Title: PSI 2017/2018 - Machine Learning for Many Body Physics - Lecture 13

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Abstract:

QUANTUM STATE TOMOGRAPHY

Setting up:

- N degrees of freedom, 2d local Hilbert space
- Reference basis $|\vec{\sigma}\rangle = |\sigma_1, \dots, \sigma_N\rangle \quad \sigma_j = 0,1$
- Observations, $\{\hat{O}_k\}$, $\overline{O}_k^{exp}$

ce

QST: find a state $|\psi\rangle$ such that

$$\langle\psi|\hat{O}_x|\psi\rangle = \langle\hat{O}_x\rangle \sim \bar{O}^{exp}$$

PURE STATE: the target $|\psi_T\rangle = \sum_\sigma \psi(\sigma)|\sigma\rangle$ 2^N coefficients ce

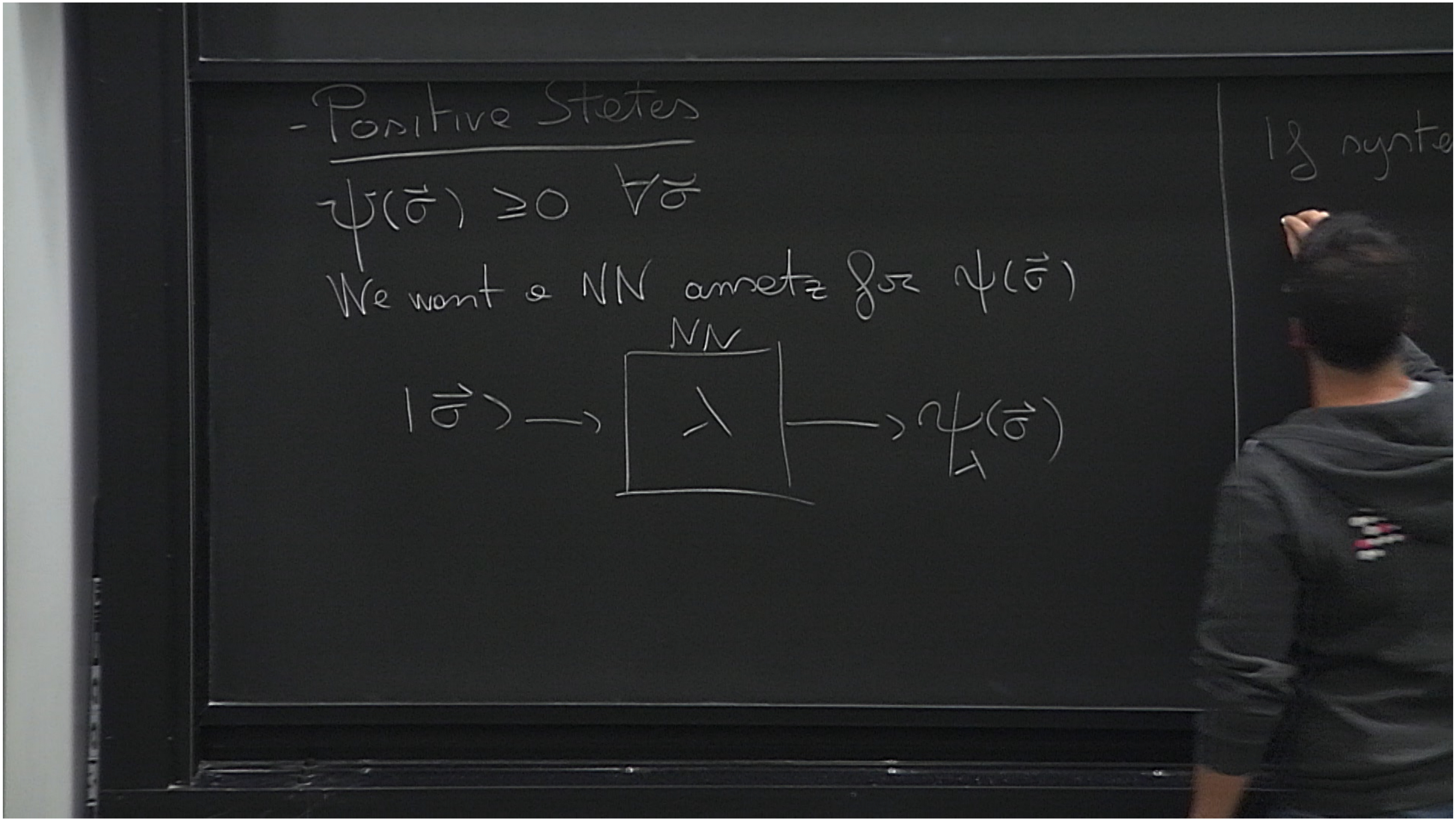
QST: find a state $|\psi\rangle$ such that

