Title: Hidden Physics in Strongly Correlated Systems: The 2D Hubbard model, and Some Unparticle Fun

Date: Apr 10, 2018 03:30 PM

URL: http://pirsa.org/18040052

Abstract: <p>When performing numerical simulation of many-body systems we work with the goal of computing some set of averaged observables. This process has a weakness in that, when our simulation is complete, we have no ability to determine what was the key physics that gave our result. In this talk, I'll give a simple example, a gap in a single-particle density of states, and explain how we can use two-particle quantities to recover key 'hidden' information. I'll then back up these claims with numerical results for the 2D Hubbard model. For the second half of my talk I'll hope to keep your attention by introducing a parallel but MUCH more whimsical idea, unparticle physics. I'll at least TRY to explain what an unparticle is and how it might impact strongly correlated condensed matter systems.</p>

Hidden Physics in Strongly Correlated Systems:

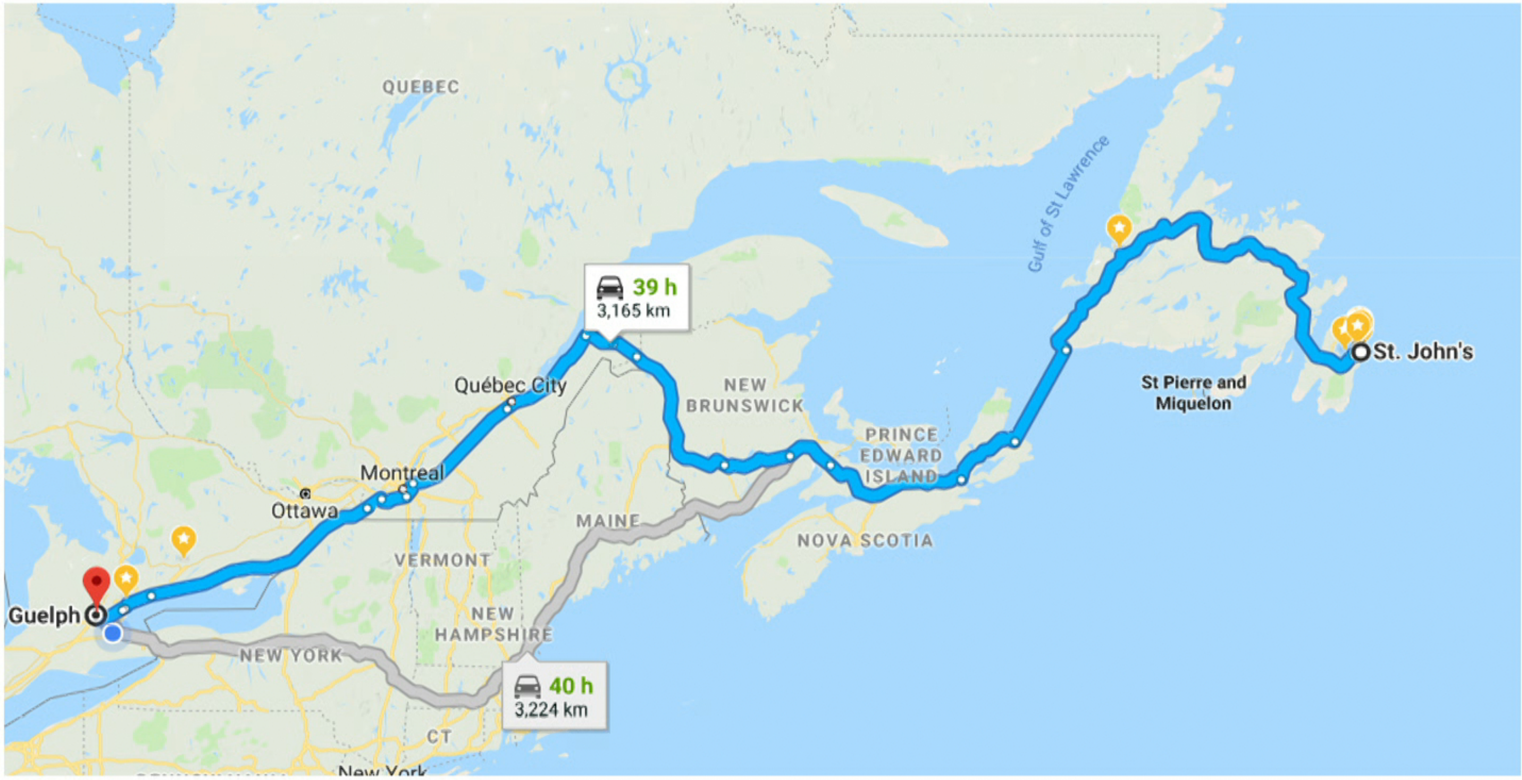
The 2D Hubbard model, and some Unparticle Fun.

James P.F. LeBlanc

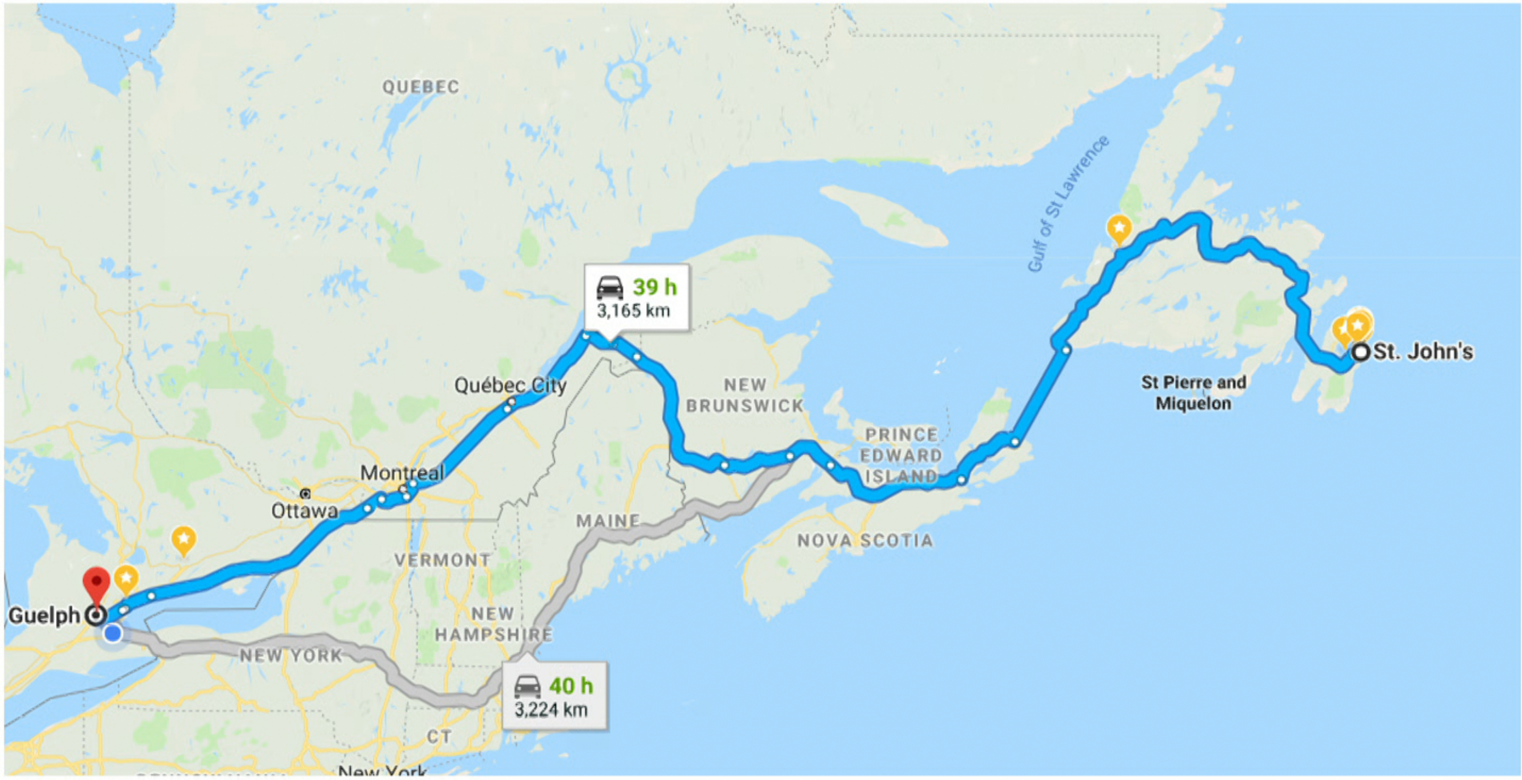
Memorial University of Newfoundland, Newfoundland, Canada

