Title: Microcanonical thermodynamics in general physical theories

Date: Apr 06, 2018 02:30 PM

URL: http://pirsa.org/18040091

Abstract: Microcanonical thermodynamics studies the operations that can be performed on systems with well-defined energy. So far, this approach has been applied to classical and quantum systems. Here we extend it to arbitrary physical theories, proposing two requirements for the development of a general microcanonical framework. We then formulate three resource theories, corresponding to three different choices of basic operations. We focus on a class of physical theories, called sharp theories with purification, where these three sets of operations exhibit remarkable properties. In these theories, a necessary condition for thermodynamic transitions is given by a suitable majorisation criterion. This becomes a sufficient condition in all three resource theories if and only if the dynamics allowed by the theory satisfy a condition that we call "unrestricted reversibility". Under this condition, we derive a duality between the resource theory of microcanonical thermodynamics and the resource theory of pure bipartite entanglement.

Microcanonical thermodynamics in general physical theories

Carlo Maria Scandolo

Department of Computer Science, University of Oxford

Workshop "Observers in quantum and foil theories"

Introduction

- Work in collaboration with Giulio Chiribella [Chiribella & CMS '17]

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This allows us to capture the informational and operational aspects of thermodynamics.

Resource theories: an agent-based approach

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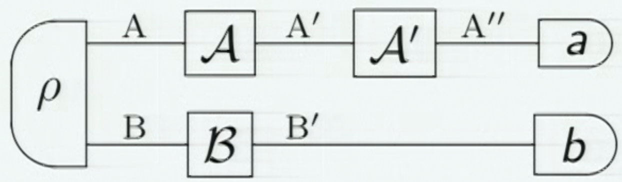
This preorder yields the allowed thermodynamic transitions.

This is very useful for describing microscopic thermodynamics.

How to address general physical theories

- We use OPTs, a variant of GPTs based on circuits [Chiribella et al., Hardy].



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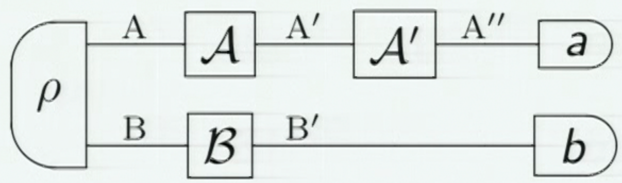

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A circuit with no external wires represents a (joint) probability.

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- We define real vector spaces spanned by states and effects. We assume they're finite-dimensional.
- We can define coarse-graining and purity through sums.

Purity

A transformation $\mathcal{T}$ is pure if $\mathcal{T} = \sum_i \mathcal{T}_i$ implies $\mathcal{T}_i = p_i \mathcal{T}$, where $\{p_i\}$ is a probability distribution.

Outline

1. Microcanonical thermodynamics
   - General setting
   - Resource theories
2. Sharp theories with purification

Section 1

Microcanonical thermodynamics

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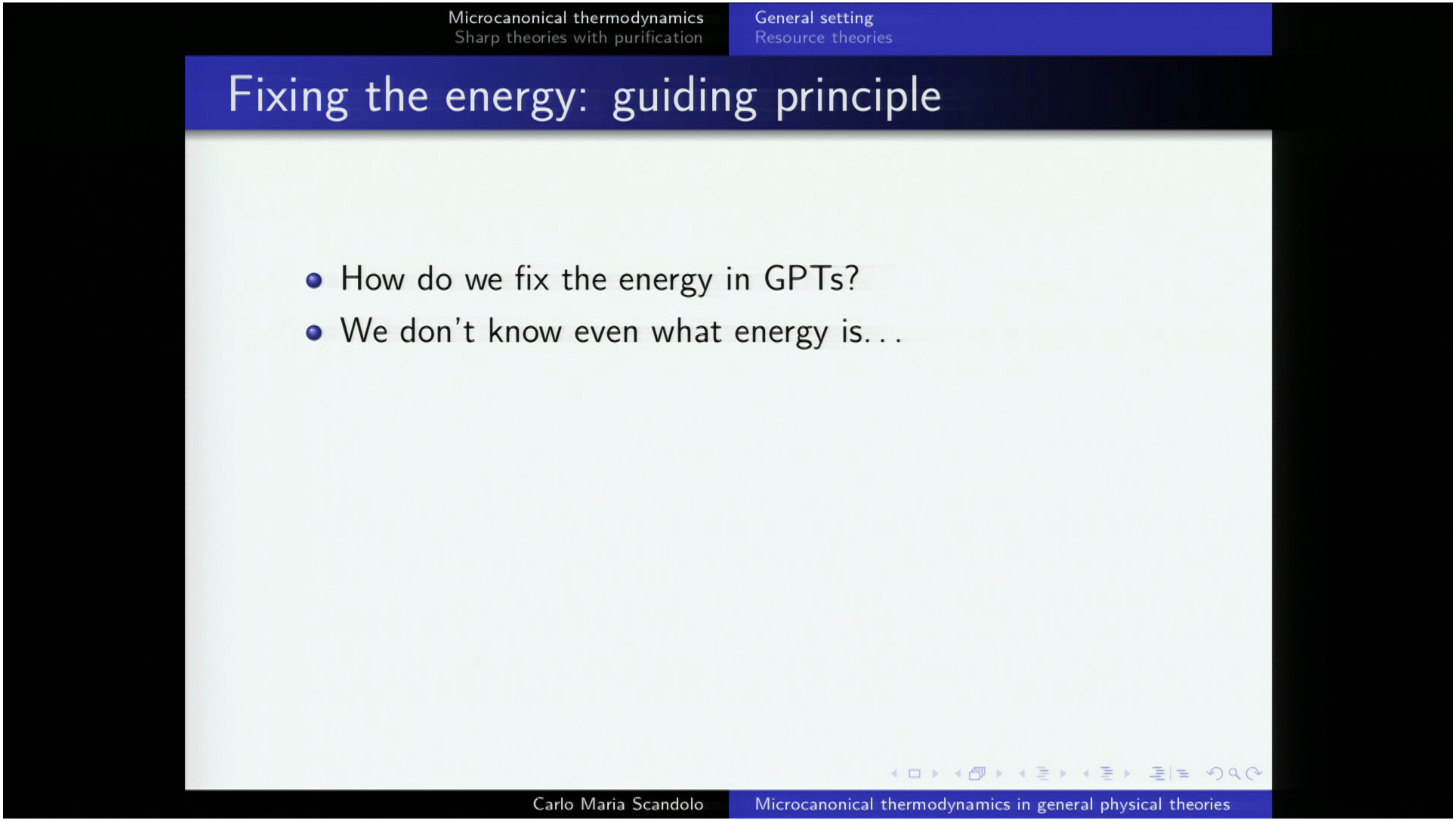
So far studied only in classical and quantum theory, we want to extend it to arbitrary physical theories.

