

Title: Constraints on Interacting Massive High Spins

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Abstract: <p>There seem to be no good examples of UV complete theories that have low-lying massive higher spin states isolated by a large gap, despite the relative ease of constructing effective field theories describing such states. We discuss constraints from analytic dispersion relations and subluminality of eikonal scattering that may help to explain this and provide insight into the possible interactions of massive higher spins.</p>

Constraints on Massive Higher Spins

Kurt Hinterbichler (Case Western)

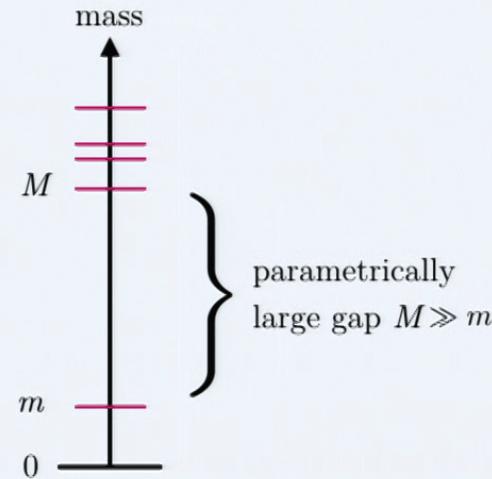
Perimeter, Apr. 10, 2018

arxiv: 1708.05716 w/ Austin Joyce and Rachel Rosen

arxiv: 1803:xxxxx w/ James Bonifacio

Isolated massive spinning particles?

Is it possible to have a theory
with a spectrum like this:



Spin 0, 1/2: Yes (pseudo Goldstones)

Spin 1, 3/2: Yes (spontaneously broken weakly coupled gauge theory/SUGRA)

Spin ≥ 2 : ?



Common lore says No: a massive higher spin always comes
with more states at parametrically the same mass

Isolated massive spinning particles?

Examples:

Kaluza Klein theory:

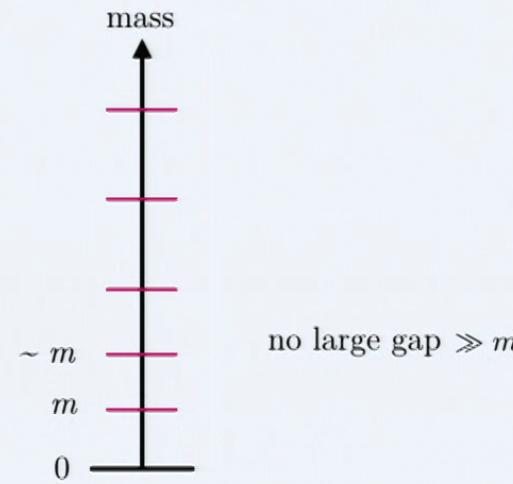
towers of spin ≤ 2 $m^2 \sim \lambda_{\text{laplacian}}$

Confining gauge theory

towers of all spins $m^2 \sim \Lambda_{\text{QCD}}^2$

String theory

towers of all spins $m^2 \sim \frac{1}{\alpha'}$



Isolated massive spinning particles?

Can there be “elementary” particles with spin ≥ 2 ?

Are there hadrons with Compton wavelength \gg intrinsic size ?

Could the graviton have a small Hubble-scale mass?

$$V(r) \sim \frac{1}{r} e^{-mr}, \quad m \sim H$$

IR modification scale
↓

Isolated massive spinning particles?

If such isolated massive particles are possible, there must exist an *effective field theory* (EFT) for them with a cutoff parametrically larger than the mass:

$$\Lambda \gg m$$

If such an EFT doesn't exist: problem solved

If it does exist: must figure out if it can be UV completed

Our approach: look for such EFTs and find obstructions to UV completion.

Massive spin-2

Massive spin 2 particle: 5 degrees of freedom (as opposed to 2 for massless helicity 2)

Fierz-Pauli action:

Fierz, Pauli (1939)

$$\mathcal{L} = \left[-\frac{1}{2} \partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda} \partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu} \partial_\nu h + \frac{1}{2} \partial_\lambda h \partial^\lambda h \right] - \frac{1}{2} m^2 (h_{\mu\nu} h^{\mu\nu} - h^2) + \frac{1}{M_P} h_{\mu\nu} T^{\mu\nu}$$



Einstein-Hilbert (massless) part.

Gauge symmetry: $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$



Mass term breaks gauge symmetry.

Fierz-Pauli tuning ensures 5 D.O.F.

Equations of motion: $(\square - m^2)h_{\mu\nu} = 0, \quad \partial^\mu h_{\mu\nu} = 0, \quad h = 0$

Helicity components

Stückelberg fields: $h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + 2 \partial_\mu \partial_\nu \phi$

$$h_{\mu\nu} \xrightarrow[\text{relativistic limit } m \rightarrow 0]{\quad} \begin{cases} h_{\mu\nu} \sim \text{helicity } \pm 2 & 2 \text{ DOF} \\ A_\mu \sim \text{helicity } \pm 1 & 2 \text{ DOF} \\ \phi \sim \text{helicity } 0 & 1 \text{ DOF} \end{cases}$$

Canonically normalize: $A_\mu \sim \frac{1}{m} \hat{A}_\mu, \quad \phi \sim \frac{1}{m^2} \hat{\phi}$

massless limit

Diagonalize kinetic terms: $h_{\mu\nu} = h'_{\mu\nu} + \hat{\phi} \eta_{\mu\nu}$

$$\mathcal{L}_{m=0}(h') - \frac{1}{2} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} - 3 \partial_\mu \hat{\phi} \partial^\mu \hat{\phi} + \frac{1}{M_P} h'_{\mu\nu} T^{\mu\nu} + \frac{1}{M_P} \hat{\phi} T$$

Interaction terms

$$\frac{M_P^2}{2} \int d^4x \left[(\sqrt{-g}R) - \sqrt{-g} \frac{1}{4} m^2 V(g, h) \right],$$

$$V(g, h) = V_2(g, h) + V_3(g, h) + V_4(g, h) + V_5(g, h) + \dots,$$

$$V_2(g, h) = \langle h^2 \rangle - \langle h \rangle^2,$$

$$V_3(g, h) = +c_1 \langle h^3 \rangle + c_2 \langle h^2 \rangle \langle h \rangle + c_3 \langle h \rangle^3,$$

$$V_4(g, h) = +d_1 \langle h^4 \rangle + d_2 \langle h^3 \rangle \langle h \rangle + d_3 \langle h^2 \rangle^2 + d_4 \langle h^2 \rangle \langle h \rangle^2 + d_5 \langle h \rangle^4,$$

$$\begin{aligned} V_5(g, h) = & +f_1 \langle h^5 \rangle + f_2 \langle h^4 \rangle \langle h \rangle + f_3 \langle h^3 \rangle \langle h \rangle^2 + f_4 \langle h^3 \rangle \langle h^2 \rangle + f_5 \langle h^2 \rangle^2 \langle h \rangle \\ & + f_6 \langle h^2 \rangle \langle h \rangle^3 + f_7 \langle h \rangle^5, \end{aligned}$$