SIMONS FOUNDATION

Memorial University of Newfoundland

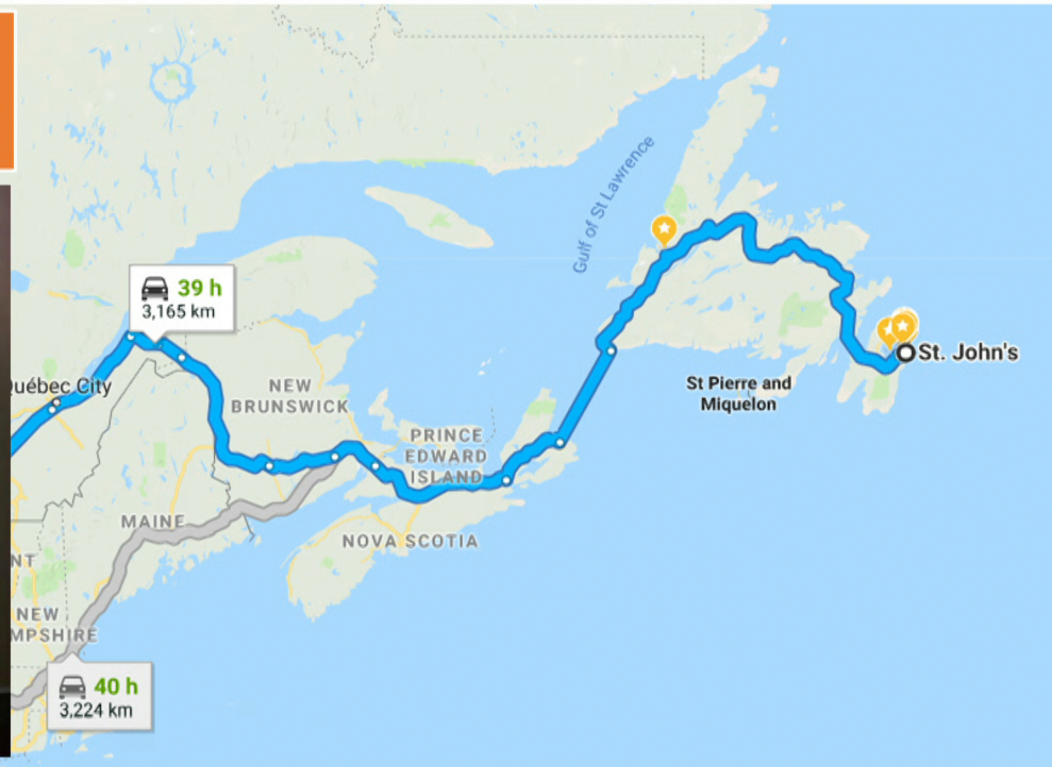


Memorial University of New-fun-land



Memorial University of New-fun-land

Just like Ontario
Very mild winters



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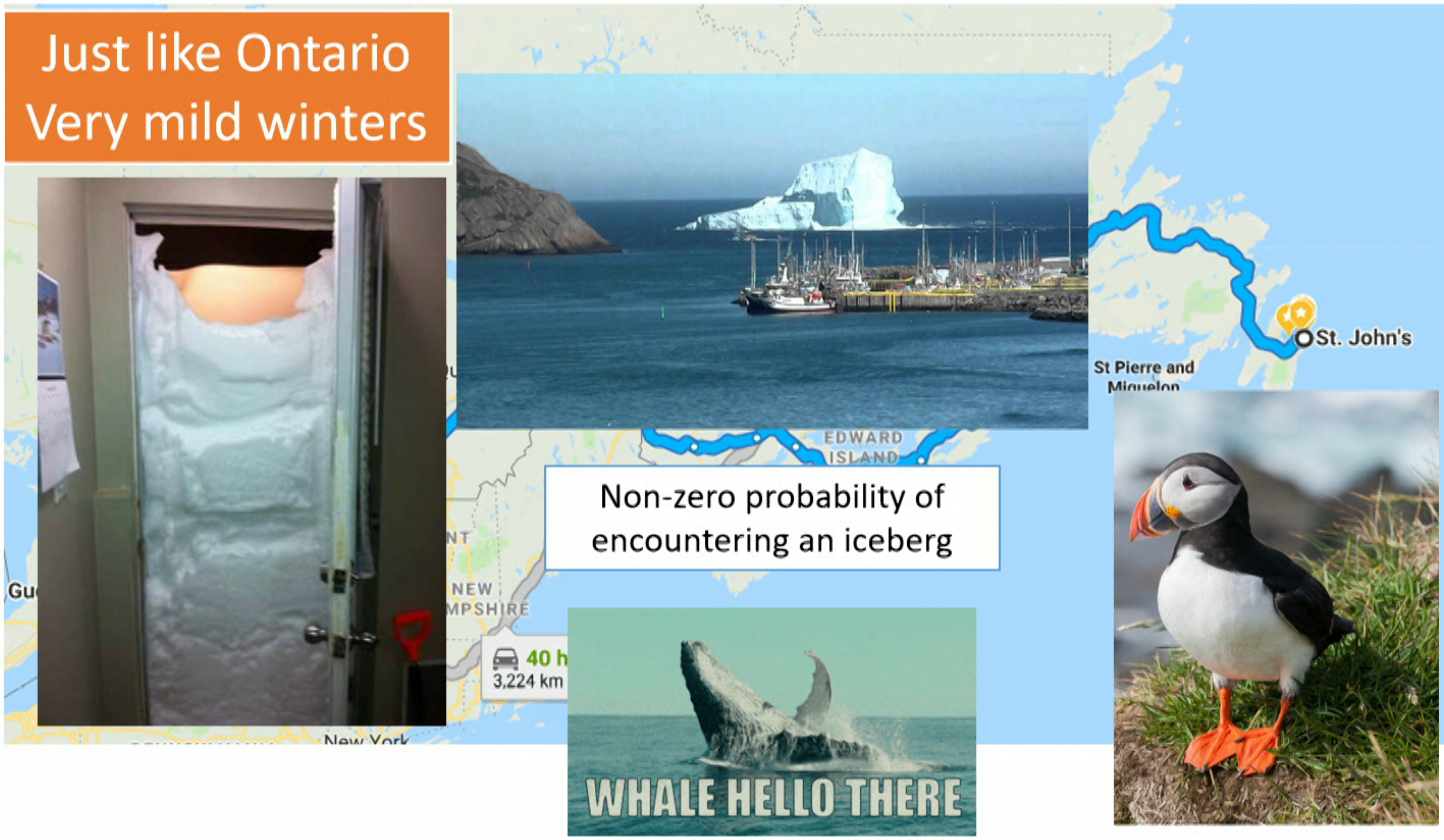
Non-zero probability of encountering an iceberg



Memorial University of New-fun-land

Just like Ontario
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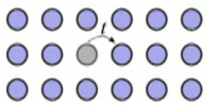
Non-zero probability of encountering an iceberg

Overview

Part 1: “Towing the Party Line”

Strongly Correlated Systems:
The 2D Hubbard model



$$H = \sum_{k\sigma}(\epsilon_k - \mu)c^{+}_{k\sigma}c_{k\sigma} + U\sum_{i} n_{i\uparrow}n_{i\downarrow}$$

Quick Primer to Many-Body Physics

Two Particle Properties

- Uncovering ‘Hidden’ information
  Pseudogap phase

- Neutron and NMR studies from two-particle susceptibilities

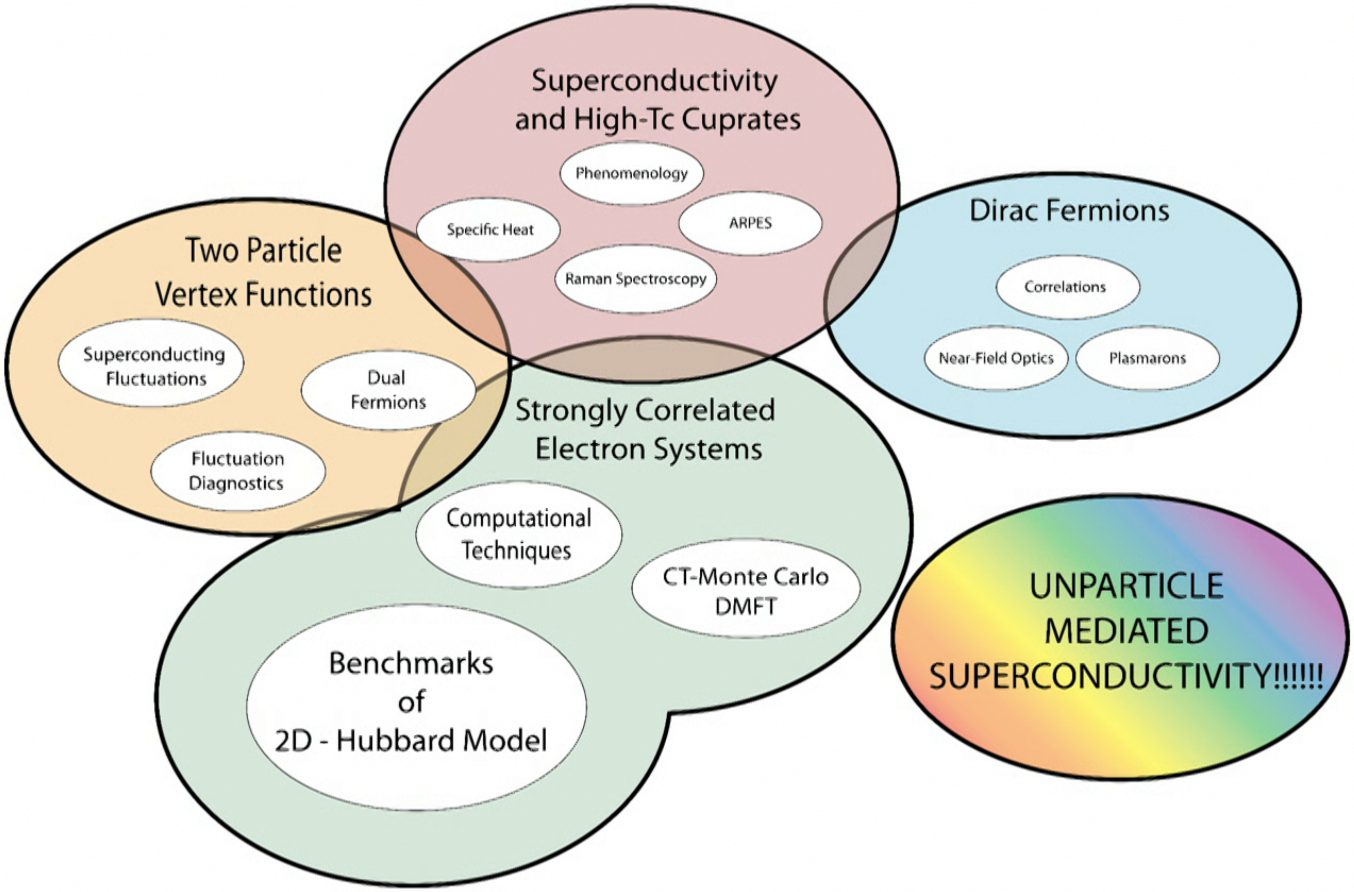
Part 2: “Unparticle Nonsense”

1) Story Time

2) Scale Invariance and ‘Hidden’ Unparticles

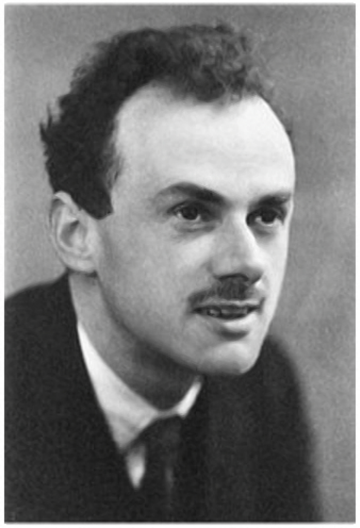
3) Unparticle Mediated Superconductivity

Disclaimer – I'm all over the place



Dirac – The Many Electron Problem

Proceedings of the Royal Society of London.
Series A, Containing Papers of a Mathematical and Physical Character, Vol. 123, No. 792 (1929)



"The underlying physical laws necessary for the mathematical theory of a large part of physics and the whole of chemistry are thus completely known, and the difficulty is only that the exact application of these laws leads to equations much too complicated to be soluble. "

"It therefore becomes desirable that approximate practical methods of applying quantum mechanics should be developed, which can lead to an explanation of the main features of complex atomic systems without too much computation."

Introduction: Many-electron Problem

How does one solve a problem, such as electrons in a solid, which has 10^{23} constituent particles?

The four fundamental interactions of nature

https://en.wikipedia.org/wiki/Fundamental_interaction

Interaction	Gravitation	Weak	Electromagnetic	Strong	
		(Electroweak)		Fundamental	Residual
Acts on:	Mass - Energy	Flavor	Electric charge	Color charge	Atomic nuclei
Particles experiencing:	All	Quarks, leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Not yet observed (Graviton hypothesised)	$W^+ W^- Z^0$	γ (photon)	Gluons	Mesons

More is Different (Anderson Science 1972 – just after quarks)

The result of the complex system, is **fundamentally** different from a handful of particles.

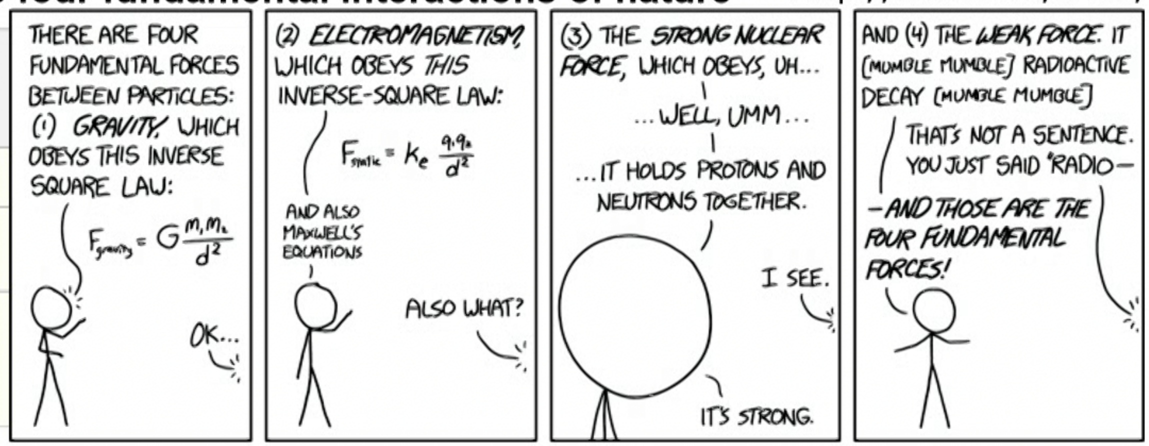
Searching for emergent behaviour

Introduction: Many-electron Problem

How does one solve a problem, such as electrons in a solid, which has 10^{23} constituent particles?

The four fundamental interactions of nature http://xkcd.com/1489/

Interaction	
Acts on:	
Particles experiencing:	
Particles mediating:	



More is Different (Anderson Science 1972 – just after quarks)

The result of the complex system, is **fundamentally** different from a handful of particles.

Searching for emergent behaviour

Collaboration on the Many Electron Problem

SIMONS FOUNDATION

The Simons Collaboration on the Many Electron Problem brings together a group of scientists focused on developing new ways to solve the quantum mechanical behavior of systems comprised of many interacting electrons, with the goal of revolutionizing our ability to calculate and understand the properties of molecules and solids important in chemistry, physics and everyday life.

FLATIRON INSTITUTE — Center for Computational Quantum Physics

CCQ's mission is to develop the concepts, theories, algorithms and codes needed to solve the quantum many-body problem and to use the solutions to predict the behavior of materials and molecules of scientific and technological interest.

ALPSCore: Core libraries of ALPS project

Save yourself some time!

Website: http://alpscore.org/
(Based on Algorithms and Libraries for Physics Simulations, alps.comp-phys.org)

ALPSCore package

params
(Access to paramer file and command line)

accumulators
(Gather statistical data with error estimates)

archive
(Save to, restore from HDF5 files)

Monte Carlo scheduler
(Distribute work via MPI, set termination criteria)

Green's Functions
(Manipulate Green's functions)

Tutorials
(E.g., Ising model MC simulation)

Matsubara vs Real Axis Formalism

- Need reliable tools for analytic continuation $i\omega_n \rightarrow \omega$

Maxent
https://github.com/CQMP/Maxent

- Specifically tuned to analytically continue Green's functions and Bosonic Response functions
- Minimal Complication (ALPSCore, EIGEN3)
  - 10 minutes
- **arXiv:1606.00368**

Scientific Rigour

Working on challenging problems means you can't get the 'right' answer.

