Title: PSI 2017/2018 - Scattering Amplitudes in QFT & String Theory - Lecture 6

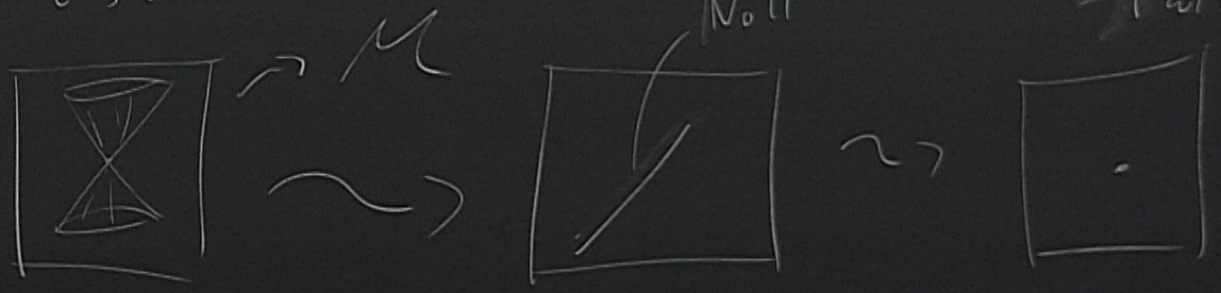
Date: Apr 16, 2018 11:30 AM

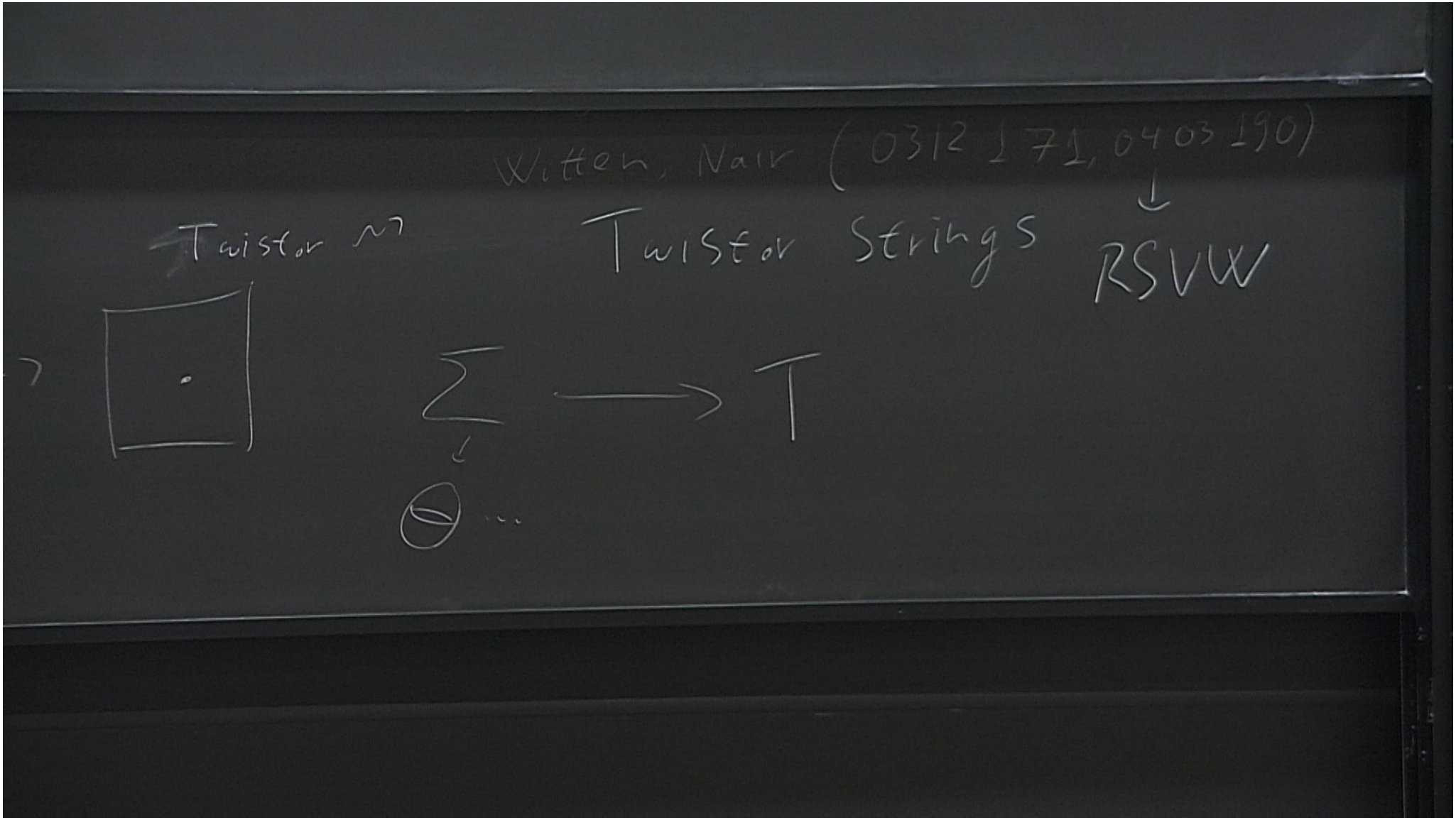
URL: http://pirsa.org/18040040

Abstract:

Introduction to Ambitwistor Strings

(ambi)twistor




Witten, Nair (0312171, 0403190)
Twistor ~?
Twistor Strings
RSVW
Σ
T

Scatt. Amp

$$\mathcal{A} = \int d\mu \; \hat{\mathcal{I}} \; \pi S(E_i^{\vec{k}})$$

refined scattering eq.

