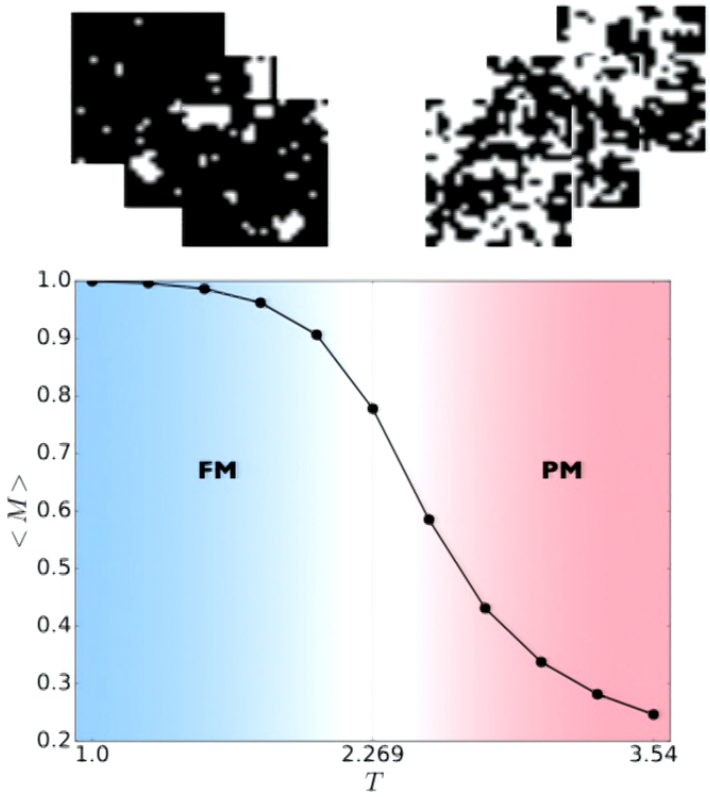
Title: PSI 2017/2018 - Machine Learning for Many Body Physics - Lecture 14

Date: Apr 24, 2018 09:00 AM

URL: http://pirsa.org/18040065

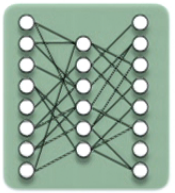
Abstract:

Learning thermodynamics of the Ising model



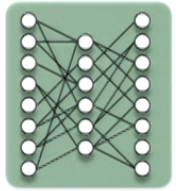
$$p_{\text{low}}(\boldsymbol{\sigma}) \propto \mathrm{e}^{-\beta_{\text{low}} H(\boldsymbol{\sigma})}$$

$$p_{\text{nn}} \simeq p_{\text{low}}$$

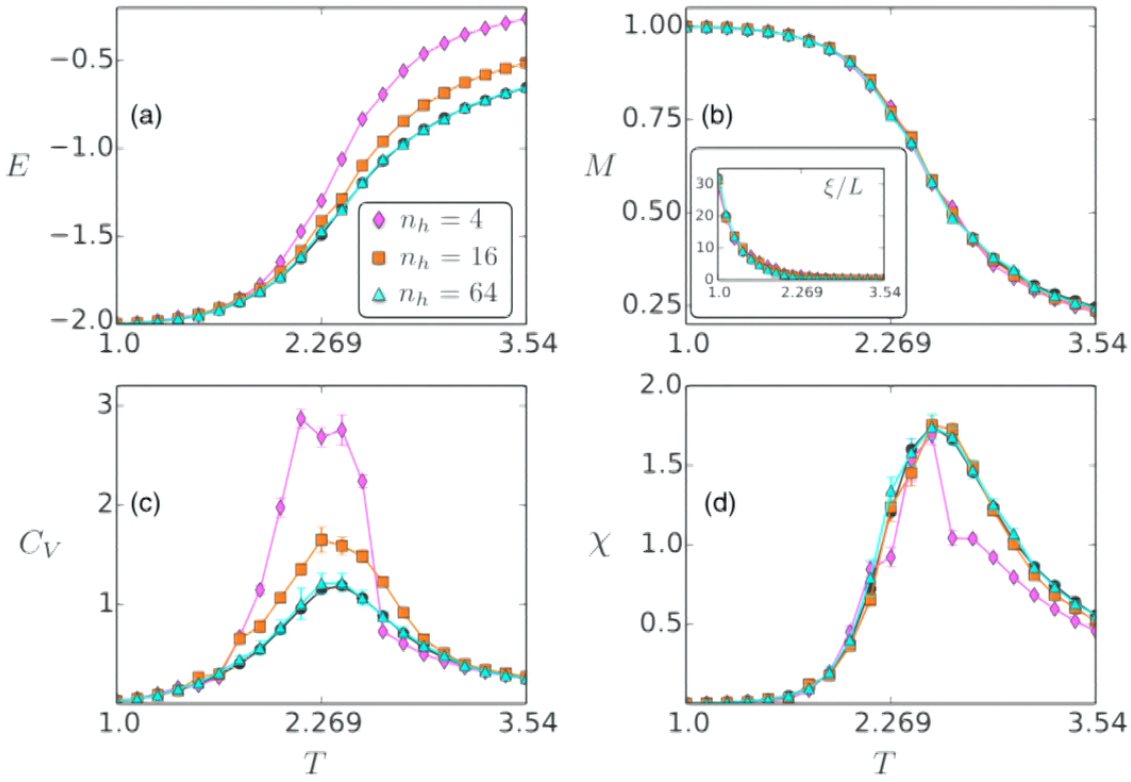


$$p_{\text{high}}(\boldsymbol{\sigma}) \propto \mathrm{e}^{-\beta_{\text{high}} H(\boldsymbol{\sigma})}$$

$$p_{\text{nn}} \simeq p_{\text{high}}$$



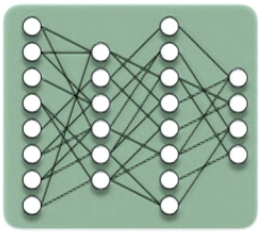
Learning thermodynamics of the Ising model

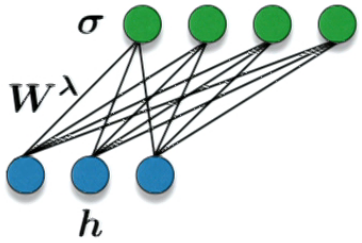


GT and Melko, *Physical Review B 2016* (1606.02718)

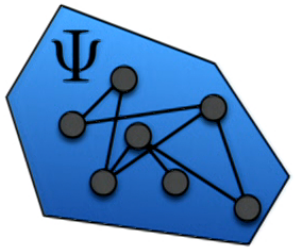
NN-QST of positive wavefunctions

$(\boldsymbol{\lambda})$

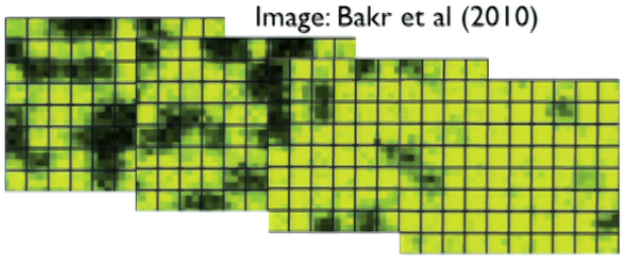
$|\boldsymbol{\sigma}\rangle$ →  → $\psi_{\boldsymbol{\lambda}}(\boldsymbol{\sigma}) = \sqrt{\frac{p_{\boldsymbol{\lambda}}(\boldsymbol{\sigma})}{Z_{\boldsymbol{\lambda}}}}$



Restricted Boltzman Machine: $p_{\boldsymbol{\lambda}}(\boldsymbol{\sigma}) = \frac{1}{Z_{\boldsymbol{\lambda}}} \sum_{\boldsymbol{h}} \mathrm{e}^{-E_{\boldsymbol{\lambda}}(\boldsymbol{\sigma},\boldsymbol{h})}$



→



$P_{\mathrm{data}}(\boldsymbol{\sigma}) = |\Psi(\boldsymbol{\sigma})|^2$

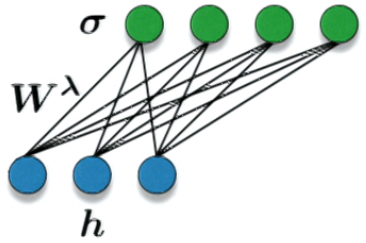
Training: tune the parameters $(\boldsymbol{\lambda})$ to achieve: $|\psi_{\boldsymbol{\lambda}}(\boldsymbol{\sigma})|^2 \sim P_{\mathrm{data}}(\boldsymbol{\sigma})$

GT et al, *Nature Physics 2017* (1703.05334)

