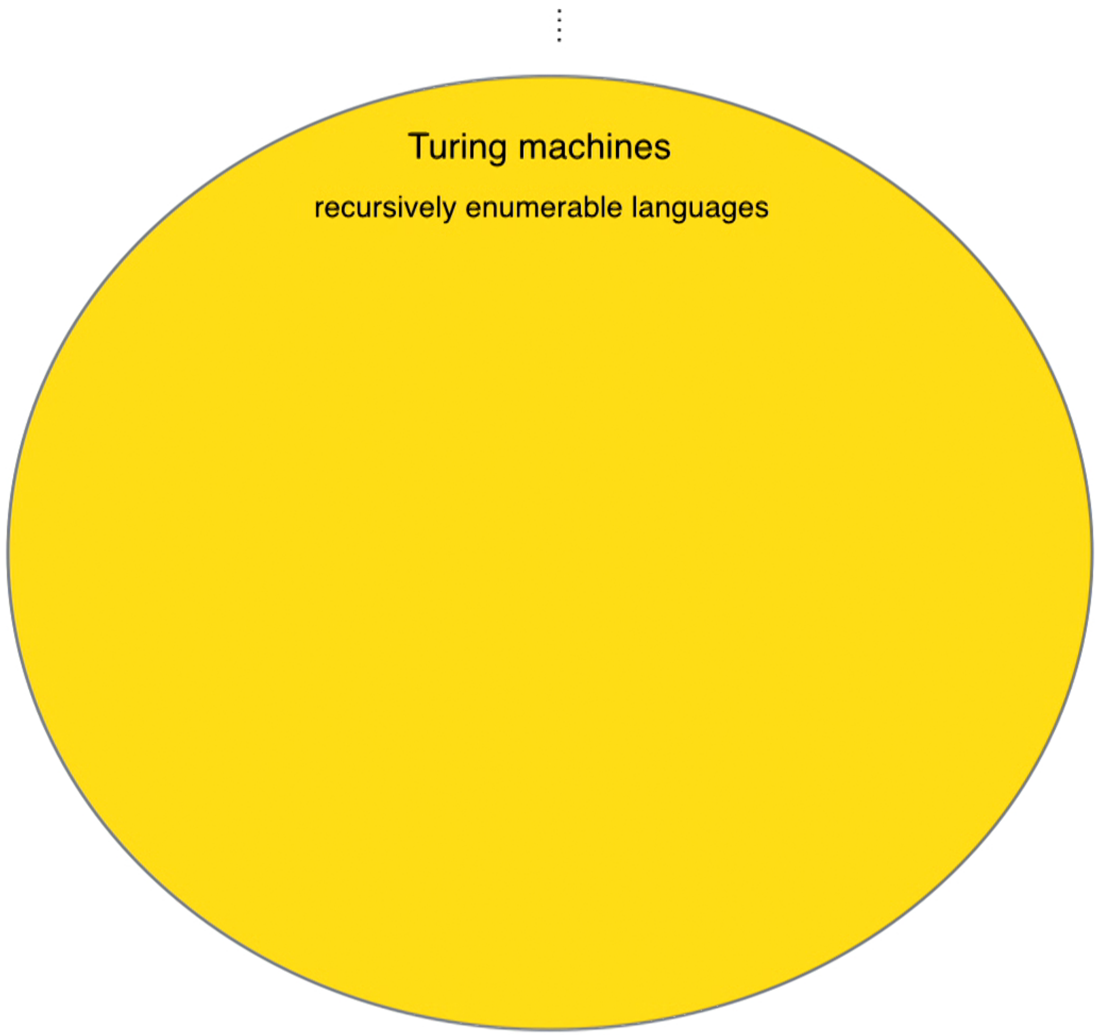
- Consider a machine that accepts or rejects strings

- The language associated to a machine is the set of strings accepted by that machine

