

Title: Universality at large transverse spin in defect CFT

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Abstract:

We study the spectrum of local operators living on a defect in a generic conformal field theory, and their coupling to the local bulk operators. We establish the existence of universal accumulation points in the spectrum at large s , s being the charge of the operators under rotations in the space transverse to the defect. Our tools include a formula that inverts the bulk to defect OPE and is analytic in s , analogous to the Caron-Huot formula for the four-point function. Some important assumptions are made in deriving this result: we comment on them.

Universality at large transverse spin in defect

w. M. Lemos, P. Liend

Overview

1. Intro
2. Inversion of the defect OPE
3. The large s sector
4. An example
5. Conclusions

$$\phi(x_1)\phi(x_2)$$

$$\phi(x^2, x) =$$

inverse spin in defect CFT

v. M. Lemos, P. Liendo, S. Sarkar

$$\phi(x_1) \phi(x_2) = c_{\phi\phi\theta} |x_{12}|^{-2\Delta_\phi + \Delta - \bar{J}} x_{12}^{m_1} \dots x_{12}^{m_s} \Theta_{m_1 \dots m_s}(x_2) + \dots$$

$$\phi(x^a, x) = b_{\phi\hat{\theta}} |x^a|^{-\Delta_\phi + \hat{\Delta} - s} x^{i_1} \dots x^{i_s} \hat{\Theta}_{i_1 \dots i_s}(x^a)$$

Universality of large transverse spin in defect CFT

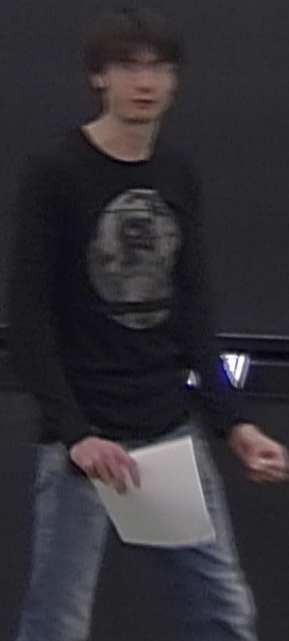
w. M. Lemos, P. Liendo, S. Sarker

Overview

1. Intro
2. Inversion of the defect OPE
3. The large $\Rightarrow$ sector
4. An example
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$$\phi(x_1)\phi(x_2) = c_{\phi\phi G} |x_{12}|^{-\Delta_{\phi\phi} + \Delta - J} x_{1,0}^{n_1} \dots x_{1,2}^{n_2} \mathcal{O}_{n_1 \dots n_2}(x_2) + \dots$$

$$\phi(x^2, x) = b_{\phi\hat{G}} |x|^{\Delta_{\phi\hat{G}} - \Delta - S} x^{i_1} \dots x^{i_S} \hat{\mathcal{O}}_{i_1 \dots i_S}(x^2)$$



Universality at large transverse spin in defect CFT

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Overview

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$$\phi(x_1)\phi(x_2) = c_{\phi\phi G} |x_{12}|^{-\Delta_\phi + \hat{\Delta} - J} x_{12}^{i_1} \dots x_{12}^{i_5} \mathcal{O}_{\mathcal{H}_1, \mathcal{H}_2}(x_2) + \dots$$

$$\phi(x^2, x) = b_{\phi\hat{G}} |x|^{\Delta_\phi + \hat{\Delta} - S} x^{i_1} \dots x^{i_5} \hat{\mathcal{O}}_{i_1 \dots i_5}(x^2)$$

defect CFT

, P. Liendo, S. Sarkar

$$\phi(x_1) \phi(x_2) = c_{\phi\phi\theta} |x_{12}|^{-2\Delta_\phi + \Delta - J} x_{12}^{m_1} \dots x_{12}^{m_e} \Theta_{m_1 \dots m_e}(x_2) + \dots$$

$$\phi(x^2, x) = b_{\phi\hat{\theta}} |x^2|^{-\Delta_\phi + \hat{\Delta} - S} x^{i_1} \dots x^{i_s} \hat{\Theta}_{i_1 \dots i_s}(x^2)$$

$$\tau = \Delta - J = \begin{cases} \frac{d-2}{2} & J=0 \\ d-2 & J>0 \end{cases}$$

4. An example
5. Conclusions



$$p + q = d$$

4. An example
5. Conclusions



$$p + q = d$$

$$\underset{\substack{\uparrow \\ \text{cons trans}}}{\text{SO}(p+1, 1)} \times \underset{\substack{\uparrow \\ \text{rotations}}}{\text{SO}(q)} \text{ transverse}$$

4. An example
5. Conclusions



$$p + q = d$$

$$SO(p+1, 1) \times SO(q)$$

↗ transverse
 ↘ rotations

$$(\hat{\Delta}, \hat{\Gamma}, \hat{\Sigma})$$

4. An example
5. Conclusions



$$p + q = d$$

$$SO(p+1, 1) \times SO(q)$$

↗ transverse
 ↘ rotations

~~$(\hat{\Delta}, \hat{\Delta}, \hat{\Delta})$~~

Universality at large transverse spin in defect

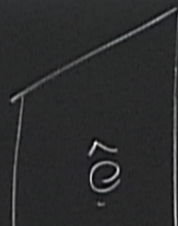
w. M. Lemos, P. Liend

Overview

1. Intro
2. Inversion of the defect OPE
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$$\phi(x_1)\phi(x_2)$$

$$\text{defect OPE: } \phi(x^2, x) =$$



$$p + q = d$$

$$SO(p+1, 1) \times \underbrace{SO(q)}_{\text{transverse}}$$

defect CFT

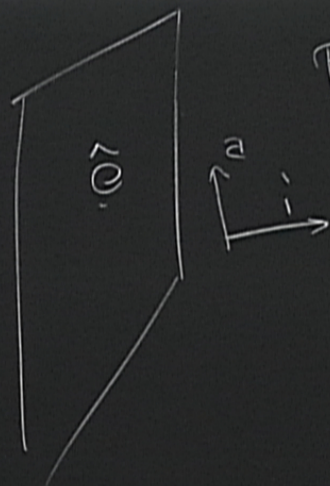
, P. Liendo, S. Sarkar

$$\phi(x_1) \phi(x_2) = c_{\phi\phi\phi} |x_{12}|^{-2\Delta_\phi + \Delta - J} x_{12}^{m_1} \dots x_{12}^{m_\ell} \phi_{m_1 \dots m_\ell}(x_2) + \dots$$

$$\phi(x^2, x) = b_{\phi\hat{\phi}} |x|^{-\Delta_\phi + \hat{\Delta} - S} x^{i_1} \dots x^{i_s} \hat{\phi}_{i_1 \dots i_s}(x^2)$$

$$\tau = \Delta - J = \begin{cases} \frac{d-2}{2} & J=0 \\ d-2 & J>0 \end{cases}$$

4. An example
5. Conclusions



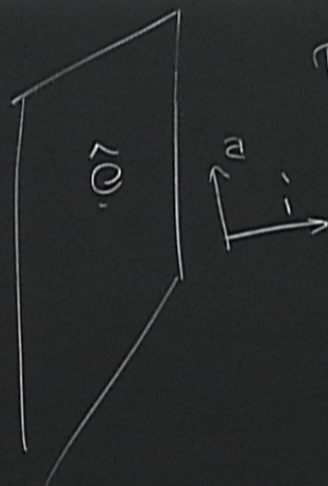
$$p + q = d$$

$$SO(p+1, 1) \times SO(q)$$

transverse
rotations

~~$(\hat{a}, \hat{i}, \hat{j})$~~

4. An example
5. Conclusions



$$p + q = d$$

$$SO(p+1, 1) \times SO(q)$$

(The first factor $SO(p+1, 1)$ is associated with "transverse" transformations, indicated by a curved arrow pointing to it.)
 (The second factor $SO(q)$ is associated with "rotations", indicated by a curved arrow pointing to it.)
 Below the product, the expression $(\hat{\Delta}, \hat{J}, \hat{S})$ is written and crossed out with a large 'X'. A curved arrow points from this crossed-out expression to the text "transverse spin".)

