Title: Wavefunction branches as a foundation for constructing foil theories

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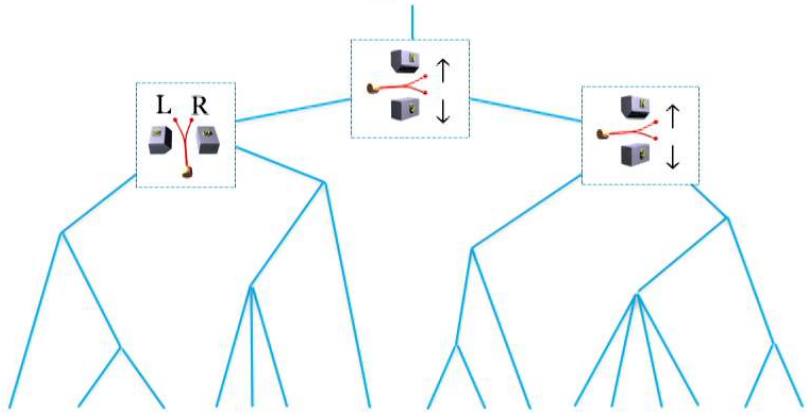
Abstract:

Wavefunction branches
as a foundation for
constructing foil theories

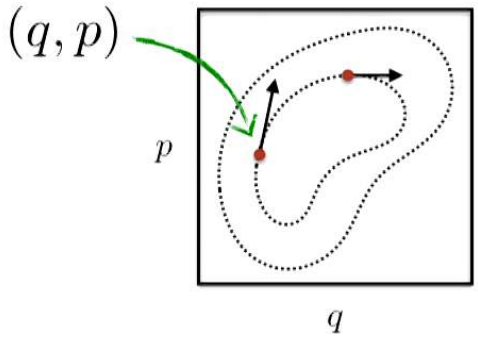


C. Jess Riedel

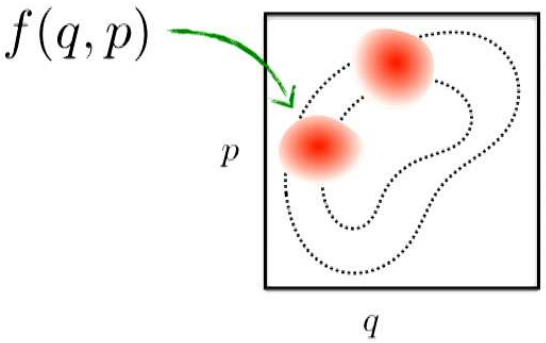
arXiv:1608.05377 [PRL 118, 120402 (2017)]
and drawing heavily from
A. Kent, arXiv:1204.5961 [PRA 87, 022105 (2013)]

Classical probability

- Classically we can discuss dynamics without information/probability



$$(\dot{q},\dot{p}) = \{H,(q,p)\} \equiv \left(\frac{\partial H}{\partial p}, -\frac{\partial H}{\partial q}\right)$$



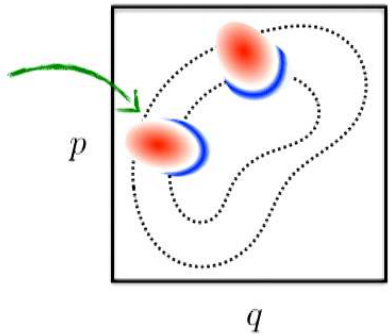
$$\partial_t f = \{H,f\} \equiv \frac{\partial f}{\partial q}\frac{\partial H}{\partial p} - \frac{\partial f}{\partial q}\frac{\partial H}{\partial p}$$

- Classical probability (e.g., Kolmogorov axioms) is *also* an island in theory space…

Quantum information inseparable from dynamics

- Unlike the classical case, there is no quantum *dynamics* independent of quantum *information*

$$\int \mathrm{d}\Delta q\, e^{ip\Delta q} \langle q + \frac{\Delta q}{2} | \rho | q - \frac{\Delta q}{2} \rangle = W(q,p)$$