$\lambda_\alpha(\beta)$

$\tilde{\lambda}_\alpha(\beta)$

$D = 4$

$E^R_{\alpha\dot{\alpha}} = 0 \iff P_{\alpha\dot{\alpha}}(z) = \lambda_\alpha(z)\,\tilde{\lambda}_{\dot{\alpha}}(z)$

$\Uparrow$

$P^2(z) = 0 \quad \forall z \in \Sigma$

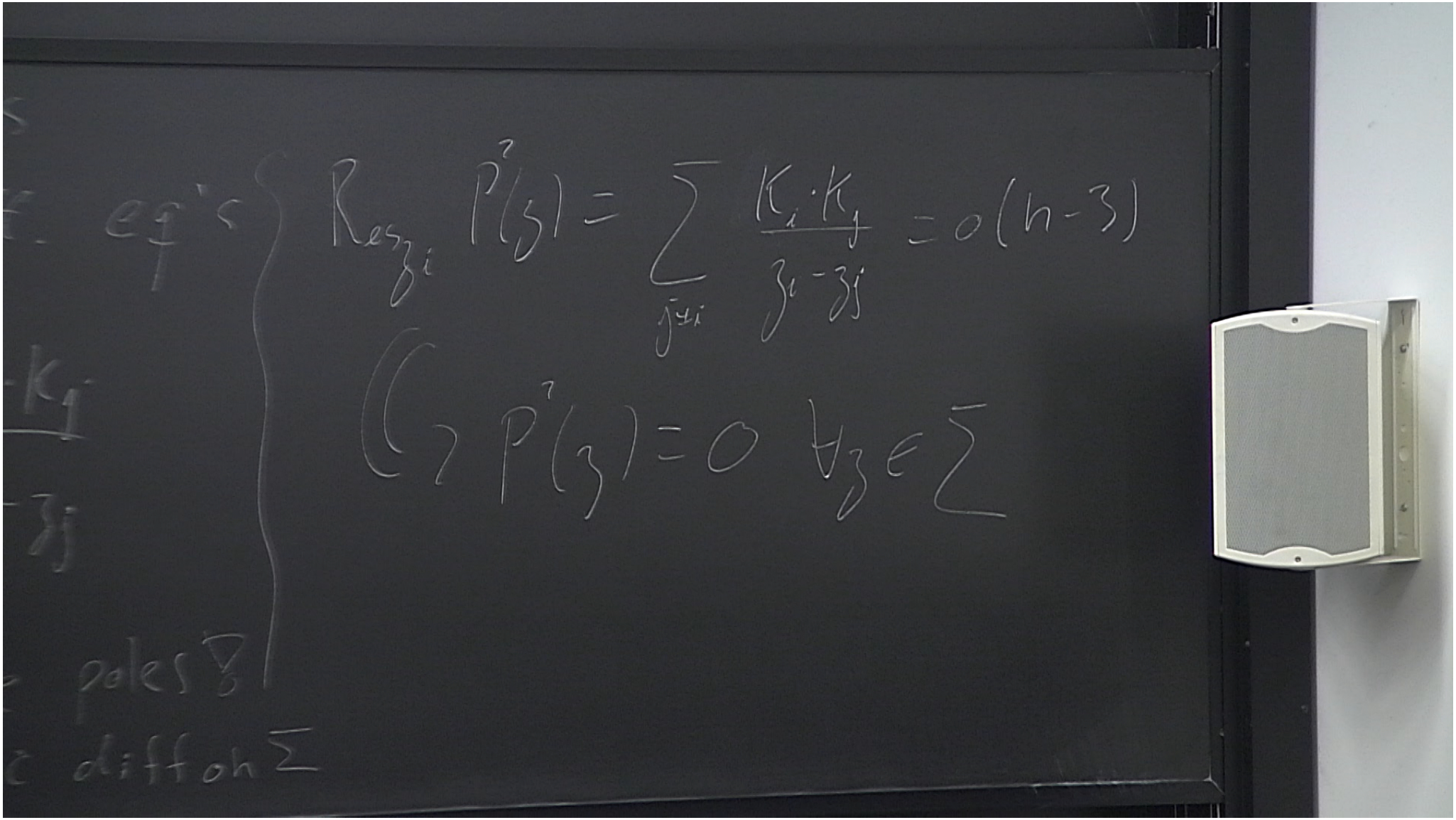
Generalization to any D → CHY-formulas and scatt. eq's