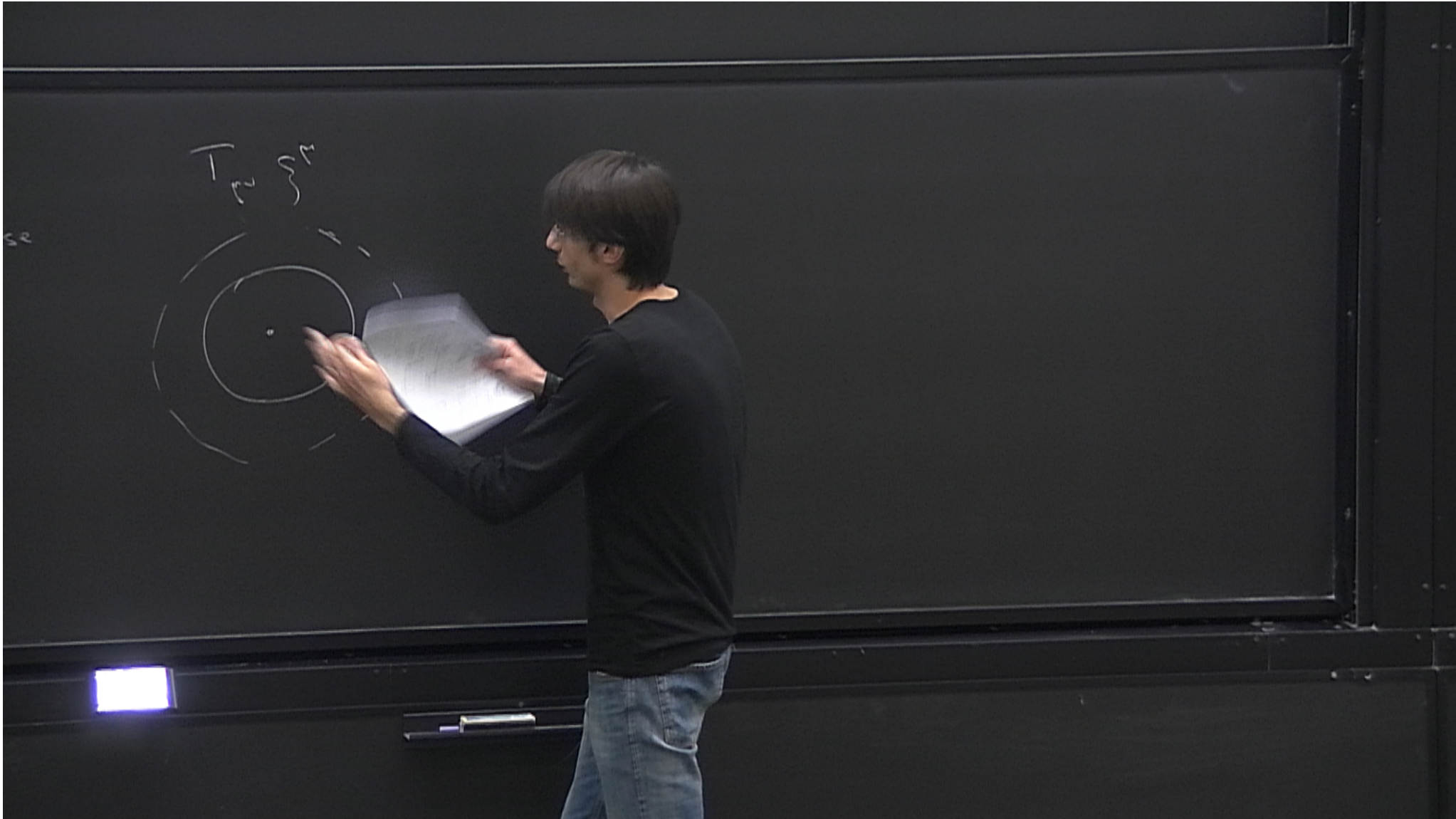
T_{μ}^{μ}

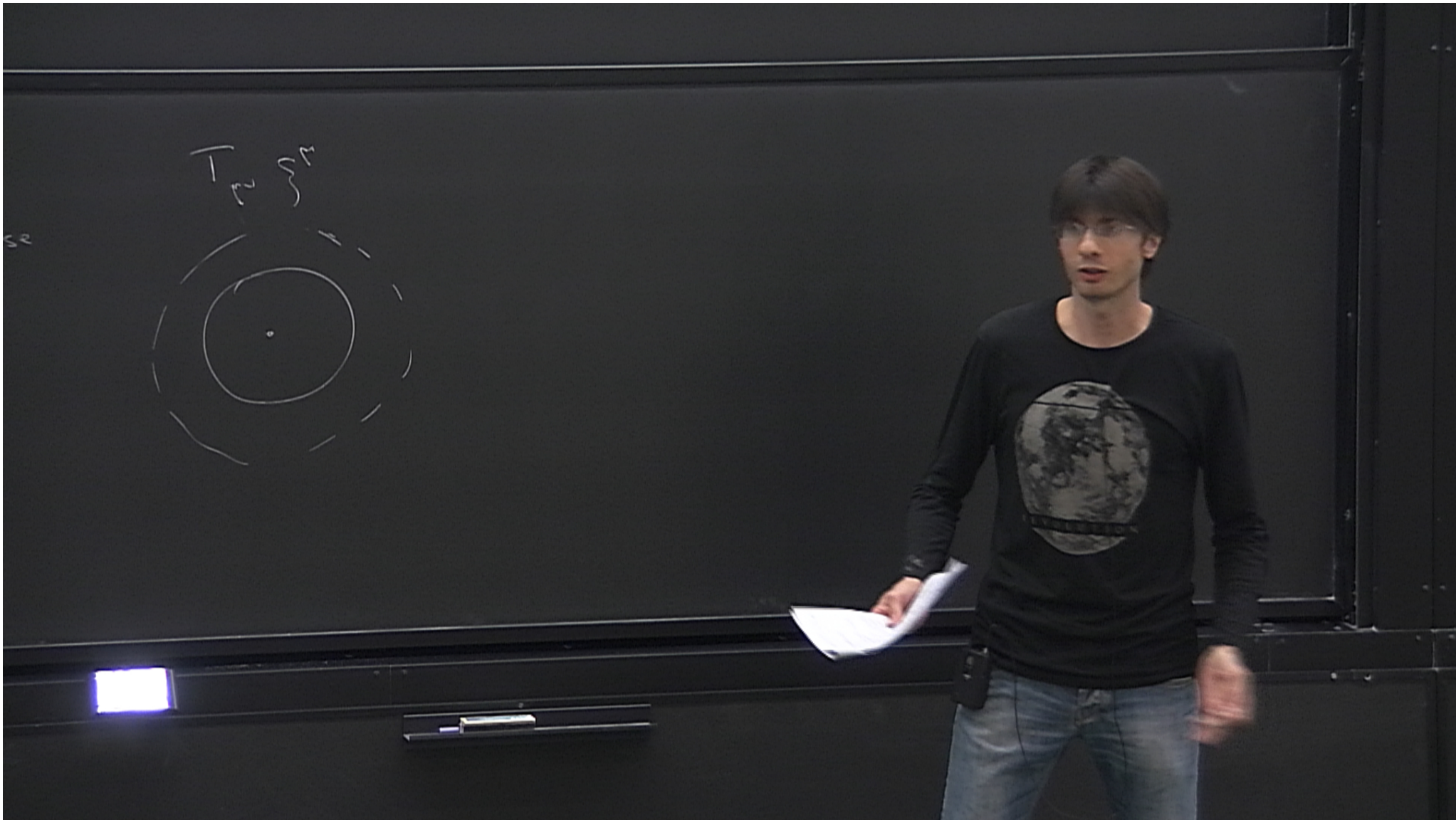
52

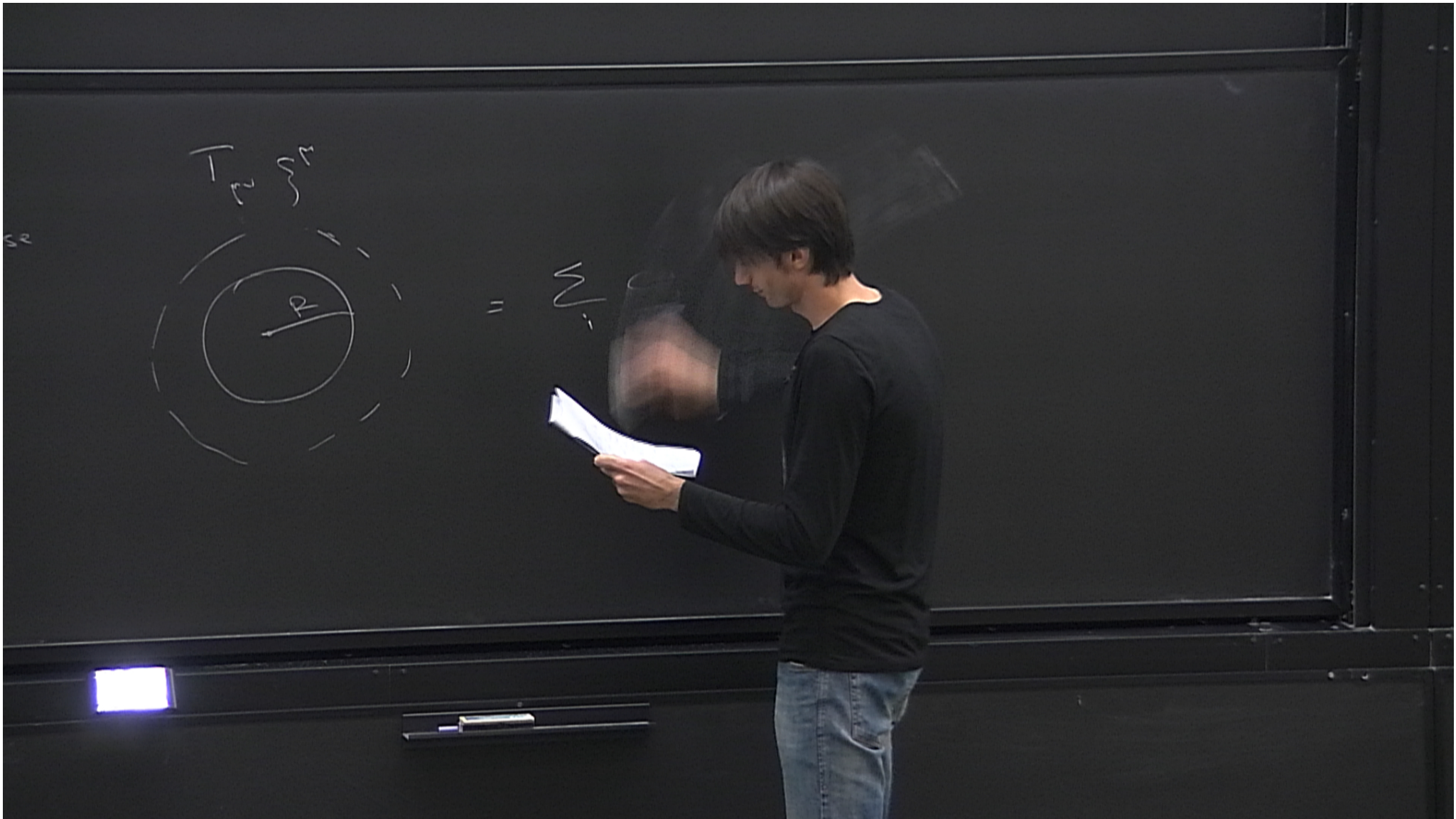
$T_m \{^m$

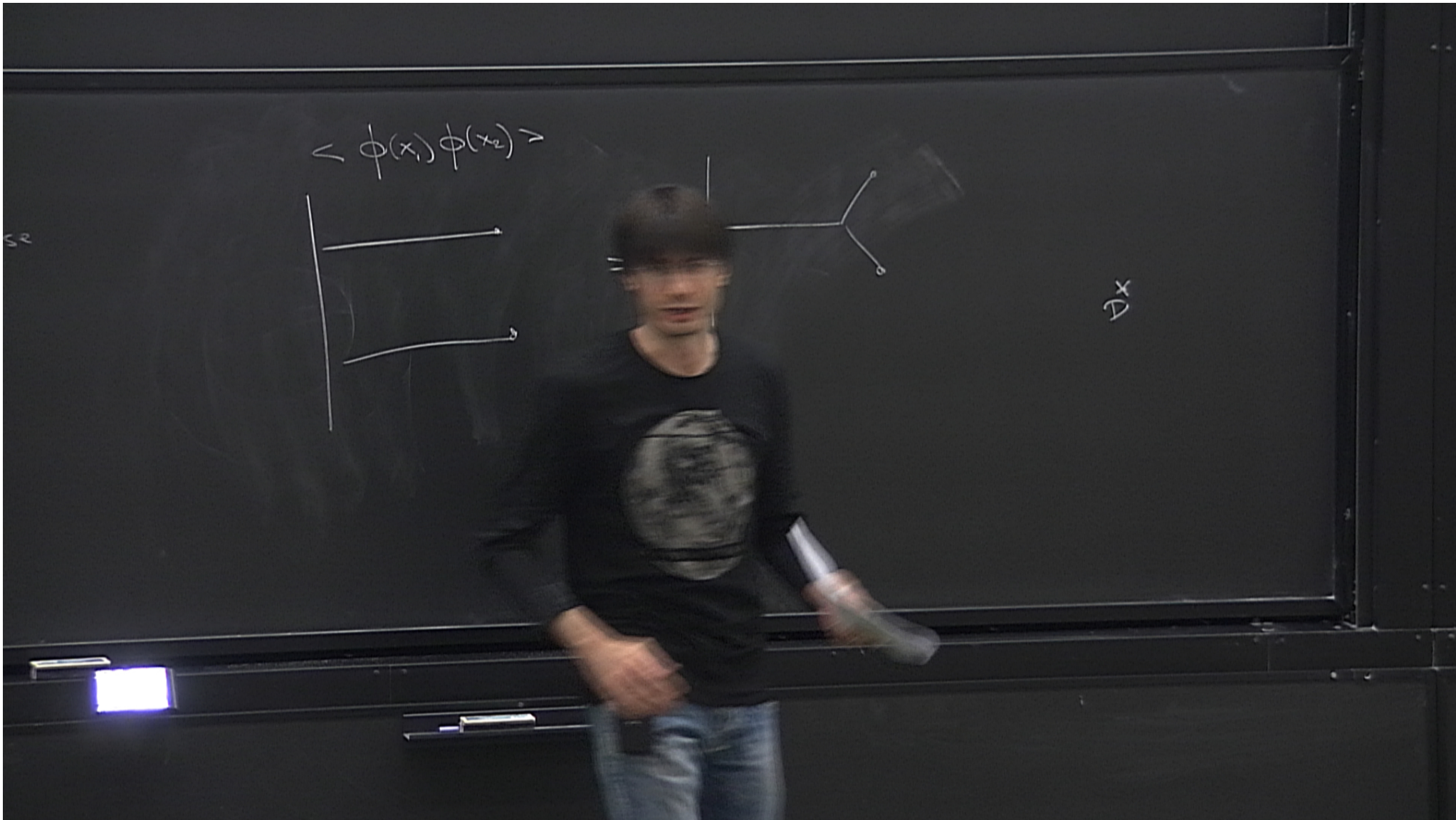
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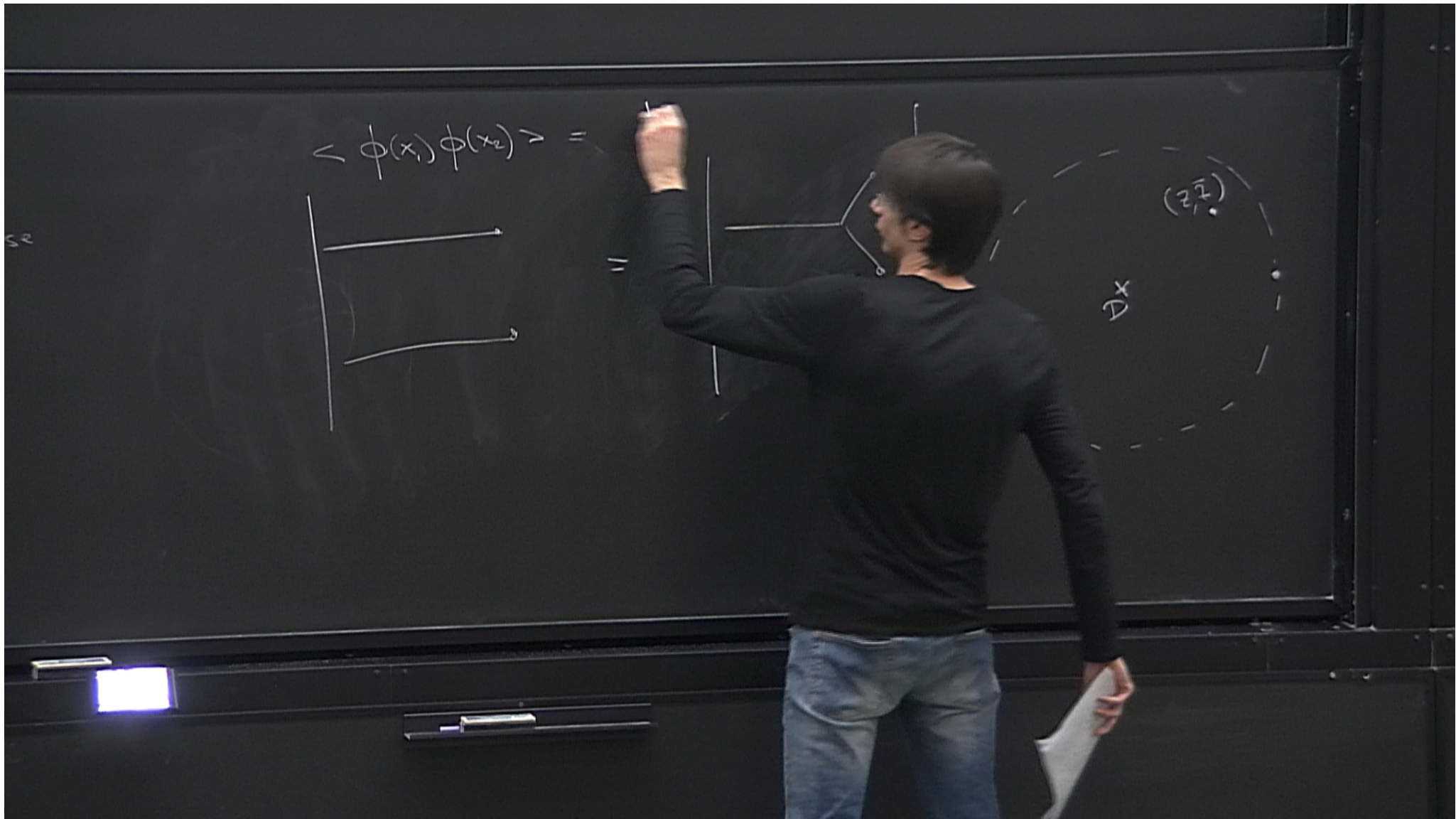




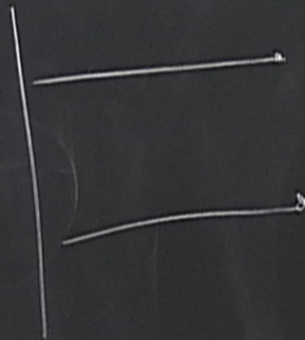




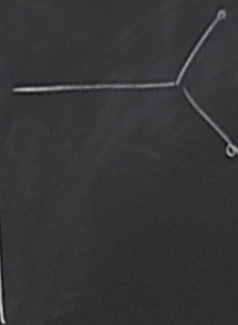




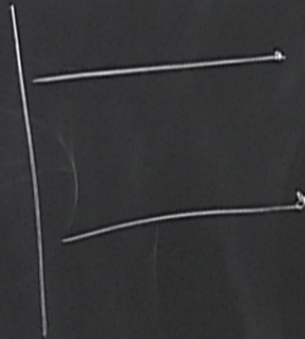
$$\langle \phi(x_1) \phi(x_2) \rangle = \frac{1}{(z\bar{z})^{\Delta\phi/2}} g(z, \bar{z})$$



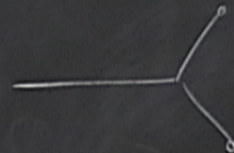
=



$$\langle \phi(x_1) \phi(x_2) \rangle = \frac{1}{(z\bar{z})^{\Delta\phi/2}} g(z, \bar{z})$$



=



$$g(z, \bar{z}) = \sum_{\Delta, s} b_{\phi}^2 \hat{f}_{\Delta, s}(z, \bar{z}) = \left[\frac{\sqrt{z\bar{z}}}{(1-z)(1-\bar{z})} \right]^{a_{\phi}} \sum_{\Delta, s} c_{\phi} \phi$$

$$g(z, \bar{z}) = \sum_{\hat{\Delta}, s} b_{\phi\theta}^2 \hat{f}_{\hat{\Delta}, s}(z, \bar{z}) = \left[\frac{\sqrt{z\bar{z}}}{(1-z)(1-\bar{z})} \right]^{d_\phi} \sum_{\Delta, J} c_{\phi\theta} \partial_\theta f_{\Delta, J}(z, \bar{z})$$

