Title: The Logic of Physical Law

Date: Apr 09, 2018 02:00 PM

URL: http://pirsa.org/18040105

Abstract: Landauer's principle claims that "Information is Physical." Its conceptual antipode, Wheeler's "It from Bit," has since long been popular among computer scientists in the form of the Church-Turing hypothesis: All natural processes can be simulated by a universal Turing machine. Switching back and forth between the two paradigms, motivated by quantum-physical Bell correlations and the doubts they raise about fundamental space-time causality, we look for an intrinsic, physical randomness notion and find one, namely complexity, around the second law of thermodynamics. Bell correlations combined with Kolmogorov complexity in the role of randomness imply an all-or-nothing nature of the Church-Turing hypothesis: Either beyond-Turing computations are physically impossible, or they can be carried out by "devices" as simple as individual photons. This latter result demonstrates in an exemplary way the fruitful interplay between physical and informational-computational principles.

The Logic of Physical Law

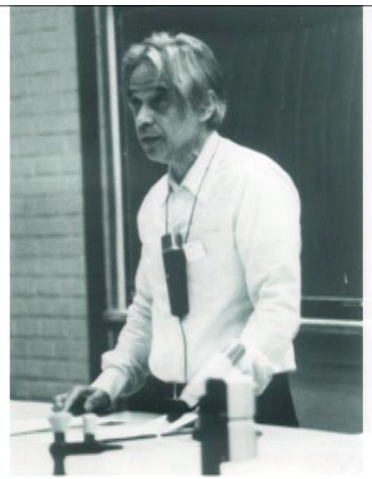
Stefan Wolf
Università della Svizzera italiana
Facoltà indipendente di Gandria

Algorithmic Information, Induction and Observers in Physics
9.4.18, 14:00
Perimeter Institute
Waterloo, ON

Logic and Physics

Ferdinand Gonseth
1936

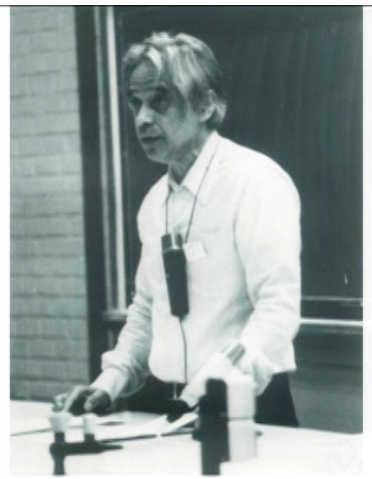
La logique est tout d'abord une science naturelle.



Logic and Physics

Ernst Specker
1961

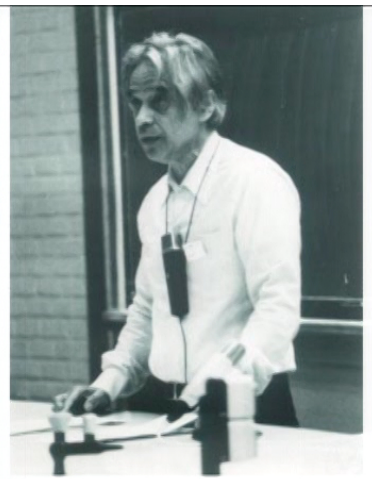
Is it possible to embed the description of any physical system into classical logic?



Logic and Physics

Ernst Specker
1961

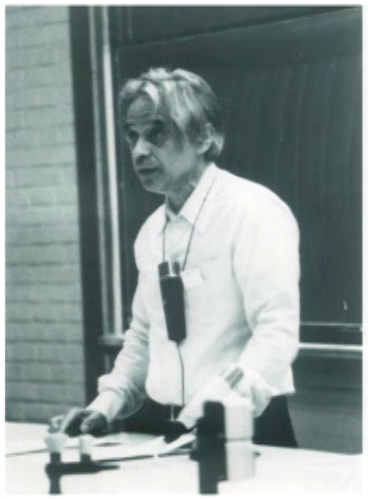
Is it possible to extend the description of a quantum-mechanical system by adding additional – *fictional* – propositions such that in the resulting domain, classical propositional logic governs?



Logic and Physics

Ernst Specker
1961

An elementary-geometric argument shows that in general, *no consistent prophecies* are possible concerning the behavior of a quantum-mechanical system.



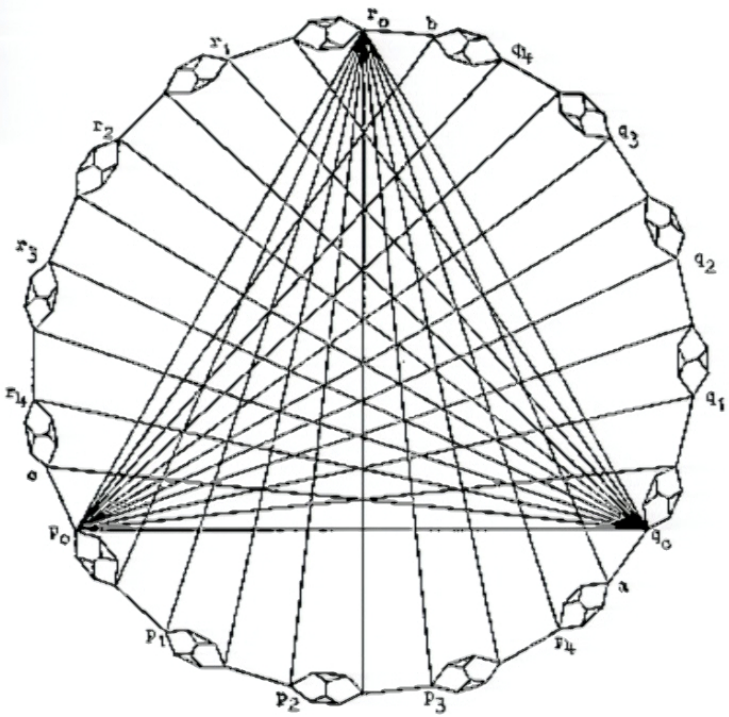
Logic and Physics

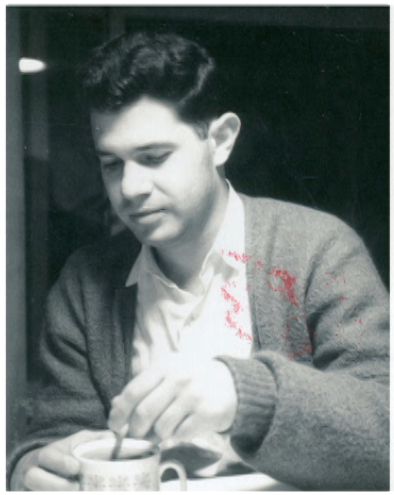
1967

Ernst Specker

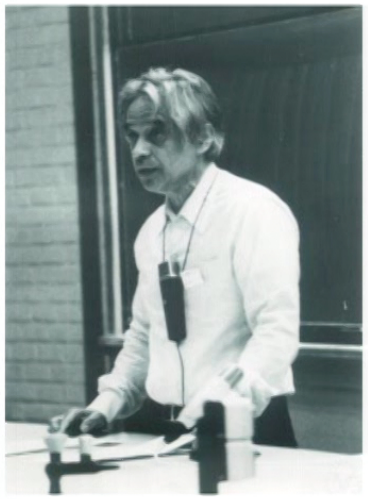
1961

Simon Kochen





Logic and Physics



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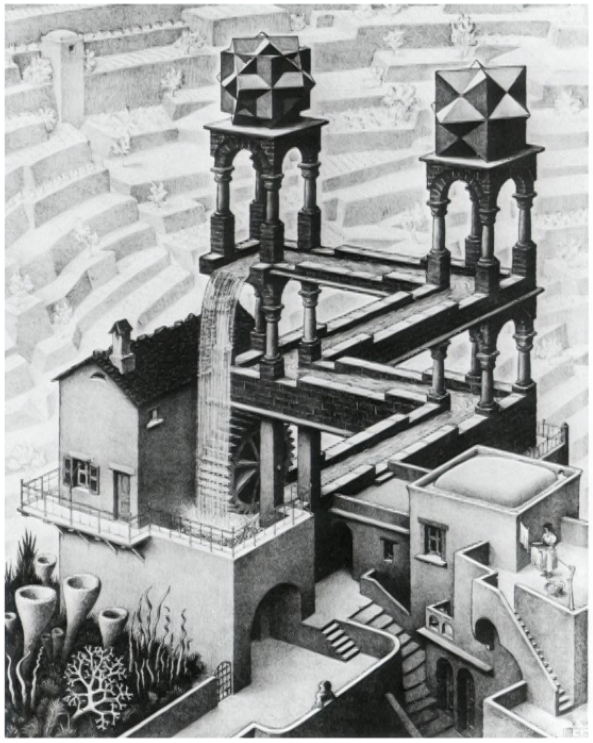
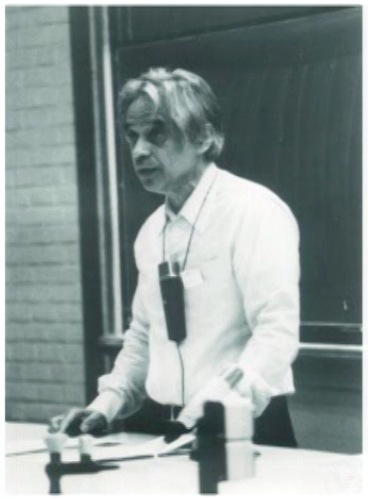


M. C. Escher

1961

Logic and Physics



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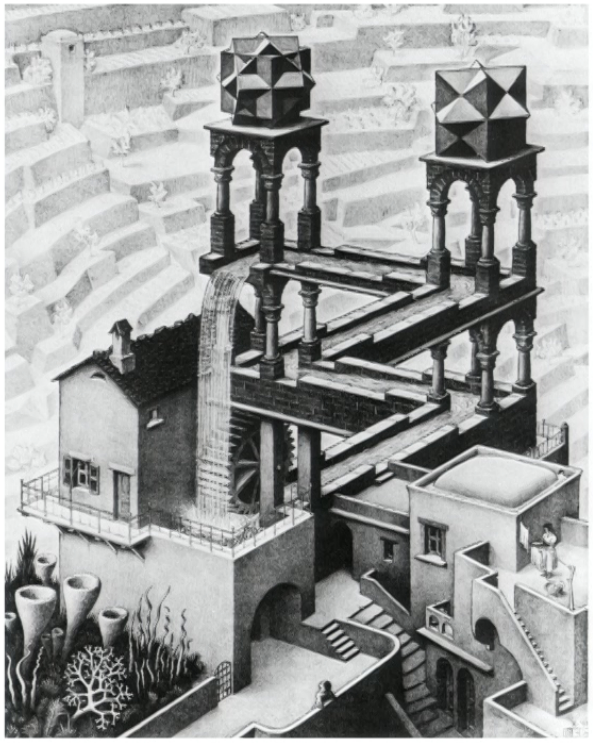

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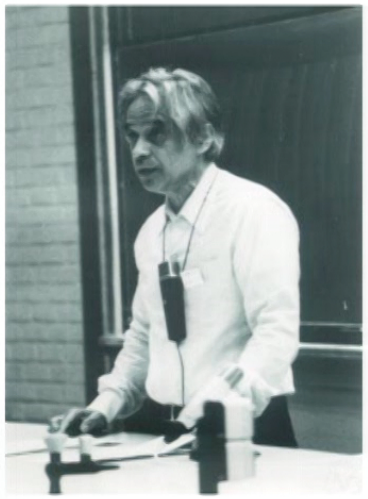
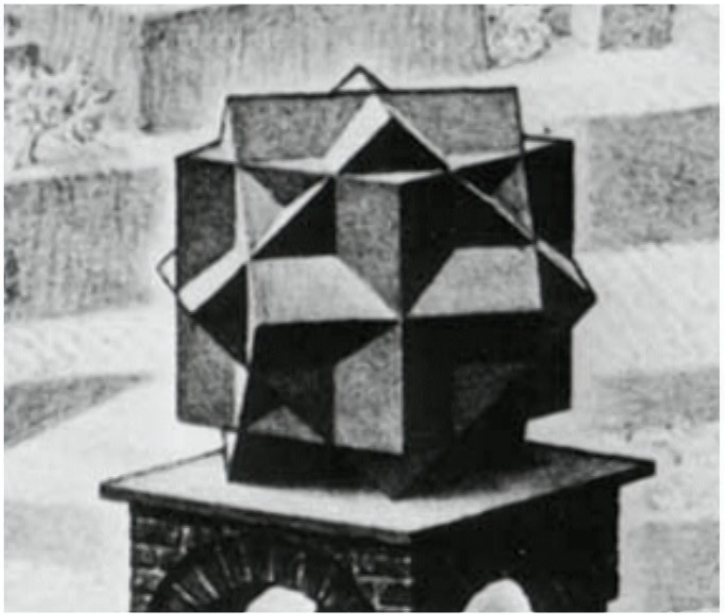


Simon Kochen



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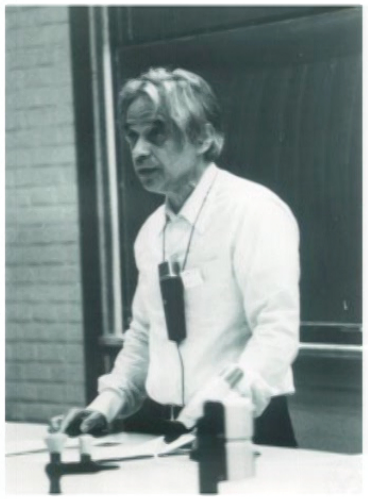

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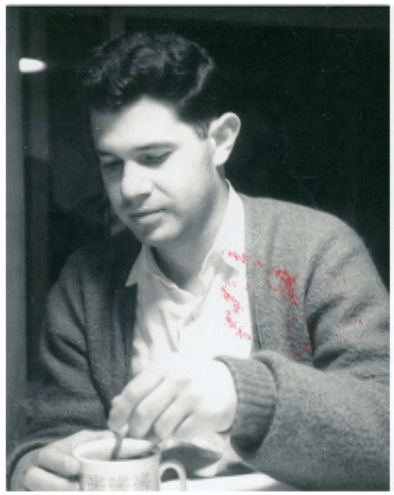

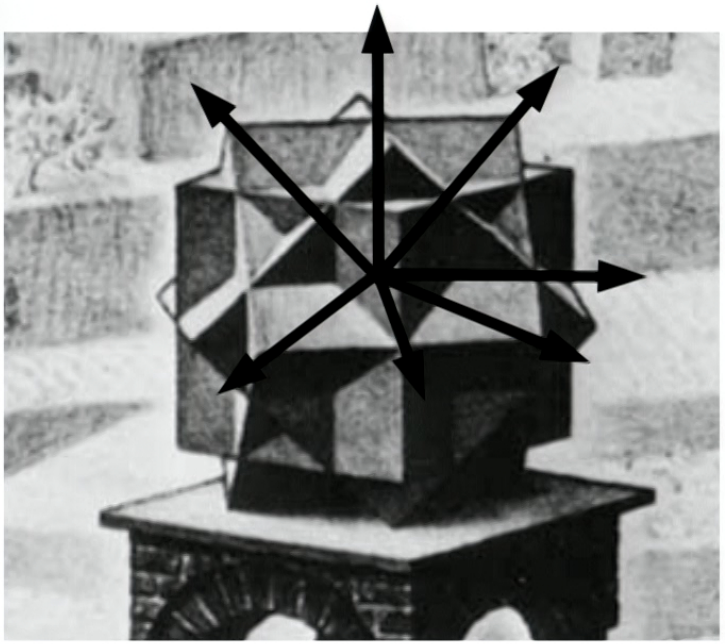

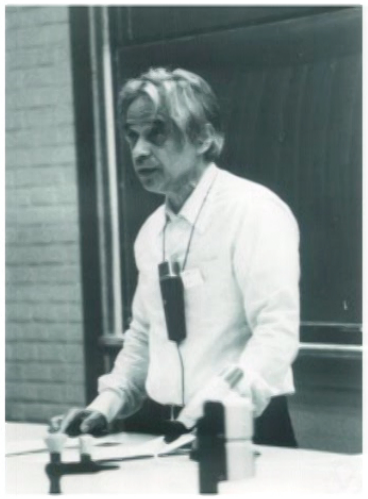

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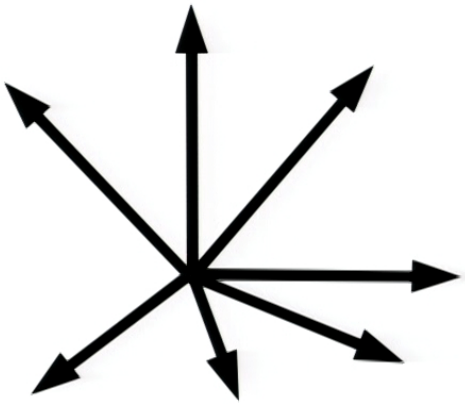

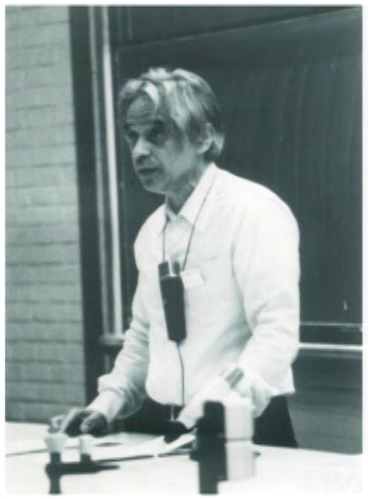

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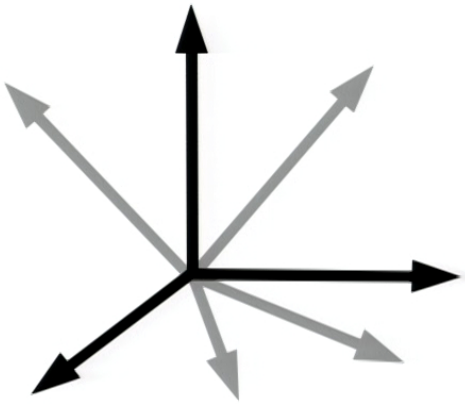

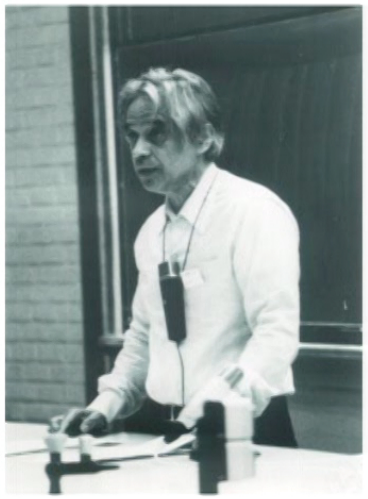

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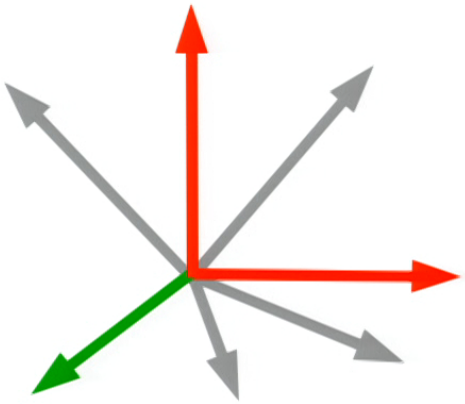

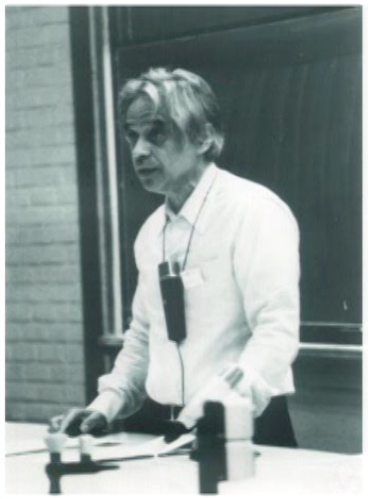

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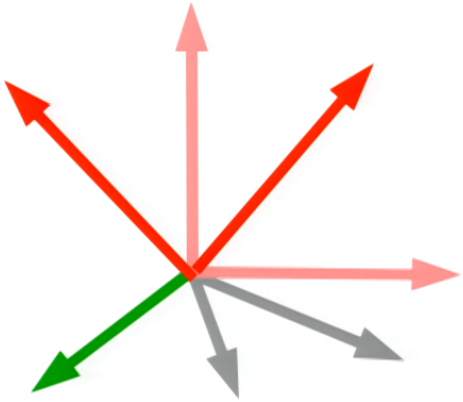
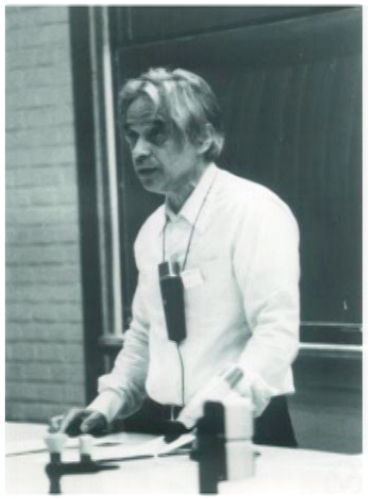


Non-Contextuality: The color of a vector is independent of the completion to a basis