Each Group chips away independently on a problem and gives their best guess

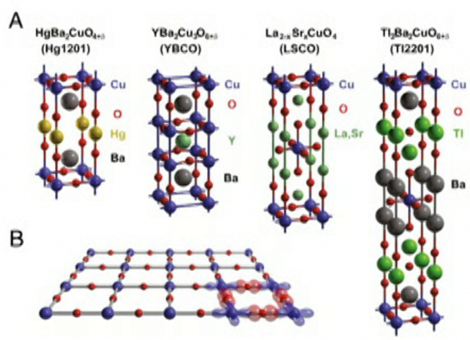
Hopefully some consensus is reached

Side Effects

Breeds mistrust

- "Those theorists don't know what they are doing"
- "Oh god, not another High-Tc talk…"

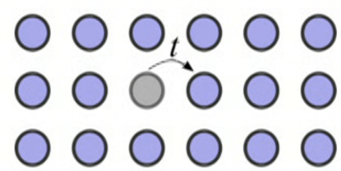
Replace Complicated Materials



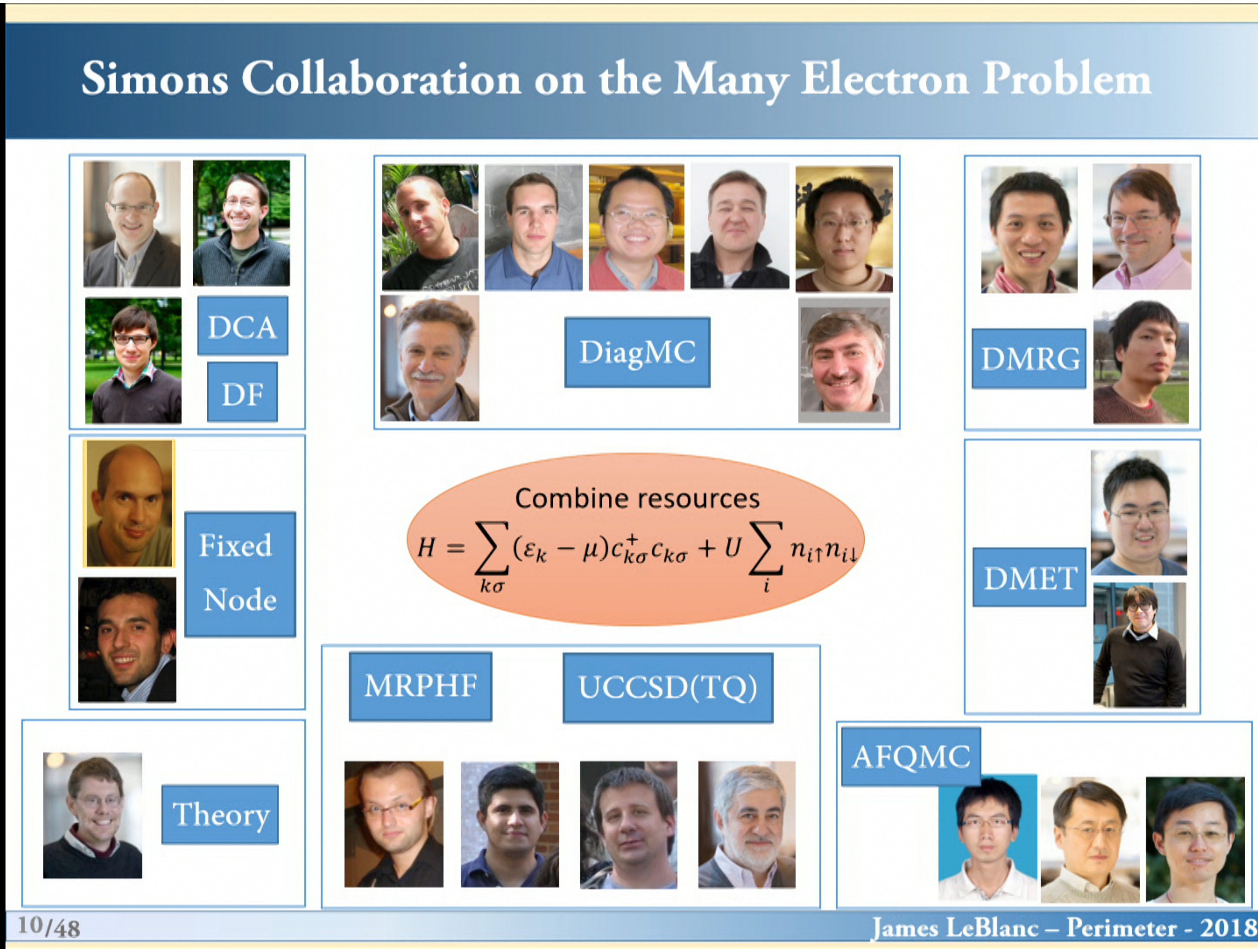
Barišić et al. PNAS 2013 July, 110 (30) 12235-12240

Simplified Lattice Models

2D Square Lattice – Hubbard Model



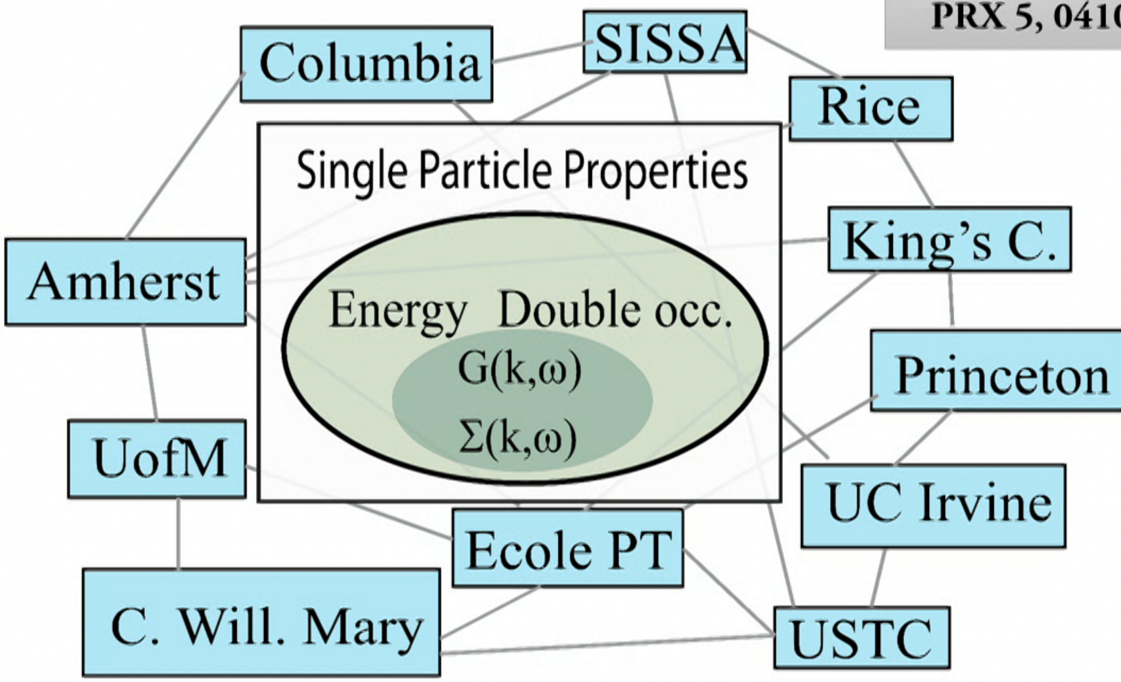
$$H = \sum_{k\sigma} (\epsilon_k - \mu) c^{+}_{k\sigma} c_{k\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$


Simons Collaboration on the Many Electron Problem
DCA
DF
DiagMC
DMRG
Fixed Node
Combine resources
$H = \sum_{k\sigma}(\varepsilon_k - \mu)c^{+}_{k\sigma}c_{k\sigma} + U\sum_{i} n_{i\uparrow}n_{i\downarrow}$
DMET
MRPHF
UCCSD(TQ)
Theory
AFQMC
10/48
James LeBlanc – Perimeter - 2018

Part 1: The 2D Hubbard Model

Simons Foundation Collaboration on the Many-Electron Problem

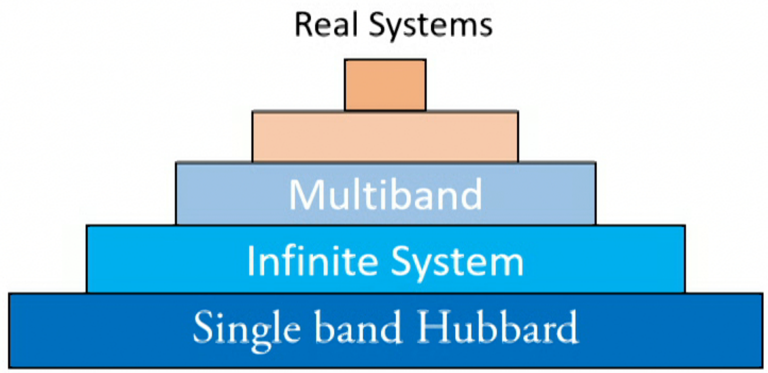
PRX 5, 041041 (2015)



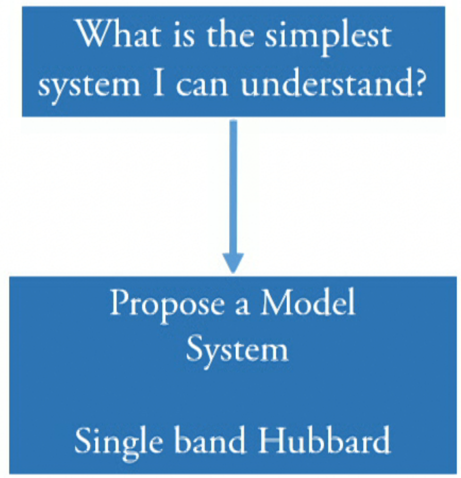
Tremendous effort to establish concensus

Stripe Phase: Science 358, 1155 (2017) Hydrogen Chain: Phys. Rev. X **7**, 031059 (2017)

First: Why would we do this?



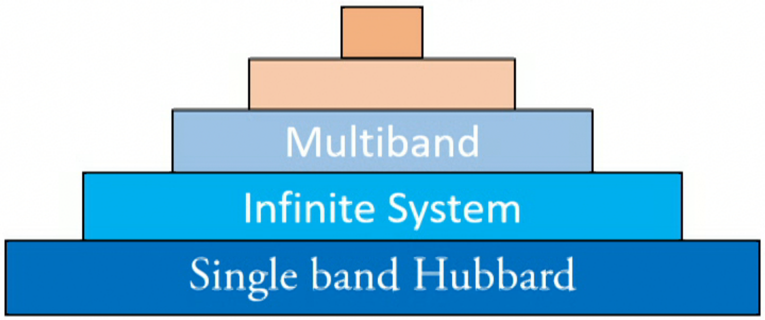
The situation for exactly solving correlated systems is very grim!



Solve the (insert expletive) out of it

First: Why would we do this?

Real Systems



The phase diagram of the single band Hubbard model is not known.

$$U, \beta, \mu$$

The situation for exactly solving correlated systems is very grim!

What is the simplest system I can understand?

