Title: Motility of the internal-external cut as a foundational principle

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Abstract:

Motility of the internal-external cut as a foundational principle

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Observers in Quantum and Foil Theories
Perimeter Institute
April 3, 2018

The principle of cut-motility:

For any physical theory purporting to have universal applicability, describing an experimental phenomenon typically involves making a cut between (i) the systems that are modeled explicitly within the theory and are the objects of interventions (preparations, measurements, transformations) (ii) the systems that make up the devices that implement these interventions.

A theory is said to satisfy the principle of cut-motility if its predictions are independent of the placement of the cut.

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The placement of the cut is a *conventional choice of the theorist,* possibly chosen for convenience but with no physical significance.

Comparable to principle of general covariance

“The dividing line between the system to be observed and the measuring apparatus is immediately defined by the nature of the problem but it obviously signifies no discontinuity of the physical process. For this reason there must, within limits, exist complete freedom in choosing the position of the dividing line.”
---W. Heisenberg

“[…] the principle of the psycho-physical parallelism is violated, so long as it is not shown that the boundary between the observed system and the observer can be displaced arbitrarily in the sense given above.”
---J. Von Neumann

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Comparable to principle of general covariance

The foundational credentials of
the principle of cut-motility

Makes evident the existence of a measurement problem in the textbook interpretation of quantum theory

The textbook interpretation of quantum theory

Representational completeness of ψ. The rays of Hilbert space correspond one-to-one with the physical states of the system.

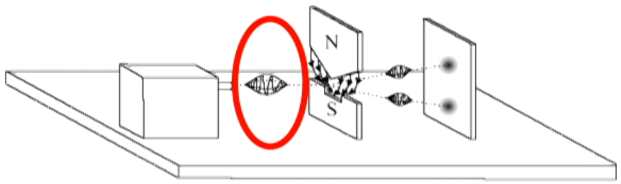
Measurement. If the Hermitian operator A with spectral projectors $\{P_k\}$ is measured, the probability of outcome k is $\langle\psi|P_k|\psi\rangle$. These probabilities are objective -- indeterminism.

Evolution of isolated systems. It is unitary, $|\psi\rangle \to U|\psi\rangle = e^{-\frac{i}{\hbar}Ht}|\psi\rangle$ therefore deterministic and continuous.

Evolution of systems undergoing measurement. If Hermitian operator A with spectral projectors $\{P_k\}$ is measured and outcome k is obtained, the physical state of the system changes discontinuously,

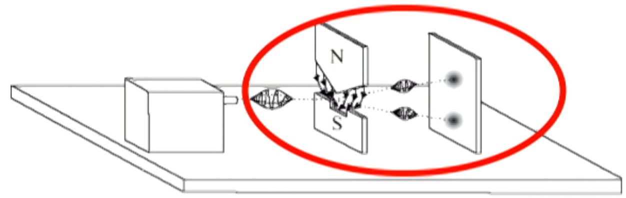
$$|\psi\rangle \to |\psi_k\rangle = \frac{P_k|\psi\rangle}{\sqrt{\langle\psi|P_k|\psi\rangle}}$$

Inconsistencies of the textbook interpretation



By the collapse postulate
(applied to the system)

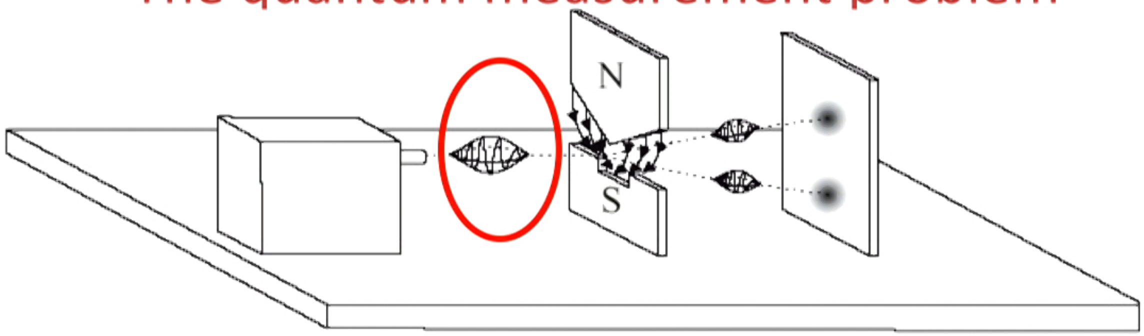
Indeterministic and discontinuous evolution



By unitary evolution postulate
(applied to isolated system that includes the apparatus)

Deterministic and continuous evolution

The quantum measurement problem



If the measurement apparatus is treated externally

$$a|\uparrow\rangle + b|\downarrow\rangle \;\rightarrow |\uparrow\rangle \text{ with probability } |a|^2$$
$$\rightarrow |\downarrow\rangle \text{ with probability } |b|^2$$

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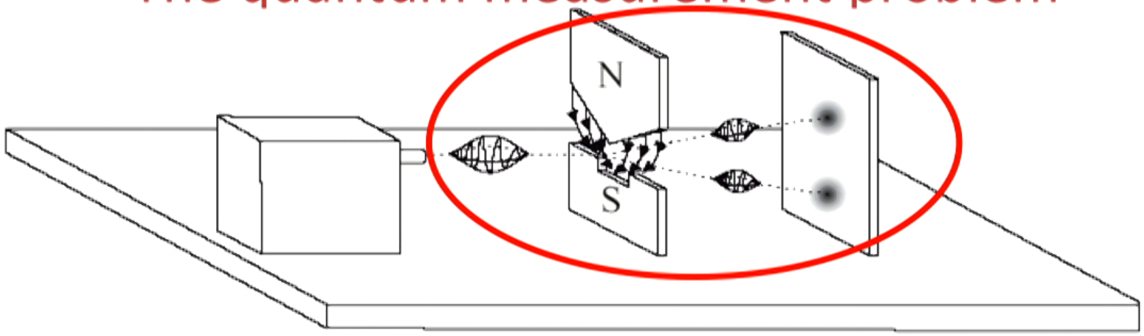

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$$a|\uparrow\rangle + b|\downarrow\rangle \;\rightarrow |\uparrow\rangle \text{ with probability } |a|^2$$
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If the measurement apparatus is treated internally

$$|\uparrow\rangle \otimes |\text{“ready”}\rangle \;\rightarrow U(|\uparrow\rangle \otimes |\text{“ready”}\rangle) = |\uparrow\rangle \otimes |\text{“up”}\rangle$$
$$|\downarrow\rangle \otimes |\text{“ready”}\rangle \;\rightarrow U(|\downarrow\rangle \otimes |\text{“ready”}\rangle) = |\downarrow\rangle \otimes |\text{“down”}\rangle$$

U is a linear operator $\quad U(a|\psi\rangle + b|\phi\rangle) = aU|\psi\rangle + bU|\phi\rangle$

$$(a|\uparrow\rangle + b|\downarrow\rangle) \otimes |\text{“ready”}\rangle \rightarrow U[a|\uparrow\rangle \otimes |\text{“ready”}\rangle + b|\downarrow\rangle \otimes |\text{“ready”}\rangle]$$
$$= a|\uparrow\rangle \otimes |\text{“up”}\rangle + b|\downarrow\rangle \otimes |\text{“down”}\rangle$$

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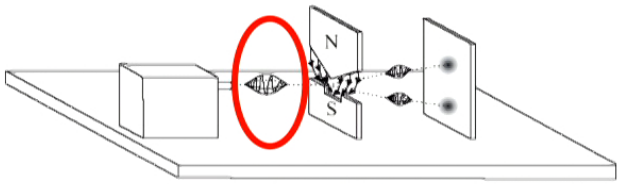
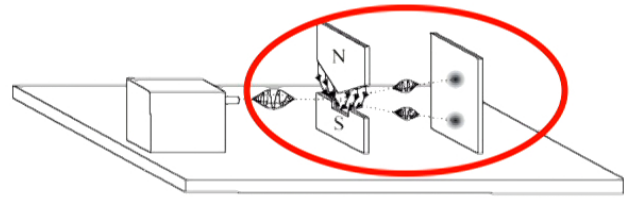




By the collapse postulate
(applied to the system)

By unitary evolution postulate
(applied to isolated system that includes the apparatus)

Indeterministic and discontinuous evolution

Determinate properties

Deterministic and continuous evolution

Indeterminate properties

The foundational credentials of
the principle of cut-motility

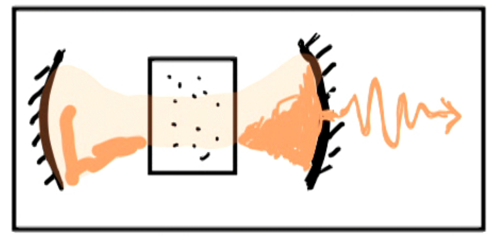
Resolves the longstanding debate about whether coherences between eigenspaces of conserved quantities are fact or fiction

Optical coherence: a convenient myth?