⋮

The effective field theory

Arkani-Hamed, Georgi and Schwartz (2003)

After replacement $h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu + 2\partial_\mu \partial_\nu \phi + \dots$ there are interaction terms:

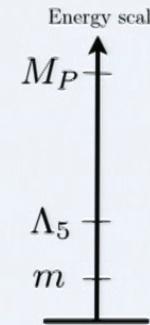
$$m^2 M_P^2 h^{n_h} (\partial A)^{n_A} (\partial^2 \phi)^{n_\phi} \sim \Lambda_\lambda^{4-n_h-2n_A-3n_\phi} \hat{h}^{n_h} (\partial \hat{A})^{n_A} (\partial^2 \hat{\phi})^{n_\phi}$$

Various strong coupling scales: $\Lambda_\lambda = (M_P m^{\lambda-1})^{1/\lambda}, \quad \lambda = \frac{3n_\phi + 2n_A + n_h - 4}{n_\phi + n_A + n_h - 2}$

The smallest scale is carried by a cubic scalar interaction:

$$\sim \frac{(\partial^2 \hat{\phi})^3}{\Lambda_5^5}, \quad \Lambda_5 = (M_P m^4)^{1/5}$$

This is the (UV) strong coupling scale of the theory

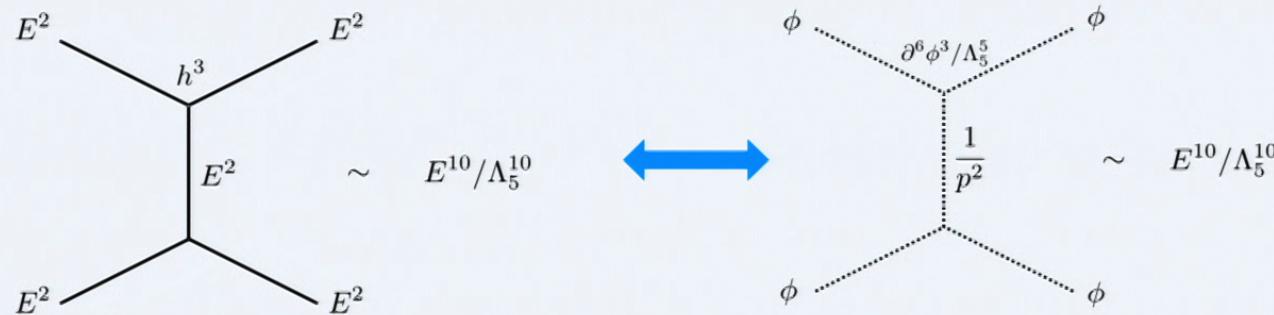


The effective field theory

$2 \rightarrow 2$ amplitudes grows like E^{10}

Polarization tensors: $\epsilon_{\mu\nu}^T \rightarrow \sim 1$ $\epsilon_{\mu\nu}^V \rightarrow \sim \frac{E}{m}$ $\epsilon_{\mu\nu}^S \rightarrow \sim \frac{E^2}{m^2}$

Propagator: $P_{\mu\nu,\rho\sigma} \rightarrow \sim \frac{1}{p^2} \frac{p_\mu p_\nu p_\rho p_\sigma}{m^4} \sim \frac{E^2}{m^4}$



The effective field theory

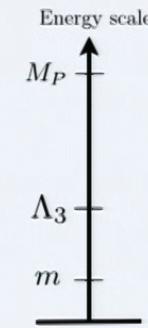
Arkani-Hamed, Georgi and Schwartz (2003)
Creminelli, Nicolis, Pappuchi, Trincherini (2005)

insight: Can choose the interactions, order by order in h , so that the scalar self-interactions appear in total derivative combinations.

2 parameter family of ways to do this.

The leading operators are now:

$$\sim \frac{\hat{h}(\partial^2 \hat{\phi})^n}{M_P^{n+1} m^{2n+2}} \quad \Lambda_3 \equiv (M_P m^2)^{1/3}$$



The Λ_3 theory

de Rham, Gabadadze (2010)

Leading operators:

$$\frac{1}{2} \hat{h}_{\mu\nu} \mathcal{E}^{\mu\nu,\alpha\beta} \hat{h}_{\alpha\beta} - \frac{1}{2} \hat{h}^{\mu\nu} \left[-4X_{\mu\nu}^{(1)}(\hat{\phi}) + \frac{4(6c_3 - 1)}{\Lambda_3^3} X_{\mu\nu}^{(2)}(\hat{\phi}) + \frac{16(8d_5 + c_3)}{\Lambda_3^6} X_{\mu\nu}^{(3)}(\hat{\phi}) \right] + \frac{1}{M_P} \hat{h}_{\mu\nu} T^{\mu\nu}$$

2 parameters



Galileon operators:

$$X_{\mu\nu}^{(0)} = \eta_{\mu\nu} \quad (\Pi_{\mu\nu} \equiv \partial_\mu \partial_\nu \phi)$$

$$X_{\mu\nu}^{(1)} = [\Pi] \eta_{\mu\nu} - \Pi_{\mu\nu}$$

$$X_{\mu\nu}^{(2)} = ([\Pi]^2 - [\Pi^2]) \eta_{\mu\nu} - 2[\Pi] \Pi_{\mu\nu} + 2\Pi_{\mu\nu}^2$$

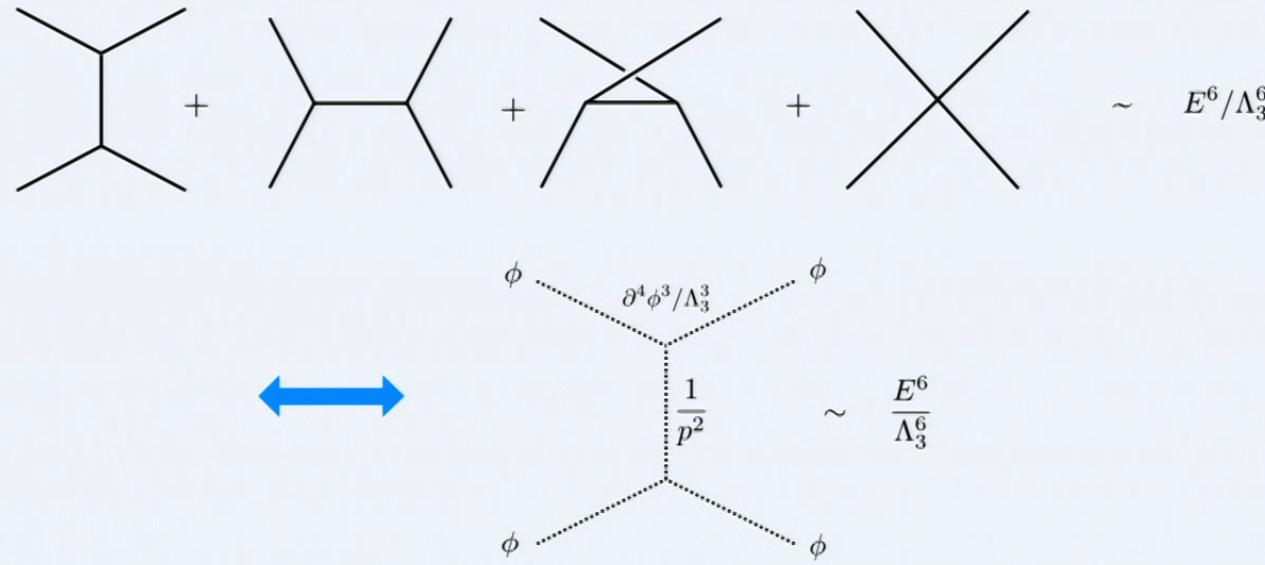
$$X_{\mu\nu}^{(3)} = ([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]) \eta_{\mu\nu} - 3([\Pi]^2 - [\Pi^2]) \Pi_{\mu\nu} + 6[\Pi] \Pi_{\mu\nu}^2 - 6\Pi_{\mu\nu}^3$$