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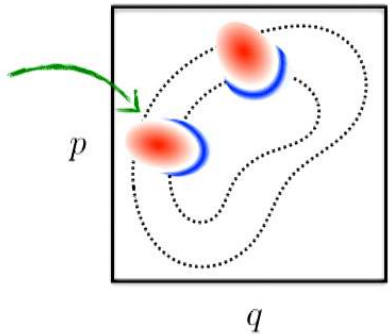


$$\partial_t W = \overbrace{\{\{H, W\}\}}^{\text{Moyal bracket}} \equiv 2H \sin\left(\frac{\overleftarrow{\partial}_q \overrightarrow{\partial}_p - \overleftarrow{\partial}_p \overrightarrow{\partial}_q}{2}\right) W$$

$$\Leftrightarrow \partial_t \rho = i[H, \rho]$$

Quantum state :: Classical PDF

- The quantum state is analogous to a classical probability distribution, not a classical point in phase space
  - Number of parameters is exponential in number of degrees of freedom
  - Operationally used for computing expectation value of any variable
- *Mathematically*, at least, we are Ψ-epistemic

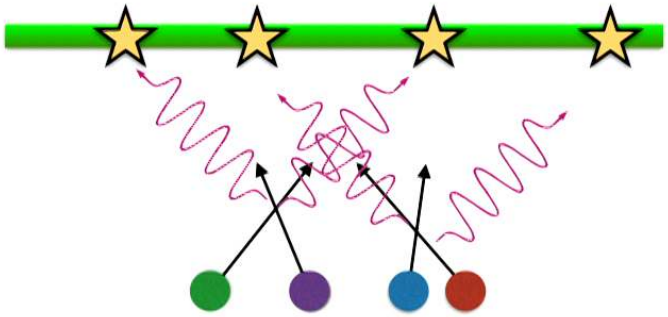
Classical probabilities from quantum mechanics

- To obtain predictions from unitarily evolving quantum state, we must declare the classical outcomes Λ
  - This choice of "beables" forms our sample space $\{\Lambda\}$ — the set of things we can reason about logically/probabilistically
  - Allows for a classical probability distribution $\{p(\Lambda)\}$
  - Accessible observables are classical functions on that space
  - Sufficient conditions for self-consistent classical probabilities are known:"consistency" in the consistent histories formalism
- Examples: Bohm, Kent, Copenhagen / consistent histories, branches

My jam

Kent late-time photodetection PDF

- Beables taken to be results of hypothetical measurements of electromagnetic field at future infinity



arXiv:1311.0249, 1411.2957, 1608.04805

- Strength: Manifestly Lorentz covariant.
- Drawback: Epiphenomenal. Not rigorously defined. Ad-hoc.

Copenhagen / Consistent-histories PDF

- Beables taken to be outcomes of *measurements* made at discrete times

$$\Lambda(t_n) = (\lambda_1, \ldots, \lambda_n)$$

- *Iterated* Copenhagen:

$$p_{\lambda_1}(t_1) = \mathrm{Tr}\left[\rho\, P^{(1)}_{\lambda_1}(t_1)\right] \qquad \rho \to \rho^{(1)} = \frac{P^{(1)}\rho P^{(1)}}{\mathrm{Tr}\left[P^{(1)}\rho P^{(1)}\right]}$$

$$p_{\lambda_2}(t_2) = \mathrm{Tr}\left[\rho^{(1)}\, P^{(2)}_{\lambda_2}(t_2)\right] \qquad \rho^{(1)} \to \rho^{(2)} = \frac{P^{(2)}\rho^{(1)} P^{(2)}}{\mathrm{Tr}\left[P^{(2)}\rho^{(1)} P^{(2)}\right]}$$

$$\vdots \qquad\qquad \vdots$$

Branches PDF

- Beables taken to be outcomes of *amplification events* at discrete times

To be discussed shortly

$$\Lambda(t_n) = (\lambda_1, \ldots, \lambda_n)$$

$$p_\Lambda(t_n) = \mathrm{Tr}\left[\rho\, P^{(1)}_{\lambda_1}(t_1) \cdots P^{(n)}_{\lambda_n}(t_n)\right]$$

Identical *form* as Copenhagen

- Strength: Potentially recovers anthropocentric measurements as special case of amplification events

- Drawback: Not yet defined / shown to exist

Why care about beables?