  L(M) = {x accepted by M}


Turing machines
recursively enumerable languages


⋮
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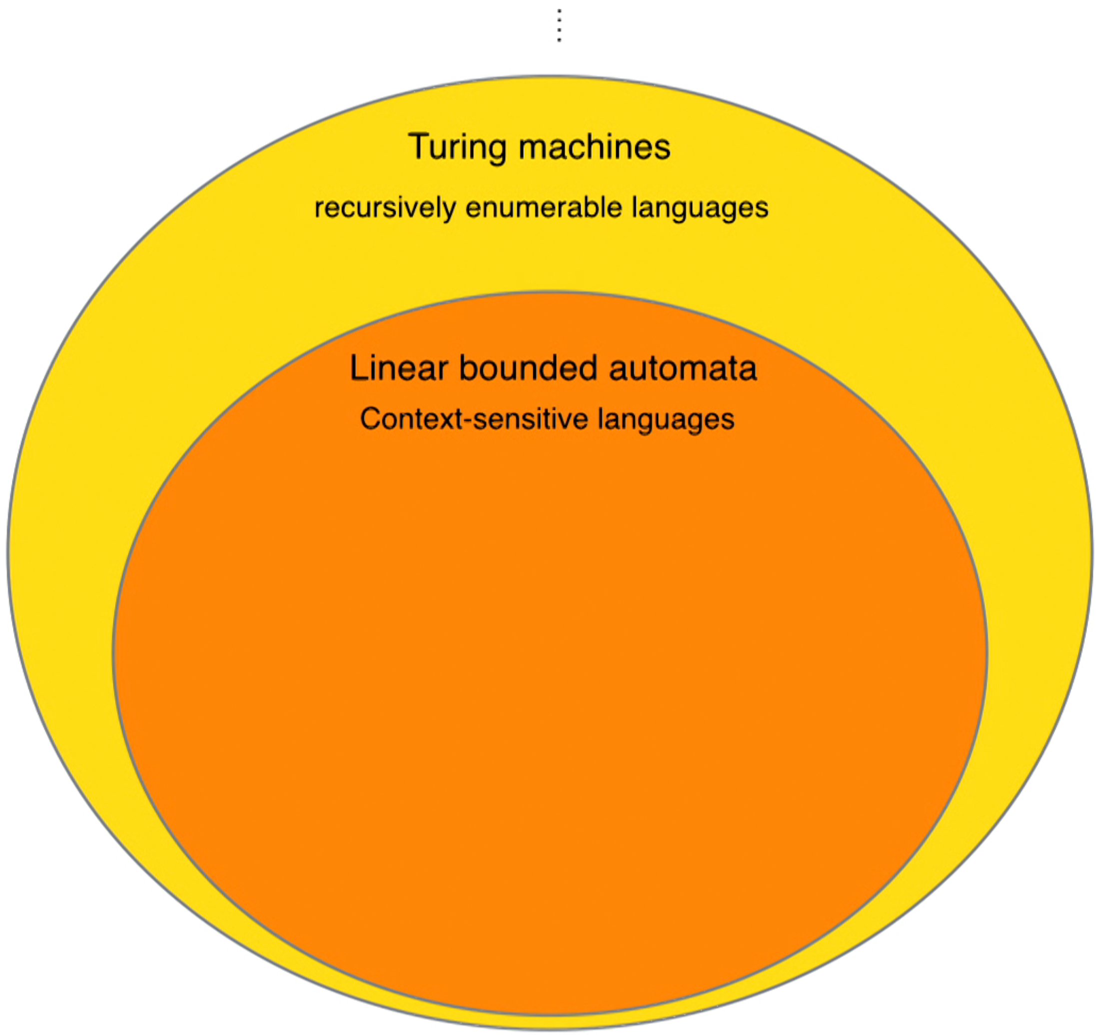
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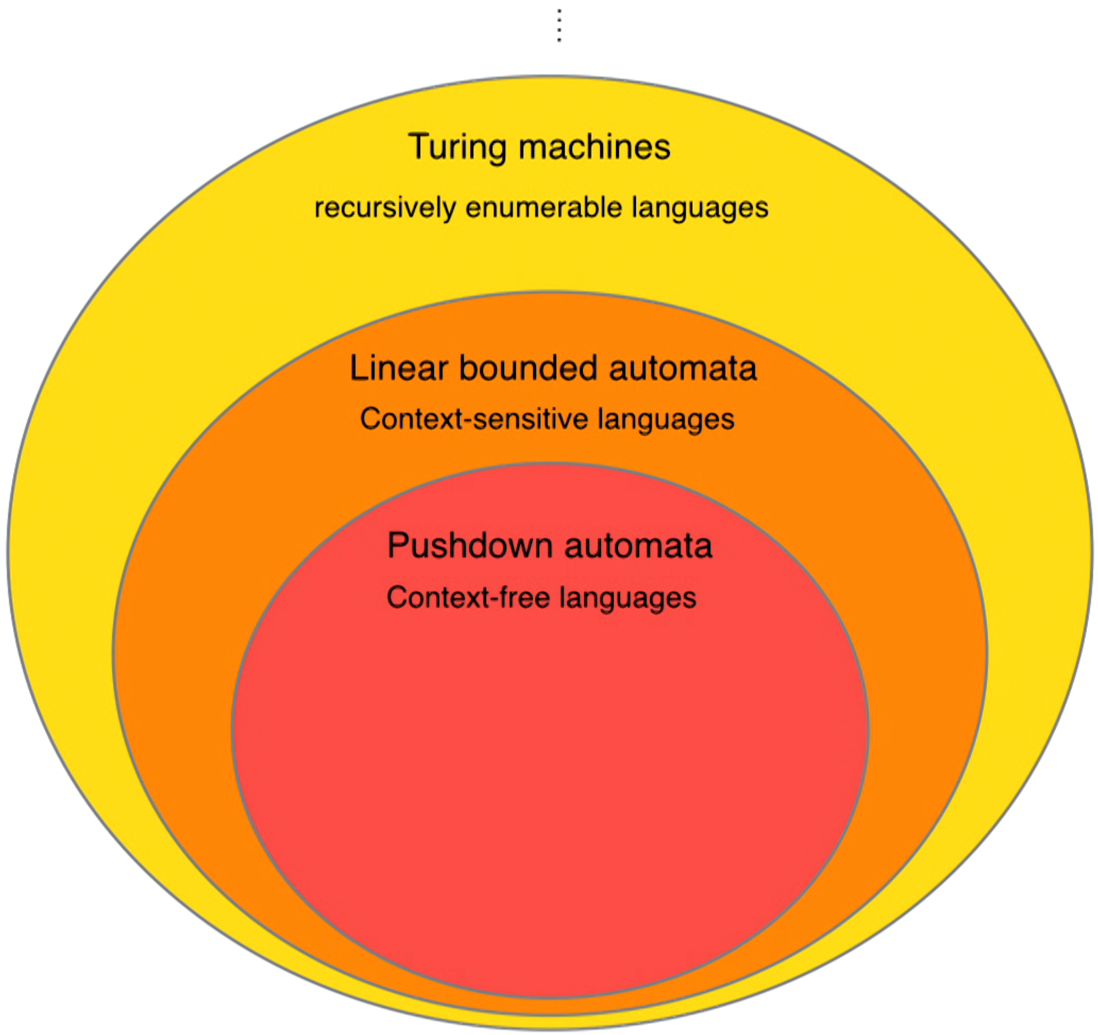
Turing machines