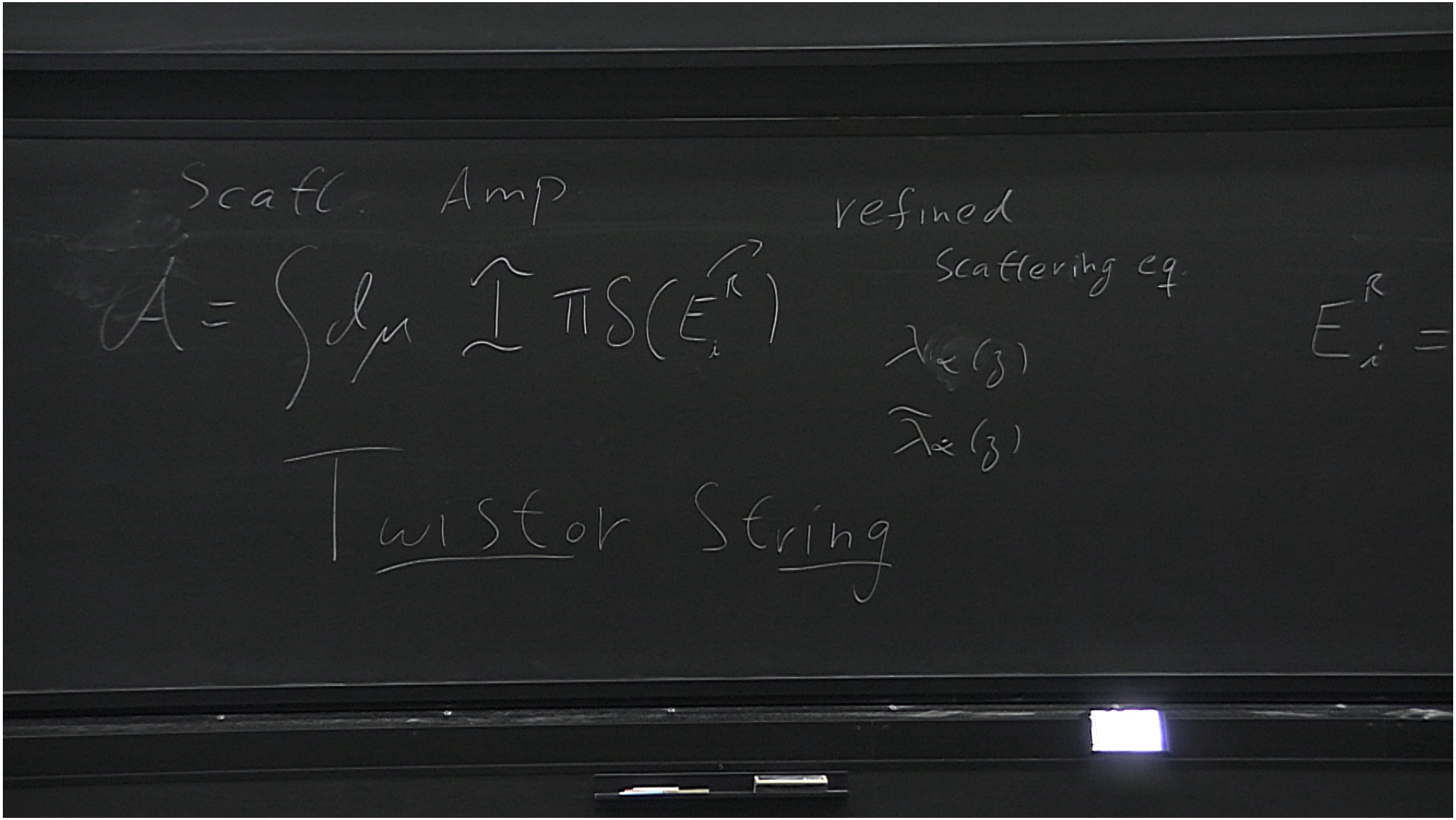
$$\mathcal{A} = \int d\mu \, \mathcal{I} \prod \delta(E_i) \quad ; \quad E_i = \sum_{j \neq i} \frac{k_i \cdot k_j}{z_i - z_j}$$