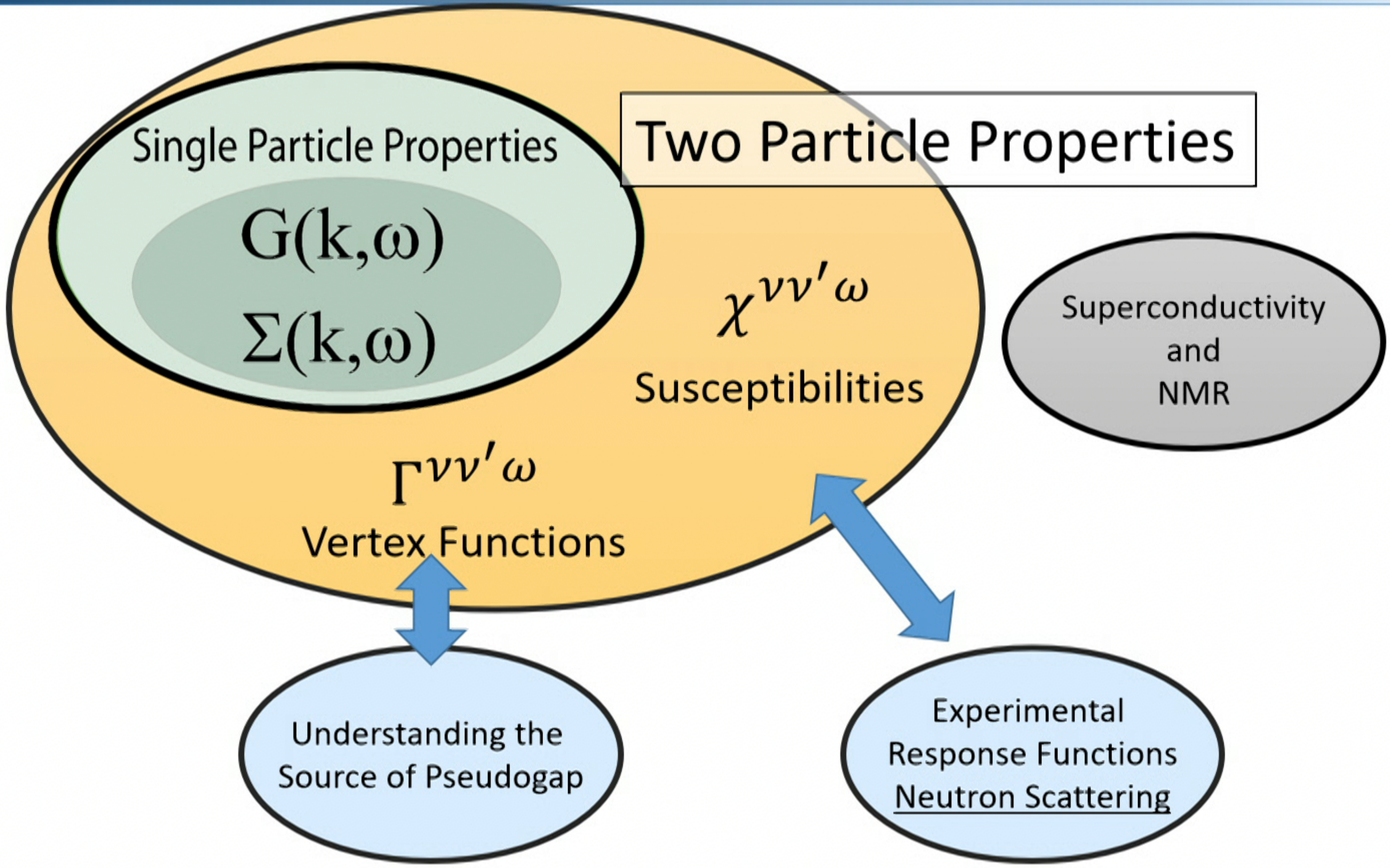
↓

Propose a Model System

Single band Hubbard

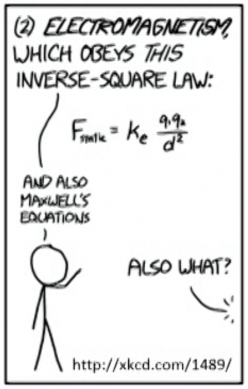
Solve the (insert expletive) out of it

Objectives



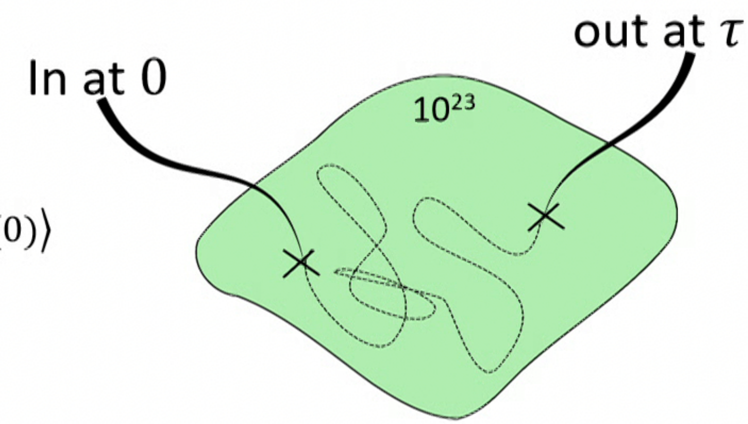
Single Particle Properties
G(k,ω)
Σ(k,ω)
Two Particle Properties
$\chi^{\nu\nu'\omega}$
Susceptibilities
$\Gamma^{\nu\nu'\omega}$
Vertex Functions
Superconductivity
and
NMR
Understanding the
Source of Pseudogap
Experimental
Response Functions
Neutron Scattering

Many-Body Theory in Condensed Matter



Single Particle Properties
Green's Function $G(\mathrm{r}, t)$

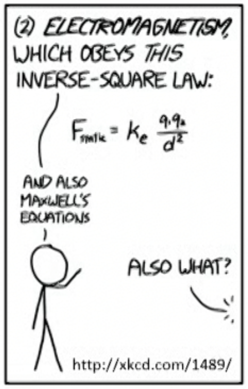
$$G_\sigma(r-r',\tau) = \langle c_{r\sigma}(\tau) c^{+}_{r'\sigma}(0)\rangle$$



Repeat this process again and again to build a probability of being in a particular state at time τ

Resulting Green's function (propagator) contains density of states.

Many-Body Theory in Condensed Matter



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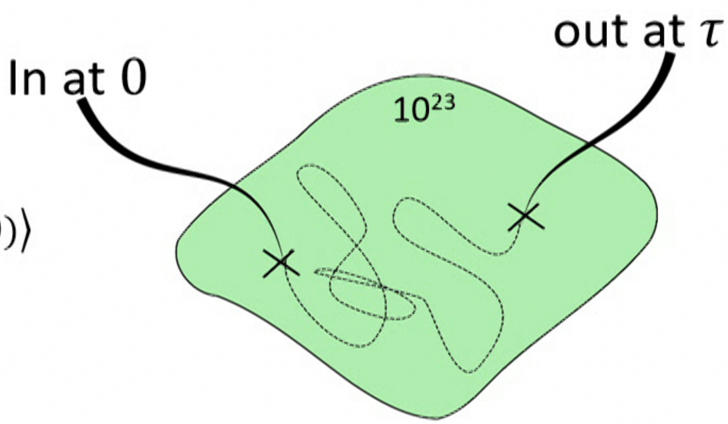

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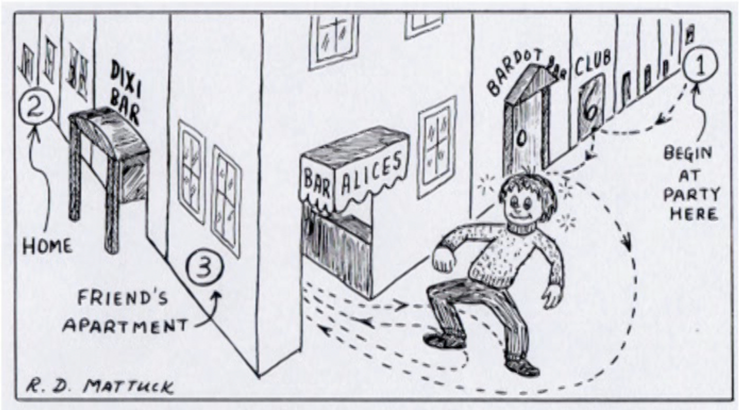
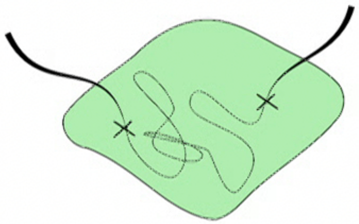
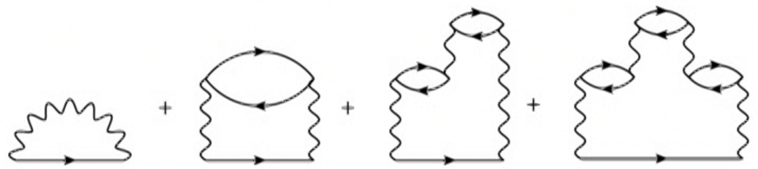


Fig. 1.1 *Propagation of Drunken Man*

Feynman Diagrams



Perturbation theory in powers of interaction

$\Sigma(k, \omega) =$  $+ \cdots$

Basic Building Blocks

G — Interacting Green's Function (k, ω)

G_0 — Non-Interacting Green's Function (k, ω)

V — Coulomb Potential → U → ●

All possible permutations and expansion orders must be summed

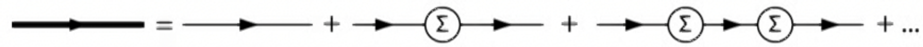
$$\sum_{\{perm,i\}} \sum_{n=0}^{\infty} \frac{1}{n!} U^n [F_n^i(G_0)]$$

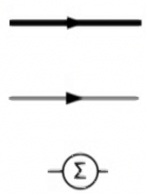
If interaction is weak – expansion order small

If interaction is strong – expansion order large

Self Energy – Dyson Equation

Basic Building Blocks



G		Interacting Green's Function
G_0		Non-Interacting Green's Function
Σ		Self Energy

Single Particle Self Energy is the difference between an interacting and non-interacting system's energy

$$\Sigma(\mathrm{k},\omega) = G_0^{-1}(k,\omega) - G^{-1}(k,\omega)$$

Any problem of interactions (at the single particle level) boils down to:
What is the self energy?

Many-Body Theory

Single Particle

G: Green's Function

Σ: Self Energy

Sufficient for density of states, and spectral function → STM, ARPES

Two Particle

Γ: Vertex Function

χ: Susceptibility → Optical Conductivity, Raman Spectroscopy, NMR, Neutron Scattering

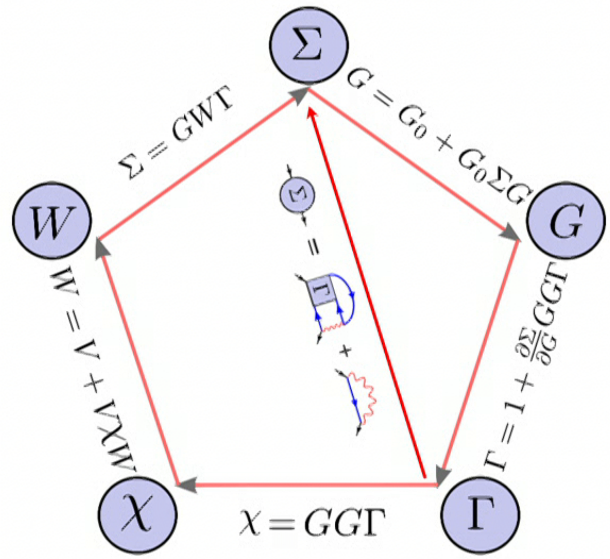
Necessary for phase transitions

Necessary for optical experiments

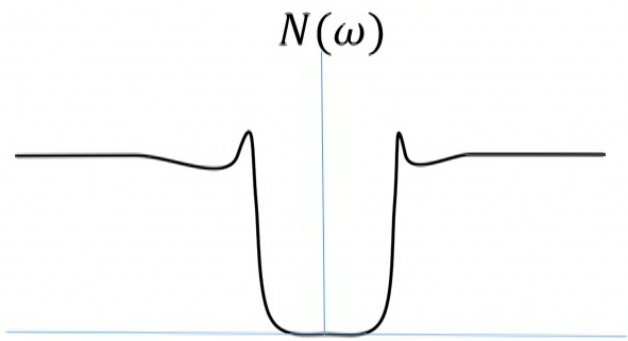
W: Screened Potential

Single particle and two-particle properties related

Hedin Equations



A thought experiment



"I've measured the density of states of material 'X'.
What does it mean?!?!?"

Realistic viewpoint
- There is a gap
- The gap has an amplitude

What does this actually tell us?

Strong Coupling models
- It's an insulator!

Cuprates
- I bet it's a superconductor!

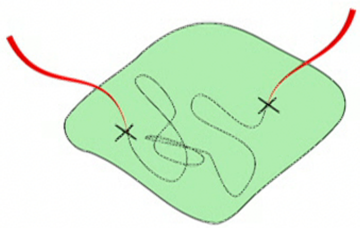
Hybridization Gap
- Some ordering!

This same issue arises in numerical work.

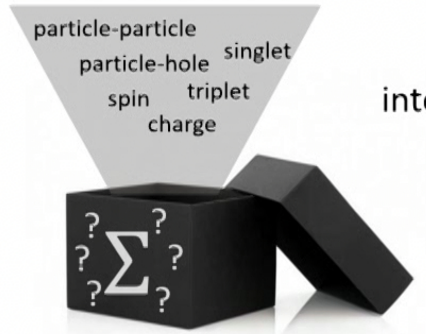
Two-particle properties can directly account for lost information.

