Title: PSI 2017/2018 - Scattering Amplitudes in QFT & String Theory - Lecture 14

Date: Apr 24, 2018 11:30 AM

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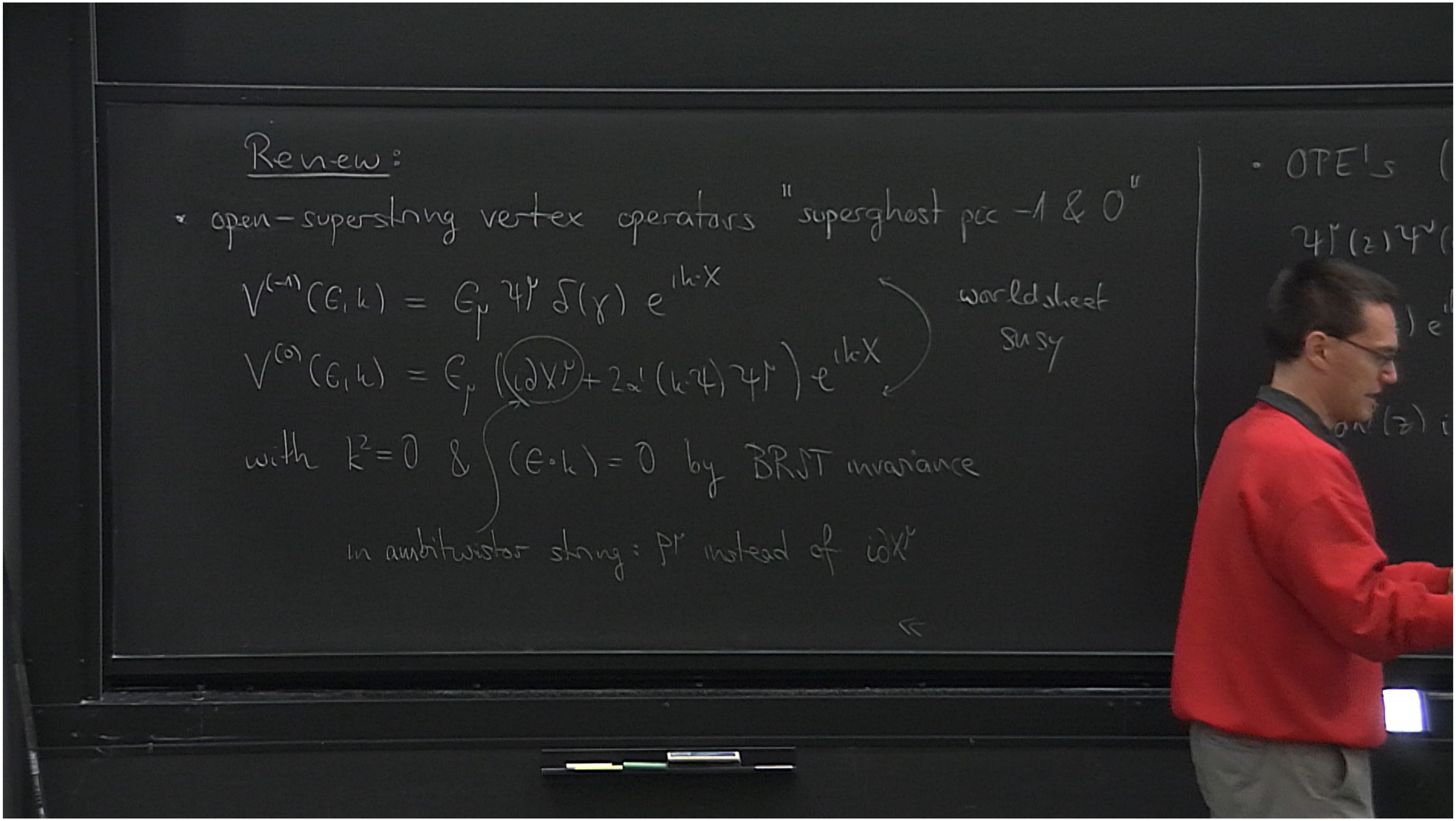
Abstract:

$$R_{ijkl} = \left(\frac{\partial \bar{x}^a}{\partial x^i}\right)\left(\frac{\partial \bar{x}^b}{\partial x^j}\right)\left(\frac{\partial \bar{x}^c}{\partial x^k}\right)\left(\frac{\partial \bar{x}^d}{\partial x^l}\right)\bar{R}_{abcd} + K_{ik}K_{jl} - K_{il}K_{jk} \quad \text{[EUCLIDEAN]}$$

$$R^{(3)}_{ijkl} = \delta^\mu_i \delta^\nu_j \delta^\rho_k \delta^\sigma_l R^{(4)}_{\mu\nu\rho\sigma} - K_{ik}K_{jl} + K_{il}K_{jk} \quad \text{[LORENTZIAN, ADM]}$$