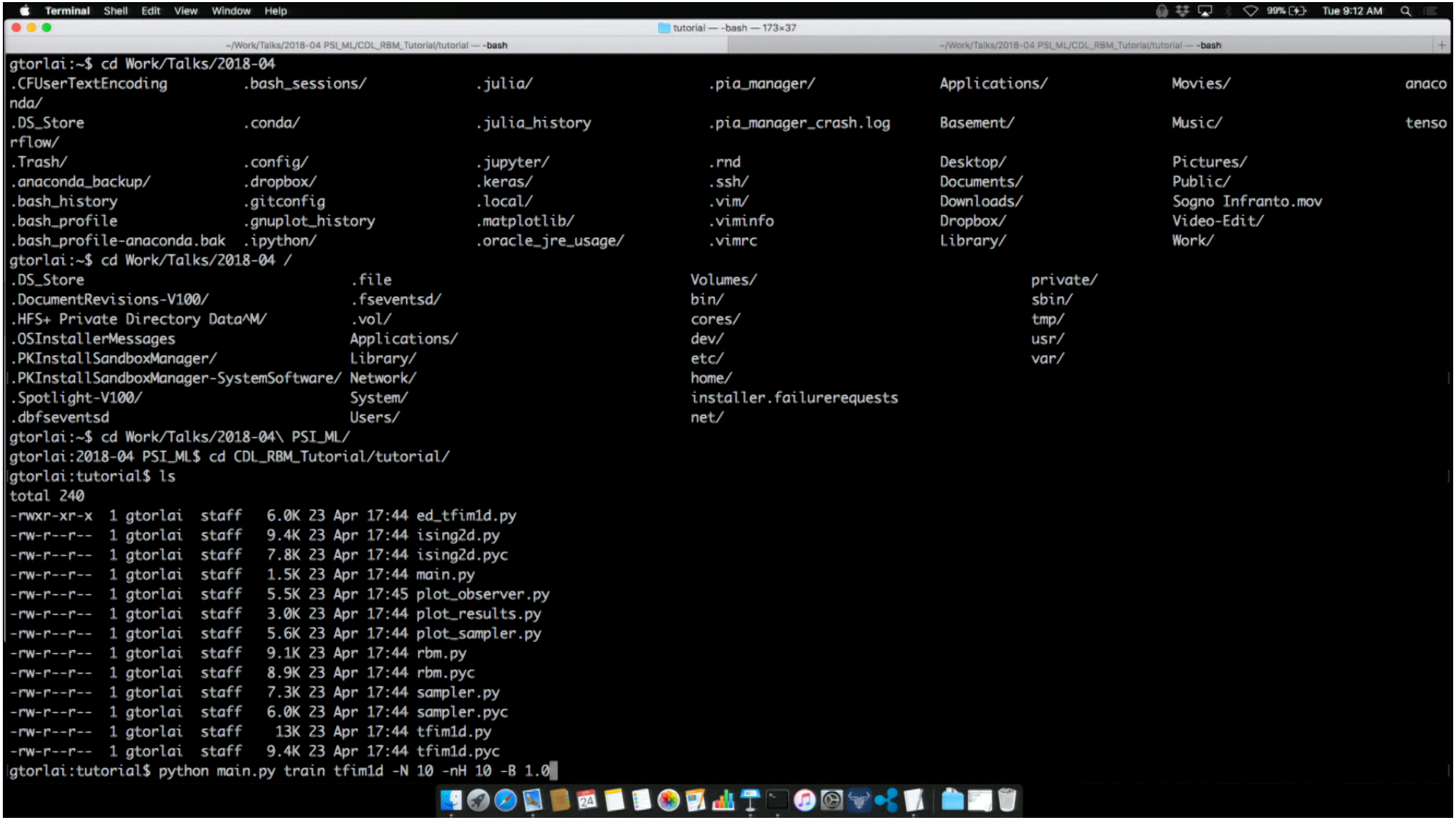
NN-QST of positive wavefunctions

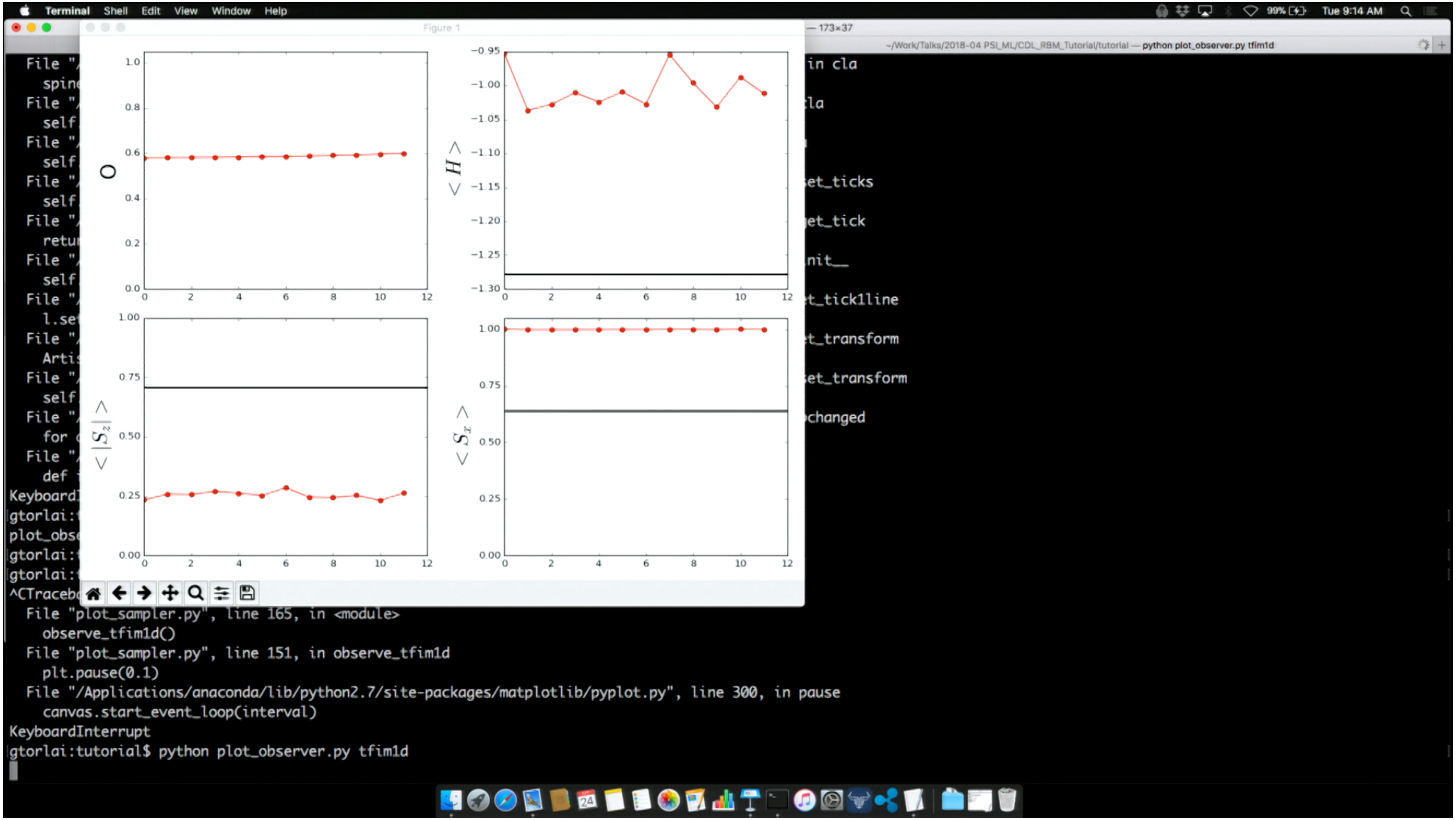
Ground states of local Hamiltonian

$$\psi_{\lambda}(\sigma) = \sqrt{\frac{p_{\lambda}(\sigma)}{Z_{\lambda}}}$$



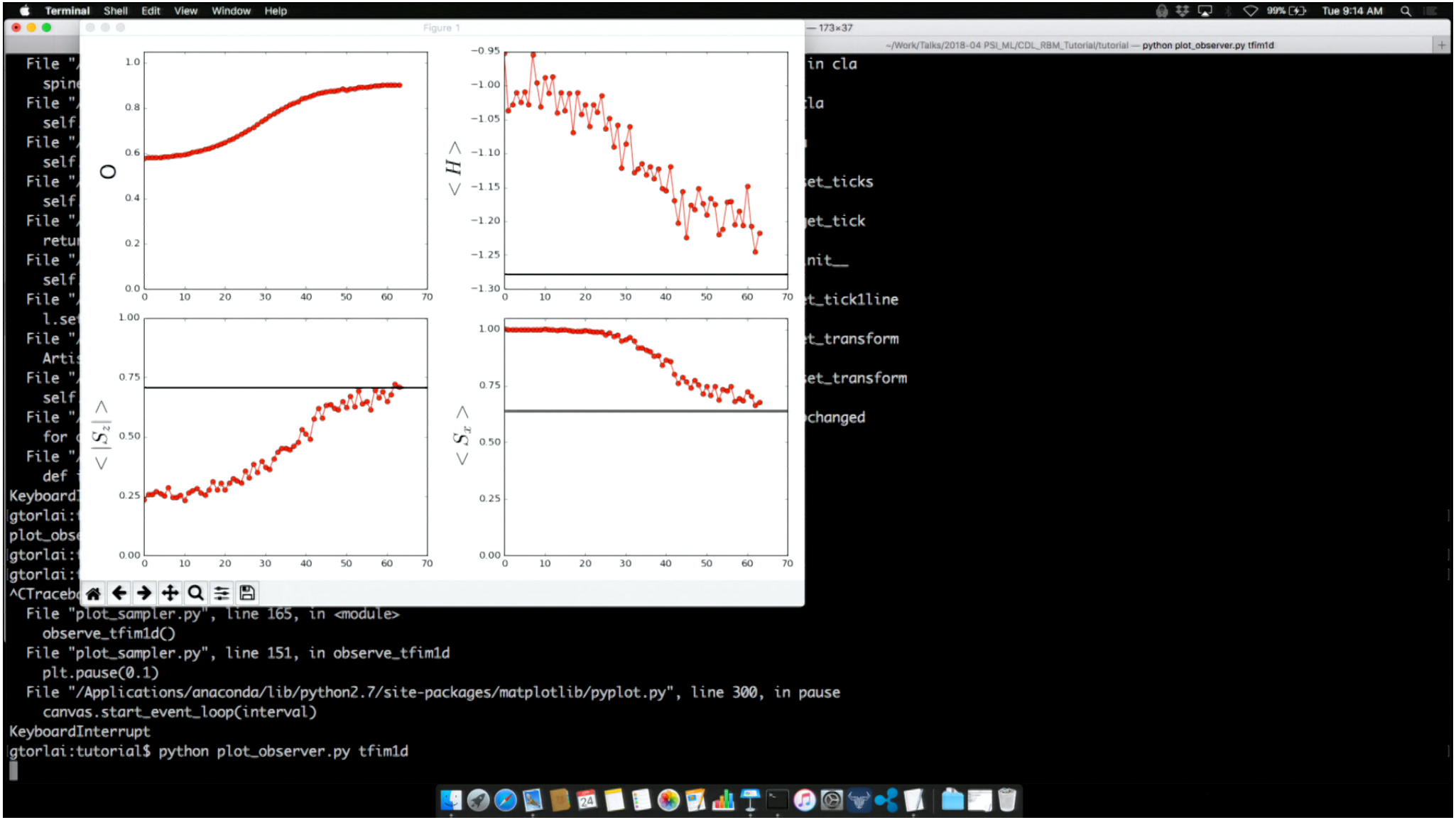
GT et al, *Nature Physics 2017* (1703.05334)