recursively enumerable languages
Linear bounded automata
Context-sensitive languages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Turing machines
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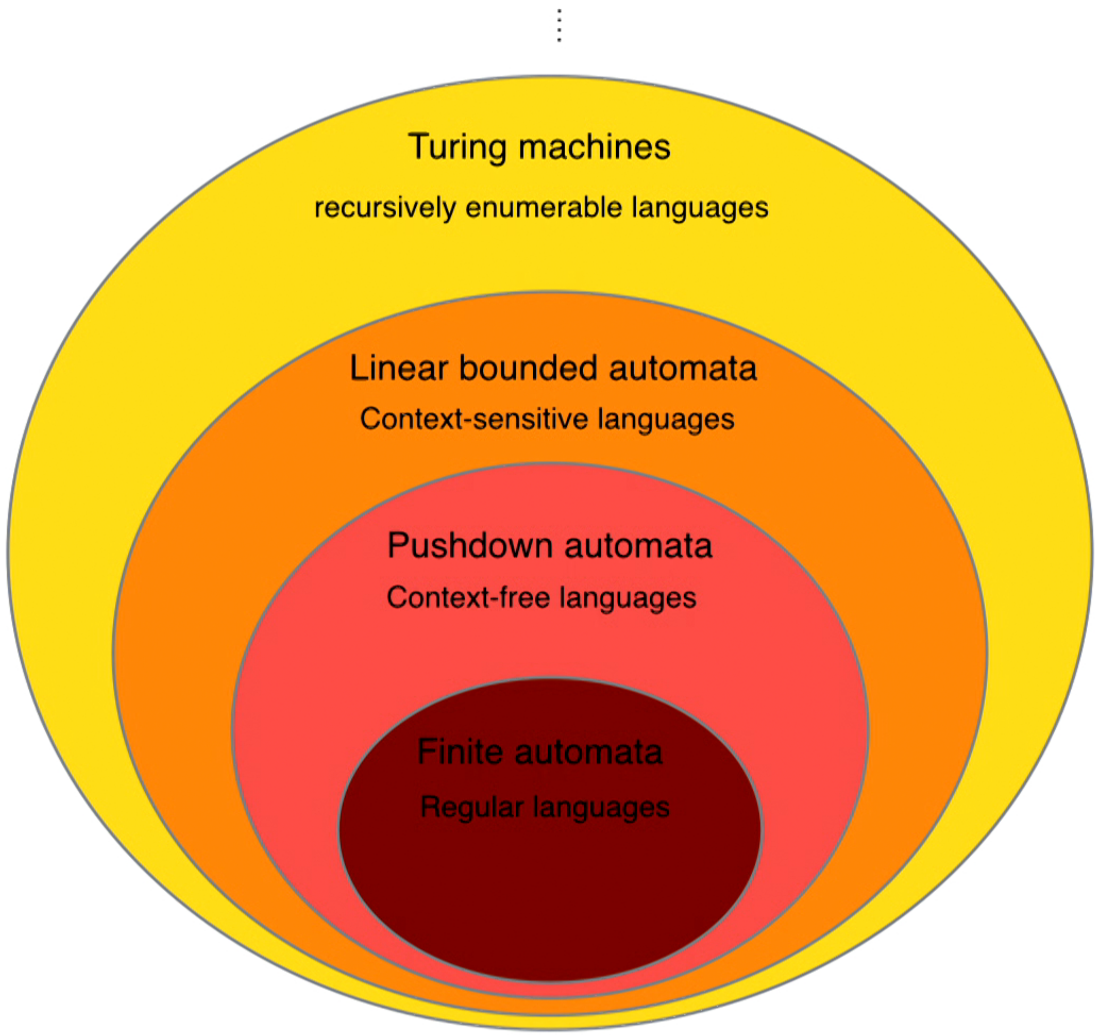
Pushdown automata
Context-free languages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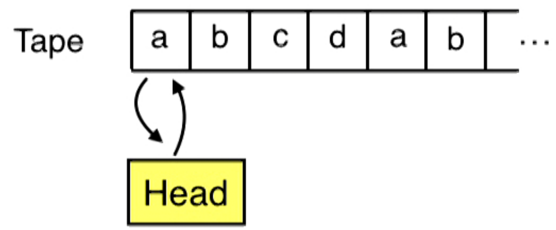
Turing machines
recursively enumerable languages
Linear bounded automata
Context-sensitive languages
Pushdown automata
Context-free languages
Finite automata
Regular languages