- LIE DERIVATIVE, GAUGE TRANSFORMATIONS
- EXTRINSIC CURVATURE, ADM
- PERTURBATION THEORY


Review:
• open-superstring vertex operators "superghost pic −1 & 0"
$V^{(-1)}(\epsilon,k) = \epsilon_\mu \psi^\mu \delta(\gamma)\, e^{ik\cdot X}$
$V^{(0)}(\epsilon,k) = \epsilon_\mu \left(i\partial X^\mu + 2\alpha'(k\cdot\psi)\psi^\mu \right) e^{ik\cdot X}$
worldsheet susy
with $k^2=0$ & $(\epsilon\cdot k)=0$ by BRST invariance
in ambitwistor string: P^μ instead of $i\partial X^\mu$
• OPE's

Review:

• open-superstring vertex operators "superghost pic −1 & 0"

$$V^{(-1)}(\epsilon, k) = \epsilon_\mu \psi^\mu \delta(\gamma)\, e^{ik\cdot X}$$

$$V^{(0)}(\epsilon, k) = \epsilon_\mu \left(i\partial X^\mu + 2\alpha' (k\cdot\psi)\psi^\mu \right) e^{ik\cdot X}$$

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• OPE's (

$\psi^\mu(z)\psi^\nu($

$i\partial X^\mu(z)\, e^{i}$

$i\partial X^\mu(z)\, i$

1 & 0"

