Title: PSI 2017/2018 - Scattering Amplitudes in QFT & String Theory - Lecture 13

Date: Apr 23, 2018 11:30 AM

URL: http://pirsa.org/18040045

Abstract:

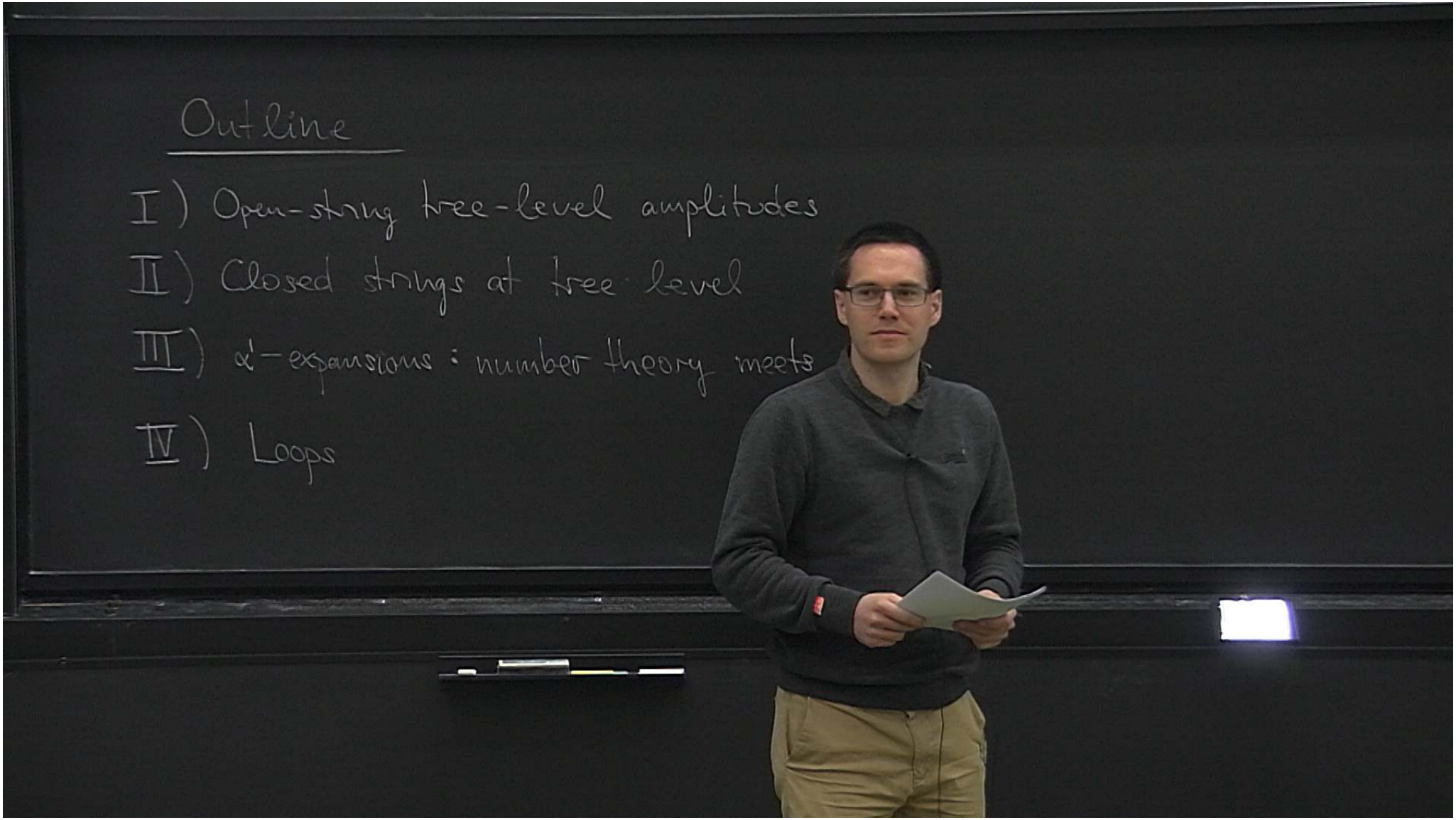
Outline

I) Open-string tree-level amplitudes

II) Closed strings at tree level

III) α'-expansions : number theory meets strings

IV) Loops


Outline
I) Open-string tree-level amplitudes
II) Closed strings at tree level
III) α'-expansions : number theory meets
IV) Loops

Goals

- string-theory derivation of BCJ & double copy
- understand close connections with CHY
- echo of double copy at all orders in α' (tree-lv)
- fun mathematics relevant to Feynman integrals

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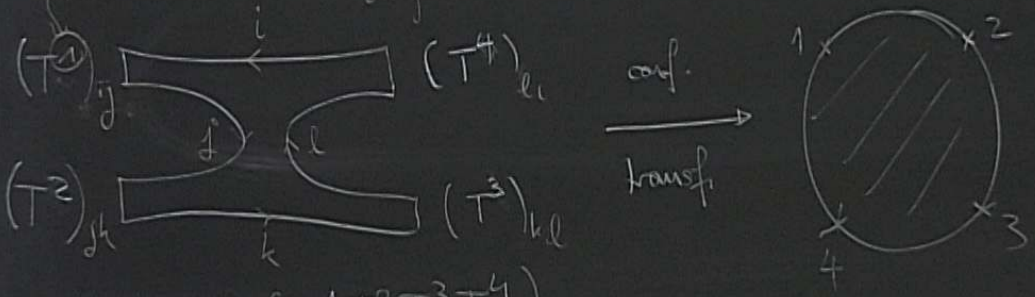
- string-theory derivation of BCJ & double copy
- understand close connections with CHY
- echo of double copy at all orders in α' (tree-lv)
- fun mathematics relevant to Feynman integrals
- glimpse into loop-lv topics (UV softness, SUSY cancellations)