$$\hat{f}_{\hat{\Delta}, s}(z, \bar{z}) = \hat{f}_{\hat{\Delta}}(r) \hat{g}_s(\cos\theta)$$

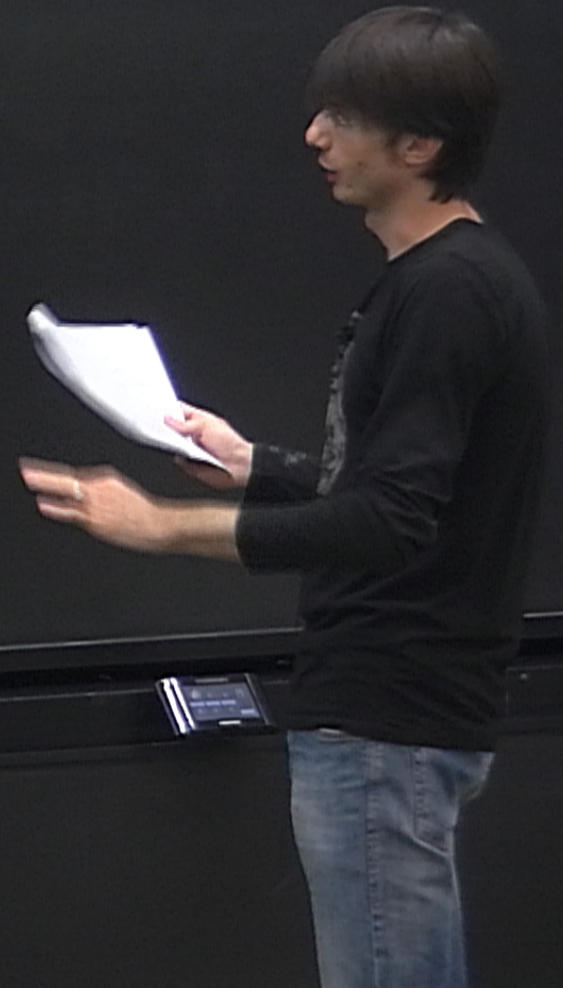
2. Inversion of the defect OPE

2. Inversion of the defect OPE

$$(\hat{\Delta}, s) \rightarrow \hat{\Delta}(s)$$

2. Inversion of the defect OPE

- $(\hat{\Delta}, s) \rightarrow \hat{\Delta}(s), b_{\phi} \hat{c}(s)$
- $\frac{1}{s}$ expansion (computable)



$$g(z, \bar{z}) = \sum_{\Delta, s} b_{\phi\theta}^2 \hat{f}_{\Delta, s}(z, \bar{z}) = \left[\frac{\sqrt{z\bar{z}}}{(1-\theta)(1-\bar{\theta})} \right]^{4\phi} \sum_{\Delta, s} c_{\phi\theta} a_{\theta} f_{\Delta, s}(z, \bar{z})$$

$$\hat{f}_{\Delta, s}(z, \bar{z}) = \hat{f}_{\Delta}(r) \hat{g}_s(\cos\theta)$$

$$\hat{g}_s(\cos\theta) = C_s^{(\frac{q}{2}-1)}(\cos\theta) \quad \checkmark$$

$$\hat{f}_{\hat{A}}(r) \quad \rightsquigarrow \quad \psi_{\hat{A}}(r) = \frac{1}{2} \left(f_{\hat{A}}(r) + \# f_{p-\hat{A}}(r) \right)$$

