Title: PSI 2017/2018 - Scattering Amplitudes in QFT & String Theory - Lecture 15

Date: Apr 25, 2018 11:30 AM

URL: http://pirsa.org/18040047

Abstract:

- "factorized" or "double-copy" form of n-pt open superstring amplitudes

## Review:

- disk integrals of Parke-Taylor form

$$Z(\rho(1,2,\dots,n)\,|\,1,2,\dots,n) = (-2\alpha')^{n-3} \int_{D(\rho(1,2,\dots,n))} \frac{dz_1 \dots dz_n}{\mathrm{vol}\, SL_2(\mathbb{R})} \frac{\mathcal{I}_n}{(1,2,\dots,n)}$$

- BCJ relations from $\int \frac{\partial}{\partial z_j} (\mathcal{I}_n \dots) = 0$

$$\sum_{j=2}^{n-1} k_1 \cdot k_{23\dots j}\, Z(\rho\,|\,2,3,\dots,j,1,j+1,\dots,n) = 0$$

matrix inverse to $m(\cdot|\cdot)$ in "magical basis"

$$S_0[\alpha|\beta]_1 = m^{-1}(1,\alpha,n,n-1 \,|\, 1,\beta,n-1,n)$$

entries $\sim (k\cdot k)^{n-3}$ are <u>local</u>, i.e. no poles in any $k^2_{ij\ldots}$

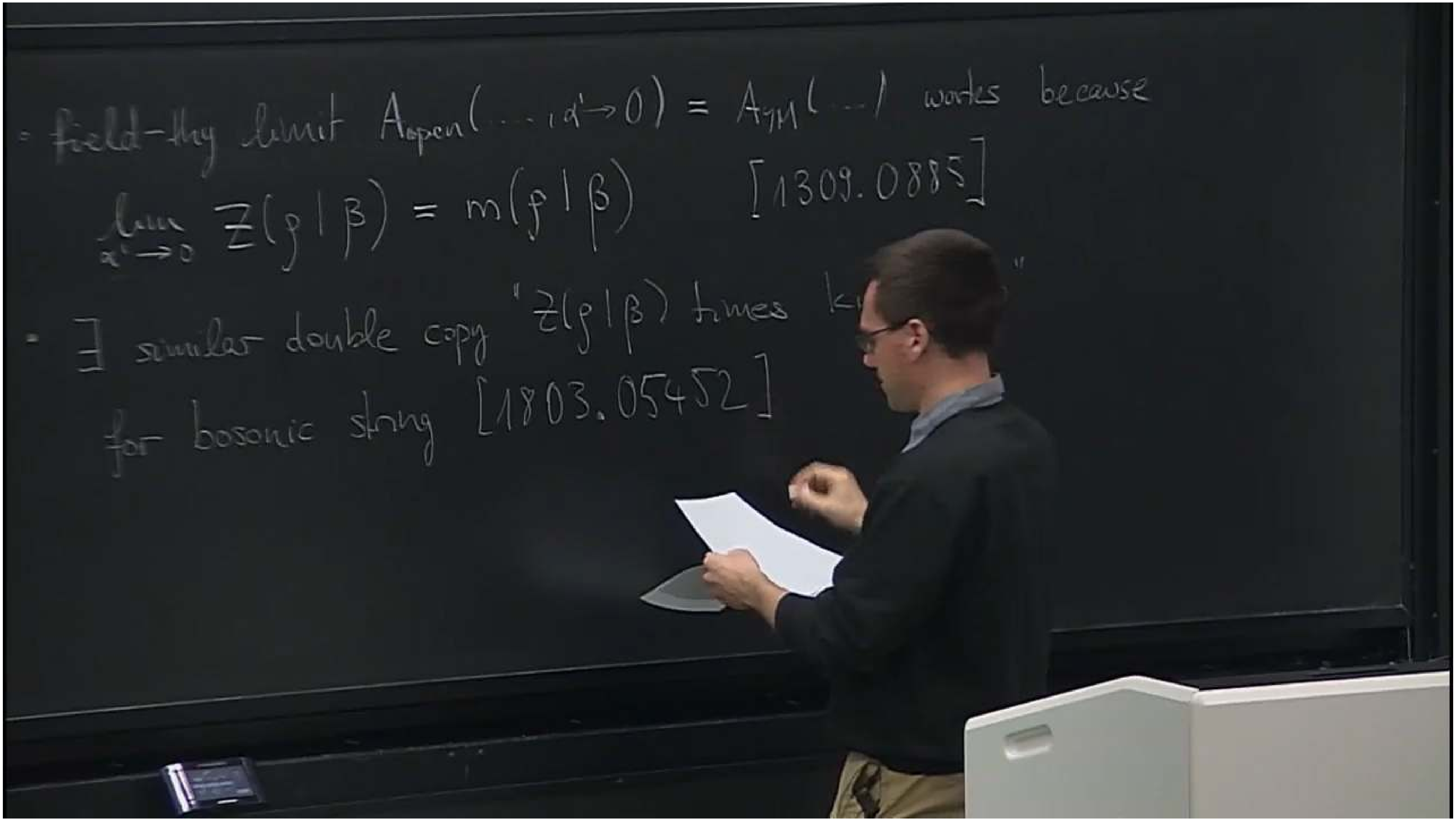
simple recursion $S_0[2|2]_1 = -k_1\cdot k_2$ and

$$S_0[A,j|B,j,C]_1 = -k_j\cdot(k_1+k_B)\,S_0[A|B,C]_1$$

$A$: $a_1,a_2,\ldots,a_m$ $B$: $b_1,b_2,\ldots,b_p$ $k_B$: $k_{b_1}+k_{b_2}+\ldots+k_{b_p}$