Turing machine



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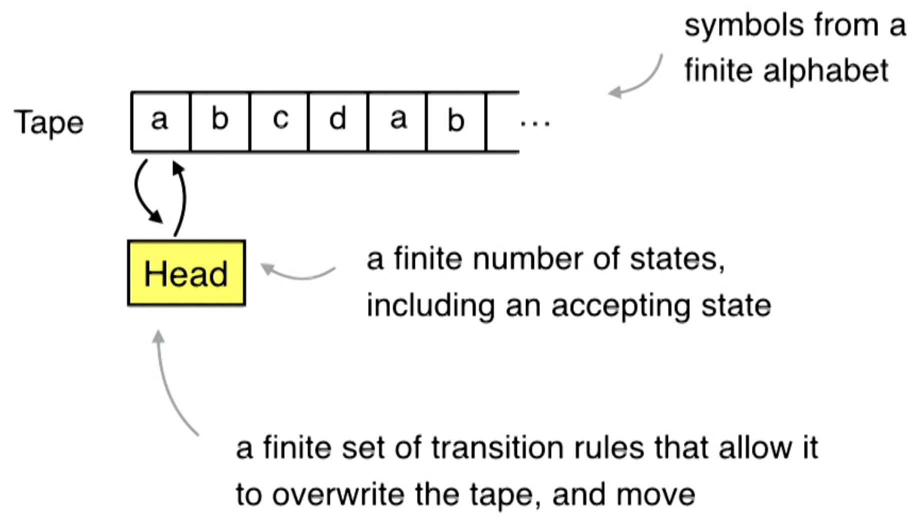
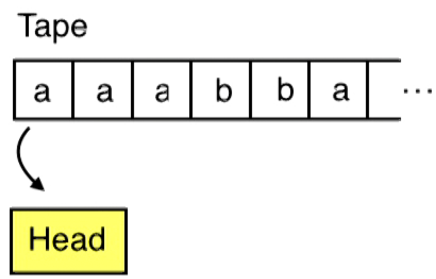


Finite automata

- A finite automaton is a read-only Turing machine



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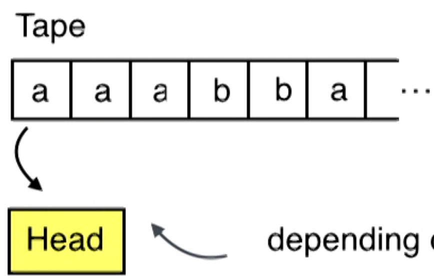

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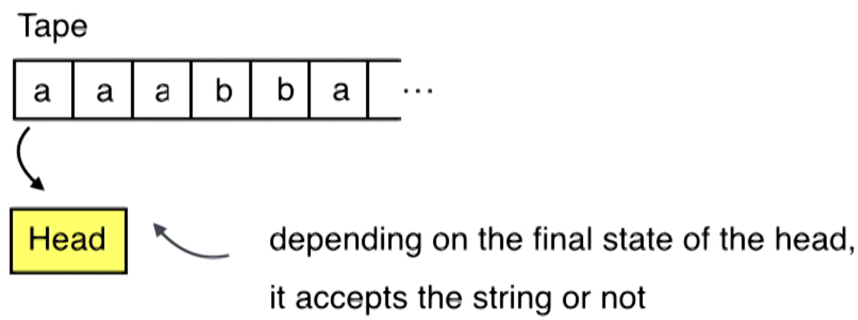

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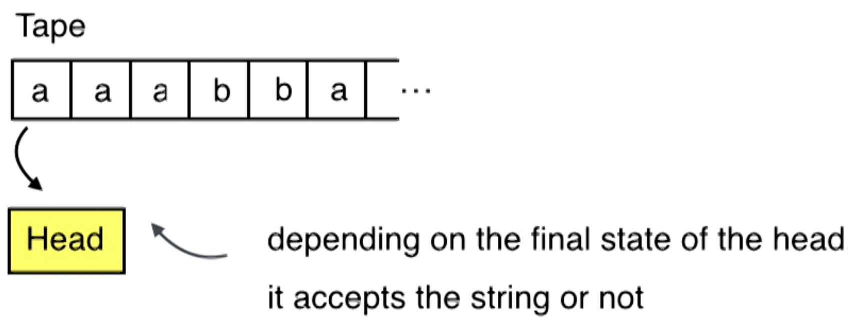

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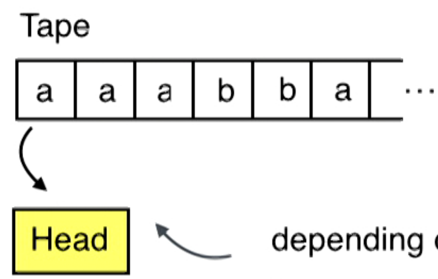