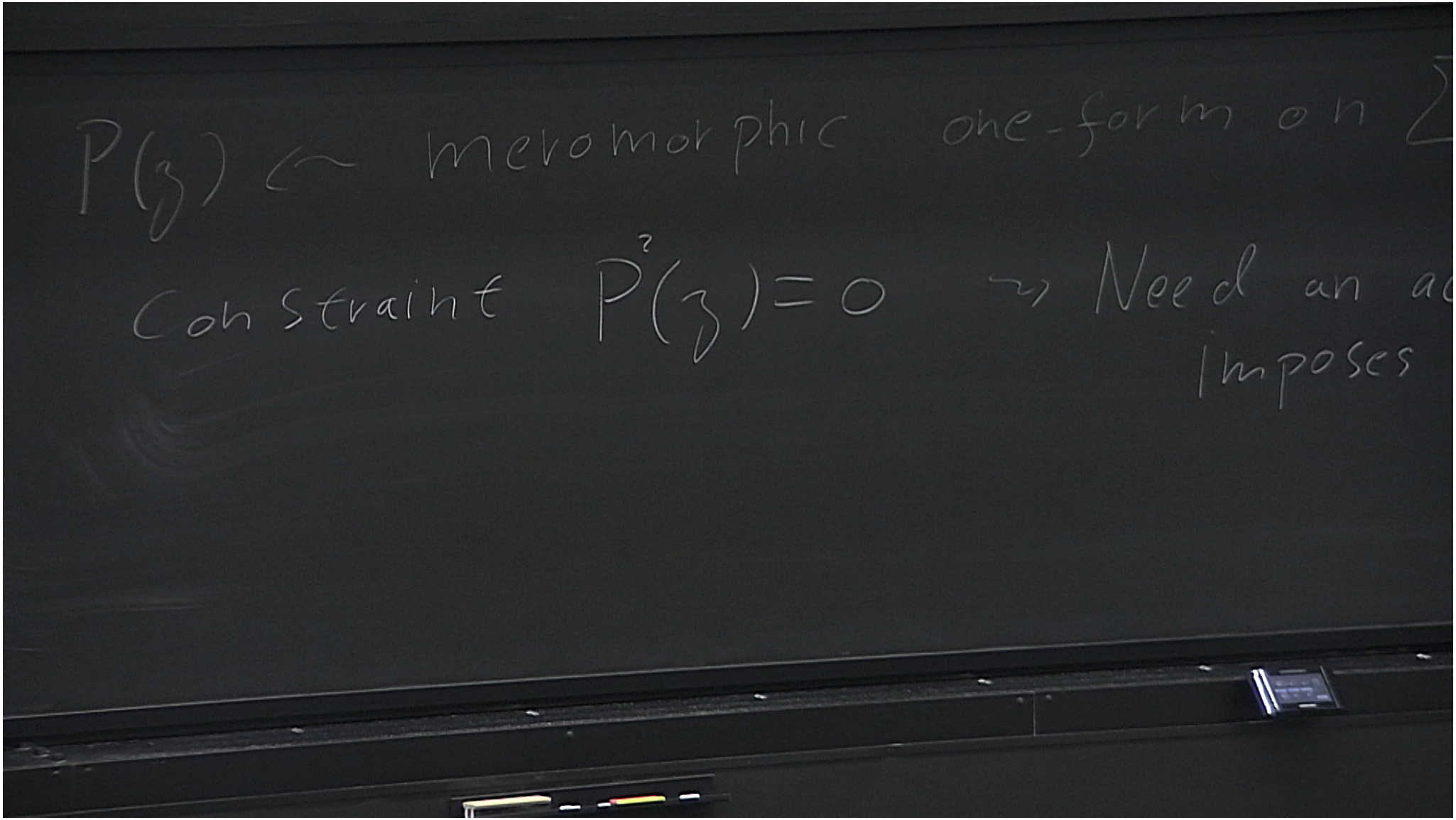
$$P(z) = \sum \frac{k_i}{z - z_i} \, , \quad k_i^2 = 0 \; \forall i$$

$$P^2(z) = \sum \frac{k_i \, k_j \, (dz)^2}{(z - z_i)(z - z_j)} \rightsquigarrow \text{only simple poles} \; \forall$$