⋮

The Λ_3 theory

$2 \rightarrow 2$ amplitudes now grow like: E^6

Cancellation between exchange and contact diagrams:



Pseudo-linear theory

KH: 1305.7227

Another way to achieve E^6

$$\mathcal{L} = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h\partial^\lambda h - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$

Also 2 parameters

$$\begin{aligned} &+ \lambda_3 \frac{m^2}{M_p} \eta^{\mu_1\nu_1 \dots \mu_3\nu_3} h_{\mu_1\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} \\ &+ \lambda_4 \frac{m^2}{M_p^2} \eta^{\mu_1\nu_1 \dots \mu_4\nu_4} h_{\mu_1\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} h_{\mu_4\nu_4} \\ &+ \frac{1}{M_p} \eta^{\mu_1\nu_1 \dots \mu_4\nu_4} \partial_{\mu_1} \partial_{\nu_1} h_{\mu_2\nu_2} h_{\mu_3\nu_3} h_{\mu_4\nu_4} \end{aligned}$$

Can we do any better?

KH, James Bonifacio (to appear)

Is there any way to do better than E^6 ?

$$\begin{aligned}\mathcal{L} \sim & (\partial h)^2 + h^2 \\ & + h^3 + \partial^2 h^3 + \partial^4 h^3 + \dots \\ & + h^4 + \partial^2 h^4 + \partial^4 h^4 + \dots \\ & \vdots\end{aligned}$$

Field redefinitions \rightarrow put fields on shell: transverse, traceless, $\square \rightarrow -m^2$

Classify all on-shell cubic and quartic vertices

cubic vertices

Polarization tensors:

$$\epsilon_{\mu_1 \dots \mu_s} \rightarrow z_{\mu_1} z_{\mu_2} \dots z_{\mu_s} , \quad z^2 = 0$$

No on-shell non-trivial functions of momenta:

$$p_1^\mu + p_2^\mu + p_3^\mu = 0 \quad \Rightarrow \quad p_1 \cdot p_2 = \frac{1}{2} (m_1^2 + m_2^2 - m_3^2) , \text{ etc.}$$

$$\mathcal{A}_3 \sim z_{12}^{n_{12}} z_{13}^{n_{13}} z_{23}^{n_{23}} z p_{12}^{m_{12}} z p_{23}^{m_{23}} z p_{31}^{m_{31}}$$

$$n_{12} + n_{13} + m_{12} = s_1,$$

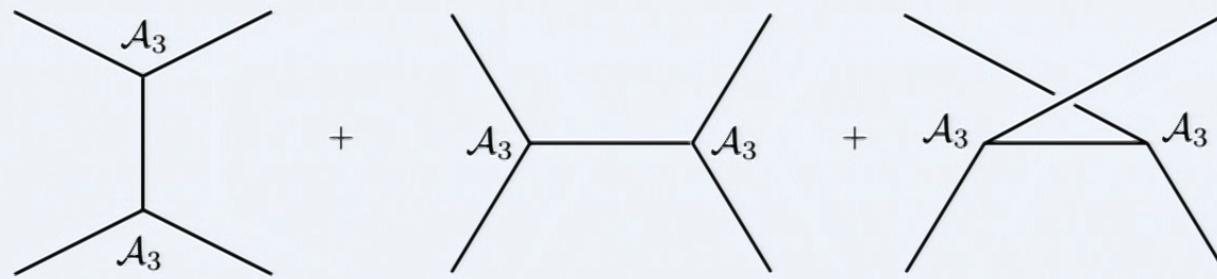
$$n_{12} + n_{23} + m_{23} = s_2,$$

$$n_{13} + n_{23} + m_{31} = s_3.$$

Finite number of solutions. \rightarrow On-shell cubic amplitudes nailed down by Lorentz invariance.

Best possible scaling

Build the exchange diagrams:



Finite number of cubic vertices \rightarrow finite number of exchange diagrams \rightarrow bounded growth with energy

$$\mathcal{A}_{\text{exchange}} \sim E^{\#}$$

Best possible scaling

KH, James Bonifacio (to appear)

Classify all analytic quartic amplitudes (contact terms):

2 independent invariants made of momenta (2 Mandelstams)

$$p_{12}^{k_{12}} p_{13}^{k_{13}} z_{12}^{n_{12}} z_{13}^{n_{13}} z_{14}^{n_{14}} z_{23}^{n_{23}} z_{24}^{n_{24}} z_{34}^{n_{34}} z p_{13}^{m_{13}} z p_{14}^{m_{14}} z p_{21}^{m_{21}} z p_{24}^{m_{24}} z p_{31}^{m_{31}} z p_{32}^{m_{32}} z p_{42}^{m_{42}} z p_{43}^{m_{43}}$$

unconstrained

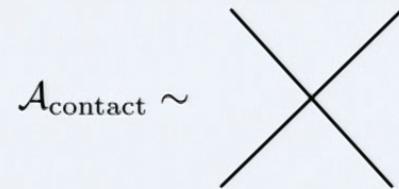
$$n_{12} + n_{13} + n_{14} + m_{13} + m_{14} = s_1,$$

$$n_{12} + n_{23} + n_{24} + m_{21} + m_{24} = s_2,$$

$$n_{13} + n_{23} + n_{34} + m_{31} + m_{32} = s_3,$$

$$n_{14} + n_{24} + n_{34} + m_{42} + m_{43} = s_4.$$

This is the contact diagram:

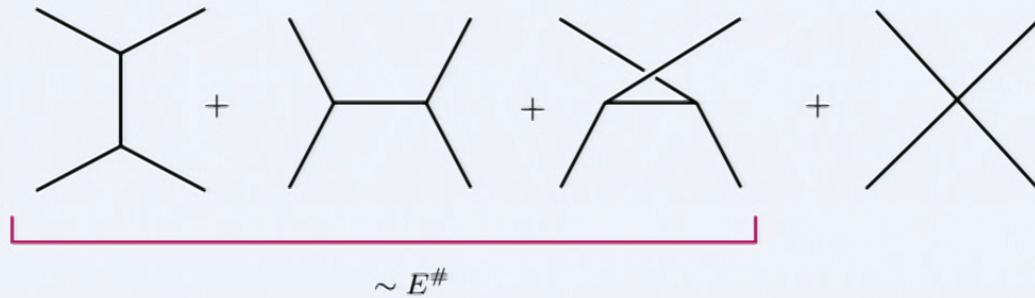


Best possible scaling

KH, James Bonifacio (to appear)

Try to cancel off highest energy scaling of exchange diagrams, working down:

$$\mathcal{A}_4 = \mathcal{A}_{\text{exchange}} + \mathcal{A}_{\text{contact}}$$



Result: Best possible scaling is E^6
Only theories that achieve this are Λ_3 theory and pseudo-linear

Best possible scalings

KH, James Bonifacio (to appear)

Best scaling for spin-1:

$$E^4 \quad \Lambda_2 \sim (M_P m)^{1/2}$$

allow additional scalar \rightarrow can achieve $E^0 \rightarrow$ Higgs mechanism

Best scaling for spin-2:

$$E^6 \quad \Lambda_3 \sim (M_P m^2)^{1/3}$$

allow additional scalar+vector \rightarrow no simple gravitational Higgs mechanism

Christensen, Stefanus (2014)

Nima Arkani-Hamed, Huang, Huang (2017)

Conjecture for higher spins:

$$\mathcal{A}_4 \sim \begin{cases} E^{3s} & s \text{ even}, \\ E^{3s+1} & s \text{ odd}. \end{cases}$$

$$\Lambda_{\max} = \begin{cases} \Lambda_{\frac{3s}{2}} & s \text{ even}, \\ \Lambda_{\frac{3s+1}{2}} & s \text{ odd}. \end{cases} \quad \Lambda_n \equiv (M_p m^{n-1})^{1/n}$$

Best possible scalings

Massive higher spins coupled to gauge fields:

Massive higher spin coupled to U(1): $\Lambda_{U(1)} \sim \frac{m}{e^{1/(2s-1)}}$ Porrati, Rahman (2008)

$$\mathcal{A} \sim E^{2(2s-1)}$$

Massive higher spin coupled to gravity: $\Lambda_{\text{gravity}} \sim \Lambda_{2s-1} = (M_p m^{2s-2})^{1/(2s-1)}$

$$\mathcal{A} \sim E^{2(2s-1)}$$

KH, James Bonifacio (to appear)

Weak gravity conjecture: $e \gtrsim \frac{m}{M_P}$

$$\Lambda_{U(1)} \quad \longleftrightarrow \quad \Lambda_{\text{gravity}}$$
$$e = \frac{m}{M_P}$$

Dispersion relations

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi (2006)

UV constraints on EFT coefficients

Forward amplitude: $\mathcal{A}(s) \equiv \mathcal{A}(s, t = 0)$

$$f \equiv \frac{1}{2\pi i} \oint_{\Gamma} ds \frac{\mathcal{A}(s)}{(s - \mu^2)^{n+1}}$$

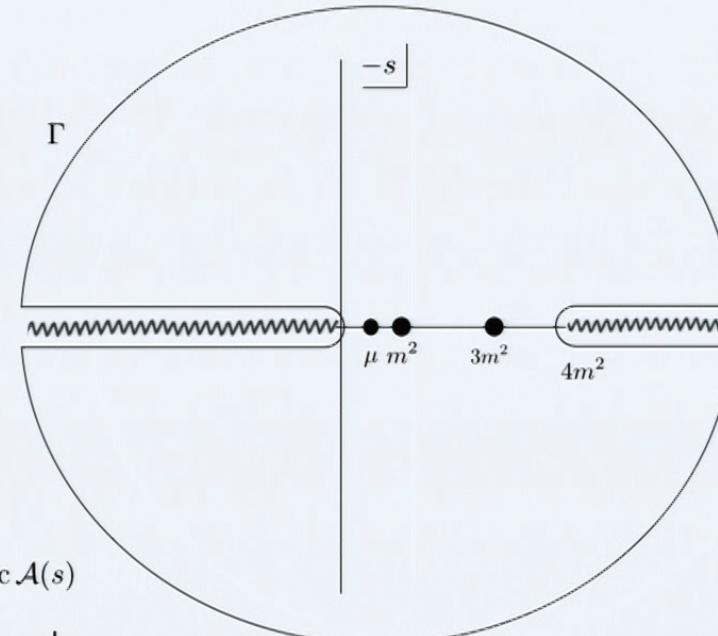
Froissart bound kills semi-circle:

$$|\mathcal{A}(s)| \lesssim \mathcal{O}(s \ln^2 s)$$

$$f = \frac{1}{2\pi i} \int_{4m^2}^{\infty} ds \left[\frac{1}{(s - \mu)^{n+1}} + \frac{1}{(s - 4m^2 + \mu)^{n+1}} \right] \text{disc } \mathcal{A}(s)$$

optical theorem \downarrow

$$= \frac{1}{\pi} \int_{4m^2}^{\infty} ds \left[\frac{1}{(s - \mu)^{n+1}} + (-1)^n \frac{1}{(s - 4m^2 + \mu)^{n+1}} \right] s \sqrt{1 - \frac{4m^2}{s}} \sigma(s) \quad \rightarrow \quad f > 0$$



positive (for n even) positive

Dispersion relations

Deform contour:

$$f = \frac{1}{2\pi i} \oint_{\Gamma'} ds \frac{\mathcal{A}(s)}{(s - \mu^2)^{n+1}}$$

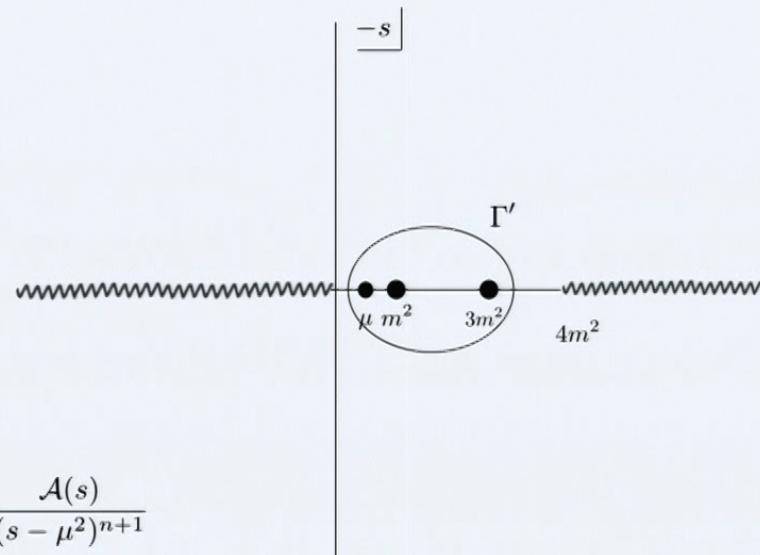
$$f = \underset{s \rightarrow \mu^2}{\text{res}} \frac{\mathcal{A}(s)}{(s - \mu^2)^{n+1}} + \underset{s \rightarrow m^2}{\text{res}} \frac{\mathcal{A}(s)}{(s - \mu^2)^{n+1}} + \underset{s \rightarrow 3m^2}{\text{res}} \frac{\mathcal{A}(s)}{(s - \mu^2)^{n+1}}$$