1.1 Introductory example: 4pt disk integrals

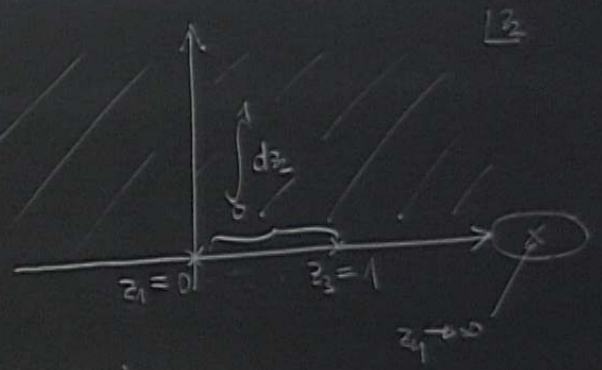
Chan-Paton matrices (T^a) $\longleftrightarrow$ color d.o.f.

adjoint — (anti-)fundamental $U(N)$



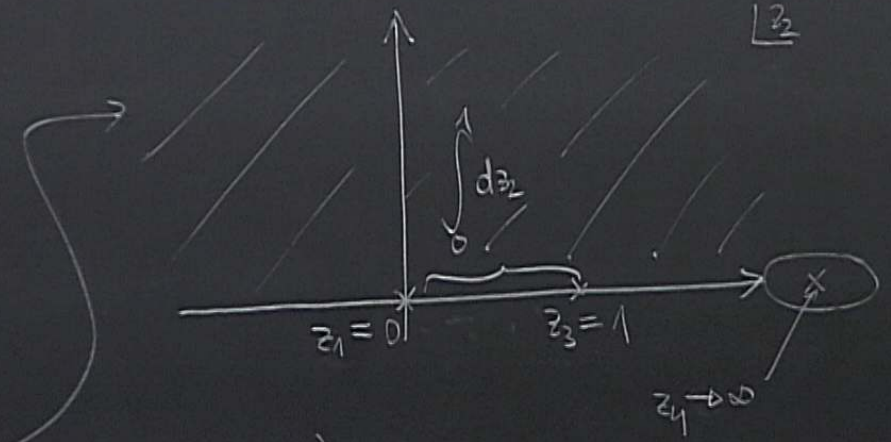
$\Rightarrow \operatorname{tr}(T^1 T^2 T^3 T^4)$

conf. transf.



mod out by $SL_2(\mathbb{R})$

fix $(z_1, z_3, z_4) = (0, 1, \infty)$



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Color decomposition

$$M_{open}(1,2,\dots,n;\alpha') = \sum_{\rho \in S_{n-1}} tr(T^1 T^{\rho(2)} \cdots T^{\rho(n)})$$

$$\times A_{open}(1, \rho(2), \dots, \rho(n); \alpha')$$

$\pm(\dots)$ order interlocked with int. domain

$$A_{open}(1,2,\dots,n) = \int (\dots)$$

$$D(1,2,\dots,n) = \{-\infty < z_1 < z_2 < \dots < z_n < \infty\}$$

Typical 4pt disk integral

$$Z_{toy} = \int_0^1 \frac{dz_2}{z_2} |z_2|^{s_{12}} |1-z_2|^{s_{23}}$$

- dim'less Mandelstams $s_{ij} = 2\alpha' k_i \cdot k_j$
- Koba Nielsen factor

$$J_n = J_n(z_j, s_{ij}) = \frac{\left\langle \prod_{j=1}^{n} e^{ik_j x(z_j)} \right\rangle}{\delta^D\left(\sum_{i=1}^{n} k_i\right)} = \prod_{i<j} |z_i - z_j|^{s_{ij}}$$

[check: if ext. states are massless $z_a \to \infty$, drop the $|z_a - z_b|^{-s_{ab}}$ with $b = \{1, 2, \ldots n\} \setminus \{a\}$]

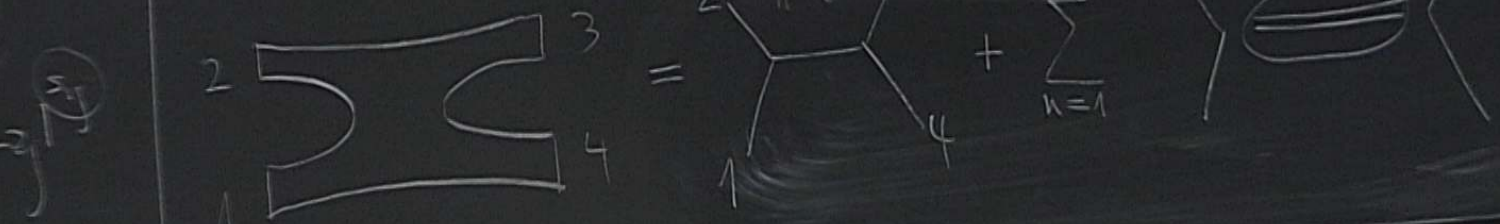
a) state-exchange newpoint

$$Z_{toy} = \int_0^1 dz_2 \; z_2^{s_{12}-1} \sum_{n=0}^{\infty} \binom{s_{23}}{n} (-z_2)^n$$

binomial: deg n in s_{23}

$$= \sum_{n=0}^{\infty} \frac{(-)^n \binom{s_{23}}{n}}{s_{12}+n}$$

propagators for massive: $-(k_1+k_2)^2 = \frac{n}{\alpha'}$ mass for n



Typical 4pt disk integral

$$Z_{toy} = \int_0^1 \frac{dz_2}{z_2} |z_2|^{s_{12}} |1-z_2|^{s_{23}}$$

external $m^2 = 0$

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ext. states are massless

a) state-exchan

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$$= \sum_{n=0}^{\infty} \frac{(-)^n}{s_{12}+n}$$

$\Rightarrow \quad \mathrm{tr}\left(T^1 T^2 T^3 T^4\right)$ 4

b) Effective-field-thy viewpoint

Taylor-expansion in α' $\longleftrightarrow$ "integrate out massive modes"

Riemann zeta fct.