- "factorized" or "double-copy" form of n-pt open-superstring amplitudes

$$A_{open}(\tau(1,2,\ldots,n);\alpha') = \sum_{\alpha,\beta\in S_{n-3}} Z(\tau(1,2,\ldots,n)\,|\,1,\alpha,n,n-1)$$
$$\times\, S_0[\alpha|\beta]\, A_{YM}(1,\beta,n-1,n)$$


· field-thy limit $A_{open}(\ldots, \alpha' \to 0) = A_{YM}(\ldots)$ works because
$\lim_{\alpha' \to 0} Z(\rho|\beta) = m(\rho|\beta)$ [1309.0885]
· ∃ similar double copy "$Z(\rho|\beta)$ times k
for bosonic string [1803.05452]

1.5 Monodromy relations

- $Z(\rho|\beta)$ @ fix $\rho$ $\Rightarrow$ $(n-3)!$ independent PT's $\beta$

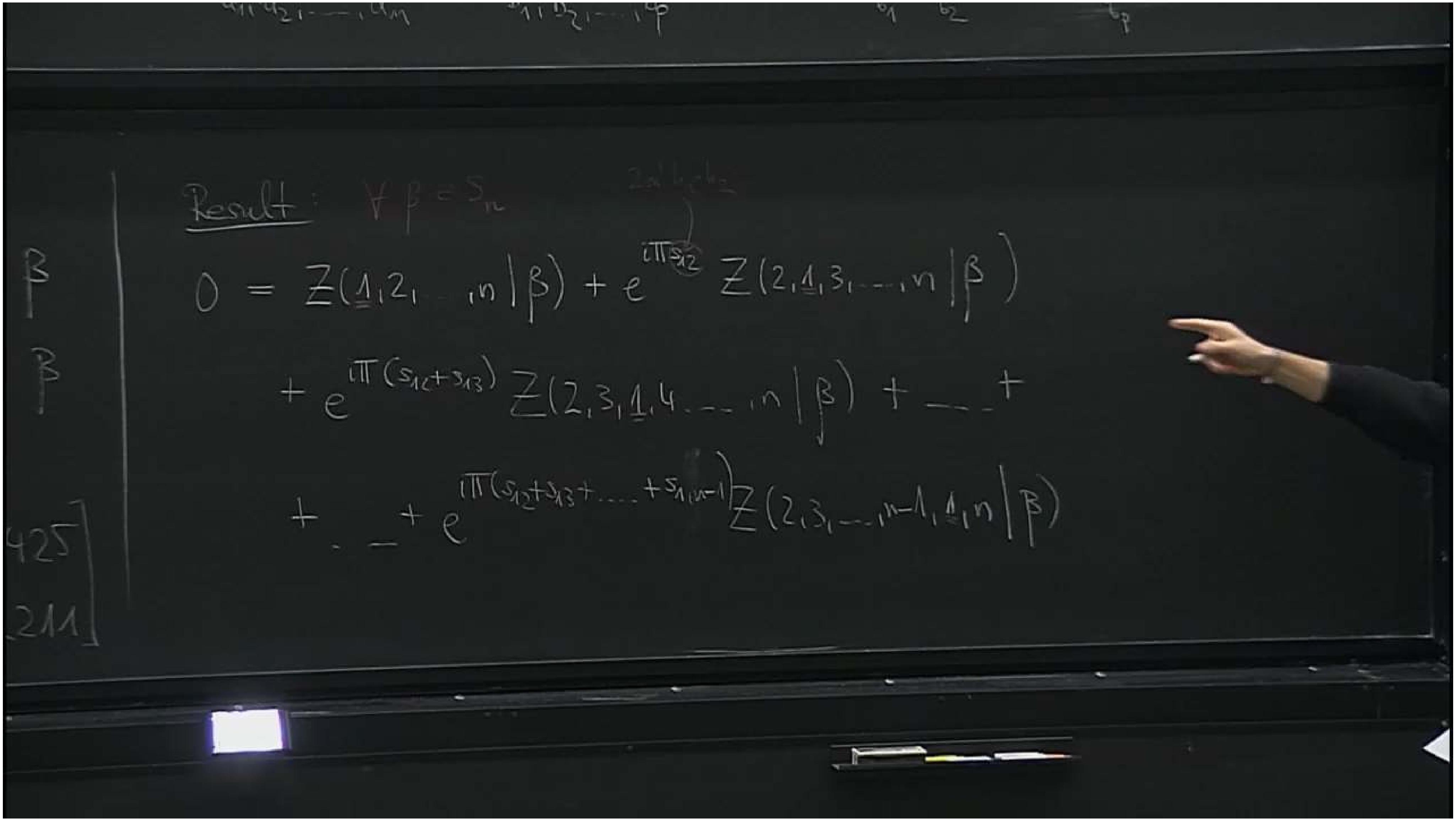
  $\rightsquigarrow$ how many independent cycles $\rho$ @ fixed $\beta$