$$\langle\psi|\hat{O}_x|\psi\rangle = \langle\hat{O}_x\rangle \sim \bar{O}^{exp}$$

PURE STATE: the target $|\psi_T\rangle = \sum_{\sigma} \psi(\sigma)|\sigma\rangle$ $\quad 2^N$ coefficients

2 ingredient: • REPRESENTATION: we need a compact representation of $|\psi\rangle$

• RECONSTRUCTION


- Positive States
$\psi(\vec{\sigma}) \geq 0 \quad \forall \vec{\sigma}$
We want a NN ansatz for $\psi(\vec{\sigma})$
NN
λ
$|\vec{\sigma}\rangle \longrightarrow \boxed{\lambda} \longrightarrow \psi_\lambda(\vec{\sigma})$

O_k^{exp}

representation of $|\psi\rangle$

• RECONSTRUCTION

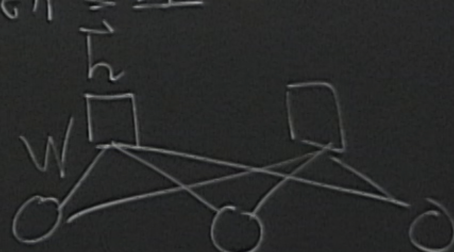
$\psi(\vec\sigma)$

$(\vec\sigma)$

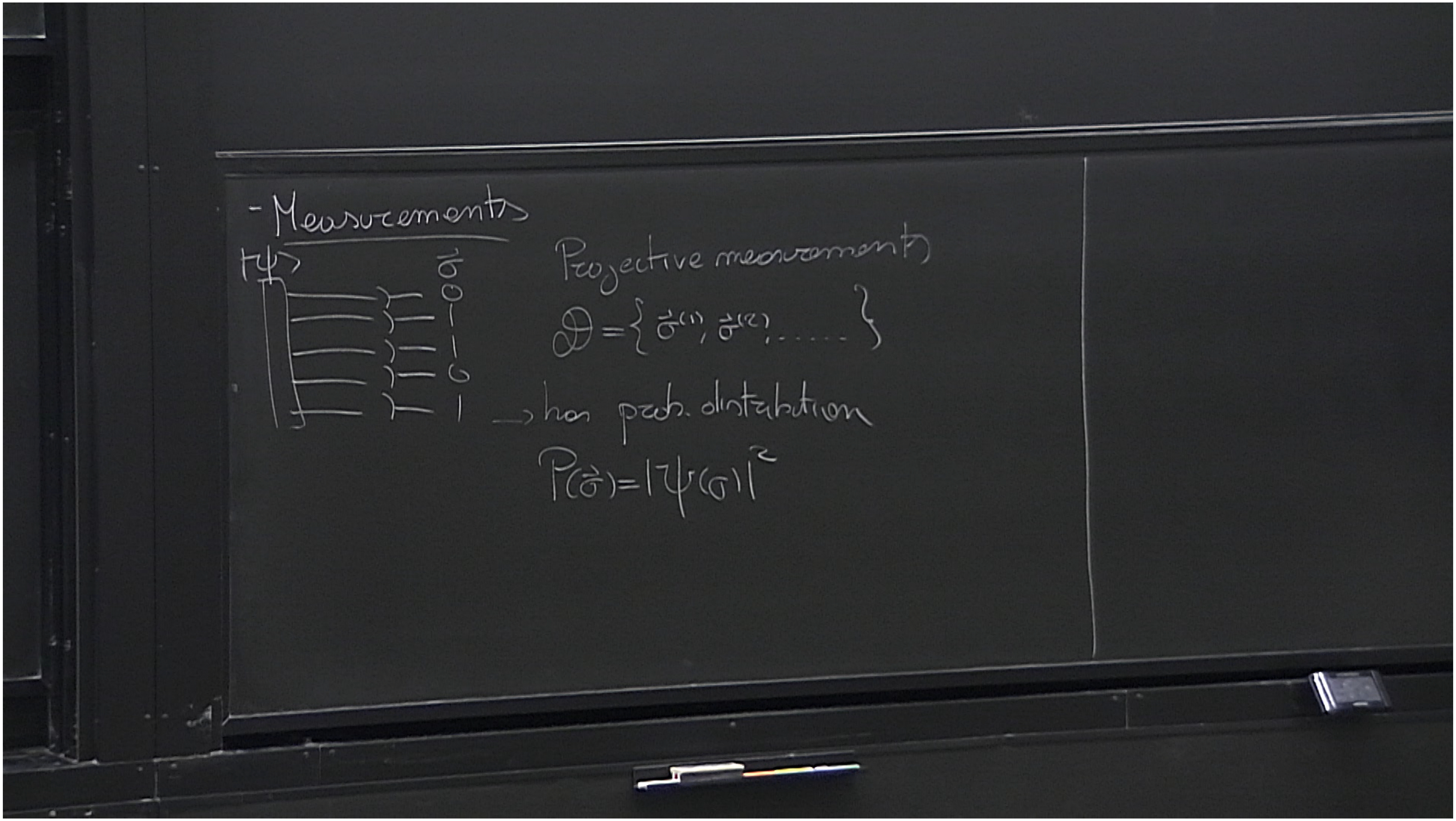
If system is in some state $|\psi\rangle$, the probability of finding $|\vec\sigma\rangle$ is $P(\vec\sigma) = |\langle\vec\sigma|\psi\rangle|^2 = |\psi(\sigma)|^2$

$$\psi_\lambda(\vec\sigma) = \sqrt{\frac{P_\lambda(\vec\sigma)}{Z_\lambda}} \quad , \; Z_\lambda = \sum_\sigma P_\lambda(\sigma) \Rightarrow \sum_\sigma |\psi_\lambda(\vec\sigma)|^2 = 1$$

$$P_\lambda(\sigma) = \sum_h p(\sigma,h) = e^{\underbrace{\vec b\,\vec\sigma}_{\text{MF}} + \underbrace{\sum \log\left(1+e^{W^{(i)}\vec\sigma + c^{(i)}}\right)}_{\text{Correlations}}}$$



$\lambda = \{W, b, c\}$


- Measurements
|ψ⟩
σ⃗
0
1
1
0
1
Projective measurements
𝒪 = { σ⃗⁽¹⁾, σ⃗⁽²⁾, ... }
→ has prob. distribution
P(σ⃗) = |ψ(σ)|²

- KL-Training

Goal: Find λ^* for which $|\psi_\lambda(\sigma)|^2 \sim P(\vec{\sigma}) = |\psi(\vec{\sigma})|^2$

then $\psi_\lambda(\vec{\sigma}) \sim \psi(\vec{\sigma})$.