$$\zeta_n = \sum_{k=1}^{\infty} \frac{1}{k^n}$$

$$Z_{toy} \overset{!}{=} \frac{\Gamma(s_{12})\Gamma(1+s_{23})}{\Gamma(1+s_{12}+s_{23})} = \frac{1}{s_{12}} - \zeta_2 s_{23} - \zeta_3 s_{13} s_{23} + O(s^3)$$

Euler beta

$\Rightarrow \quad \mathrm{tr}\left(T^1 T^2 T^3 T^4\right)$

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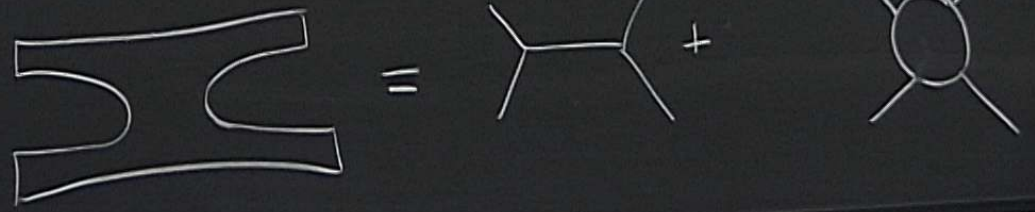
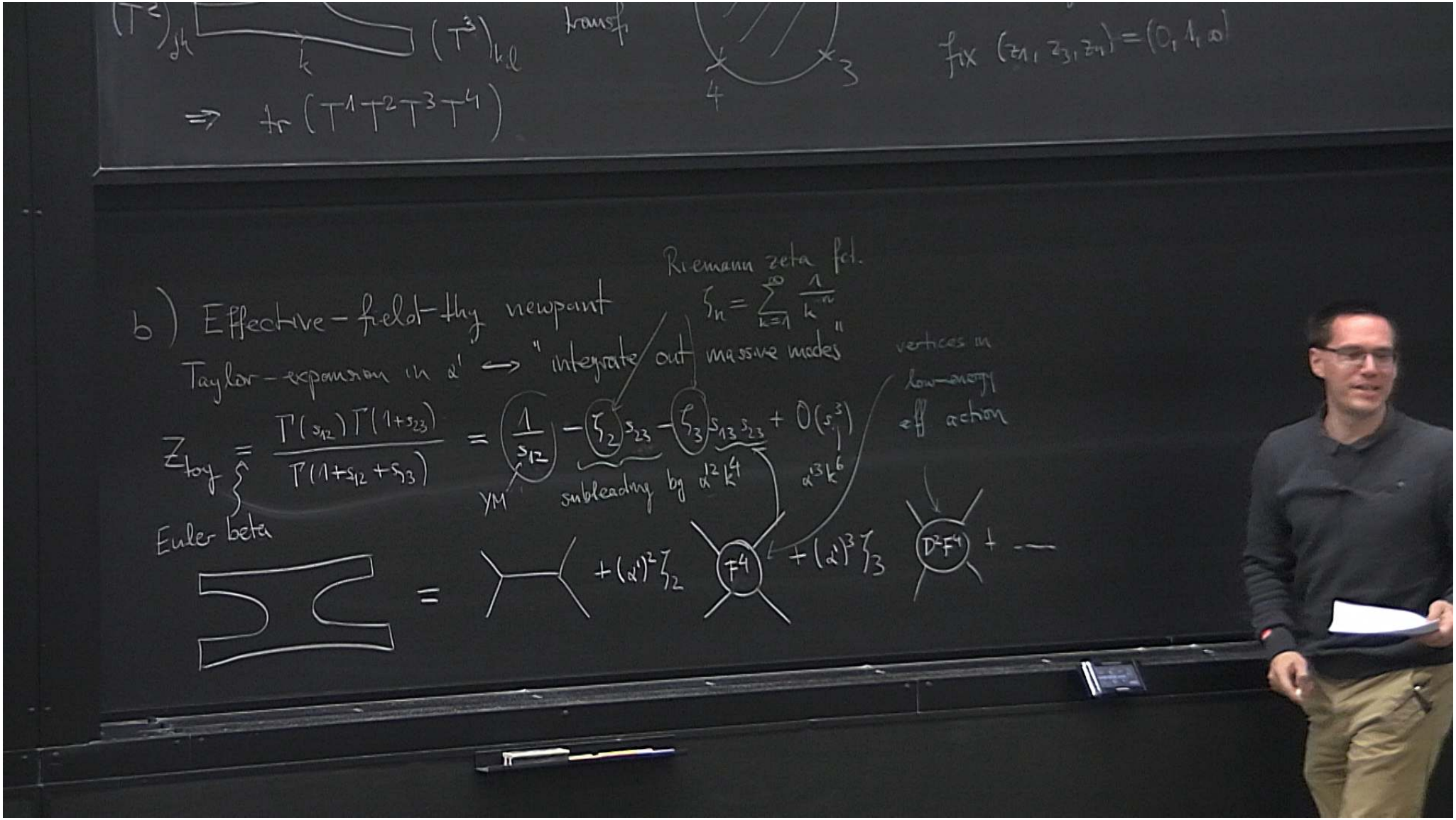
Euler beta