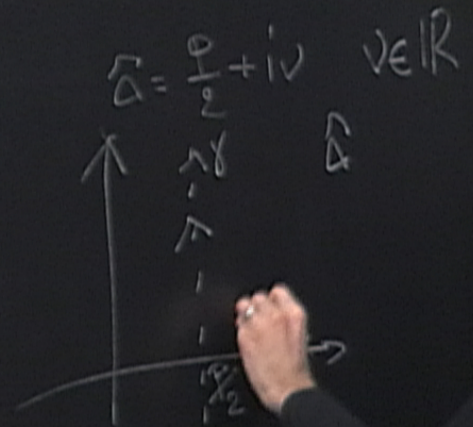
$$\hat{g}_s(\cos\theta) = C_s^{(\frac{q}{2}-1)}(\cos\theta) \quad \checkmark$$

$$\hat{f}_{\hat{a}}(r)$$

$\rightsquigarrow$

$$\psi_{\hat{a}}(r) = \frac{1}{2} (f_{\hat{a}}(r) + \# f_{p-\hat{a}}(r))$$

$$g(z, \bar{z}) = \sum_{s=0}^{\infty} \int_{\gamma} \frac{d\hat{a}}{2\pi i} b(\hat{a}, s) \hat{g}_s(\cos\theta) \psi_{\hat{a}}(r)$$

$$\hat{a} = \frac{p}{2} + iv \quad v \in \mathbb{R}$$


$$\hat{f}_A(r)$$

$$\hat{f}_A(r)$$

$$b(\hat{\alpha}) \sim$$

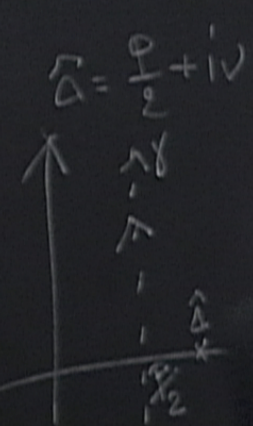
$$- \frac{b^2_{\Delta_1}}{\hat{\Delta} - \hat{b}_{\Delta_1}} \quad \hat{\Delta}_{\Delta_1} \in \mathbb{Z}$$

$$\hat{g}_s(\cos\theta) = C_s^{\left(\frac{q}{2}-1\right)}(\cos\theta) \quad \checkmark$$

$$\hat{f}_{\hat{a}}(r) \rightsquigarrow \psi_{\hat{a}}(r) = \frac{1}{2} \left(f_{\hat{a}}(r) + f_{p-\hat{a}}(r) \right)$$

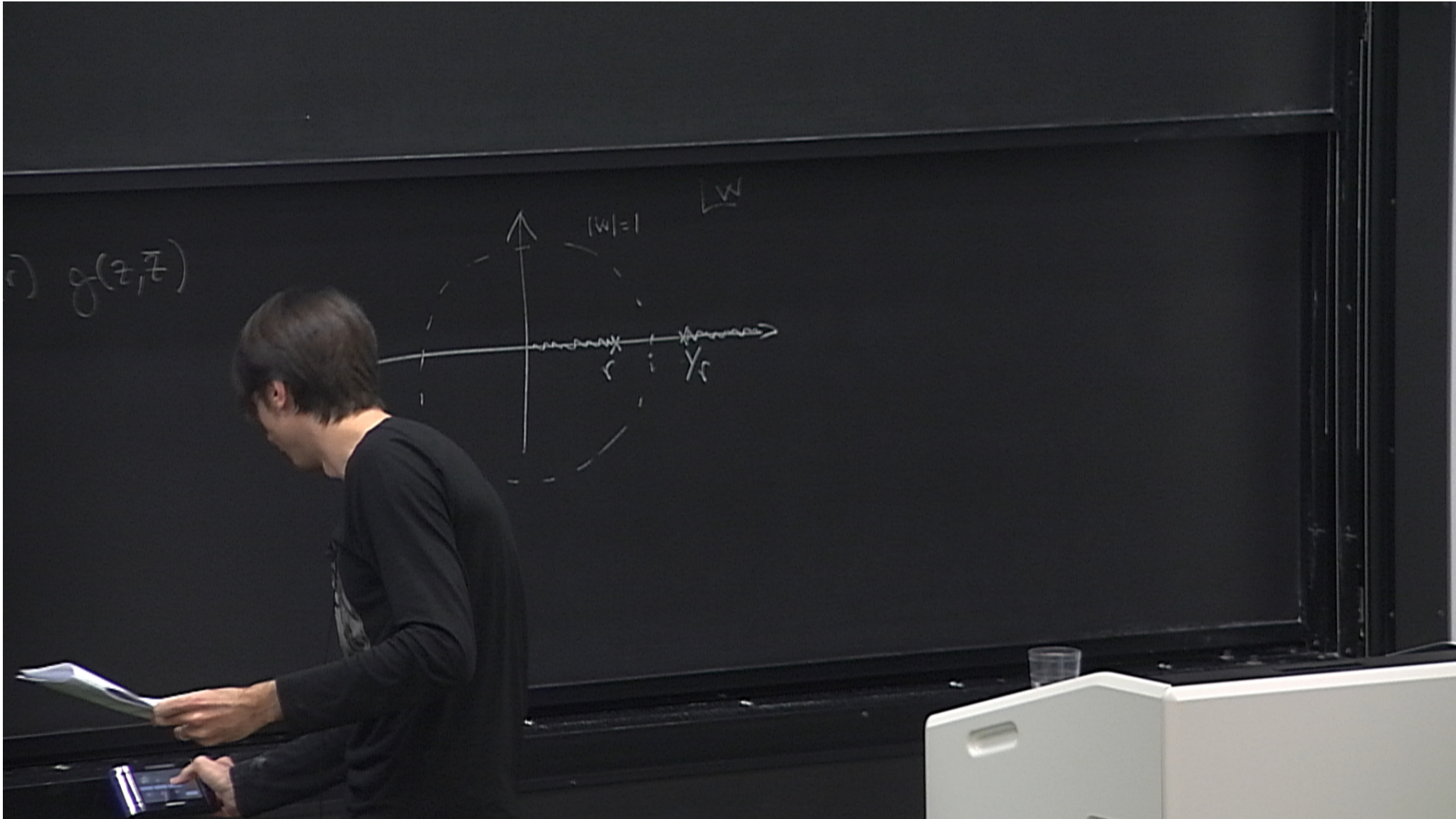
$$g(z, \bar{z}) = \sum_{s=0}^{\infty} \int_{\gamma} \frac{d\hat{a}}{2\pi i} b(\hat{a}, s) \hat{g}_s(\cos\theta) \psi_{\hat{a}}(r)$$

$$b(\hat{a}, s) \sim - \frac{b_{\hat{a}, s}^2}{\hat{a} - \hat{a}_s} \quad \hat{a}_s \in \mathbb{Z}$$

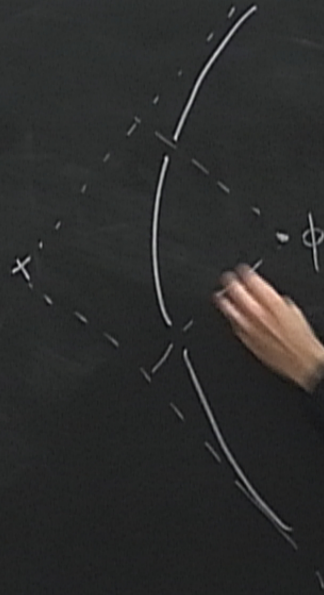


$$b(\hat{\Delta}, s) = \oint_{|w|=1} \frac{dw}{2\pi i w} \int_a^1 dr \, \mu(r, w) \hat{g}_s(w) \psi_{\hat{\Delta}}(r) g(z, \bar{z})$$

$$b(\hat{\Delta}, s) = \oint_{|w|=1} \frac{dw}{2\pi i w} \int_a^1 dr \, \mu(r, w) \underbrace{\hat{g}_s(w)}_{w^s + w^{-s}} \psi_{\hat{\Delta}}(r) g(z, \bar{z})$$