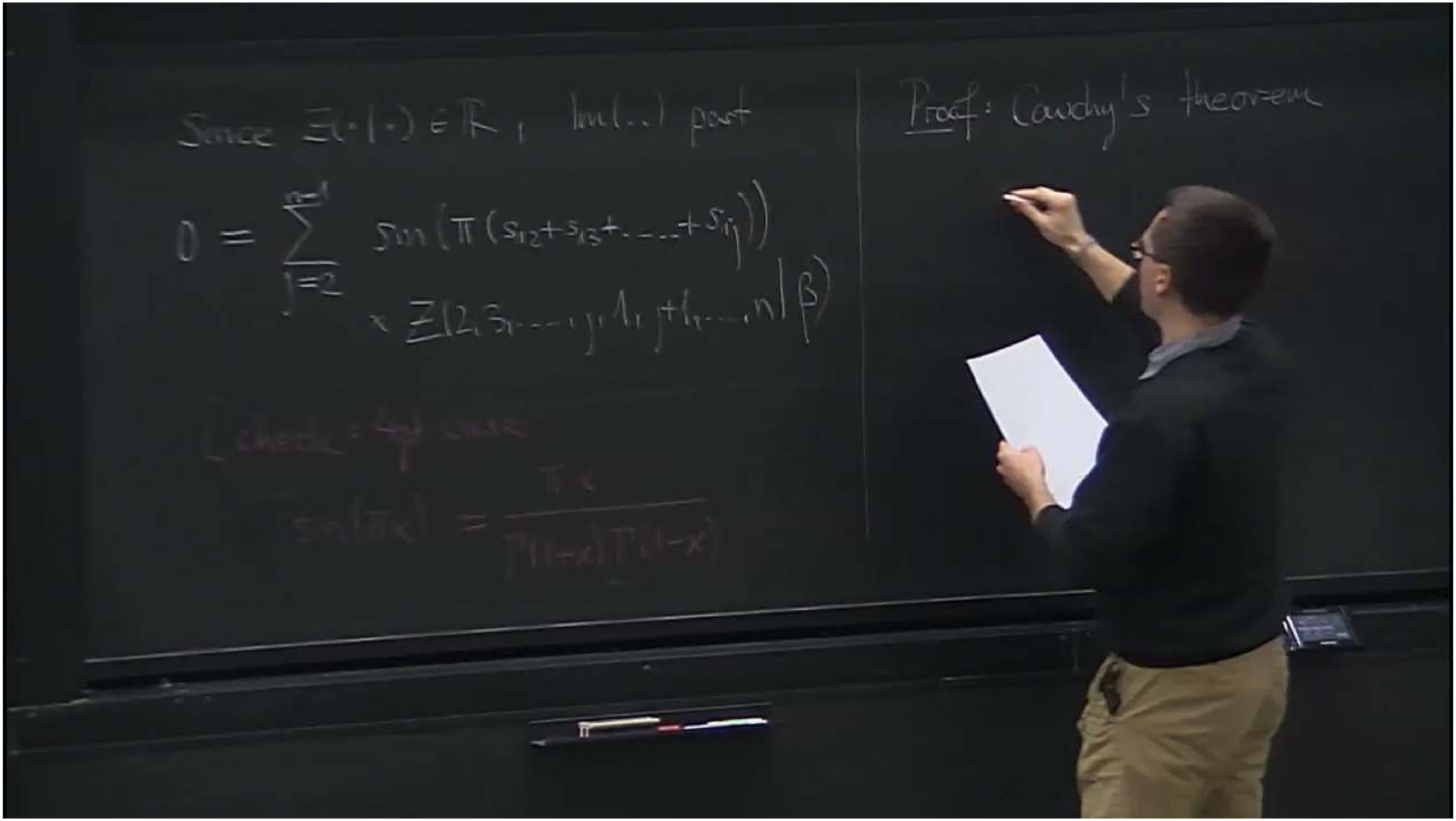
- will see first string-theory derivation of

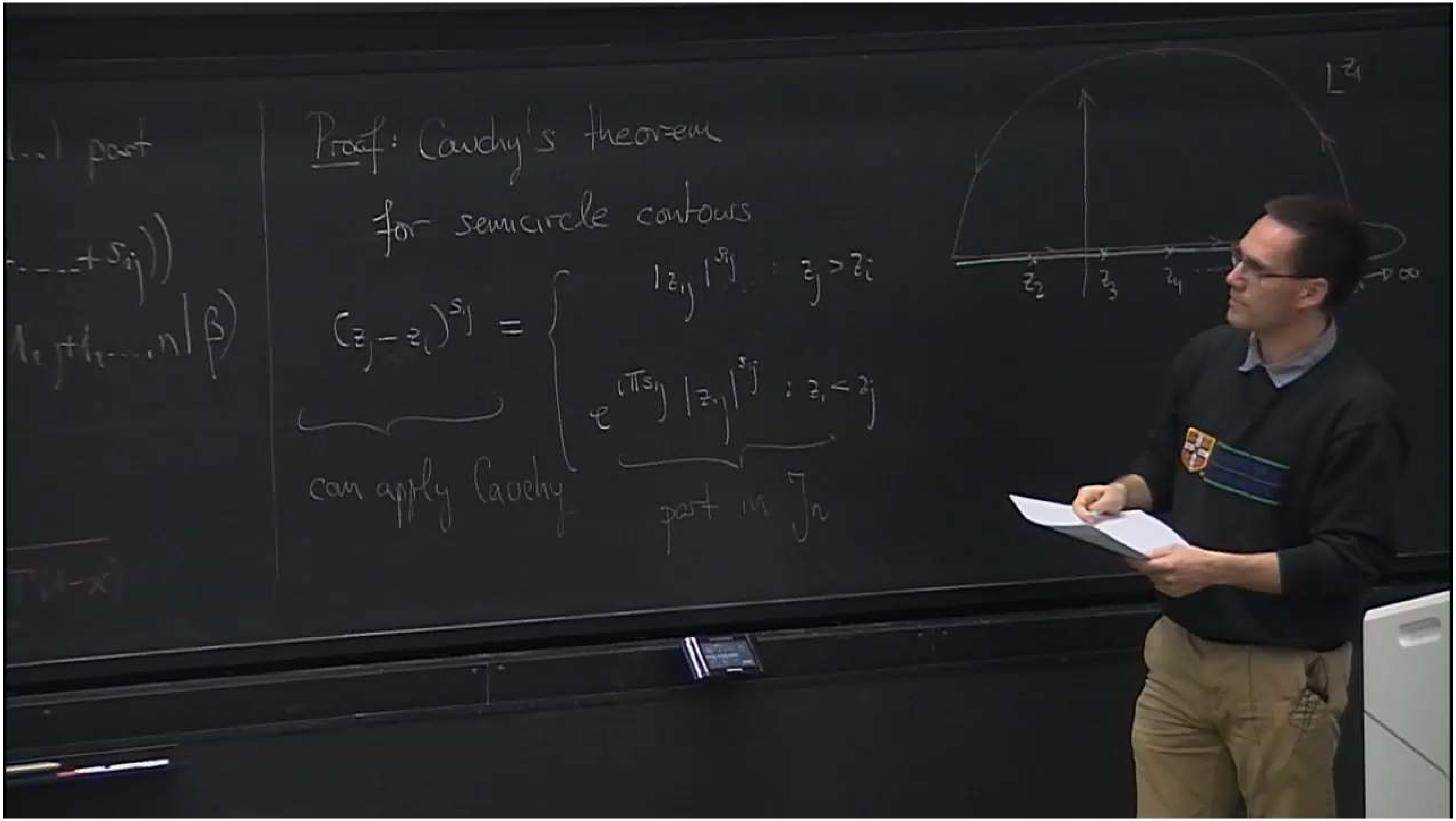
  BCJ relations in YM $\begin{bmatrix} 0907.1425 \\ 0907.2211 \end{bmatrix}$