K. Molmer, Phys. Rev. A. 55, 3195 (1997)

Standard assumption for field: $\rho = |\alpha\rangle\langle\alpha|$

$$|\alpha\rangle = \sum_{n=0}^{\infty} \frac{e^{-|\alpha|^2/2}\alpha^n}{\sqrt{n!}}|n\rangle$$



coherence is **fact**

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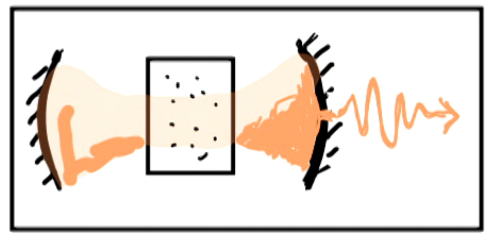

coherence is **fact**

But if we quantize the atoms in the gain medium, and

- assume incoherent mixture of energy eigenstates
- apply energy conservation

$\rightarrow$ For a given n, atoms and field evolve to an entangled state

$$|e\rangle|n\rangle \rightarrow a(t)|e\rangle|n\rangle + b(t)|g\rangle|n+1\rangle$$

$$\rho = |a(t)|^2\,|n\rangle\langle n| + |b(t)|^2\,|n+1\rangle\langle n+1|$$

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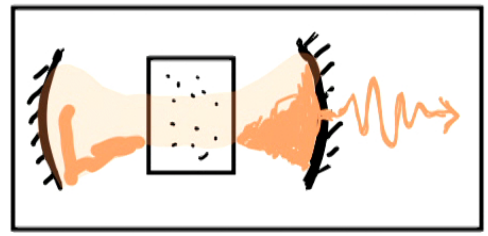


coherence is **fact**

But if we quantize the atoms in the gain medium, and

- assume incoherent mixture of energy eigenstates (thermal state)
- apply energy conservation

$$\rho = \sum_{n=0}^{\infty} p_n|n\rangle\langle n| \quad \text{where} \quad p_n = \frac{e^{-|\alpha|^2}|\alpha|^{2n}}{n!}$$

Thus, coherence is **fiction**!

Other areas in which the coherence controversy has arisen

Nonlocality of a single photon
D. M. Greenberger, M. A. Horne and A. Zeilinger, Phys. Rev. Lett. 75 (1995) 2064.
L. Hardy, Phys. Rev. Lett. 75 (1995) 2065.

Bose-Einstein condensation
J. Javanainen and S. M. Yoo, Phys. Rev. Lett. 76 (1996) 161.
W. Hoston and L. You, Phys. Rev. A 53 (1996) 4254.
S. M. Yoo, J. Ruostekoski and J. Javanainen, J. Mod. Opt. 44 (1997) 1763.
Y. Castin and J. Dalibard, Phys. Rev. A 55 (1997) 4330.

Superconductivity
P. W. Anderson, in The Lesson of Quantum Theory, eds. J. D. Boer, E. Dal, O. Ulfbeck (Elsevier, Amsterdam, 1986), pp. 2333.
R. Haag, Il Nuovo Cimento XXV (1962) 2695.
D. Kershaw and C. H. Woo, Phys. Rev. Lett. 33 (1974) 918.

Whether there are superselection rules for charge, baryon number, etc.
G. C. Wick, A. S. Wightman and E. P. Wigner, Phys. Rev. 88 (1952) 101.
Y. Aharonov and L. Susskind, Phys. Rev. 155 (1967) 1428.

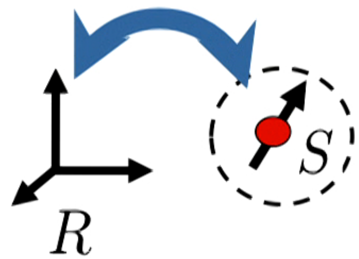
The proposed resolution

Bartlett, Rudolph, and Spekkens, Int. J. Quantum Information 4, 17 (2006)

Quantum states only describe the properties of a system **relative to** some external system

Consequently, whether or not coherences are applicable depends on the external system to which one is comparing

What does it mean to say that the spin is up along the z axis?

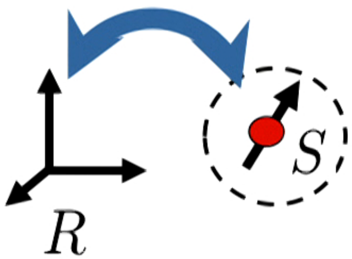


It means spin up relative to another physical system, such as gyroscopes in the lab, that define the z axis (i.e. act as a Cartesian reference frame)

What does it mean to say that
a mode has a particular phase?

Implicated RF treated externally

$$\rho_S \in \mathcal{L}(\mathcal{H}_S)$$



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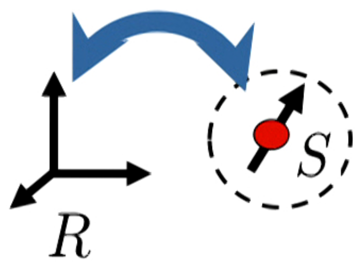
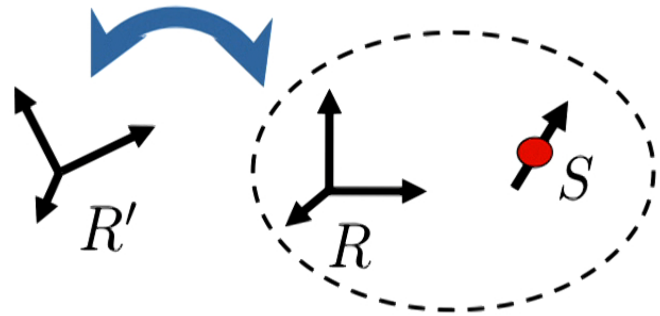


Implicated RF treated internally

$$\sigma_{RS} \in \mathcal{L}(\mathcal{H}_R) \otimes \mathcal{L}(\mathcal{H}_S)$$



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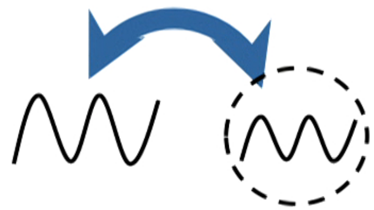

It means that it has that phase relative to another physical system, such as another oscillator in the lab (i.e. one that acts as a phase reference)

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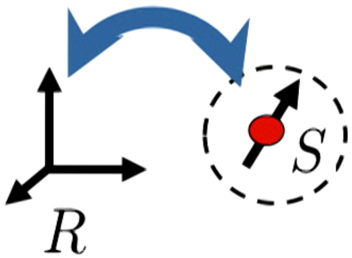

So, the two states need not be the same

Implicated RF treated internally

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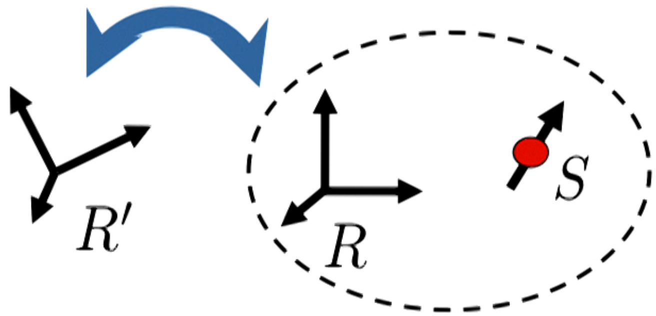

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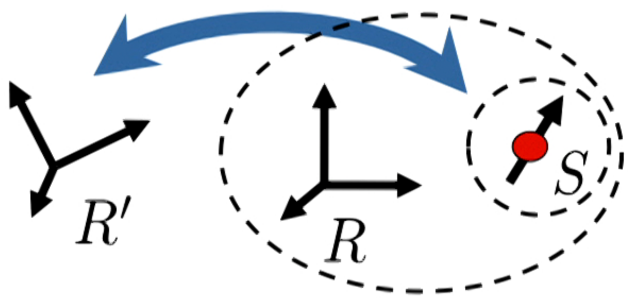

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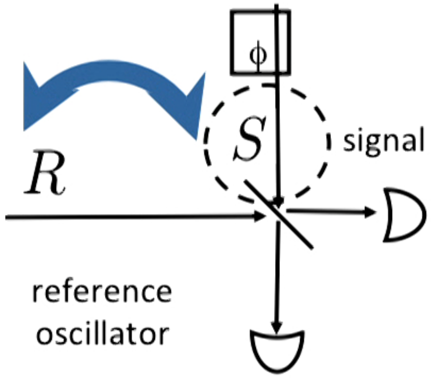