Fixing the energy: guiding principle

- How do we fix the energy in GPTs?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In quantum theory the states in the eigenspace with energy E are those that $P_E \rho P_E = \rho$, with P_E projector on E.

It's just a linear constraint on the state space.

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Composition of constrained systems

The constraints must be applied to each system *individually* even after they're composed.

$$\begin{cases} (\mathcal{L}_{\mathrm{A}} \otimes \mathcal{I}_{\mathrm{B}})(\rho_{\mathrm{AB}}) = 0 \\ (\mathcal{I}_{\mathrm{A}} \otimes \mathcal{L}'_{\mathrm{B}})(\rho_{\mathrm{AB}}) = 0 \end{cases}$$

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The energy of the two individual systems stays fixed.

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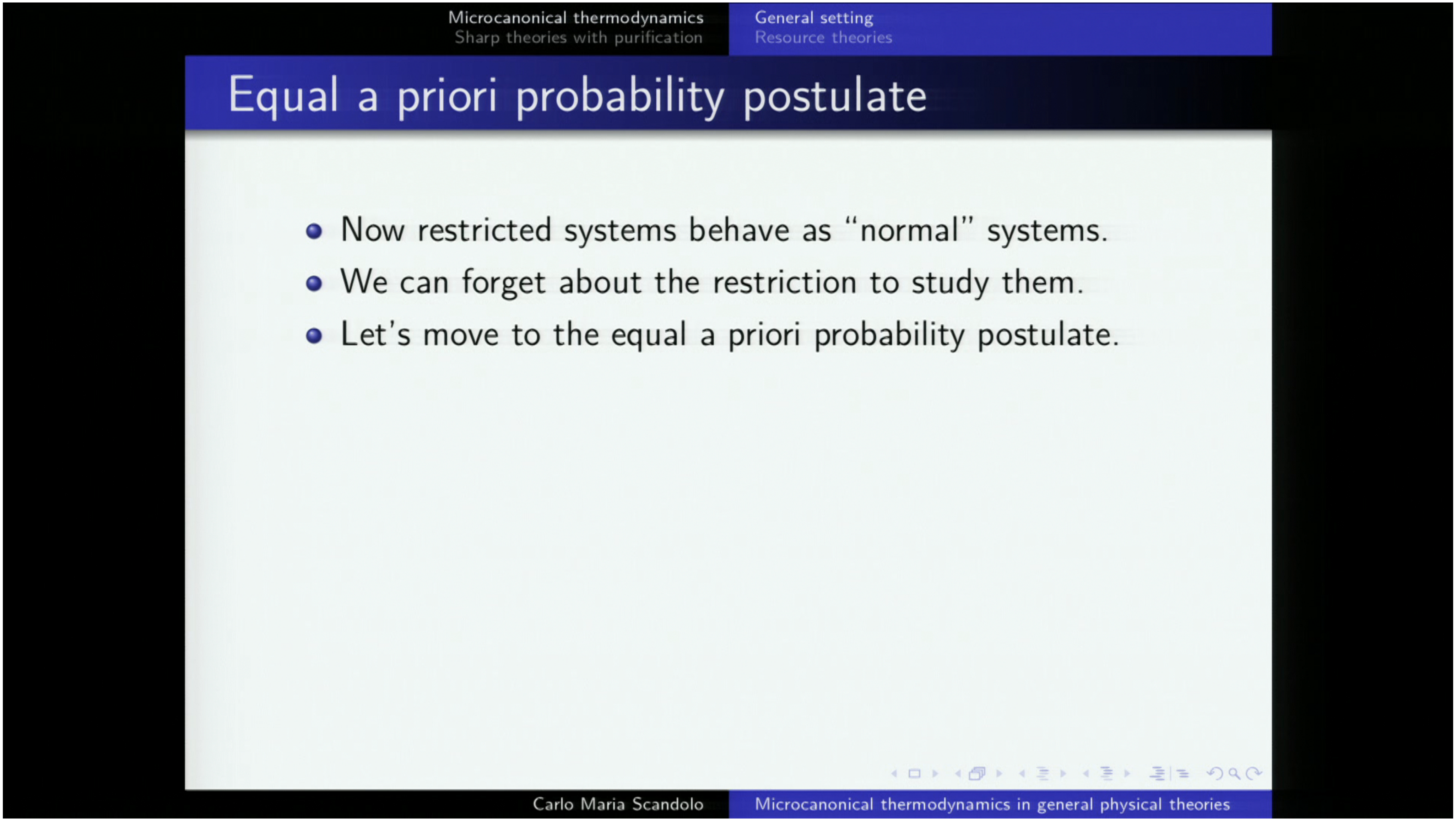