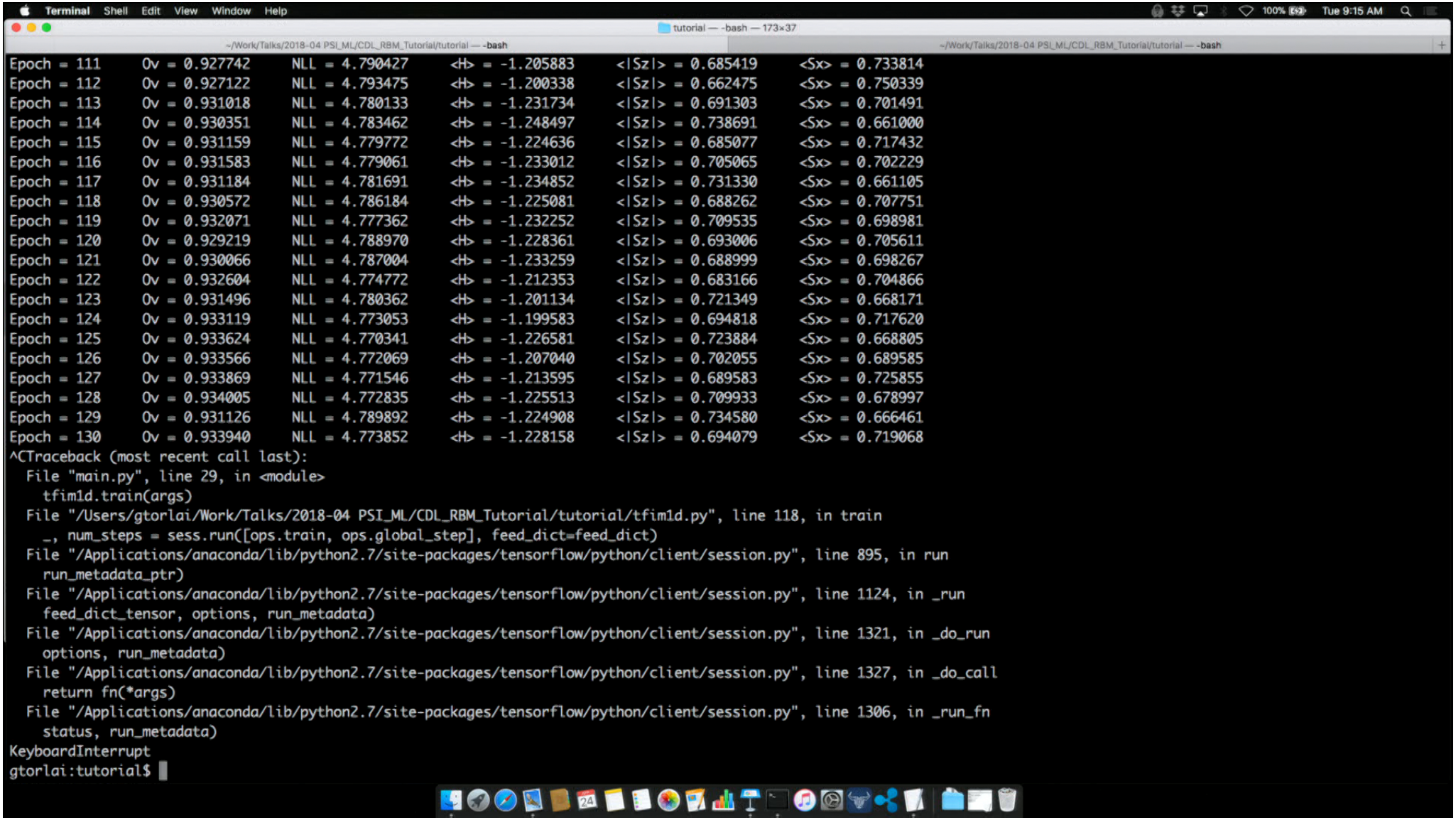

Terminal Shell Edit View Window Help
99% Tue 9:12 AM
tutorial — -bash — 173×37
~/Work/Talks/2018-04 PSI_ML/CDL_RBM_Tutorial/tutorial — -bash
~/Work/Talks/2018-04 PSI_ML/CDL_RBM_Tutorial/tutorial — -bash
gtorlai:~$ cd Work/Talks/2018-04
.CFUserTextEncoding .bash_sessions/ .julia/ .pia_manager/ Applications/ Movies/ anaconda/
.DS_Store .conda/ .julia_history .pia_manager_crash.log Basement/ Music/ tensorflow/
.Trash/ .config/ .jupyter/ .rnd Desktop/ Pictures/
.anaconda_backup/ .dropbox/ .keras/ .ssh/ Documents/ Public/
.bash_history .gitconfig .local/ .vim/ Downloads/ Sogno Infranto.mov
.bash_profile .gnuplot_history .matplotlib/ .viminfo Dropbox/ Video-Edit/
.bash_profile-anaconda.bak .ipython/ .oracle_jre_usage/ .vimrc Library/ Work/
gtorlai:~$ cd Work/Talks/2018-04 /
.DS_Store .file Volumes/ private/
.DocumentRevisions-V100/ .fseventsd/ bin/ sbin/
.HFS+ Private Directory Data^M/ .vol/ cores/ tmp/
.OSInstallerMessages Applications/ dev/ usr/
.PKInstallSandboxManager/ Library/ etc/ var/
.PKInstallSandboxManager-SystemSoftware/ Network/ home/
.Spotlight-V100/ System/ installer.failurerequests
.dbfseventsd Users/ net/
gtorlai:~$ cd Work/Talks/2018-04\ PSI_ML/
gtorlai:2018-04 PSI_ML$ cd CDL_RBM_Tutorial/tutorial/
gtorlai:tutorial$ ls
total 240
-rwxr-xr-x 1 gtorlai staff 6.0K 23 Apr 17:44 ed_tfim1d.py
-rw-r--r-- 1 gtorlai staff 9.4K 23 Apr 17:44 ising2d.py
-rw-r--r-- 1 gtorlai staff 7.8K 23 Apr 17:44 ising2d.pyc
-rw-r--r-- 1 gtorlai staff 1.5K 23 Apr 17:44 main.py
-rw-r--r-- 1 gtorlai staff 5.5K 23 Apr 17:45 plot_observer.py
-rw-r--r-- 1 gtorlai staff 3.0K 23 Apr 17:44 plot_results.py
-rw-r--r-- 1 gtorlai staff 5.6K 23 Apr 17:44 plot_sampler.py
-rw-r--r-- 1 gtorlai staff 9.1K 23 Apr 17:44 rbm.py
-rw-r--r-- 1 gtorlai staff 8.9K 23 Apr 17:44 rbm.pyc
-rw-r--r-- 1 gtorlai staff 7.3K 23 Apr 17:44 sampler.py
-rw-r--r-- 1 gtorlai staff 6.0K 23 Apr 17:44 sampler.pyc
-rw-r--r-- 1 gtorlai staff 13K 23 Apr 17:44 tfim1d.py
-rw-r--r-- 1 gtorlai staff 9.4K 23 Apr 17:44 tfim1d.pyc
gtorlai:tutorial$ python main.py train tfim1d -N 10 -nH 10 -B 1.0

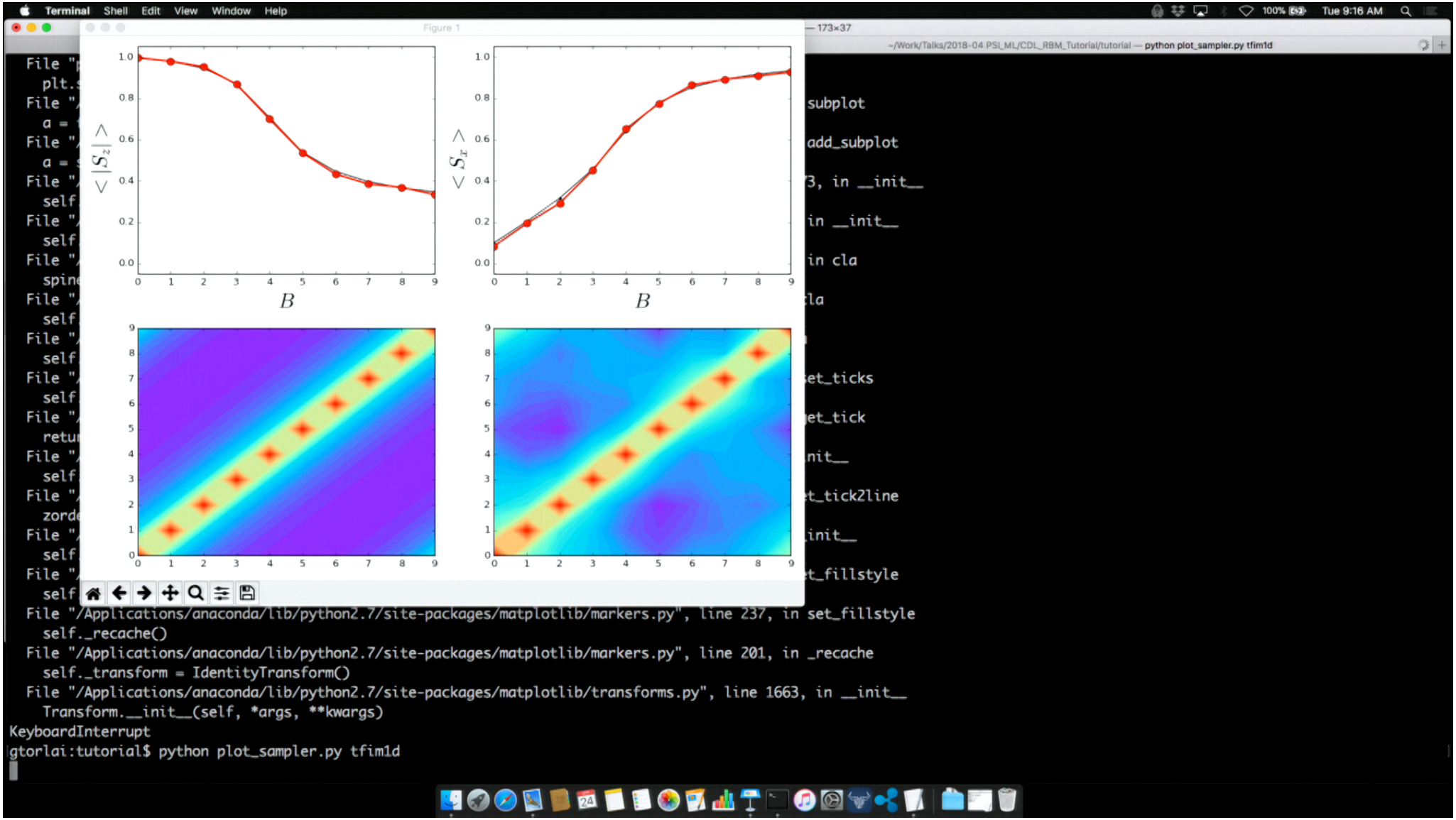

Terminal Shell Edit View Window Help
Tue 9:14 AM
Figure 1
O
$\langle H \rangle$
$\langle |S_z| \rangle$
$\langle S_x \rangle$
File "plot_sampler.py", line 165, in <module>
observe_tfim1d()
File "plot_sampler.py", line 151, in observe_tfim1d
plt.pause(0.1)
File "/Applications/anaconda/lib/python2.7/site-packages/matplotlib/pyplot.py", line 300, in pause
canvas.start_event_loop(interval)
KeyboardInterrupt
gtorlai:tutorial$ python plot_observer.py tfim1d

$$\alpha = \frac{n_H}{N} \qquad H = -\sum_{i=1}^{N} \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z - h \sum_{i=1}^{N} \hat{\sigma}_i^x$$

$$O = |\langle \psi | \psi \rangle| \xrightarrow[h_c = 1.0]{F \uparrow \quad P \uparrow} h$$