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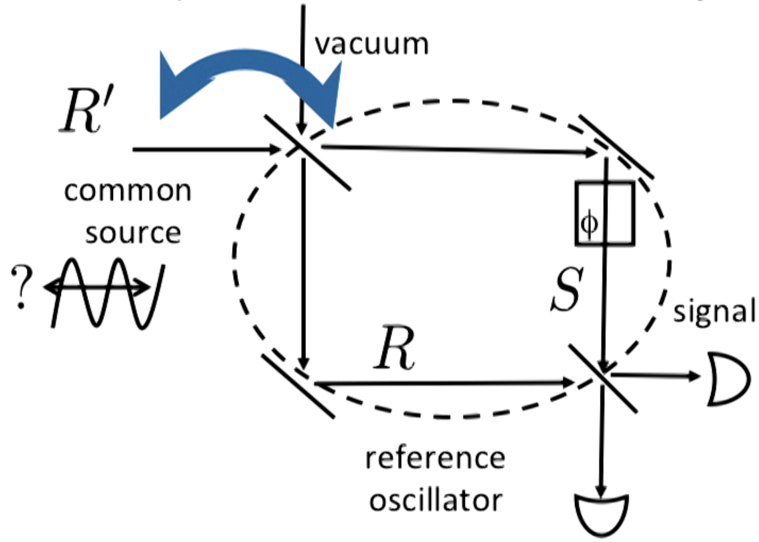

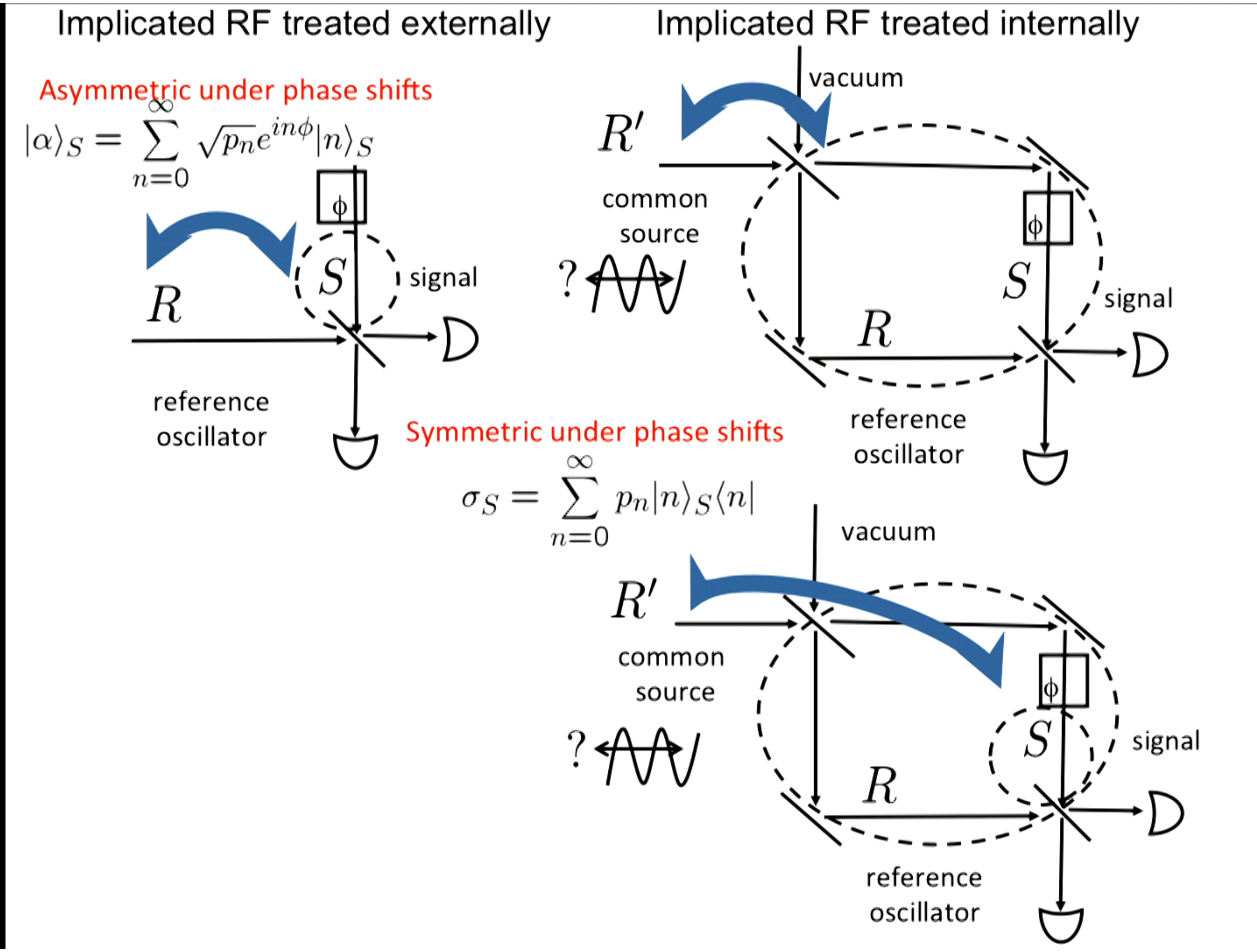
Implicated RF treated externally
Implicated RF treated internally
Asymmetric under phase shifts
$|\alpha\rangle_S = \sum_{n=0}^{\infty} \sqrt{p_n} e^{in\phi} |n\rangle_S$
ϕ
S
signal
R
reference
oscillator
vacuum
R′
common
source
?
ϕ
S
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The foundational credentials of
the principle of cut-motility

Clarifies what is the correct definition of the free operations in certain resource theories

The resource theory of speakable coherence

Speakable versus unspeakable coherence

Marvian and RWS, PRA 94, 052324 (2016)

$$|\psi\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad |\phi\rangle = \frac{|0\rangle + |2\rangle}{\sqrt{2}}$$

What should be taken as the free operations?

"Incoherence-preserving operations"

A generalization of the proposal of:
Baumgratz, Cramer, and Plenio, PRL 113, 140401 (2014)

$$\rho \in \mathcal{I} \implies \mathcal{E}(\rho) \in \mathcal{I}$$

where $\mathcal{I}$ = set of incoherent states $\rho = \sum_l p_l |l\rangle\langle l|$

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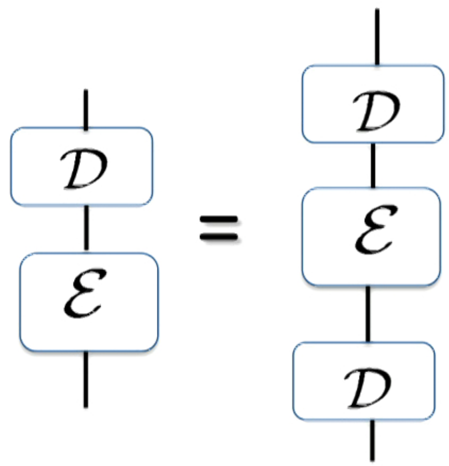
Equivalently,

$$\mathcal{E} \circ \mathcal{D} = \mathcal{D} \circ \mathcal{E} \circ \mathcal{D}$$

where

$$\mathcal{D}(\cdot) \equiv \sum_l |l\rangle\langle l|(\cdot)|l\rangle\langle l|$$



What should be taken as the free operations?

"Dephasing-covariant operations"

Chitambar and Gour, PRA 94, 052336 (2016)
Marvian and RWS, PRA 94, 052324 (2016)

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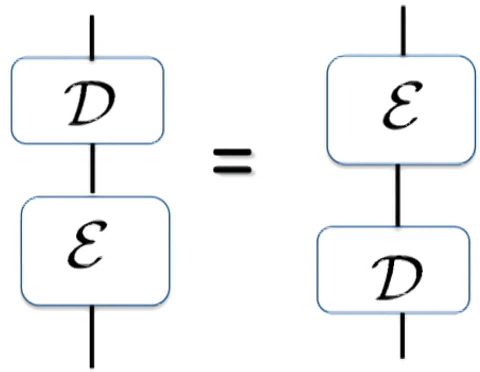


Dephasing covariant ⊂ Incoherence-preserving

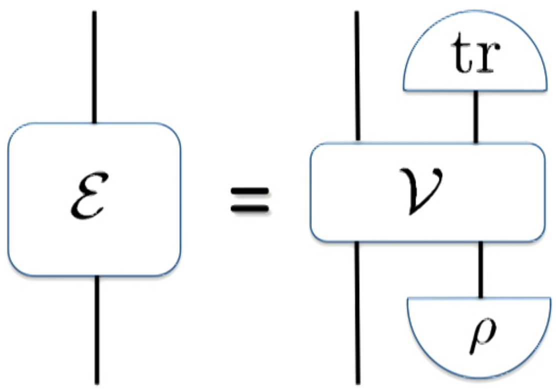
E.g.

$$\mathcal{E}(\rho) = |0\rangle\langle 0|\mathrm{Tr}(|+\rangle\langle +|\rho) + |1\rangle\langle 1|\mathrm{Tr}(|-\rangle\langle -|\rho)$$

Two proposals agree on what are the free unitaries and the free states

$$\rho = \sum_l p_l |l\rangle\langle l|$$

$$V = \sum_l e^{i\theta_l} |\pi(l)\rangle\langle l|$$



Theorem:
If ρ and V are incoherent, then $\mathcal{E}$ is dephasing-covariant

The category of incoherence-preserving operations is not cut-motile

The foundational credentials of the principle of cut-motility

Solves the Maxwell's demon challenge to the second law

C. H. Bennett, Int. J. Theor. Phys. 21, 905 (1982)

The foundational credentials of
the principle of cut-motility

Resolves an apparent challenge to Curie's principle involving quantum collapse

RWS, http://pirsa.org/16060060/

The foundational credentials of the principle of cut-motility

- Makes evident the existence of a measurement problem in the textbook interpretation of quantum theory
- Resolves the longstanding debate about whether coherences between eigenspaces of conserved quantities are fact or fiction
- Clarifies what is the correct definition of the free operations in certain resource theories
- Solves the Maxwell's demon challenge to the second law (Bennett)
- Resolves an apparent challenge to Curie's principle involving quantum collapse
- Further prospects: Debates about background independence? Debates about real versus complex field?