d sheet

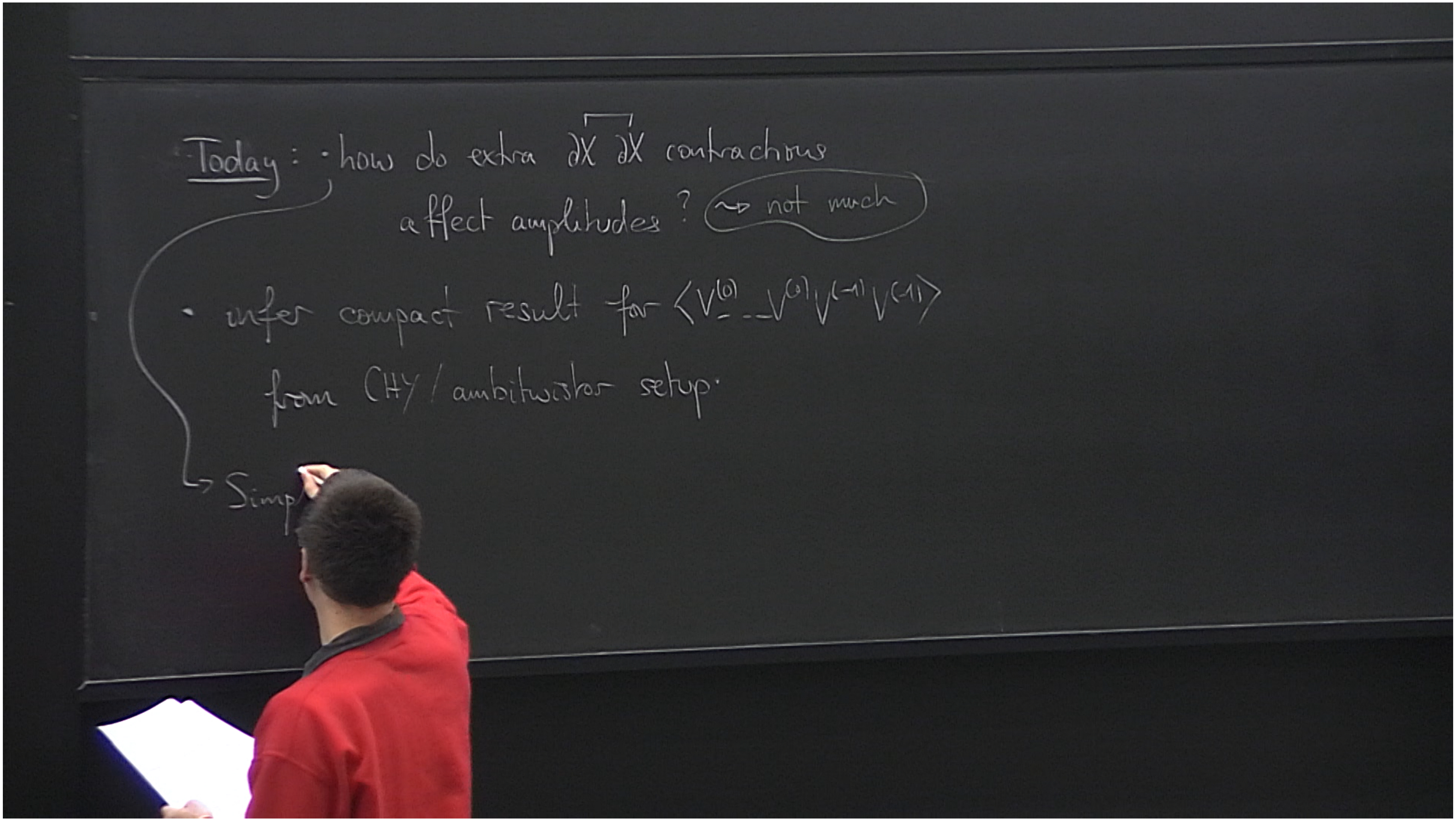
sy

• OPE's (slightly different form as yesterday)

$$\psi^\mu(z)\psi^\nu(w) \sim \frac{\eta^{\mu\nu}}{z-w}, \qquad \bar\psi^\mu(\bar z)\bar\psi^\nu(\bar w) \sim \frac{\eta^{\mu\nu}}{\bar z - \bar w}$$

$$i\partial X^\mu(z)\, e^{ik\cdot X}(w) \sim \frac{2\alpha' k^\mu}{z-w} e^{ik\cdot X} \leftarrow \text{cf. } P^\mu(z) = dz \sum_{j=1}^{n} \frac{k_j^\mu}{z-z_j} \ \checkmark$$

$$i\partial X^\mu(z)\, i\partial X^\nu(w) \sim \frac{2\alpha' \eta^{\mu\nu}}{(z-w)^2} \leftarrow \text{no analogue for } P^\mu(z)P^\nu(w)$$


Today: · how do extra $\partial X \, \partial X$ contractions
affect amplitudes? $\rightsquigarrow$ not much
· infer compact result for $\langle V^{(0)} \ldots V^{(0)} V^{(-1)} V^{(-1)} \rangle$
from CHY / ambitwistor setup.
↳ Simp

Today: · how do extra $\partial X\, \partial X$ contractions affect amplitudes? $\rightsquigarrow$ not much

· infer compact result for $\langle V^{(0)} \ldots V^{(0)} V^{(-1)} V^{(-1)} \rangle$ from CHY / ambitwistor setup.

$\hookrightarrow$ Simple testing grounds: $(\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot \epsilon_4)$ kinematics @ 4pt's

$$\left\langle V_1^{(0)} V_2^{(0)} V_3^{(-1)} V_4^{(-1)} \right\rangle \Big|_{(\epsilon_1\cdot\epsilon_2)(\epsilon_3\cdot\epsilon_4)} = \epsilon^1_\mu \, \epsilon^2_\nu \, \epsilon^3_\lambda \, \epsilon^4_\rho \, \left\langle \delta(\gamma_3)\, \delta(\gamma_4) \right\rangle$$

$$\left\langle \left[i\partial X_1^\mu + 2\alpha'(k_1\cdot\psi_1)\psi_1^\mu \right] \left[i\partial X_2^\nu + 2\alpha'(k_2\cdot\psi_2)\psi_2^\nu \right] \psi_3^\lambda \, \psi_4^\rho \, \prod_{j=1}^{4} e^{ik_j\cdot X(z_j)} \right\rangle \Big|_{(\epsilon_1\cdot\epsilon_2)(\epsilon_3\cdot\epsilon_4)}$$

$$-(2\alpha')^2 \, k_1\cdot k_2 \, \eta^{\mu\nu}/z_{12}^2 \qquad \eta^{\lambda\rho}/z_{34}$$

Integration by parts in $(z_1, z_3, z_4) \to (0, 1, \infty)$ frame

$$\underbrace{0 \cong \frac{\partial}{\partial z_2}}_{J_n \text{ suppresses bdy terms since } \lim_{z_i \to z_j} |z_i - z_j|^{s_{ij}} = 0} \left(\frac{|z_2|^{s_{12}} |1 - z_2|^{s_{23}}}{z_2} \right) = \left(\underbrace{\frac{s_{12} - 1}{z_2^2}}_{\text{what we have}} - \underbrace{\frac{s_{23}}{z_2(1 - z_2)}}_{\text{what we want: } SL_2(\mathbb{R})\text{-fixed } (1,2,3,4)^{-1}} \right) |z_2|^{s_{12}} |1 - z_2|^{s_{23}}$$

in ambitwistor string: P^μ instead of $i\partial X^\mu$

$$\Rightarrow \quad \langle V_1^{(0)} V_2^{(0)} V_3^{(-1)} V_4^{(-1)} \rangle \Big|_{(\epsilon_1\epsilon_2)(\epsilon_3\epsilon_4)} = \frac{-s_{23}\, J_4}{(1,2,3,4)} \quad \mathrm{mod}\ \frac{\partial}{\partial z_2}(\ldots)$$