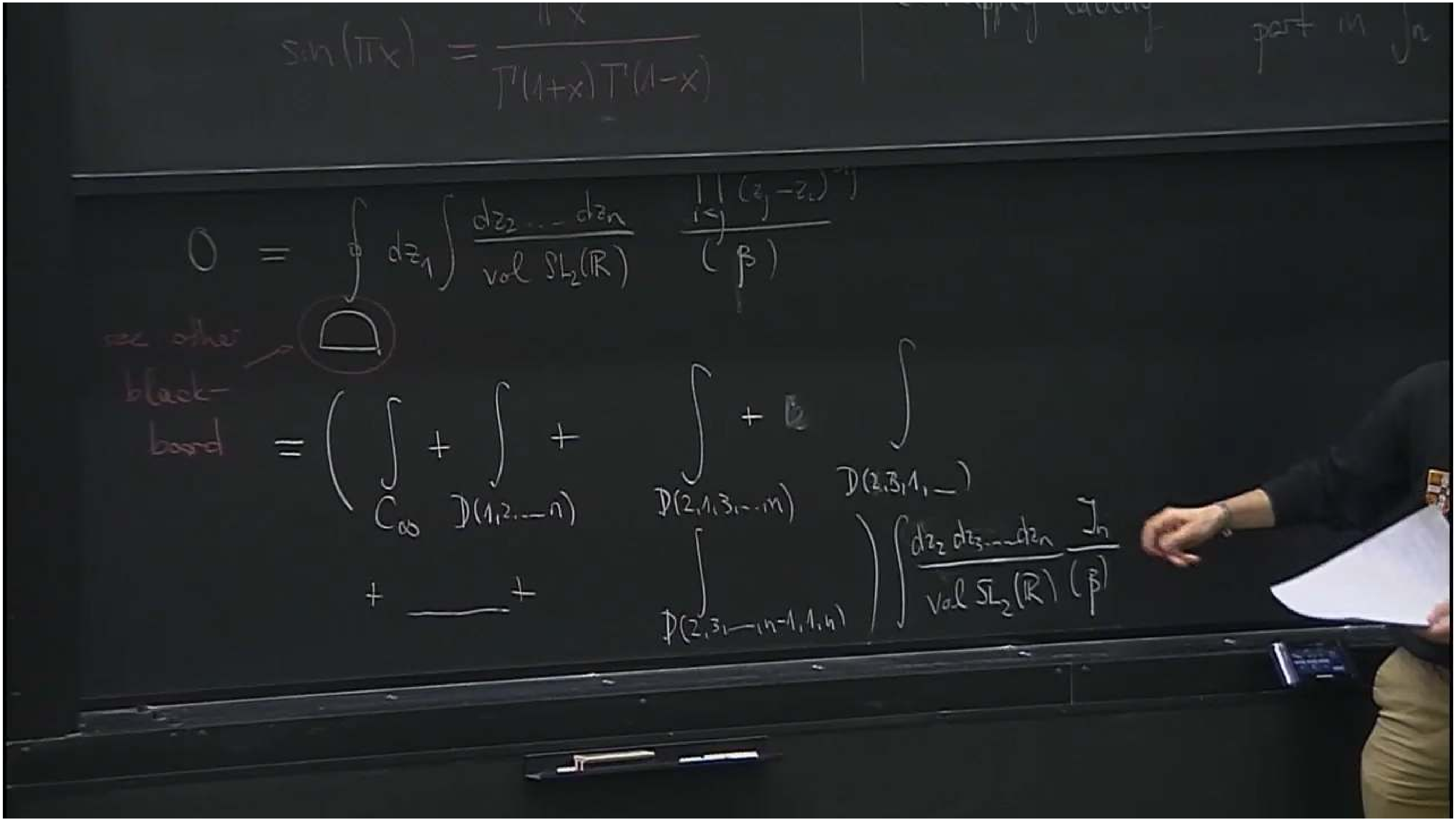
Result

$0 =$


Result:
$\forall \beta \in S_n$
$0 = Z(1,2,\dots,n|\beta) + e^{i\pi s_{12}} Z(2,1,3,\dots,n|\beta)$
$+ e^{i\pi(s_{12}+s_{13})} Z(2,3,1,4,\dots,n|\beta) + \dots +$
$+ \dots + e^{i\pi(s_{12}+s_{13}+\dots+s_{1,n-1})} Z(2,3,\dots,n-1,1,n|\beta)$