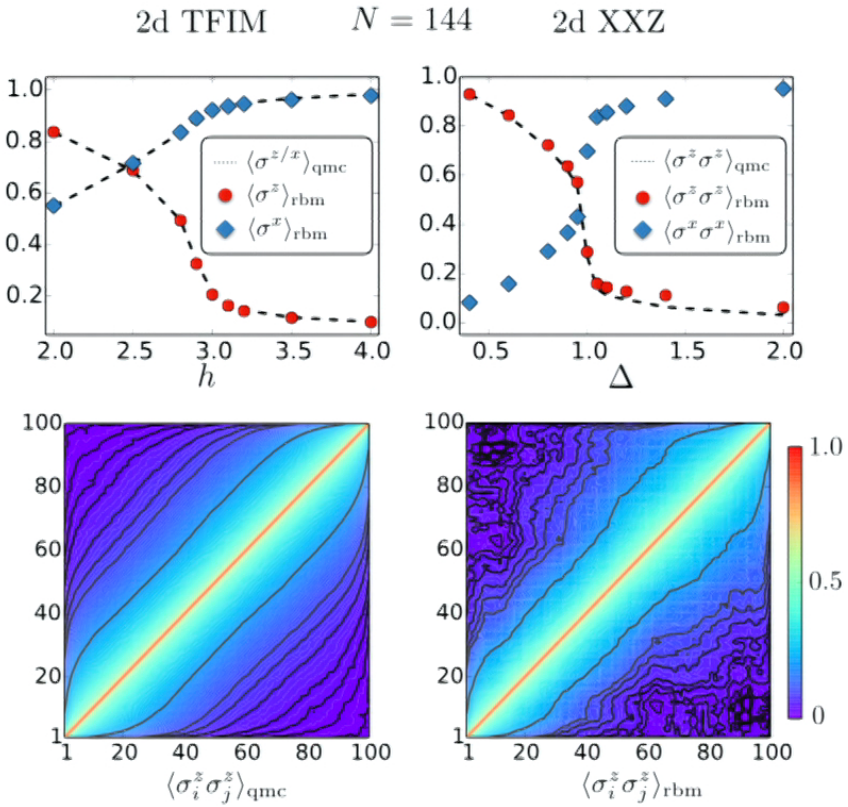

Terminal Shell Edit View Window Help
Tue 9:14 AM
Figure 1
~/Work/Talks/2018-04 PSI_ML/CDL_RBM_Tutorial/tutorial — python plot_observer.py tfim1d
O
$\langle H \rangle$
$\langle |S_z| \rangle$
$\langle S_x \rangle$
File "plot_sampler.py", line 165, in <module>
observe_tfim1d()
File "plot_sampler.py", line 151, in observe_tfim1d
plt.pause(0.1)
File "/Applications/anaconda/lib/python2.7/site-packages/matplotlib/pyplot.py", line 300, in pause
canvas.start_event_loop(interval)
KeyboardInterrupt
gtorlai:tutorial$ python plot_observer.py tfim1d


Terminal Shell Edit View Window Help
Tue 9:15 AM
tutorial — -bash — 173×37
~/Work/Talks/2018-04 PSI_ML/CDL_RBM_Tutorial/tutorial — -bash
~/Work/Talks/2018-04 PSI_ML/CDL_RBM_Tutorial/tutorial — -bash
Epoch = 111 Ov = 0.927742 NLL = 4.790427 <H> = -1.205883 <|Sz|> = 0.685419 <Sx> = 0.733814
Epoch = 112 Ov = 0.927122 NLL = 4.793475 <H> = -1.200338 <|Sz|> = 0.662475 <Sx> = 0.750339
Epoch = 113 Ov = 0.931018 NLL = 4.780133 <H> = -1.231734 <|Sz|> = 0.691303 <Sx> = 0.701491
Epoch = 114 Ov = 0.930351 NLL = 4.783462 <H> = -1.248497 <|Sz|> = 0.738691 <Sx> = 0.661000
Epoch = 115 Ov = 0.931159 NLL = 4.779772 <H> = -1.224636 <|Sz|> = 0.685077 <Sx> = 0.717432
Epoch = 116 Ov = 0.931583 NLL = 4.779061 <H> = -1.233012 <|Sz|> = 0.705065 <Sx> = 0.702229
Epoch = 117 Ov = 0.931184 NLL = 4.781691 <H> = -1.234852 <|Sz|> = 0.731330 <Sx> = 0.661105
Epoch = 118 Ov = 0.930572 NLL = 4.786184 <H> = -1.225081 <|Sz|> = 0.688262 <Sx> = 0.707751
Epoch = 119 Ov = 0.932071 NLL = 4.777362 <H> = -1.232252 <|Sz|> = 0.709535 <Sx> = 0.698981
Epoch = 120 Ov = 0.929219 NLL = 4.788970 <H> = -1.228361 <|Sz|> = 0.693006 <Sx> = 0.705611
Epoch = 121 Ov = 0.930066 NLL = 4.787004 <H> = -1.233259 <|Sz|> = 0.688999 <Sx> = 0.698267
Epoch = 122 Ov = 0.932604 NLL = 4.774772 <H> = -1.212353 <|Sz|> = 0.683166 <Sx> = 0.704866
Epoch = 123 Ov = 0.931496 NLL = 4.780362 <H> = -1.201134 <|Sz|> = 0.721349 <Sx> = 0.668171
Epoch = 124 Ov = 0.933119 NLL = 4.773053 <H> = -1.199583 <|Sz|> = 0.694818 <Sx> = 0.717620
Epoch = 125 Ov = 0.933624 NLL = 4.770341 <H> = -1.226581 <|Sz|> = 0.723884 <Sx> = 0.668805
Epoch = 126 Ov = 0.933566 NLL = 4.772069 <H> = -1.207040 <|Sz|> = 0.702055 <Sx> = 0.689585
Epoch = 127 Ov = 0.933869 NLL = 4.771546 <H> = -1.213595 <|Sz|> = 0.689583 <Sx> = 0.725855
Epoch = 128 Ov = 0.934005 NLL = 4.772835 <H> = -1.225513 <|Sz|> = 0.709933 <Sx> = 0.678997
Epoch = 129 Ov = 0.931126 NLL = 4.789892 <H> = -1.224908 <|Sz|> = 0.734580 <Sx> = 0.666461
Epoch = 130 Ov = 0.933940 NLL = 4.773852 <H> = -1.228158 <|Sz|> = 0.694079 <Sx> = 0.719068
^CTraceback (most recent call last):
File "main.py", line 29, in <module>
tfim1d.train(args)
File "/Users/gtorlai/Work/Talks/2018-04 PSI_ML/CDL_RBM_Tutorial/tutorial/tfim1d.py", line 118, in train
_, num_steps = sess.run([ops.train, ops.global_step], feed_dict=feed_dict)
File "/Applications/anaconda/lib/python2.7/site-packages/tensorflow/python/client/session.py", line 895, in run
run_metadata_ptr)
File "/Applications/anaconda/lib/python2.7/site-packages/tensorflow/python/client/session.py", line 1124, in _run
feed_dict_tensor, options, run_metadata)
File "/Applications/anaconda/lib/python2.7/site-packages/tensorflow/python/client/session.py", line 1321, in _do_run
options, run_metadata)
File "/Applications/anaconda/lib/python2.7/site-packages/tensorflow/python/client/session.py", line 1327, in _do_call
return fn(*args)
File "/Applications/anaconda/lib/python2.7/site-packages/tensorflow/python/client/session.py", line 1306, in _run_fn
status, run_metadata)
KeyboardInterrupt
gtorlai:tutorial$

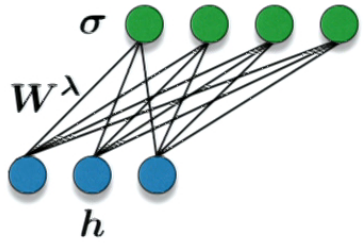

Terminal Shell Edit View Window Help
Tue 9:16 AM
Figure 1
$\langle |S_z| \rangle$
$\langle S_x \rangle$
B
B
File "/Applications/anaconda/lib/python2.7/site-packages/matplotlib/markers.py", line 237, in set_fillstyle
self._recache()
File "/Applications/anaconda/lib/python2.7/site-packages/matplotlib/markers.py", line 201, in _recache
self._transform = IdentityTransform()
File "/Applications/anaconda/lib/python2.7/site-packages/matplotlib/transforms.py", line 1663, in __init__
Transform.__init__(self, *args, **kwargs)
KeyboardInterrupt
gtorlai:tutorial$ python plot_sampler.py tfim1d

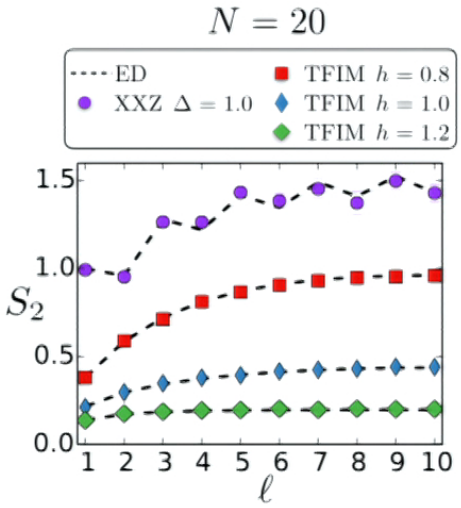
NN-QST of positive wavefunctions

Ground states of local Hamiltonian



$$\psi_{\boldsymbol{\lambda}}(\boldsymbol{\sigma}) = \sqrt{\frac{p_{\boldsymbol{\lambda}}(\boldsymbol{\sigma})}{Z_{\boldsymbol{\lambda}}}}$$

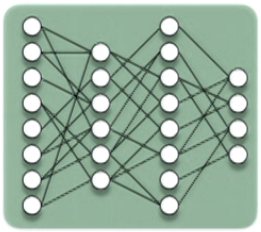


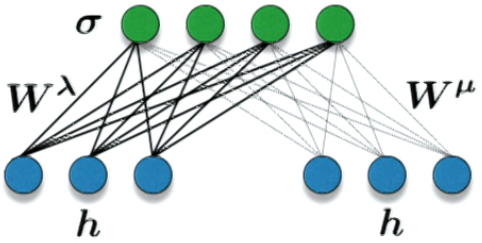


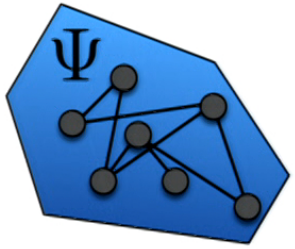
GT et al, *Nature Physics 2017* (1703.05334)

NN-QST of complex wavefunctions

$(\boldsymbol{\lambda}, \boldsymbol{\mu})$

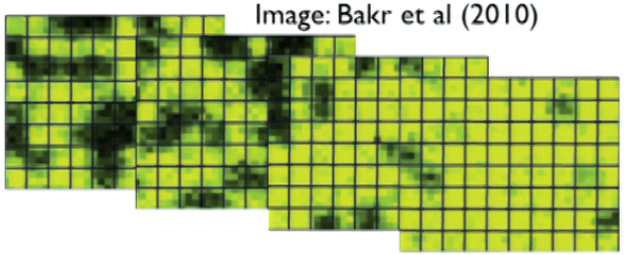
$|\boldsymbol{\sigma}\rangle$ →  → $\psi_{\boldsymbol{\lambda}\boldsymbol{\mu}}(\boldsymbol{\sigma}) = \sqrt{\frac{p_{\boldsymbol{\lambda}}(\boldsymbol{\sigma})}{Z_{\boldsymbol{\lambda}}}} \mathrm{e}^{i \log p_{\boldsymbol{\mu}}(\boldsymbol{\sigma})/2}$