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Microcanonical state

Ideally

$$\chi_E = \int_E \psi \, \mathrm{d}\psi,$$

ψ deterministic pure.

Some technical aspects

We want χ to be invariant under reversible dynamics (equilibrium state).

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Reversible channels

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It exists in finite dimension, it's inherited from the Haar measure of the group of reversible channels [Chiribella & CMS '17].

Uniqueness of the invariant probability measure [Chiribella & CMS '17]

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The invariant probability measure is unique if and only if the action of reversible channels on deterministic pure states is transitive.

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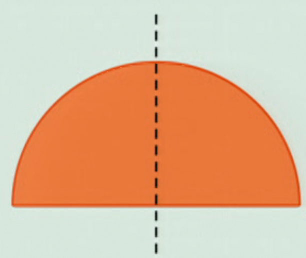
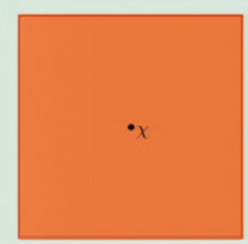
Examples





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But this is exactly the transitivity condition!

Approaching the equilibrium

It's always possible to reach the equilibrium through random fluctuations.

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- Indeed, take the transformation $\mathcal{T} = \int \mathcal{U}\, \mathrm{d}\mathcal{U}$ ($\mathrm{d}\mathcal{U}$ normalized Haar measure)

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Random reversible (RaRe) evolution.

Composing microcanonical systems

Since microcanonical states are provided "for free", it's natural to impose the following requirement:

The product of two microcanonical states is the microcanonical states of the composite system

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True in the thermodynamic limit (cf. textbook statistical mechanics).

Resource theories for microcanonical thermodynamics

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Three choices of free operations:

1. RaRe
2. noisy
3. unital

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The RaRe resource theory *doesn't* need free states...
nor can we derive them from the structure of free operations.

Noisy operations [Horodecki et al., Chiribella & CMS '17]

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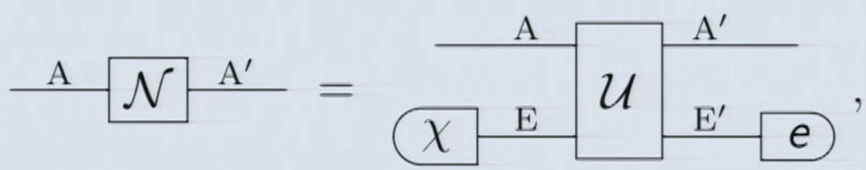

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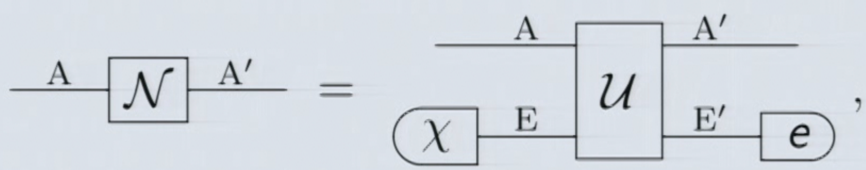

and their topological closure [Shor].

They respect the principle of cut motility (cf. Rob's talk).