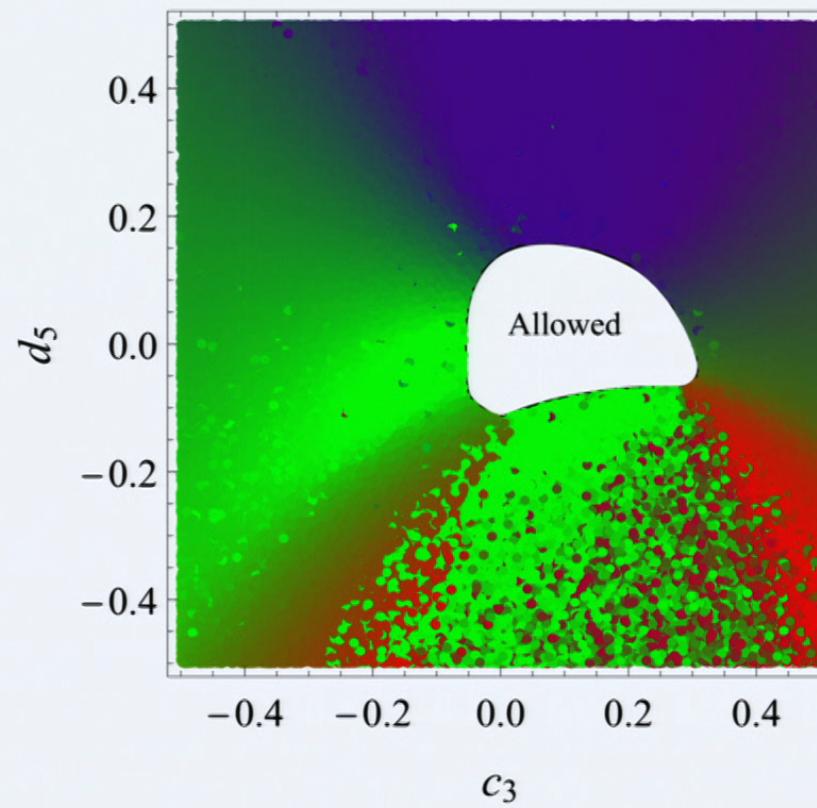
exchange poles: tree-level in EFT

$$= - \left(\underset{s \rightarrow \infty}{\text{res}} \frac{\mathcal{A}(s)}{(s - \mu^2)^{n+1}} \right)_{\text{EFT,tree}} > 0$$

← Constrains coefficients of leading E^{2n} terms in tree-level of EFT



dRGT theory allowed island



Pseudo-linear not allowed

James Bonifacio, KH, Rachel Rosen (I607.06084)

$$\mathcal{L}_{\text{int}} = \frac{1}{M_p} \lambda_1 \mathcal{L}_{2,3} + \frac{m^2}{M_p} \lambda_3 \mathcal{L}_{0,3} + \frac{m^2}{M_p^2} \lambda_4 \mathcal{L}_{0,4}$$

Forward amplitude: $\mathcal{A}_{\text{forward}} \sim \frac{E^4}{M_P^2 m^2}$

$$f(TTTT)_- = \frac{\lambda_1^2}{m^2 M_p^2}$$

$$f(VVVV)_+ = -\frac{15\lambda_1^2 + 13\lambda_1\lambda_3 + 5\lambda_3^2}{12 m^2 M_p^2} \quad \rightarrow \text{No allowed region}$$

$$f(SSSS) = -\frac{5\lambda_1^2 + 6\lambda_1\lambda_3 + \lambda_3^2 + 2\lambda_4}{9 m^2 M_p^2}$$

\vdots

Superluminality constraints

Another traditional constraint on EFTs:

Superluminality of small fluctuations on non-trivial Lorentz-violating backgrounds (e.g. Velo-Zwanziger problem)

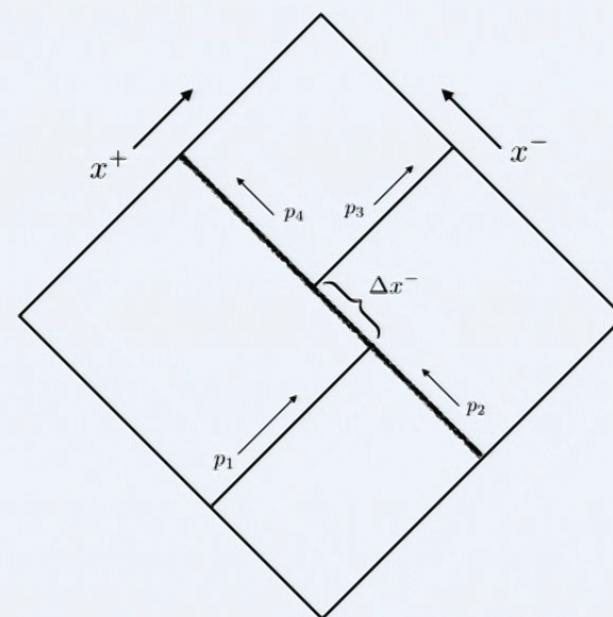
Less problematic: superluminality in the S-matrix

Eikonal scattering:

high-energy, fixed impact parameter:

$$s/t \rightarrow \infty$$

Camanho, Edelstein, Maldacena, Zhiboedov (2016)



Eikonal limit

$$\begin{aligned} \mathcal{A}_{\text{eikonal}} &= \text{I} + \text{II} + \text{III} + \text{IV} + \dots \\ &\quad + \text{V} + \text{VI} + \text{VII} + \dots \\ &\quad \vdots \qquad \vdots \\ &= e^{\left(\text{I} \right)} \end{aligned}$$

$$i\mathcal{A}_{\text{eikonal}} = 4p^- p^+ \int d^2\mathbf{b} e^{i\mathbf{b}\cdot\mathbf{q}} \left(e^{i\delta(\mathbf{b})} - 1 \right) \quad , \quad \delta(\mathbf{b}) = \frac{1}{4p^- p^+} \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{q}} \mathcal{A}_0(\mathbf{q})$$