Analogous CHY / ambitwistor computation

Same result in PT(_) representation

$$\frac{\mathrm{Pf}'\,\Psi^{34}_{34}}{z_3-z_4}\Big|_{(\epsilon_1\epsilon_2)(\epsilon_3\epsilon_4)} = \frac{-s_{12}}{z_{12}^2\, z_{34}^2} = \frac{-s_{23}}{(1,2,3,4)} \quad \mathrm{mod}\ E_2$$

no "1−"

$$\Rightarrow \left\langle V_1^{(0)} V_2^{(0)} V_3^{(-1)} V_4^{(-1)} \right\rangle \Big|_{(\epsilon_1\epsilon_2)(\epsilon_3\epsilon_4)} = \frac{-s_{23} J_4}{(1,2,3,4)} \bmod \frac{\partial}{\partial z_2}(\ldots)$$

Same result in PT(_) representation mod E_2

IBP takes the role of scattering equations

Analogous CHY / ambitwistor computation

$$\frac{\mathrm{Pf}' \Psi^{34}_{34}}{z_3 - z_4}\Big|_{(\epsilon_1\epsilon_2)(\epsilon_3\epsilon_4)} = \frac{-s_{12}}{z_{12}^2 z_{34}^2} = \frac{-s_{23}}{(1,2,3,4)}$$

Bosonic string: no $\psi\psi$ in $V^{bos}(\epsilon, k) = i\partial X^\mu \epsilon_\mu e^{ikX}$

$$\Rightarrow \left\langle \prod_{j=1}^{4} V_j^{bos}(z_j) \right\rangle \Big|_{(\epsilon_1\cdot\epsilon_2)(\epsilon_3\cdot\epsilon_4)} = \frac{J_4}{z_{12}^2 z_{34}^2}$$

$$\Longrightarrow \langle V_1^{(0)} V_2^{(0)} V_3^{(-1)} V_4^{(-1)} \rangle \Big|_{(\epsilon_1\cdot\epsilon_2)(\epsilon_3\cdot\epsilon_4)} = \frac{-s_{23} I_4}{(1,2,3,4)} \quad \text{mod } \frac{\partial}{\partial z_2}(\ldots)$$

Same result in PT(...) representation

IBP takes the role of scattering equations

Analogous CHY / ambitwistor computation

$$\frac{\mathrm{Pf}'\,\Psi^{34}_{34}}{z_3 - z_4}\Big|_{(\epsilon_1\cdot\epsilon_2)(\epsilon_3\cdot\epsilon_4)} = \frac{-s_{12}}{z_{12}^2\, z_{34}^2} = \frac{-s_{23}}{(1,2,3,4)} \quad \text{mod } E_2$$

tachyon pole

Bosonic string: no $\psi\psi$ in $V^{bos}(\epsilon, k) = i\partial X^\mu \epsilon_\mu e^{ik\cdot X}$

$$\Longrightarrow \langle \prod_{j=1}^4 V_j^{bos}(z_j) \rangle \Big|_{(\epsilon_1\cdot\epsilon_2)(\epsilon_3\cdot\epsilon_4)} = \frac{I_4}{z_{12}^2\, z_{34}^2} = \frac{1}{1 - s_{12}} \quad \left(\text{superstring result}\right)$$