Since Ξ(·|·) ∈ ℝ, Im(..) part
0 = Σ_{j=2}^{n-1} sin(π(s12+s13+...+s1j))
× Ξ(2,3,...,j,1,j+1,...,n|β)
[check: 4pt case
sin(πx) = πx / Γ(1+x)Γ(1-x)
Proof: Cauchy's theorem


Proof: Cauchy's theorem
for semicircle contours
can apply Cauchy
part in $J_n$


sin(πx) = πx / Γ(1+x)Γ(1−x)
part in J_n
0 = ∮ dz_1 ∫ dz_2 ... dz_n / vol SL_2(R) · ∏_{i<j} (z_j − z_i)^{s_{ij}} / (β)
see other black-board
= ( ∫_{C_∞} + ∫_{D(1,2,...,n)} + ∫_{D(2,1,3,...,n)} + ∫_{D(2,3,1,...)}
+ ____ + ∫_{D(2,3,...,n−1,1,n)} ) ∫ dz_2 dz_3 ... dz_n / vol SL_2(R) · J_n / (β)

$$0 = \oint dz_1 \int \frac{dz_2 \cdots dz_n}{\mathrm{vol}\, SL_2(\mathbb{R})} \; \frac{\prod_{i<j} (z_j - z_i)^{s_{ij}}}{(\beta)}$$

see other black-board

differ by phases

$$= \Big( \cancel{\int_{C_\infty}} + 1 \int_{D(1,2,\ldots,n)} + e^{i\pi s_{12}} \int_{D(2,1,3,\ldots,n)} + e^{i\pi(s_{12}+s_{13})} \int_{D(2,3,1,\ldots)}$$

$$+ \cdots + e^{i\pi(s_{12}+s_{13}+\cdots+s_{1,n-1})} \int_{D(2,3,\ldots,n-1,1,n)} \Big) \int \frac{dz_2\, dz_3 \cdots dz_n}{\mathrm{vol}\, SL_2(\mathbb{R})} \; \frac{J_n}{(\beta)}$$

drops out, integrand decays @ $\infty$

Claim follows b

differ by phases

Claim follows by identifying $\int_{D(\rho)} (\ldots) \longrightarrow Z(\rho \mid \ldots)$

$\left( \xrightarrow[\mathrm{Im}(\ldots)]{\alpha' \to 0} B(\,)\ \text{rel's},\quad \sin(\pi s_{ij}) \to k_i \cdot k_j \right)$

Corollary: monodromy relations

$$0 = \sum_{j=1}^{n-1} e^{i\pi(s_{12}+s_{13}+\ldots+s_{1j})} A_{open}(2,3,\ldots,j,1,j+1,\ldots,n;\alpha')$$

$$\Rightarrow (n-3)!\ \text{basis, e.g.}\ \left\{ A_{open}(1,\rho,n-1,n),\ \rho \in S_{n-3} \right\}$$

BCJ relations in YM [0907.1425, 0907.2211]

## II) Closed-string tree-level amplitudes

### 2.1 Vertex operators & amplitudes

Double copy $\left( \dfrac{\text{open-string vertex op's}}{\text{plane waves}} = \hat{V} \right)$ $\qquad \bar{\epsilon}\cdot k = 0$

$\bar{\epsilon}^{\mu}$ indep. on $\epsilon^{\mu}$ as long as

$$V^{(q=0,-1)}_{\text{open}}(\epsilon,k) = \hat{V}^{(q)}(\epsilon,k)\exp(ik\cdot X)$$

$$\hookrightarrow \quad V^{(q,\bar{q})}_{\text{closed}}(\epsilon,\bar{\epsilon},k) = \hat{V}^{(q)}(\epsilon,k)\,\hat{\bar{V}}^{(\bar{q})}(\bar{\epsilon},k)\,e^{ik\cdot X}$$

• m

- $\hat{\tilde{V}}$ has $(\bar\partial X^\mu, \bar\psi^\mu)$ in place of $(\partial X^\mu, \psi^\mu)$ in $\hat{V}$

- $\epsilon^\mu \bar\epsilon^\nu = \underbrace{h^{(\mu\nu)}}_{\substack{\text{symm traceless} \\ \text{graviton}}} \oplus \underbrace{B^{[\mu\nu]}}_{\substack{\text{antisymm} \\ \text{"B-field"}}} \oplus \underbrace{\eta^{\mu\nu}_{SO(d-2)}\ \phi}_{\substack{\text{little-group} \\ \text{trace: dilaton}}}$