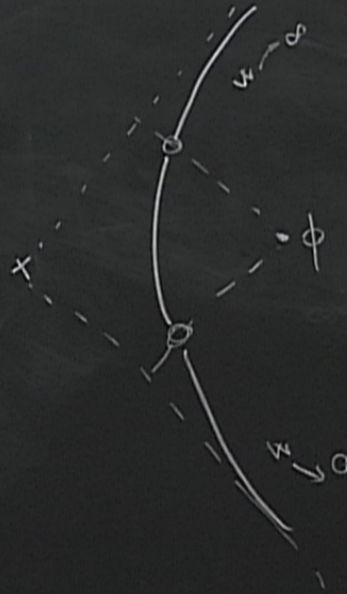
4. An example
5. Conclusions



4. An example
5. Conclusions

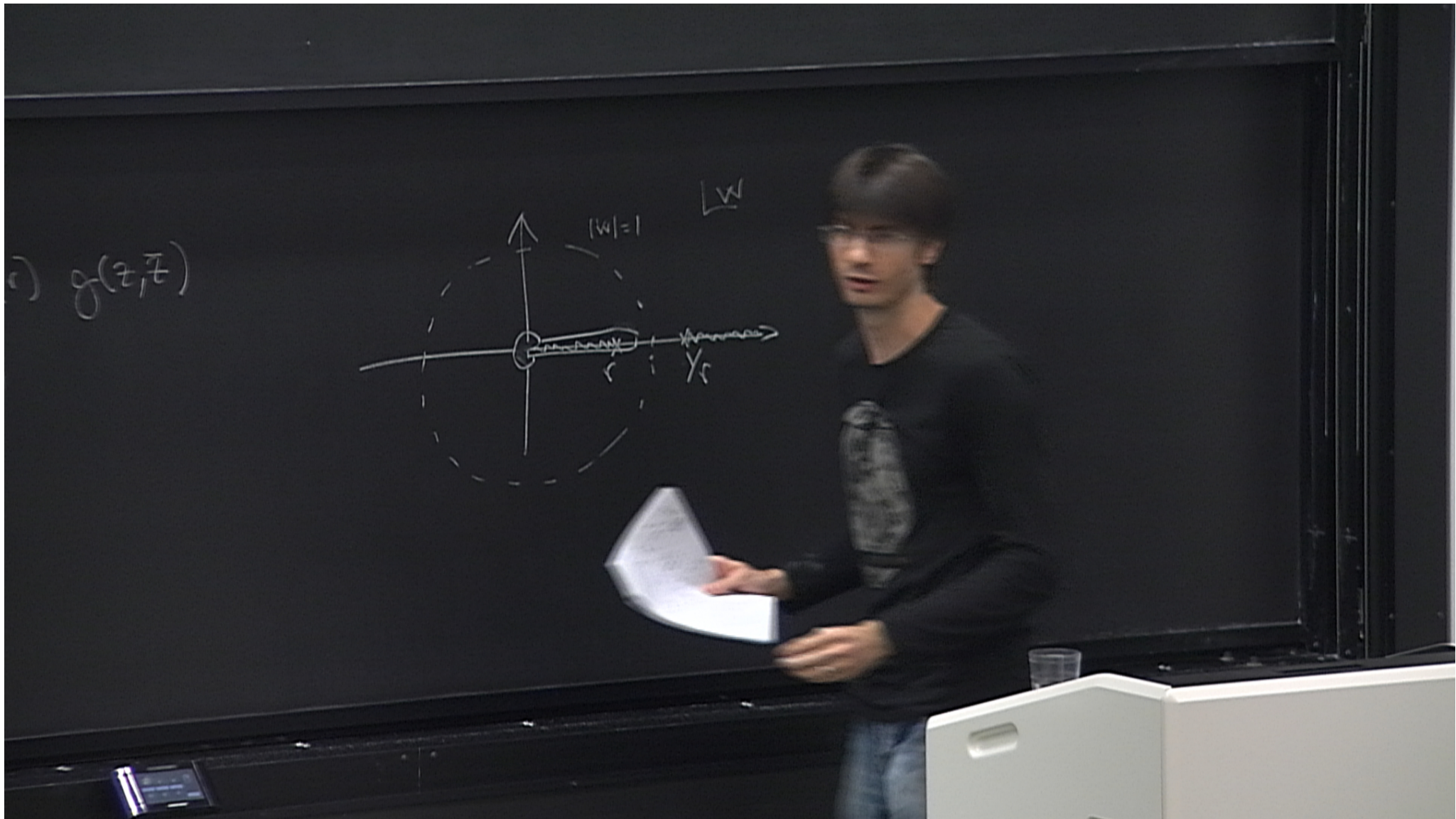


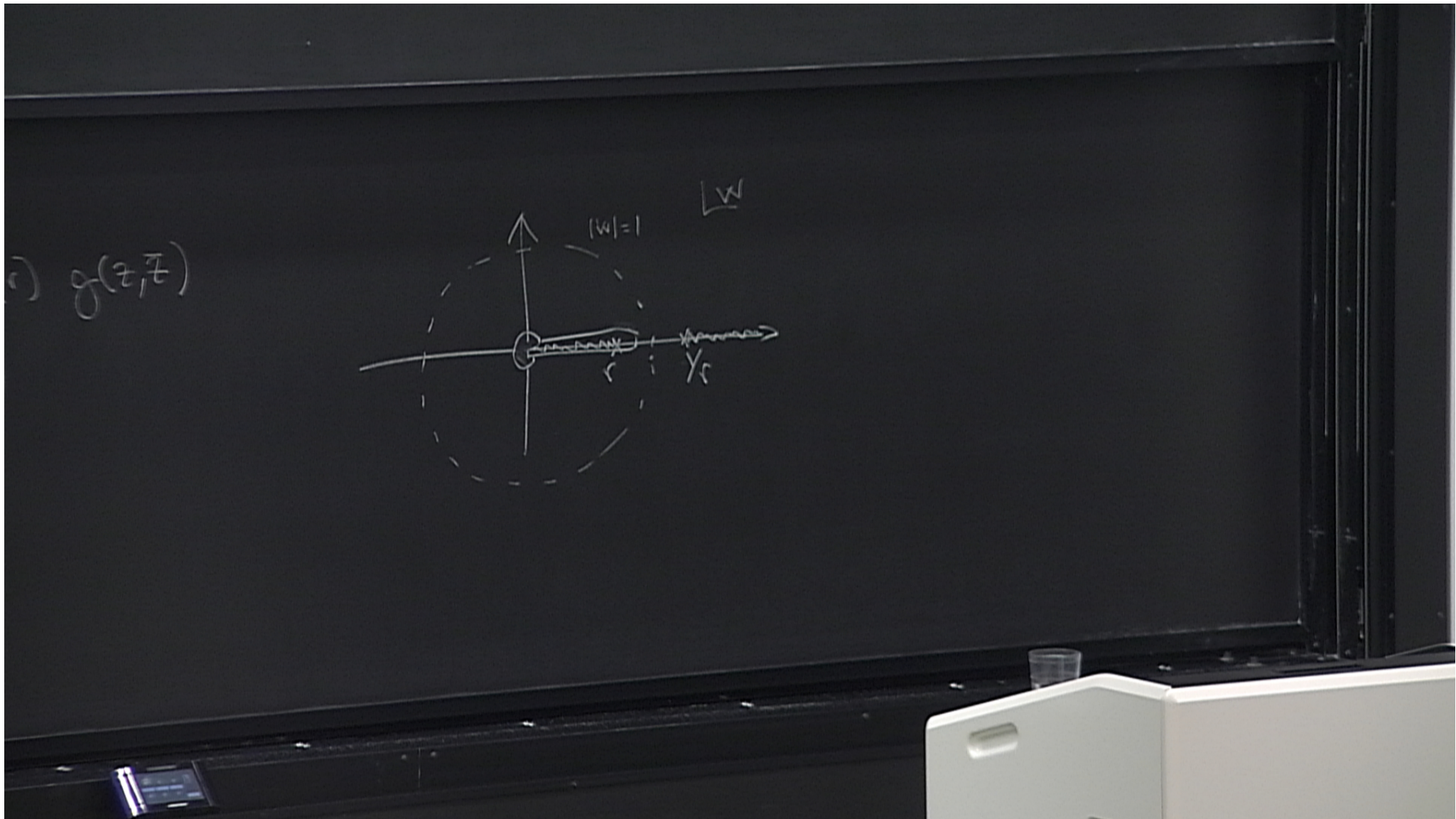
4. An example
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$$b(\hat{\Delta}, s) = \oint_{|w|=1} \frac{dw}{2\pi i w} \int_0^1 dr \, p(r, w) \hat{g}_s(w) \psi_{\hat{\Delta}}(r) g(z, \bar{z})$$

$\underbrace{\hat{g}_s(w)}_{w^s + w^{-s}}$





$$b(\hat{\Delta}, s) = \oint_{|w|=1} \frac{dw}{2\pi i w} \int_0^1 dr \, \mu(r, w) \hat{g}_s(w) \psi_{\hat{\Delta}}(r) g(z, \bar{z})$$

$$\underbrace{\hat{g}_s(w)}_{w^s + w^{-s}}$$

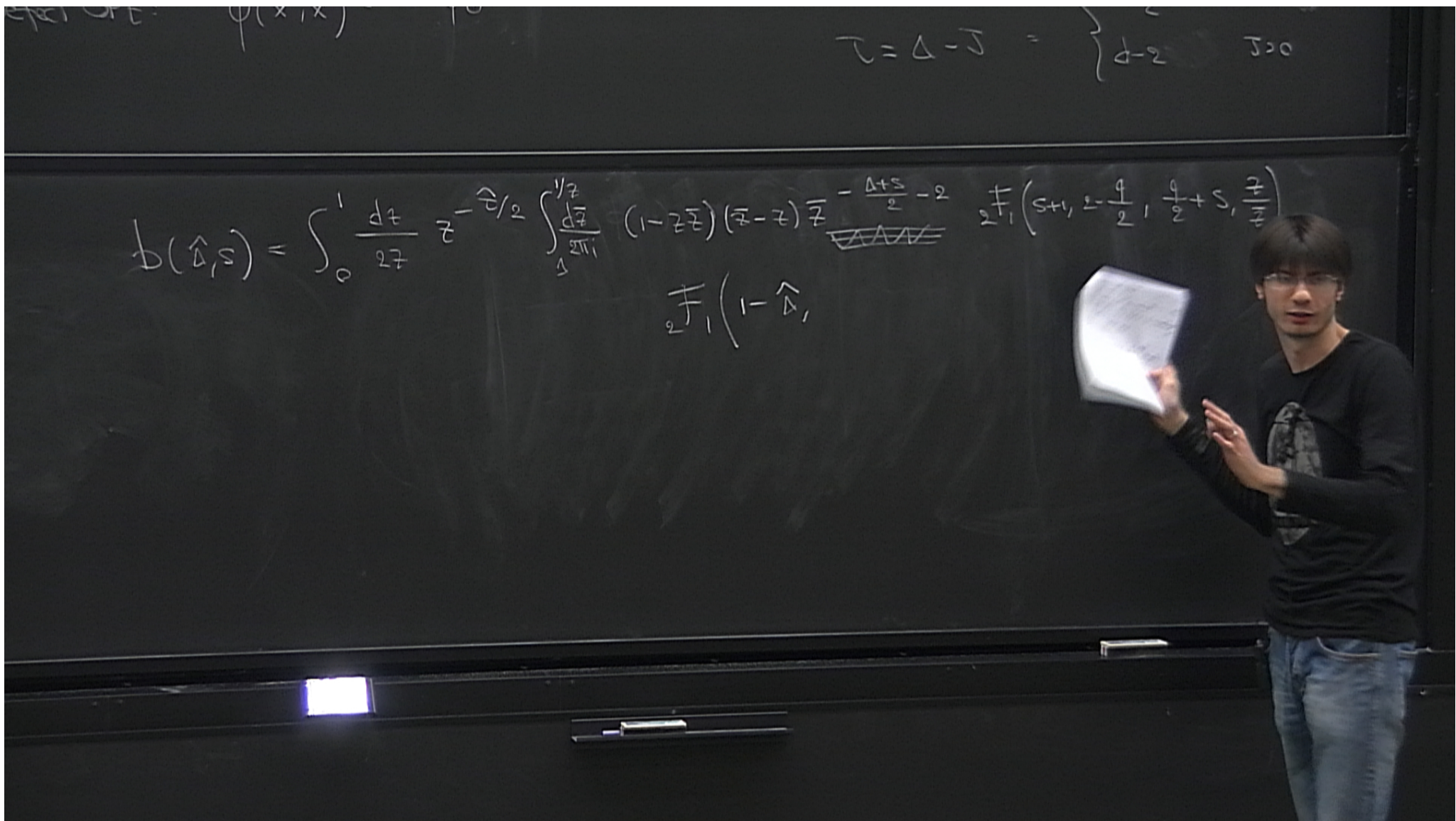
$$g(z, \bar{z}) \lesssim w^{-s} \quad w \rightarrow 0$$