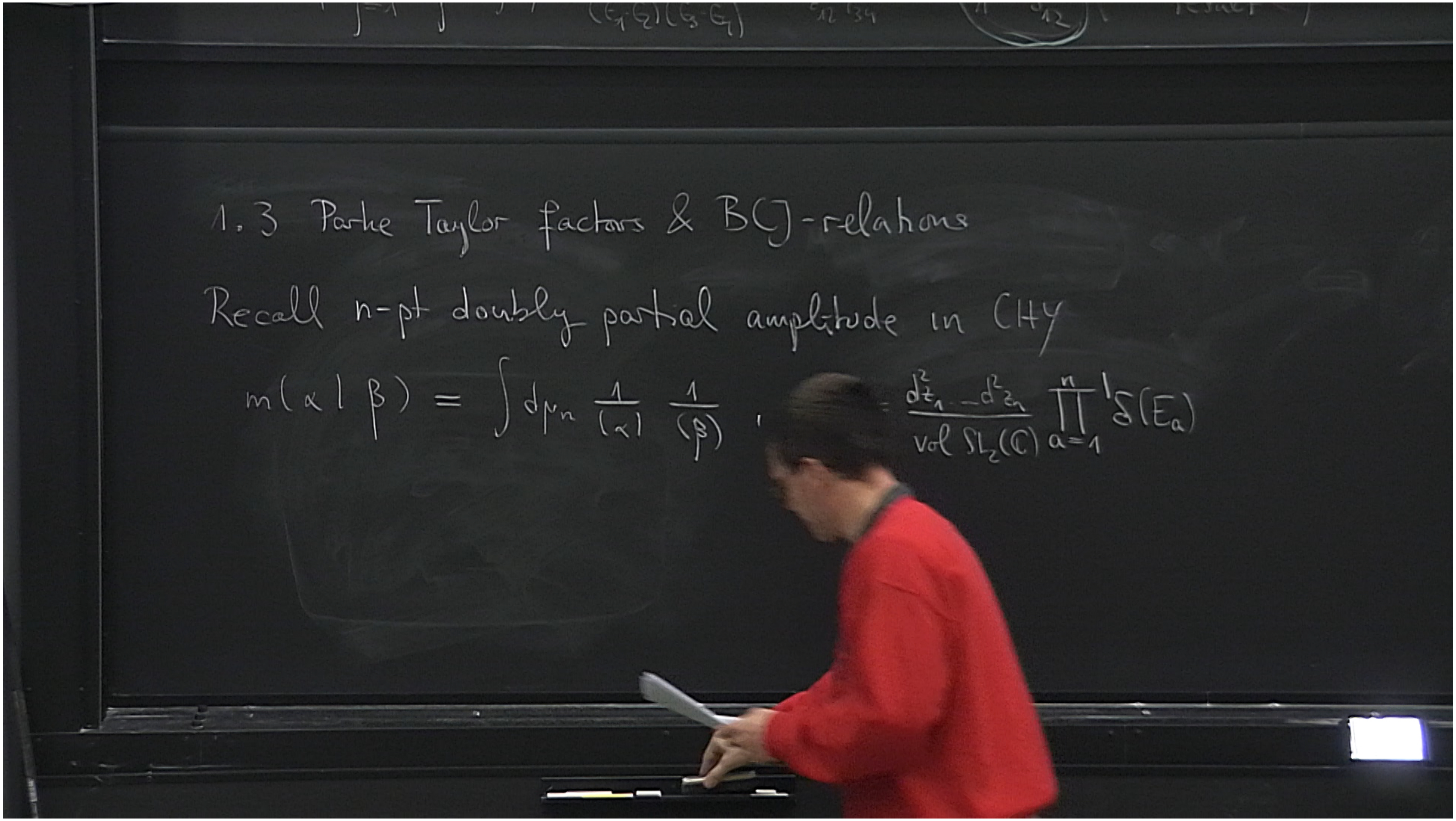
Clue: OPE difference $[\partial X \, \partial X \text{ vs. } PP]$

between superstring / ambitwistor string washes out

in Parke-Taylor form

Claim at n-pts:

$$\left\langle V_1^{(0)} V_2^{(0)} \ldots V_{n-2}^{(0)} V_{n-1}^{(-1)} V_n^{(-1)} \right\rangle \Big|_{PT(\ldots)} \bmod \frac{\partial}{\partial z_j} = (2\alpha')^{n-2} J_n \; \mathrm{Pf}'\Psi \bmod E_j$$


1.3 Parke Taylor factors & BCJ-relations
Recall n-pt doubly partial amplitude in CHY
$m(\alpha|\beta) = \int d\mu_n \frac{1}{(\alpha)} \frac{1}{(\beta)}$,
$\frac{d^2z_1 \dots d^2z_n}{\mathrm{vol}\, SL_2(\mathbb{C})} \prod_{a=1}^{n}{}' \delta(E_a)$

1.3 Parke Taylor factors & BCJ-relations

Recall n-pt doubly partial amplitude in CHY

$$m(\alpha|\beta) = \int d\mu_n \frac{1}{(\alpha)} \frac{1}{(\beta)}, \quad d\mu_n = \frac{d^2z_1 \dots d^2z_n}{\mathrm{vol}\, SL_2(\mathbb{C})} \prod_{a=1}^{n}{}' \delta(E_a)$$

The matrix rank of $m(\alpha|\beta)$ is $(n-3)!$

$\hookrightarrow$ property of a single PT $\frac{1}{(\alpha)}$ @ support of E

$$\sum_{j=2}^{n-1}$$

(E_a)