What the principle of cut-motility implies for how to define epistemically restricted classical theories

Classical theory	Statistical theory for the classical theory	Epistemically restricted theory for the classical theory
Mechanics	Liouville mechanics	Epistemically restricted Liouville mechanics = Clifford subtheory of quantum mechanics

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There are two senses in which this scheme defines foil theories

A fact about operational quantum theory:

Jointly-measurable observables = a commuting set of observables (relative to matrix commutator)

Continuous degrees of freedom

Configuration space: $\mathsf{R}^n \ni (x_1, x_2, \ldots, x_n)$

Phase space: $\Omega \equiv \mathsf{R}^{2n} \ni (x_1, p_1, x_2, p_2, \ldots, x_n, p_n) \equiv m$

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Functionals on phase space: $F : \Omega \to \mathrm{R}$

$$X_k(m) = x_k$$
$$P_k(m) = p_k$$

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Poisson bracket of functionals:

$$[F, G](m) \equiv \sum_{i=1}^{n} \left(\frac{\partial F}{\partial X_i}\frac{\partial G}{\partial P_i} - \frac{\partial F}{\partial P_i}\frac{\partial G}{\partial X_i}\right)(m)$$

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The linear functionals / canonical variables are:

$$F = a_1 X_1 + b_1 P_1 + \cdots + a_n X_n + b_n P_n \qquad a_1, b_1, \ldots, a_n, b_n \in \mathbb{R}$$

Discrete degrees of freedom $\mathbb{Z}_d = \{0, 1, \ldots, d-1\}$

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$$[F, G](m) \equiv \sum_{i=1}^{n} (F[m + e_{x_i}] - F[m])(G[m + e_{p_i}] - G[m]) \\ -(F[m + e_{p_i}] - F[m])(G[m + e_{q_i}] - G[m])$$

A canonically conjugate pair $[F, G] = 1$

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e.g. $\{X_1, P_1\}, \{X_2, P_2\}$, and $\{X_1 + X_2, P_1\}$

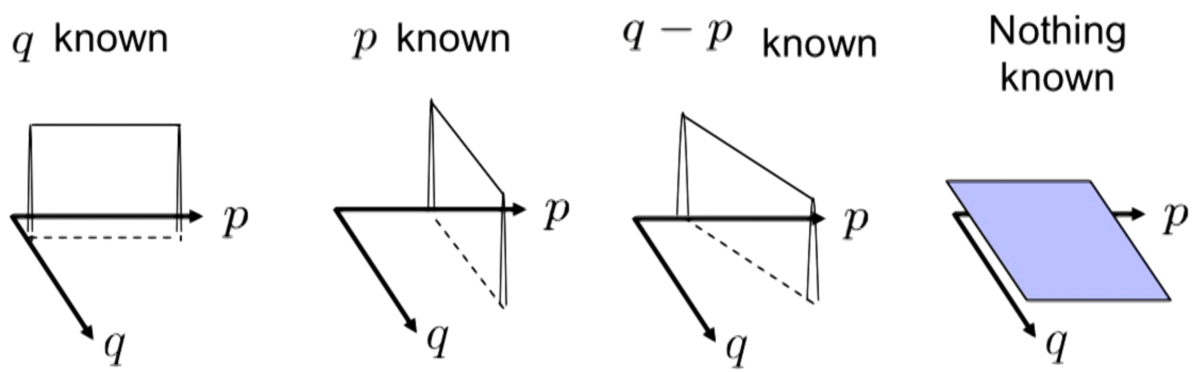
A commuting pair $[F, G] = 0$

e.g. $\{X_1, X_2\}, \{X_1, P_2\}$, and $\{X_1 - X_2, P_1 + P_2\}$

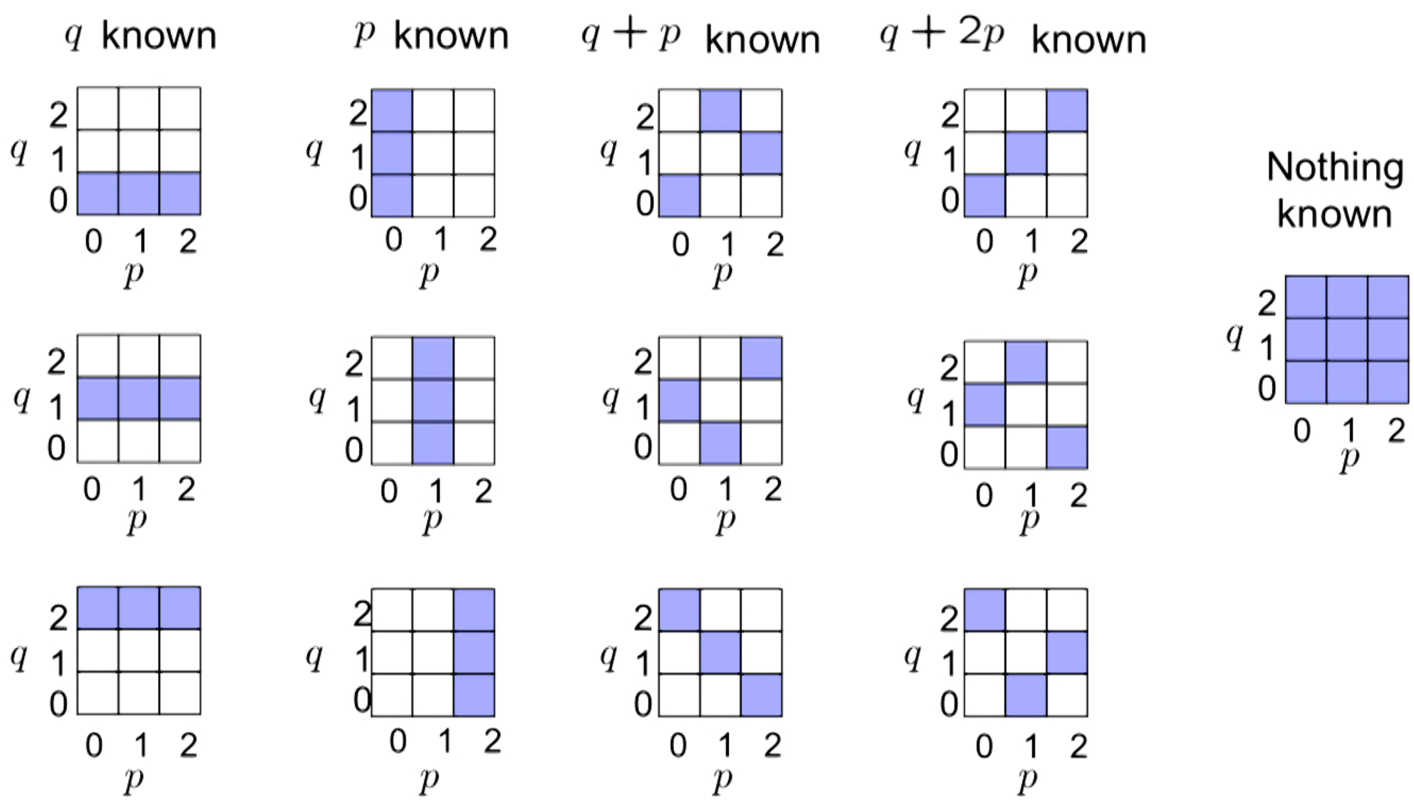
The principle of classical complementarity:

An observer can only have knowledge of the values of a commuting set of canonical variables and is maximally ignorant otherwise.