YM

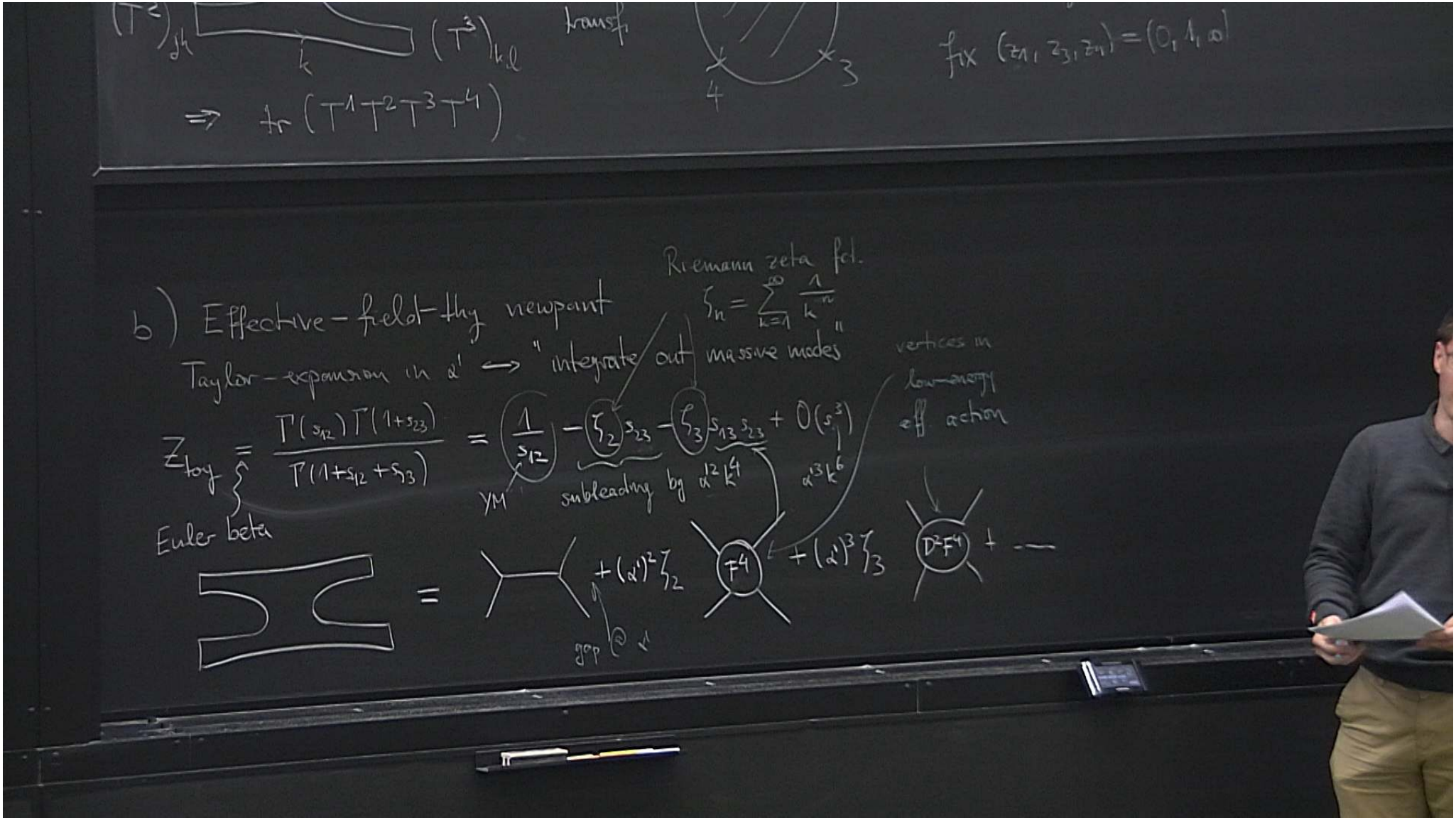
subleading by $\alpha'^2 k^4$

$\alpha'^3 k^6$





$(T^2)_{jk}$
$(T^3)_{kl}$
transf.
fix $(z_1, z_3, z_4) = (0, 1, \infty)$
$\Rightarrow \mathrm{tr}(T^1 T^2 T^3 T^4)$
Riemann zeta fct.
$\zeta_n = \sum_{k=1}^{\infty} \frac{1}{k^n}$
b) Effective-field-thy viewpoint
Taylor-expansion in α' $\longleftrightarrow$ "integrate out massive modes"
vertices in low-energy eff action