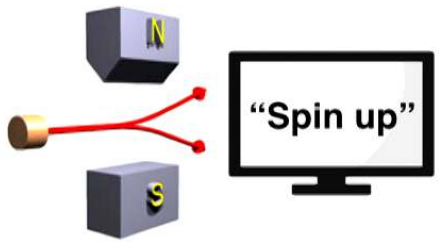
- Well, beables help us expand the island
  - Beables are ugly, but will give us a foundation on which to build
  - Parameterized post-Newtonian (PPN) formalism for checking general relativity has 10 free parameters, but only small submanifold is plausible — still very useful
- However, given an evolving quantum state, the possible beable sample spaces appear to be unlimited
  - No obvious preferred beables *even if* foil theories constructed from a choice of beables are experimentally distinguishable from vanilla quantum mechanics

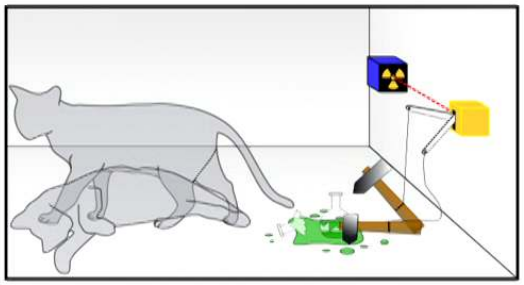
Why **wavefunction branches** as the beables?

- Tentative, vague claim: there exist a minimal choice of beables
  - Essentially: simplest set of beables that contains all possible Copenhagen measurement outcomes
- Key idea: measurements are about amplification and amplification is a naturally occurring process

Macroscopic superpositions are everywhere

- Macroscopic superpositions are not just rare situations created by carefully designed equipment

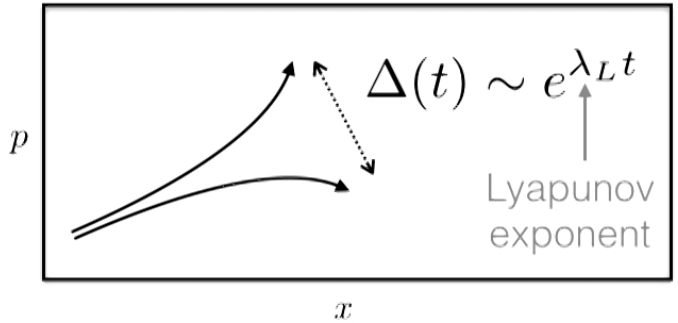


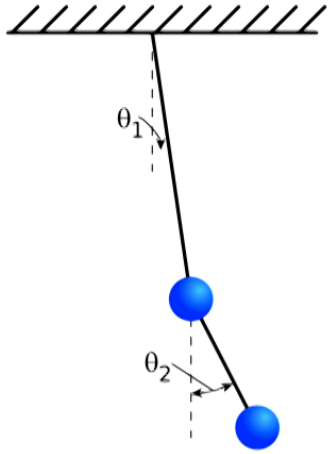


- Macroscopic superpositions are created generically by macroscopic chaotic systems
  - Which is pretty much everything

Macroscopic superpositions are everywhere

- Even minimal uncertainty wavepackets are stretched over the entire available phase space after several multiples of the Lyapunov time





- Not eliminated by classical limit

$$T^{\text{macro}}_{\text{superpos}} \sim \lambda_L^{-1} \ln \frac{S_0}{\hbar}$$

Scale of system's action

Macroscopic superpositions decohere

- Macroscopic superpositions are continuously decohered by their environment as they are created