Unital channels [Landau & Streater, Chiribella & CMS '17]

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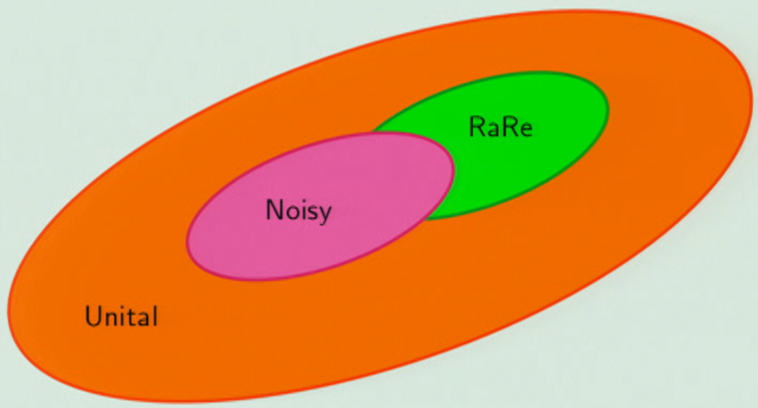
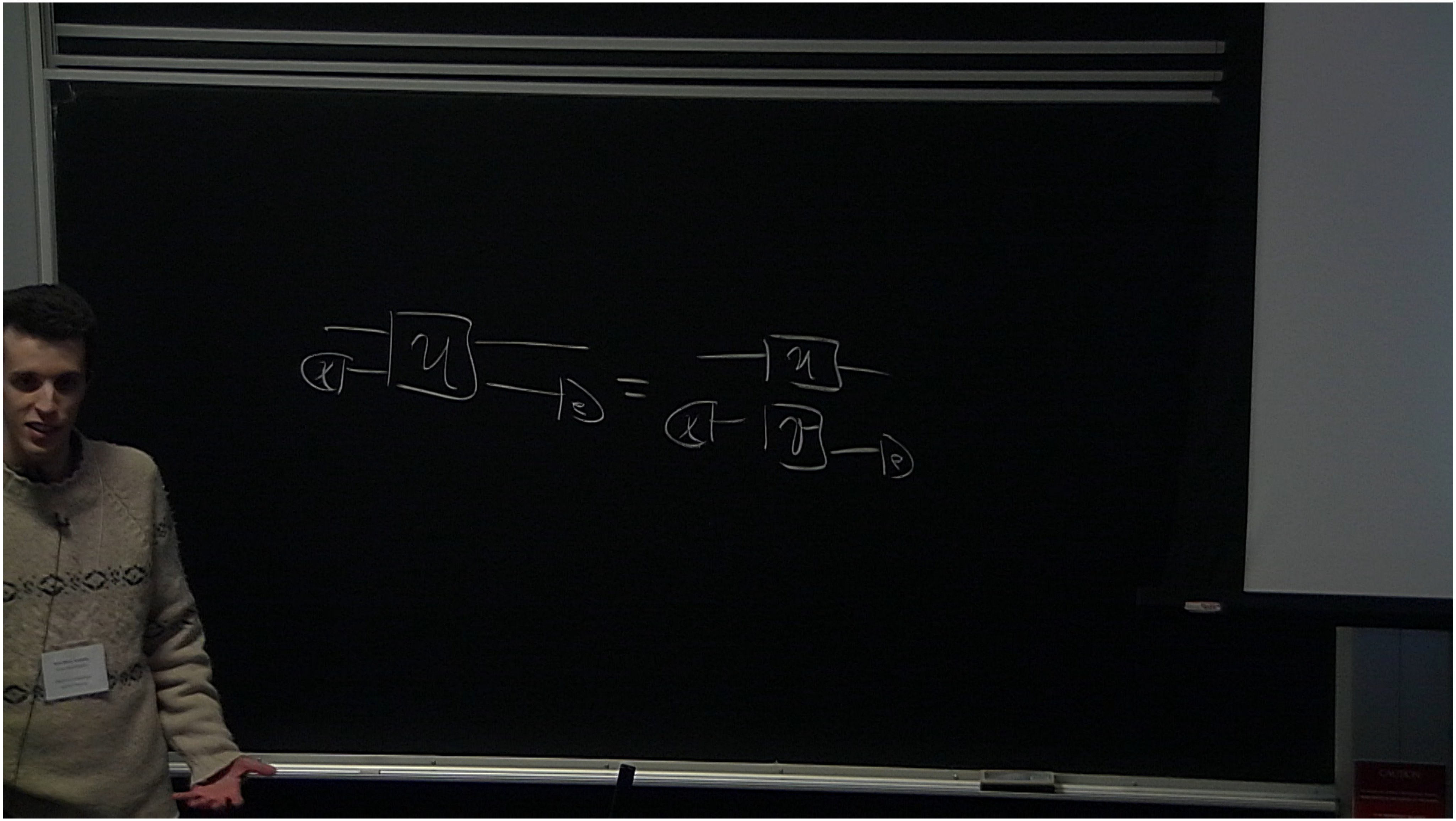
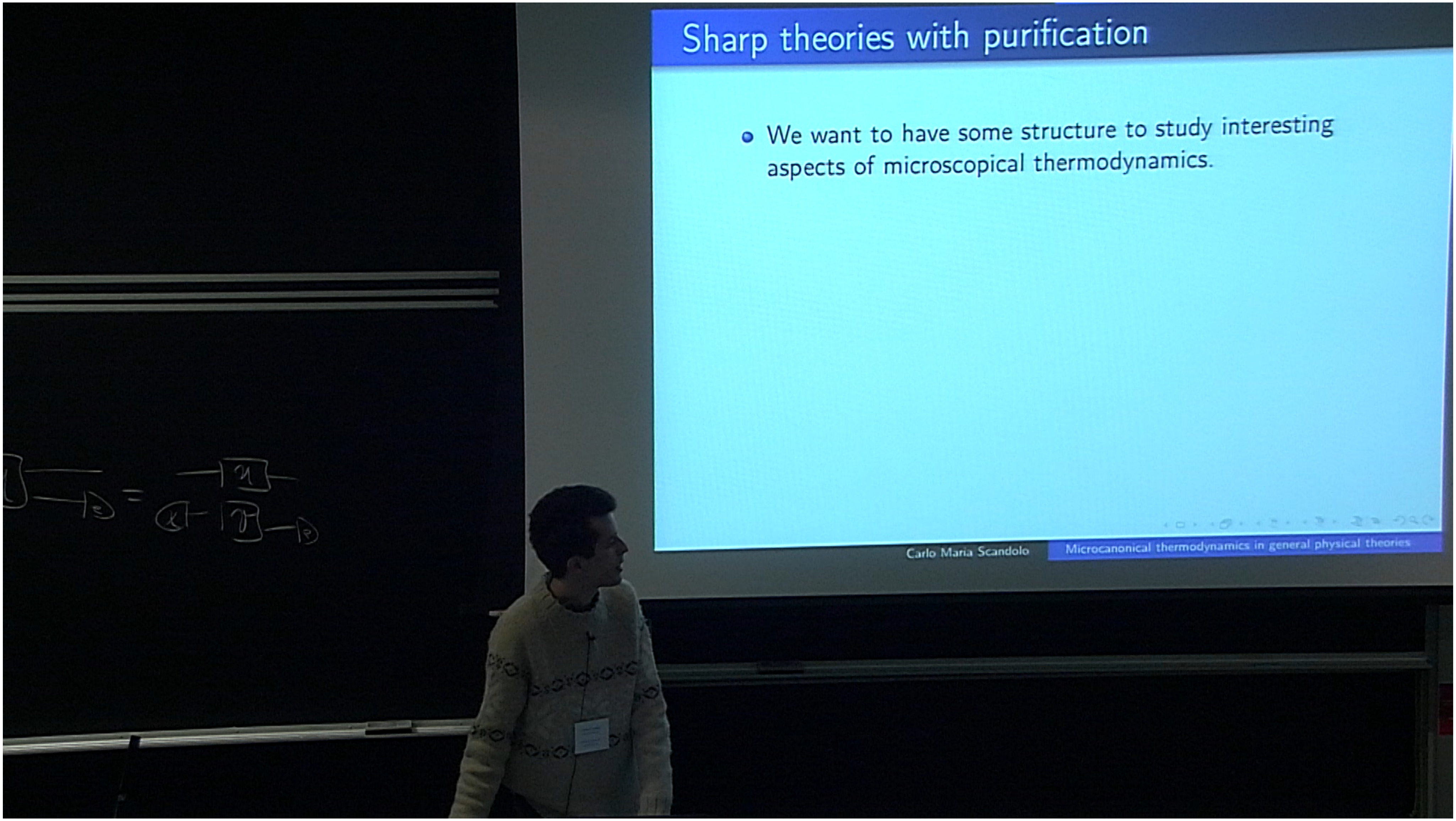
Inclusions