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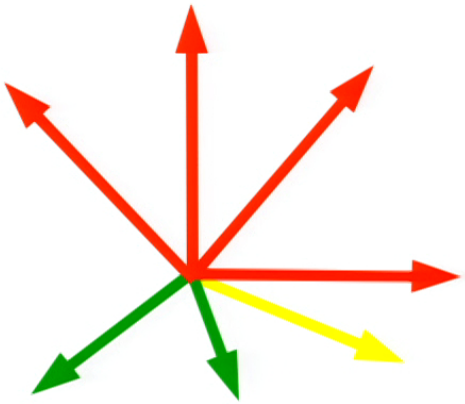
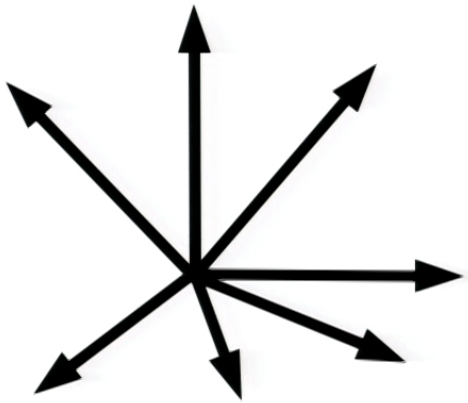


joint with
Renato Renner

Logic and Physics



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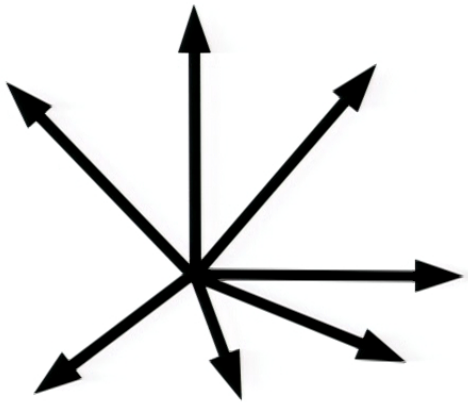

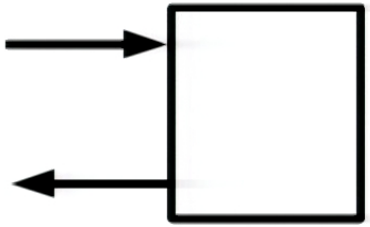

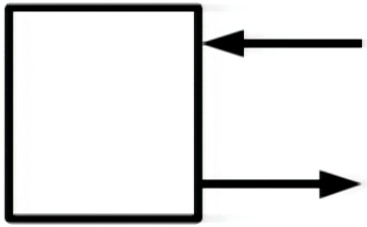

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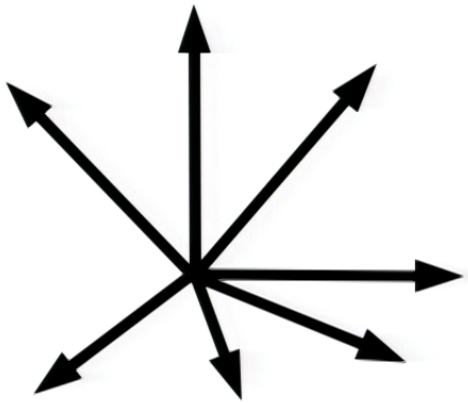

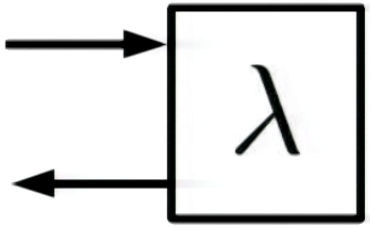

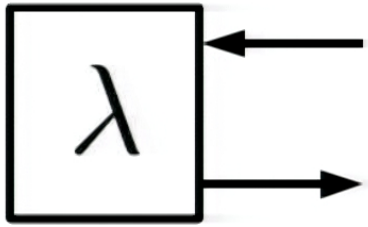

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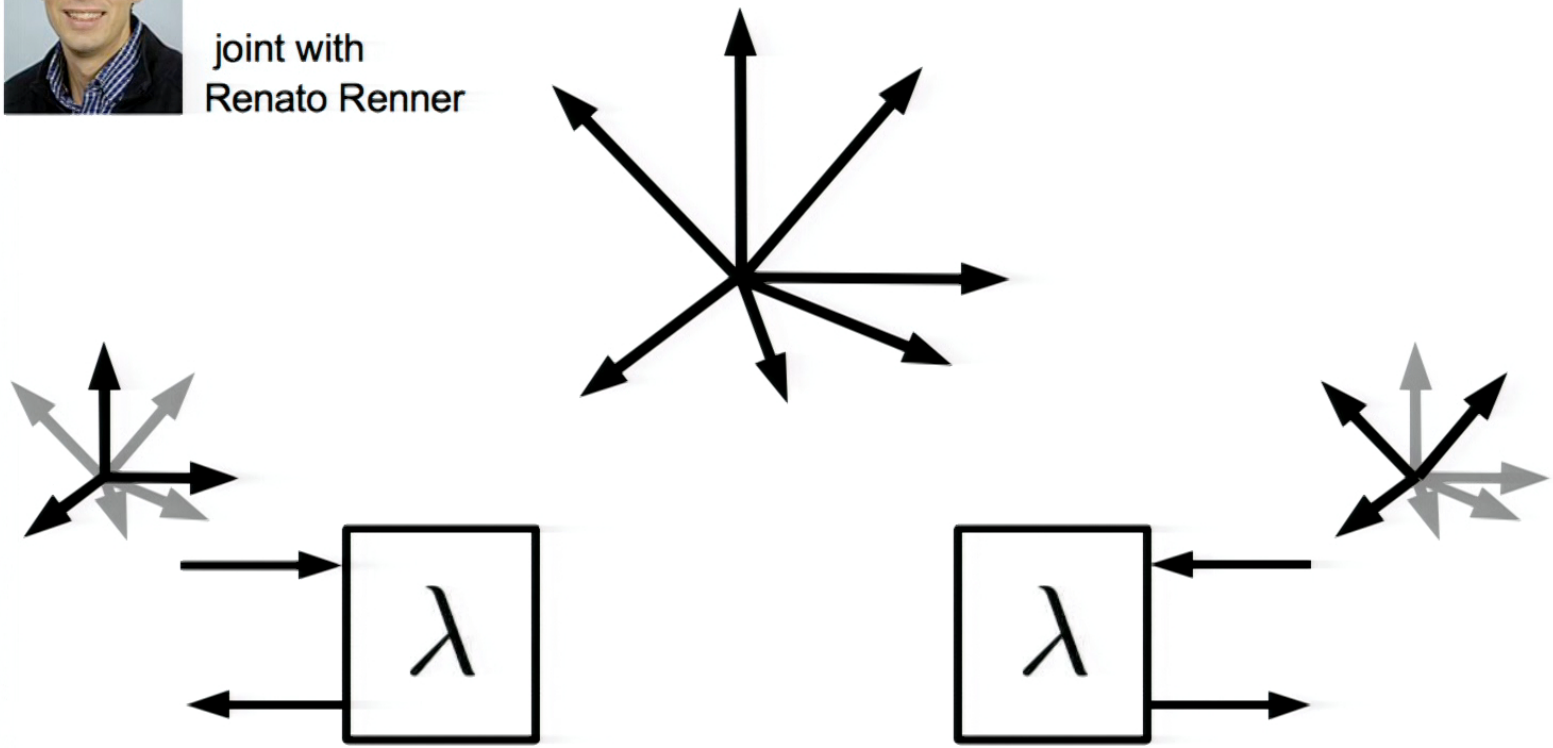
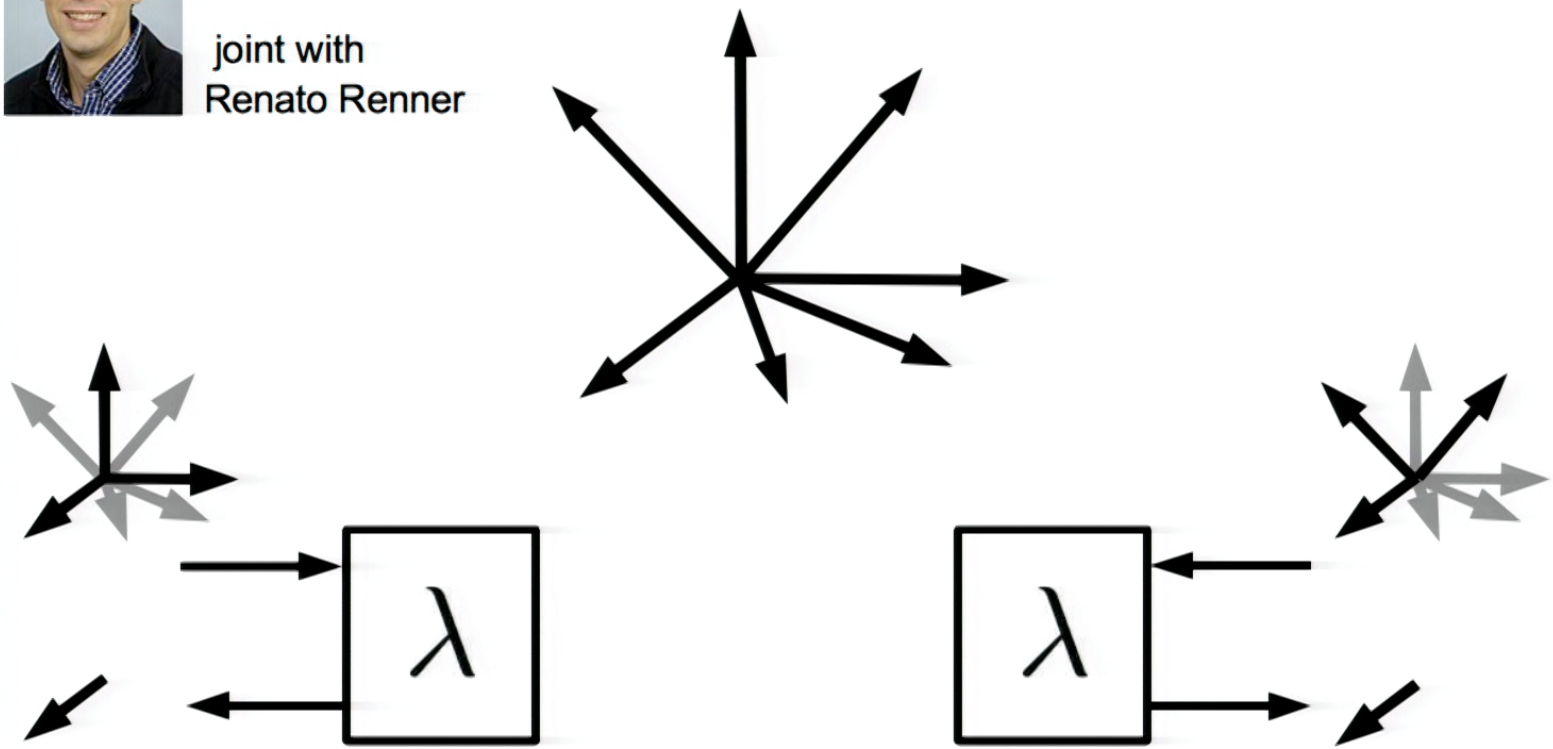


Logic and Physics

joint with
Renato Renner



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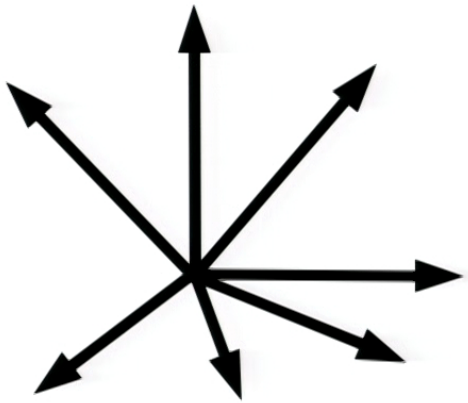

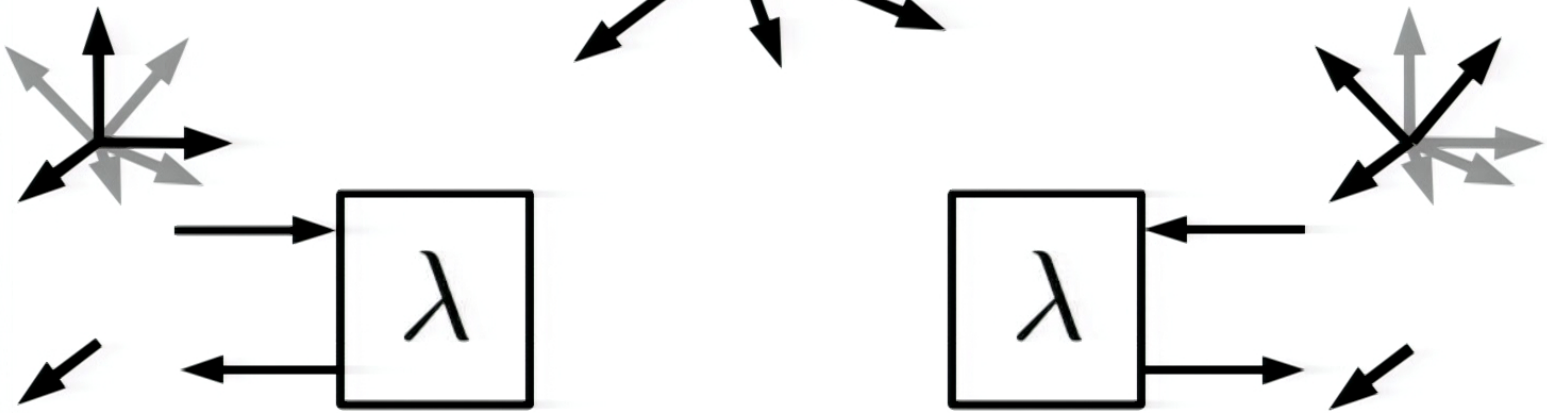

Winning Condition:
The output vectors are not orthogonal

joint with
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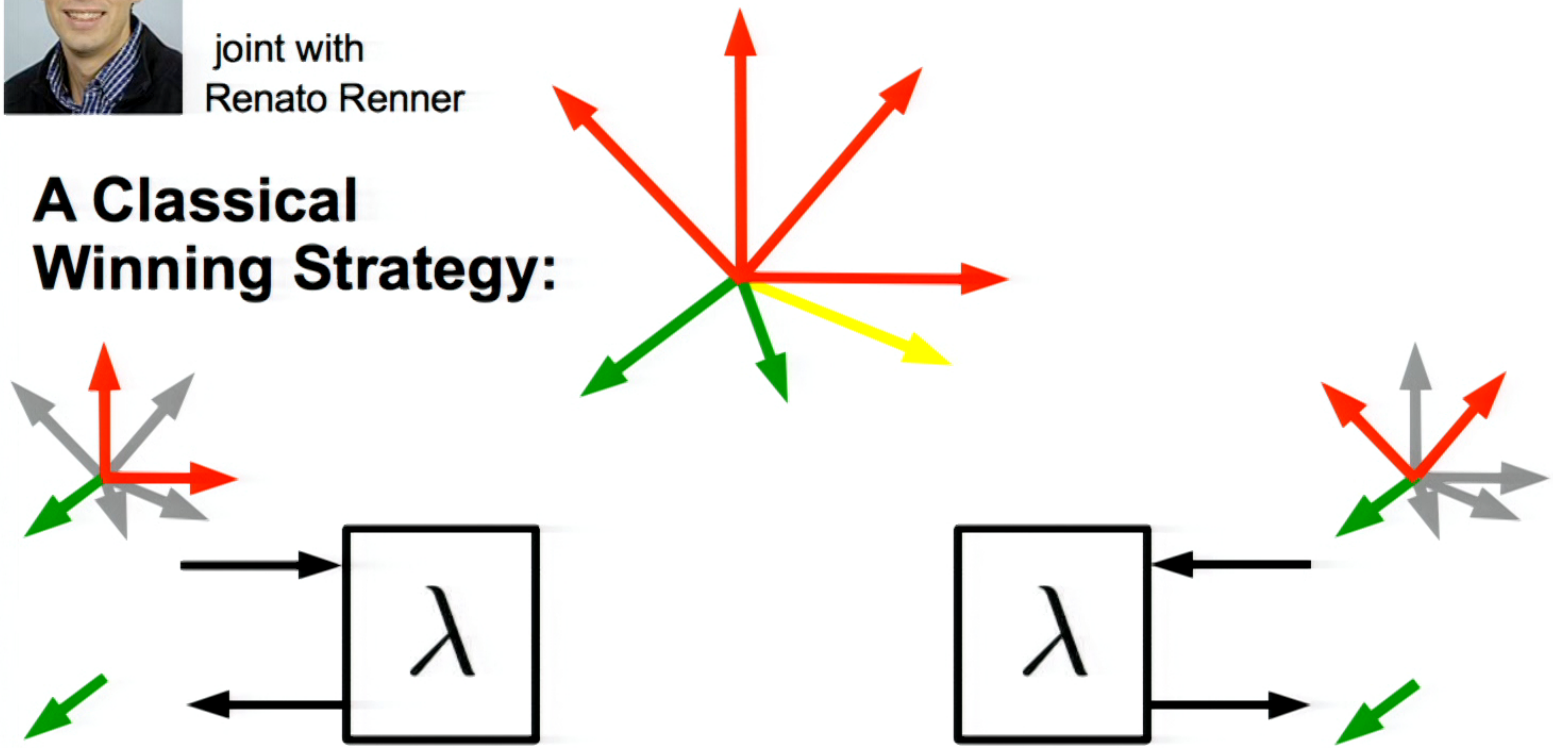
Logic and Physics

A Classical Winning Strategy:



Winning Condition:
The output vectors are not orthogonal

Logic and Physics

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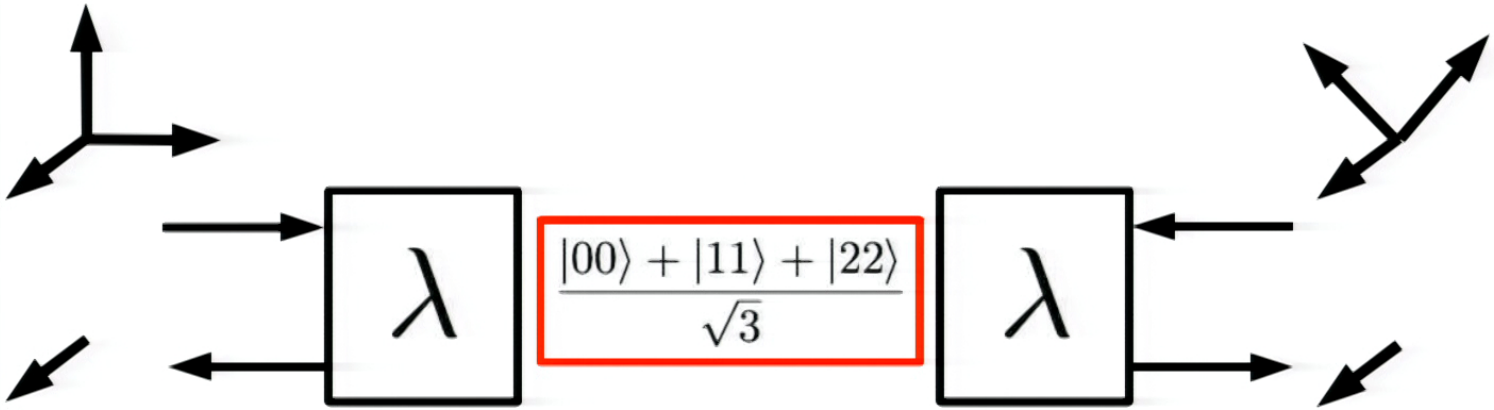
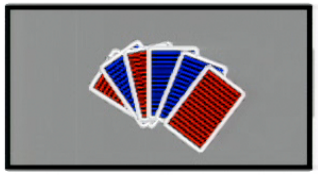
A Quantum Winning Strategy:

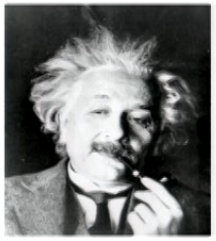


Winning Condition:
The output vectors are not orthogonal

Non-Local Correlations

The EPR Paradox





Albert Einstein

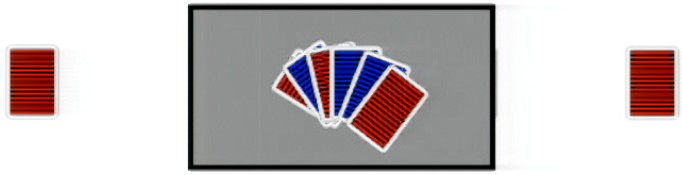
Boris Podolsky

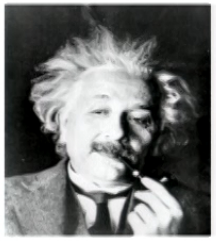
Nathan Rosen

1935

Non-Local Correlations

The EPR Paradox





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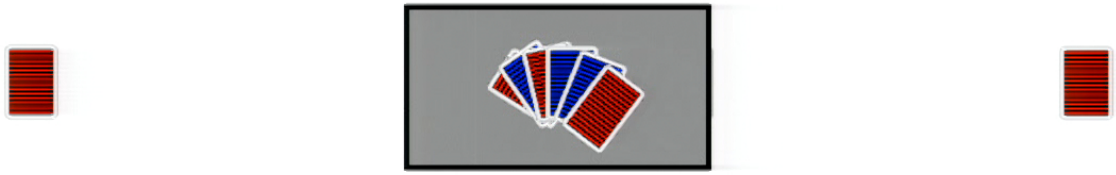
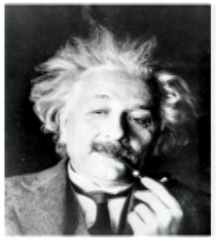




Albert Einstein

Boris Podolsky

Nathan Rosen

1935

Non-Local Correlations

~~"Cards must have pre-determined colors"~~

John Bell

1964

This is impossible...