$$KL(P \,\|\, |\psi_\lambda|^2) = -\sum_\sigma P(\sigma) \log|\psi_\lambda(\sigma)|^2 + \cancel{\mathcal{H}(P)}$$

$$= -\frac{1}{|D|} \sum_k \log|\psi_\lambda(\sigma)|^2$$

$$= \log Z_\lambda - \frac{1}{|D|} \sum_k \log P_\lambda(\vec{\sigma})$$

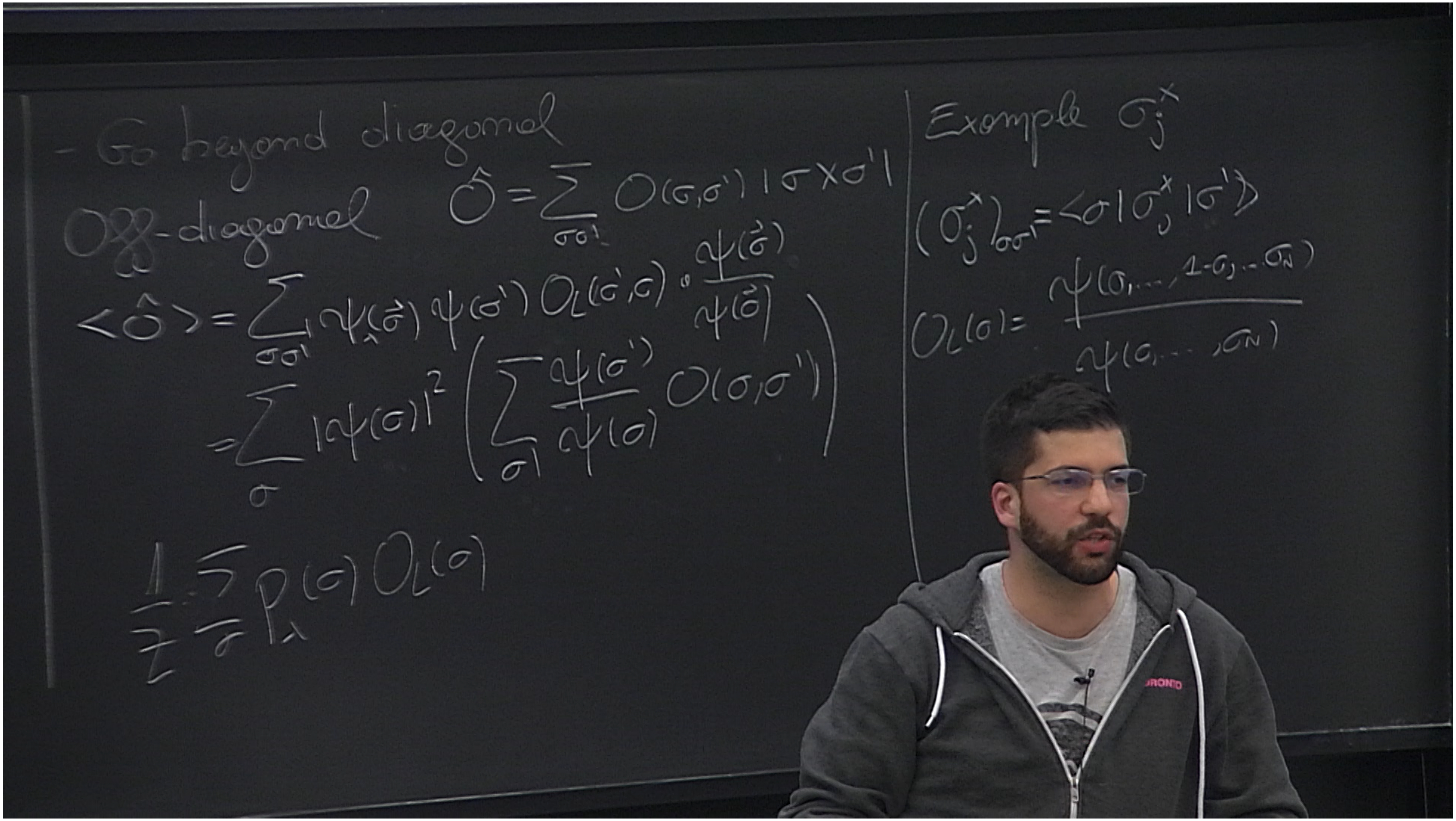
- Evaluation

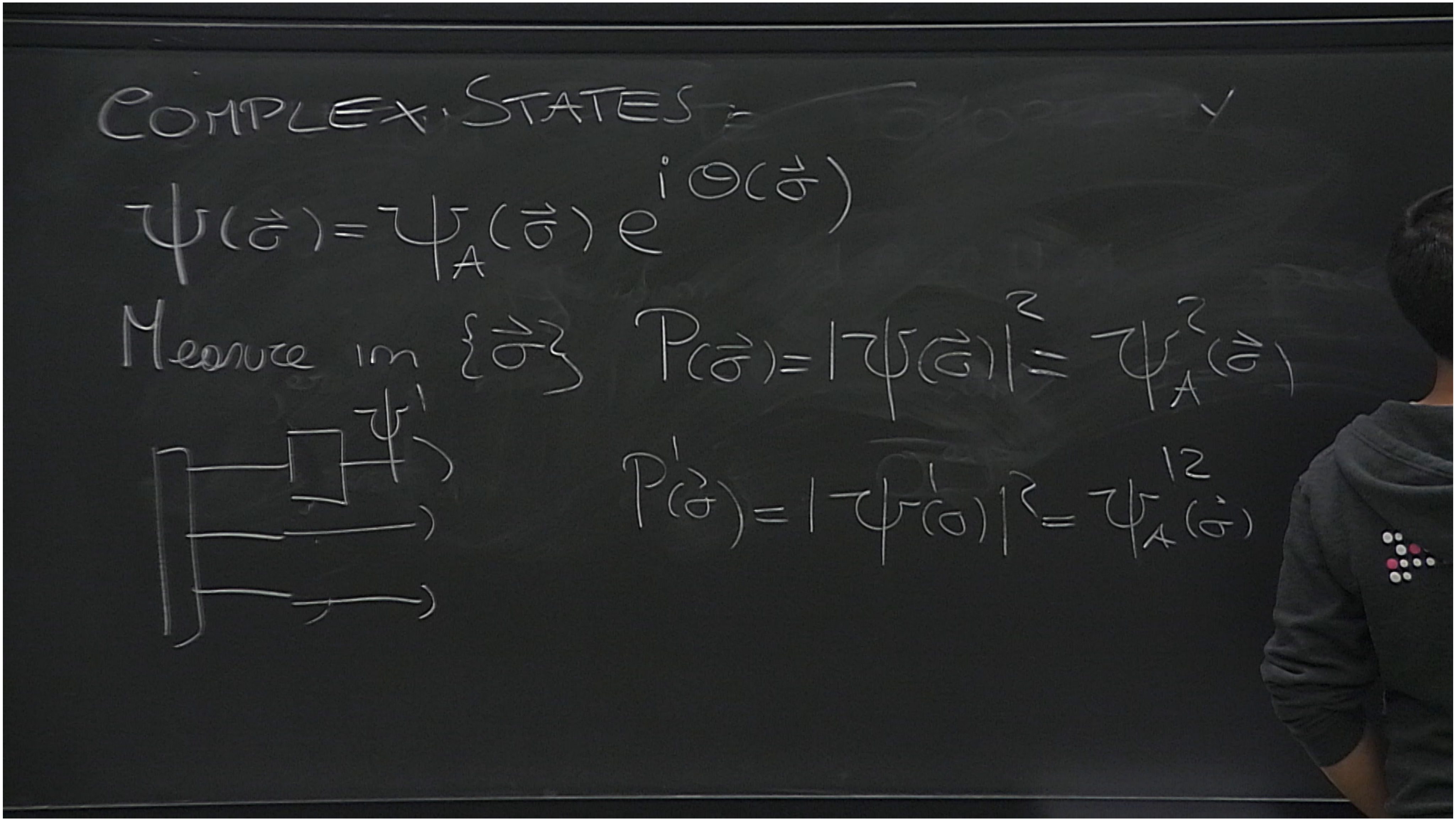
Compare Observables (diagonal) $\hat{O} = \sum_{\sigma} O(\sigma)|\sigma\rangle\langle\sigma|$