$Z_{toy} = \frac{\Gamma(s_{12})\Gamma(1+s_{23})}{\Gamma(1+s_{12}+s_{13})} = \frac{1}{s_{12}} - \zeta_2 s_{23} - \zeta_3 s_{13} s_{23} + O(s^3)$
YM
subleading by $\alpha'^2 k^4$
$\alpha'^3 k^6$
Euler beta
$= \quad + (\alpha')^2 \zeta_2 \quad F^4 \quad + (\alpha')^3 \zeta_3 \quad D^2F^4 \quad + \ldots$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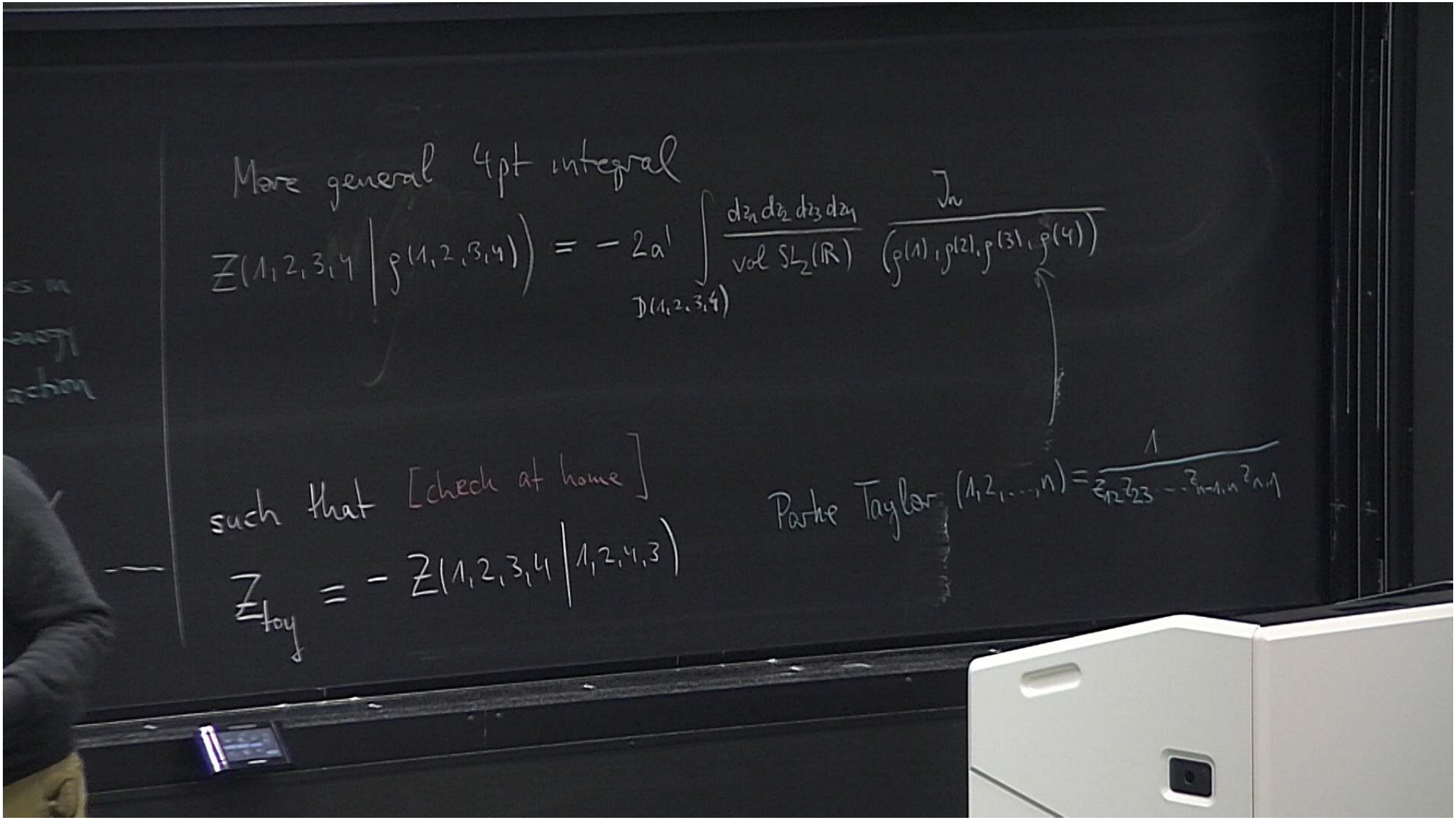
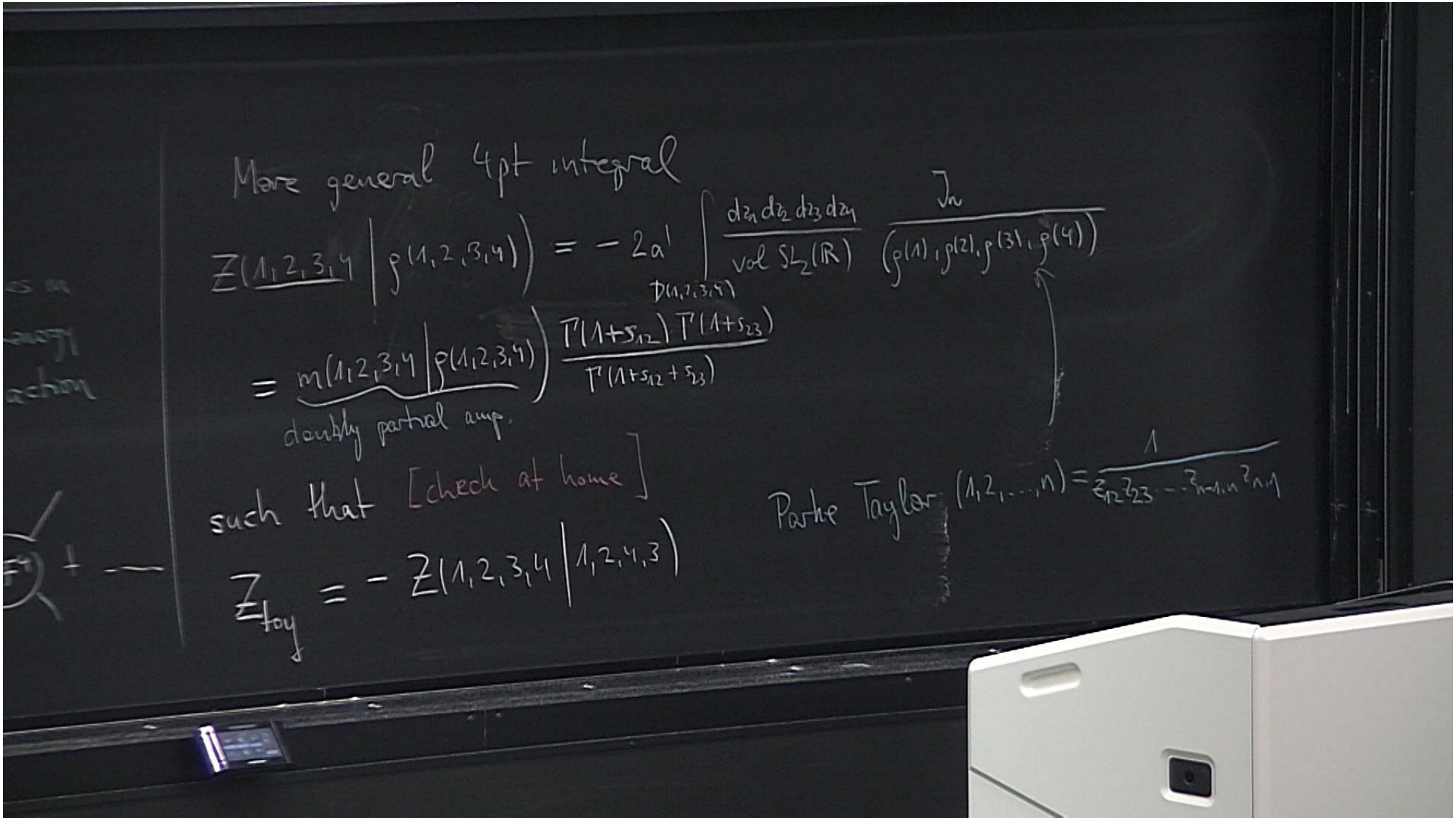
D^2F^4
+ ...
gap @ 4