$V$

symm traceless graviton

antisymm "B-field"

little-group trace: dilaton

- no OPE contractions between $\hat{V}$ & $\hat{\bar{V}}$

$\Rightarrow$ tree-lv. correlators factorize

$$\alpha'_{closed} = 4\alpha'_{open} = 2$$

- tree-lv. amplitudes

$$M_{closed}(1,2,\dots,n,4\alpha') = \frac{1}{\pi^{n-3}(2\alpha')^{n-1}}$$

$$\times \int_{\mathbb{C}^n \setminus \{z_i = z_j\}} \frac{d^2 z_1\, d^2 z_2 \dots d^2 z_n}{\mathrm{vol}\, SL_2(\mathbb{C})} \left\langle \left( \prod_{j=1}^{n-2} V_j^{(0,0)}(z_j) \right) V_{n-1}^{(-1,-1)}(z_{n-1})\, V_n^{(-1,-1)}(z_n) \right\rangle$$

$$= \left(\frac{2\alpha'}{\pi}\right)^{n-3} \int \frac{d^2 z_1\, d^2 z_2 \dots d^2 z_n}{\mathrm{vol}\, SL_2(\mathbb{C})} |J_n|^2$$

$$\times \left| \sum_{\alpha,\beta \in S_{n-3}} A_{YM}(1,\alpha,n-1,n) \frac{S_0[\alpha|\beta]_1}{(1,\beta,n,n-1)} \right|^2$$

→ what is the complex co

- $A_{YM} \to \bar{A}_{YM}$ with $\epsilon^\mu \to \bar{\epsilon}$

...e-lv. amplitudes

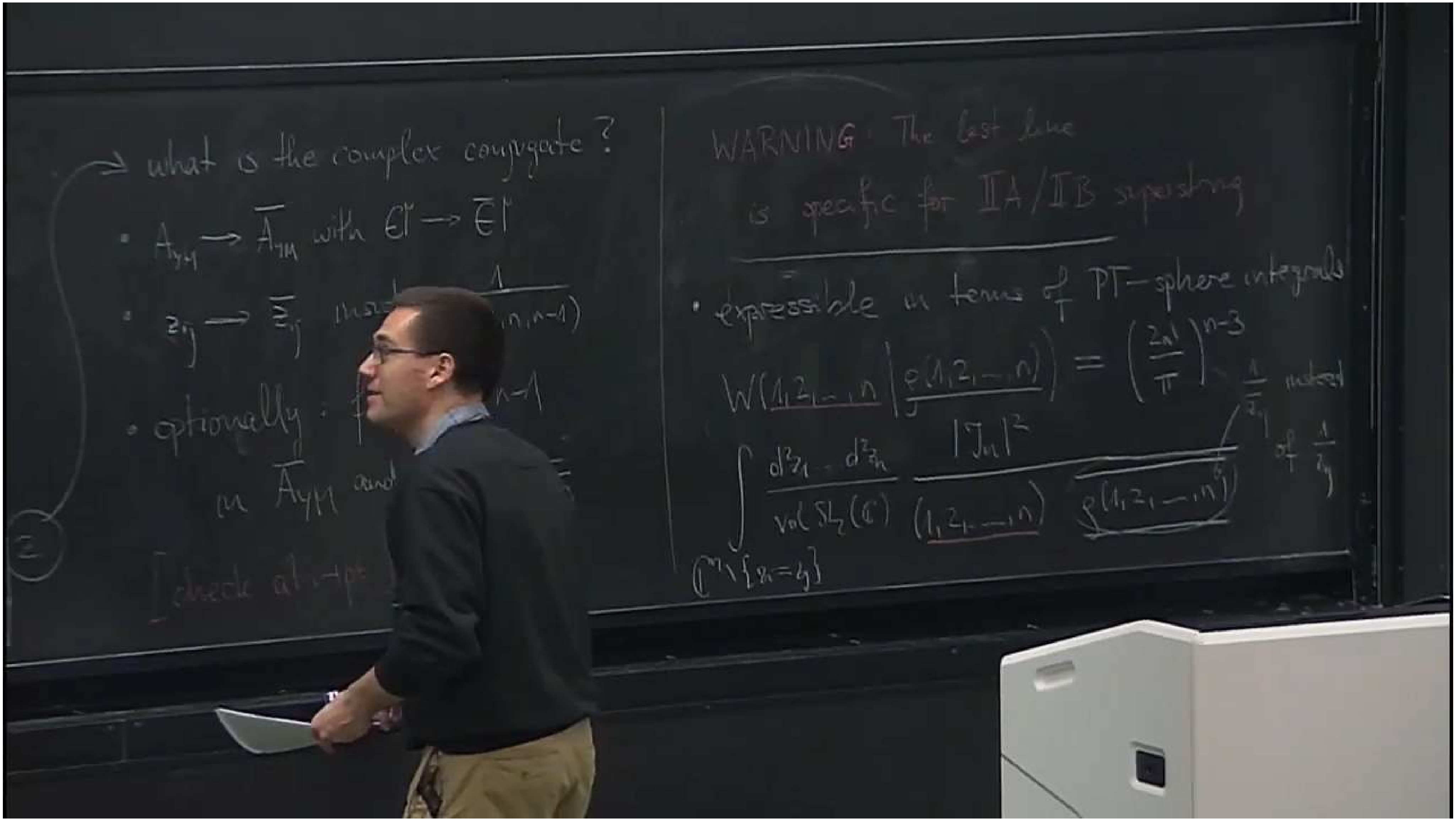
$$A_{\text{closed}}(1,2,\dots,n;\alpha') = \frac{1}{\pi^{n-3}(2\alpha')^{n-1}} \int_{\mathbb{C}^n \setminus \{z_i = z_j\}} \frac{d^2z_1\, d^2z_2 \dots d^2z_n}{\text{vol } SL_2(\mathbb{C})} \left\langle \left( \prod_{j=1}^{n-2} V_j^{(0,0)}(z_j) \right) V_{n-1}^{(-1,-1)}(z_{n-1}) V_n^{(-1,-1)}(z_n) \right\rangle$$

$$= \left(\frac{2\alpha'}{\pi}\right)^{n-3} \int \frac{d^2z_1\, d^2z_2 \dots d^2z_n}{\text{vol } SL_2(\mathbb{C})} |\mathcal{J}_n|^2$$

$$\times \left| \sum_{\alpha,\beta \in S_{n-3}} A_{YM}(1,\alpha,n-1,n) \frac{S_0[\alpha|\beta]_1}{(1,\beta,n,n-1)} \right|^2$$

→ what is the complex conjugate?