Non-Local Correlations

... if such behavior occurs

John Bell

1964

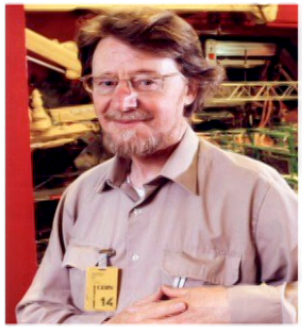
Non-Local Correlations

John Bell

1964

... if such behavior occurs

Non-Local Correlations



John Bell

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... if such behavior occurs

Non-Local Correlations

John Bell

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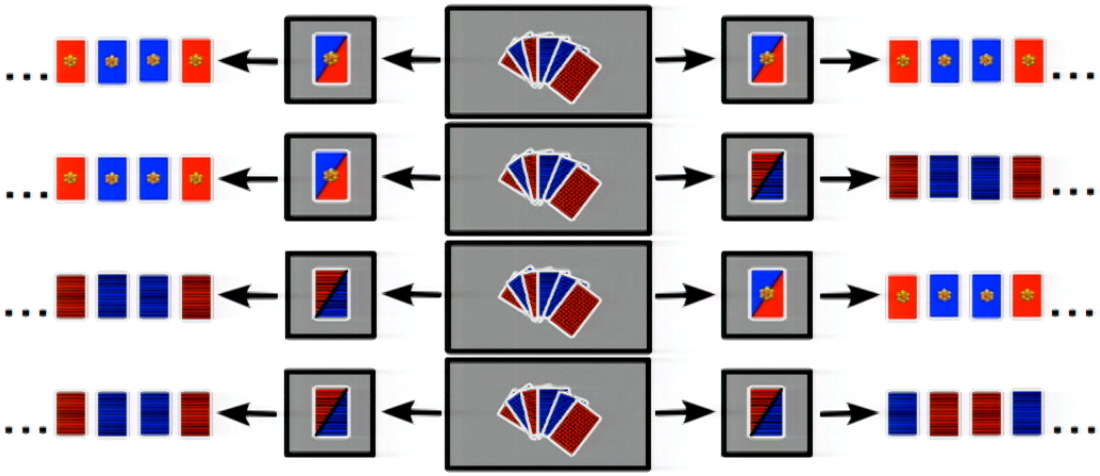
Non-Local Correlations



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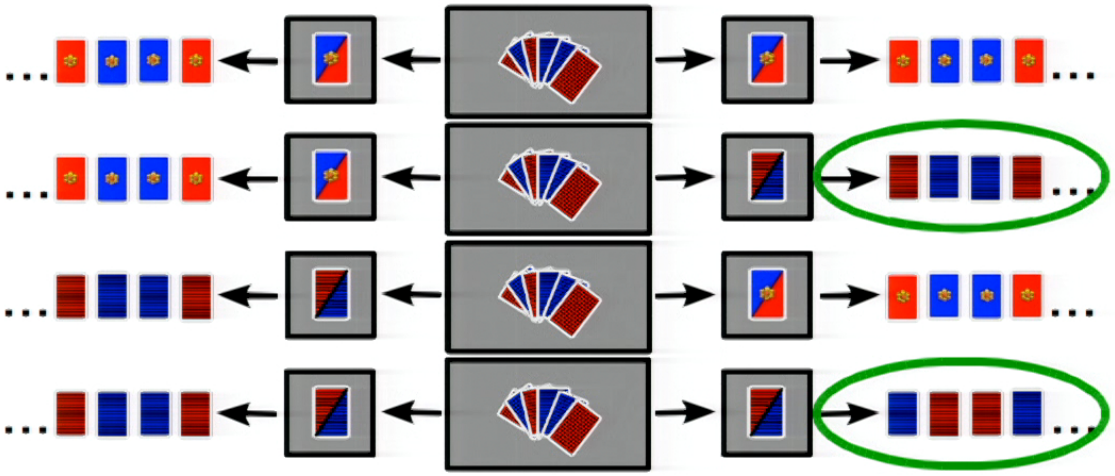
Non-Local Correlations

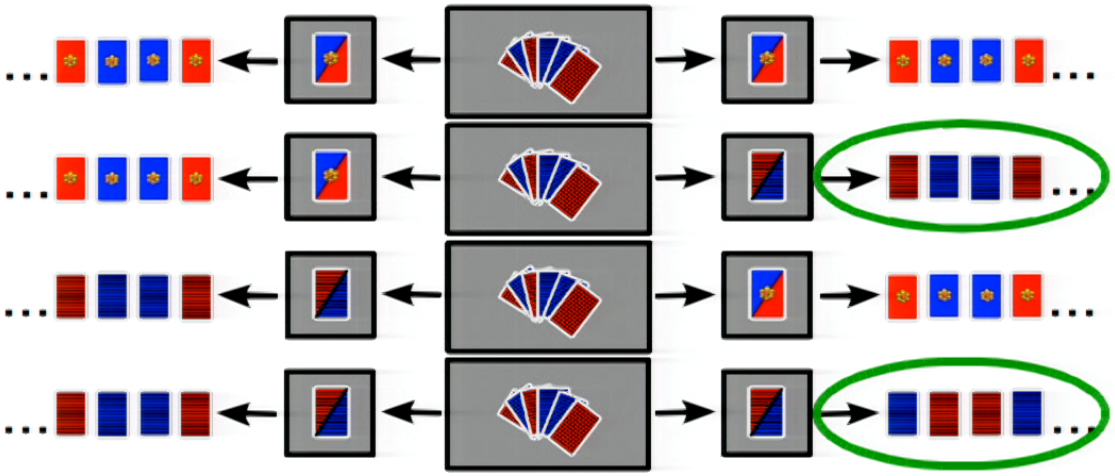


John Bell

1964

Colored cards: 75%

Non-Local Correlations



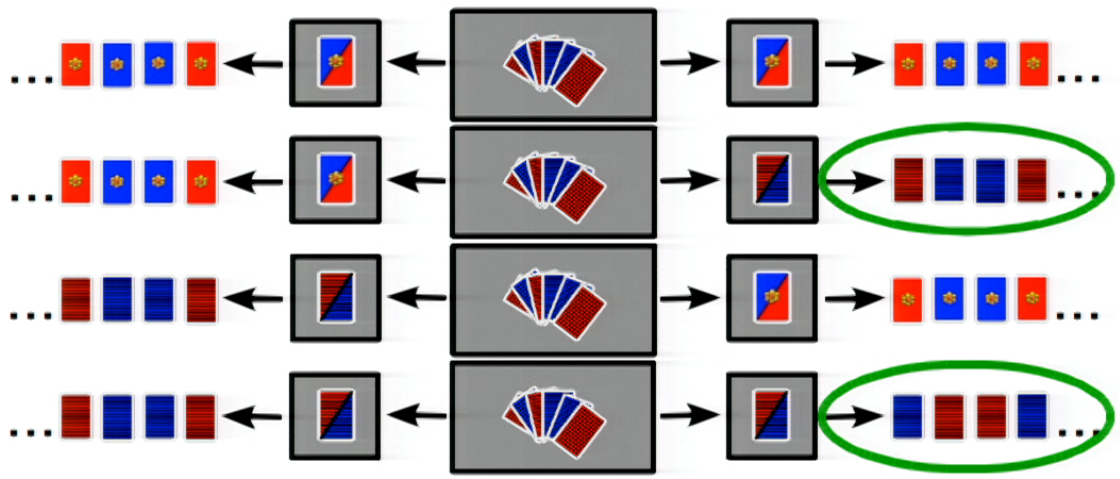
John Bell

1964

Colored cards: 75%

Photons: ~85% !

Non-Local Correlations



John Bell

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Colored cards: 75%

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Spontaneous correlations

= "Non-locality"...

1994

Non-Local Correlations

Sandu Popescu

Daniel Rohrlich

$$X \oplus Y = A \odot B$$

John Bell

1964

Colored cards: 75%

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Spontaneous correlations

= "Non-locality"...

1994

Non-Local Correlations

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John Bell

1964

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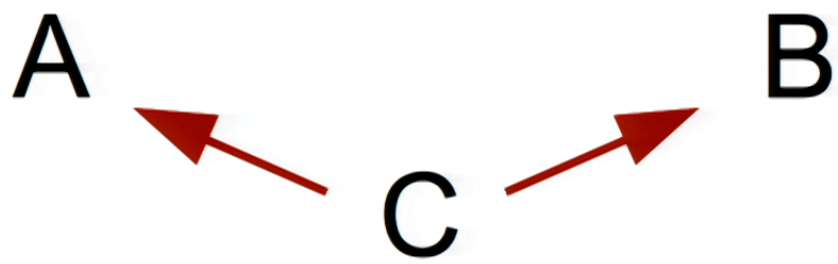
Spontaneous correlations

= "Non-locality"... Do they talk?

Explaining Correlations in a Causal Structure

1) Common Cause



2) Influence

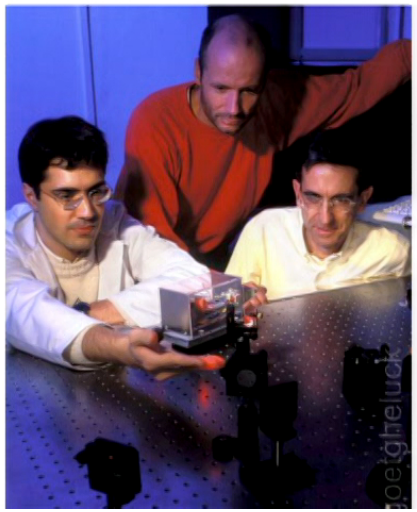
1956

Hans Reichenbach

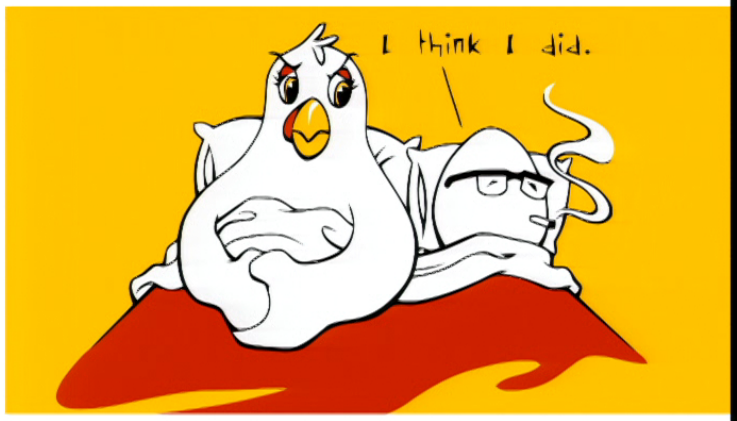
Can Communication Explain the Correlations?

The "before-before" experiment

A ⇄ B

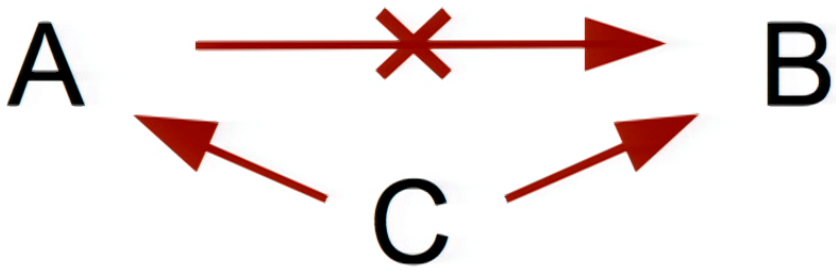


Stefanov
Suarez
Zbinden



Can Communication Explain the Correlations?

Finite-speed influences ruled out by
transitivity of non-locality



joint with

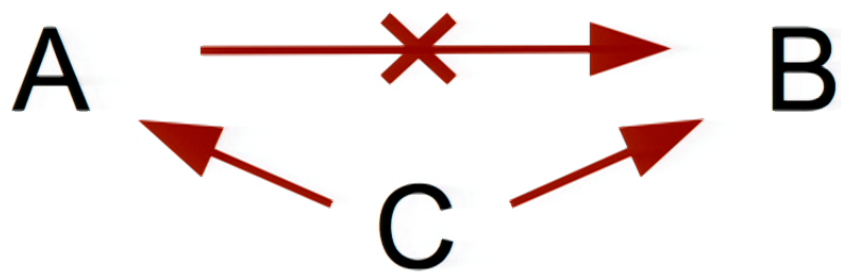
Sandro Coretti

Esther Hänggi

2011

Can Communication Explain the Correlations?

Finite-speed influences ruled out by
transitivity of **quantum** non-locality



Jean-Daniel Bancal

Tomer Barnea

Nicolas Gisin

Yeong-Cherng Liang

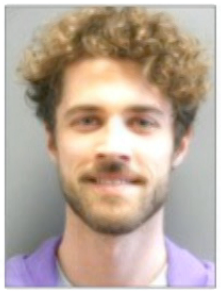
2013

Can Communication Explain the Correlations?

$$A \xrightarrow[\text{hidden influences}]{\textit{instantaneous}} B$$

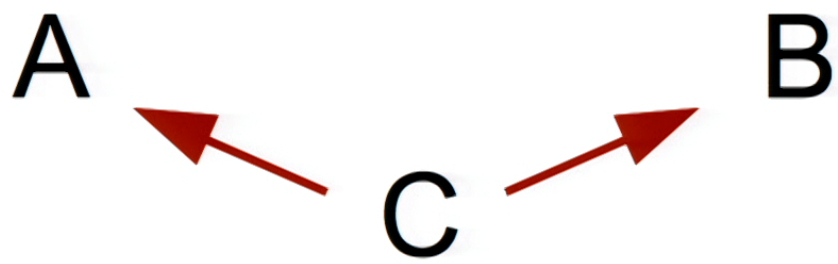
Also unsatisfactory: *Fine-tuning* required

Rob Spekkens 2012 Christopher Wood



Explaining Correlations in a Causal Structure

1) Common Cause



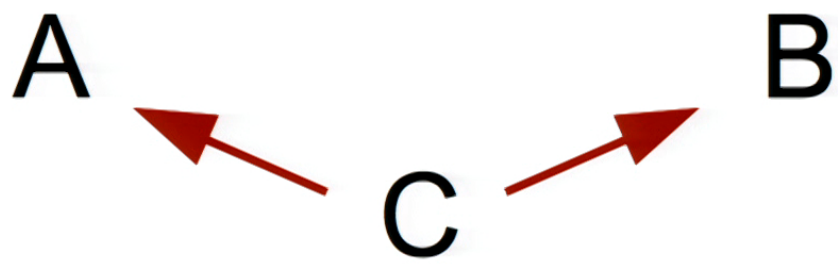
2) Influence

1956

Hans Reichenbach

Explaining Correlations in a Causal Structure

1) Common Cause



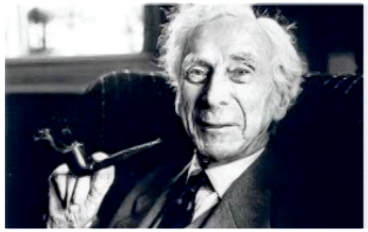
2) Influence

3) Drop the Causal Structure?

1956

Hans Reichenbach

Explaining Correlations in a Causal Structure



Bertrand Russell

1913

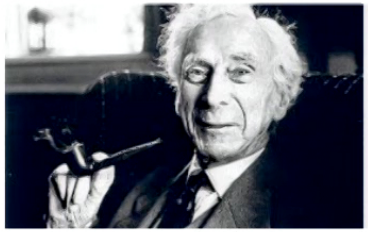
The law of causality is a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm

Drop the Causal Structure?

1956

Hans Reichenbach

Explaining Correlations in a Causal Structure



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1913

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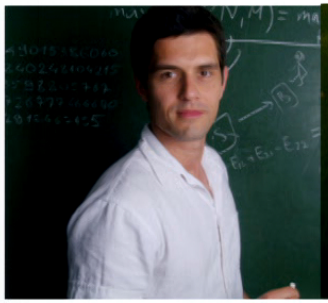
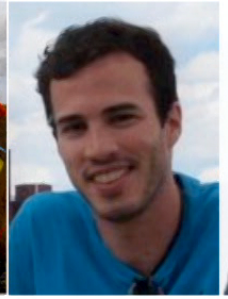
Oreshkov



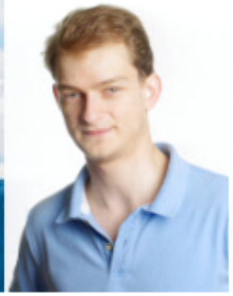
Costa

Brukner

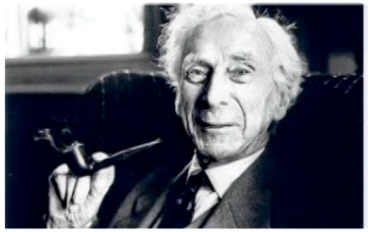
Baumeler



Feix



Explaining Correlations in a Causal Structure



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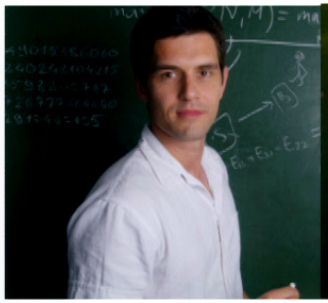

Costa

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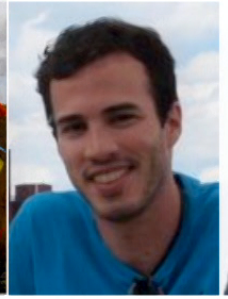
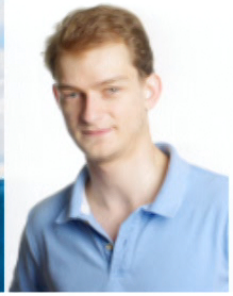
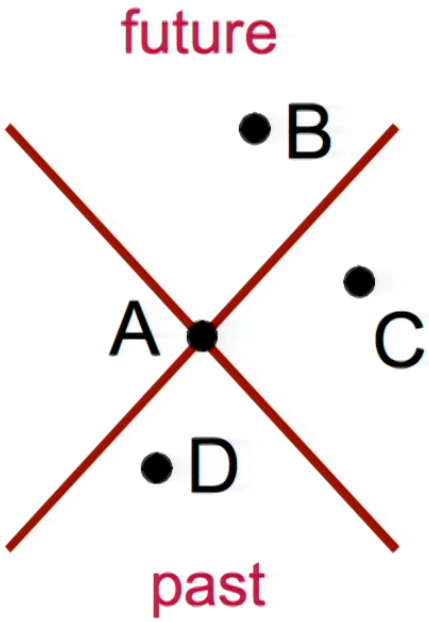


Feix



Causality and Randomness



A is free if it is independent of C and D (of all except its future)

Colbeck

Renner

Randomness

How to define it intrinsically?

Randomness

How to define it *physically*?

"Information Is Physical"

1961

Rolf Landauer

Randomness

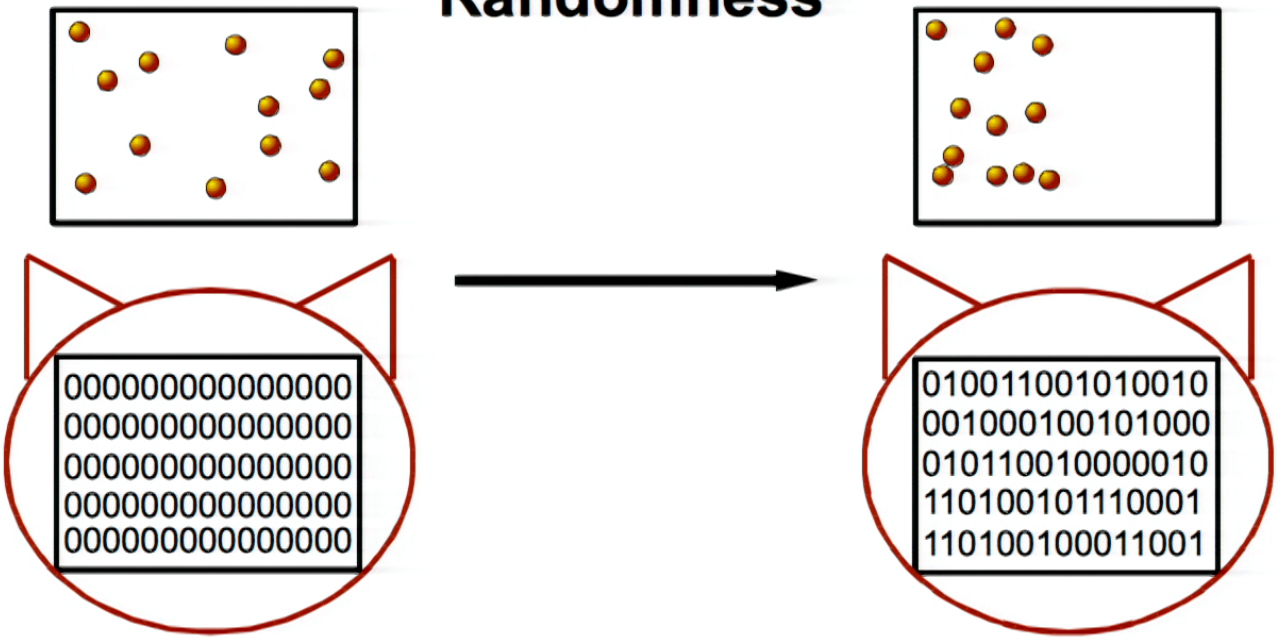
How to define it *physically*?

