

Title: TBA

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Abstract:

Inadequacy of <sup>epistemic</sup> modal logic in quantum settings  
at Xiv: 1804.01106 Nuriya Norgaliyeva, LdR

Kripke structures

Syntax

Propositions

Possible worlds & interpretations

Agents' knowledge

CAUTION  
DO NOT TOUCH THE BOARD SURFACE  
OR THE BOARD FRAME

Kripke structures

Syntax

$\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$

Propositions

$\emptyset$

Possible worlds & interpretations

Agents' knowledge

Inadequacy of <sup>epistemic!</sup> modal logic in quantum settings  
arXiv: 1804.01106 Nuriya Nurgalieva, LdR

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Alice  $\phi_A$ : "Alice wants a beer"  
Bob  $\phi_B$ : Bob "  
Chris  $\phi_C$ : Chris "

# Inadequacy of <sup>epistemic</sup> modal logic in quantum settings

at XIV: 1804.01106

Nuriya Nurgalieva, LdR

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$$\phi = \phi_A \wedge \phi_B \wedge \phi_C$$

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$\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$

Propositions

$\phi, K_B \phi_A, K_B \neg K_A \phi$

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$\Sigma \ni s \quad \pi: \Sigma \times \Phi \rightarrow \{\text{true, false}\}$

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$K_i \text{ in } \Sigma \quad (s, t) \in R_i$

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$\Sigma \ni s \quad \pi: \Sigma \times \Phi \rightarrow \{\text{true, false}\}$

$\pi(s, \phi): s \models \phi$

Agent

knowledge

$\Sigma \ni (s, t) \in R_i$

# Kripke structures

## Syntax

$\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$

## Propositions

$\phi \quad K_B \phi_A, K_B \neg K_A \phi$

Possible worlds & interpretations

$\Sigma \ni s \quad \pi: \Sigma \times \Phi \rightarrow \{\text{true}, \text{false}\}$

$\pi(s, \phi): s \models \phi \quad \text{vs.} \quad s \models \neg \phi$

Agents' knowledge

$K_i \text{ in } \Sigma \quad (s, t) \in R_i$

Axioms of knowledge

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Propositions

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Possible worlds & interpretations

$\Sigma \ni s \quad \pi: \Sigma \times \Phi \rightarrow \{true, false\}$

$\pi(s, \phi): s \models \phi \quad K_s: \models \phi$

Agents' knowledge

$K_i \text{ in } \Sigma \quad (\langle s, t \rangle) \in K_i$

actual world  $\leftarrow$

Knowledge generalization rule

3 Truth axiom

# Kripke structures

Syntax

$\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$

Propositions

$\phi, K_B \phi_A, K_B \neg K_A \phi$

Possible worlds & interpretations

$\Sigma \ni s, \pi: \Sigma \times \mathcal{F} \rightarrow \{\text{true, false}\}$

$\pi(s, \phi)$

Agents' knowledge

$K_i$  in  $\Sigma$  (s, t)  
actual world  $\leftarrow$

$s \models K_i \phi : \forall t \in (s, t) \in K_i$   
 $\models \phi$

$\pi_i \text{ in } \sum (\sum_t) \in K_i$   
actual world  $\leftarrow$   $\rightarrow$  possible from i's persp.

## Axioms of knowledge

1. Distribution axiom

$$S \models (K_i \phi \wedge K_i (\phi \Rightarrow \psi)) \Rightarrow S \models K_i \psi$$

2. Knowledge generalization rule

3. Truth axiom

$\pi_i \text{ in } \sum (\sum_t) \in K_i$   
actual world  $\leftarrow$   $\rightarrow$  possible from i's persp.

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$$\forall s \ S \models \phi \Rightarrow \models K_i \phi \quad \forall i$$

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3. Truth axiom  $\Leftrightarrow \models K_j K_i \phi, \text{ etc}$

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$$(\forall s \ S \models \phi) \Rightarrow \forall s \ S \models K_i \phi \quad \forall i$$