- $A_{YM} \to \bar{A}_{YM}$ with $\epsilon \to \bar{\epsilon}$
- $z_i \to \bar{z}_i$ inside $\frac{1}{(1,\beta,n,n-1)}$
- optionally: flip meaning of $A_{YM}$ and $\frac{1}{(1,\beta,n,n-1)}$


what is the complex conjugate ?
optionally
WARNING: The last line
is specific for IIA/IIB superstring
expressible in terms of PT-sphere integrals
instead

$$\times \left| \sum_{\alpha,\beta \in S_{n-3}} A_{YM}(1,\alpha,n-1,n) \, \frac{\cdots}{(1,\beta,n,n-1)} \right|$$

[check at 4pt]

reproduce $M_{GR}$ @ $\alpha' \to 0$ since

$$\lim_{\alpha' \to 0} \overbrace{W(\beta | \rho)}^{\text{symm.}} = m(\beta|\rho) = \lim_{\alpha' \to 0} \overbrace{Z(\beta|\rho)}^{\text{not symm}}$$

[check using $n-1 \leftrightarrow n$ flip in cc part of above $|\ldots|^2$]

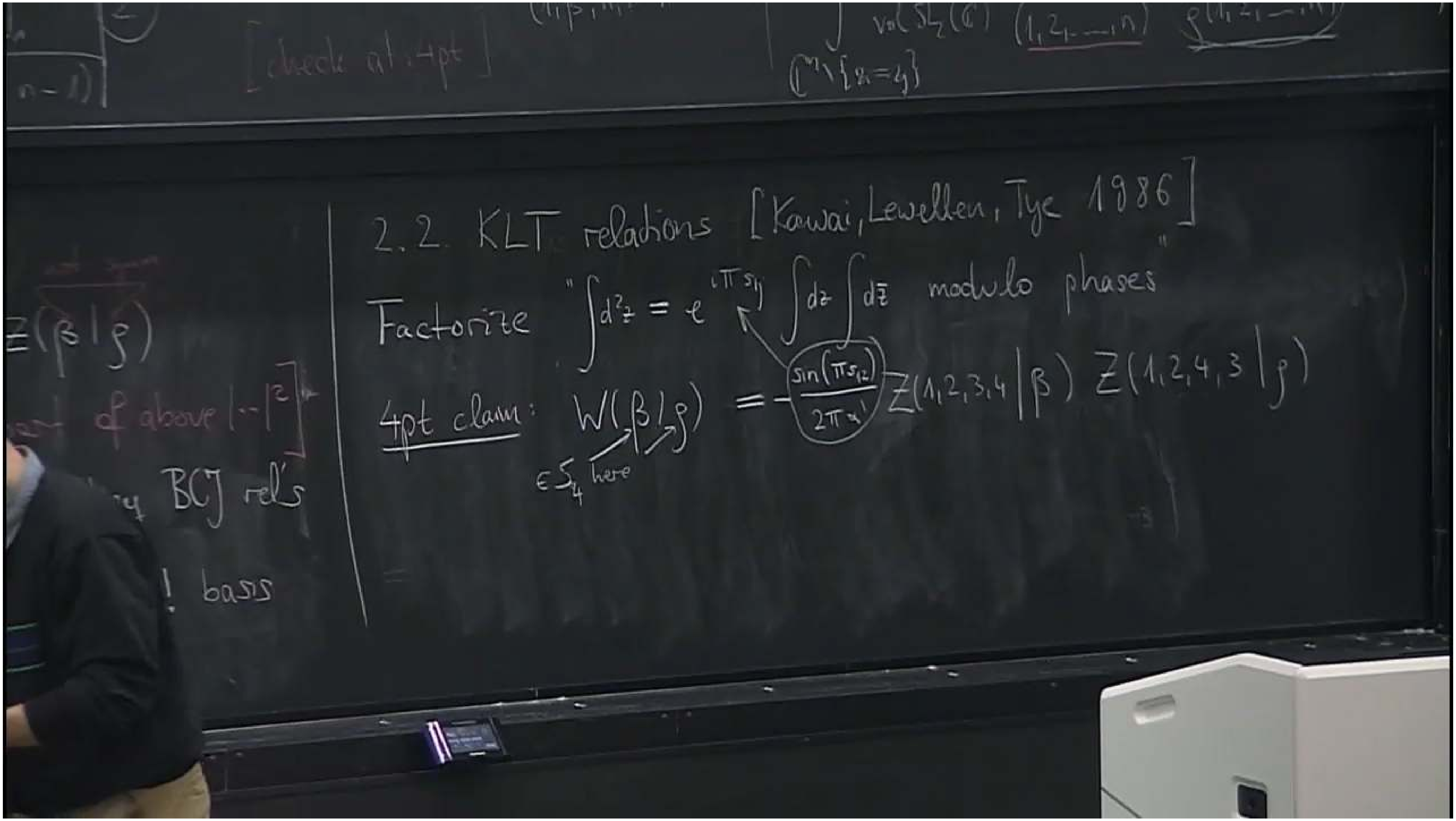
by discarding $\int \frac{\partial}{\partial z_j}$ & $\int \frac{\partial}{\partial \bar{z}_j}$, $W(\beta|\rho)$ obey BCJ rel's separately in both $\beta$ & $\rho$ $\Rightarrow$ $(n-3)! \times (n-3)!$ basis