More general 4pt integral
$Z(1,2,3,4 \mid \rho(1,2,3,4)) = -2a' \int_{\mathcal{D}(1,2,3,4)} \frac{dz_1 dz_2 dz_3 dz_4}{\mathrm{vol}\, SL_2(\mathbb{R})} \frac{\mathcal{I}_n}{(\rho(1), \rho(2), \rho(3), \rho(4))}$
such that [check at home]
$Z_{toy} = -Z(1,2,3,4 \mid 1,2,4,3)$
Parke Taylor $(1,2,\ldots,n) = \frac{1}{z_{12} z_{23} \cdots z_{n-1,n} z_{n,1}}$


More general 4pt integral
$Z(1,2,3,4 \mid \rho(1,2,3,4)) = -2\alpha' \int_{\mathcal{D}(1,2,3,4)} \frac{dz_1\, dz_2\, dz_3\, dz_4}{\mathrm{vol}\, SL_2(\mathbb{R})} \frac{\mathcal{I}_n}{(\rho(1), \rho(2), \rho(3), \rho(4))}$
$= m(1,2,3,4 \mid \rho(1,2,3,4)) \frac{\Gamma(1+s_{12})\,\Gamma(1+s_{23})}{\Gamma(1+s_{12}+s_{23})}$
doubly partial amp.
such that [check at home]
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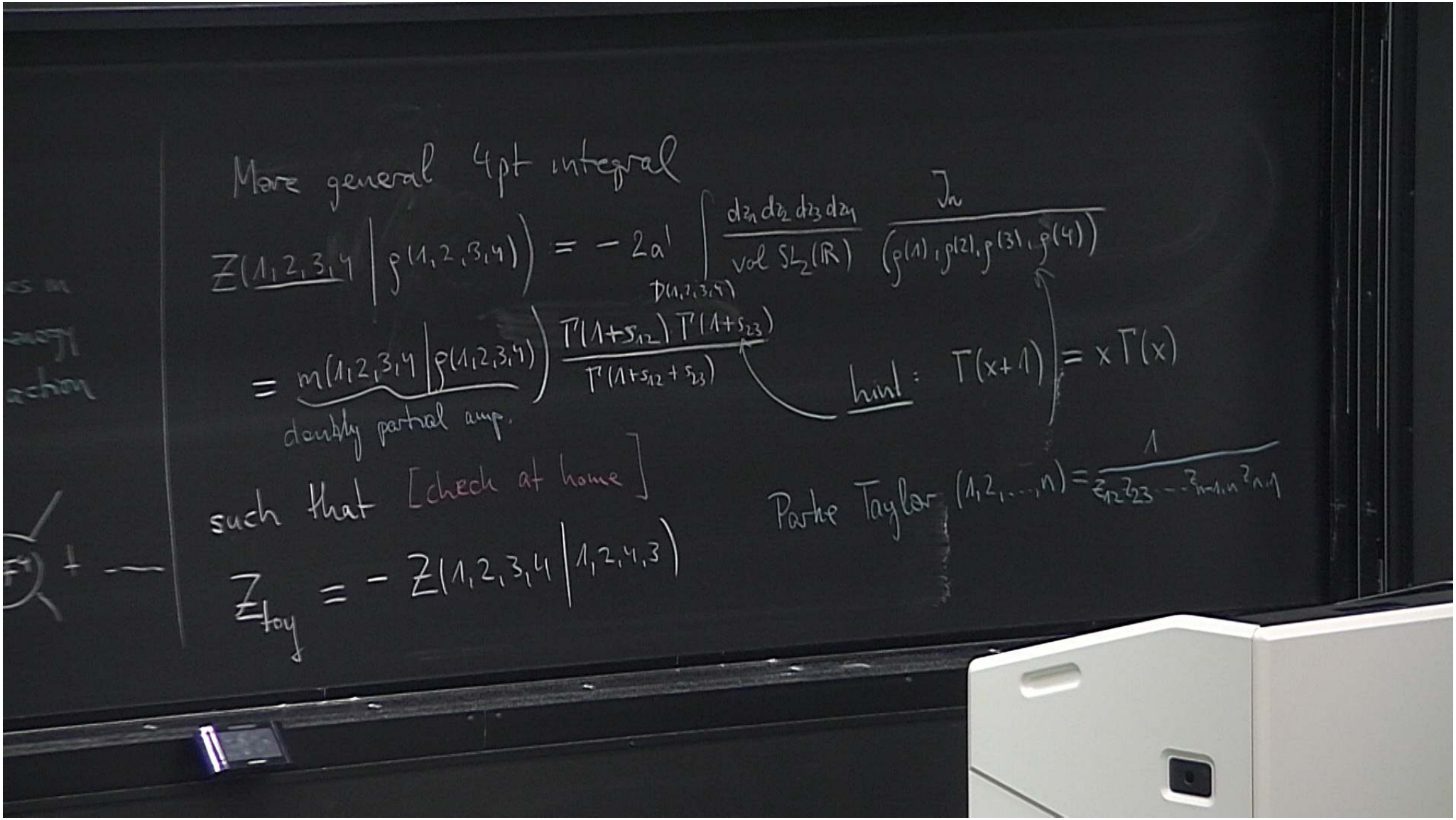
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$= \underbrace{m(1,2,3,4 \mid \rho(1,2,3,4))}_{\text{doubly partial amp.}} \frac{\Gamma(1+s_{12})\Gamma(1+s_{23})}{\Gamma(1+s_{12}+s_{23})}$
hint: $\Gamma(x+1) = x\Gamma(x)$
such that [check at home]
$Z_{toy} = -Z(1,2,3,4 \mid 1,2,4,3)$
Parke Taylor $(1,2,\ldots,n) = \frac{1}{z_{12} z_{23} \cdots z_{n-1,n} z_{n,1}}$