 →

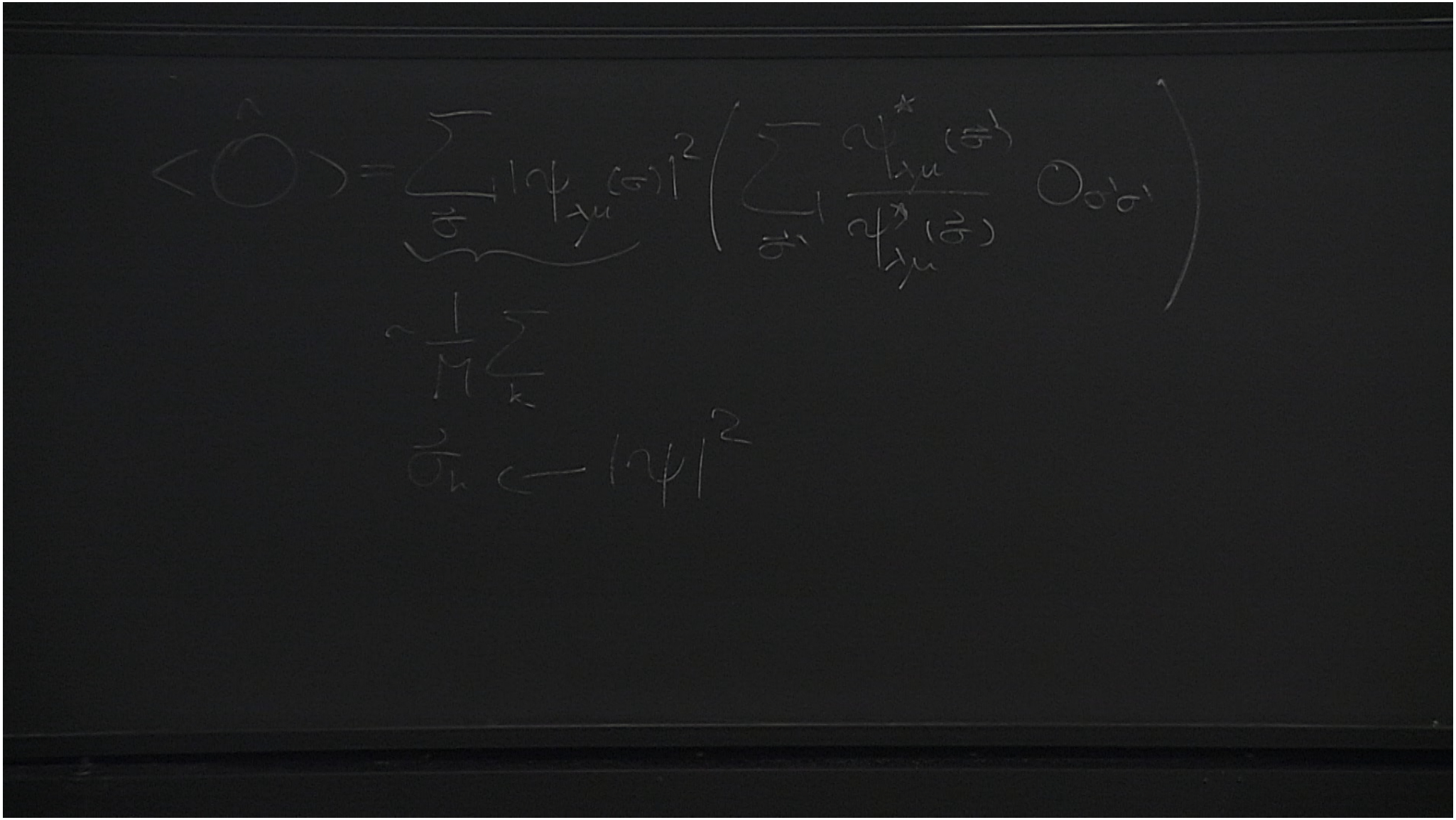
Image: Bakr et al (2010)



$P_{\mathrm{data}}(\boldsymbol{\sigma}^{\boldsymbol{b}}) = |\Psi(\boldsymbol{\sigma}^{\boldsymbol{b}})|^2$

Training: tune the parameters $(\boldsymbol{\lambda}, \boldsymbol{\mu})$ to achieve: $|\psi_{\boldsymbol{\lambda}\boldsymbol{\mu}}(\boldsymbol{\sigma}^{\boldsymbol{b}})|^2 \sim P_{\mathrm{data}}(\boldsymbol{\sigma}^{\boldsymbol{b}})$ in any basis $\{\boldsymbol{\sigma}^{\boldsymbol{b}}\}$

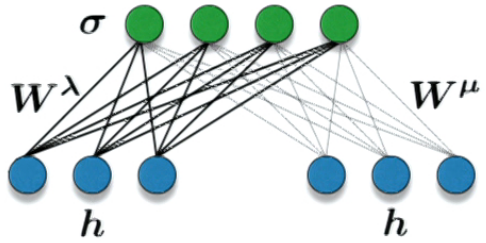
GT et al, *Nature Physics 2017* (1703.05334)


$\langle \hat{O} \rangle = \sum_{\sigma} |\psi_{\lambda\mu}(\sigma)|^2 \left(\sum_{\sigma'} \frac{\psi^*_{\lambda\mu}(\sigma')}{\psi^*_{\lambda\mu}(\sigma)} O_{\sigma'\sigma} \right)$
$\sim \frac{1}{M} \sum_k$
$\sigma_k \leftarrow |\psi|^2$

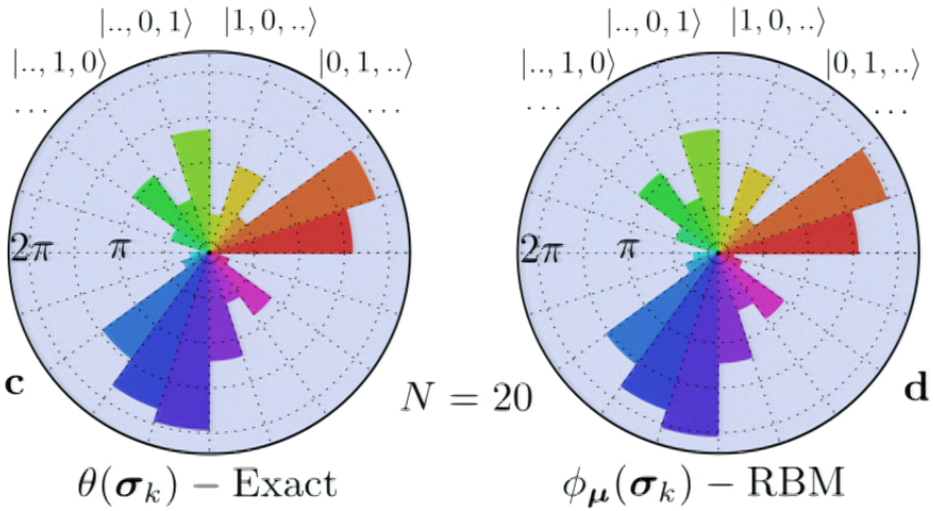
NN-QST of complex wavefunctions

W state

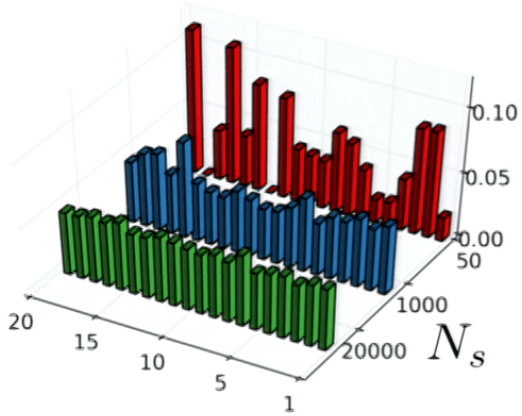
$$|\Psi_W\rangle = \frac{1}{\sqrt{N}}(e^{i\theta_0}|100\ldots0\rangle + e^{i\theta_1}|010\ldots0\rangle + \cdots + e^{i\theta_{N-1}}|000\ldots1\rangle)$$



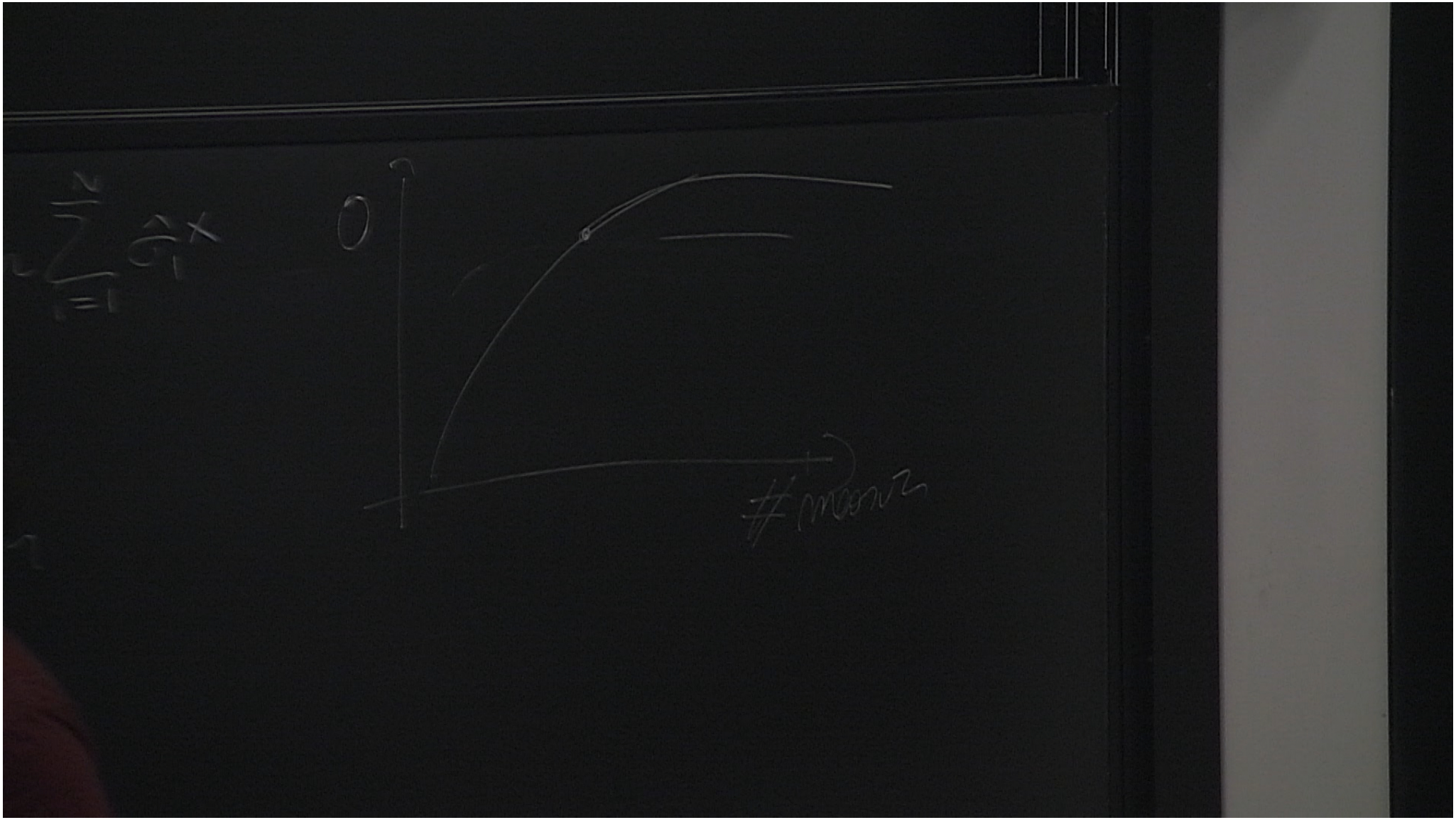
Phases



Amplitudes



GT et al, *Nature Physics 2017* (1703.05334)