Valid epistemic states for one canonical system



Valid epistemic states for one canonical system



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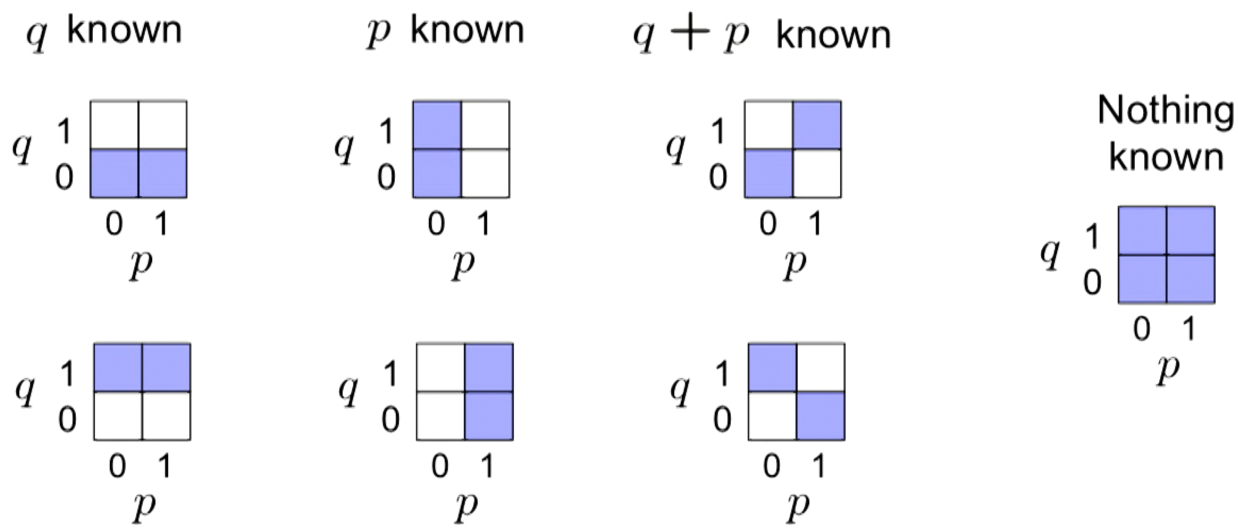


Valid reversible transformations

Those that preserve the Poisson bracket / symplectic inner product:

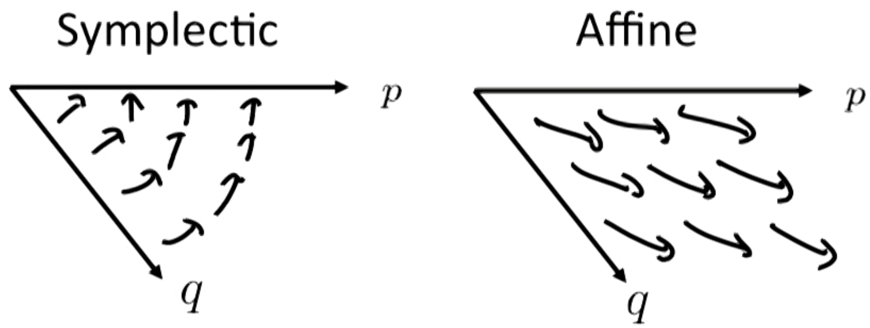
The group of symplectic affine transformations (Clifford group)

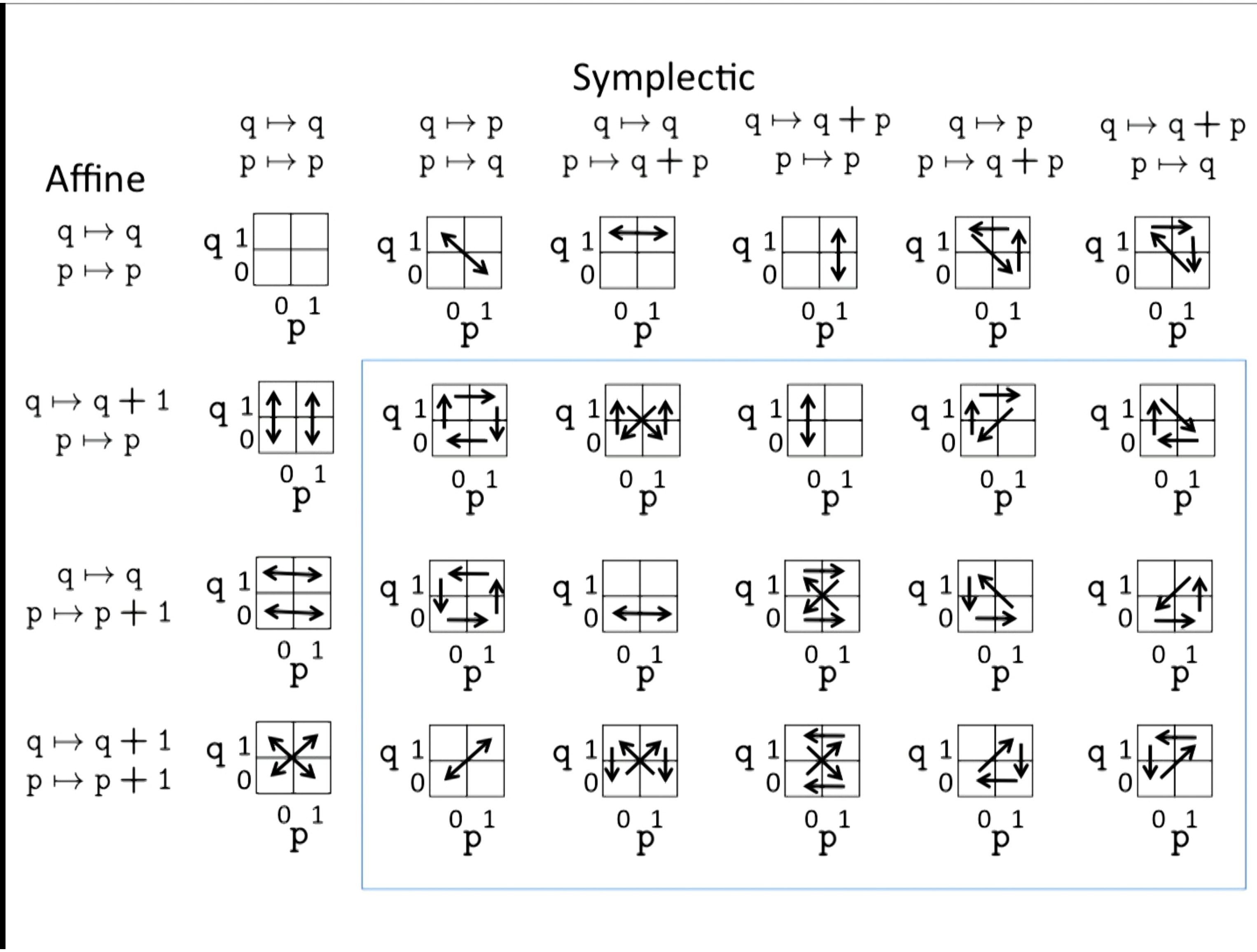
for $m \in \Omega$

$$m \mapsto Sm + a$$

where $[Su, Sv] = [u, v]$ Symplectic

and $a \in \Omega$ Affine (Heisenberg-Weyl)


Symplectic
p
q
Affine
p
q


Symplectic
Affine
q ↦ q
p ↦ p
q ↦ p
p ↦ q
q ↦ q
p ↦ q + p
q ↦ q + p
p ↦ p
q ↦ p
p ↦ q + p
q ↦ q + p
p ↦ q
q ↦ q + 1
p ↦ p
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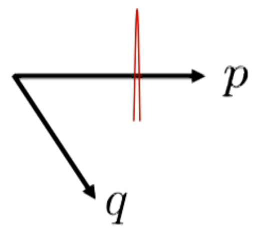
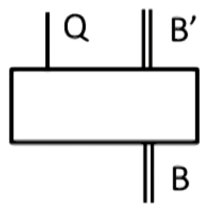
p ↦ p + 1

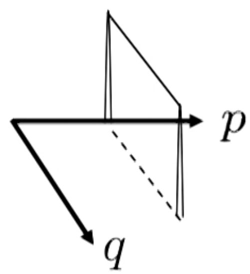
Collapse Rule in Epistemically Restricted Liouville mechanics

Measure Q_B find q



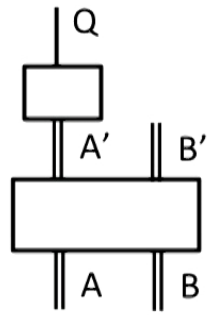
But this would violate the epistemic restriction!