Loss of information

Recall definition of self energy



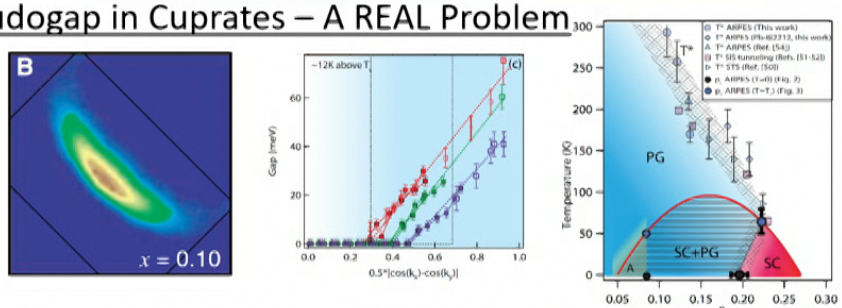
What were the MOST important interactions?



Details of the interactions are lost

Which interactions dominated?

Pseudogap in Cuprates – A REAL Problem



d-wave gap in single particle spectra

It's origin is preformed pairs!

It's origin is Mott physics!

Charge density wave!
Spin Density wave!
.... And so on

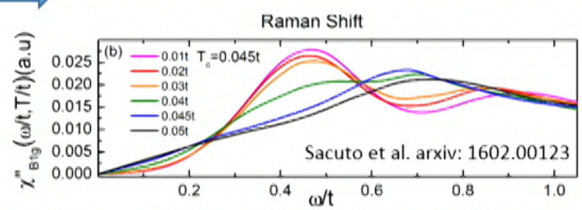
K. Shen et al Science **307** 901 (2005)   Vishik et al. PNAS 109 (45) 18332-18337 (2012)

Retrieving Lost Information

Typical Procedure

Supplement single particle properties with two-particle probes

STM, ARPES → Optical Conductivity, Raman Spectroscopy, NMR, Neutron Scattering



Generate Analogous Quantities

G: Green's Function
Σ: Self Energy

→ χ: Susceptibility

Talk about Susceptibilities

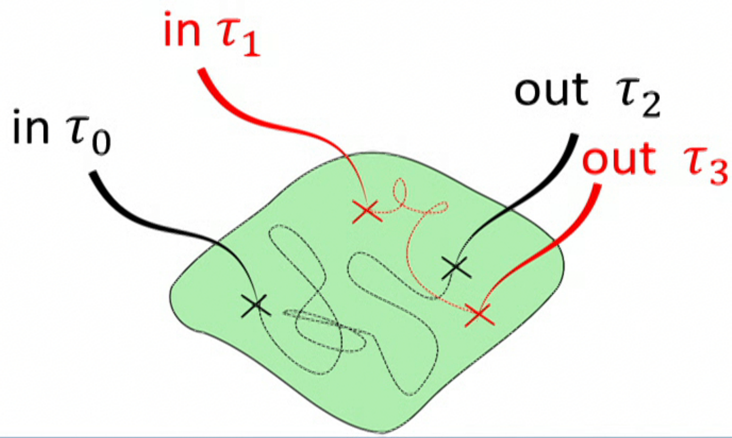
TOPIC OF PART 1

Simple description of interactions in a system: 2D Hubbard, Square Lattice

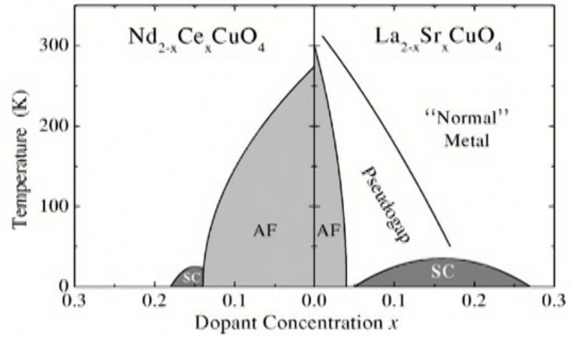
Kinetic — Potential

$$H = -\sum_{\{i,j\},\sigma} t_{ij}(c^{\dagger}_{i\sigma}c_{j\sigma} + c^{\dagger}_{j\sigma}c_{i\sigma}) + U\sum_{i} n_{i\uparrow}n_{i\downarrow}$$

Two-particle (4-point) Green's functions

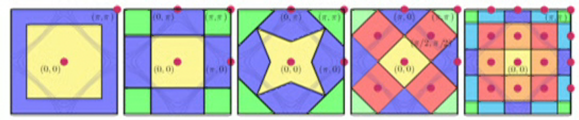


Contain more information of phase transitions and long ranged correlations

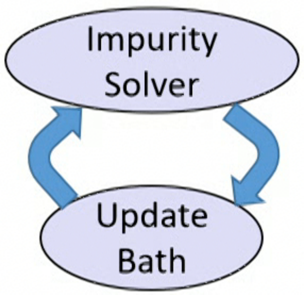


Approximation: χ from DCA

DCA Results
-Discrete or limited K resolution (K′, Q in χ)



Recall Dynamical Mean-Field Theory



- Iterate DCA loop to convergence
- Measure 4-point Green's function
- Approximate lattice vertex by cluster measurement

DCA Exact in $N_c \to \infty$ Limit

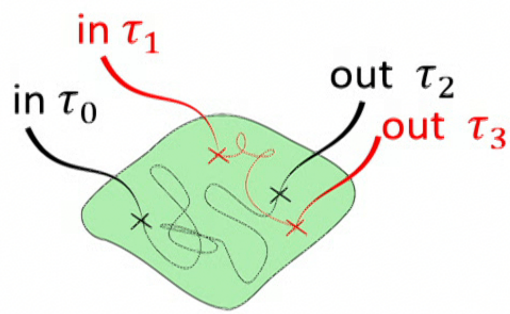
Restrict to 8-site clusters

Wick's decomposition

$$G_4^{\sigma\sigma'}(\nu,k;\nu',k';\omega,q) \to \sum G_2^{\sigma}(\nu;\nu+\omega)G_2^{\sigma'}(\nu'+\omega;\omega)$$

Generalized Susceptibility

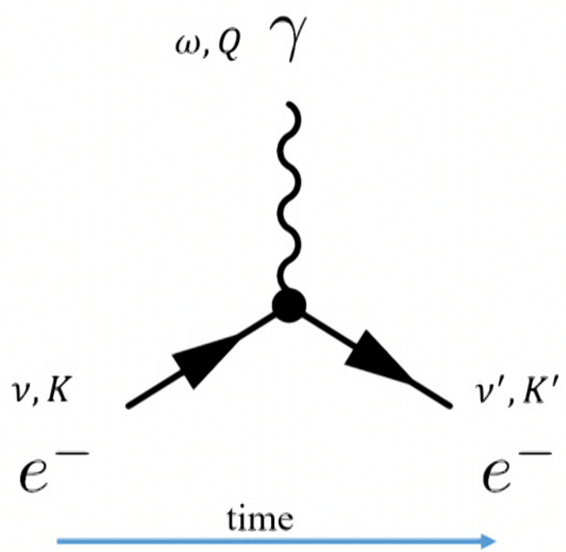
$$\chi_{\sigma_1,\sigma_2,\sigma_3} = G_4(\tau_1,\tau_2,\tau_3) - G_2(\tau_1,\tau_2)G_2(\tau_3,0)$$



We need a vertex!

Electron scatters with photon. Precisely HOW does it scatter?

Scattering Problem



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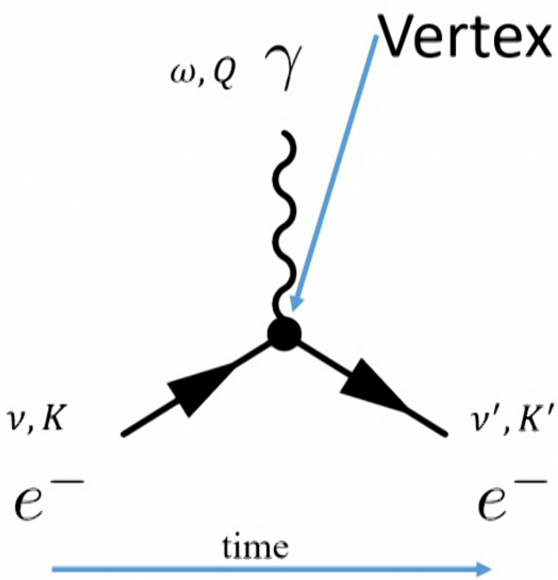

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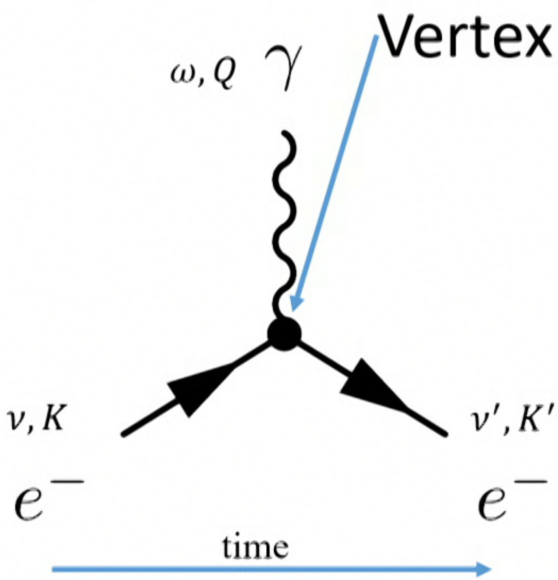


Single Scattering Event $\alpha = \frac{e^2}{4\epsilon_0 \hbar c} = \frac{1}{137}$

Vertex $\propto$ Fine Structure Constant

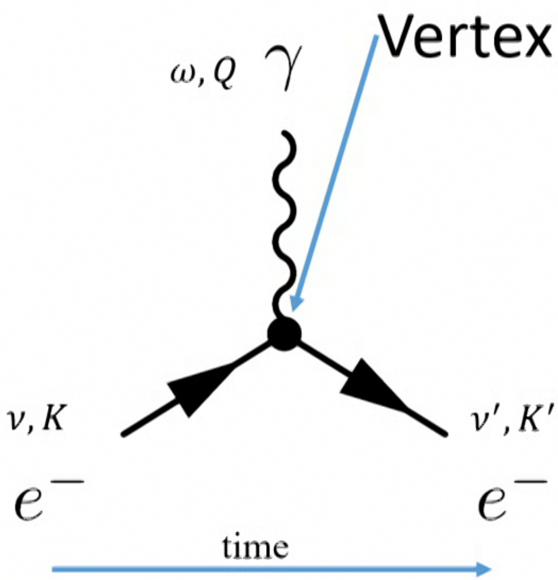
System of 10^{23} particles $\alpha \rightarrow \alpha_{eff} \equiv \Gamma$

Vertex function, Γ, determines if a particle can scatter from one state to another (pauli exclusion).

We need a vertex!

Electron scatters with photon. Precisely HOW does it scatter?

Scattering Problem



Single Scattering Event $\alpha = \frac{e^2}{4\epsilon_0 \hbar c} = \frac{1}{137}$

Vertex ∝ Fine Structure Constant

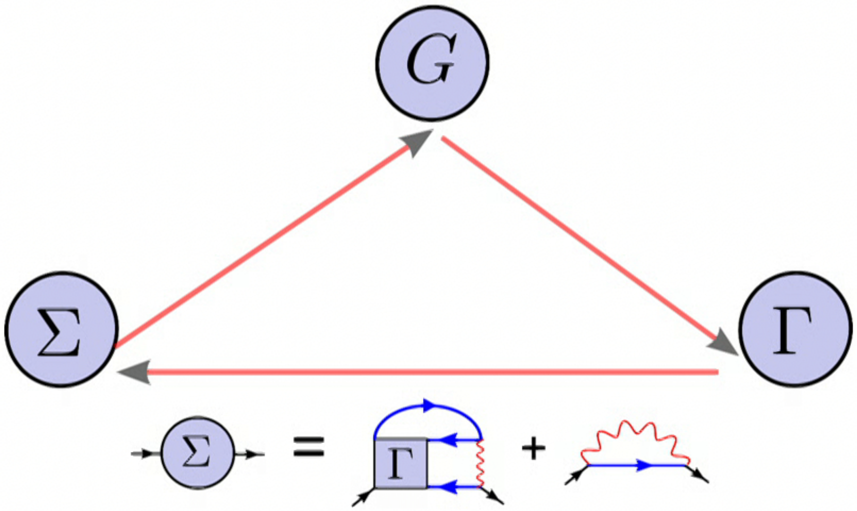
System of 10^{23} particles $\alpha \rightarrow \alpha_{eff} \equiv \Gamma$

Vertex function, Γ, determines if a particle can scatter from one state to another (pauli exclusion).