Section 2

Sharp theories with purification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- Focus on a class of theories where purity and reversibility play a fundamental level.
- Every description of states and dynamics can be reduced to pure states and reversible evolutions.

A strong version of the cut motility (cf. Rob's talk)!

They include: real and complex quantum theory, Spekkens' toy model, and more exotic theories [Chiribella & CMS '16].

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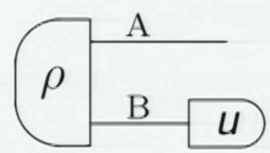

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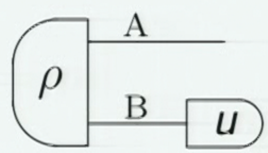

Thermodynamic meaning: discarding a system.

Purity Preservation

Purity Preservation [Chiribella & Scandolo '15a]

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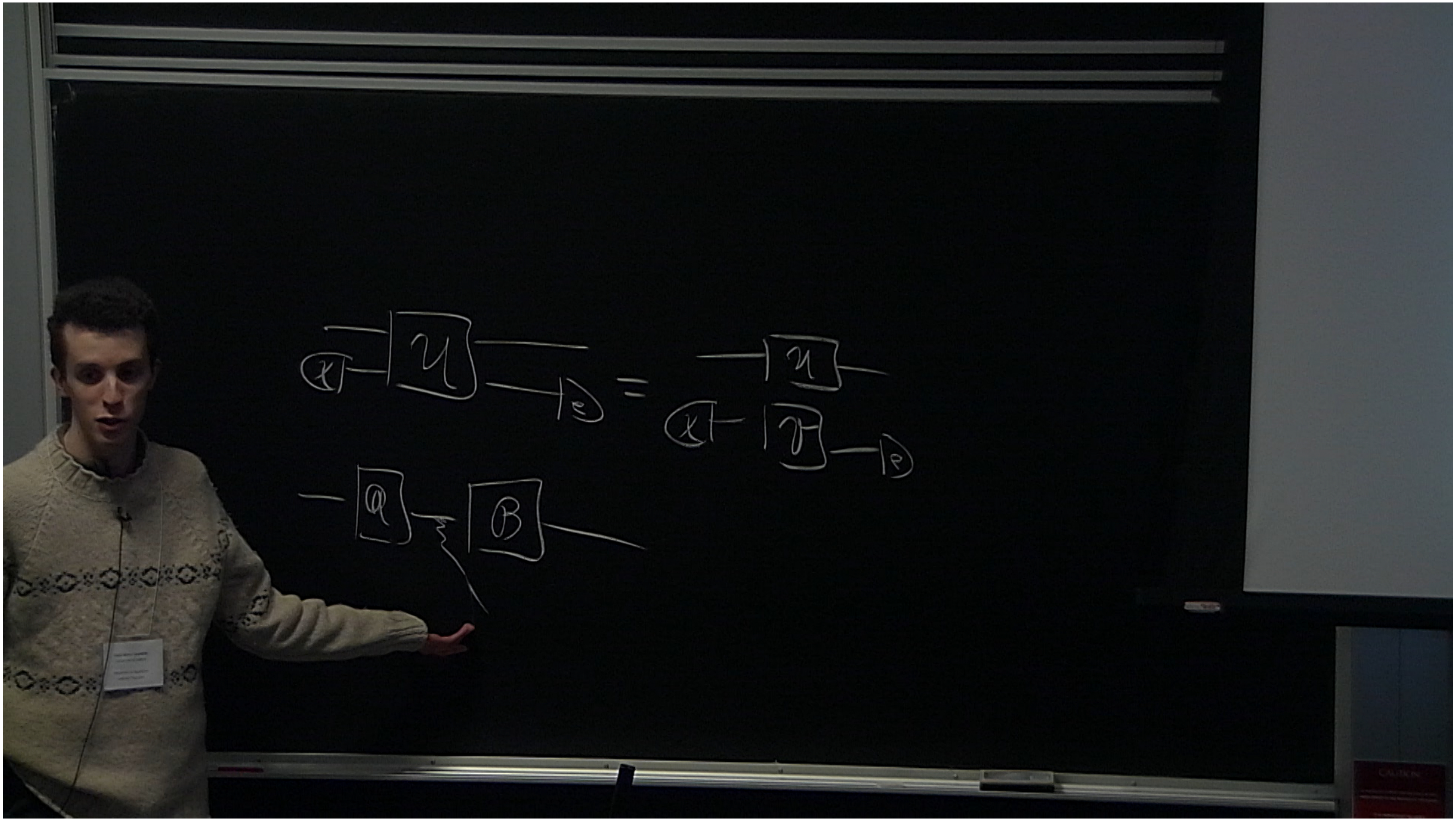
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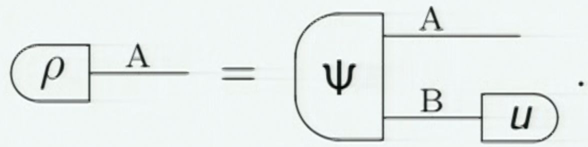
For every system, there exists at least one pure effect *a* occurring with probability 1 on some state ρ.

- We can think of *a* as part of a yes/no test to check an *elementary property* of the system.
- Pure Sharpness guarantees that every system has at least one elementary property.

Purification [Chiribella et al.]