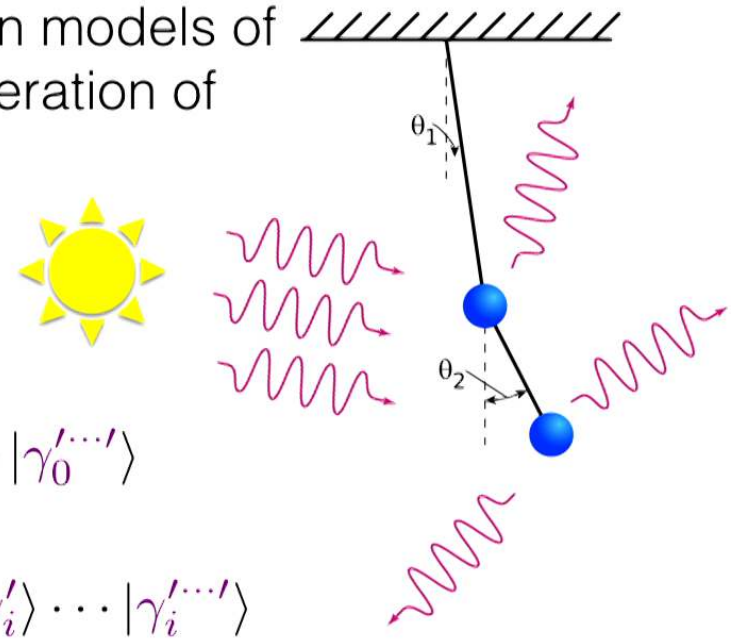
- A generic feature seen in models of decoherence is the generation of GHZ-like correlations

"Redundant records"

Different classical trajectories

$$|\Psi^0\rangle = \left[\sum_i |S_i\rangle\right] |\gamma_0\rangle|\gamma_0'\rangle\cdots|\gamma_0'^{\cdots\prime}\rangle$$

$$\rightarrow |\Psi\rangle = \sum_i |S_i\rangle|\gamma_i\rangle|\gamma_i'\rangle\cdots|\gamma_i'^{\cdots\prime}\rangle$$



Sketching branches

- Ultimately, we seek to precisely define wavefunction branches “out there in the real world”
- Today we describe a precise but imperfect definition on a lattice
- First will gather some imprecise desiderata
- Then will present a scale-dependent precise definition
- Scale-independent definition with required properties is future research

Wavefunction branch desiderata

- Branches are time-dependent orthogonal decomposition of many-body wavefunction

$$|\Psi^{(t)}\rangle = \sum_i |\Psi_i^{(t)}\rangle \qquad \langle\Psi_i^{(t)}|\Psi_j^{(t)}\rangle = 0, \quad i \neq k$$

- Coherent superposition indistinguishable from corresponding incoherent mixture for correlators of local observables

$$\langle \mathcal{O}_1 \cdots \mathcal{O}_m \rangle_\Psi = \langle \mathcal{O}_1 \cdots \mathcal{O}_m \rangle_\rho \qquad \rho = \sum_i |\Psi_i\rangle\langle\Psi_i|$$

Scale-dependent definition

- Define a **redundant** set of observable at scale L to be a set of m>2 observables local to disjoint regions with size $O(L)$…

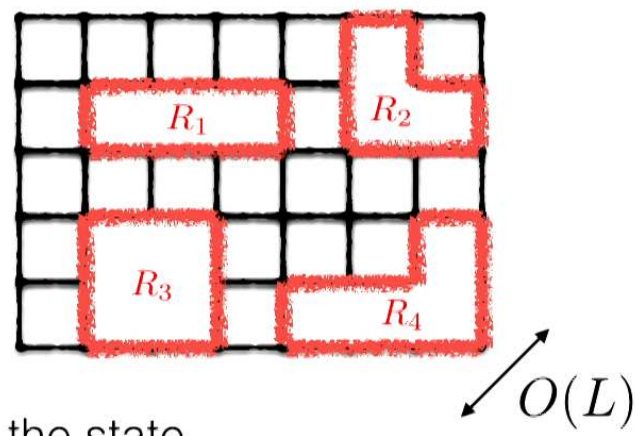
Eigenvalues

z-th observable

z-th region

$$\mathcal{O}^{(z)} = \sum_j \lambda_j P_j^{R_z}$$

Orthogonal projectors onto eigenspaces



…that are classically correlated on the state

$$P_j^{R_1}|\Psi\rangle = P_j^{R_2}|\Psi\rangle = P_j^{R_3}|\Psi\rangle = \cdots$$

$$\rho_{j,R_z}^{(R_z')} = \mathrm{Tr}_{\overline{R_z'}}\left[P_j^{(R_z)} |\psi\rangle\langle\psi| P_j^{(R_z)} \right]$$