- DATA : $\langle \hat{O} \rangle_{DATA} = \frac{1}{|D|} \sum_{k} O(\vec{\sigma})$

- MODEL $\langle \hat{O} \rangle_{RBM} = \langle \psi_\lambda | \hat{O} | \psi_\lambda \rangle = \sum_{\sigma} |\psi_\lambda(\sigma)|^2 O(\vec{\sigma})$

$$= \frac{1}{Z} \sum p_\lambda(\sigma)\, O(\sigma) \simeq \frac{1}{M} \sum_{n} O_n(\vec{\sigma})$$


- Go beyond diagonal
Off-diagonal $\hat{O} = \sum_{\sigma\sigma'} O(\sigma,\sigma') |\sigma\rangle\langle\sigma'|$
$\langle \hat{O} \rangle = \sum_{\sigma\sigma'} \psi_\lambda(\vec{\sigma}) \psi(\sigma') O_L(\sigma',\sigma) \cdot \frac{\psi(\vec{\sigma})}{\psi(\vec{\sigma})}$
$= \sum_{\sigma} |\psi(\sigma)|^2 \left(\sum_{\sigma'} \frac{\psi(\sigma')}{\psi(\sigma)} O(\sigma,\sigma') \right)$
$\frac{1}{Z} \sum_{\sigma} P_\lambda(\sigma) O_L(\sigma)$
Example σ_j^x
$(\sigma_j^x)_{\sigma\sigma'} = \langle \sigma | \sigma_j^x | \sigma' \rangle$
$O_L(\sigma) = \frac{\psi(\sigma_1, \ldots, 1-\sigma_j, \ldots \sigma_N)}{\psi(\sigma_1, \ldots, \sigma_N)}$


COMPLEX STATES
$\psi(\vec{\sigma}) = \psi_A(\vec{\sigma})\, e^{i\theta(\vec{\sigma})}$
Measure in $\{\vec{\sigma}\}$
$P(\vec{\sigma}) = |\psi(\vec{\sigma})|^2 = \psi_A^2(\vec{\sigma})$
ψ'
$P'(\vec{\sigma}) = |\psi'(\vec{\sigma})|^2 = \psi_A'^2(\vec{\sigma})$