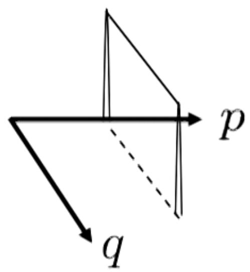


Collapse Rule in Epistemically Restricted Liouville mechanics

Measure $Q_{A'}$

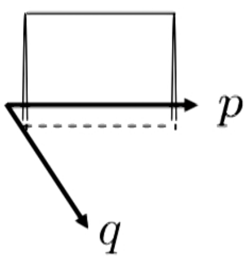


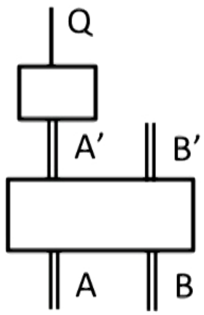
$$H_{AB} = P_A Q_B$$



Collapse Rule in Epistemically Restricted Liouville mechanics

Measure $Q_{A'}$





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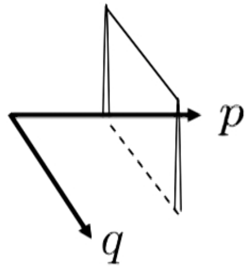
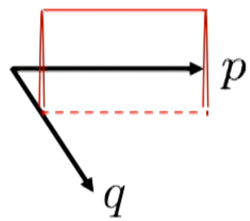
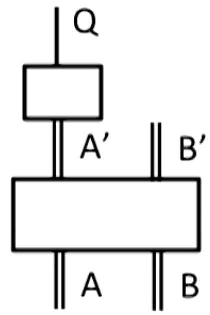
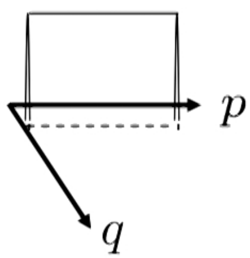


Collapse Rule in Epistemically Restricted Liouville mechanics

Measure $Q_{A'}$ find q







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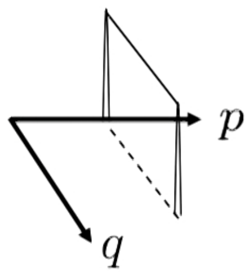
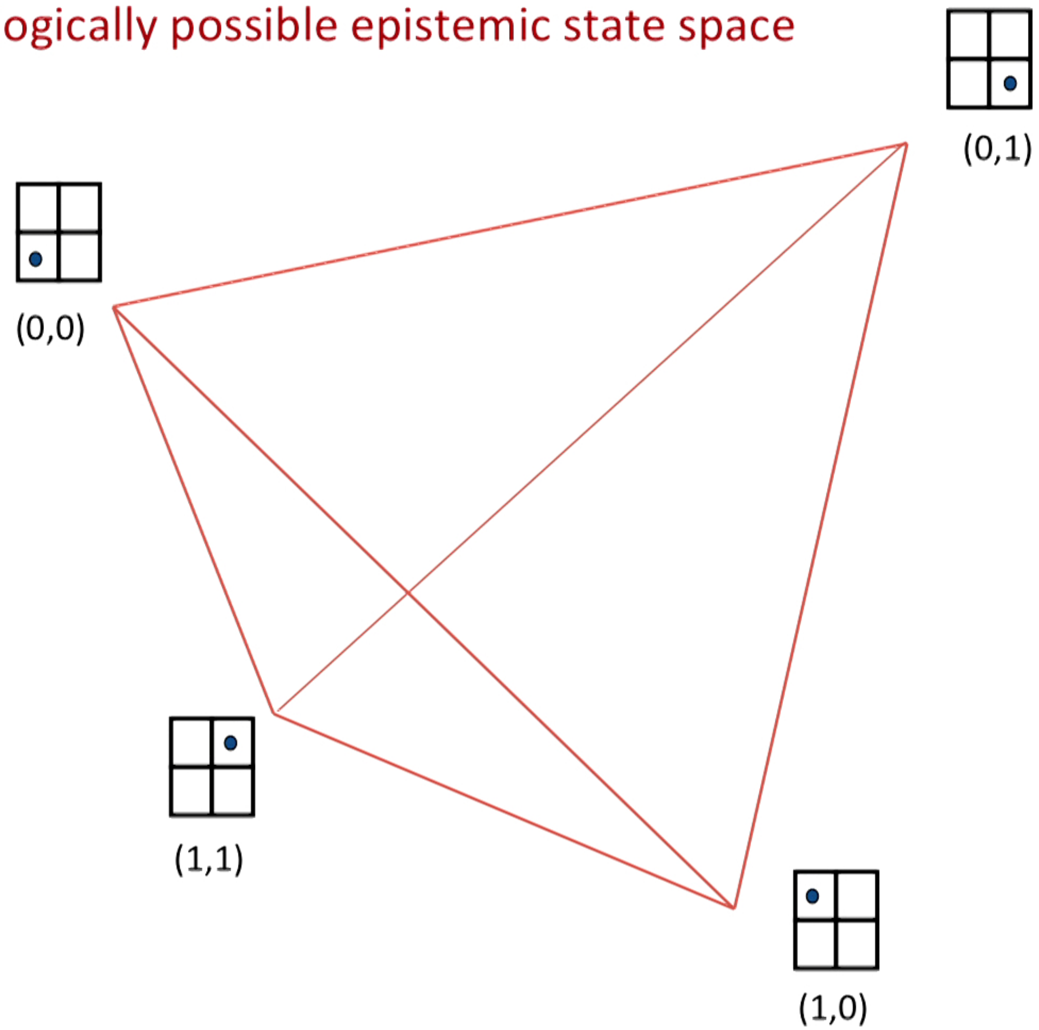
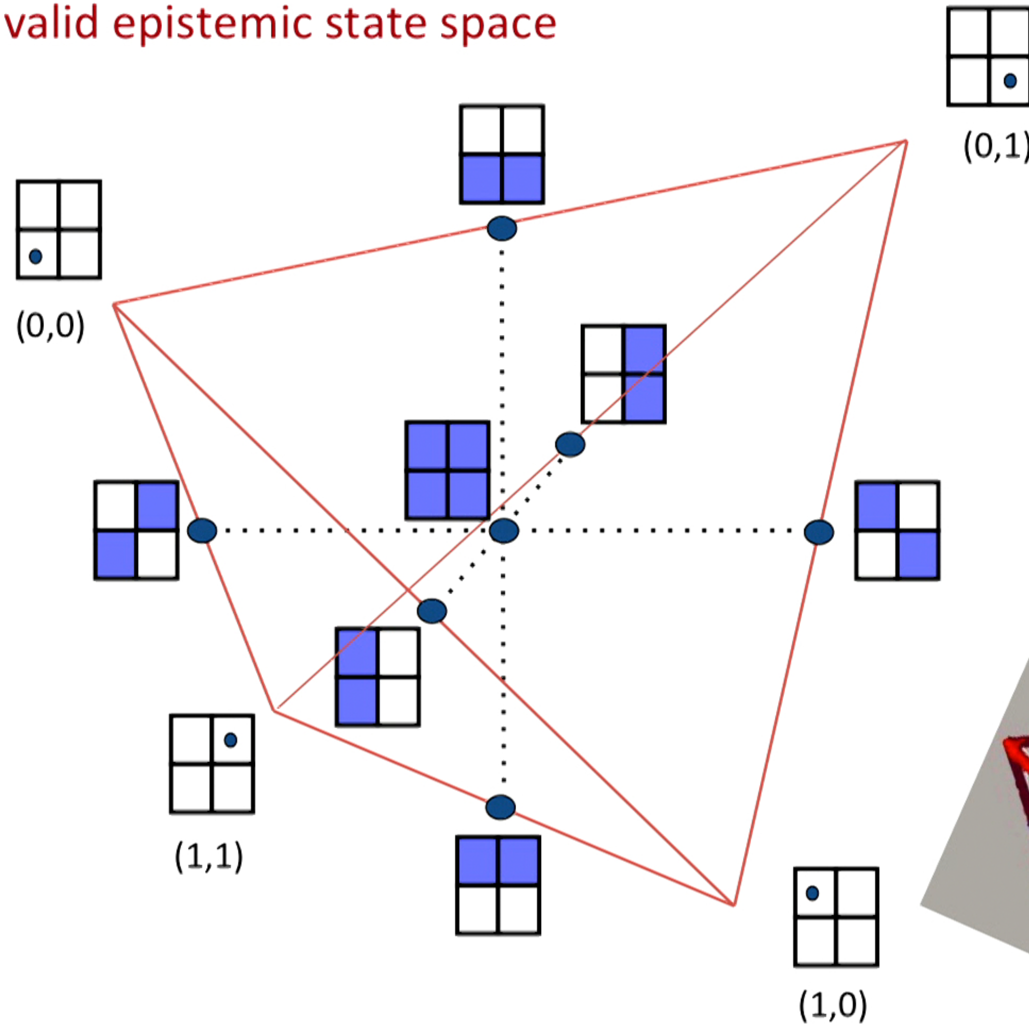
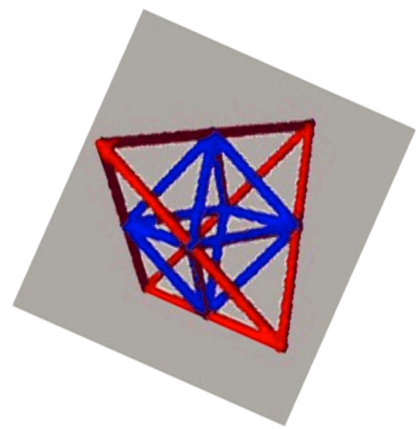