1. Every state ρ_A can be purified: there exists a pure state Ψ_{AB} such that



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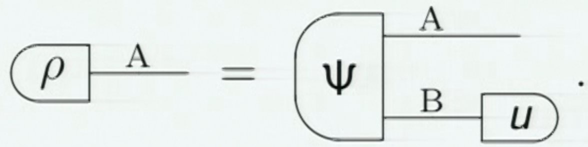
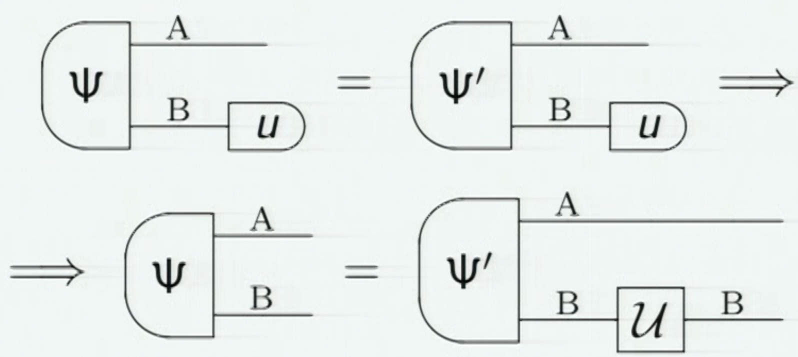


2. Purifications of the same state differ by a reversible transformation $\mathcal{U}$ on the purifying system:



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- Classical theory *doesn't* satisfy Purification.
- However, it admits an extension to a sharp theory with purification.
- Some of the systems look like classical single systems.
- But now composition brings in entanglement.

Coherent composition of two bits

$$\rho_{\mathrm{AB}} = p\rho_{\mathrm{even}} \oplus (1-p)\,\rho_{\mathrm{odd}},$$

where

$$\mathrm{even} = \mathrm{Span}\left\{|0\rangle\,|0\rangle\,,|1\rangle\,|1\rangle\right\}$$

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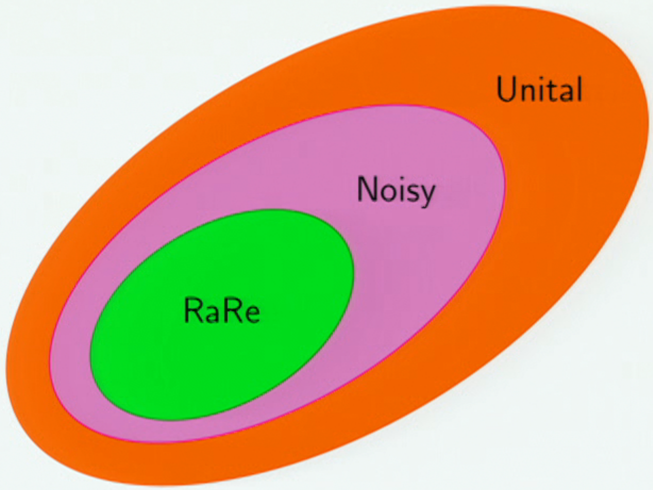
χ is diagonalized as $\frac{1}{d}\sum_{i=1}^{d}\alpha_i$, $\{\alpha_i\}_{i=1}^{d}$ pure maximal set.

New inclusions [Chiribella & CMS '17]



This means that RaRe convertibility is the strongest, i.e. the hardest to satisfy.

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Proposition ([Chiribella & CMS '17])

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Majorisation is a necessary condition for convertibility under *all* resource theories.

Doubled quantum theory [Chiribella & CMS '17]

- (Non-trivial) systems are pairs of isomorphic Hilbert spaces $\mathrm{A} = (\mathcal{H}_0, \mathcal{H}_1)$

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- (Non-trivial) systems are pairs of isomorphic Hilbert spaces $\mathrm{A} = (\mathcal{H}_0, \mathcal{H}_1)$
- States of the form $\rho = p\rho_0 \oplus (1-p)\,\rho_1$
- Composition:

$$\mathcal{H}_0^{\mathrm{AB}} = \left(\mathcal{H}_0^{\mathrm{A}} \otimes \mathcal{H}_0^{\mathrm{B}}\right) \oplus \left(\mathcal{H}_1^{\mathrm{A}} \otimes \mathcal{H}_1^{\mathrm{B}}\right)$$

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Overall "parity" superselection rule.

There are states for which $\mathbf{p} \succeq \mathbf{q}$ but there's no RaRe channel connecting them.

Three equivalent axioms in sharp theories with purification

Permutability [Hardy]

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Reversible Controllability [Lee & Selby]

Every control-reversible transformation can be implemented reversibly.

Sufficiency of majorization [Chiribella & CMS '17]

We call these axioms "*unrestricted reversibility*".