$$\mathrm{Tr}\left[\rho_{j,R_z}^{(R_z')} \rho_{k,R_z}^{(R_z')} \right] = 0$$

for $j \neq k$

Scale-dependent definition

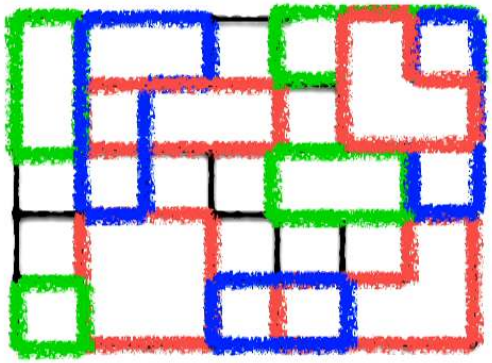
arXiv:1608.05377

- Can be shown that *all* redundant sets of observables at same scale necessarily commute

z_a-th and z_b-th regions

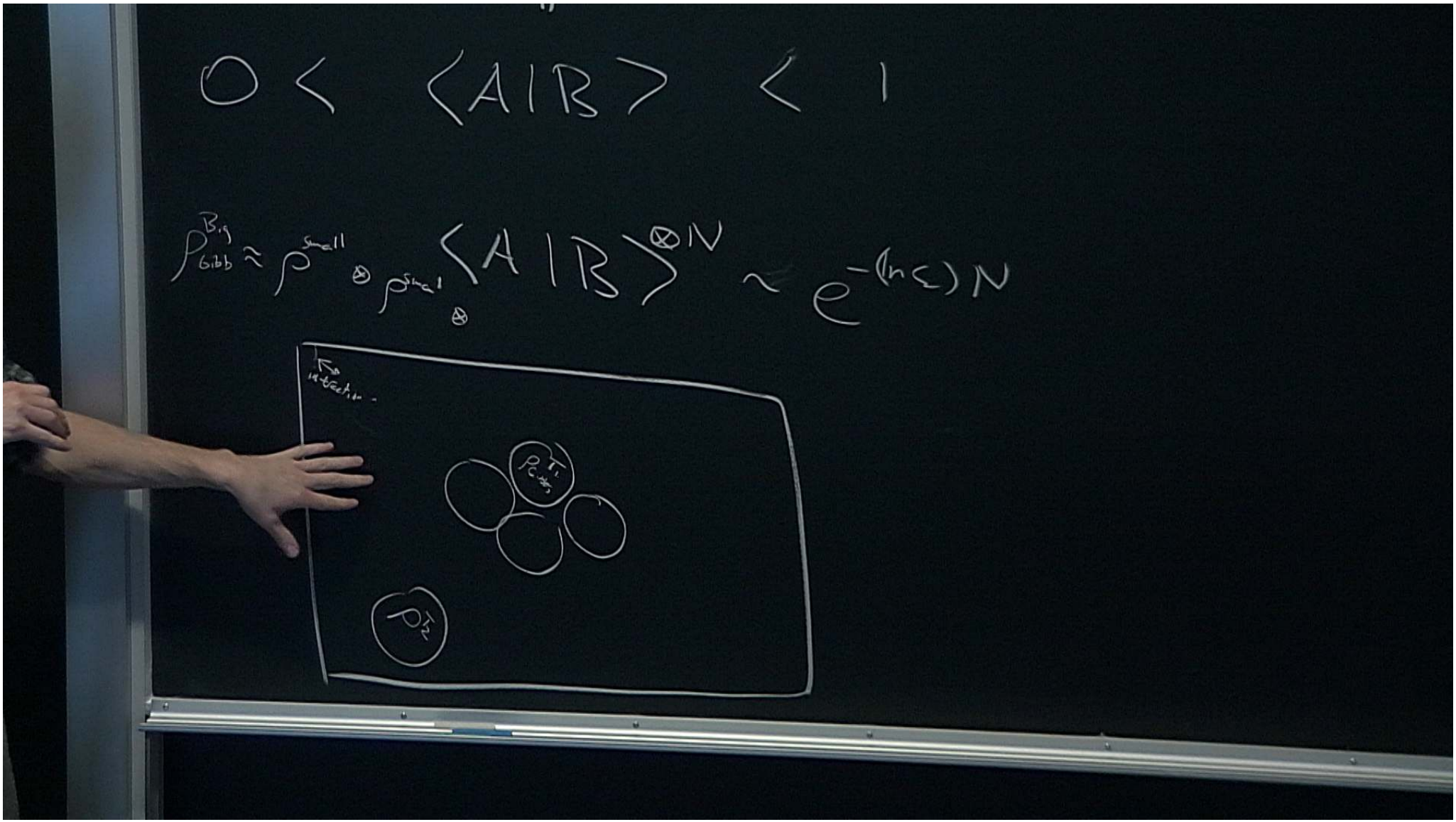
$$\left[\mathcal{O}_a^{(z_a)}, \mathcal{O}_b^{(z_b)}\right] |\Psi\rangle = 0$$