The logically possible epistemic state space

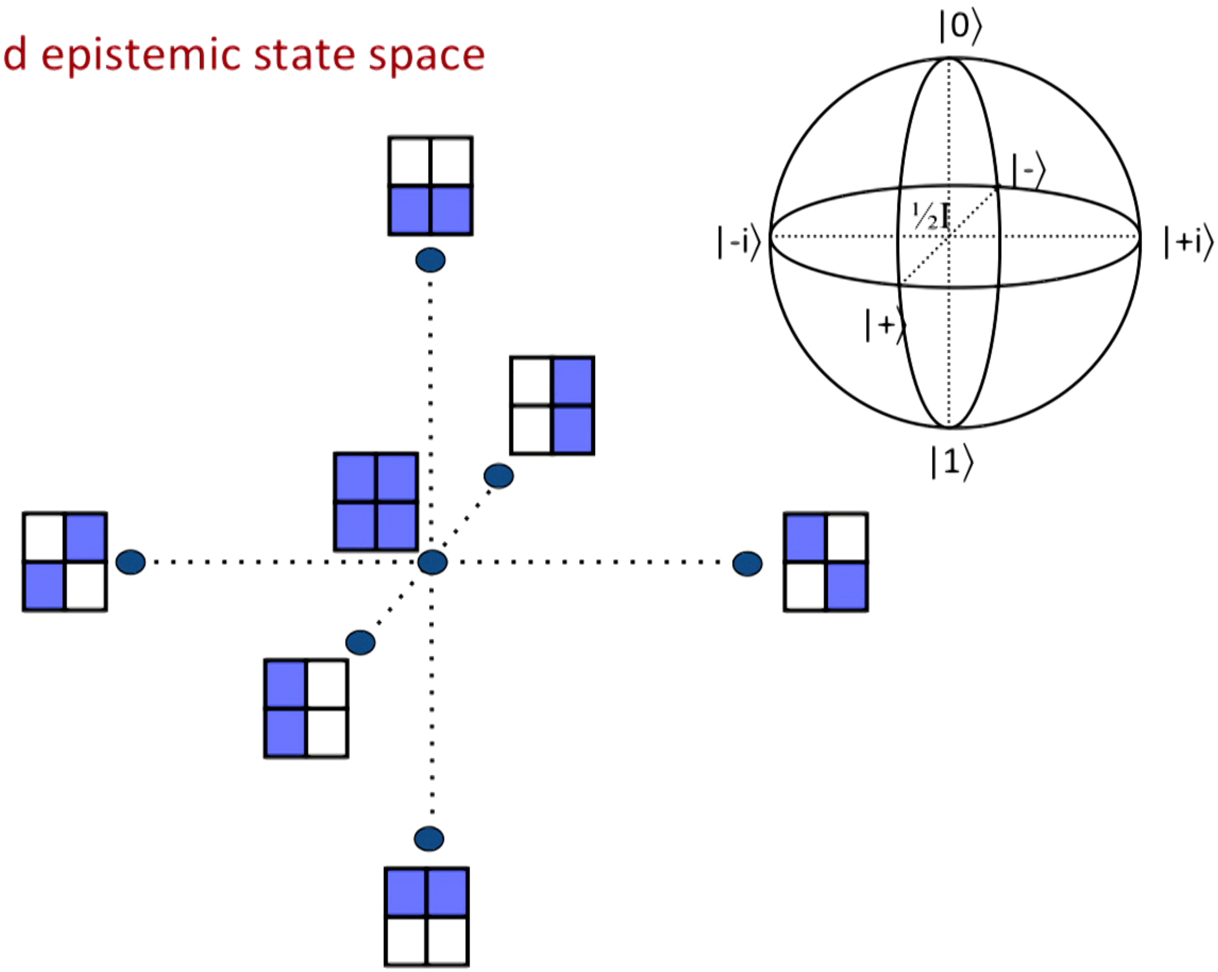


The valid epistemic state space

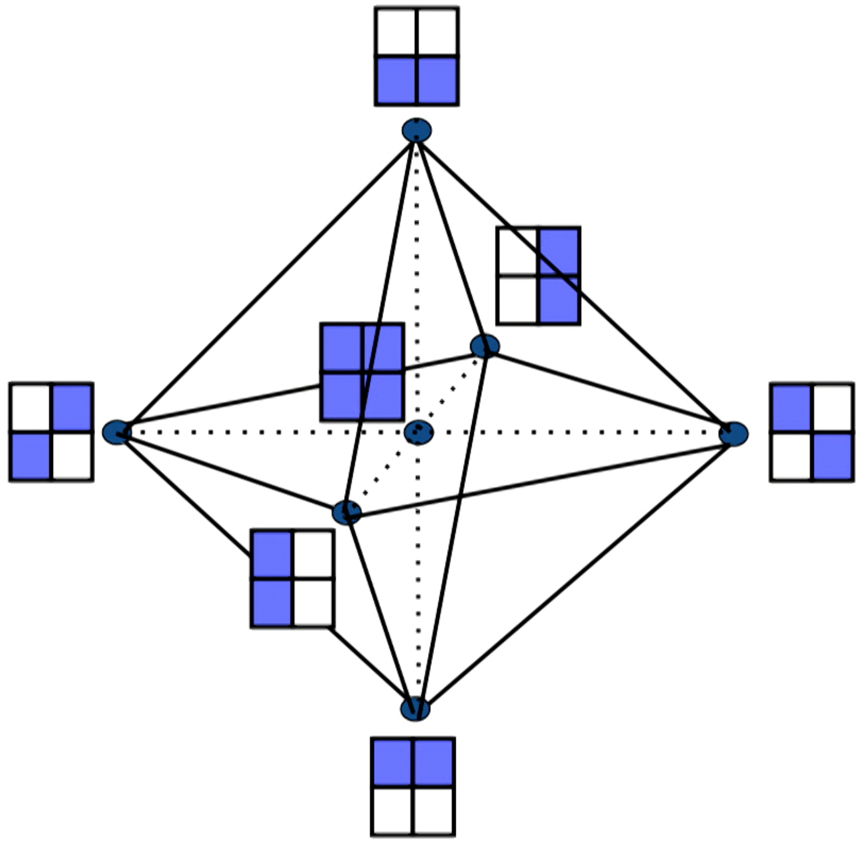




The valid epistemic state space



A common suggestion: Why not take the convex closure of the state space?



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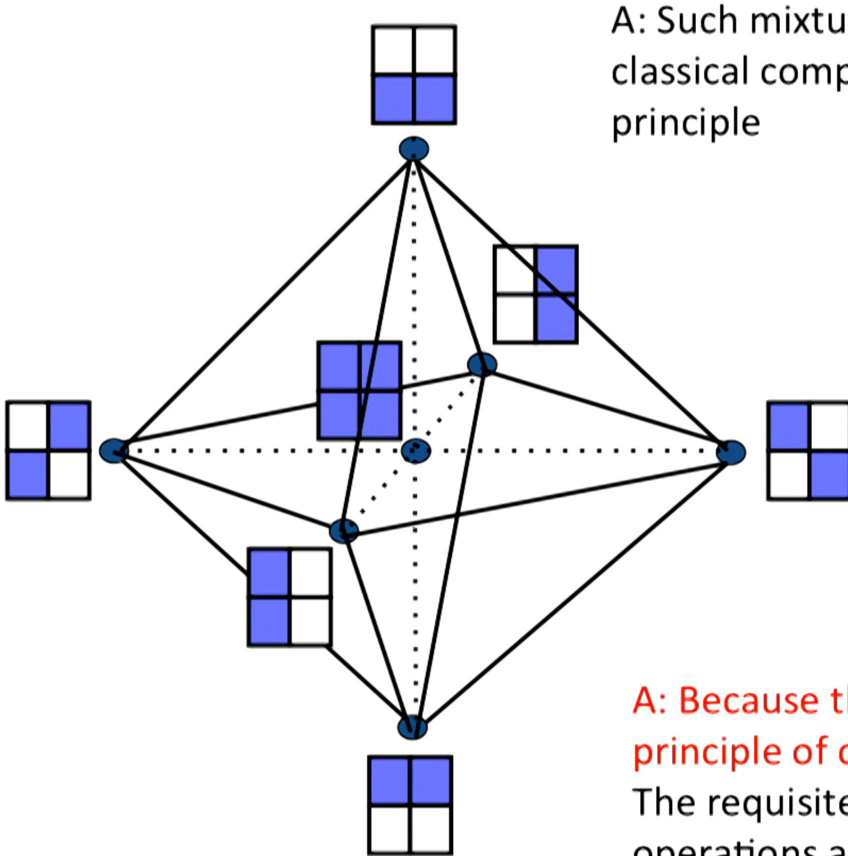

A: Such mixtures don't satisfy the classical complementarity principle

A: Because this would violate the principle of cut-motility!
The requisite dice and controlled operations are not available