00000.....000 is *not* random

"Information Is Physical"

1961
Rolf Landauer

Randomness



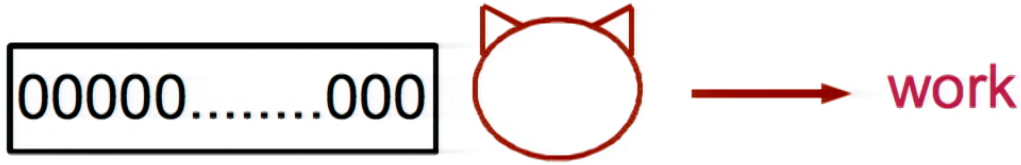
"Information Is Physical"

Charles H. Bennett

Rolf Landauer

Randomness

How to define it intrinsically?



"Information Is Physical"

Charles H. Bennett

Rolf Landauer

Randomness

How to define it intrinsically?

00000.....000 → work kTN ln 2

N

31415926535.. → work kTN ln 10

N



Edward Fredkin
Tommaso Toffoli

Randomness

How to define it intrinsically?

00000.....000 → work kTN ln 2

N

31415926535.. → work kTN ln 10

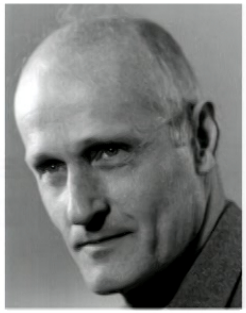
N

most strings S → **no** work: **random**

N

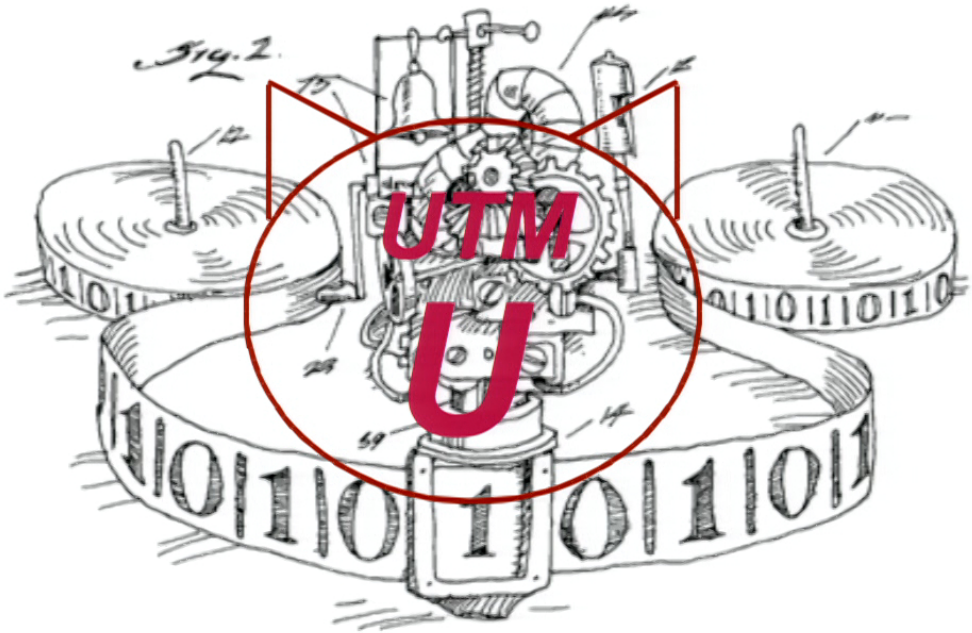
Church-Turing Hypothesis

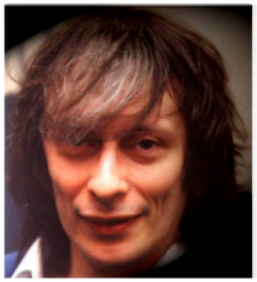
Alan Turing
1936



Stephen Kleene
1943

Alonzo Church



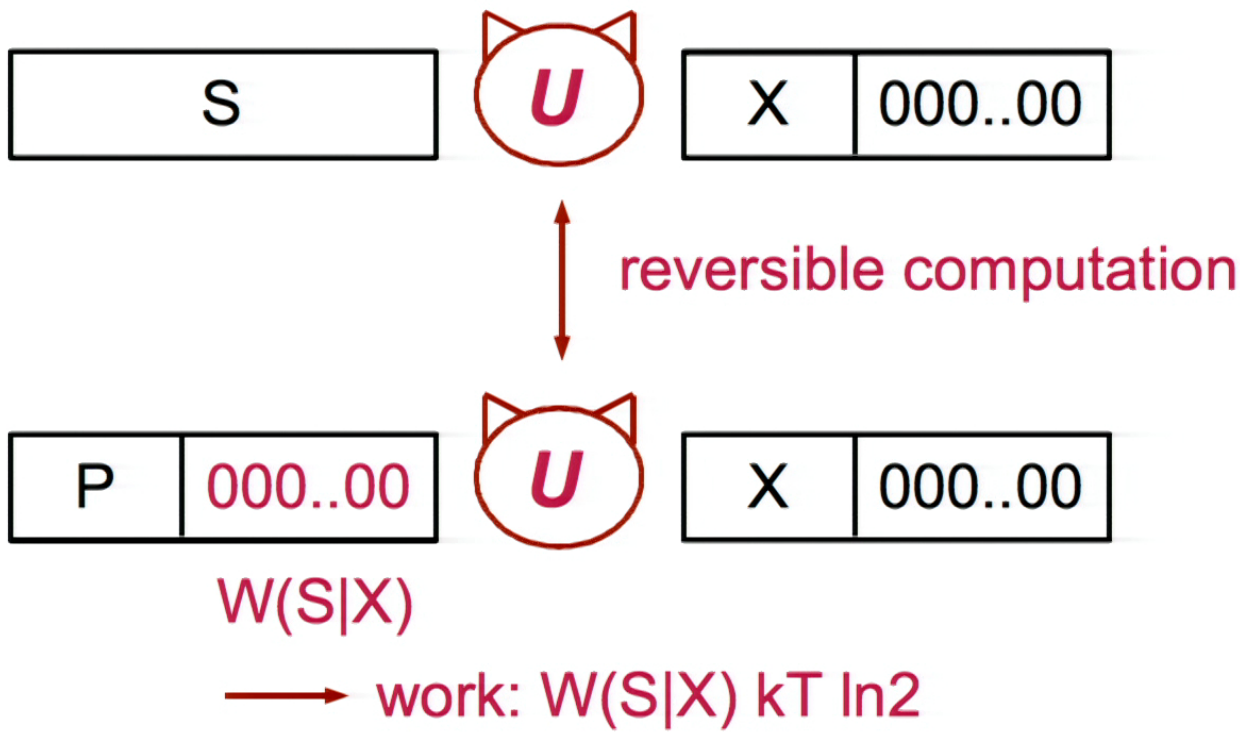


David Deutsch

Work Extraction

Work Extraction

... is data compression



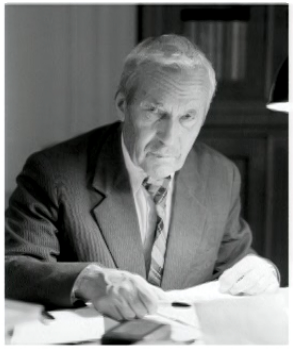
Bound on the Work Value

$$W(S|X) \leq \mathrm{len}(S) - K_{\mathcal{U}}(S|X)$$

length of the shortest program for U to compute S given X

Charles H. Bennett

Andrei Kolmogorov



Work Extraction

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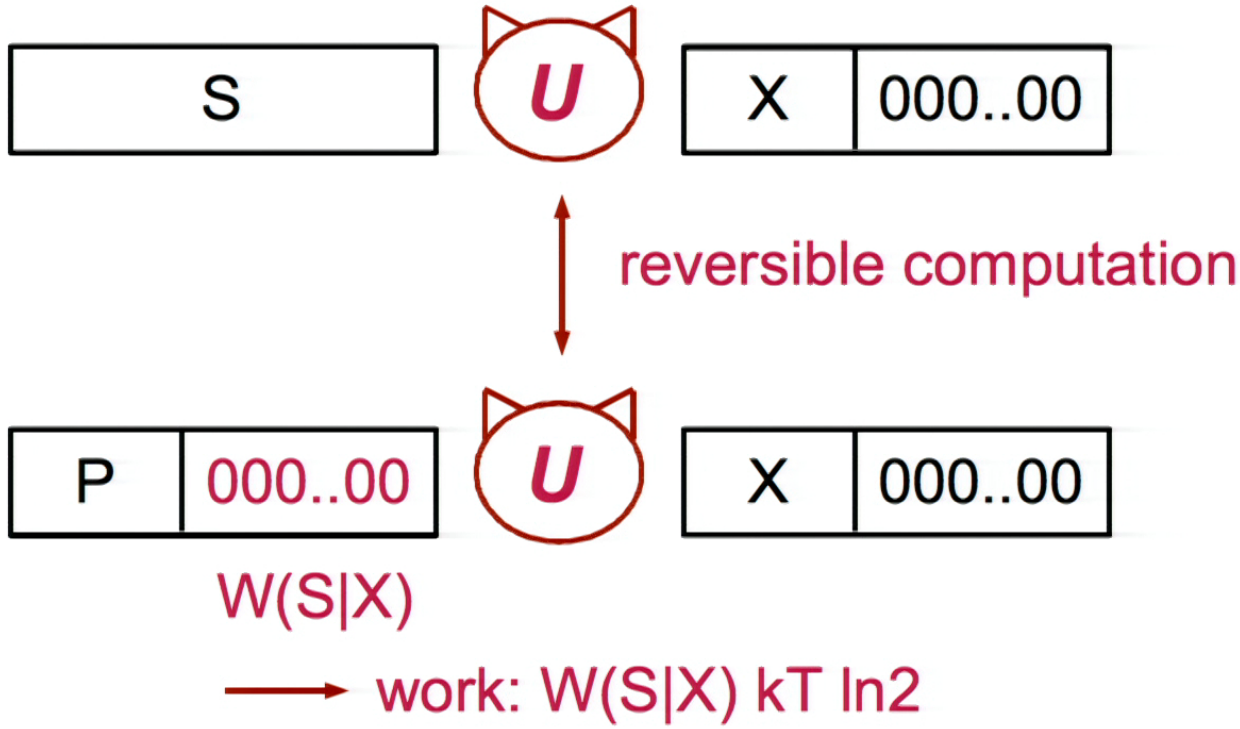

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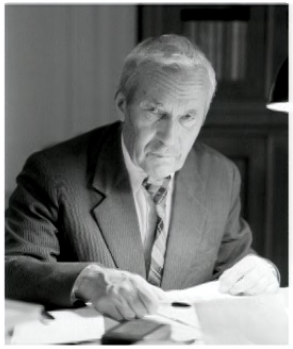


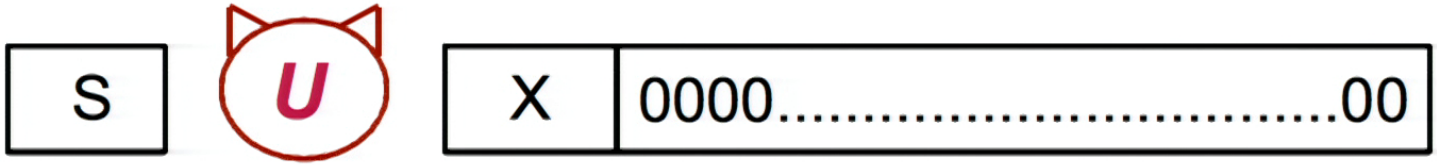
Bounds on the Work Value

Upper bound:

$$W(S|X) \leq \mathrm{len}(S) - K_{\mathcal{U}}(S|X)$$

Charles H. Bennett

Tight if $K_{\mathcal{U}}(S|X) \approx 0$:



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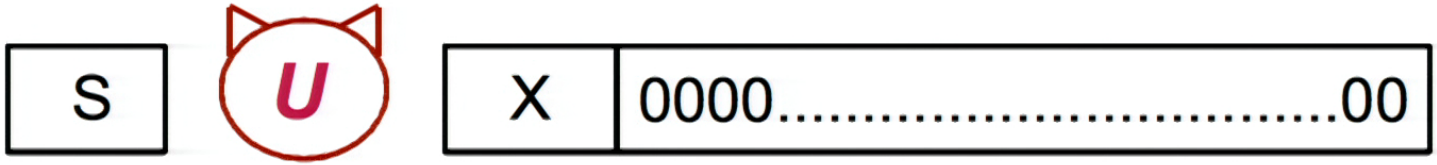


X is a program for S: Compute an extra copy

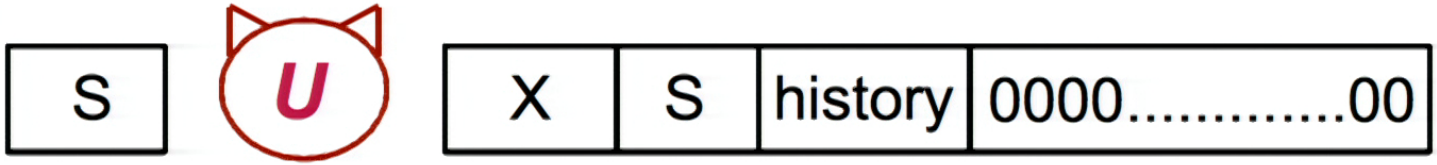
Bounds on the Work Value

Upper bound:

$$W(S|X) \leq \mathrm{len}(S) - K_{\mathcal{U}}(S|X)$$

Charles H. Bennett

Tight if $K_{\mathcal{U}}(S|X) \approx 0$:



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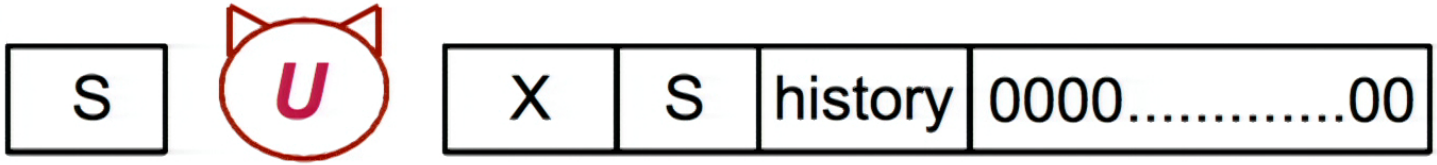


XOR the new S to the original one

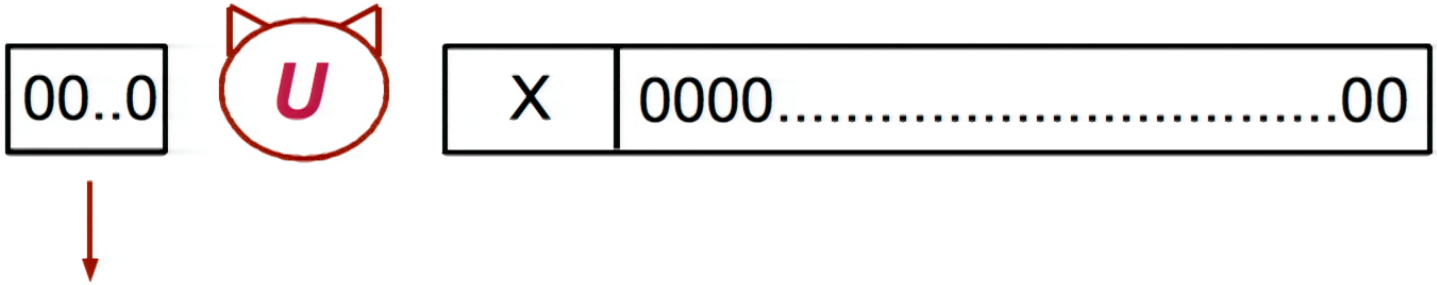
Bounds on the Work Value

Upper bound:

$$W(S|X) \leq \mathrm{len}(S) - K_{\mathcal{U}}(S|X)$$

Charles H. Bennett

Tight if $K_{\mathcal{U}}(S|X) \approx 0$:



work: len(S) kT ln2

Bounds on the Work Value

Upper bound:

$$W(S|X) \leq \mathrm{len}(S) - K_{\mathcal{U}}(S|X)$$

Charles H. Bennett

Tight if $K_{\mathcal{U}}(S|X) \approx 0$:

In accordance with entropic results if knowledge consists of a program for *S*

Dahlsten
Rieper
Renner
Vedral

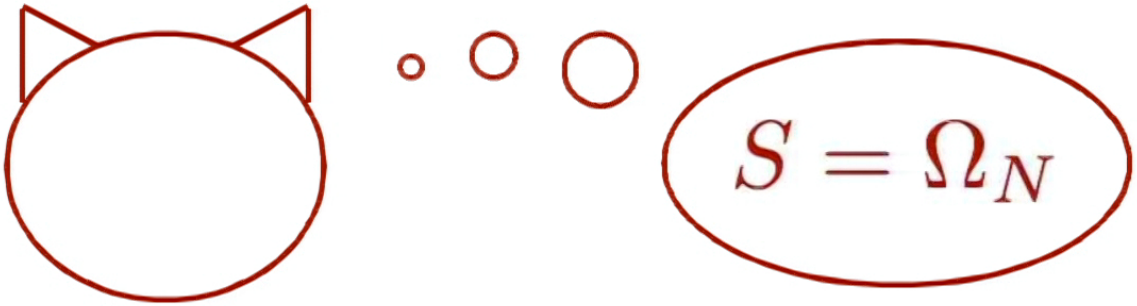
Bounds on the Work Value

Upper bound:

$$W(S|X) \leq \mathrm{len}(S) - K_{\mathcal{U}}(S|X)$$

Charles H. Bennett

What if the knowledge is *not* constructive?



Bounds on the Work Value

Upper bound:

$$W(S|X) \leq \mathrm{len}(S) - K_{\mathcal{U}}(S|X)$$

Charles H. Bennett

Lower bound:

C *compression algorithm* with helper, *i.e.*,

$C : S||X \mapsto C(S,X)||X$ reversible:

$$W(S|X) \geq \mathrm{len}(S) - \mathrm{len}(C(S,X))$$

Wojciech Zurek

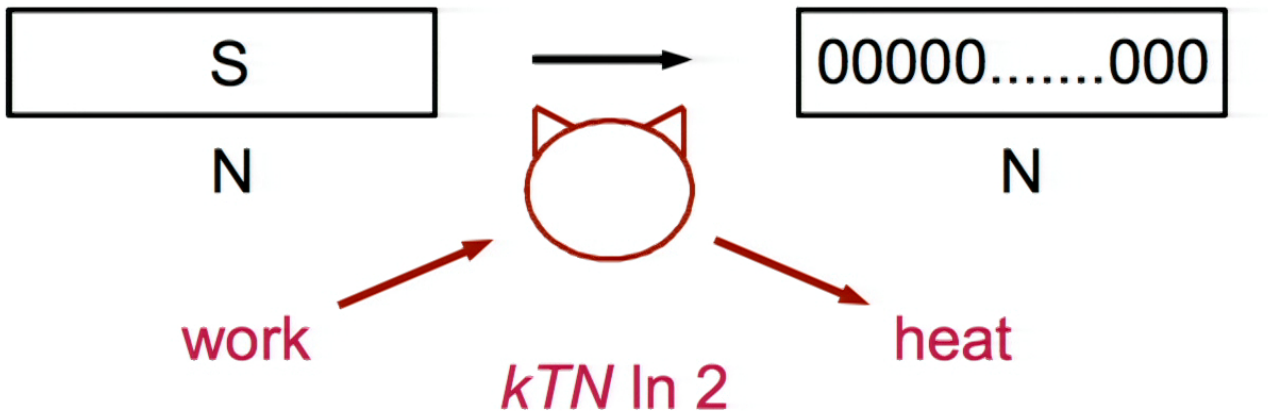
Erasure Cost

Landauer's principle

1961
Rolf Landauer

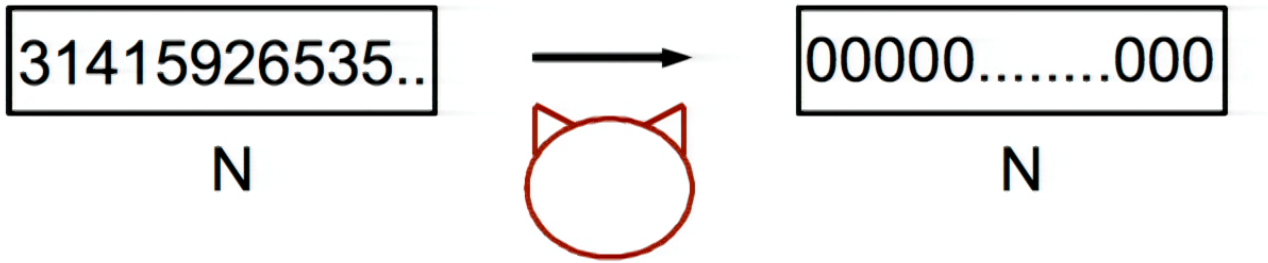
Erasure Cost

Landauer's principle

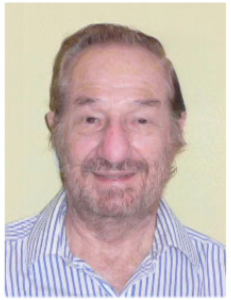


1961
Rolf Landauer

Erasure Cost



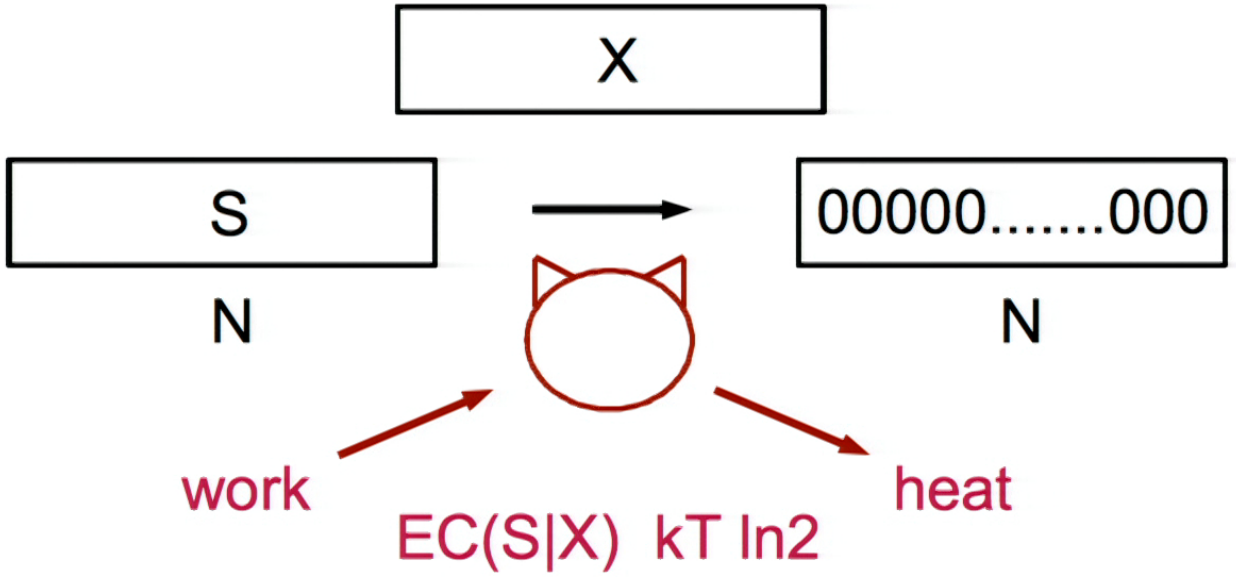
reversible: no work required



Edward Fredkin
Tommaso Toffoli
1982

1961
Rolf Landauer

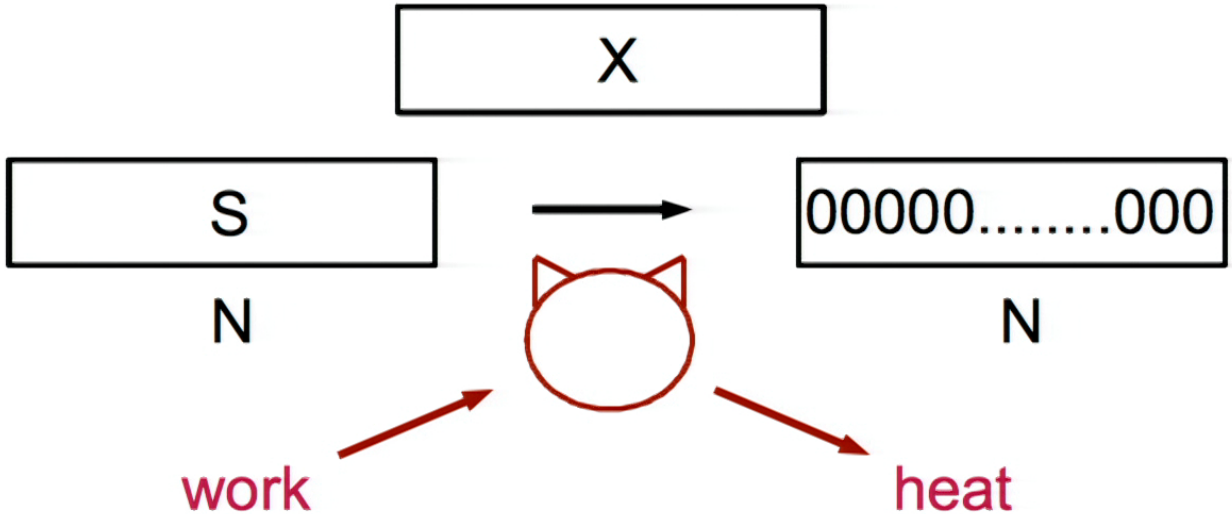
Erasure Cost



$$EC(S|X) + W(S|X) = len(S)$$

1961
Rolf Landauer

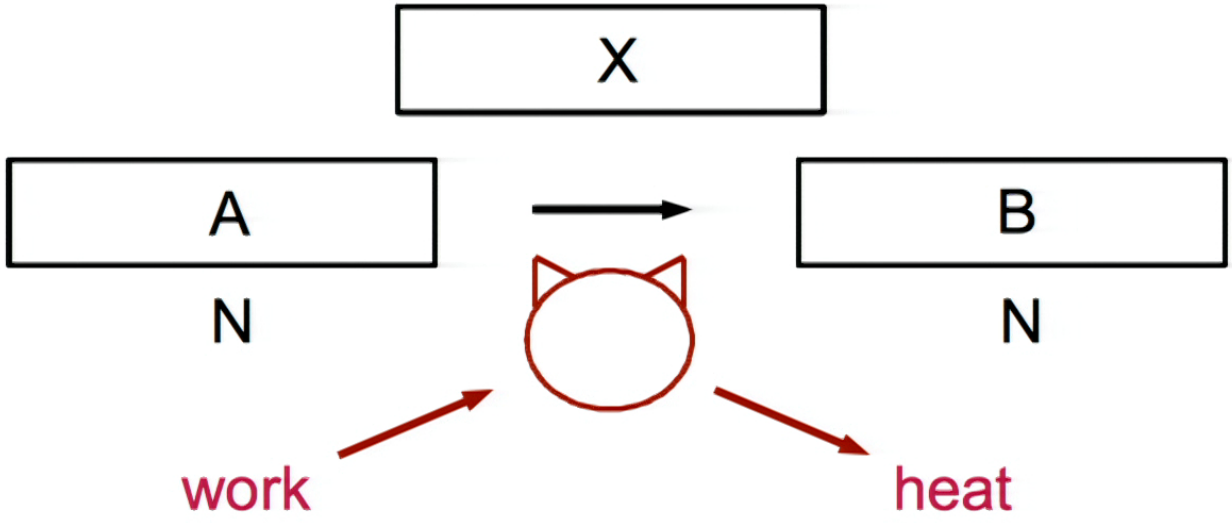
Landauer's Principle, *revisited*



$$K(S|X) \leq EC(S|X) \leq len(C(S|X))$$

1961
Rolf Landauer

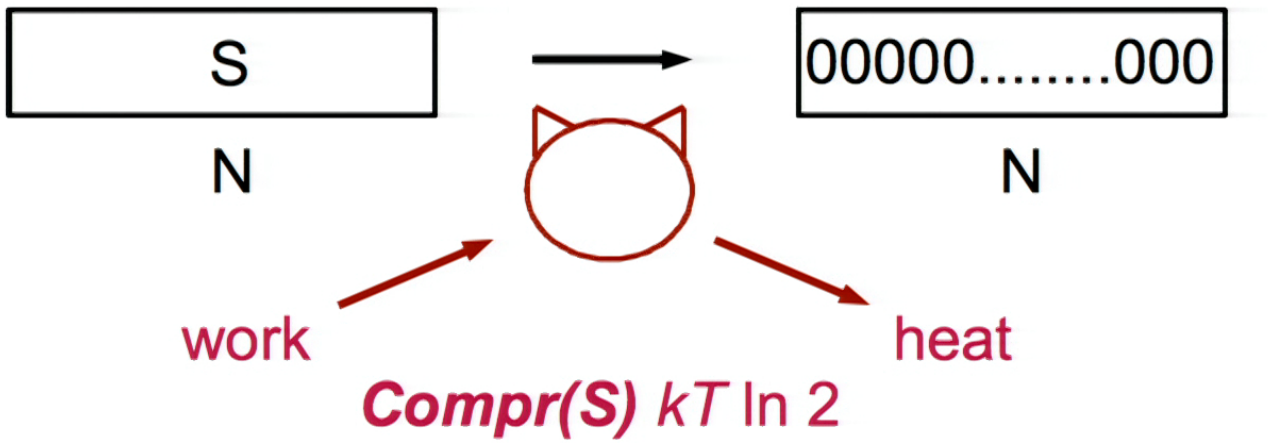
Landauer's Principle, *generalized*



$$\mathrm{Cost}(A \to B|X) \geq K(A|X) - \mathrm{len}(C(B|X))$$

1961
Rolf Landauer

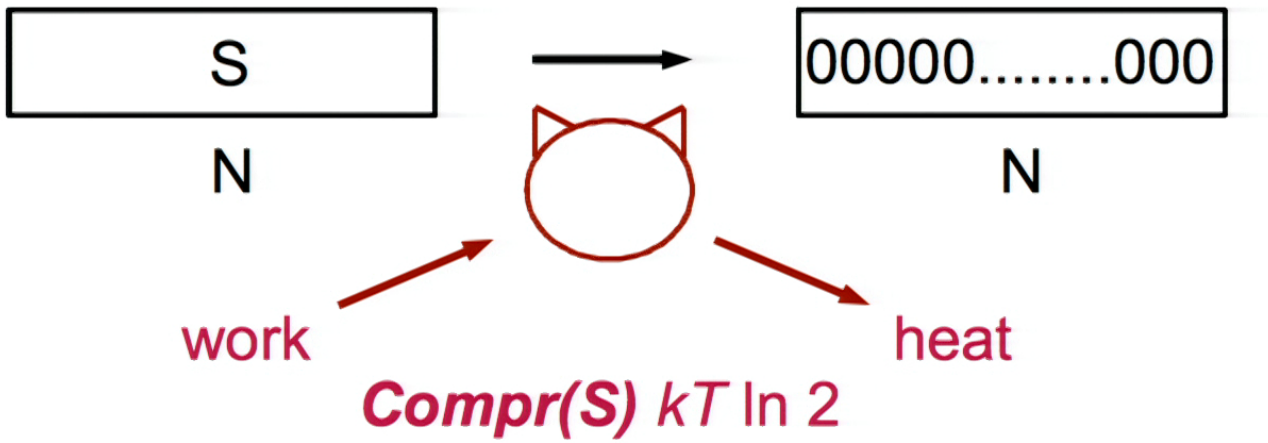
Landauer's Principle, *revisited*



Intrinsic, physical
***randomness** measure*

1961
Rolf Landauer

Landauer's Principle, *revisited*



Thermodynamical price
for ***logical*** irreversibility?

1961
Rolf Landauer

Landauer's Principle, *revisited*

"It from Bit"

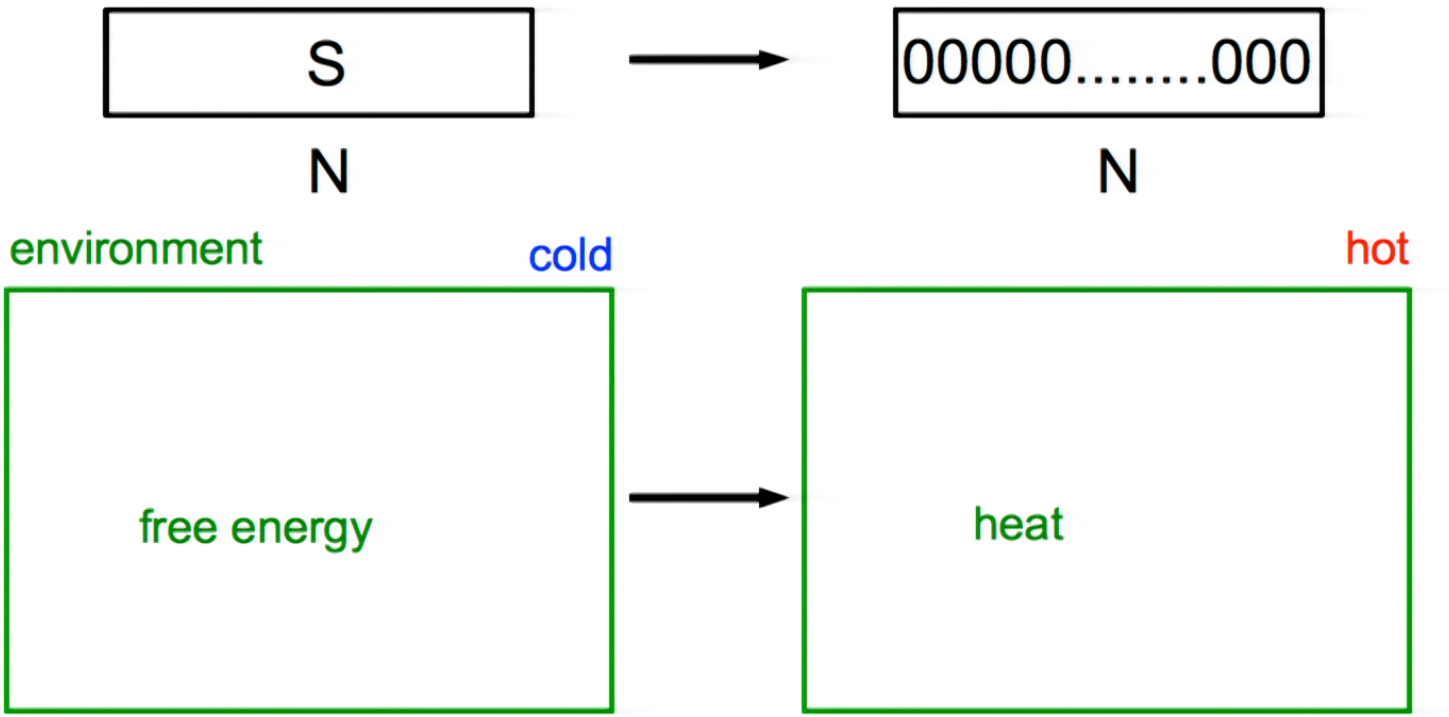
John Archibald Wheeler

1989

Every *it* — every particle, every field of force, even the spacetime continuum itself — derives its function, its meaning, its very existence entirely — from *bits.*

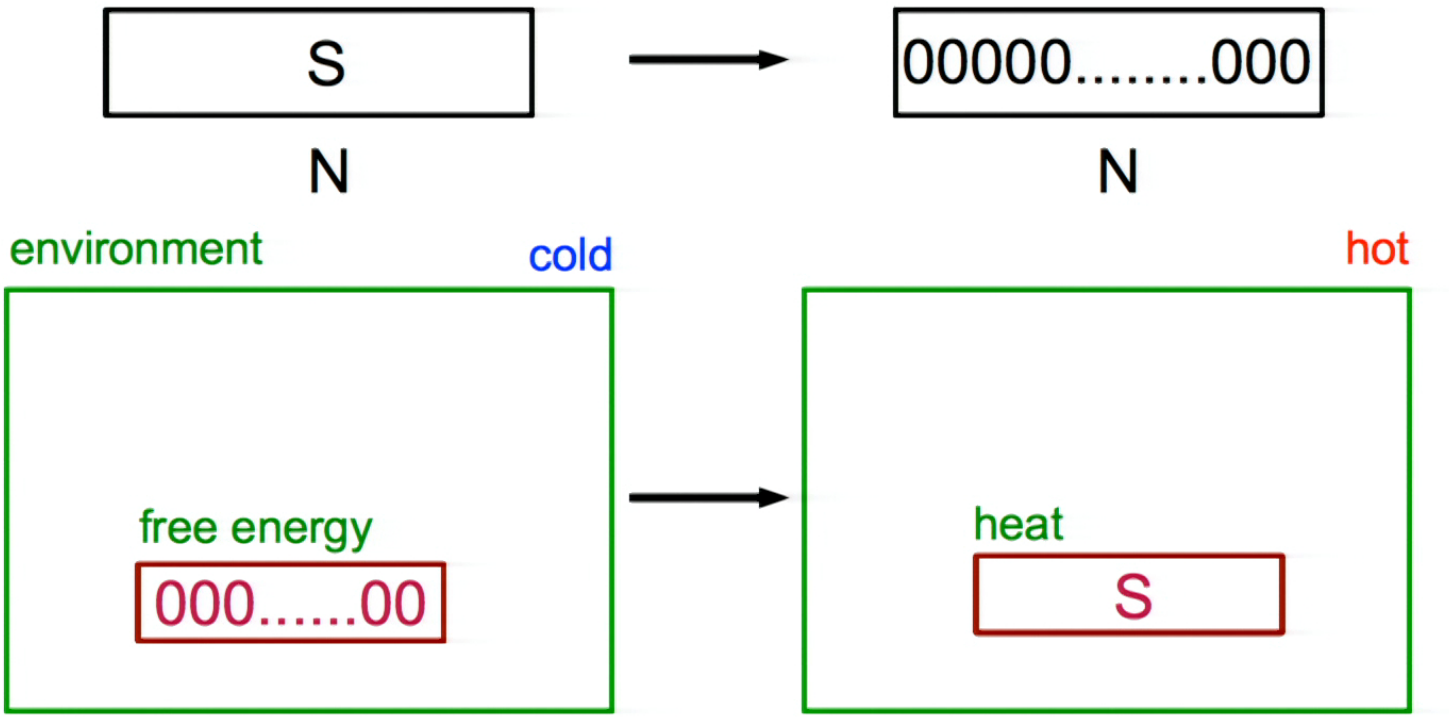
Landauer's Principle, *revisited*

Compensation in environment ...



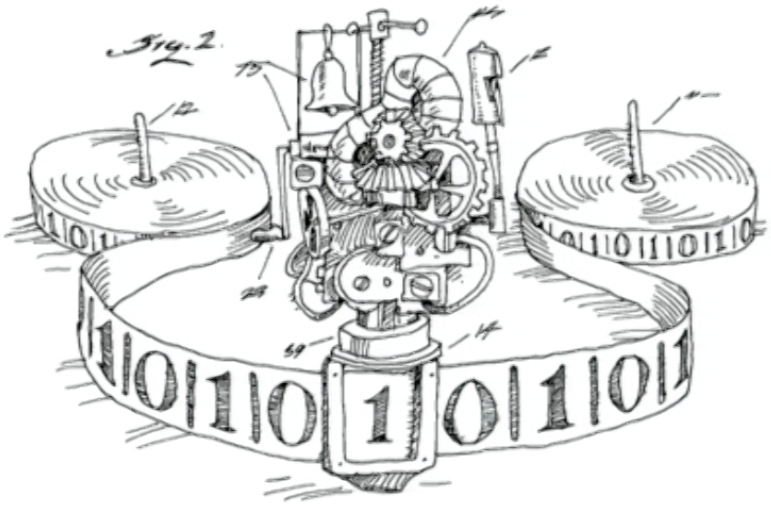
Landauer's Principle, *revisited*

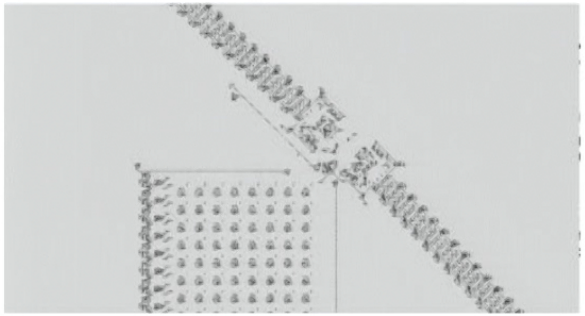
Compensation in environment must be such that ***overall map is logically reversible***