a-th and b-th redundant sets



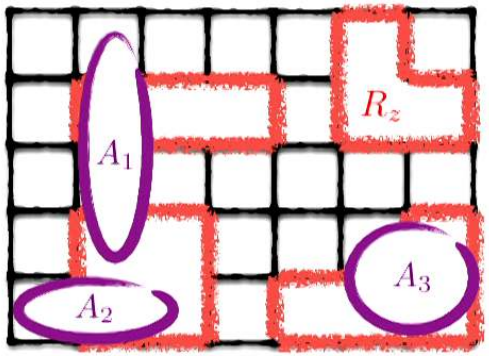
- Define **branches** at scale L to be the simultaneous eigenstates:

$$|\Psi\rangle = \sum_{i=(i_1, i_2, \cdots)} |\Psi_i\rangle \qquad \mathcal{O}_a^{(z_a)} |\Psi_i\rangle = \lambda_{i_a}^{(a)} |\Psi_i\rangle$$


0 < ⟨A|B⟩ < 1
ρ^{B,q}_{Gibb} ≈ ρ^{small} ⊗ ρ^{small} ⊗ ⟨A|B⟩^{⊗N} ~ e^{-(ln c)N}

Estimate correlators by sampling branches

- When expanding correlator of local operators at same scale, off-diagonal terms connecting different branches vanish



$$\langle A_1 \cdots A_{m-1} \rangle_\psi = \sum_{i,j} \langle \Psi_i | A_1 \cdots A_{m-1} | \Psi_j \rangle$$

Branches are eigenstates

$$= \sum_{i,j} \langle \Psi_i | P_{i_a}^{(R_z)} A_1 \cdots A_{m-1} | \Psi_j \rangle$$

For *some* region in redundant set

$$= \sum_{i,j} \langle \Psi_i | A_1 \cdots A_{m-1} P_{i_a}^{(R_z)} | \Psi_j \rangle$$

Off-diag terms annihilate

$$= \sum_{i} \langle \Psi_i | A_1 \cdots A_{m-1} | \Psi_i \rangle$$

- Can get accurate estimate of correlator with fixed-size sample, even with exponentially large number of branches

$$\langle A_1 \cdots A_{m-1} \rangle_\psi = \sum_{i \in S} \langle \Psi_i | \langle A_1 \cdots A_{m-1} | \Psi_i \rangle$$

Beable-guided foil theories (BGFT)

arXiv:1204.5961

- General quantum theory with beables takes form

$$\underbrace{|\psi(t)\rangle}_{\text{quantum state}} = e^{iHt}|\psi(0)\rangle \qquad \underbrace{\{\Lambda(t)\}}_{\text{beables (samples space)}} \qquad \underbrace{\{p_\Lambda(|\psi(t)\rangle)\}}_{\text{probability measure}}$$

- Now allow dynamics of either/both quantum state and beables to be changed
  - Manifestly self-consistent because we define accessible observables to be functions of beables

$$\langle\psi|\hat{A}|\psi\rangle \neq \langle A\rangle = \sum_\Lambda A(\Lambda)p_\Lambda$$