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4. $\mathbf{p} \succeq \mathbf{q}$

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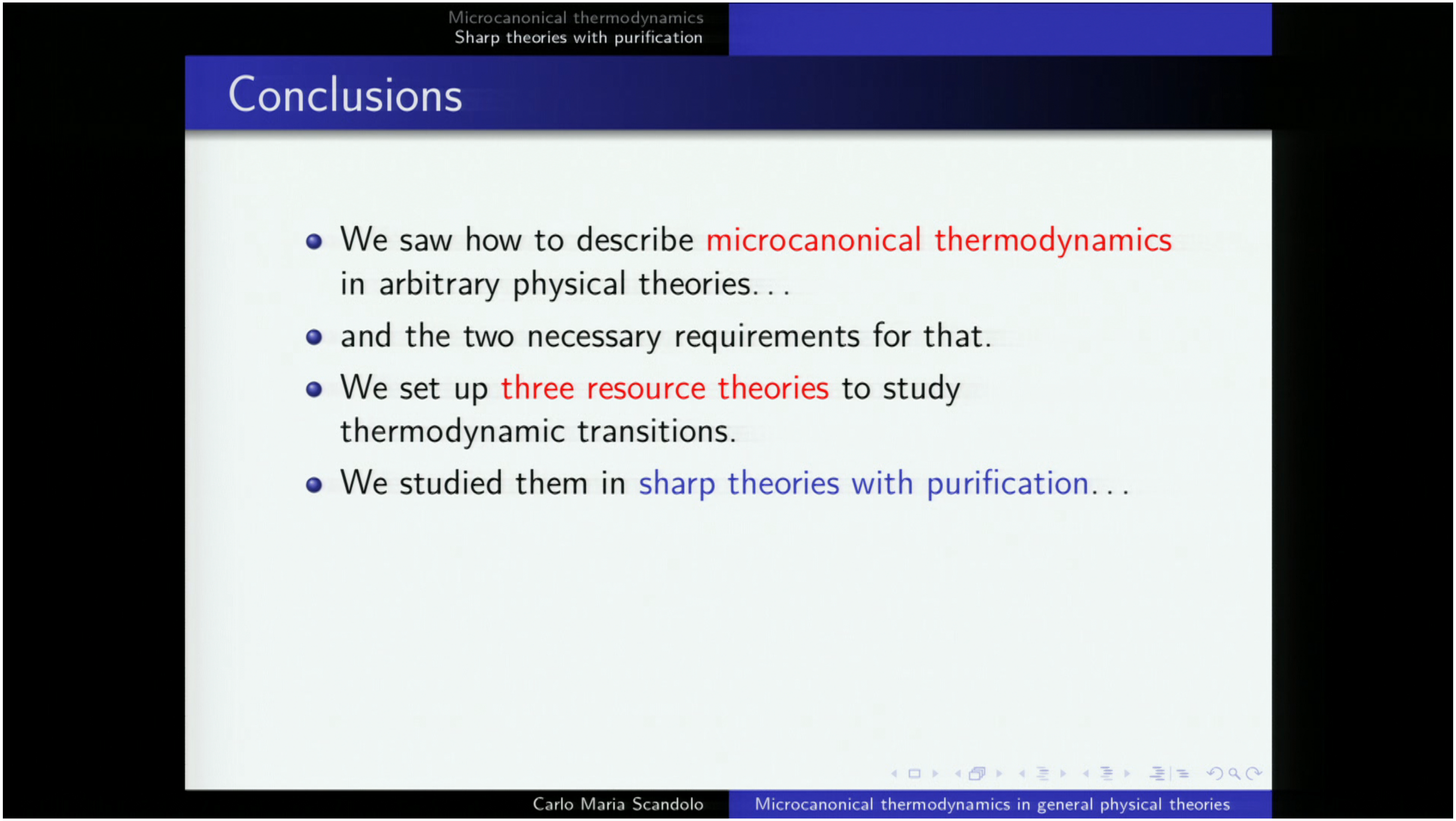
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Microcanonical thermodynamics in general physical theories

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- and discovered some remarkable thermodynamic properties these theories show.
- Finally we saw how a thermodynamic requirement influences the dynamics of the underlying theory.

References

- H Barnum, M P Müller, C Ududec, *New J. Phys.* **16** (12), 123029 (2014).
- G Chiribella, G M D'Ariano, P Perinotti, *Phys. Rev. A* **81** (6), 062348 (2010).
- G Chiribella, CMS, *EPJ Web Conf.* **95**, 03003 (2015).
- G Chiribella, CMS, *New J. Phys.* **17** (10), 103027 (2015).
- G Chiribella, CMS, *EPTCS* **195**, 96–115 (2015).
- G Chiribella, CMS, arXiv:1608.04459 [quant-ph] (2016).
- G Chiribella, CMS, *New J. Phys.* **19** (12), 123043 (2017).
- L Hardy, arXiv:1104.2066 [quant-ph] (2011).
- M Horodecki et al., *Phys. Rev. Lett.* **90** (10), 100402 (2003).
- L J Landau, R F Streater, *Linear Algebra Appl.* **193**, 107–127 (1993).
- C M Lee, J H Selby, *New J. Phys.* **18** (3), 033023 (2016).
- P Shor, *Structure of Unital Maps and the Asymptotic Quantum Birkhoff Conjecture*, presentation (2010).
- A Uhlmann, *Wiss. Z. Karl-Marx Univ. Leipzig* **20**, 633–637 (1971).