Time delay: $\Delta x^- = \frac{1}{p^-} \delta$

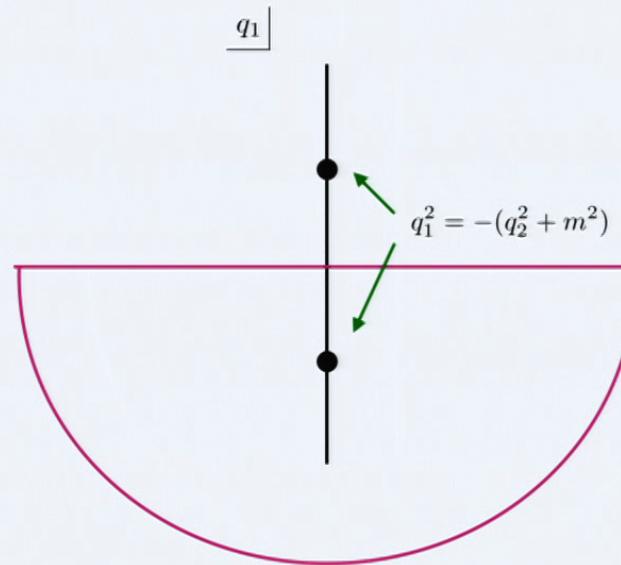
Eikonal

Camanho, Edelstein, Maldacena, Zhiboedov (2016)

Eikonal phase depends only on *on-shell* three point amplitudes:

$$\delta(\mathbf{b}) = \frac{1}{4p^- p^+} \int \frac{d^2 \mathbf{q}}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{q}} \mathcal{A}_0(\mathbf{q})$$

$$\mathcal{A}_0(\mathbf{q}) \sim \frac{1}{\mathbf{q}^2 + m^2}$$



Example: constraints on higher curvature terms

Camanho, Edelstein, Maldacena, Zhiboedov (2016)

$$M_P^2 \int d^4x \sqrt{-g} \left[R + \frac{1}{M^2} (R^2 + \dots) \right] \quad M \ll M_P$$

$$= \int d^4x \left[\partial^2 h^2 + \frac{1}{M_P} \partial^2 h^3 + \dots + \frac{1}{M^2 M_P} \partial^4 h^3 + \dots \right]$$

$$\text{EFT Strong coupling scale: } \Lambda \sim (M_P M^2)^{1/3} \quad M \ll \Lambda \ll M_P$$

Example: tree-level string theory

$$M_P^2 \int d^4x \sqrt{-g} \left[R + \frac{1}{M^2} (R^2 + \dots) + \frac{1}{M^4} (R^3 + \dots) + \dots \right]$$

$$\Lambda \sim (M_P M^n)^{1/n+1} \rightarrow M$$

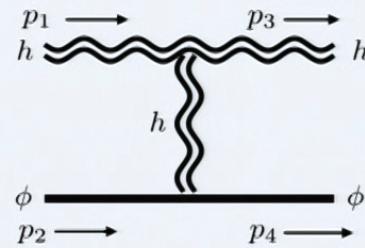
Example: constraints on higher curvature terms

Cubic vertices:

\mathcal{A}_1	$(p_1 \cdot z_3 \ z_1 \cdot z_2 + p_3 \cdot z_2 \ z_1 \cdot z_3 + p_2 \cdot z_1 \ z_2 \cdot z_3)^2$	$\sqrt{-g} R _{(3)}$
\mathcal{A}_2	$p_1 \cdot z_3 \ p_2 \cdot z_1 \ p_3 \cdot z_2 \ (p_1 \cdot z_3 \ z_1 \cdot z_2 + p_3 \cdot z_2 \ z_1 \cdot z_3 + p_2 \cdot z_1 \ z_2 \cdot z_3)$	$\sqrt{-g} (R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2) _{(3)}$
\mathcal{A}_3	$(p_1 \cdot z_3)^2 (p_2 \cdot z_1)^2 (p_3 \cdot z_2)^2$	$\sqrt{-g} R_{\rho\sigma}^{\mu\nu} R_{\alpha\beta}^{\rho\sigma} R_{\mu\nu}^{\alpha\beta} _{(3)}$

$D=4$: no \mathcal{A}_2 , 1 additional parity violating amplitudes

Example: constraints on higher curvature terms



Helicities

$$\frac{1}{s} \delta^{\lambda, \lambda'}$$

eigenvalues

$$\sim (+) \frac{1}{M_P^2} \ln(b) \pm \frac{1}{M_P^2 M^2 b^2} \pm \frac{1}{M_P^2 M^4 b^4}$$

Einstein-Hilbert

Gauss-Bonnet

(Riemann)³

$$\delta > 0 \rightarrow \text{Gauss-Bonnet and (Riemann)³ must vanish}$$

If not \rightarrow new physics comes in at the scale M

Massless higher spins

KH,Austin Joyce, Rachel Rosen (1712.10021)

Cubic vertices:

Spin-1

$$\mathcal{A}_{\text{YM}} \sim (p_1 \cdot z_3)(z_1 \cdot z_2) + (p_3 \cdot z_2)(z_1 \cdot z_3) + (p_2 \cdot z_1)(z_2 \cdot z_3)$$

$$\mathcal{A}_{F^3} \sim (p_1 \cdot z_3)(p_2 \cdot z_1)(p_3 \cdot z_2)$$

Spin-2

$$\text{Einstein-Hilbert} \sim (\mathcal{A}_{\text{YM}})^2$$

$$\text{Gauss-Bonnet} \sim \cancel{(\mathcal{A}_{\text{YM}})(\mathcal{A}_{F^3})} \text{ vanishes in } D=4$$

$$(\text{Riemann})^3 \sim (\mathcal{A}_{F^3})^2$$

Spin- s

$$(\mathcal{A}_{\text{YM}})^s$$

$$\cancel{(\mathcal{A}_{\text{YM}})^{s-1} (\mathcal{A}_{F^3})}$$

$$\cancel{(\mathcal{A}_{\text{YM}})^{s-2} (\mathcal{A}_{F^3})^2}$$

vanish in $D=4$

\vdots

$$\cancel{(\mathcal{A}_{\text{YM}}) (\mathcal{A}_{F^3})^{s-1}}$$

$$(\text{linear curvature})^3 \sim (\mathcal{A}_{F^3})^s$$

Massless higher spins

KH,Austin Joyce, Rachel Rosen (1712.10021)

Spin- s vertices:	$(\mathcal{A}_{\text{YM}})^s$	$(\mathcal{A}_{F^3})^s$
gauge symmetry:	deforms	does not deform (linear)
Consistency/locality at quartic order (4 particle test) Benincasa, Cachazo (2007)	✗	✓
Eikonal constraints	✓	✗  $\delta(b) = \pm \alpha^2 \frac{s^{s-1}}{2^{3s+2} \pi} \frac{(8s-2)!!}{b^{4s}}.$