Complexity	MegaBytes vs. GigaBytes
Each GF has ν, K	10x10 grid in K, 1024 points in ω
	$G(k, \omega) \approx$1.6MB
Each vertex has $\nu, \nu', \omega, K, K', Q$	$\Gamma^{\nu,\nu',\omega}_{K,K',Q} \approx$ 16,000 TB

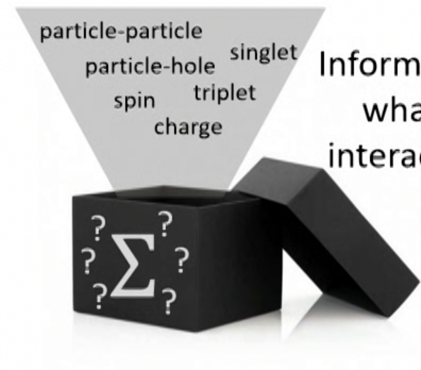
Self Energy from a Two-Particle Vertex

Equation of Motion Approach



From 2-particle result, get back to single-particle properties

$\Gamma^{\nu\nu'\omega}$



Information about what type of interaction is lost

Use vertex to get some of this information back

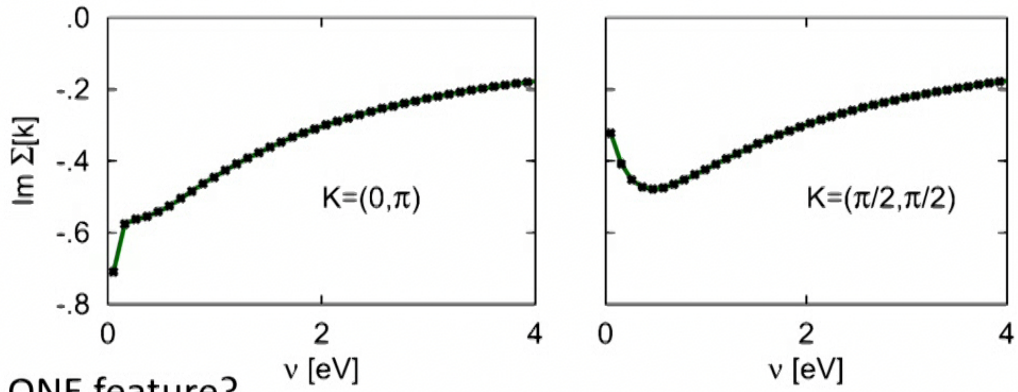
Single Particle Pseudogap

Collaboration with O. Gunnarsson, T. Schäfer, **J.P.F. LeBlanc**, E. Gull, J. Merino, G. Sangiovanni, G. Rohringer, and A. Toschi

Self Energy in antinodal direction begins to see insulating behaviour

Example: $U/t = 7,\ \langle n \rangle = 0.94\ ,\beta t = 15$

Define 'insulating' behaviour

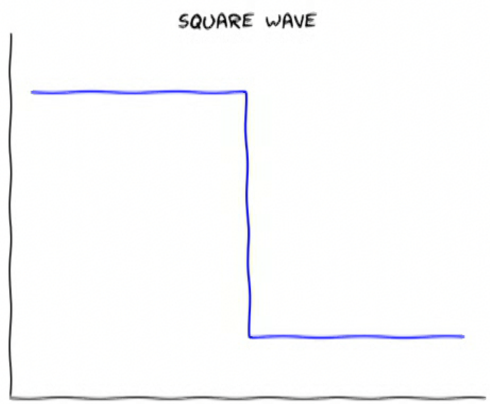


What causes this ONE feature?

What causes the specific contributions to the self energy?

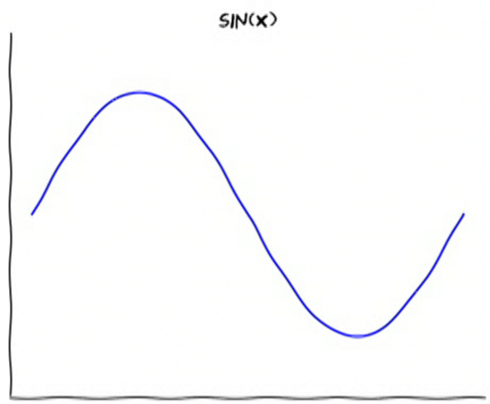
PRL **114**, 236402 (2015)

Basis Representation



- Very simple function
  - Representable with only a few parameters
    - Consider representing in a different basis
      - Fourier Basis

$$f(x) = \frac{4}{\pi} \sum_{n=1,3,5\cdots}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right)$$



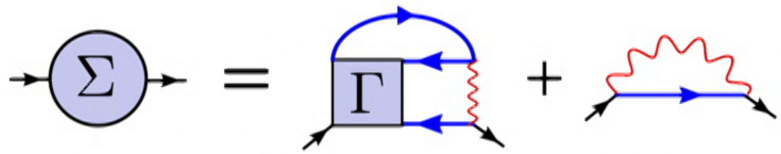
- sin(x) function
  - Complicated function
- In Fourier Basis
  - Defined by a single number

Choice of basis does not impact the result

Finding a GOOD basis, tells you something about the problem itself

Self Energy from Two-Particle Object

Collaboration with O. Gunnarsson, T. Schäfer, **J.P.F. LeBlanc**, E. Gull, J. Merino, G. Sangiovanni, G. Rohringer, and A. Toschi



Choice of Vertex Basis cannot impact Σ

Spin	Charge
$\Gamma_{sp} = \Gamma_{\uparrow\uparrow} - \Gamma_{\uparrow\downarrow}$	$\Gamma_{ch} = \Gamma_{\uparrow\uparrow} + \Gamma_{\uparrow\downarrow}$

(Also singlet, triplet, particle-particle…)

Equivalent once summed over internal indices

$$\Sigma(k) = \frac{U}{\beta^2 N}\sum_{k',Q}\Gamma_{sp}(k,k';Q)G(k')G(k'+Q)G(k+Q)$$
$$= \frac{U}{\beta^2 N}\sum_{k',Q}\Gamma_{ch}(k,k';Q)G(k')G(k'+Q)G(k+Q)$$

$$\Sigma(k,\omega_n) = \sum_Q \Sigma(k,\omega_n,Q)$$

Fluctuation Diagnostics

Partial summation over internal indices provides window into the source of interactions

1. Examine ω_n and Q dependence
2. Examine Basis dependence

PRL 114, 236402 (2015)

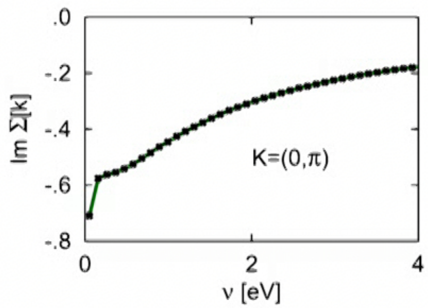
Applications of Vertex Functions from DCA

Collaboration with O. Gunnarsson, T. Schäfer, **J.P.F. LeBlanc**, E. Gull, J. Merino, G. Sangiovanni, G. Rohringer, and A. Toschi

Fluctuation Diagnostics

No variation of system parameters required

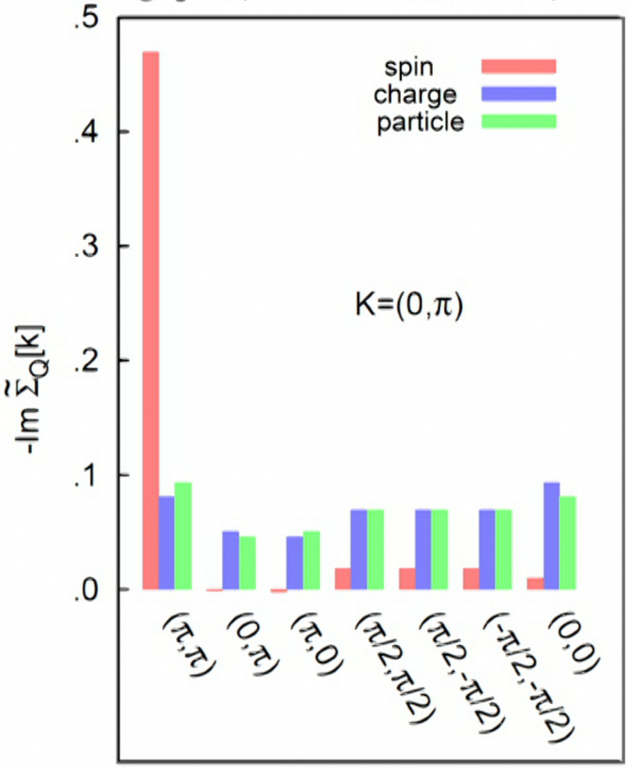
Contribution to single frequency



Conclusions:
Spin excitations are a better basis for representing interactions

The specific feature in Σ which gives rise to a pseudogap, is dominated by $Q = (\pi, \pi)$ spin fluctuations.

Pseudogap: $U/t = 6.4,\ \langle n \rangle = 1\ ,\beta = 30$



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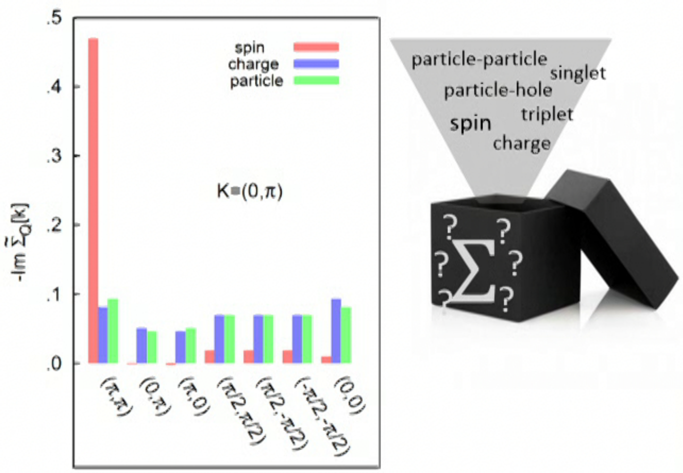
Summary

Fluctuation Diagnostics



Use Two-Particle Properties to isolate dominant contributions to the self energy

Pseudogap → $Q = (\pi,\pi)$ AFM Fluct

Experimental Response Functions

Spin and Charge Susceptibilities

Neutrons

NMR

RIXS

$\chi^{\nu\nu'\omega}$ $\Gamma^{\nu\nu'\omega}$

Susceptibilities Vertex Functions

Explore doping and temperature dependence

NMR: **Chen,** Nature Comm. **8** 14986 (2017)

PRL **114**, 236402 (2015)

Part 2: Unparticle Physics and the Joy of Exploring Nonsensical Ideas

J.P.F. LeBlanc

Memorial University of Newfoundland, NFLD, Canada

MEMORIAL UNIVERSITY

NSERC CRSNG

compute canada | calcul canada

SIMONS FOUNDATION

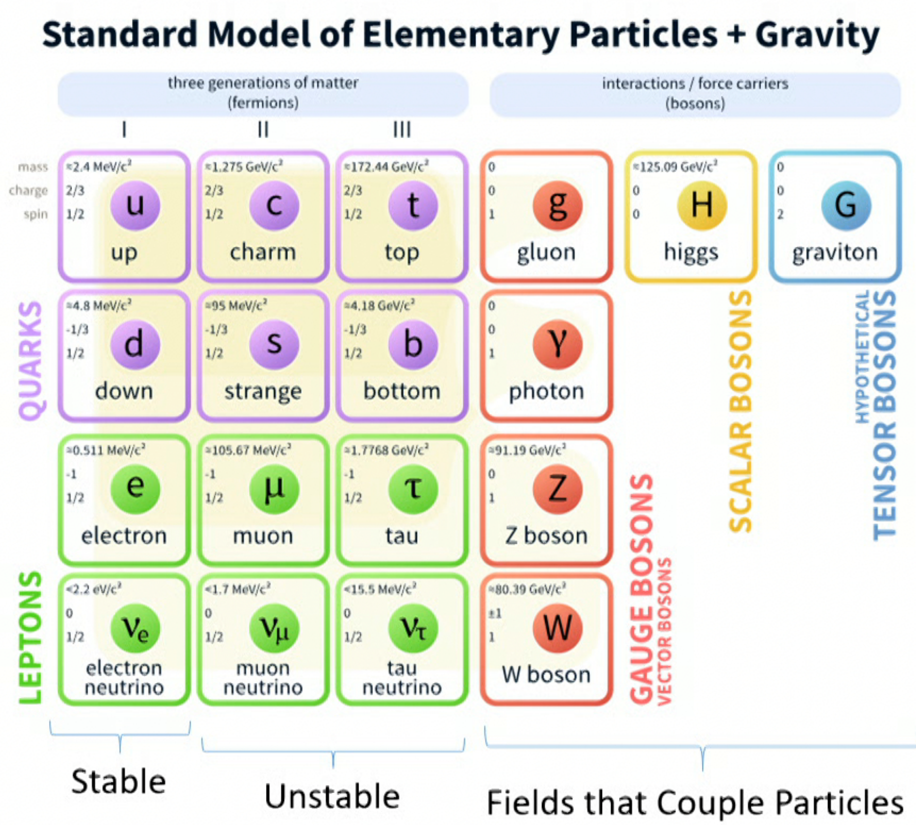
Overview

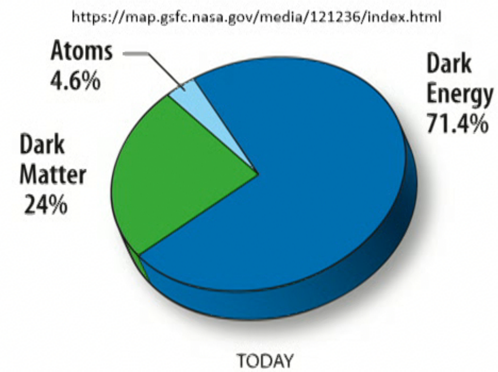
1) Story Time

2) Scale Invariance and 'Hidden' Unparticles

3) Unparticle 'stuff' and role of superconductivity

The Universe of Particles





Most of the known universe is not comprised of particles

The ongoing dark matter/ dark energy mystery

Our story begins – In the year… 2013

Adolfo Grushin
(Spanish/Russian)