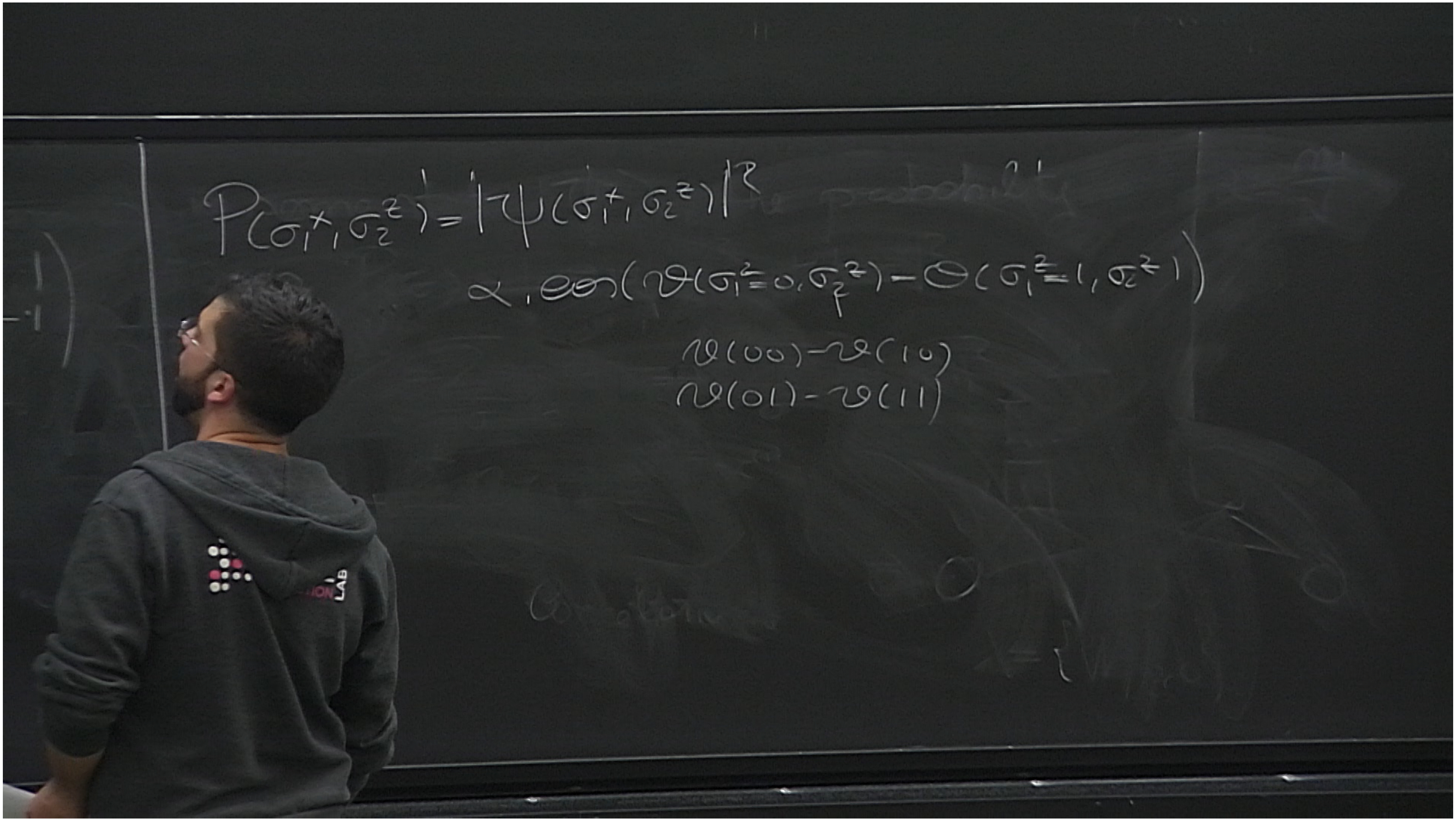
$$|\psi'\rangle = H_1 \otimes \mathbb{1}_2 \, |\psi\rangle$$

$$|\psi'\rangle = \sum_{\sigma_1^x \sigma_2^z} \psi(\sigma_1^x, \sigma_2^z) \, |\sigma_1^x, \sigma_2^z\rangle$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\langle \sigma_1^x \sigma_2^z | \psi\rangle = \sum_{\sigma_1^z} \overbrace{\langle \sigma_1^x | \sigma_1^z\rangle}\, \psi(\sigma_1^z, \sigma_2^z)$$

$$= \psi(\sigma_1^z = 0, \sigma_2^z) + (1 - 2\sigma_1^x)\, \psi(\sigma_1^z = 1, \sigma_2^z)$$


$P(\sigma_1^x, \sigma_2^z) = |\psi(\sigma_1^x, \sigma_2^z)|^2$
$\propto \cos(\vartheta(\sigma_1^z = 0, \sigma_2^z) - \vartheta(\sigma_1^z = 1, \sigma_2^z))$
$\vartheta(00) - \vartheta(10)$
$\vartheta(01) - \vartheta(11)$

$$\psi_{\lambda\mu}(\vec{\sigma}) = \sqrt{\frac{p_\lambda(\vec{\sigma})}{Z_\lambda}}\, e^{i\varphi_\mu(\vec{\sigma})}$$

$$\varphi_\mu = \log p_\mu(\vec{\sigma})$$