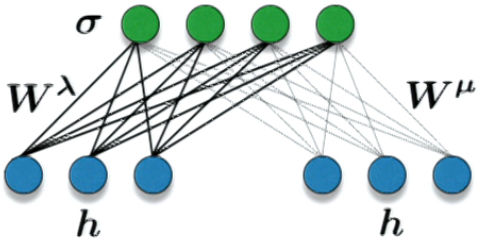


NN-QST of complex wavefunctions

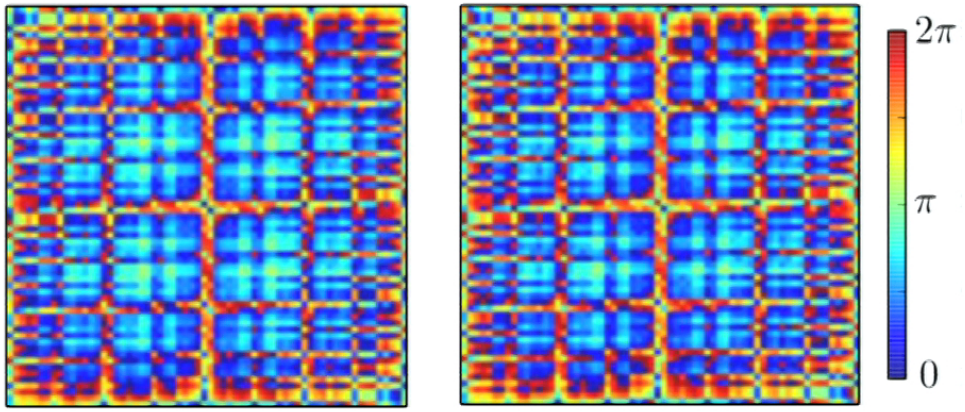
Quench dynamics

$$|\Psi(t)\rangle = \mathrm{e}^{it\mathcal{H}}|\Psi_0\rangle \qquad |\Psi_0\rangle = |\rightarrow,\ldots,\rightarrow\rangle$$

$$\mathcal{H} = \sum_{ij} J_{ij}\sigma_i^z\sigma_j^z - h\sum_i \sigma_i^x \qquad J_{ij} = |i-j|^{-\gamma}$$

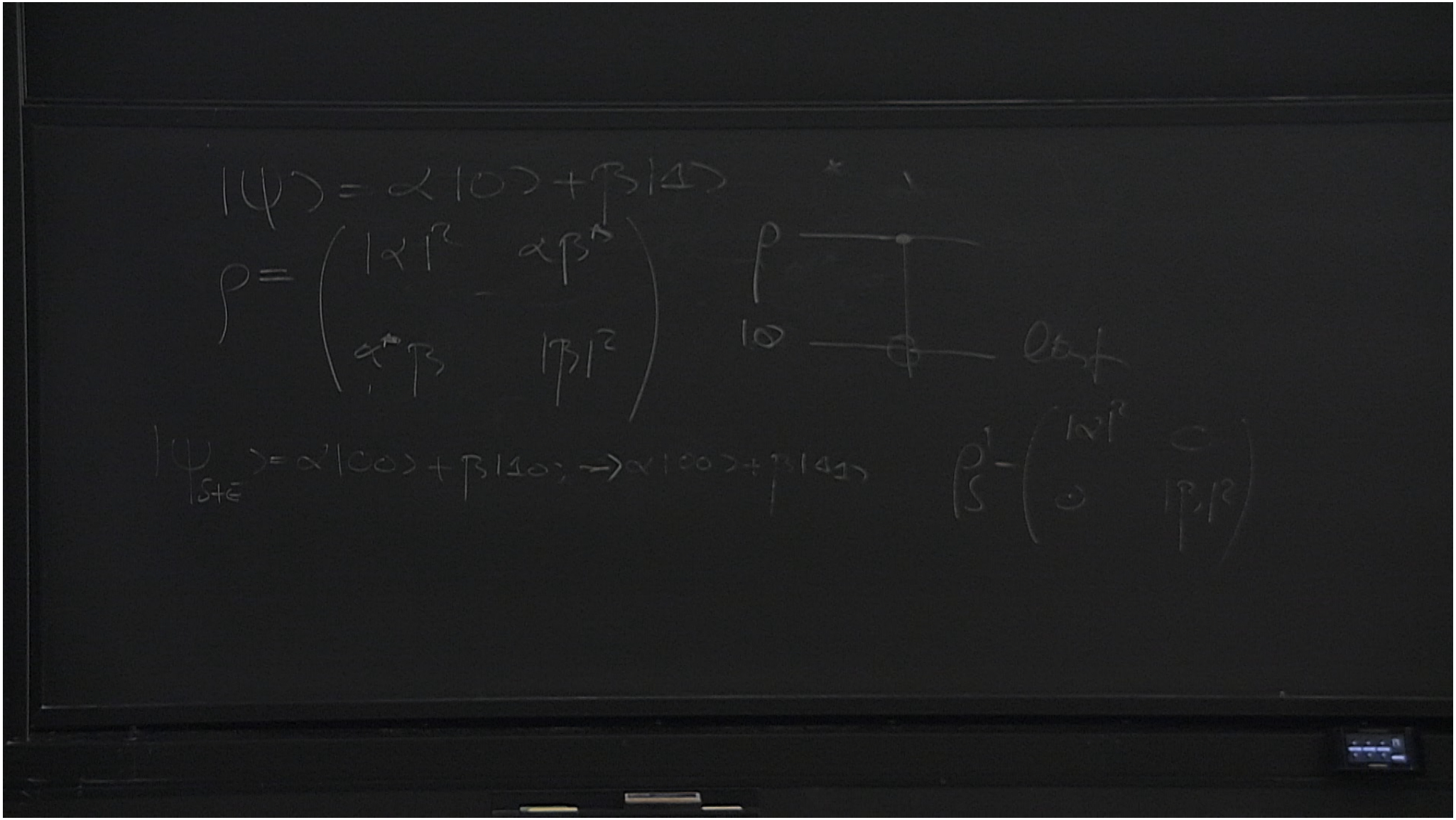


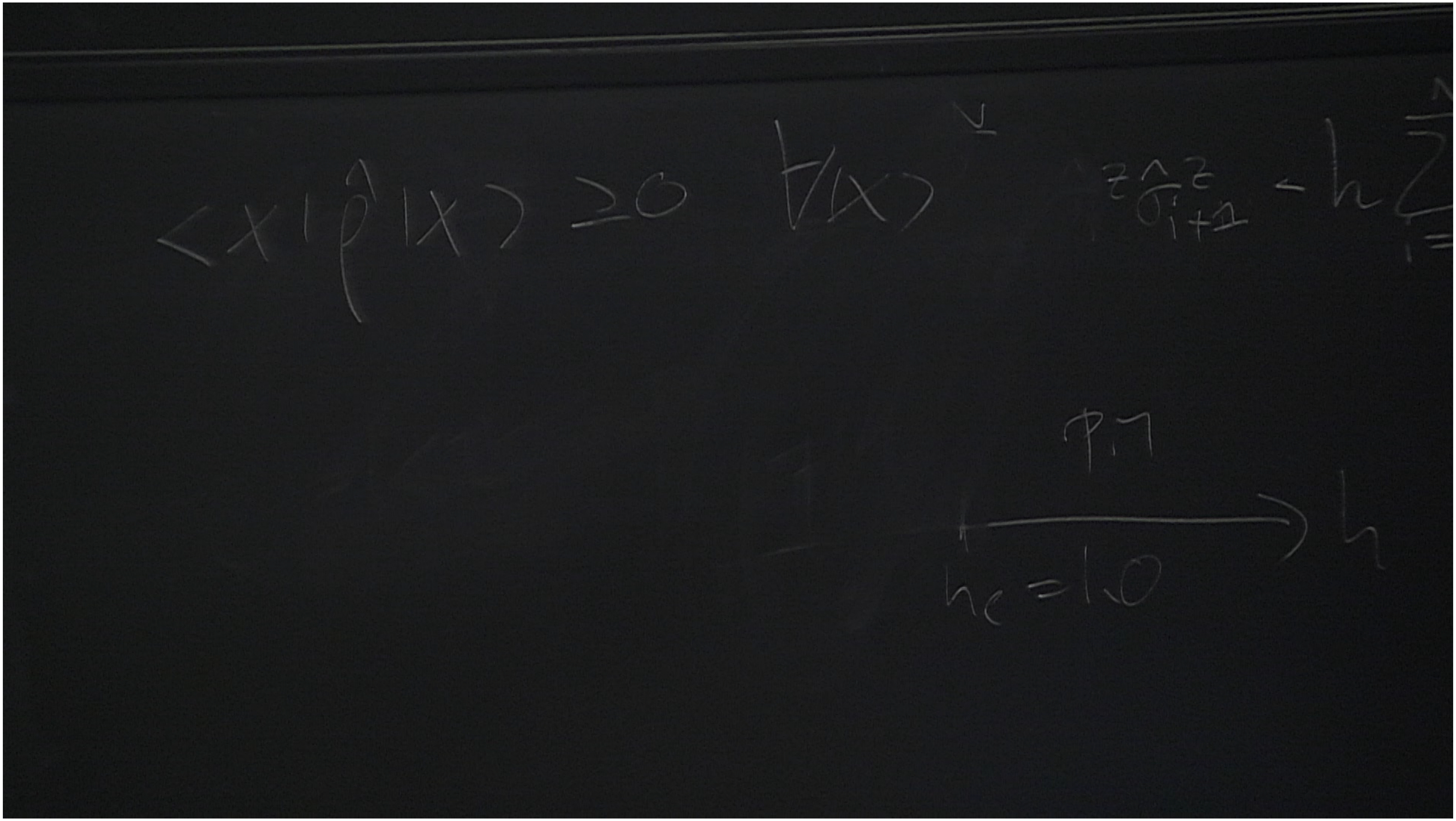
Phases



$\theta(\boldsymbol{\sigma}_k)$ – Exact $\qquad$ $\phi_{\boldsymbol{\mu}}(\boldsymbol{\sigma}_k)$ – RBM