$\rightsquigarrow$ Quadratic diffon

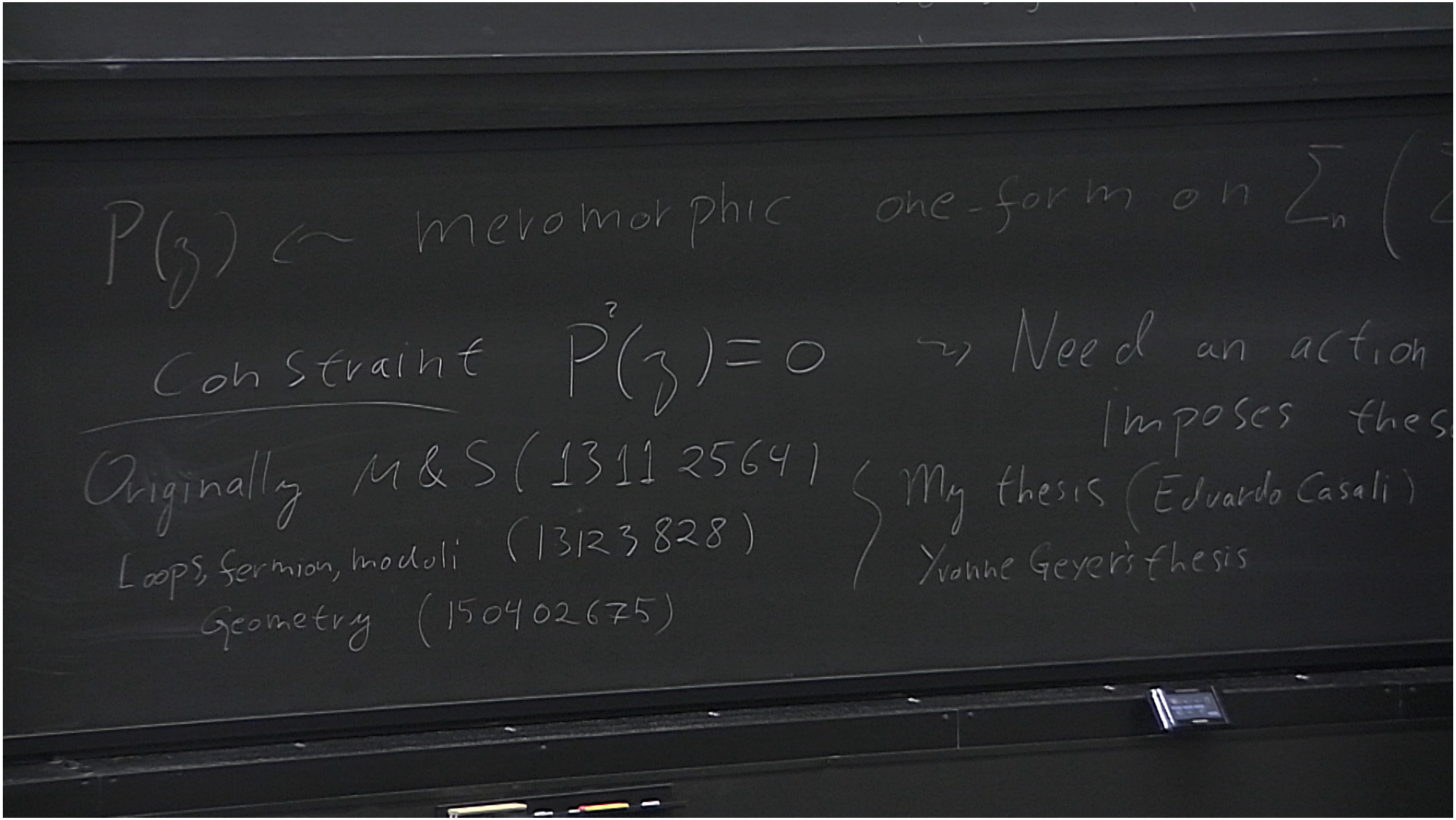

eq's
$\mathrm{Res}_{z_i} P^2(z) = \sum_{j \neq i} \frac{K_i \cdot K_j}{z_i - z_j} = o(n-3)$
$\oint_\gamma P^2(z) = 0 \quad \forall z \in \Sigma$
K_i
z_j
poles Σ
diff on Σ

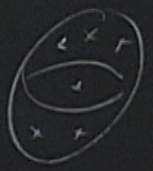

Scatt. Amp
refined
scattering eq.
Twistor String


P(z) ~ meromorphic one-form on Σ
Constraint P²(z) = 0 ⇝ Need an ac
imposes

omorphic one-form on Σ_n ($\Sigma_n \simeq$ )