The Second Law, *logical*

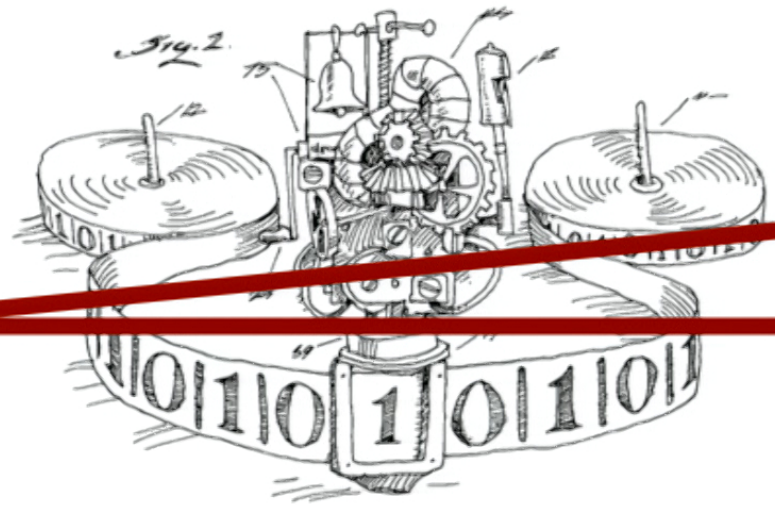
logical reversibility





The Second Law, *logical*

logical reversibility $\Longrightarrow$ K(tape) **(essentially) non-decreasing**



probabilistic

deterministic

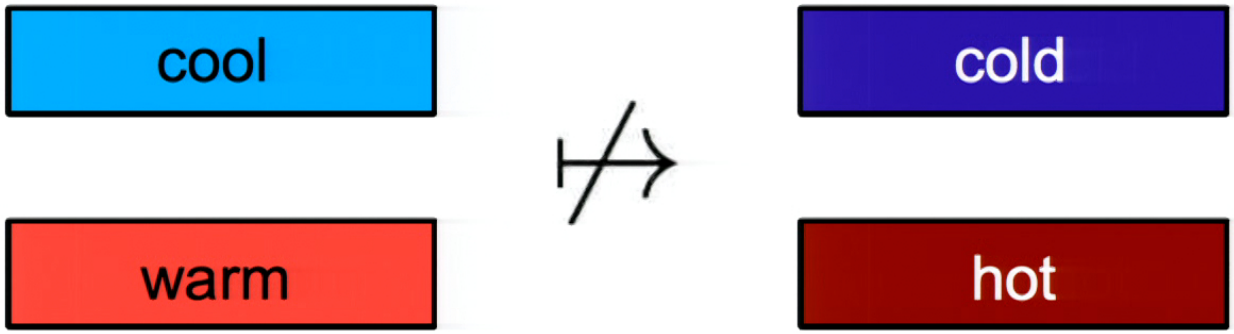
The Second Law, *logical*

logical reversibility $\Longrightarrow$

001000101000	$\not\Longrightarrow$	000000100000
1110110111101		1111110111111

The Second Law, *logical*

logical reversibility $\Longrightarrow$ **Clausius**-like second law



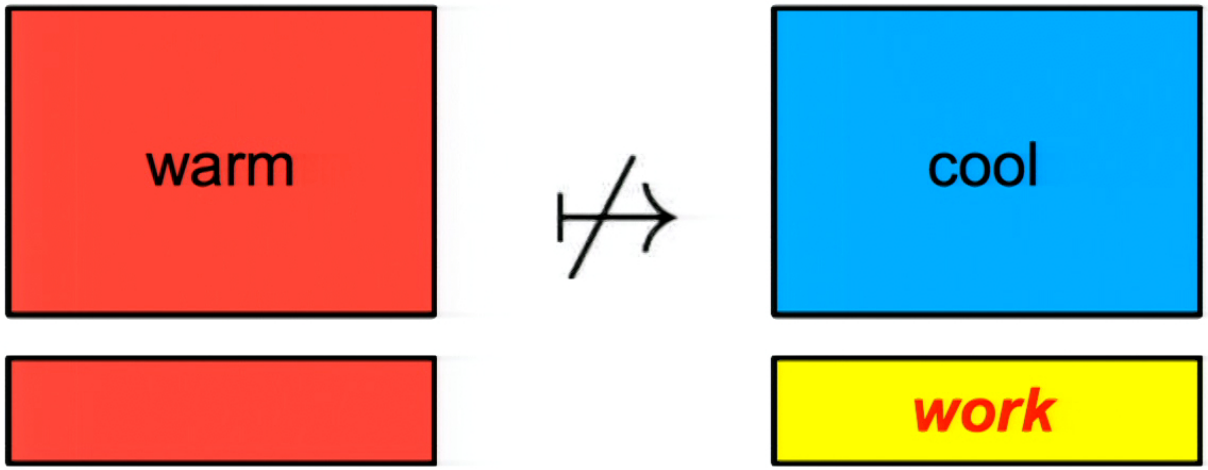
The Second Law

logical reversibility $\Longrightarrow$

1101001111011 1011011110011 1010110111101 1101101011011	$\not\Longrightarrow$	001110010101 011110101000 101111000010 000010110011
1110110111101		111111111111111

The Second Law

logical reversibility $\Longrightarrow$ **Kelvin**-like second law



Back to Non-Local Correlations

John Bell 1964

Arthur Fine 1982

$$X \oplus Y = A \odot B$$

1994
Sandu Popescu

Daniel Rohrlich

John Bell 1964

Back to Non-Local Correlations

Probabilistic reasoning

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Back to Non-Local Correlations

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John Bell 1964

Back to Non-Local Correlations

Probabilistic reasoning

no-signaling:

Arthur Fine 1982

$$X \oplus Y = A \odot B$$

1994
Sandu Popescu

Daniel Rohrlich

John Bell 1964

Back to Non-Local Correlations

Probabilistic reasoning

Arthur Fine 1982

$$X \oplus Y = A \odot B$$

1994
Sandu Popescu

Daniel Rohrlich

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Back to Non-Local Correlations

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$$X \oplus Y = A \odot B$$

1994
Sandu Popescu

contradiction

Daniel Rohrlich

John Bell 1964

Back to Non-Local Correlations

Probabilistic reasoning

Arthur Fine 1982

Consequence of non-signaling:

If all (A,B) combinations are possible...

1994
Sandu Popescu

Daniel Rohrlich

John Bell 1964

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Probabilistic reasoning

Arthur Fine 1982

Consequence of non-signaling:

If all (A,B) combinations are possible...

... then X and Y must be *perfectly random*

1994
Sandu Popescu

Daniel Rohrlich

Device-Independent Quantum Cryptography

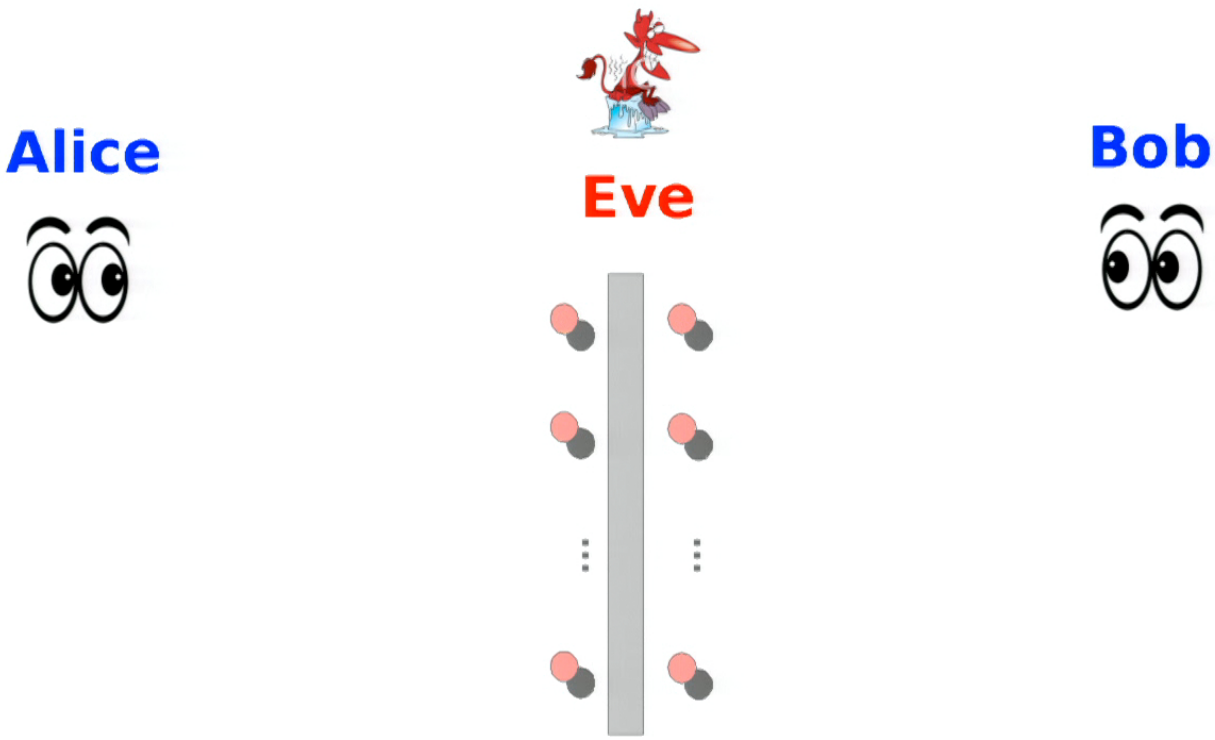
Alice

Eve

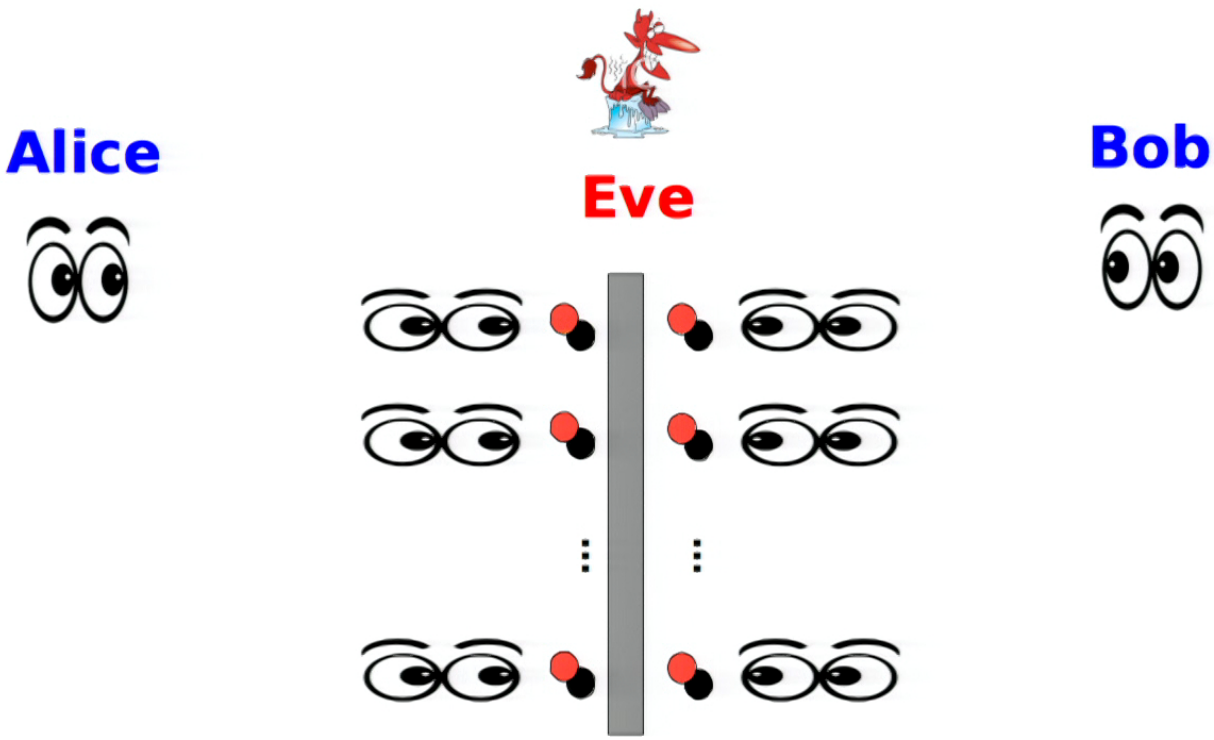
Bob

Device-Independent Quantum Cryptography

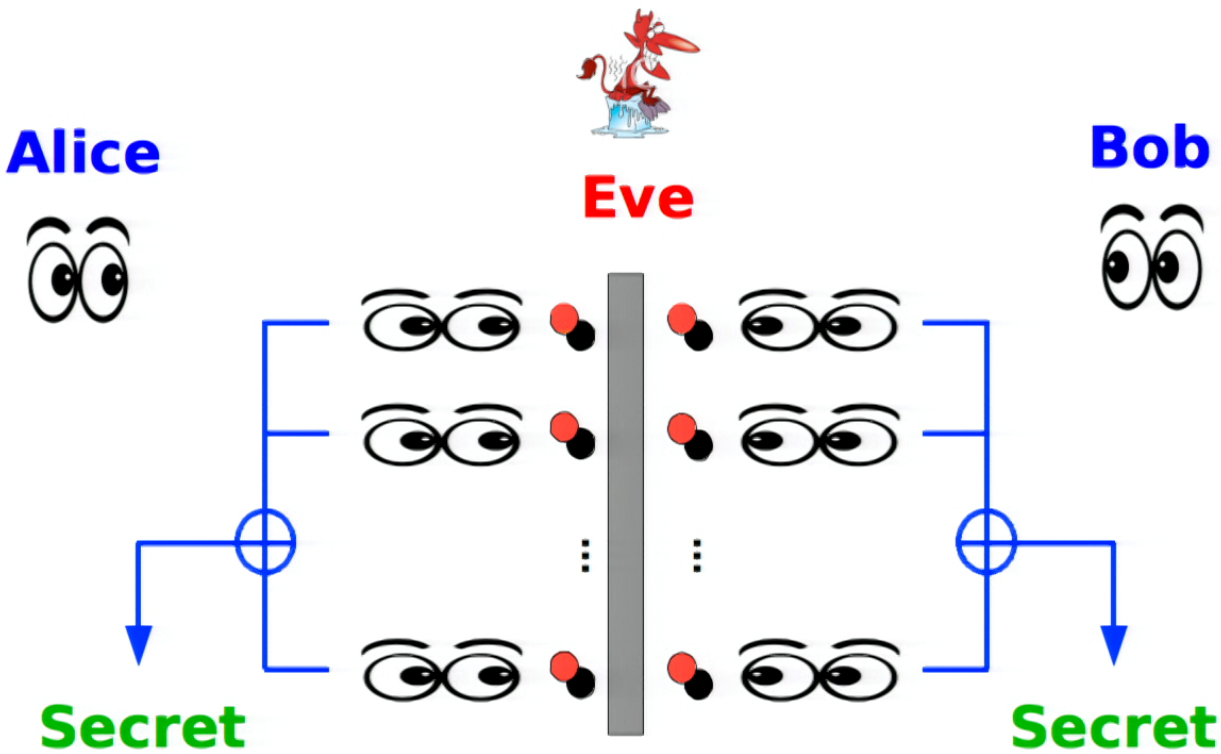
Device-Independent Quantum Cryptography



Device-Independent Quantum Cryptography



Device-Independent Quantum Cryptography



John Bell 1964

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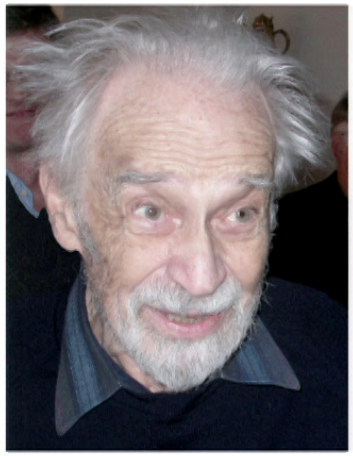
Daniel Rohrlich



Back to Non-Local Correlations

Ernst Specker
1961

This is related to the scholastic speculations on the *infuturabili*, *i.e.*, the question whether divine omniscience extends to what *would have happened if something had happened that did not happen*.

Back to Non-Local Correlations

Factual-only reasoning

$$x_i \oplus y_i = a_i \odot b_i$$

Back to Non-Local Correlations

Factual-only reasoning

***(a,b) incompressible*:**

$$\frac{K(a^n, b^n)}{2n} \longrightarrow 1$$

for $n \longrightarrow \infty$ and $a^n := (a_1, \ldots, a_n)$

$$x_i \oplus y_i = a_i \odot b_i$$

Back to Non-Local Correlations

Factual-only reasoning

no-signaling

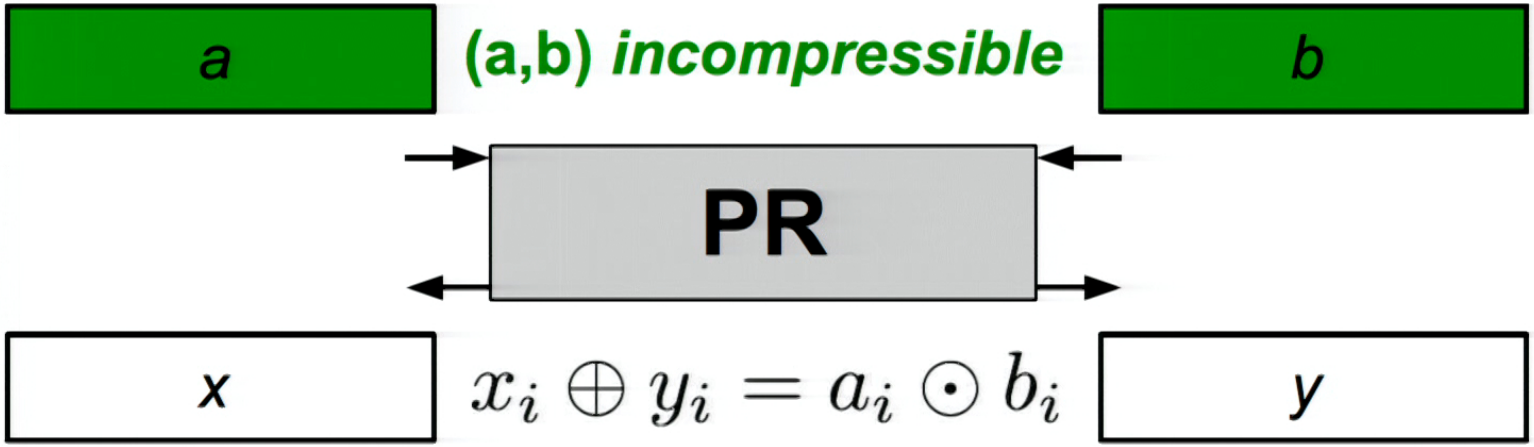
$$\frac{K(x^n|a^n)}{K(x^n|a^n,b^n)} \longrightarrow 1$$