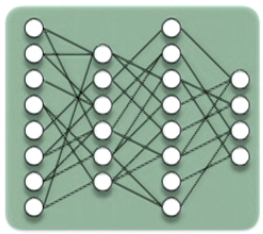
GT et al, *Nature Physics 2017* (1703.05334)

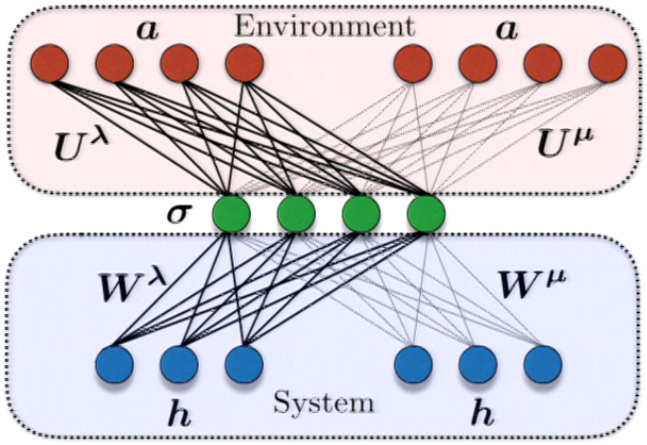

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
$\rho = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$
ρ
$|0\rangle$
Obs
$|\psi_{S+E}\rangle = \alpha|00\rangle + \beta|10\rangle \rightarrow \alpha|00\rangle + \beta|11\rangle$
$\rho_S' = \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix}$


⟨x|ρ̂|x⟩ ≥ 0 ∀|x⟩
P.M
h_c = 1.0
h

Mixed states

$(\boldsymbol{\lambda}, \boldsymbol{\mu})$

$|\boldsymbol{\sigma}\rangle, |\boldsymbol{\sigma}'\rangle \longrightarrow$  $\longrightarrow \rho_{\boldsymbol{\lambda}\boldsymbol{\mu}}(\boldsymbol{\sigma}, \boldsymbol{\sigma}') = \langle \boldsymbol{\sigma} | \boldsymbol{\rho}_{\boldsymbol{\lambda}\boldsymbol{\mu}} | \boldsymbol{\sigma}' \rangle$



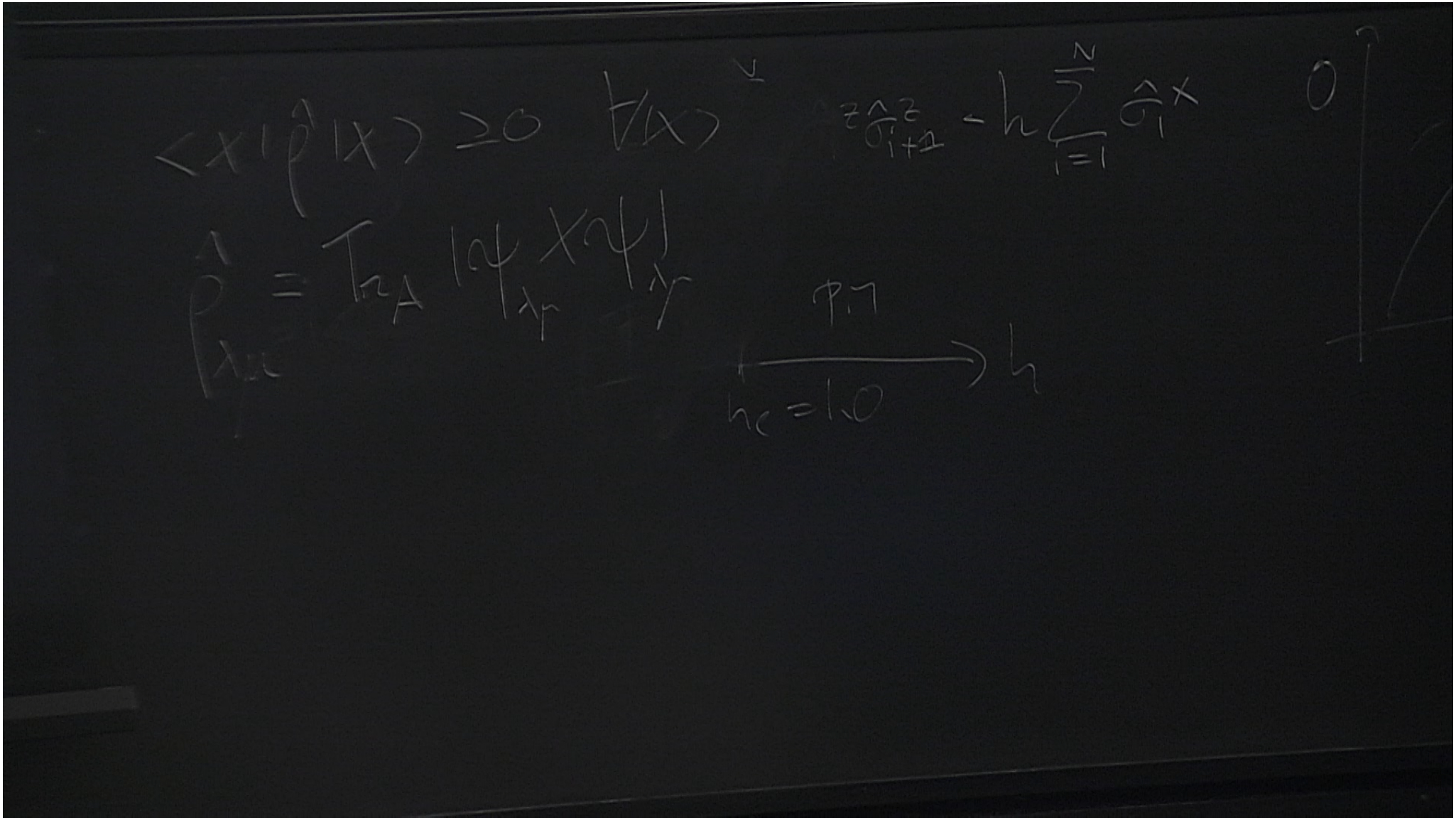
Latent space purification

$$\boldsymbol{\rho}_{\boldsymbol{\lambda}\boldsymbol{\mu}}^{\boldsymbol{\sigma}\oplus\boldsymbol{a}} = |\psi_{\boldsymbol{\lambda}\boldsymbol{\mu}}\rangle\langle\psi_{\boldsymbol{\lambda}\boldsymbol{\mu}}|$$

$$|\psi_{\boldsymbol{\lambda}\boldsymbol{\mu}}\rangle = \sum_{\boldsymbol{\sigma}\boldsymbol{a}} \psi_{\boldsymbol{\lambda}\boldsymbol{\mu}}(\boldsymbol{\sigma}, \boldsymbol{a}) |\boldsymbol{\sigma}\rangle \otimes |\boldsymbol{a}\rangle$$

$$\rho_{\boldsymbol{\lambda}\boldsymbol{\mu}}(\boldsymbol{\sigma}, \boldsymbol{\sigma}') = \sum_{\boldsymbol{a}} \psi_{\boldsymbol{\lambda}\boldsymbol{\mu}}(\boldsymbol{\sigma}, \boldsymbol{a}) \psi^*_{\boldsymbol{\lambda}\boldsymbol{\mu}}(\boldsymbol{\sigma}', \boldsymbol{a}) = Z_{\boldsymbol{\lambda}}^{-1} \mathrm{e}^{\gamma_{\boldsymbol{\lambda}\boldsymbol{\mu}}(\boldsymbol{\sigma}) + \gamma^*_{\boldsymbol{\lambda}\boldsymbol{\mu}}(\boldsymbol{\sigma}') + \Pi_{\boldsymbol{\lambda}\boldsymbol{\mu}}(\boldsymbol{\sigma}, \boldsymbol{\sigma}')}$$

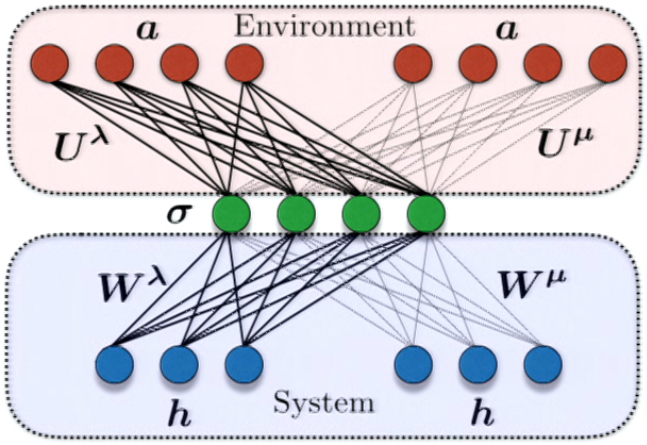
GT and Melko, *prerint 2018* (1801.09684)

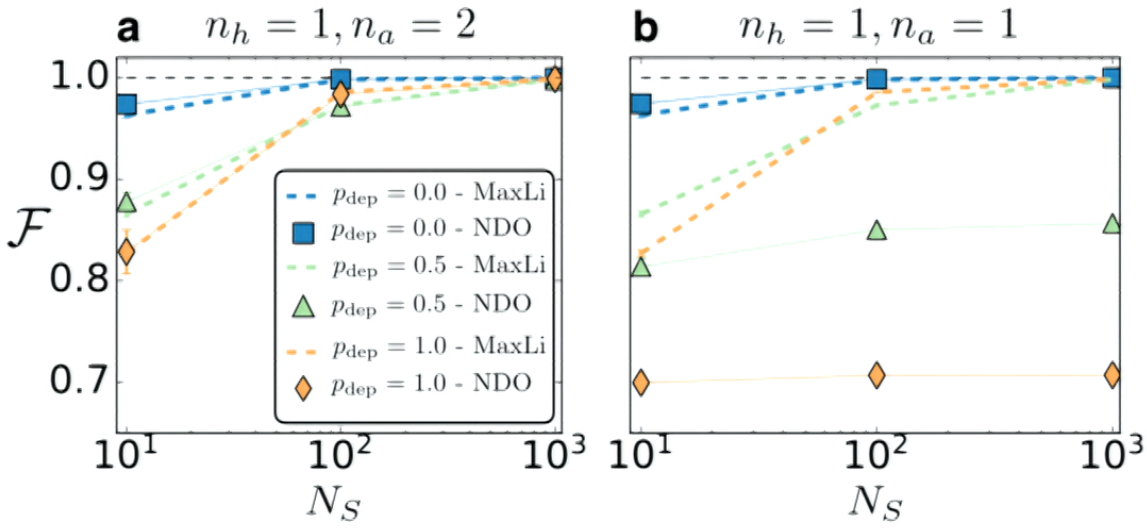

$\langle x|\rho|x\rangle \geq 0$ $\forall |x\rangle$
$\hat{\rho}_{AB} = \mathrm{Tr}_A |\psi\rangle\langle\psi|$
$h_c = 1.0$
h

Mixed states

$$|\Psi\rangle = \frac{1}{2}\left(|00\rangle + |11\rangle\right)$$

$$\boldsymbol{\rho} = (1 - p_{\text{dep}})|\Psi\rangle\langle\Psi| + p_{\text{dep}}\frac{\boldsymbol{I}}{4}$$