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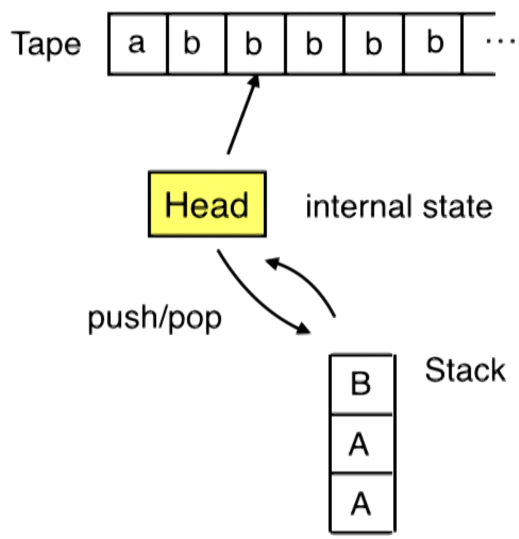
E.g. recognize L = the set of strings that contain abba

- Every finite language is regular.

Pushdown automata

- It is a non-deterministic finite automaton with a stack



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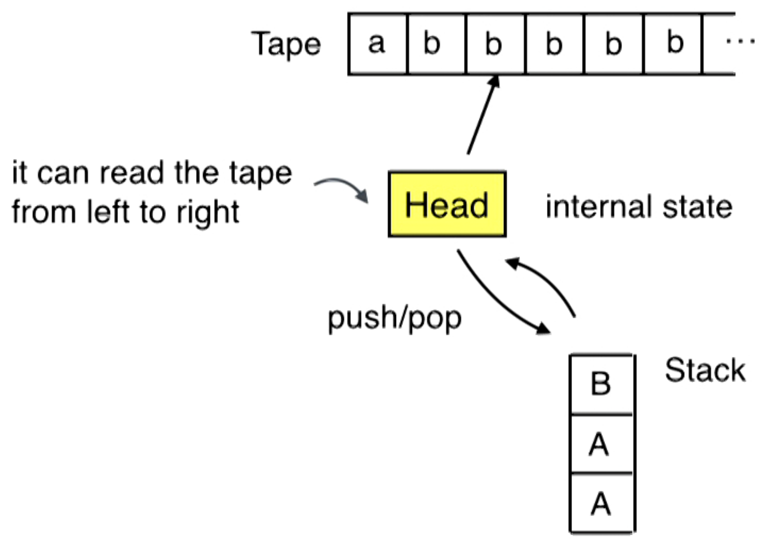

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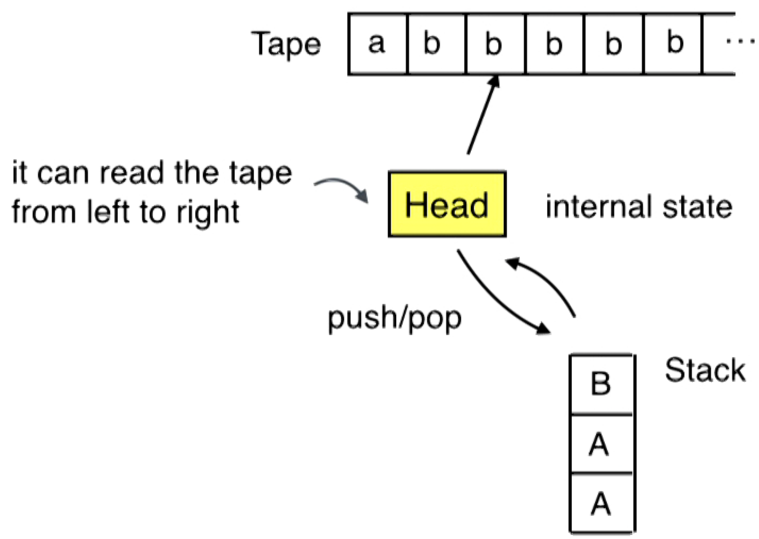