1.2. Superstrings vs ambitwistor strings

In contr

Worldsheet action of RNS superstring

$$S_{\text{matter}}[X,\psi,\bar{\psi}] = \frac{1}{2\pi\alpha'}\int_{\Sigma_n} d^2z\ \partial X_\mu \bar{\partial} X^\mu + \frac{1}{2\pi}\int_{\Sigma_n} d^2z \left\{ \psi_\mu \bar{\partial}\psi^\mu + \bar{\psi}_\mu \partial\bar{\psi}^\mu \right\}$$

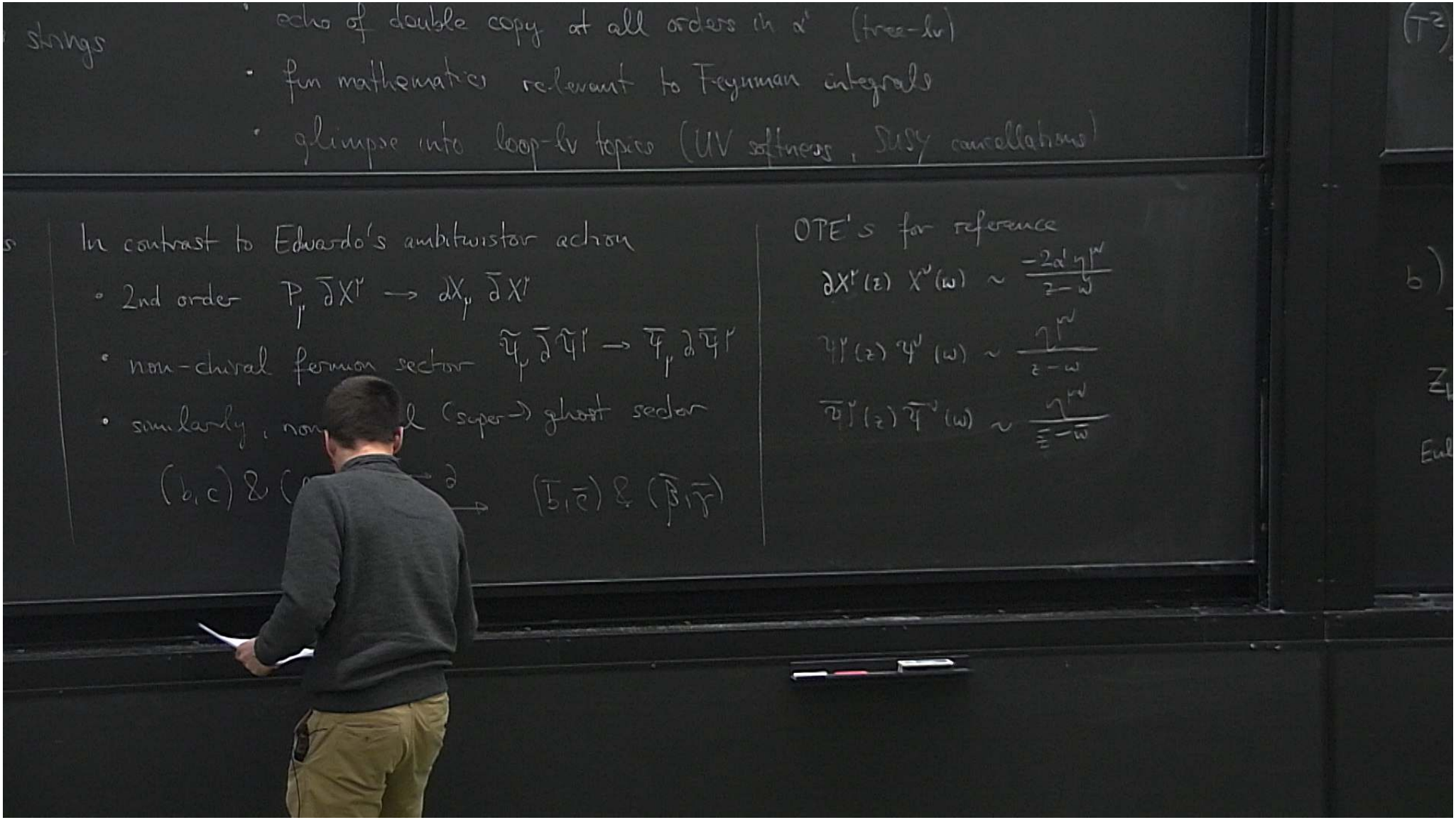
tor strings

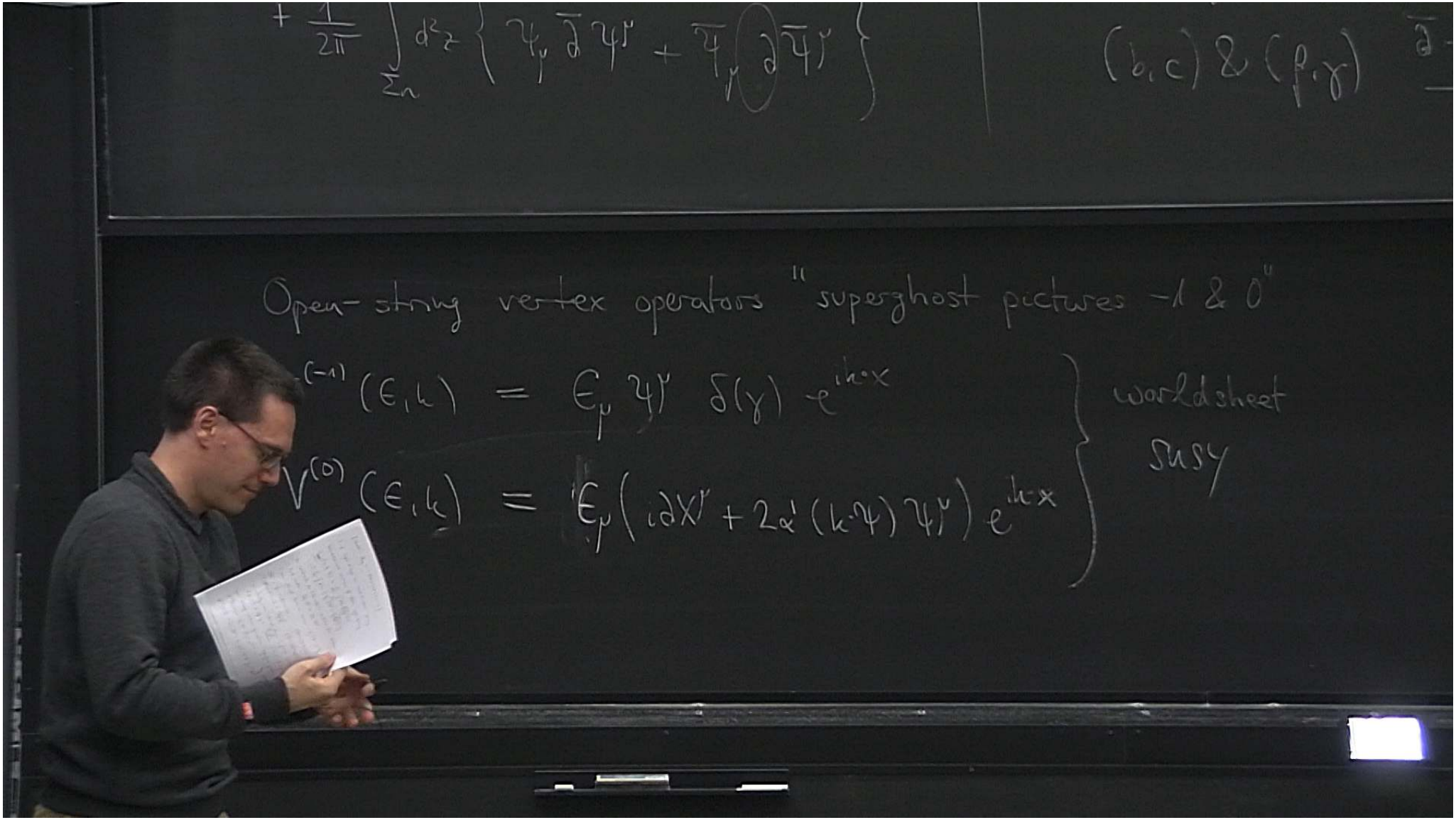
Supersting

In contrast to Eduardo's ambitwistor actions

- 2nd order $P_\mu \bar{\partial} X^\mu \to \partial X_\mu \bar{\partial} X^\mu$
- non-chiral fermion sector $\tilde{\psi}_\mu \bar{\partial} \tilde{\psi}^\mu \to \tilde{\psi}_\mu \partial \tilde{\psi}^\mu$
- similarly, non-chiral (super-) ghost sector

$$(b,c) \,\&\, (\beta,\gamma) \xrightarrow{\bar{\partial} \to \partial} (\tilde{b},\tilde{c}) \,\&\, (\tilde{\beta},\tilde{\gamma})$$


strings
echo of double copy at all orders in α' (tree-lv)
fun mathematics relevant to Feynman integrals
glimpse into loop-lv topics (UV softness, SUSY cancellations)
In contrast to Edwardo's ambitwistor action
2nd order $P_\mu \bar\partial X^\mu \to \partial X_\mu \bar\partial X^\mu$
non-chiral fermion sector $\tilde\psi_\mu \bar\partial \tilde\psi^\mu \to \bar\psi_\mu \partial \bar\psi^\mu$
similarly, non
(super-) ghost sector
(b,c) & $(\bar b,\bar c)$ & $(\bar\beta,\bar\gamma)$
OPE's for reference