$_a = 0$

$$\sum_{j=2}^{n-1} \frac{k_1\cdot k_{23\ldots j}}{(2,3,\ldots,j,1,j+1,\ldots,n)} = 0 \mod E_a$$

$k_2+k_3+\ldots+k_j$

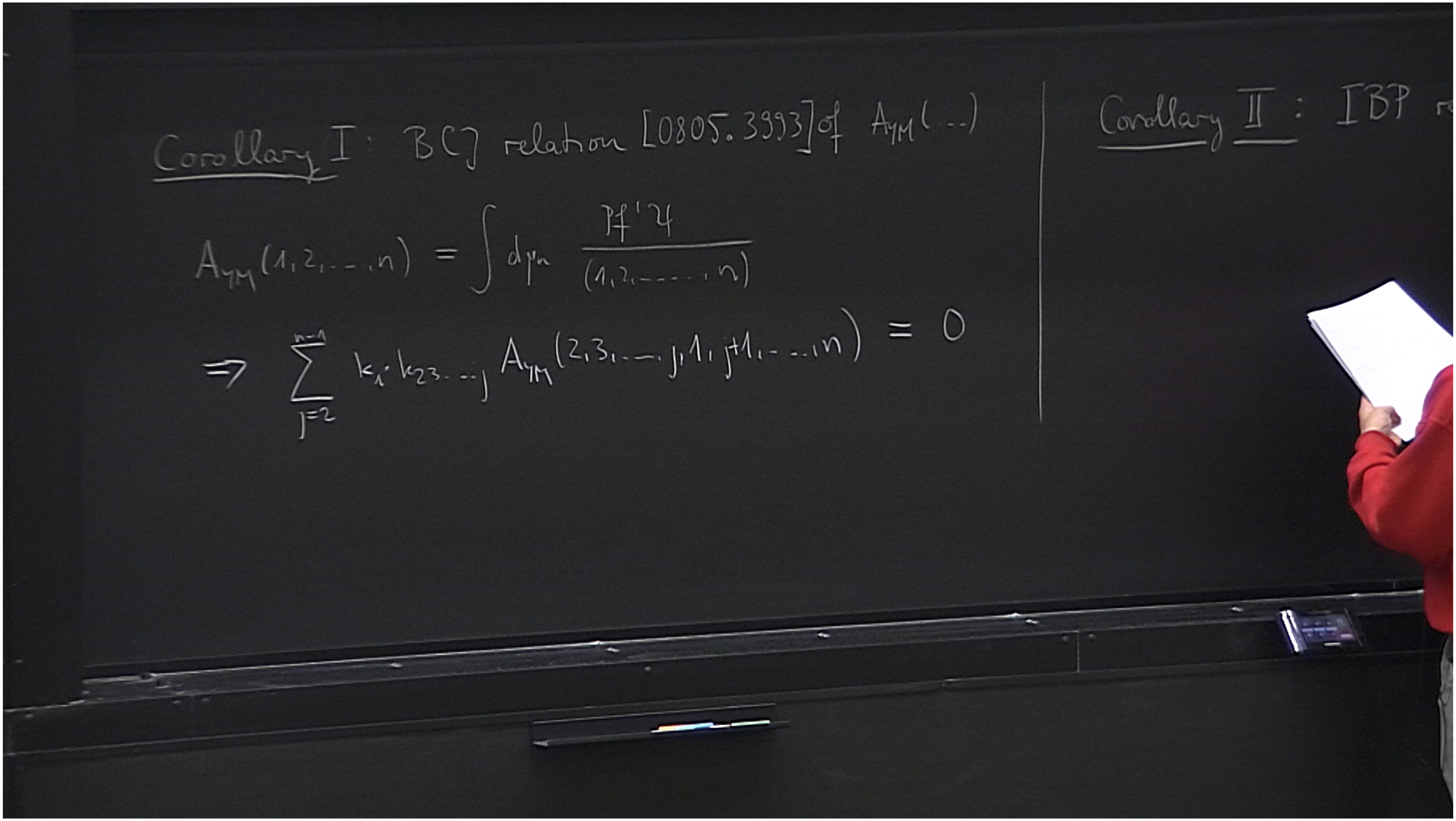
[Reading homework: study appendix B.4 of 1304.7267]

Same is true for disk integrals

$$\frac{\partial}{\partial z_j} \mathcal{I}_n = \mathcal{I}_n \sum_{i\neq j} \frac{s_{ij}}{z_{ji}} = \mathcal{I}_n E_j$$