$$b(\hat{\Delta}, s) = \# \oint_{|w|=1} \frac{dw}{2\pi i w} \int_0^1 dr \, \rho(r, w) \hat{g}_s(w) \psi_{\hat{\Delta}}(r) g(z, \bar{z})$$

$$g(z, \bar{z}) \lesssim w^{-s_*} \quad w \rightarrow 0$$

$s > s_*$ the circle can be dropped

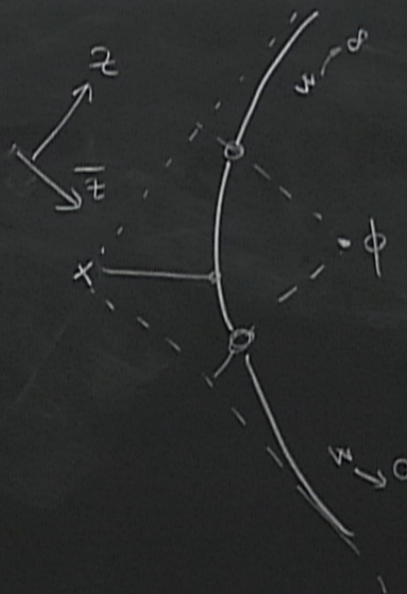
$$w^s + w^{-s}$$



$$\begin{aligned}
 & \tau = \Delta - J = \begin{cases} d-2 & J > 0 \end{cases} \\
 & b(\hat{\Delta}, s) = \int_0^1 \frac{dz}{2z} z^{-\hat{\Delta}/2} \int_{\frac{1}{2\pi i}}^{\frac{1}{2}} \frac{d\bar{z}}{\bar{z}} (1-z\bar{z})(\bar{z}-z) \bar{z}^{-\frac{\Delta+s}{2}-2} {}_2F_1\left(s+1, z-\frac{1}{2}, \frac{1}{2}+s, \frac{z}{\bar{z}}\right) \\
 & {}_2F_1\left(1-\hat{\Delta}, \dots\right)
 \end{aligned}$$

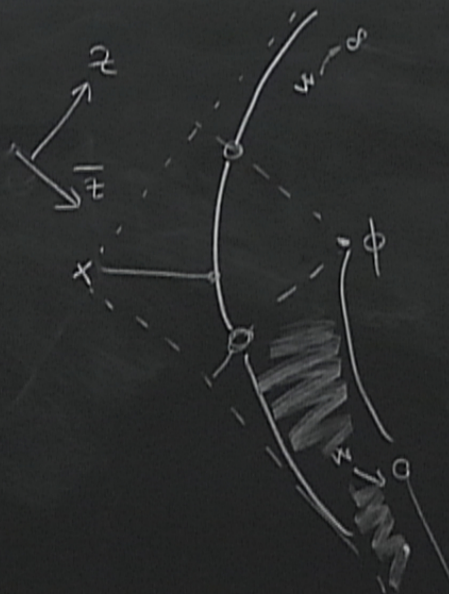
$$\tau = \Delta - J = \begin{cases} d-2 & J > 0 \end{cases}$$

$$b(\hat{\Delta}, s) = \int_0^1 \frac{dz}{2z} z^{-\hat{\Delta}/2} \int_{\frac{1}{2\pi i}}^{\frac{1}{2}} \frac{d\bar{z}}{2\pi i} (1-z\bar{z})(\bar{z}-z)\bar{z}^{-\frac{\Delta+s}{2}-2} {}_2F_1\left(s+1, z-\frac{1}{2}, \frac{1}{2}+s, \frac{z}{\bar{z}}\right) \\ {}_2F_1\left(1-\hat{\Delta}, 1-\frac{p}{2}, 1+\frac{p}{2}-\hat{\Delta}, z\bar{z}\right) \text{disc}_{\bar{z}=1} g(z, \bar{z})$$



$$b(\hat{\Delta}, s) = \int_0^1 \frac{d}{2}$$

4. An example
5. Conclusions



$$b(\hat{\Delta}, \varepsilon) = \int_0^1 \frac{d}{2}$$

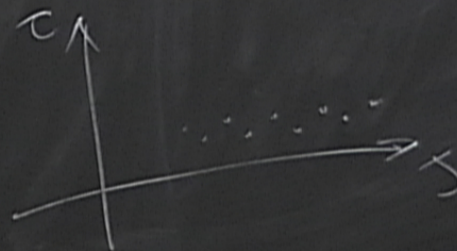
$$b(\hat{\Delta}, s) = \int_0^1 \frac{dz}{2z} z^{-\hat{\Delta}/2} \int_{\frac{1}{2\pi i}}^{\frac{1}{2}} \frac{d\bar{z}}{2\pi i} (1 - z\bar{z}) (\bar{z} - z) \bar{z}^{-\frac{\Delta+s}{2}-2} {}_2F_1\left(s+1, z-\frac{1}{2}\right) \\ {}_2F_1\left(1-\hat{\Delta}, 1-\frac{p}{2}, 1+\frac{p}{2}-\hat{\Delta}, z\bar{z}\right)$$

3. The large s sector

$$= \int_0^1 \frac{dz}{2z} z^{-\hat{c}/2} \int_1^{1/z} \frac{d\bar{z}}{2\pi i} (1-z\bar{z})(\bar{z}-z)\bar{z}^{-\frac{\Delta+s}{2}-2} {}_2F_1\left(s+1, 2-\frac{q}{2}, \frac{q}{2}+s, \frac{z}{\bar{z}}\right)$$

$${}_2F_1\left(1-\hat{\Delta}, 1-\frac{p}{2}, 1+\frac{p}{2}-\hat{\Delta}, z\bar{z}\right) \text{disc}_{\bar{z}=1} g(z, \bar{z})$$

ge s sector



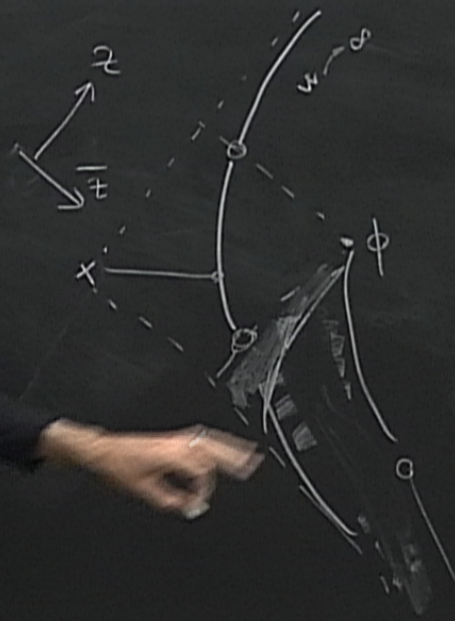
$$= \int_0^1 \frac{dz}{2z} z^{-\frac{\hat{c}}{2}} \int_1^{\frac{1}{2}} \frac{d\bar{z}}{2\pi i} (1-z\bar{z})(\bar{z}-z)\bar{z}^{-\frac{\Delta+s}{2}-2} {}_2F_1\left(s+1, 2-\frac{q}{2}, \frac{q}{2}+s, \frac{z}{\bar{z}}\right)$$

$${}_2F_1\left(1-\hat{\Delta}, 1-\frac{p}{2}, 1+\frac{p}{2}-\hat{\Delta}, z\bar{z}\right) \text{disc}_{\bar{z}=1} g(z, \bar{z})$$

ge s sector



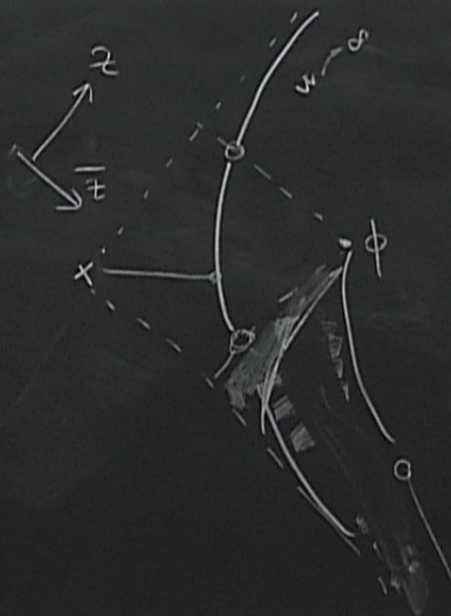
4. ...
5. Conclusions



$$b(\hat{\Delta}, s) = \int_0^1 \frac{d}{2}$$

3. The large s

4. Example
5. Conclusions



$$b(\hat{\Delta}, s) = \int_0^1 \frac{d}{2}$$

3. The large s

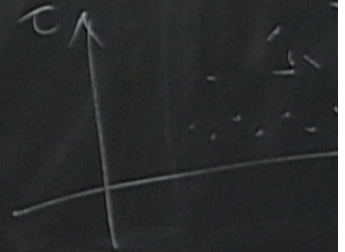
$$b(\hat{\Delta}, s) = \int_0^1 \frac{dz}{2z} z^{-\hat{\Delta}/2} \int_{-1}^{1/2} \frac{d\bar{z}}{2\pi i} (1-z\bar{z})(\bar{z}-z)\bar{z}^{-\frac{\Delta+s}{2}-2} {}_2F_1\left(s+1, \frac{1}{2}-\frac{q}{2}\right)$$

$${}_2F_1\left(1-\hat{\Delta}, 1-\frac{p}{2}, 1+\frac{p}{2}-\hat{\Delta}, z\bar{z}\right)$$