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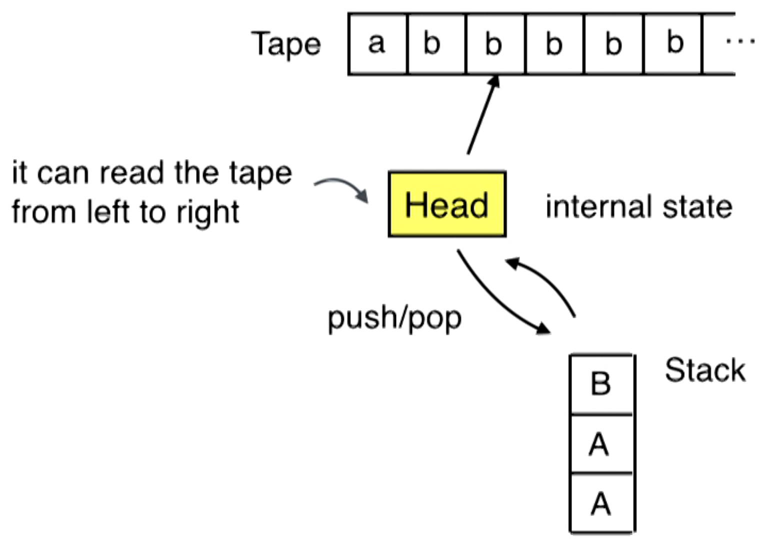

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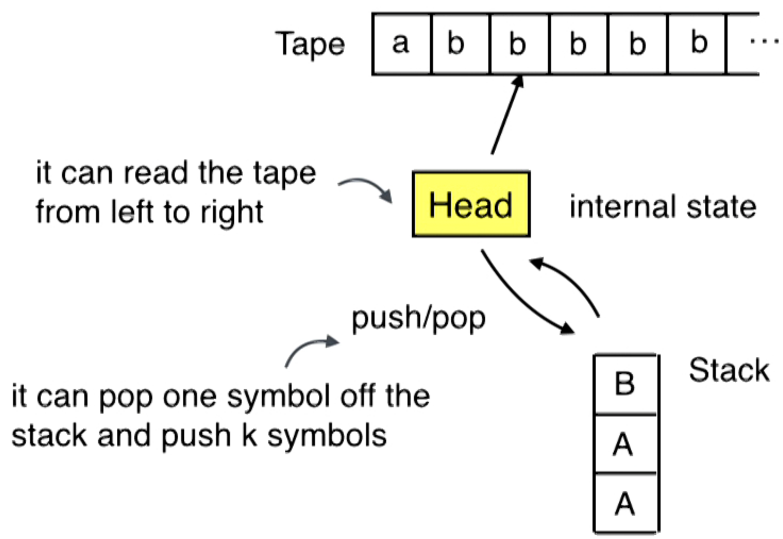

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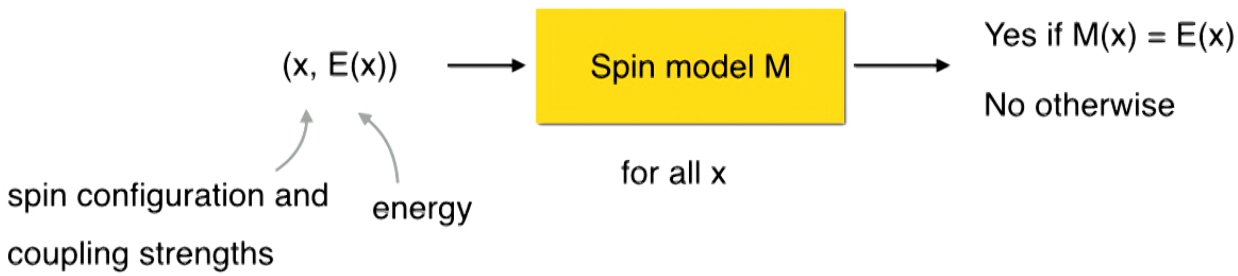
  E.g. the language containing all strings of balanced parantehesis: (), (()), ()(), (()(())), ...