No extra $\partial/\partial z_j$ action in context of Parke-Taylors

$$\sum_{j=2}^{n-1} \frac{\mathcal{I}_n\, k_1\cdot k_{23\ldots j}}{(2,3,\ldots,j,1,j+1,\ldots,n)} = 0 \mod \frac{\partial}{\partial z_a}$$


Corollary I: BCJ relation [0805.3993] of $A_{YM}(\ldots)$
$A_{YM}(1,2,\ldots,n) = \int d\mu_n \frac{\mathrm{Pf}'\Psi}{(1,2,\ldots,n)}$
$\Rightarrow \sum_{j=2}^{n-1} k_1 \cdot k_{23\ldots j} A_{YM}(2,3,\ldots,j,1,j+1,\ldots,n) = 0$
Corollary II: IBP

Corollary II : IBP relations of disk integral

$$\boxed{Z(\rho(1,2,\ldots,n)\,|\,\underline{1,2,\ldots,n}) = (-2\alpha')^{n-3} \int_{D(\rho(1,2,\ldots,n))} \frac{dz_1 dz_2 \ldots dz_n}{\mathrm{vol}\, SL_2(\mathbb{R})} \frac{\mathcal{J}_n}{(1,2,\ldots,n)}}$$

"BCJ properties of PT is inert under extension by α'"

$$\Rightarrow \sum_{j=2}^{n-1} \underbrace{k_1 \cdot k_{23\ldots j}}\; Z(\rho\,|\,\underline{2,3,\ldots,j,1},j+1,\ldots,n) = 0$$

independent on ρ

[check that BCJ rel's are obeyed by 4pt $Z(\cdot|\cdot)$ from last lecture]

1.4. Double-copy form of open-superstring amplitudes

Insert CHY

BCJ basis expansion of A_{YM}

$$A_{YM}(\tau(1,2,\ldots,n)) = \sum_{\alpha,\beta \in S_{n-3}} m(\tau(1,2,\ldots,n) \,|\, 1,\alpha,n,n-1)\; S_0[\alpha|\beta]_1 \; A_{YM}(1,\beta,n-1,n)$$