Beable-guided foil theories (BGFT)

arXiv:1204.5961

- *Not* necessarily epiphenomenal — Non-trivial interaction between quantum state and beables
- Consider, for instance, class of BGFTs obtained by slightly modifying probabilities based on the beable

$$p_\Lambda(|\psi\rangle) \to \tilde{p}_\Lambda(|\psi\rangle, \Lambda)$$

  - Could "tip the scales" in any way we want, but can always choose change small enough to be consistent with observations to date
- Examples…

Valentini-Bohm BGFT

- Valentini generalized de Broglie-Bohm mechanics to allow for "quantum nonequilibrium"
- Same quantum state evolution: $|\psi(t)\rangle = e^{iHt}|\psi(0)\rangle$
- Same space of beables: $\Lambda(t) = \{x_1(t), x_2(t), \ldots\}$
- Begin in nonequilibrium:

$$p_\Lambda(0) \neq \mathrm{Tr}\left[\rho(0)\big(|x_1\rangle\langle x_1| \otimes |x_2\rangle\langle x_2| \otimes \ldots\big)\right]$$

- Evolve beables as before: $\dot{x}_n = \frac{1}{m}\mathrm{Im}\frac{\partial_{x_n}\psi}{\psi}$

Ghirardi–Rimini–Weber BGFT

- GRW introduced “objective collapse”
- Same quantum state evolution *between collapses*:

$$|\psi(t_n)\rangle = e^{iH(t_n - t_{n-1})}|\psi_*(t_{n-1})\rangle$$

- Beables are collapse (spacetime) events, stochastic function of quantum state:

$$\Lambda(t) = \{(x_1, t_1), (x_2, t_2), \ldots\}$$

(Bell’s “flash ontology”)

- Quantum state collapses onto Gaussian at event

$$|\psi_*(t_n)\rangle \propto |e^{(\hat{x}-x_n)^2/2\sigma^2}\rangle\langle e^{(\hat{x}-x_n)^2/2\sigma^2}| \cdot |\psi(t_n)\rangle$$

Branches BGFT

- Assume we are able to find a definition of branches

$$|\psi(t)\rangle = e^{iHt}|\psi(0)\rangle \qquad \{|\psi_\Lambda(t)\rangle\}$$

$$= \sum_\Lambda |\psi_\Lambda(t)\rangle \qquad \langle\psi_{\Lambda'}(t)|\psi_\Lambda(t)\rangle = \delta_{\Lambda,\Lambda'} p_\Lambda$$

- We could then easily implement many modifications to quantum mechanics that were hard to make consistent
  - Nonlinear evolution: $\langle\psi_{\Lambda'}(t)|\psi_\Lambda(t)\rangle \not\propto \delta_{\Lambda,\Lambda'}$
  - Alternative probability measures: $\tilde{p}_\Lambda = f(p_\Lambda, \Lambda)$

Branches BGFT

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- We could then easily implement many modifications to quantum mechanics that were hard to make consistent
  - New pseudo-forces (dark matter, dark energy, etc.)
  - Non-trivial future boundary conditions: gr-qc/9304023

$$p_\Lambda = \mathrm{Tr}\left[P^{(n)}_{\lambda_n} \cdots P^{(1)}_{\lambda_1} \rho^{(\mathrm{past})} P^{(1)}_{\lambda_1} \cdots P^{(n)}_{\lambda_n} \rho^{(\mathrm{future})}\right]$$

Quick summary slide

- Beables are generally epiphenomenal…
  - …but beable-guided foil theories are observationally distinguishable from quantum mechanics and each other
- Wavefunction branches can potentially scrub anthropocentric measurement from quantum formalism…
  - …*if* they can be uniquely defined
- Branches are (very arguably) a *preferred* set of beables

Further reading

- Constructing beable-guided foil theories

  A. Kent, arXiv:1204.5961 [PRA 87, 022105 (2013)]

- Evidence branches can be defined objectively

  arXiv:1608.05377 [PRL 118, 120402 (2017)]

- Collaborators: (read: all the cool kids are getting into branches now)

Martin Ganahl (PI), Markus Hauru (PI), Curt von Keyserlingk (Birmingham), Noah MacAulay (UT), Ash Milstead (PI), Elliot Nelson (PI), Daniel Ranard (Stanford), Tian Wang (Caltech)