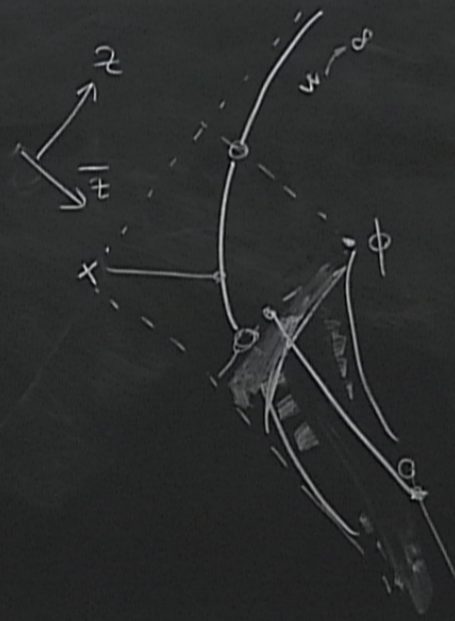
3. The large s sector

$$\hat{\Delta} = s + \Delta\phi + 2m + \mathcal{O}\left(\frac{1}{s^{1/2}}\right) \quad s \rightarrow \infty$$

$$(\partial_{\bar{z}})^m \partial_{z_1} \partial_{z_2} \phi$$



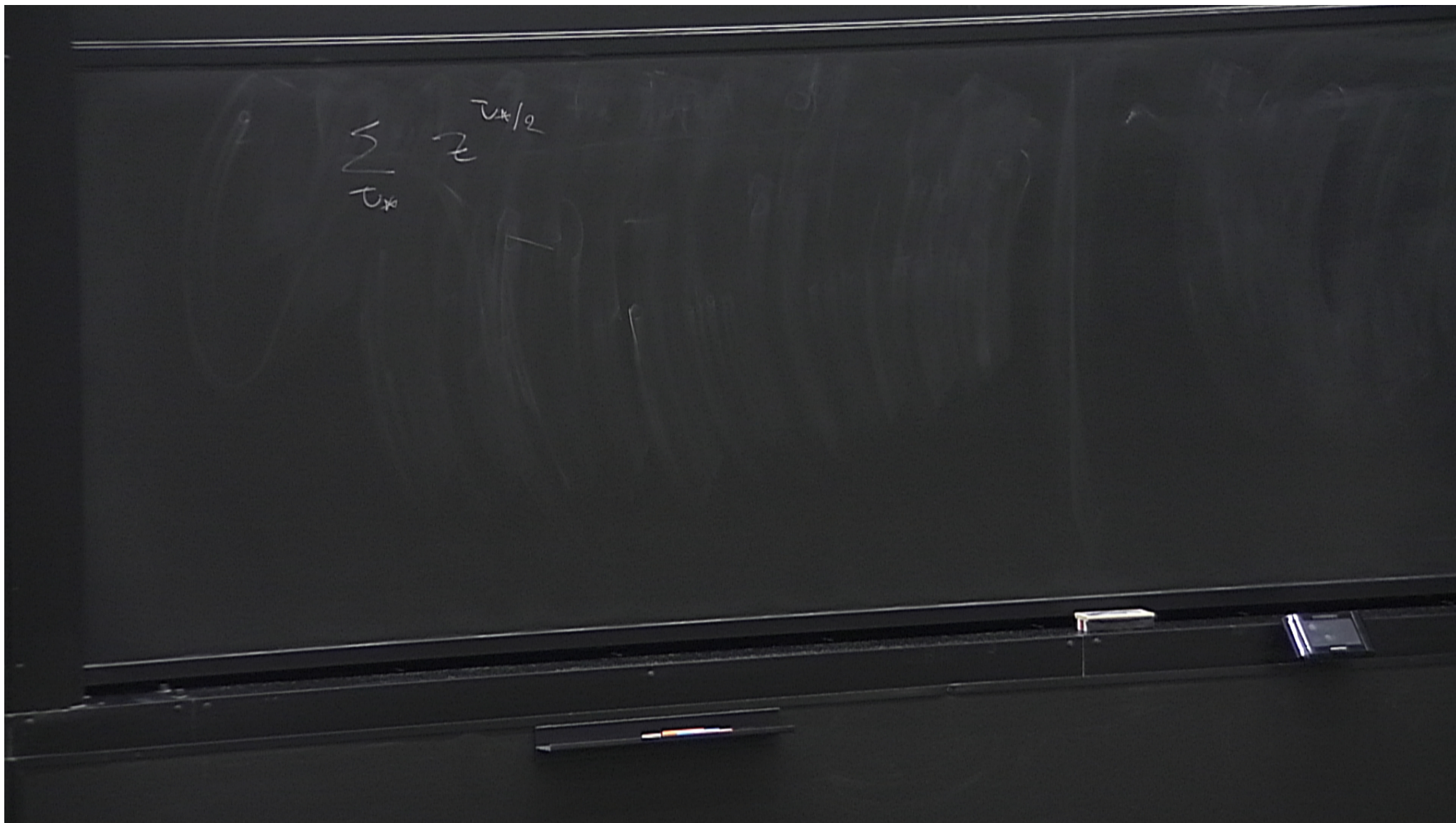
4. Example
5. Conclusions



$$L(\hat{\Delta}, s) = \int_0^1 \frac{d}{2}$$

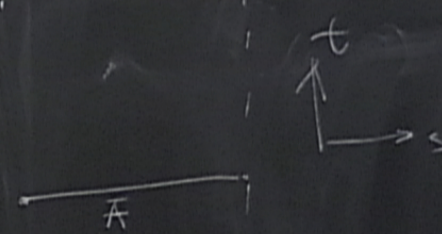
3

s s
s +



Renyi twist defect

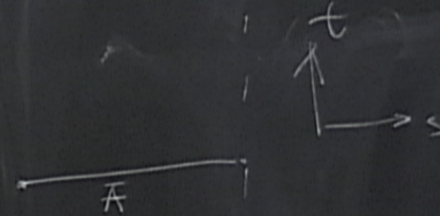
$$S_n = \frac{1}{1-n} \text{tr} \rho_A^n$$