MPI-PKS Dresden -> Berkley -> Institut Néel in Grenoble, France

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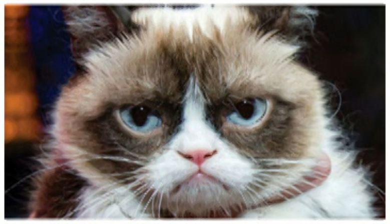

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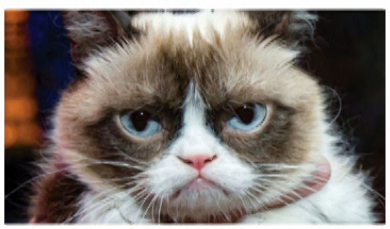

<u>MPI-PKS Dresden</u> <u>-> Berkley -> Institut Néel in Grenoble, France</u>

Knock at the door. James looks up from is terminal.

Adolfo: Hey, how's it going?

James eyes the visitor wearily.

James: Hey.

Adolfo: Have you ever heard of Unparticles?

James: No.

Adolfo: Do you think they could
cause superconductivity?

The Story Continues

PRL **98,** 221601 (2007) PHYSICAL REVIEW LETTERS week ending 1 JUNE 2007

Unparticle Physics

Howard Georgi*

Center for the Fundamental Laws of Nature, Jefferson Physical Laboratory, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 24 March 2007; revised manuscript received 20 May 2007; published 29 May 2007)

I discuss some simple aspects of the low-energy physics of a nontrivial scale invariant sector of an effective field theory—physics that cannot be described in terms of particles. I argue that it is important to take seriously the possibility that the unparticle stuff described by such a theory might actually exist in our world. I suggest a scenario in which some details of the production of unparticle stuff can be calculated. I find that in the appropriate low-energy limit, unparticle stuff with scale dimension $d_{\mathcal{U}}$ looks like a nonintegral number $d_{\mathcal{U}}$ of invisible particles. Thus dramatic evidence for a nontrivial scale invariant sector could show up experimentally in missing energy distributions.

The phrase 'Unparticle Stuff' is used repeatedly

INVISIBLE PARTICLES!

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Physics Letters B 650 (2007) 275–278

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Another odd thing about unparticle physics

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Received 21 May 2007; accepted 21 May 2007

Available online 24 May 2007

Editor: B. Grinstein

Abstract

The peculiar propagator of scale invariant unparticles has phases that produce unusual patterns of interference with Standard Model processes. We illustrate some of these effects in $e^+e^- \to \mu^+\mu^-$.

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INVISIBLE PARTICLES!

James smiles…

… decides to take a break from more important work

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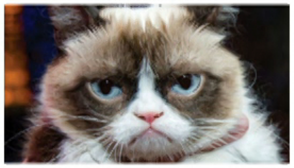


Our heroes made a pact. We would spend one day a week working on this. We chose Fridays.

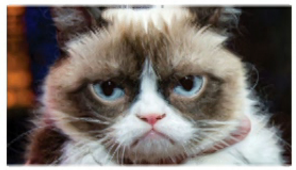
What IF unparticles could result in a superconductor?

What might that look like?

No one had EVER studied this. There were ZERO papers on it. It was 100% original!!!

I have NEVER before had so much fun doing science.

The Story Concludes



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Afternoon
Physics
Project

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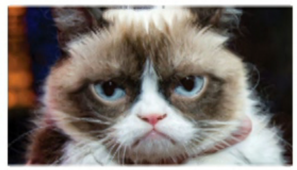

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PAPER

Unparticle mediated superconductivity

James P F LeBlanc[1,2] and Adolfo G Grushin[1]

[1] Max-Planck-Institut für Physik komplexer Systeme, D-01187 Dresden, Germany
[2] Department of Physics, University of Michigan, Ann Arbor, Michigan 48109, USA

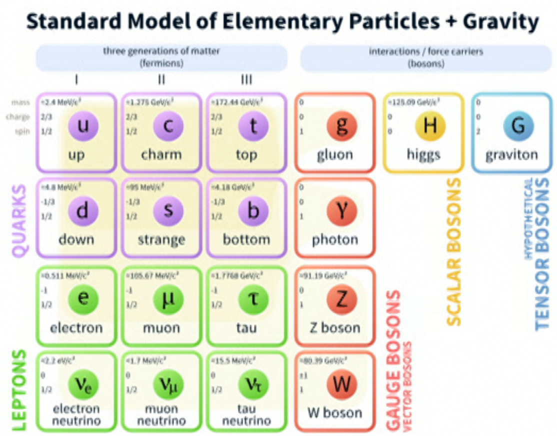
E-mail: jpfleblanc@gmail.com

Keywords: unparticle, superconductivity, coulomb

After many long Fridays of fapping away in our offices, a paper emerged.

2) So what IS an unparticle?

Particles in the standard model do not have the property of scale invariance



There might exist some 'hidden' element of our universe

- Made up of scale invariant 'stuff'
- Cannot be described as particles
- Must couple (interact) only very weakly with ordinary particles

Georgi makes a big fuss about scale invariance, so we need to understand this

Simple Scale Invariance

Any monomial, $f(x) = x^n$, is scale invariant

For any monomial, if I rescale the x-axis, there is a simple related rescaling of the y-axis.

$$f(\lambda x) = \lambda^n x^n$$

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The search for unparticles is a search for scale invariant objects

Kinetic energy of an electron

$$E_K = \frac{1}{2}mv^2 = \frac{p^2}{2m} \quad \xrightarrow{\text{AHA!!!!}} \quad E_K(2p) = 2^2 E_K(p)$$

QUESTION 1: What theory, and famous physicist RUINED this for us?

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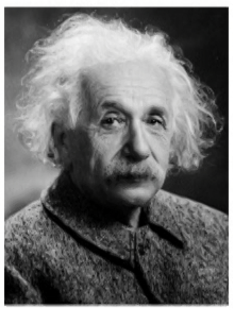

$$E^2 = p^2c^2 + m^2c^4$$

Relativistic Particles aren't scale invariant, because the mass cannot be rescaled.

QUESTION 2: Which standard model particle IS scale invariant?

So what do we know so far?

Class: Alright, if unparticles must be scale invariant they must be massless just like photons?

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Next Question: What is the density?

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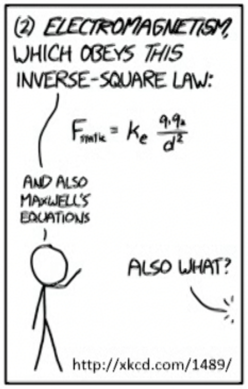
Need to know what a particle 'is'.

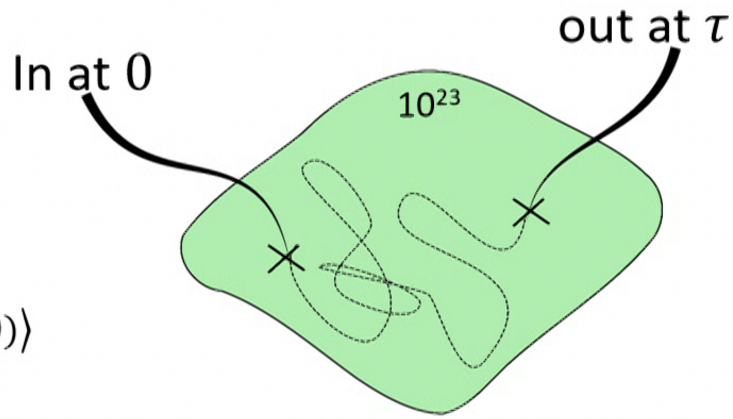
Particles in Condensed Matter



Single Particle Propagator
Green's Function $G(r,t)$

A tool to help solve problems

$$G_\sigma(r-r',\tau) = \langle c_{r\sigma}(\tau) c^+_{r'\sigma}(0) \rangle$$

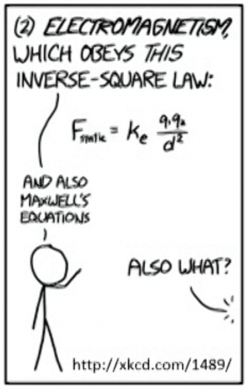


Repeat this process again and again to build a probability of being in a particular state at time τ

Resulting Green's function (propagator) contains density of states.

Fig. 1.1 *Propagation of Drunken Man*

Particles in Condensed Matter



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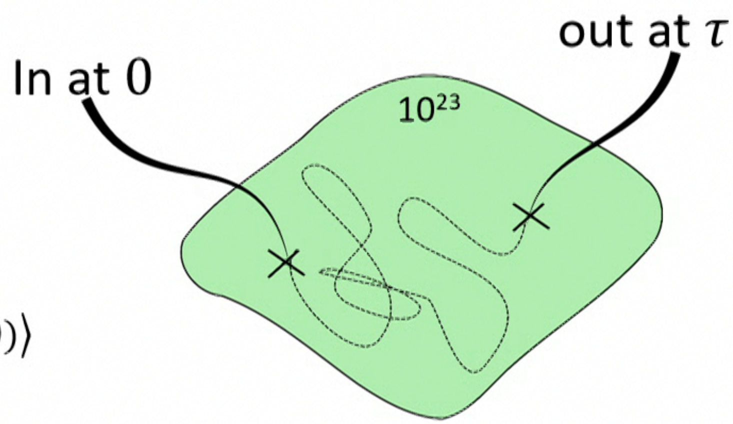

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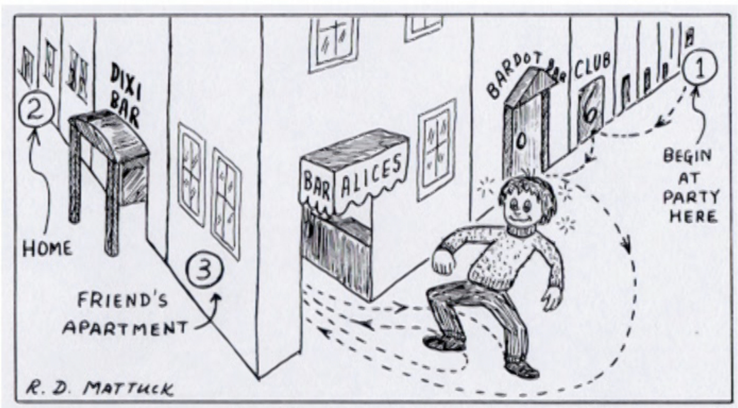


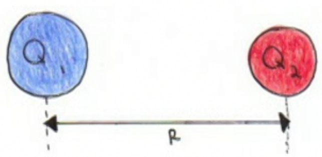
Fig. 1.1 *Propagation of Drunken Man*

A classical propagator – Scalar Field Theory

- D is the dimensionality

$$G(r) \propto \frac{1}{r^{-2+D+\eta}}$$

In 3 dimensions, $D = 3, \eta = 0$,
$G(r) = V(r) = \frac{q}{4\pi\epsilon_0}\frac{1}{r}$



$$V(r) \propto \frac{1}{r}$$

You have seen this MANY times and taken for granted some really weird facts about this simple function.