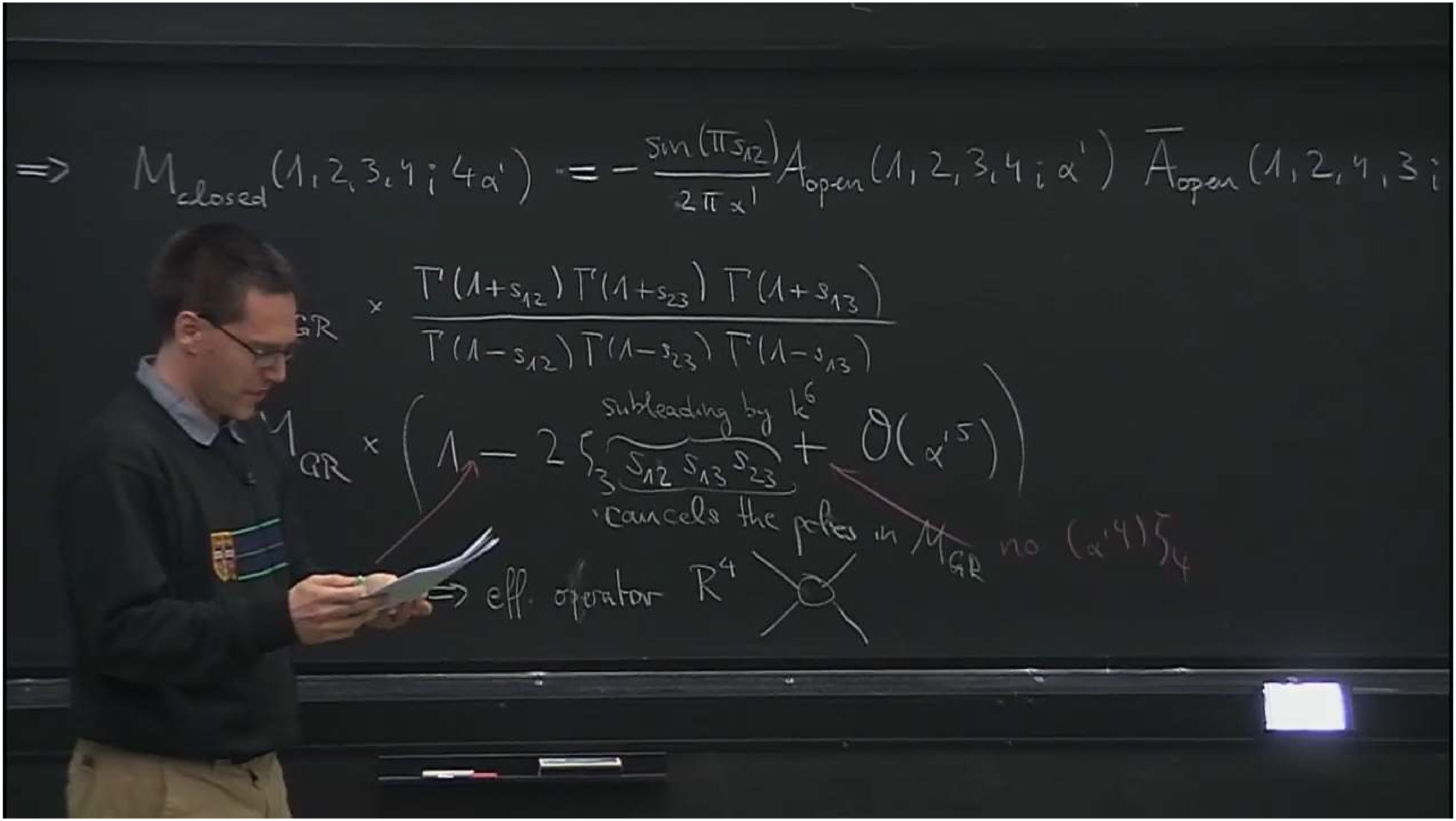
Claim follows by identif

Corollary: mono…omy

$$0 = \sum_{j=1}^{n-1} e^{i\pi(\ldots}$$

$\Rightarrow$ $(n-3)!$ basis, e.g.


[check at 4pt]
Z(β|ρ)
of above |..|²
BCJ rel's
basis
2.2. KLT relations [Kawai, Lewellen, Tye 1986]
Factorize "∫d²z = e^{iπ s_ij} ∫dz ∫dz̄ modulo phases"
4pt claim: W(β|ρ) = − sin(π s_12) / 2πα' Z(1,2,3,4|β) Z(1,2,4,3|ρ)
∈ S_4 here



$\Rightarrow M_{closed}(1,2,3,4;4\alpha') = -\frac{\sin(\pi s_{12})}{2\pi\alpha'} A_{open}(1,2,3,4;\alpha')\,\bar{A}_{open}(1,2,4,3;$
$\times \frac{\Gamma(1+s_{12})\Gamma(1+s_{23})\Gamma(1+s_{13})}{\Gamma(1-s_{12})\Gamma(1-s_{23})\Gamma(1-s_{13})}$
$M_{GR} \times \left(1 - 2\zeta_3 \overbrace{s_{12}s_{13}s_{23}}^{\text{subleading by } k^6} + \mathcal{O}(\alpha'^5)\right)$
cancels the poles in $M_{GR}$
no $(\alpha'^4)\zeta_4$
$\Rightarrow$ eff. operator $R^4$