GT and Melko, *prerint 2018* (1801.09684)

Mixed states

$$|\Psi\rangle = \frac{1}{2}\left(|00\rangle + |11\rangle\right)$$

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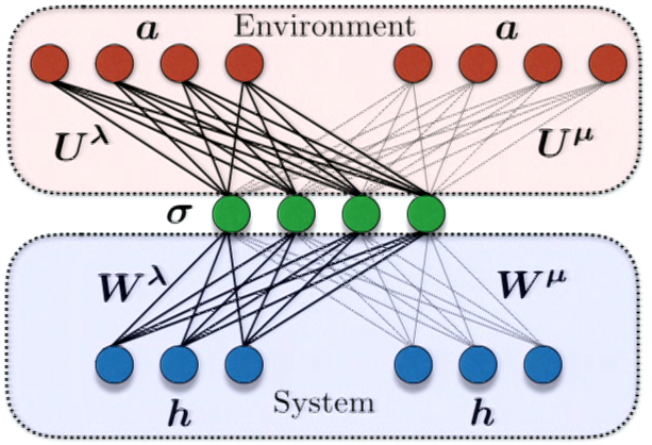
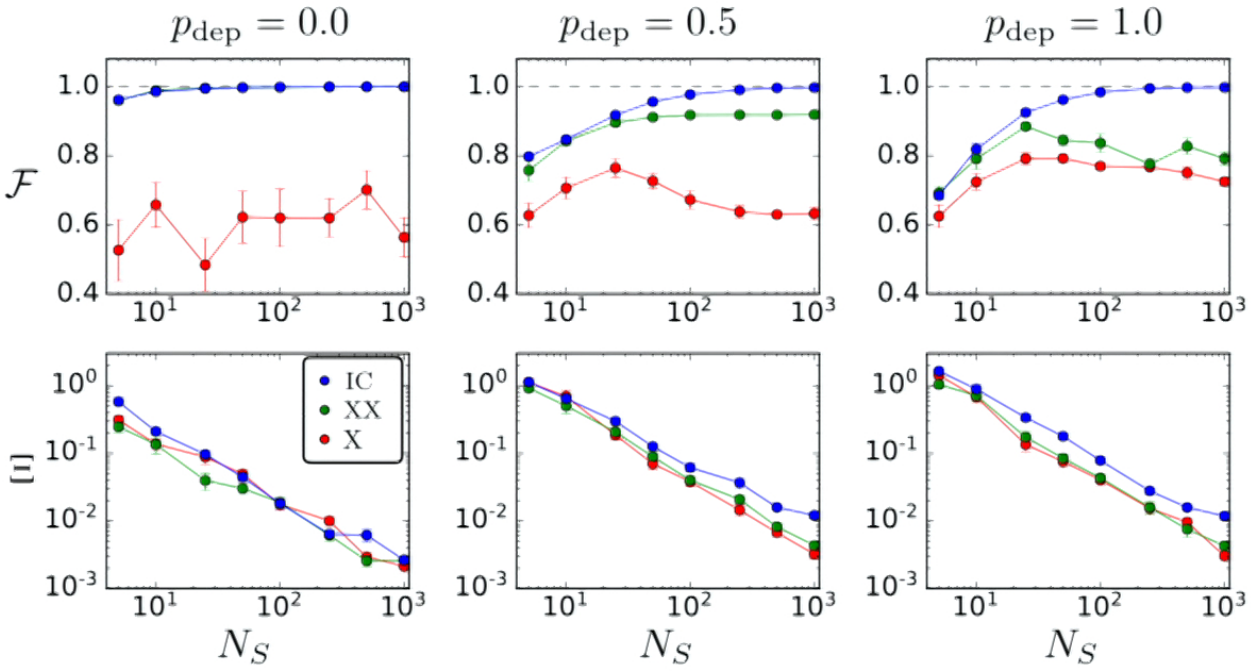
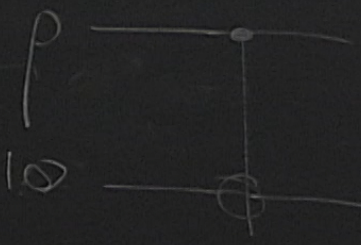




GT and Melko, *prerint 2018* (1801.09684)

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$\rho = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$



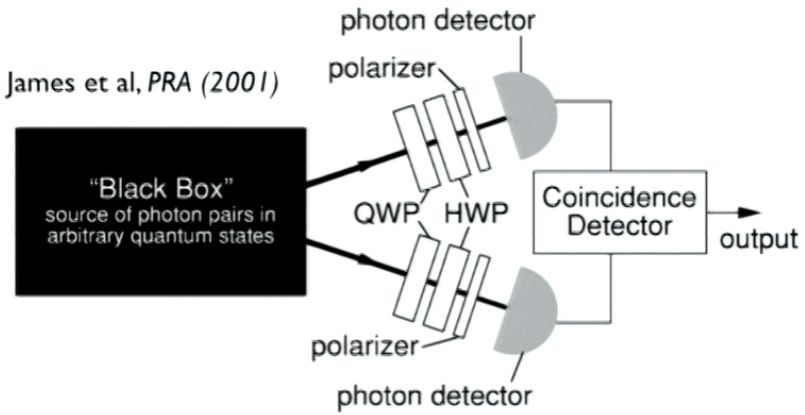
$|\psi_{S+E}\rangle = \alpha'|00\rangle + \beta|10\rangle \rightarrow \alpha|00\rangle + \beta|11\rangle$

$\rho_S' = \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix}$

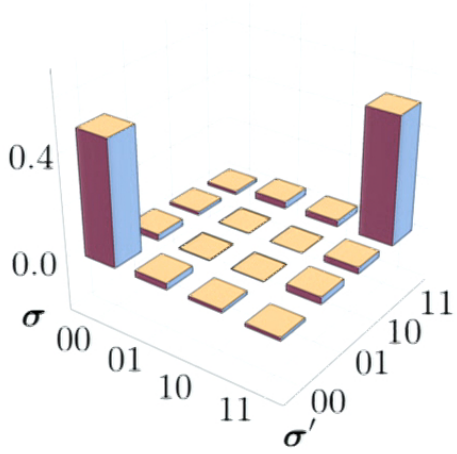
Experiments

Entangled photonic states

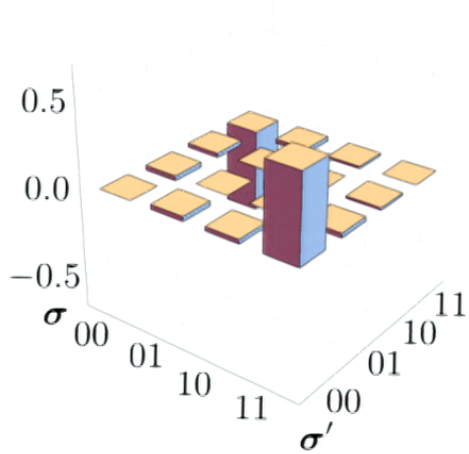
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + i|11\rangle\right)$$



$\mathrm{Re}[\rho_{\lambda^*\mu^*}]$



$\mathrm{Im}[\rho_{\lambda^*\mu^*}]$

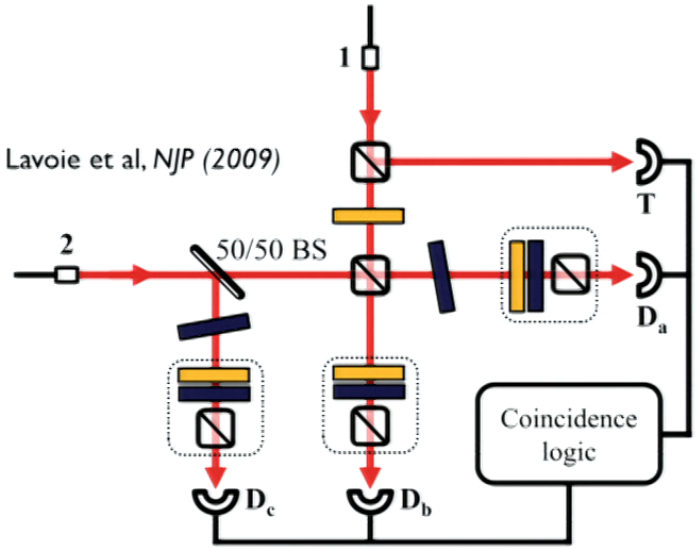


GT and Melko, *prerint 2018* (1801.09684)

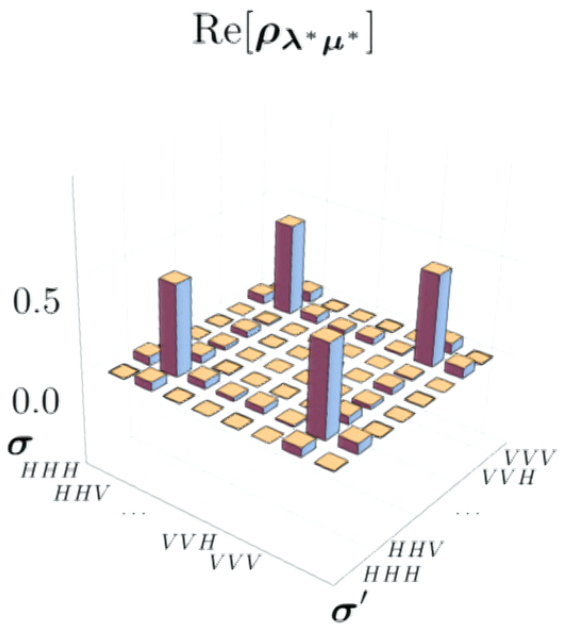
Experiments

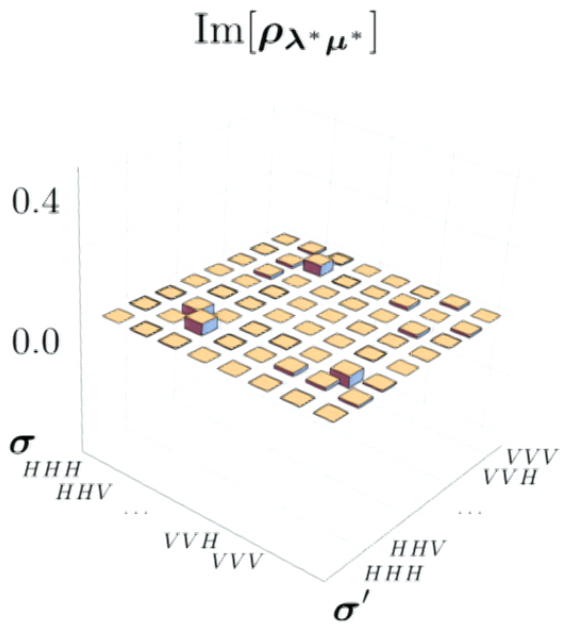
Entangled photonic states

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|HHV\rangle + |VVH\rangle\right)$$



Lavoie et al, *NJP (2009)*





Experiments

Ising-type quantum simulator