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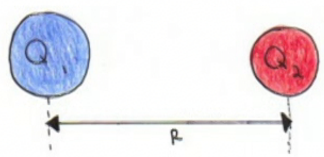

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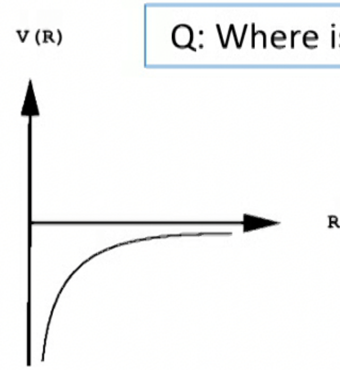
Q: Where is the particle? (R=?)



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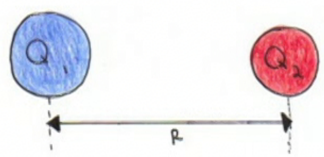

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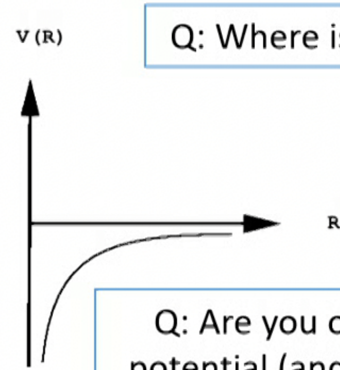

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Q: Are you ok with the fact that the potential (and propagator) is divergent where the particle 'is'?

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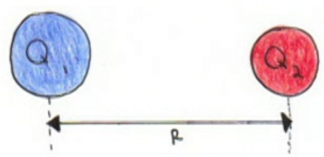

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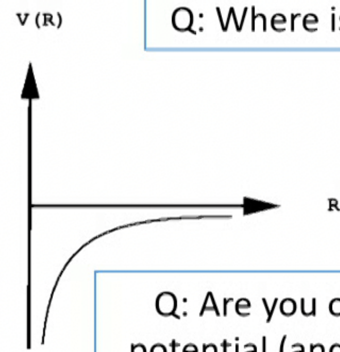

Q: Where is the particle? (R=?)

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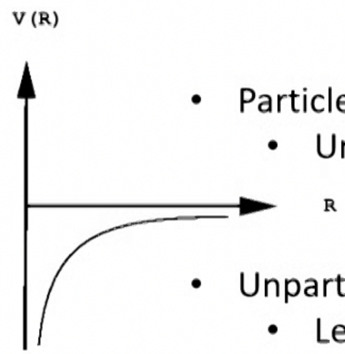
Conclusion: Particles are defined by divergences in propagators. And we are SUPER ok with those divergences.

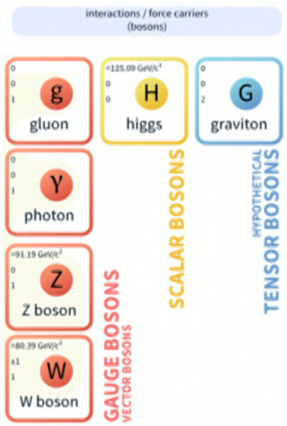
We find density by going around and adding up all the divergent points!

The Status Thus Far



- Particles are defined by divergences in potentials (propagators)
  - Unparticles might have a propagator with no divergence
- Unparticles must also be scale invariant, so they can't have mass.
  - Lets start with a mass then get rid of it

Mass, m of a bosonic field



Yukawa Potential $V(r) = \frac{e^{-mr}}{r}$

Photon, $m = 0$, $V(r) = \frac{1}{r}$, just gives coulomb interaction back!

$m = 0$ is already covered!!!

Get rid of a parameter

$$V(r) = \frac{e^{-mr}}{r}$$

Fourier Transform $r \to q$

$$V(q) = \frac{1}{q^2 + m^2}$$

The result is DEFINITELY not scale invariant unless $m = 0$

Q:How can I get rid of the mass without setting it to zero? Any thoughts?

Get rid of a parameter

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Ever hear the phrase: "You can't add apples and oranges"?
Q: Can you add apples and oranges?

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Ex. 5kg of apples, and 7kg of oranges.

We now have 12kg of 'stuff'.

We have neither broken math nor physics to do this.

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We now have 12kg of ‘stuff’.

We have neither broken math nor physics to do this.

Congrats: You’ve just added apples and oranges

Define Unparticles

We are going to take a collection of massive objects, and add them up.

$$V(q) = \frac{1}{q^2 + m^2}$$

Sum of masses doesn't depend on an individual mass

Something interesting happens if we take a specific collection of masses, $\rho(m)$.

$$V_U(q) = \int_0^\infty \frac{\rho(m)dm}{q^2 + m^2}$$

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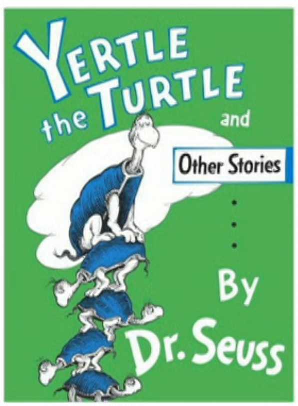


Unparticles are a tower of infinitely many particles of masses

Those masses might be distributed according to $\rho(m)$.

'Actually looks like d_U invisible particles'

All choices of d_U are scale invariant (monomial) and ill-defined mass

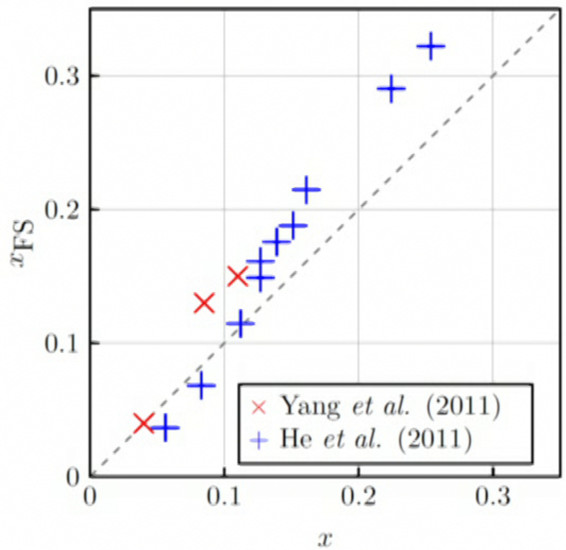
Estimates for condensed matter suggest $d_U > 4.23$

No Pole exists – Not a particle

Possible, Yes! Probable ? …. Maybe?

As theorists, we just need to prove that something physically COULD happen.

We don't need to tell you WHERE it might happen… But ok



- Put a hole in system
- Make measurement
- Find > 1 hole

Absence of Luttinger's Theorem due to Zeros in the Single-Particle Green Function

P. Phillips and Charles Kane
PRL **110**, 090403 (2013)

Unparticle Propagators Have Zeroes, and might cause a breakdown in Luttinger theorem.

Electrons have 'vanished' or become invisible, and no longer contribute to the density, and this means MORE holes

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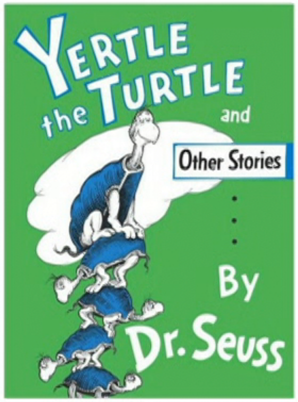

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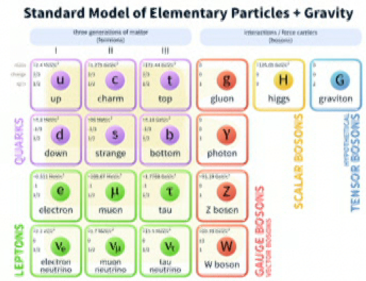
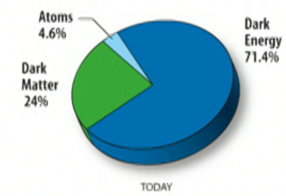
In Summary

Our picture of the universe is incomplete, and physics beyond the standard model is taken very seriously in high energy physics.





It is AMAZING to take a simple set of ideas and not violate any physics in the process, but arrive at an unexpected result!

We take a stack of massive objects

Put them a box

Masses should add…but…

The result has no density

All of physics is not solved. And some of the most important physics is emergent, in that collections of particles behave in unexpected ways.

Summary

Numerically 'Exact' Solutions to the 2D Hubbard model ⟷ Some Fun Unparticle Nonsense

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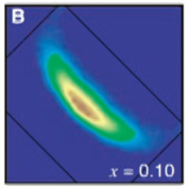
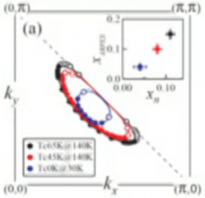
Some Fun Unparticle Nonsense





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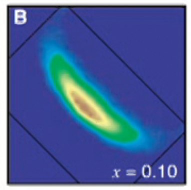

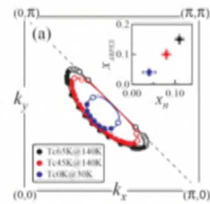

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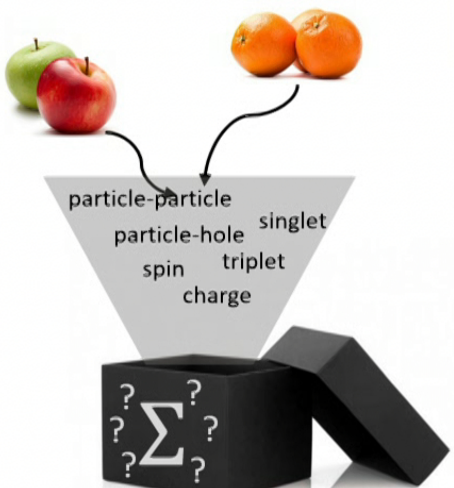

We deal with a lot of black boxes in science.

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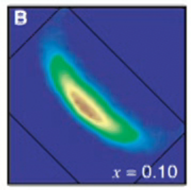

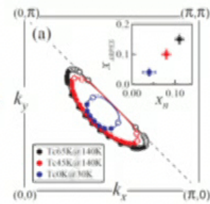

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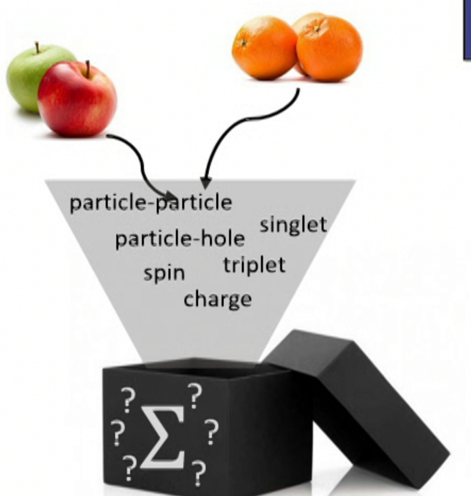
Averages, or integrating out a degree of freedom results in loss of information

Strive to systematically rule out such things.



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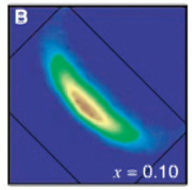

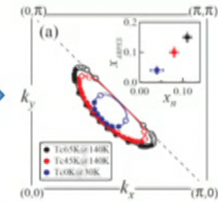

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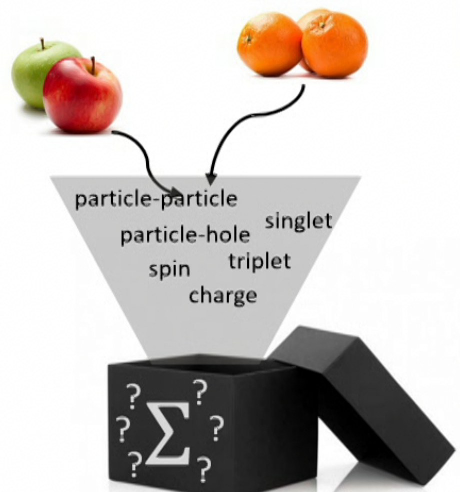

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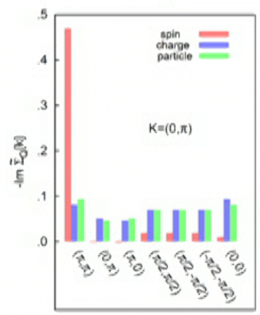

Pseudogap → $Q = (\pi, \pi)$ AFM Fluct

Fluctuation Diagnostics: PRL **114**, 236402 (2015)

Hubbard Benchmarks: PRX **5**, 041041 (2015)

NMR: Nature Communications: **8**, 14986 (2017)

Unparticle SC: NJP **17**, 033039 (2015)