Cubic massive spin-2 vertices

\mathcal{A}_1	$z_1 \cdot z_2 \ z_2 \cdot z_3 \ z_3 \cdot z_1$	$h_{\mu\nu}^3$
\mathcal{A}_2	$(p_1 \cdot z_3 \ z_1 \cdot z_2 + p_3 \cdot z_2 \ z_1 \cdot z_3 + p_2 \cdot z_1 \ z_2 \cdot z_3)^2$	$\sqrt{-g} R _{(3)}$
\mathcal{A}_3	$(p_1 \cdot z_3)^2 (z_1 \cdot z_2)^2 + (p_3 \cdot z_2)^2 (z_1 \cdot z_3)^2 + (p_2 \cdot z_1)^2 (z_2 \cdot z_3)^2$	$\delta_{\nu_1}^{[\mu_1} \delta_{\nu_2}^{\mu_2} \delta_{\nu_3}^{\mu_3} \delta_{\nu_4}^{\mu_4]} \partial_{\mu_1} \partial^{\nu_1} h_{\mu_2}^{\nu_2} h_{\mu_3}^{\nu_3} h_{\mu_4}^{\nu_4}$
\mathcal{A}_4	$p_1 \cdot z_3 \ p_2 \cdot z_1 \ p_3 \cdot z_2 \ (p_1 \cdot z_3 \ z_1 \cdot z_2 + p_3 \cdot z_2 \ z_1 \cdot z_3 + p_2 \cdot z_1 \ z_2 \cdot z_3)$	$\sqrt{-g} (R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2) _{(3)}$
\mathcal{A}_5	$(p_1 \cdot z_3)^2 (p_2 \cdot z_1)^2 (p_3 \cdot z_2)^2$	$\sqrt{-g} R^{\mu\nu}_{\rho\sigma} R^{\rho\sigma}_{\alpha\beta} R^{\alpha\beta}_{\mu\nu} _{(3)}$

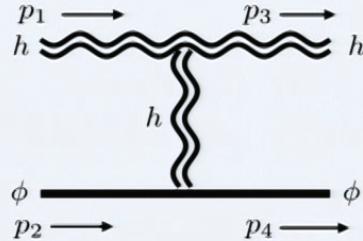
$D=4$: no \mathcal{A}_4 , 2 additional parity violating amplitudes

Cubic massive spin-2 vertices

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\mathcal{A}_2	$(p_1 \cdot z_3 \ z_1 \cdot z_2 + p_3 \cdot z_2 \ z_1 \cdot z_3 + p_2 \cdot z_1 \ z_2 \cdot z_3)^2$	$\sqrt{-g} R _{(3)}$
\mathcal{A}_3	$(p_1 \cdot z_3)^2 (z_1 \cdot z_2)^2 + (p_3 \cdot z_2)^2 (z_1 \cdot z_3)^2 + (p_2 \cdot z_1)^2 (z_2 \cdot z_3)^2$	$\delta_{\nu_1}^{[\mu_1} \delta_{\nu_2}^{\mu_2} \delta_{\nu_3}^{\mu_3} \delta_{\nu_4}^{\mu_4]} \partial_{\mu_1} \partial^{\nu_1} h_{\mu_2}^{\nu_2} h_{\mu_3}^{\nu_3} h_{\mu_4}^{\nu_4}$
\mathcal{A}_4	$p_1 \cdot z_3 \ p_2 \cdot z_1 \ p_3 \cdot z_2 \ (p_1 \cdot z_3 \ z_1 \cdot z_2 + p_3 \cdot z_2 \ z_1 \cdot z_3 + p_2 \cdot z_1 \ z_2 \cdot z_3)$	$\sqrt{-g} (R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2) _{(3)}$
\mathcal{A}_5	$(p_1 \cdot z_3)^2 (p_2 \cdot z_1)^2 (p_3 \cdot z_2)^2$	$\sqrt{-g} R^{\mu\nu}_{\rho\sigma} R^{\rho\sigma}_{\alpha\beta} R^{\alpha\beta}_{\mu\nu} _{(3)}$

$D=4$: no \mathcal{A}_4 , 2 additional parity violating amplitudes

Massive spin-2 eikonal



$$\frac{1}{s} \delta^{\lambda, \lambda'} = \begin{matrix} T & T & V & V & S \\ \begin{array}{c|ccccc} T & \frac{48(b^2 m^2 - 6) K_1(b m) \alpha_3 + K_0(b m) (b^3 (\alpha_2 - \alpha_3 + 18 \alpha_5) m^3 + 164 b \alpha_2 m)}{2 b^3 m^2 \text{Mp}^2 \pi} & 0 & \frac{K_1(b m) (b^2 m^2 (\alpha_2 - 24 \alpha_5) - 96 \alpha_5) - 48 b m K_0(b m) \alpha_3}{4 b^2 m^2 \text{Mp}^2 \pi} & 0 & \frac{K_2(b m) (\alpha_3 - 18 \alpha_5)}{2 \sqrt{3} \text{Mp}^2 \pi} \\ T & 0 & \frac{K_0(b m) (b^3 m^3 (\alpha_2 - \alpha_3 + 6 \alpha_5) - 144 b m \alpha_5) - 48 (b^2 m^2 - 6) K_1(b m) \alpha_3}{2 b^3 m^2 \text{Mp}^2 \pi} & 0 & \frac{b m K_1(b m) \alpha_3 + 48 K_2(b m) \alpha_5}{4 b m \text{Mp}^2 \pi} & 0 \\ V & \frac{K_1(b m) (b^2 m^2 (\alpha_2 - 24 \alpha_5) - 96 \alpha_5) - 48 b m K_0(b m) \alpha_3}{4 b^2 m^2 \text{Mp}^2 \pi} & 0 & \frac{K_0(b m) (\alpha_1 - 2 \alpha_2 + 24 \alpha_5) - \frac{K_1(b m) (\alpha_2 - 24 \alpha_5)}{b m}}{4 \text{Mp}^2 \pi} & 0 & \frac{K_1(b m) (2 \alpha_1 - 6 \alpha_2 + \alpha_3 + 24 \alpha_5)}{4 \sqrt{3} \text{Mp}^2 \pi} \\ V & 0 & \frac{b m K_1(b m) \alpha_3 + 48 K_2(b m) \alpha_5}{4 b m \text{Mp}^2 \pi} & 0 & \frac{K_0(b m) (\alpha_1 - 2 \alpha_2 + \alpha_3) - \frac{K_1(b m) (\alpha_2 - 24 \alpha_5)}{b m}}{4 \text{Mp}^2 \pi} & 0 \\ S & \frac{K_2(b m) (\alpha_3 - 18 \alpha_5)}{2 \sqrt{3} \text{Mp}^2 \pi} & 0 & \frac{K_1(b m) (2 \alpha_1 - 6 \alpha_2 + \alpha_3 + 24 \alpha_5)}{4 \sqrt{3} \text{Mp}^2 \pi} & 0 & \frac{K_0(b m) (2 \alpha_1 - 5 \alpha_2 + 2 (\alpha_3 + 6 \alpha_5))}{4 \text{Mp}^2 \pi} \end{array} \end{matrix}$$

$$\begin{aligned} \alpha_1 &\leftrightarrow h_{\mu\nu}^3 \\ \alpha_2 &\leftrightarrow \text{Einstein-Hilbert} \\ \alpha_3 &\leftrightarrow \text{Pseudo-linear} \\ \alpha_4 &\leftrightarrow \text{Gauss-Bonnet} \\ \alpha_5 &\leftrightarrow \text{Riemann}^3 \end{aligned}$$