for $n \longrightarrow \infty$, and symmetric

$$x_i \oplus y_i = a_i \odot b_i$$

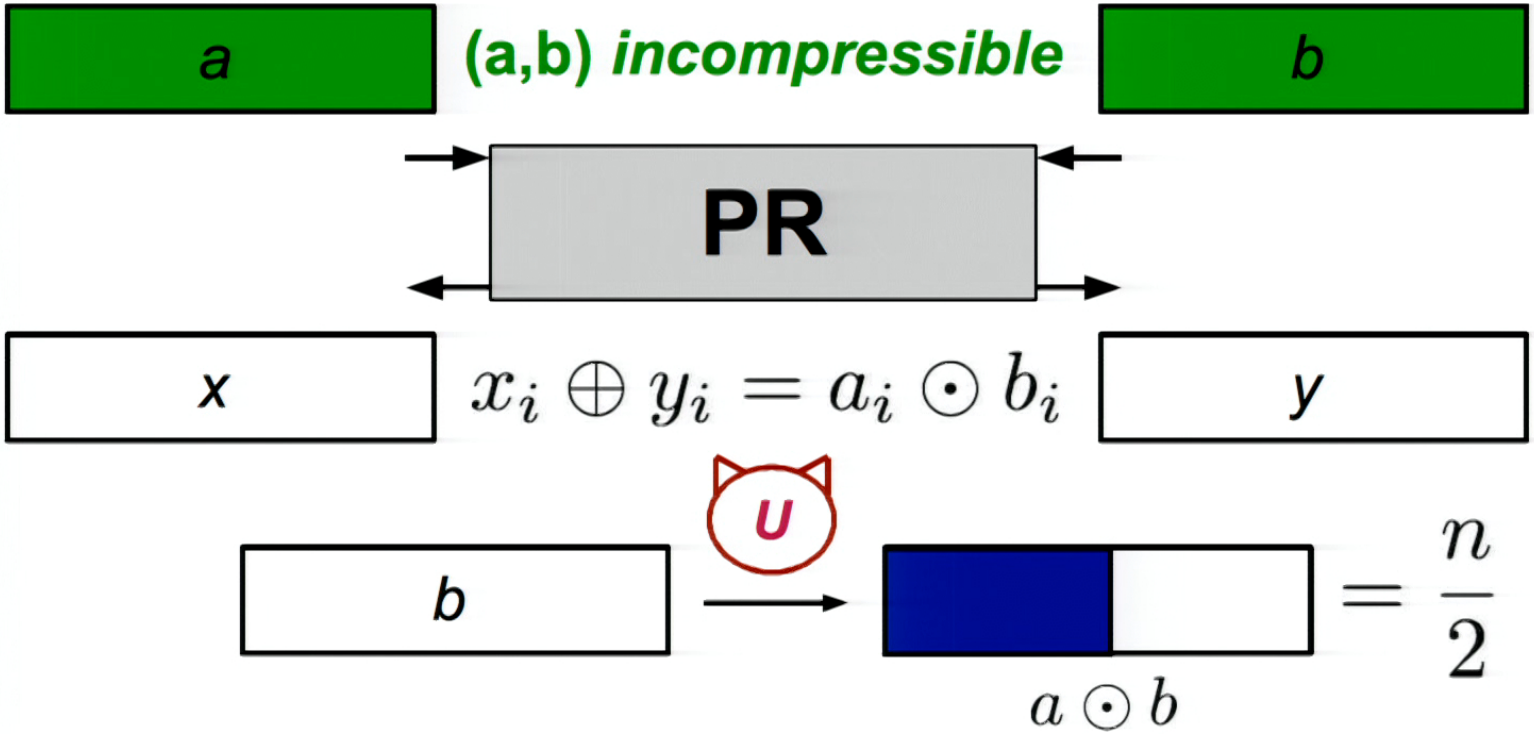
Back to Non-Local Correlations

Factual-only reasoning

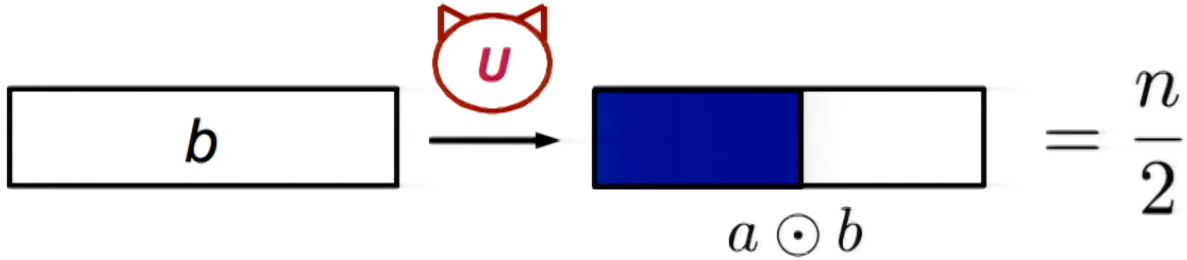


Back to Non-Local Correlations

Factual-only reasoning

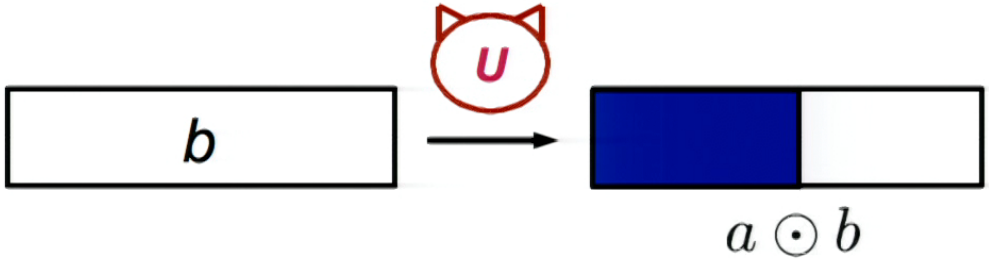


$$x_i \oplus y_i = a_i \odot b_i$$

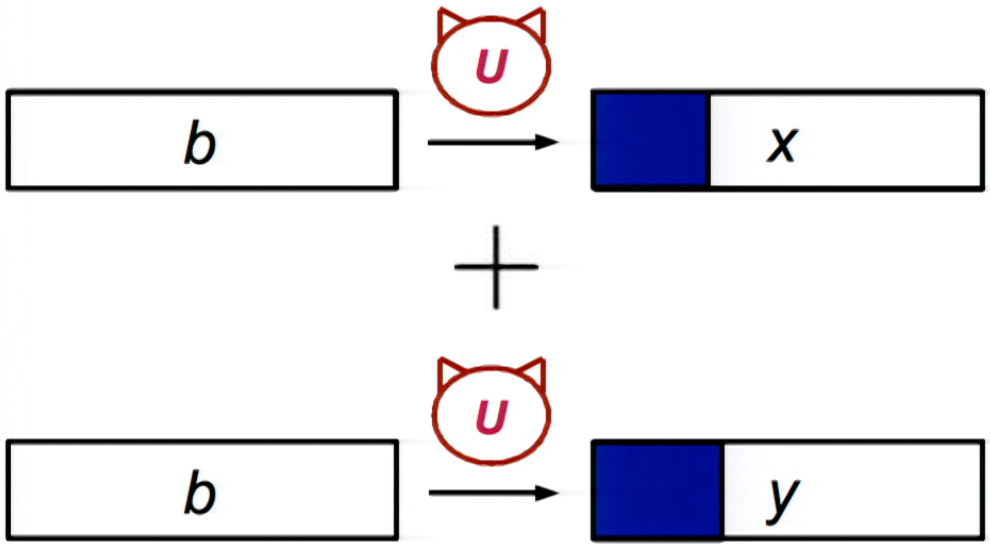


$$= \frac{n}{2}$$

$$x_i \oplus y_i = a_i \odot b_i$$

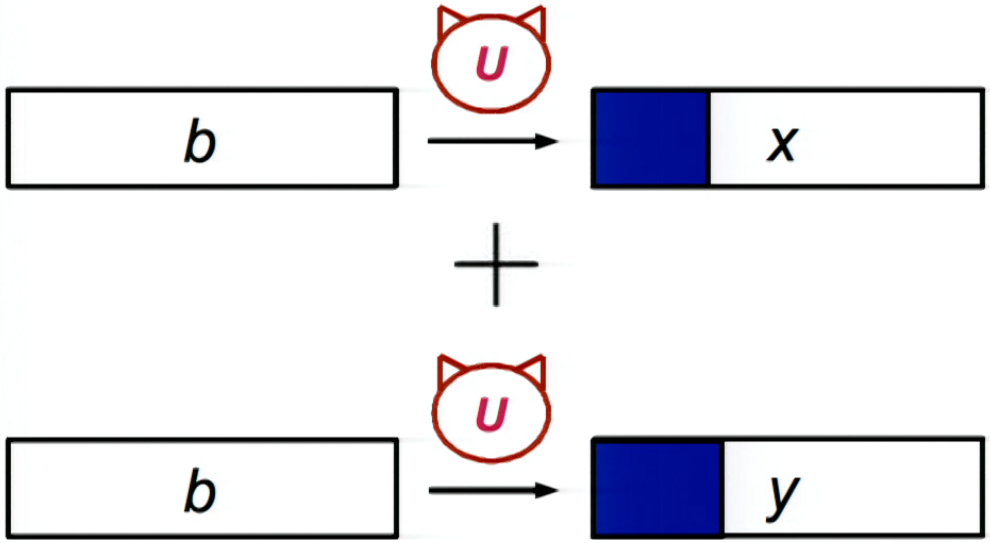


$$= \frac{n}{2}$$



$$\geq \frac{n}{2}$$

$$x_i \oplus y_i = a_i \odot b_i$$



$$\geq \frac{n}{2}$$

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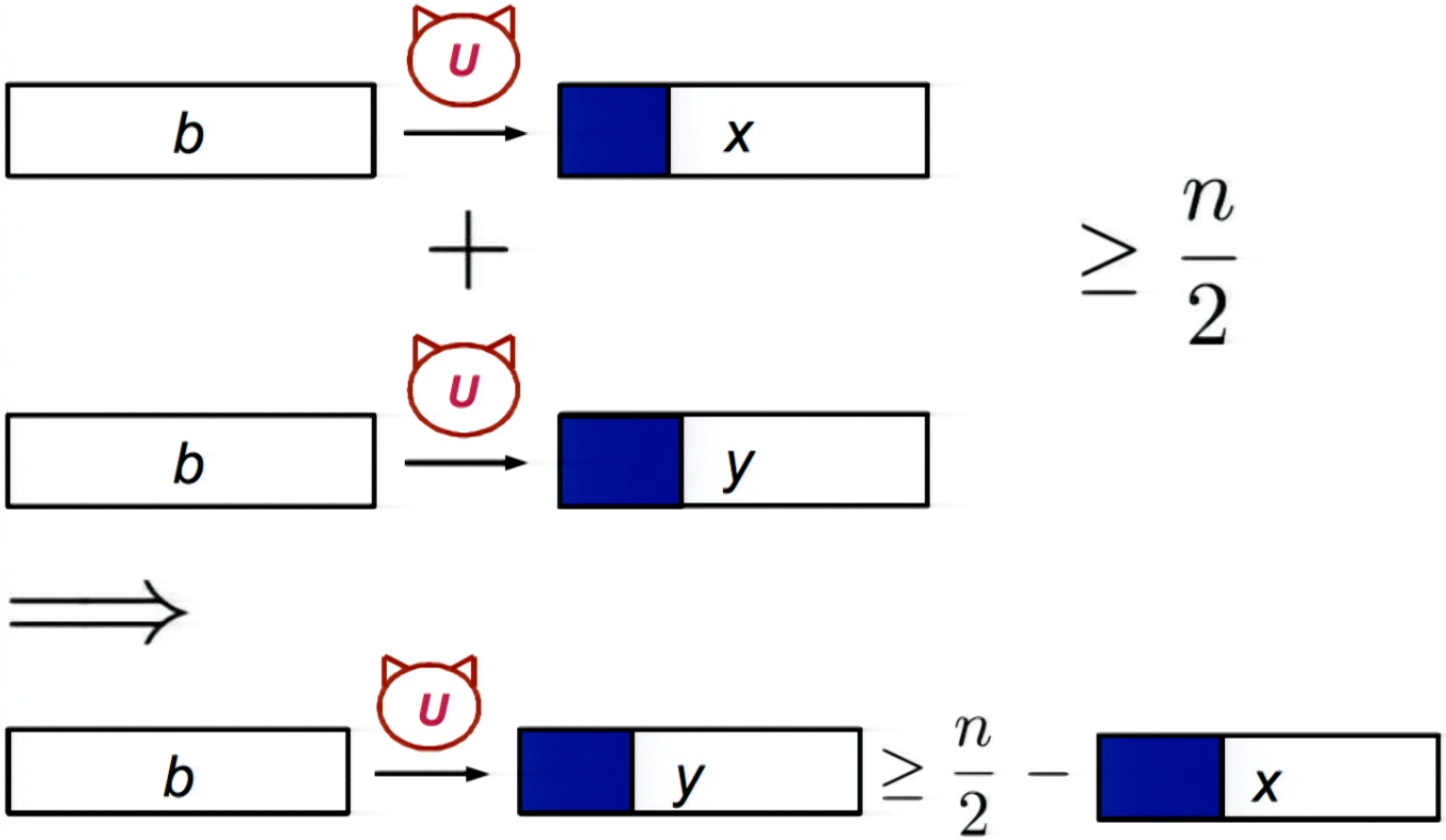
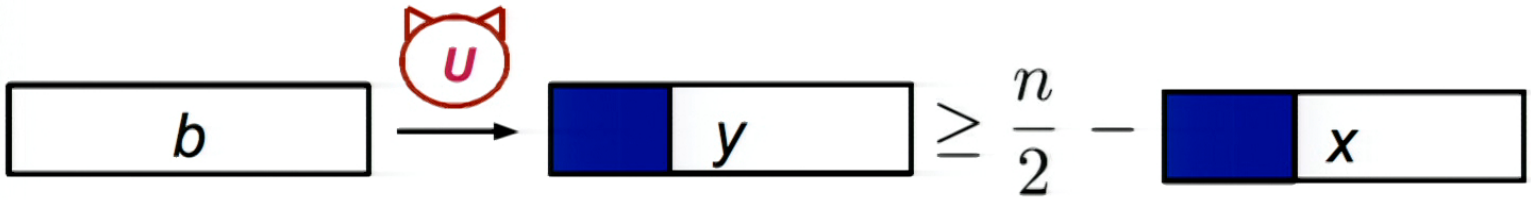
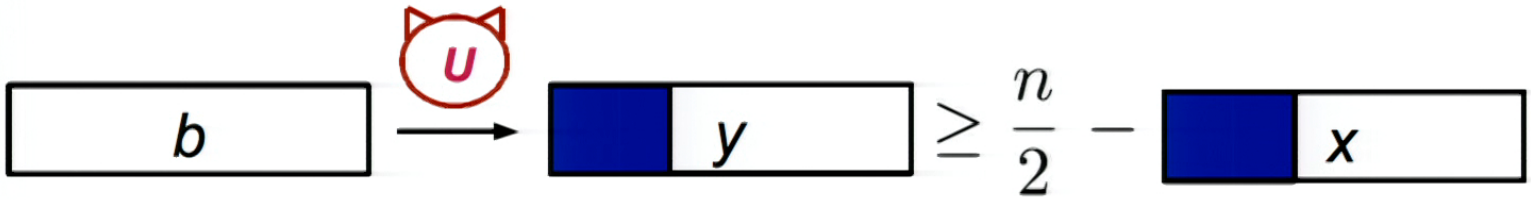
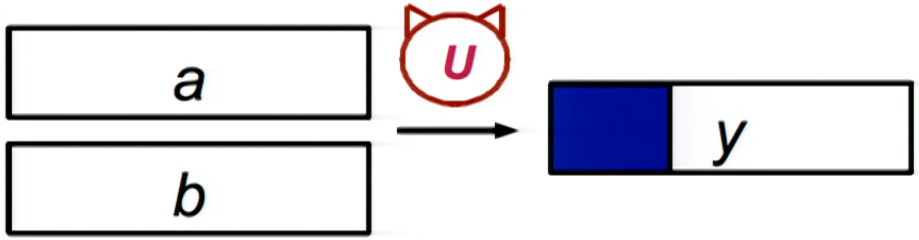
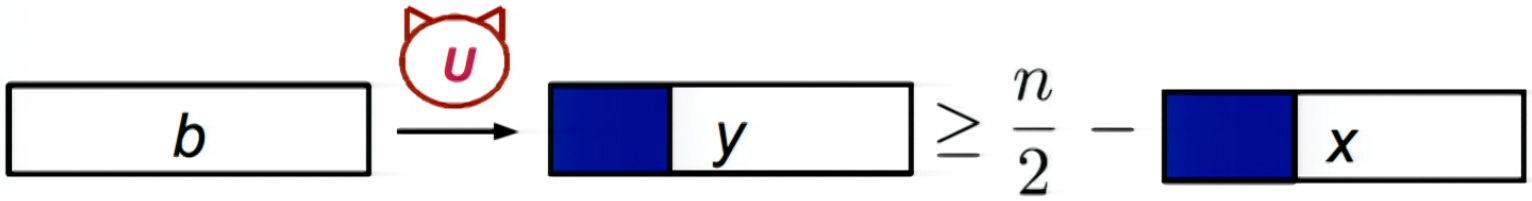



b
U
y
$\geq \frac{n}{2} -$
x

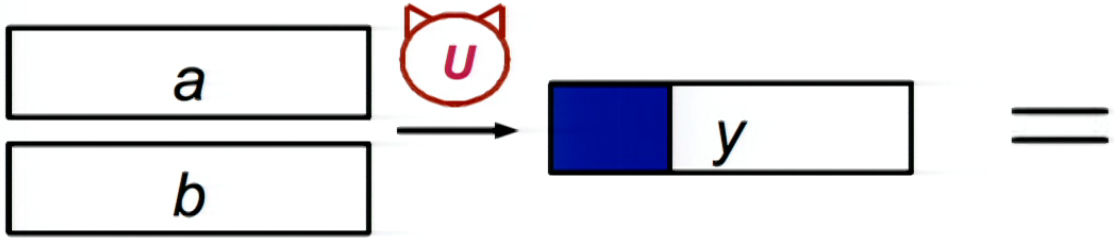


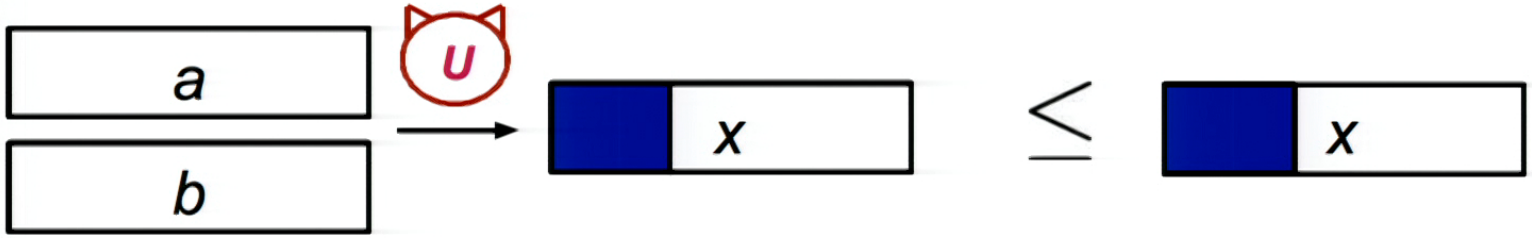
On the other hand:

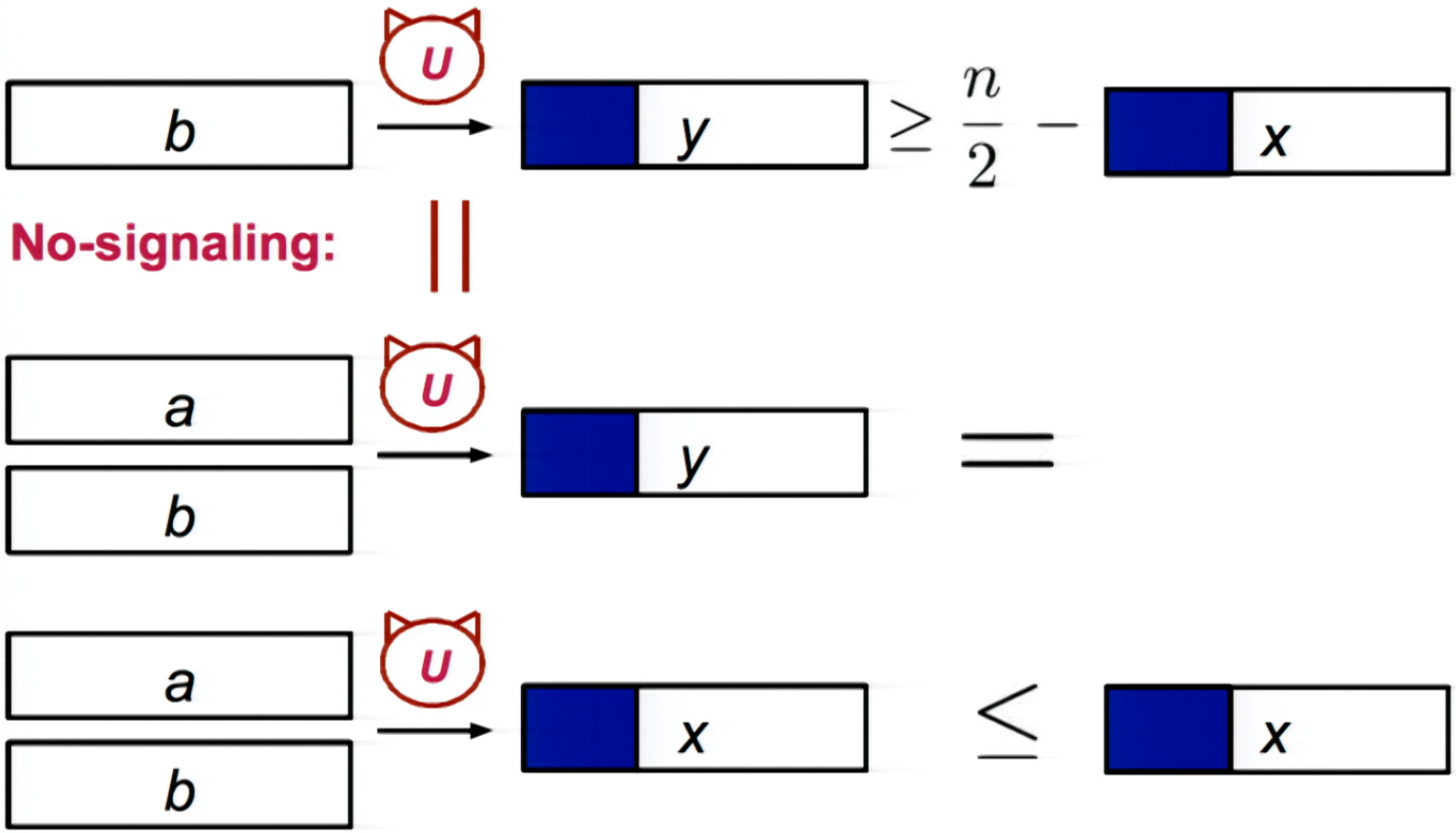


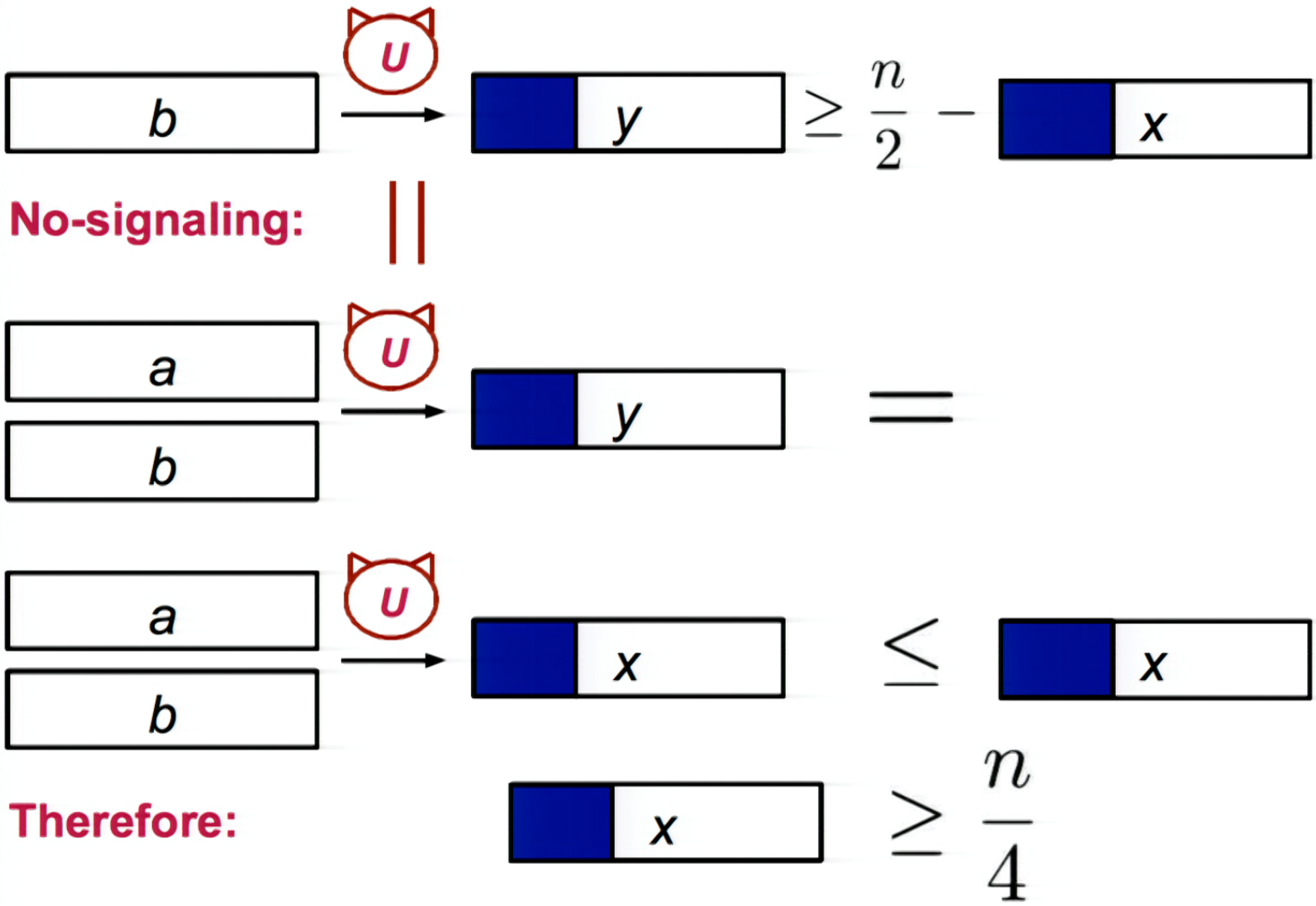


On the other hand:






b
U
y
$\geq \frac{n}{2} -$
x
No-signaling:
a
b
U
y
=
a
b
U
x
$\leq$
x


b
U
y
$\geq \frac{n}{2} -$
x
No-signaling:
a
b
U
y
=
a
b
U
x
$\leq$
x
Therefore:
x
$\geq \frac{n}{4}$

Non-Local Correlations

Factual-only reasoning

If (A,B) is incompressible...