$P^?(z) = 0$ ⇝ Need an action that imposes these conditions

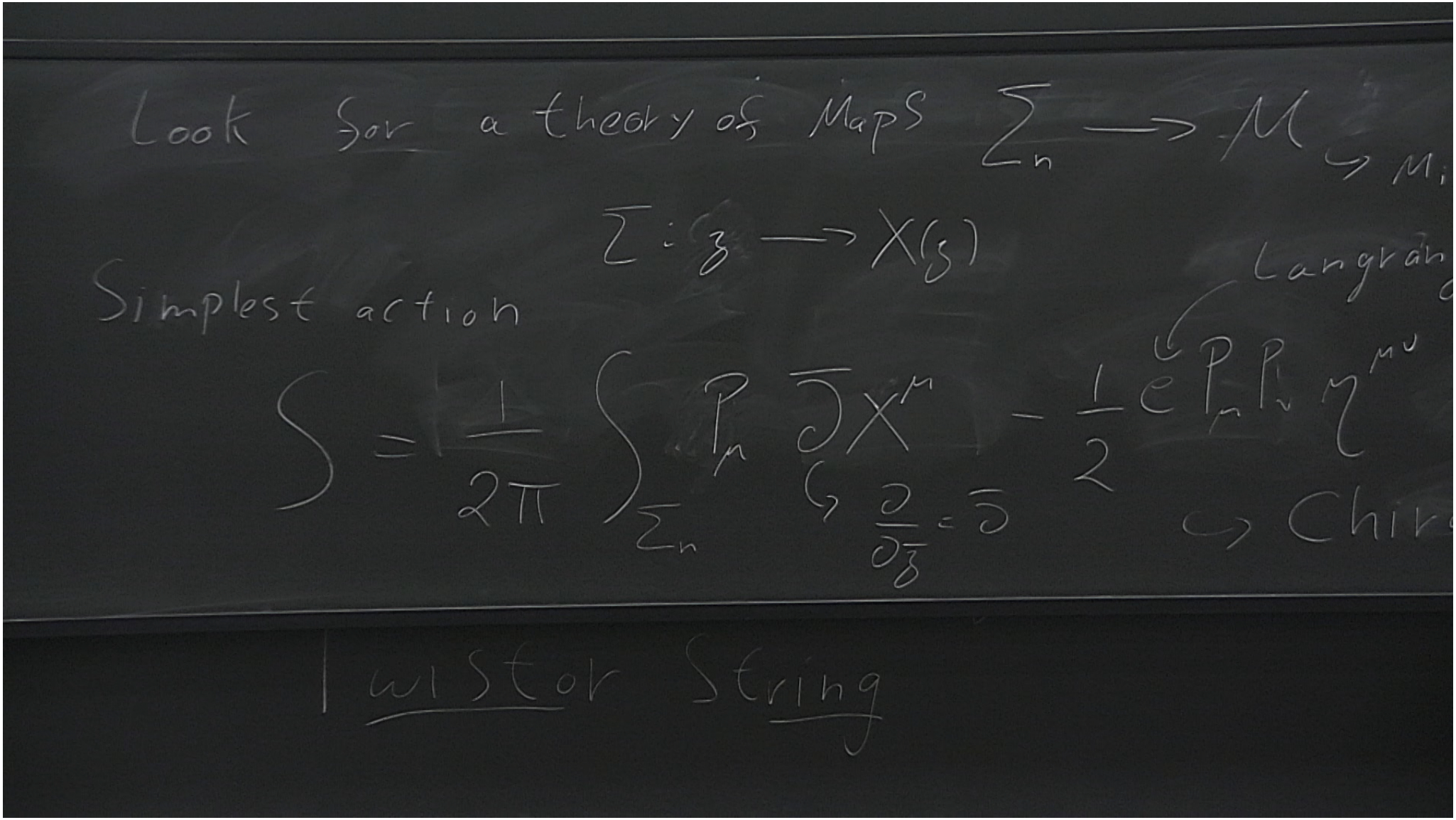

P(z) ~ meromorphic one-form on Σ_n
Constraint P²(z) = 0 ⇝ Need an action
Imposes thes
Originally M&S (1311 2564)
Loops, fermion, moduli (1312 3828)
Geometry (150402675)
My thesis (Eduardo Casali)
Yvonne Geyer's thesis

one-form on $\bar{\Sigma}_n$ ($\bar{\Sigma}_n \simeq$ )

$\rightsquigarrow$ Need an action that imposes these conditions

My thesis (Eduardo Casali)