3. Truth axiom  $\Rightarrow K_j K_i \phi, \text{ etc}$

$$S \models K_i \phi \Rightarrow S \models \phi$$

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4. Positive & negative introspection

$$\bullet s \models K_i \phi \Rightarrow s \models K_i K_i \phi$$

$$\bullet s \models \neg K_i \phi \Rightarrow s \models K_i \neg K_i \phi$$

[Context]

$$\phi = \phi_A \wedge \phi_B \wedge \phi_C$$

$t=0$

$$\models K_j (K_i \neg \phi_i \vee K_i \phi_i), \forall i, j$$

$$\models K_j (K_i \neg \phi_i \Rightarrow K_i \neg \phi)$$

$$\phi = \phi_A \wedge \phi_B \wedge \phi_C$$

$$\models \bigwedge_i (K_i \neg \phi_i \vee K_i \phi_i), \forall i, j$$

$$\models K_j (K_i \neg \phi_i \Rightarrow K_i \neg \phi)$$

$$\Leftrightarrow K_j (\neg K_i \phi \Rightarrow K_i \phi_i)$$

t=0

$$\models K_j (K_i \neg \phi_i \vee K_i \phi_i), \forall i, j$$

$$\models K_j (K_i \neg \phi_i \Rightarrow K_i \neg \phi)$$

$$\Leftrightarrow \models K_j (\neg K_i \phi \Rightarrow K_i \phi_i)$$

t=1

t=0

$$\models K_j (K_i \neg \phi_i \vee K_i \phi_i), \forall i, j$$

$$\models K_j (K_i \neg \phi_i \Rightarrow K_i \neg \phi)$$

$$\Rightarrow K_j (\neg K_i \phi \Rightarrow K_i \phi_i) \quad (\Delta)$$

t=1 s=(1,1,1)

$$s \models K_B \neg K_A \phi \wedge (\Delta) \stackrel{DA}{\Rightarrow} K_B K_A \phi_A^{\text{Truth}} \Rightarrow K_B \phi_A, K_C \phi_A$$

t=2

$$s \models K_C \neg K_B \phi \wedge K_C K_B \phi_A \wedge (\Delta) \Rightarrow K_C K_B \phi_B \Rightarrow K_C \phi_B$$

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Propositions

$\phi, K_B \phi_A, K_B \neg K_A \phi$

Possible worlds & interpretations

$\Sigma \ni s, \pi: \Sigma \times \Phi \rightarrow \{\text{true, false}\}$

$\pi(s, \phi): s \models \phi \text{ vs. } \models \phi$

Agents' knowledge

$K_i \text{ in } \Sigma, (\langle s, t \rangle) \in K_i$   
actual world  $\leftarrow$   $\rightarrow$  possible from i's persp.

$s \models K, \phi : K \vdash \phi$   
 $\models \phi$

4. Positive & negative introspection

$$\bullet s \models K_i \phi \Rightarrow s \models K_i K_i \phi$$

$$\bullet s \models \neg K_i \phi \Rightarrow s \models K_i \neg K_i \phi$$

[Context]

4. Positive & negative introspection

$$\bullet s \models K_i \phi \Rightarrow s \models K_i K_i \phi$$

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[Context]

$\Pi$ : Contexts  $\times \Sigma \times \phi$

$\Pi$  ("locally",  $s$ , " $a=0$ ") = true

$\Pi$  ("globally",  $s$ , " $100 \rangle + 111 \rangle$ ") = true

4. Positive & negative introspection

$$\bullet s \models K_i \phi \Rightarrow s \models K_i K_i \phi$$

$$\bullet s \models \neg K_i \phi \Rightarrow s \models K_i \neg K_i \phi$$

[Context]  $\begin{matrix} \{local, global\} \\ \uparrow \\ \pi \end{matrix}$

$\pi: \text{Contexts} \times \Sigma \times \phi$

$\pi(\text{"locally"}, s, \text{"a=0"}) = \text{true}$

$\pi(\text{"globally"}, s, \text{"100 > 111"}) = \text{true}$

4. Positive & negative introspection

$$\bullet s \models K_i \phi \Rightarrow s \models K_i K_i \phi$$

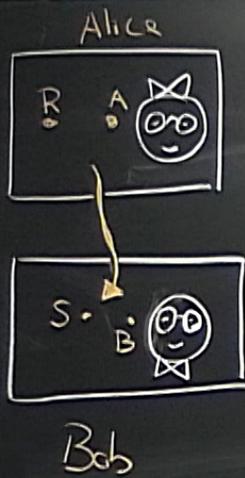
$$\bullet s \models \neg K_i \phi \Rightarrow s \models K_i \neg K_i \phi$$