$\partial X^\mu(z)\, X^\nu(w) \sim \frac{-2\alpha' \eta^{\mu\nu}}{z-w}$
$\psi^\mu(z)\, \psi^\nu(w) \sim \frac{\eta^{\mu\nu}}{z-w}$
$\bar\psi^\mu(z)\, \bar\psi^\nu(w) \sim \frac{\eta^{\mu\nu}}{\bar z-\bar w}$


$+\frac{1}{2\pi}\int_{\Sigma_n} d^2z \left\{ \psi_\mu \bar{\partial} \psi^\mu + \bar{\psi}_\mu \partial \bar{\psi}^\mu \right\}$
(b, c) & (β, γ)
Open-string vertex operators "superghost pictures −1 & 0"
$V^{(-1)}(\epsilon, k) = \epsilon_\mu \psi^\mu \, \delta(\gamma) \, e^{ik\cdot x}$
$V^{(0)}(\epsilon, k) = \epsilon_\mu \left(i\partial X^\mu + 2\alpha' (k\cdot\psi) \psi^\mu \right) e^{ik\cdot x}$
worldsheet SUSY

$$+\frac{1}{2\pi}\int_{\Sigma_n} d^2z \left\{ \psi_\mu \bar{\partial} \psi^\mu + \bar{\psi}_\mu \partial \bar{\psi}^\mu \right\}$$

(b,c) & (β,γ)

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} worldsheet SUSY

$(\gamma) \longrightarrow (b,\bar{c}) \otimes (\beta,\bar{\gamma})$

Tree amplitudes: need total superghost picture -2 (or $2g-2$, @ genus g)

$$A_{open}(1,2,\dots,n;\alpha') = \frac{(-)^{n-3}}{2\alpha'} \int_{D(1,2,\dots,n)} \frac{dz_1\, dz_2 \dots dz_n}{\mathrm{Vol}\, SL_2(\mathbb{R})} \times \left\langle V^{(0)}(z_1)\, V^{(0)}(z_2) \cdots V^{(0)}(z_{n-2})\, V^{(-1)}(z_{n-1})\, V^{(-1)}(z_n) \right\rangle$$

$\int dz$

independent on $V^{(0)} \leftrightarrow V^{(-1)}$ distribution after

incorporates $\langle c(z_1)\, c(z_j)\, c(z_k) \rangle = (z_{ij}\, z_{ik}\, z_{jk})$