Renyi twist defect

$$S_n = \frac{1}{1-n} \text{tr} \rho_A^n$$

$$S_{EE} = \lim_{n \rightarrow 1} S_n$$

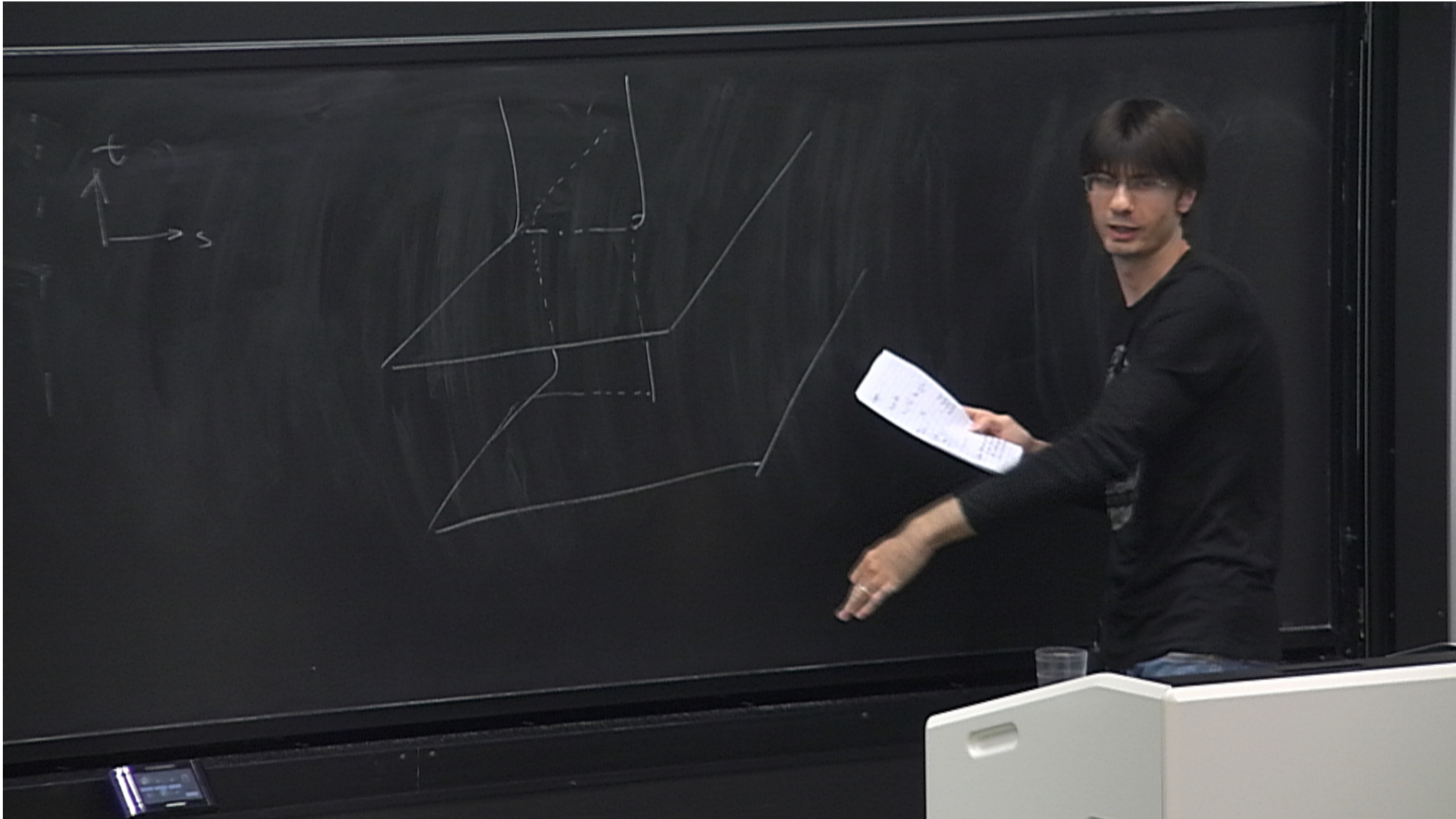


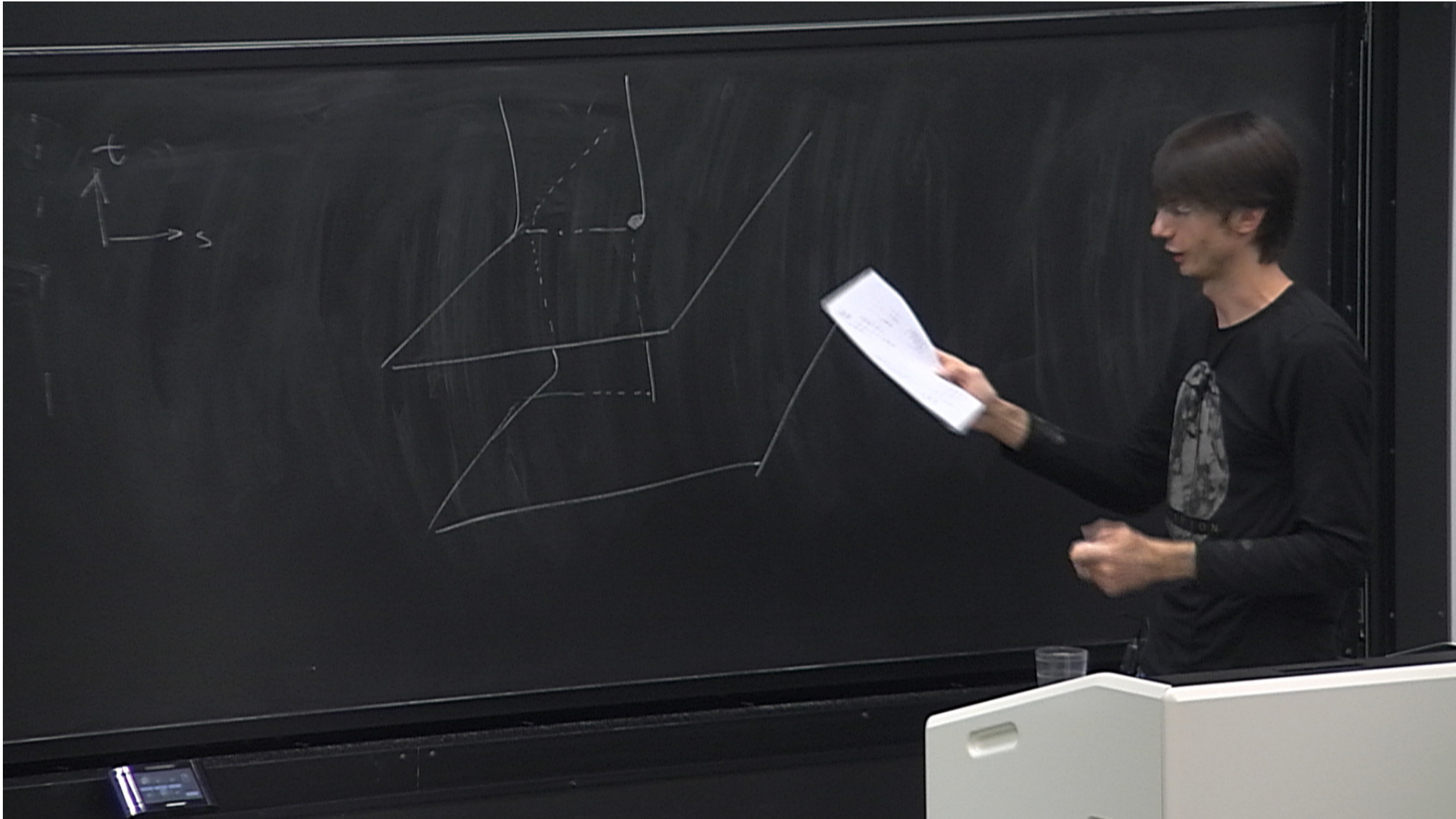
Renyi twist defect

$$S_n = \frac{1}{1-n} \text{tr} \rho_A^n$$

$$S_{EE} = \lim_{n \rightarrow 1} S_n$$







$$1) \quad \frac{(CFT)^n}{\mathbb{Z}_n}$$

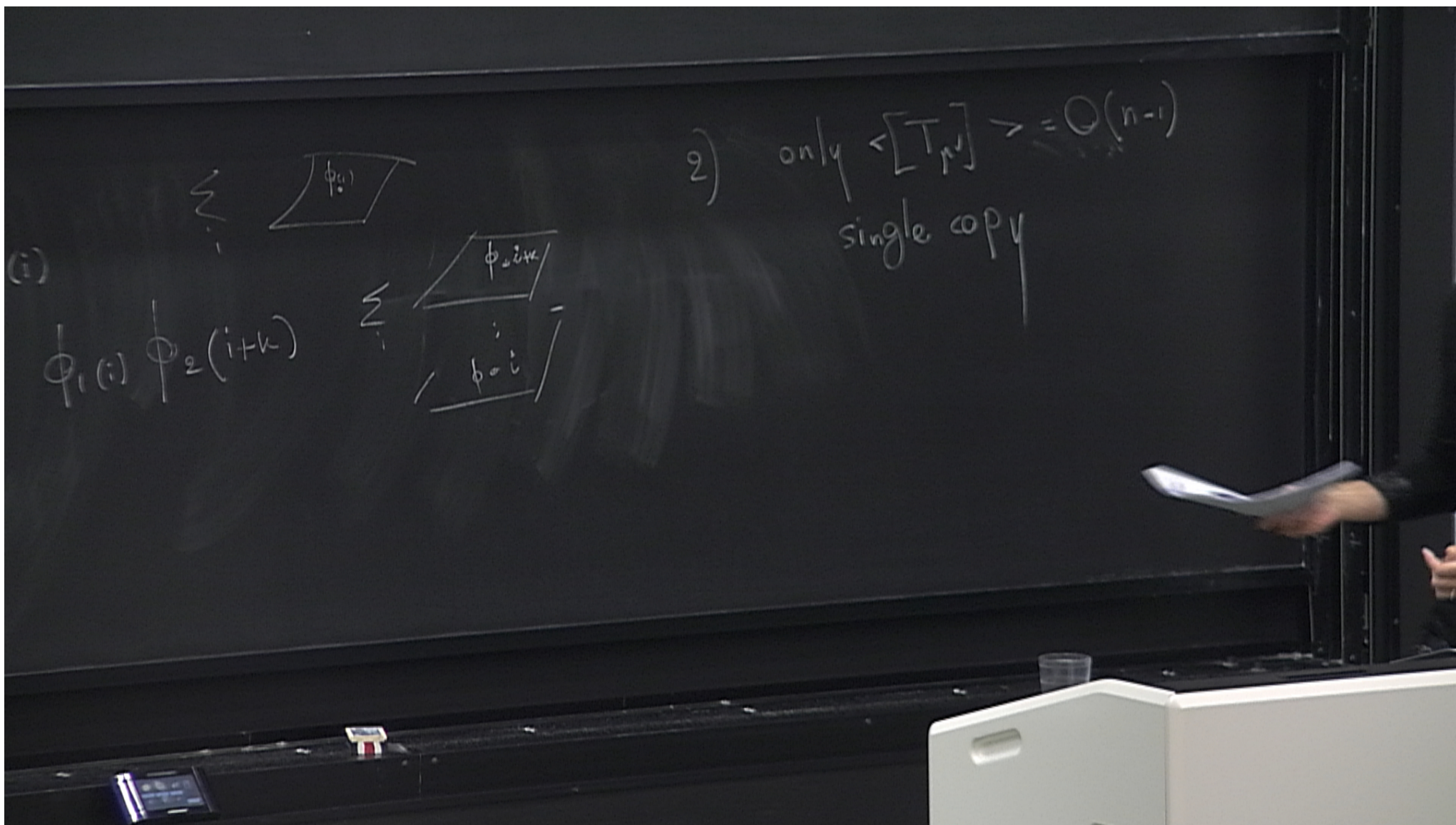
singlets

$$[\phi] = \sum_{i=1}^n \phi(i)$$

$$[\phi_1 \phi_2]_K = \sum_{i=1}^n \phi_1(i) \phi_2(i+K)$$

$$\sum_i \left[\begin{array}{c} \phi(i) \end{array} \right]$$

$$\sum_i \left[\begin{array}{c} \phi_2(i+K) \\ \vdots \\ \phi_1(i) \end{array} \right]$$



$$g(z, \bar{z}) = \sum_{\Delta, s} b_{\phi\theta}^2 \hat{f}_{\Delta, s}(z, \bar{z}) = \left[\frac{\sqrt{z\bar{z}}}{(1-\theta)(1-\bar{\theta})} \right]^{4\phi} \sum_{\Delta, s} c_{\phi\theta} \partial_\theta f_{\Delta, s}(z, \bar{z})$$

$$\hat{f}_{\Delta, s}(z, \bar{z}) = \hat{f}_{\Delta}(r) \hat{g}_s(\cos\theta) \quad \langle [\phi] [\phi] \rangle$$

$$\hat{\Delta}_s(z, \bar{z}) = \left[\frac{\sqrt{z\bar{z}}}{(1-z)(1-\bar{z})} \right]^{A_\phi} \sum_{A, J} c_{\phi\phi} \partial_\theta f_{A, J}(z, \bar{z})$$

$$\hat{\Delta}_s(z, \bar{z}) = \hat{f}_\Delta(r) \hat{g}_s(\cos\theta)$$

$$\langle [\phi] [\phi] \rangle$$

$$[\phi] \times [\phi] \sim T_{r=1} +$$

$$" 2\Delta\phi + 2m "$$

$$\hat{\Delta}_s(z, \bar{z}) = \left[\frac{\sqrt{z\bar{z}}}{(1-z)(1-\bar{z})} \right]^{4\phi} \sum_{\Delta, J} c_{\phi\phi\mathcal{O}} \mathcal{O} f_{\Delta, J}(z, \bar{z})$$

$$\hat{\Delta}_s(z, \bar{z}) = \hat{f}_{\Delta}(r) \hat{g}_s(\cos\theta) \quad \langle [\phi] [\phi] \rangle$$

$$[\phi] \times [\phi] \sim T_{r=1} +$$

~~$$2\Delta_{\phi} + 2m //$$~~

$$g(z, \bar{z}) = \sum_{\Delta, s} b_{\phi \theta}^2 \hat{f}_{\Delta, s}(z, \bar{z}) = \left[\frac{\sqrt{z\bar{z}}}{(1-z)(1-\bar{z})} \right]^{\Delta\phi} \sum_{\Delta, s} c_{\phi \theta} z_{\theta} \bar{z}_{\theta} f_{\Delta, s}(z, \bar{z})$$

$$\hat{f}_{\Delta, s}(z, \bar{z}) = \hat{f}_{\Delta}(r) \hat{g}_s(\cos\theta) \quad \langle [\phi] [\phi] \rangle$$

$$\hat{c} = \Delta\phi - (n-1) \frac{\Delta\phi}{d-1} \frac{(\Delta\phi - \frac{d}{2} + 1)_s}{(\Delta\phi)_s} + O(n-1)^2 \quad T_{\mu\nu} +$$