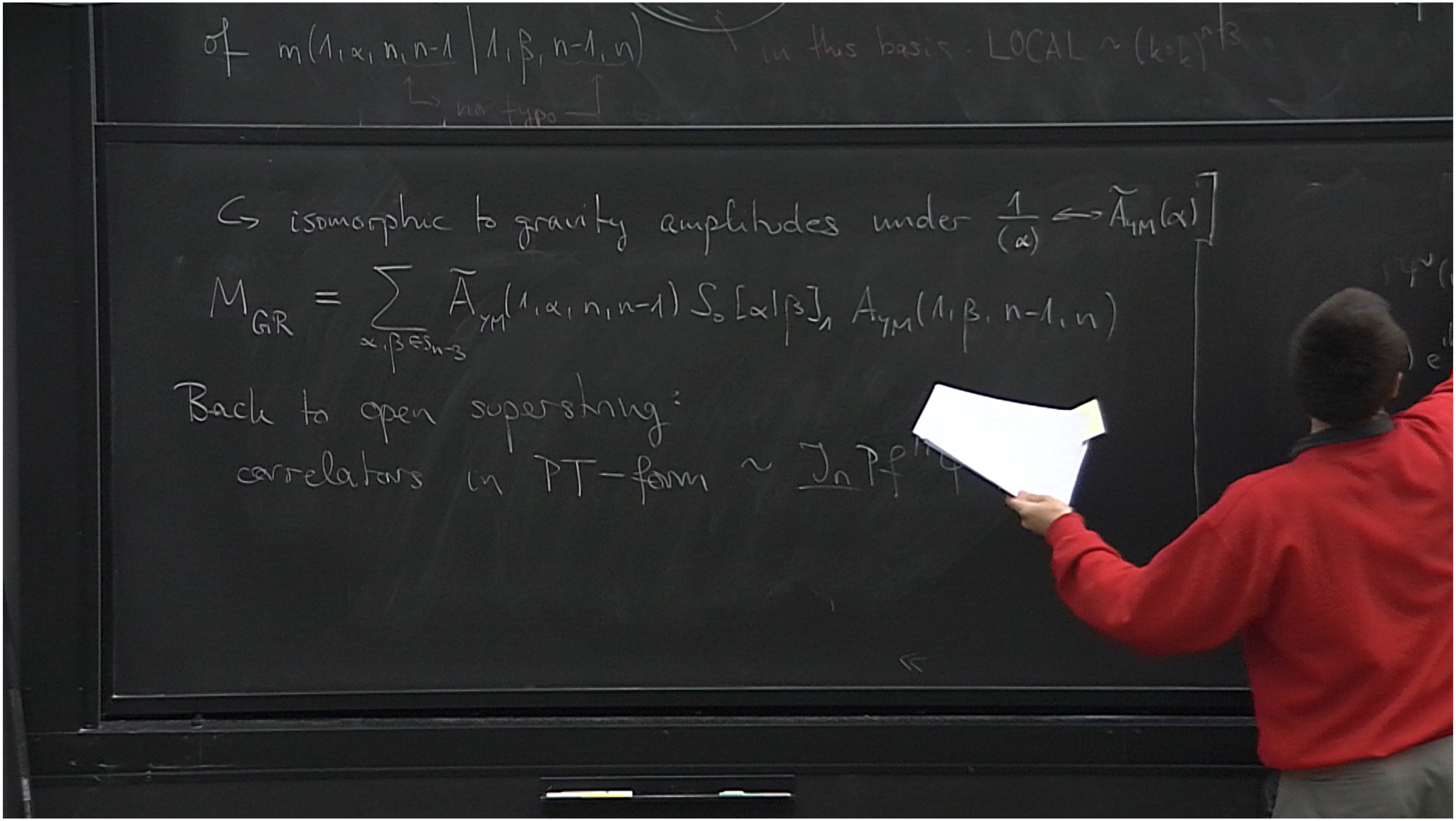
$(n-3)! \times (n-3)!$

matrix inverse

of $m(1,\alpha,n,n-1 \,|\, 1,\beta,n-1,n)$

no typo

in this basis: LOCAL $\sim (k\cdot k)^{n-3}$


of $m(1,\alpha,n,n-1 \mid 1,\beta,n-1,n)$
no typo
in this basis: LOCAL $\sim (k \cdot k)^{n-3}$
$\hookrightarrow$ isomorphic to gravity amplitudes under $\frac{1}{(\alpha)} \longleftrightarrow \tilde{A}_{YM}(\alpha)$
$M_{GR} = \sum_{\alpha,\beta \in S_{n-3}} \tilde{A}_{YM}(1,\alpha,n,n-1)\, S_0[\alpha|\beta]_1\, A_{YM}(1,\beta,n-1,n)$
Back to open superstring:
correlators in PT-form $\sim \underline{I_n}\, \mathrm{Pf}$

of $m(1,\alpha,n,n-1\,|\,1,\beta,n-1,n)$ in this basis: LOCAL $\sim (k\cdot k)^{n-3}$

↳ no typo

↳ isomorphic to gravity amplitudes under $\frac{1}{(\alpha)} \longleftrightarrow \tilde{A}_{YM}(\alpha)$ "KLT-li

$$M_{GR} = \sum_{\alpha,\beta \in S_{n-3}} \tilde{A}_{YM}(1,\alpha,n,n-1)\, S_0[\alpha|\beta]_1\, A_{YM}(1,\beta,n-1,n)$$

Back to open superstring:

correlators in PT-form $\sim \underline{I_n}\, \mathrm{Pf}'\Psi\,\big|_{PT}$

"KLT"

$A_{YM}(\alpha)]$

KLT-like formula for open superstrings

$$A_{open}(\tau(1,2,\dots,n);\alpha') = \int_{D(\tau(1,2\dots n))} \frac{dz_1\dots dz_n}{\text{vol}\, SL_2(\mathbb{R})} \frac{(-)^{n-3}}{2\alpha'} \mathcal{I}_n \,(2\alpha')^{n-2} \left(\text{Pf}'\Psi \big|_{PT} \right)$$

$$= \sum_{\alpha,\beta \in S_{n-3}} \underbrace{Z(\tau(1,2,\dots,n)\,|\,1,\alpha,n,n-1)}_{\text{all } \alpha'} \; S_0[\alpha|\beta]_1 \; \underbrace{A_{YM}(1,\beta,n-1,n)}_{\text{all polarizations}}$$

First derived from pure-spinor superstring [1106.2645, 1304.7267]

$\Longrightarrow$ valid for gauge multiplet of $D=10$ SYM & dimensional reduction

$\delta(E_a)$

$$\frac{\partial}{\partial z_j} \mathcal{I}_n = \mathcal{I}_n \sum_{i \neq j} \frac{s_{ij}}{z_{ji}} = \mathcal{I}_n E_j$$

of 1304.7267]

No extra $\partial/\partial z_j$ action in context of Parke-Taylors

$E_a = 0$

$$\sum_{j=2}^{n-1} \frac{\mathcal{I}_n \, k_1 \cdot k_{23\dots j}}{(2,3,\dots,j,1,j+1,\dots,n)} = 0 \mod \frac{\partial}{\partial z_a}$$