Massive spin-2 eikonal

diagonalize in powers of $1/b$

$$\frac{1}{s} \delta^{\lambda, \lambda'} \rightarrow \begin{pmatrix} \frac{144 \alpha_5}{b^4 m^4 M_p^2 \pi} & 0 & 0 & 0 & 0 \\ 0 & -\frac{144 \alpha_5}{b^4 m^4 M_p^2 \pi} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \xrightarrow{\text{blue arrow}} \quad \alpha_5 = 0$$

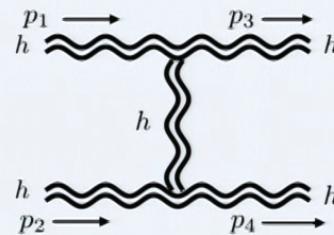
$$\frac{1}{s} \delta^{\lambda, \lambda'} \rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\alpha_3}{\sqrt{3} b^2 m^2 M_p^2 \pi} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\alpha_3}{4 b^2 m^2 M_p^2 \pi} & 0 & 0 \\ 0 & 0 & 0 & \frac{\alpha_3}{4 b^2 m^2 M_p^2 \pi} & 0 \\ \frac{\alpha_3}{\sqrt{3} b^2 m^2 M_p^2 \pi} & 0 & 0 & 0 & 0 \end{pmatrix} \quad \xrightarrow{\text{eigenvalues}} \quad \left\{ -\frac{\alpha_3}{\sqrt{3} \pi b^2 m^2 M_p^2}, \frac{\alpha_3}{\sqrt{3} \pi b^2 m^2 M_p^2}, -\frac{\alpha_3}{4 \pi b^2 m^2 M_p^2}, \frac{\alpha_3}{4 \pi b^2 m^2 M_p^2}, 0 \right\} \quad \xrightarrow{\text{blue arrow}} \quad \alpha_3 = 0$$

$$\frac{1}{s} \delta^{\lambda, \lambda'} \rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\alpha_1 - 3 \alpha_2}{2 \sqrt{3} b m M_p^2 \pi} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha_1 - 3 \alpha_2}{2 \sqrt{3} b m M_p^2 \pi} & 0 & 0 \end{pmatrix} \quad \xrightarrow{\text{eigenvalues}} \quad \left\{ -\frac{\alpha_1 - 3 \alpha_2}{2 \sqrt{3} \pi b m M_p^2}, \frac{\alpha_1 - 3 \alpha_2}{2 \sqrt{3} \pi b m M_p^2}, 0, 0, 0 \right\} \quad \xrightarrow{\text{blue arrow}} \quad \alpha_1 = 3\alpha_2$$

$$\frac{1}{s} \delta^{\lambda, \lambda'} \rightarrow \begin{pmatrix} \frac{K_0(b m) \alpha_2}{2 M_p^2 \pi} & 0 & 0 & 0 & 0 \\ 0 & \frac{K_0(b m) \alpha_2}{2 M_p^2 \pi} & 0 & 0 & 0 \\ 0 & 0 & \frac{K_0(b m) \alpha_2}{4 M_p^2 \pi} & 0 & 0 \\ 0 & 0 & 0 & \frac{K_0(b m) \alpha_2}{4 M_p^2 \pi} & 0 \\ 0 & 0 & 0 & 0 & \frac{K_0(b m) \alpha_2}{4 M_p^2 \pi} \end{pmatrix} \quad \text{non-negative}$$

Massive spin-2 eikonal constraints

Can do the same for
spin-2 - spin-2 scattering



diagonalize 25×25
phase shift matrix

$$\delta^{\lambda_1 \lambda_2, \lambda_3 \lambda_4}$$



same constraints:

$$\alpha_3 = \alpha_5 = 0, \quad \alpha_3 = 3\alpha_2$$

Allowed cubic vertex: $\mathcal{L}_3 \propto \frac{1}{2M_{\text{Pl}}} R_{\text{EH}}^{(3)} + \frac{m^2}{2M_{\text{Pl}}} h_{\mu\nu}^3$

Vertex not of this form \rightarrow new physics at m

Massive spin-2 eikonal constraints

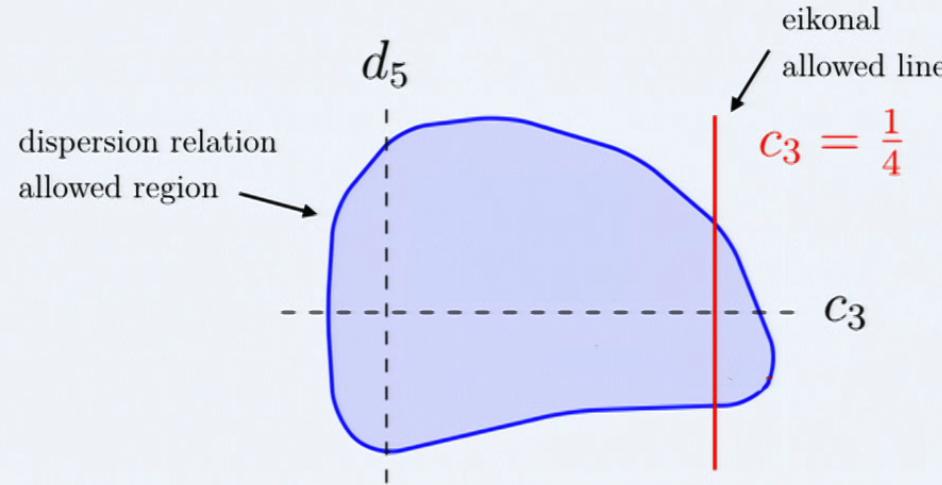
Allowed cubic vertex:

$$\mathcal{L}_3 \propto \frac{1}{2M_{\text{Pl}}} R_{\text{EH}}^{(3)} + \frac{m^2}{2M_{\text{Pl}}} h_{\mu\nu}^3 ,$$

KK reduction of Einstein-Hilbert

Conjecture: massive time delay avoided in KK theory by using this cubic vertex, not by cancellations among the KK tower

Constraints on Λ_3 theory:



Conclusions

- Eikonal scattering and dispersion relations can provide useful model independent constraints on massive theories.
- An isolated massive spin-2 is not completely ruled out.
- Going beyond leading interactions: dispersion relations beyond the forward limit, subleading corrections to the Eikonal approximation may provide more information.
- May be useful as part of a bootstrap for large N QCD.