A hamiltonian as a machine

- Recall: a spin model is a machine



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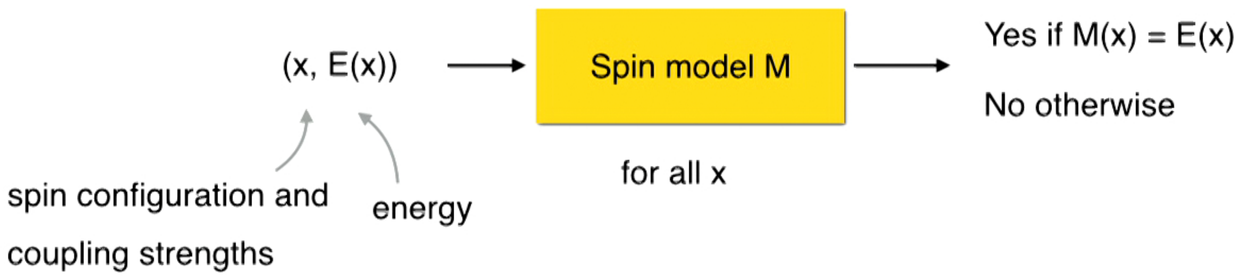

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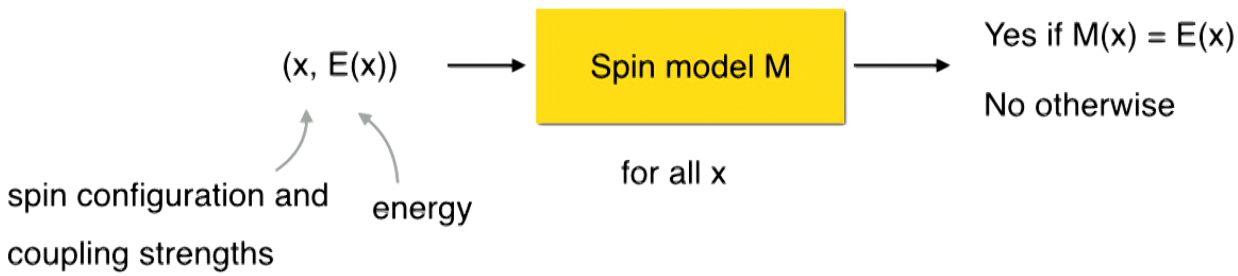

- What is the power of this machine?

  I claim that it need only be a pushdown automaton.

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If the final stack is empty, it accepts. Else, it rejects.

A hamiltonian as a machine

- Simulation between spin models translates to reduction between pushdown automata.
- Implications of our full characterization of simulations between spin models?

Summary &
Perspectives

Summary

- Universality everywhere

Summary

- Universality everywhere …. and undecidability

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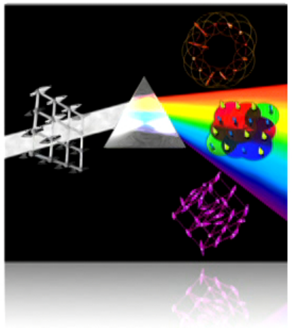
- New universality results in physics



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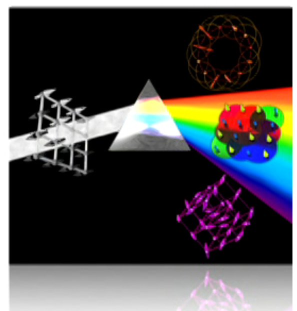

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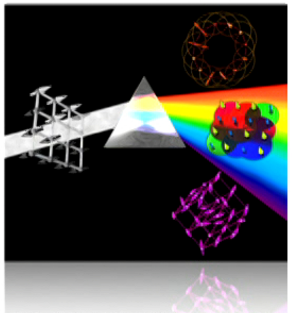

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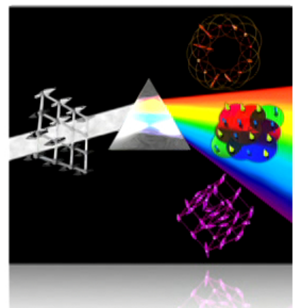

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- Universality results in physics: characterization of pushdown automata?

More perspectives

More perspectives

- Implications of these results for undecidability in physics?

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