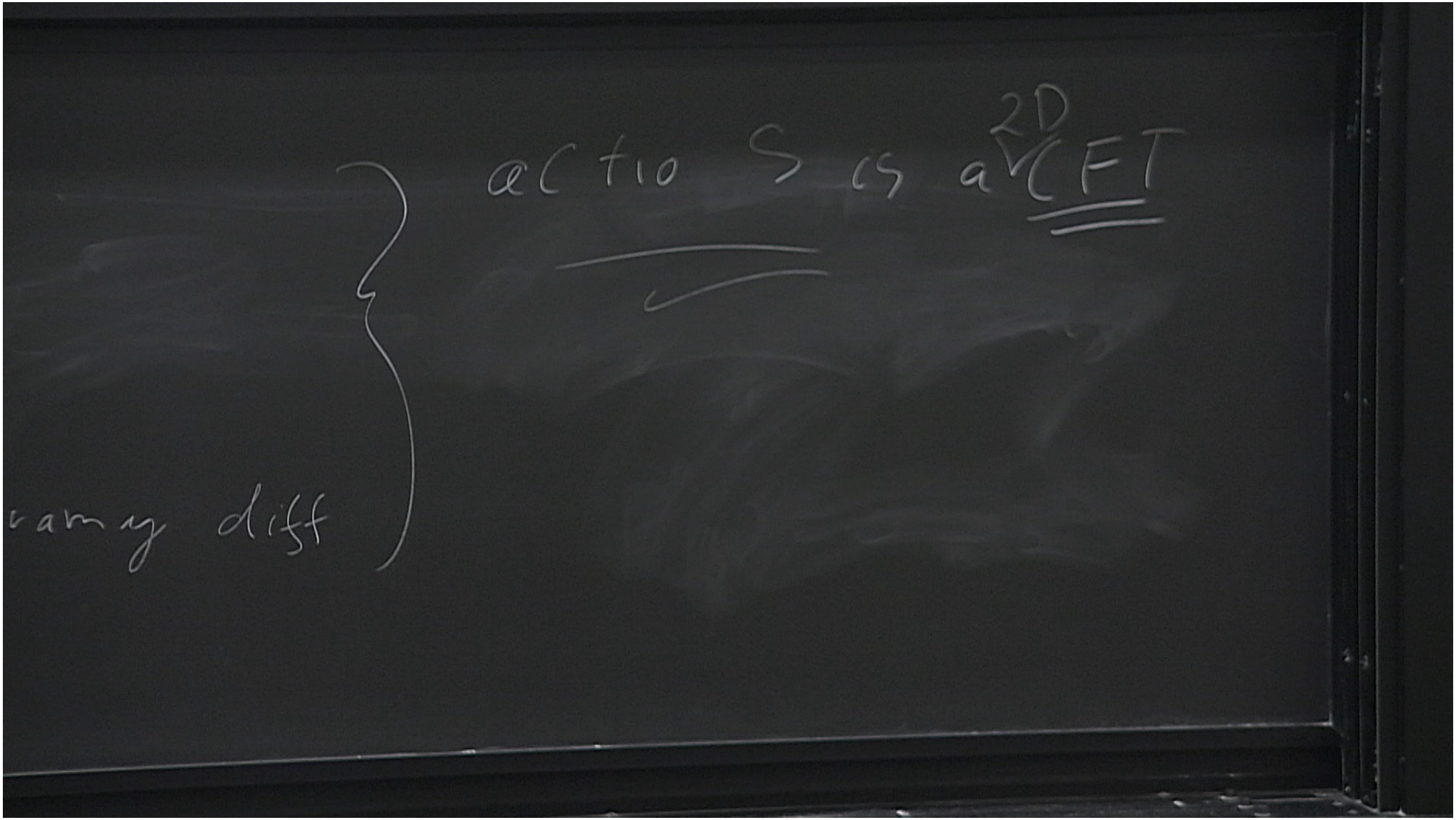
Yvonne Geyer's thesis

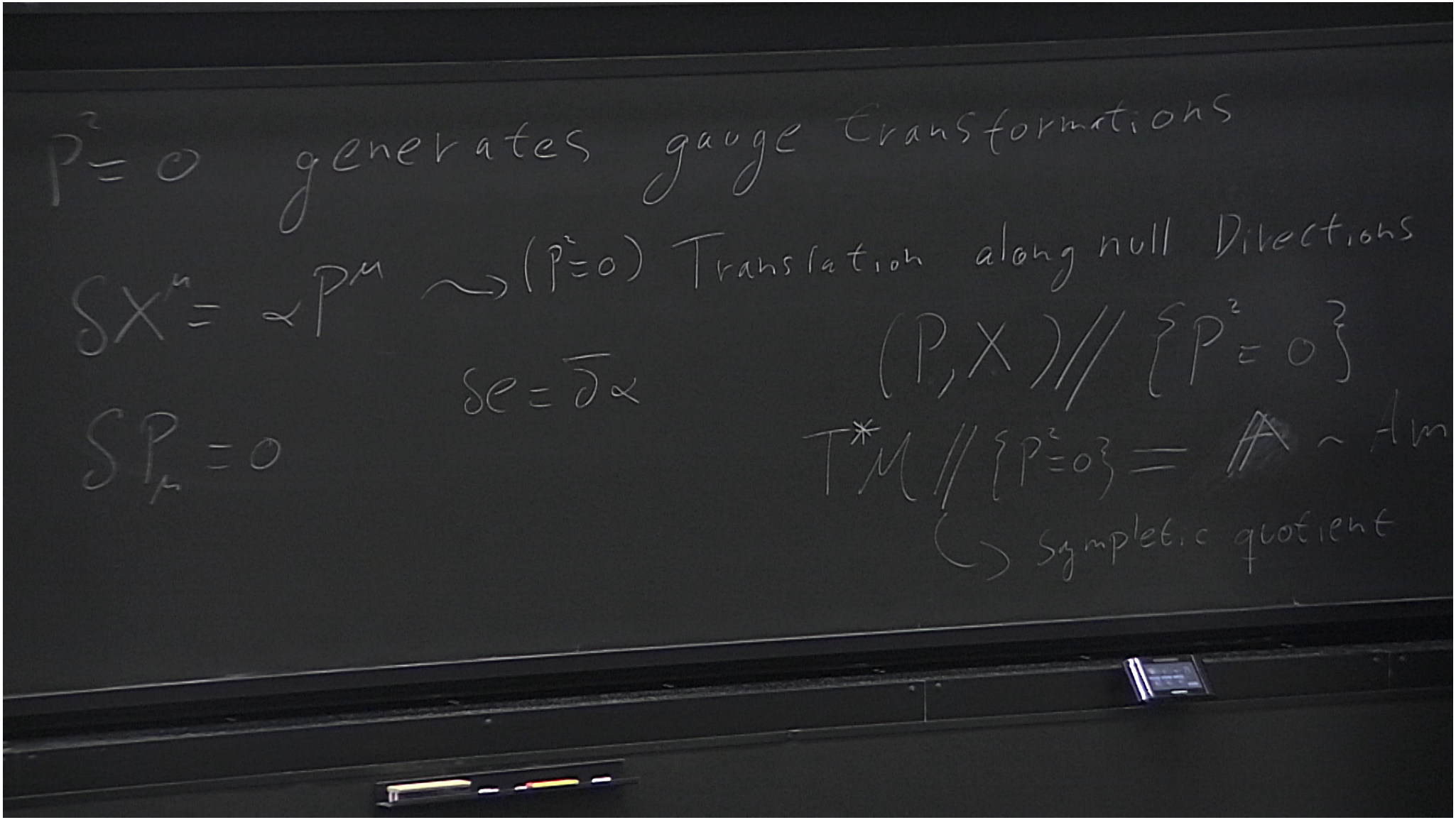

Look for a theory of Maps $\Sigma_n \longrightarrow \mathcal{M}$
$\hookrightarrow M_i$
$\Sigma : z \longrightarrow X(z)$
Simplest action
Langran
$S = \frac{1}{2\pi} \int_{\Sigma_n} P_\mu \bar{\partial} X^\mu - \frac{1}{2} e P_\mu P_\nu \eta^{\mu\nu}$
$\hookrightarrow \frac{\partial}{\partial \bar{z}} = \bar{\partial}$
$\hookrightarrow$ Chir
Twistor String