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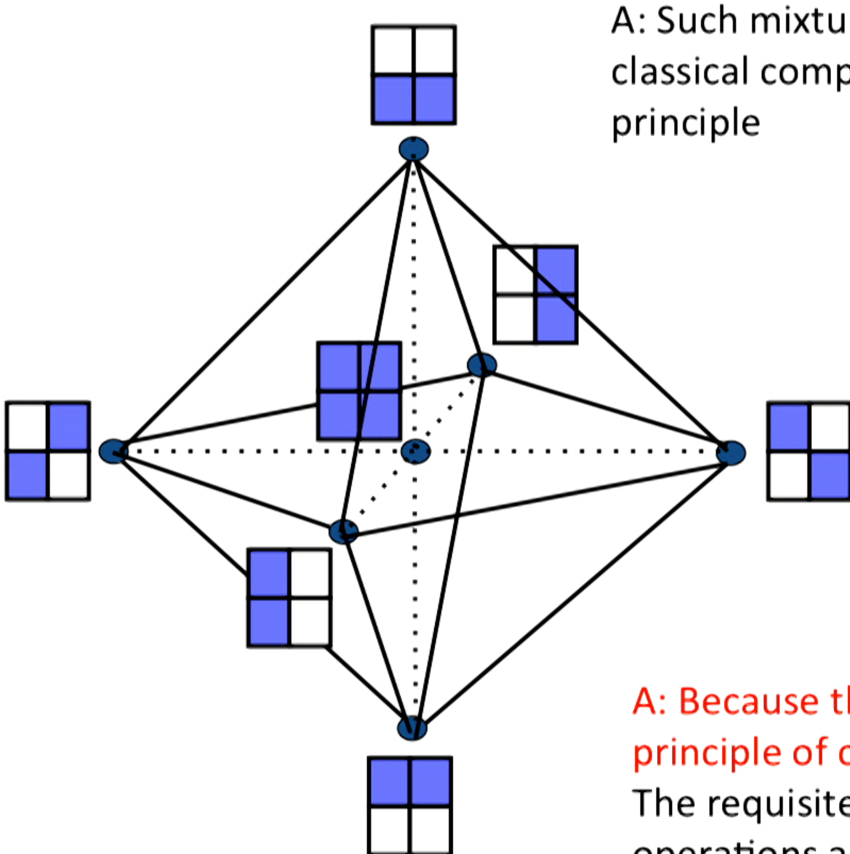


A: Such mixtures don't satisfy the classical complementarity principle

A: Because this would violate the principle of cut-motility!
The requisite dice and controlled operations are not available

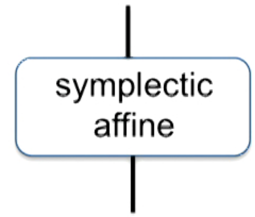
Epistemically restricted theories

- Evolving any system with a symplectic affine transformation

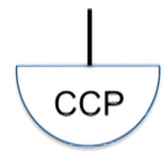
$$m \mapsto Sm + a$$

where $[Su, Sv] = [u, v]$

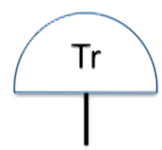
and $a \in \Omega$



- Preparing epistemic states satisfying classical complementarity principle

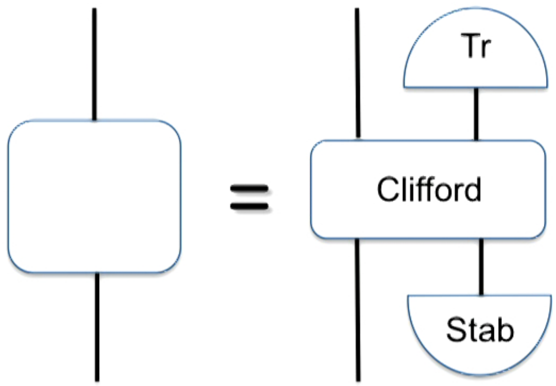


- Marginalizing over any system



Clifford subtheories of quantum theory

Assuming the principle of cut-motility, the set of all valid (possibly irreversible) transformations are those that are expressible as:



Classical theory	Statistical theory for the classical theory	Epistemically restricted theory for the classical theory
Mechanics	Liouville mechanics	Epistemically restricted Liouville mechanics = Clifford subtheory of quantum mechanics
Bits	Statistical theory of bits	Epistemically restricted statistical theory of bits (a.k.a. toy theory) $\simeq$ Clifford subtheory for qubits
Trits	Statistical theory of trits	Epistemically restricted statistical theory of trits (toy theory for trits) = Clifford subtheory for qutrits

Insight into a longstanding puzzle concerning contextuality and computation?

Classical theory	Statistical theory for the classical theory	Epistemically restricted theory for the classical theory
Mechanics	Liouville mechanics	Epistemically restricted Liouville mechanics = Clifford subtheory of quantum mechanics ADMITS OF NONCONTEXTUAL MODEL
Bits	Statistical theory of bits	Epistemically restricted statistical theory of bits (a.k.a. toy theory) $\simeq$ Clifford subtheory for qubits
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The longstanding puzzle

There seem to be explicit examples of contextuality in the Clifford subtheory of qubits

Ex: the Peres Mermin proof

And yet, any computation within the Clifford subtheory of qubits is efficiently classically simulatable by the Gottesman-Knill theorem

The longstanding puzzle

There seem to be explicit examples of contextuality in the Clifford subtheory of qubits

Ex: the Peres Mermin proof

And yet, any computation within the Clifford subtheory of qubits is efficiently classically simulatable by the Gottesman-Knill theorem

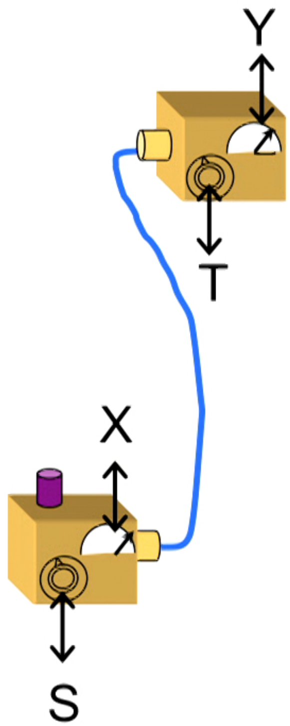
The apparent conclusion:

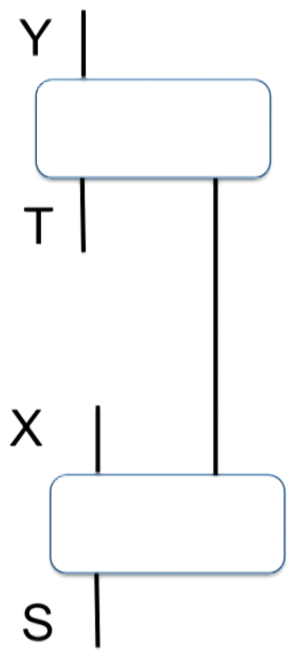
Admitting of a noncontextual model may be a sufficient condition for efficient classical simulatability, but it is not a necessary condition
i.e., contextuality is not always a resource for computation

BUT…

Consider the experimental scenario that allows one to test the principle of noncontextuality

Kunjwal and RWS, PRL 115, 110403 (2015)





P(XY|ST)

is constrained by noncontxtuality inequalities

For Peres-Mermin

Krishna, RWS, Wolfe,
NJP 19, 123031 (2017)

$X \otimes I$	$I \otimes X$	$X \otimes X$
$I \otimes Z$	$Z \otimes I$	$Z \otimes Z$
$X \otimes Z$	$Z \otimes X$	$Y \otimes Y$

9 binary-outcome measurements associated to these observables

9 binary-outcome sources preparing the mixed states onto the eigenspaces of these observables

$X \otimes I$	$I \otimes X$	$X \otimes X$
$I \otimes Z$	$Z \otimes I$	$Z \otimes Z$
$X \otimes Z$	$Z \otimes X$	$Y \otimes Y$

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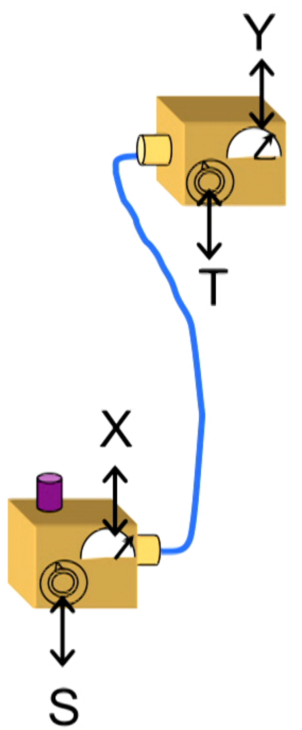

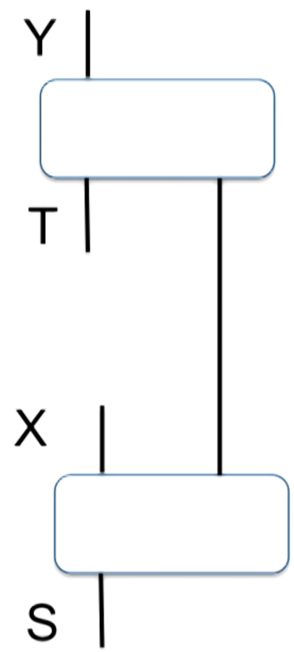

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