[Context]  $\begin{matrix} \{local, global\} \\ \uparrow \\ \pi \end{matrix}$

$$\pi: \text{Contexts} \times \Sigma \times \phi \rightarrow \{\top, \text{F}, \text{und}\}$$

$$\pi(\text{"locally"}, s, \text{"a=0"}) = \text{true}$$

$$\pi(\text{"globally"}, s, \text{"100 > 1111"}) = \text{true}$$



Ursula  
U

Wigner  
W

Protocol

$$t=0 \quad |\psi\rangle_R = \frac{1}{\sqrt{3}} |0\rangle_R + \frac{\sqrt{2}}{\sqrt{3}} |1\rangle_R$$

$$|0\rangle_S, |0\rangle_A, |0\rangle_B$$

t=1

- Alice measures R in  $\{|0\rangle_R, |1\rangle_R\}$  basis
- Alice records outcome in A

- Alice performs CH on S

$$|0\rangle_R |0\rangle_S \rightarrow |0\rangle_A |0\rangle_S$$

$$|1\rangle_R |0\rangle_S \rightarrow |1\rangle_A |1\rangle_S$$

Unitary modelling

t=1

$$\left\{ \frac{1}{\sqrt{3}} |0\rangle_R |0\rangle_A + \frac{\sqrt{2}}{\sqrt{3}} |1\rangle_R |1\rangle_A \right\}$$

$$\left\{ \frac{1}{\sqrt{3}} \begin{pmatrix} |0\rangle_R |0\rangle_A |0\rangle_S \\ + |1\rangle_R |1\rangle_A |0\rangle_S \\ + |1\rangle_R |1\rangle_A |1\rangle_S \end{pmatrix} \right\}$$

t=2

R measures S in  $\{|0\rangle_S, |1\rangle_S\}$

$$S \models K_C \rightarrow K_B \phi \wedge K_C K_B \phi_A \wedge (1) \Rightarrow K_C K_B \phi_B \Rightarrow K_C \psi_B$$

Possible worlds / Common knowledge

- of worlds that satisfy :
  - Q Born rule
  - U unitarity
  - setting of exp.

◦ "S" : agent assigned w/ single outcome

CAUTION

DO NOT TOUCH THE BOARD  
OR THE EQUIPMENT IN THE ROOM

CAUTION

$$S \models K_C \rightarrow K_B \phi \wedge K_C K_B \phi_A \wedge (1) \Rightarrow K_C K_B \phi_B \Rightarrow K_C \psi_B$$

Possible worlds / Common knowledge

of worlds that satisfy :

- Q Born rule
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• "S" : agent assigned w/ single outcome

$$K_A (a \Rightarrow w = \text{fail})$$

$$S \models K_C \rightarrow K_B \phi \wedge K_C K_B \phi_A \wedge (1) \Rightarrow K_C K_B \phi_B \Rightarrow K_C \psi_B$$

Possible worlds / Common knowledge

of worlds that satisfy

- Q Born rule
- U unitarity
- setting of exp.

• "S" : agent assigned w/ single outcome

$$\forall s, t \models K_i (a=1 \Rightarrow w = \text{fail}), K_i$$

$$K_s, s \models K_i (b=1 \Rightarrow a=1)$$

$$K_t, s \models K_i (u = \text{ok} \Rightarrow b=1)$$

CAUTION

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$$S \models K_C \rightarrow K_B \phi \wedge K_C K_B \phi_A \wedge (1) \Rightarrow K_C K_B \phi_B \Rightarrow K_C \psi_B$$

Possible worlds / Common knowledge

- of worlds that satisfy:
  - Q Born rule
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  - setting of exp.

• "S": agent assigned w/ single outcome

$$\forall s, t \models K_i (a=1 \Rightarrow w=\text{fail}), K_i \mid \forall s, K_i (U=ok \Rightarrow w=\text{fail}),$$

$$K_s, s \models K_i (b=1 \Rightarrow a=1) \quad \exists s, K_i (W=ok \wedge U=ok) \wedge K_w (U=ok \Rightarrow w=\text{fail})$$

$$K_t, s \models K_i (U=ok \Rightarrow b=1) \quad \Rightarrow K_w (W=ok \wedge w=\text{fail})$$