Non-Local Correlations

Factual-only reasoning

If (A,B) is incompressible...

... then X and Y cannot be computable

Non-Local Correlations

Factual-only reasoning

If (A,B) is incompressible...

... then X and Y cannot be computable

Non-Local Correlations

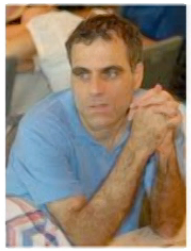
Factual-only reasoning

If (A,B) is incompressible...

... then X and Y cannot be computable
even given the respective inputs

from parallel-repetition theorem

Ran Raz 1998



Non-Local Correlations

All-or-Nothing *Feature of* ***Church-Turing*** *Thesis*

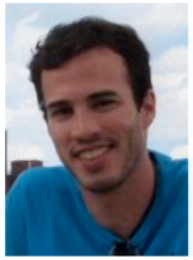
Non-Local Correlations

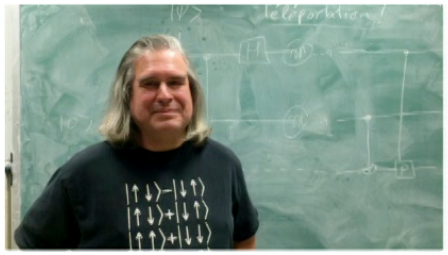
All-or-Nothing *Feature of* ***Church-Turing*** *Thesis*

Beyond TM **Beyond TM**

Quantum Violation of Chained Bell

Beyond TM **Beyond TM**





Baumeler
Bédard
Brassard
2017

Non-Local Correlations

All-or-Nothing *Feature of* ***Church-Turing*** *Thesis*

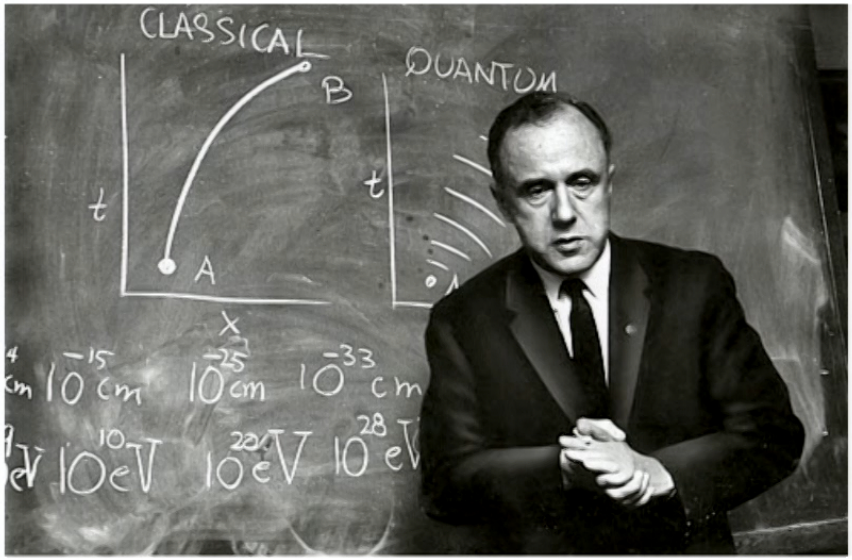
If ***some (complicated) device***
can generate uncomputable data,
then already ***single photons*** *can*

It from Bit

Epilogue

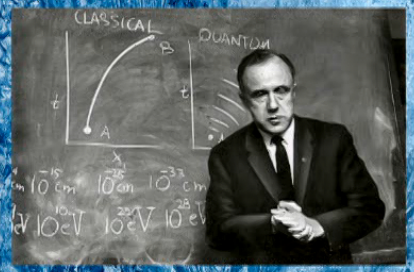
Information is Physical

Wheeler

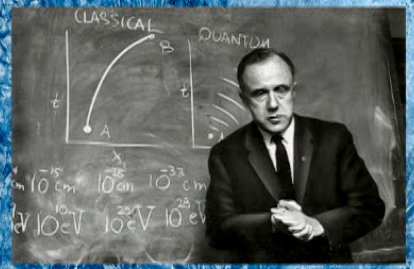


Landauer

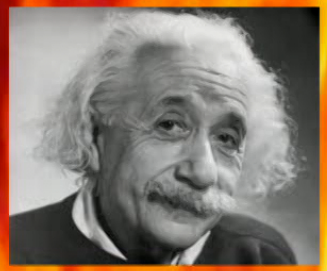
Epilogue

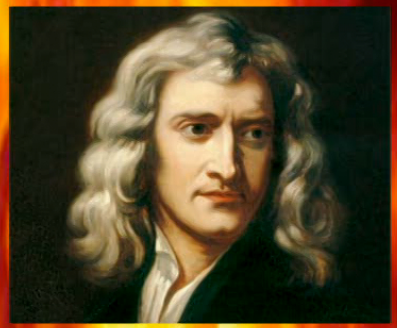


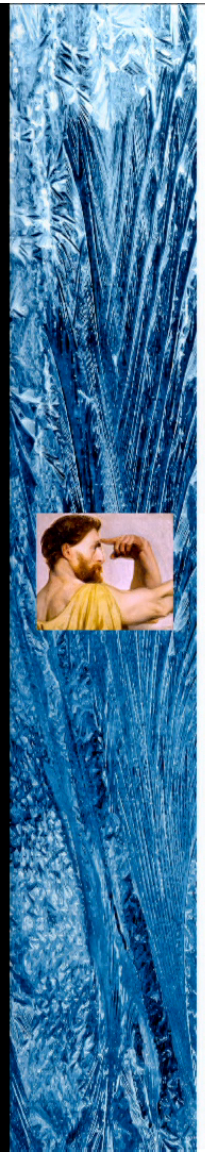
Epilogue


CLASSICAL
QUANTUM


REPUBLIQUE FRANCAISE
POSTES
18F
H.POINCARE
1854 - 1912
+5F



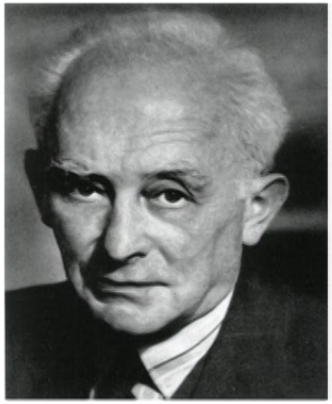




classical

Bohm

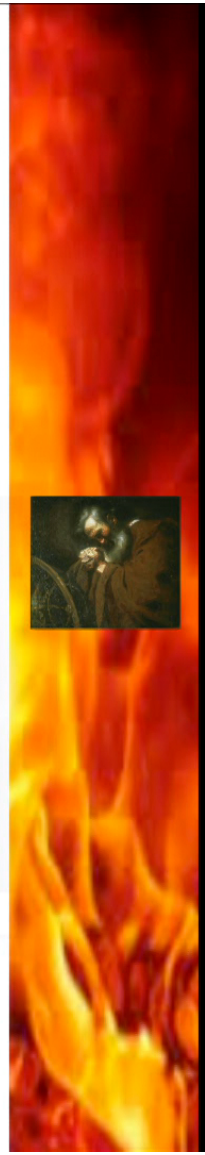
Fuchs



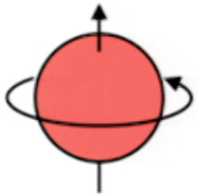
Born

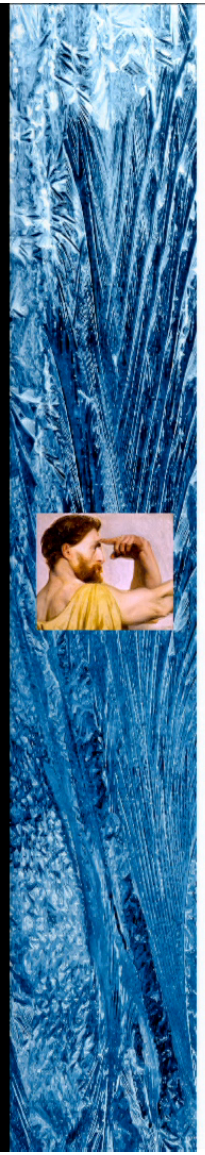
Everett

quantum



classicality

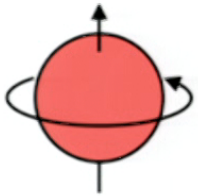


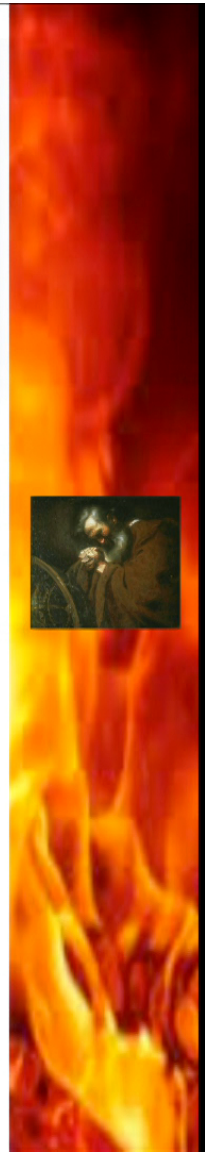


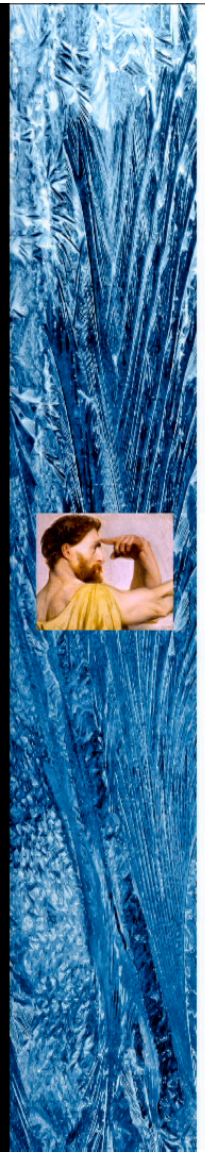
What does ***classicality*** (of information) mean physically?

"What I can publish!"

Gisin





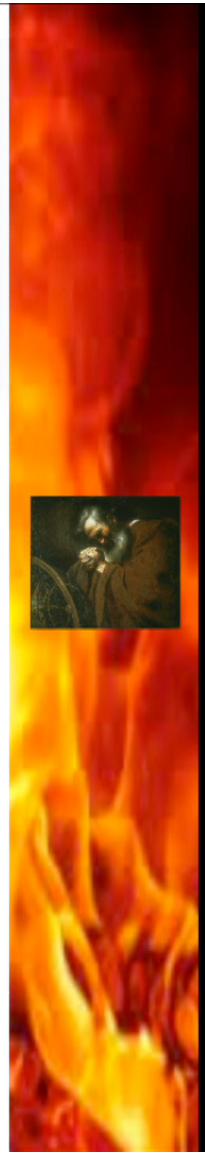


What does ***classicality*** (of information) mean physically?

"What I can publish!"

Gisin

copiable
observable
without
disturbing
objective

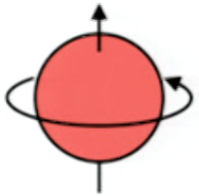


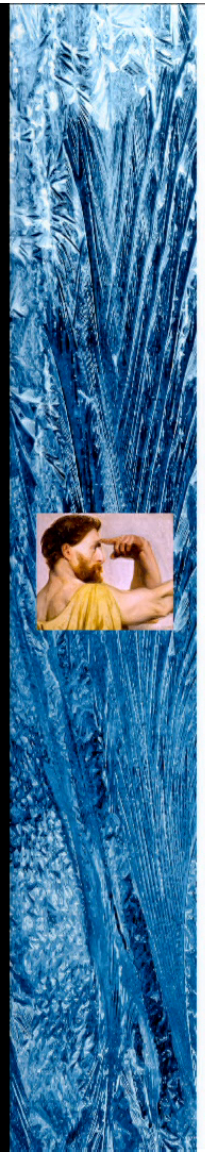
What does ***classicality*** (of information) mean physically?

"What I can publish!"

Gisin

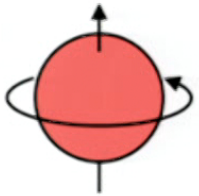
copiable
observable
without
disturbing
objective
recognizable
as such





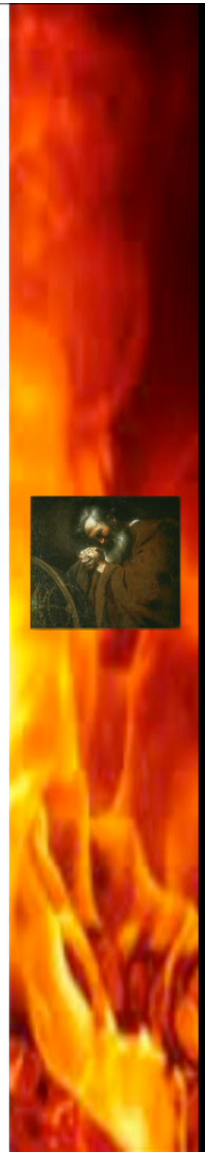
classicality
exists in the
thermodynamic
limit and is
related to:

macroscopicity
redundancy
- low erasure cost
- high work value

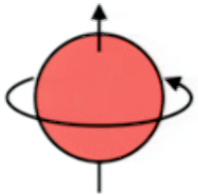


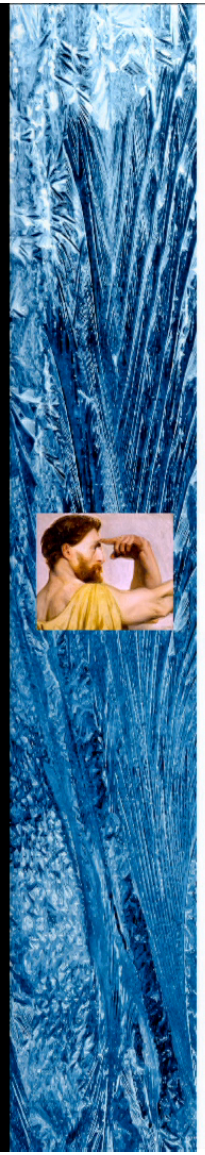
"What I can publish!"

copiable
observable
without
disturbing
objective
recognizable
as such



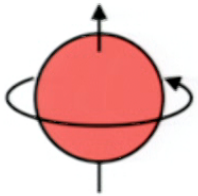
classicality
is a
thermodynamic
notion

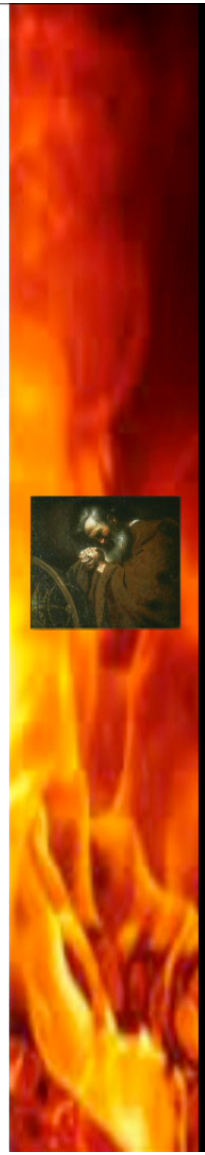




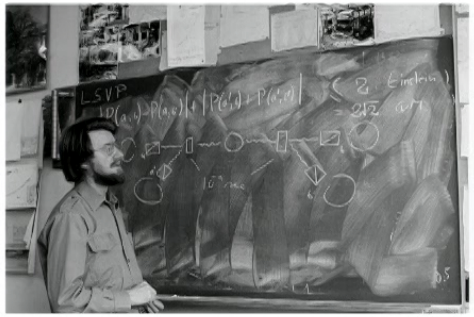
classicality
is a
thermodynamic
notion

"Bell, c'est difficile pour tout le monde"



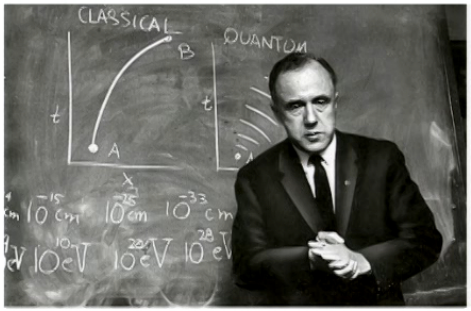


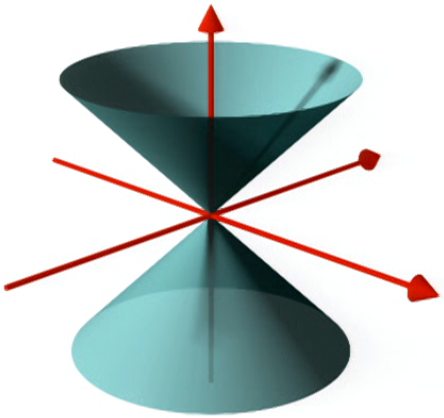
classicality
is a
thermodynamic
notion

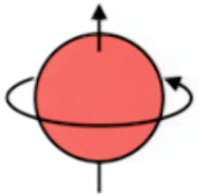


correlations
of
classical
information

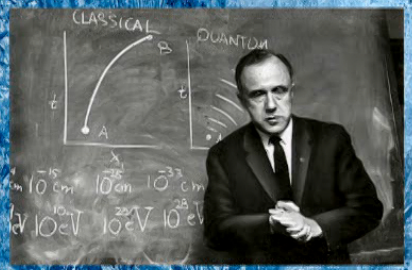
classicality
and
spacetime causality
arise in the
thermodynamic
limit ...

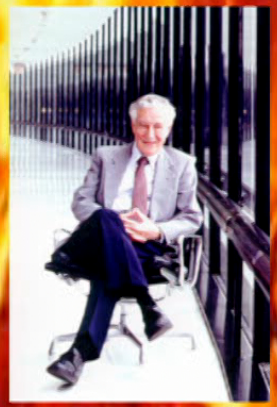


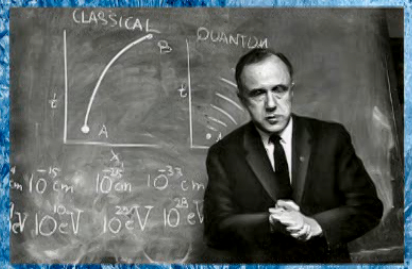




... ***together with***
– ***not prior to*** –
the ***correlated***
pieces of
information


CLASSICAL
QUANTUM
B
A

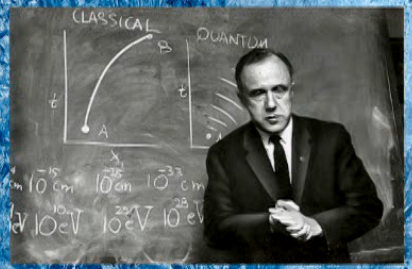




Feyerabend

1980

traditions of thought



Feyerabend 1980

Could the
richness of the discourse
and the
opposition between styles
in the end be what got closest
to "the nature of *reality*" ?

Based on collaboration and discussion with:

Mateus Araújo, Veronika Baumann, Charles Bédard, Claus Beisbart, Bänz Bessire, Gilles Brassard, Harvey Brown, Harry Buhrman, Caslav Brukner, Xavier Coiteux-Roy, Sandro Coretti, Claude Crépeau, Bora Dakic, Julien Degorre, Paul Erker, Adrien Feix, Jürg Fröhlich, Manuel Gil, Nicolas Gisin, Esther Hänggi, Arne Hansen, Marcus Huber, Alberto Montina, Samuel Ranellucci, Paul Raymond-Robichaud, Renato Renner, Louis Salvail, Benno Salwey, Martin Schüle, Sacha Schwarz, Ernst Specker, André Stefanov, Thomas Strahm, Alain Tapp, Andreas Winter, and Chris Wüthrich