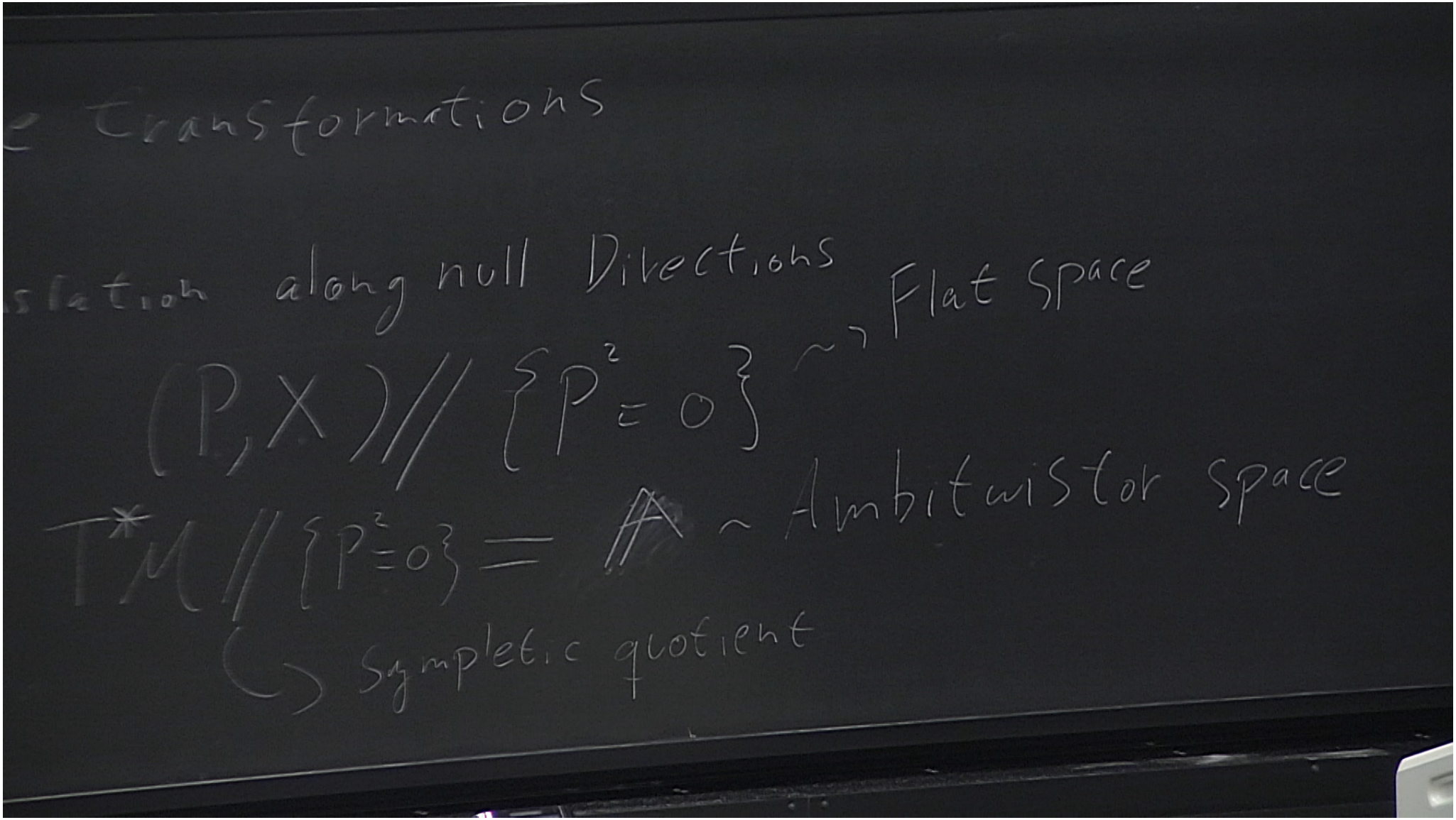
$h_p = (1,0)$ $P \in \Omega^0(\Sigma_n, K^{\sigma(dz)})$ → Section of canonical bd

$h_e = (1,-1)$ $e \in \Omega^{0,1}(\Sigma_n, T^{\sigma(\frac{\partial}{\partial \bar{z}})})$ ~ same weights as Be

$h_x = (0,0)$ $X \in \Omega^0(\Sigma_n)$


action is a 2D CFT
ramy diff


$P^2 = 0$ generates gauge transformations
$\delta X^\mu = \alpha P^\mu \rightsquigarrow (P^2 = 0)$ Translation along null Directions
$\delta e = \dot{\alpha}$
$(P, X) /\!/ \{P^2 = 0\}$
$\delta P_\mu = 0$
$T^*M /\!/ \{P^2 = 0\} = \mathbb{A}$
Symplectic quotient

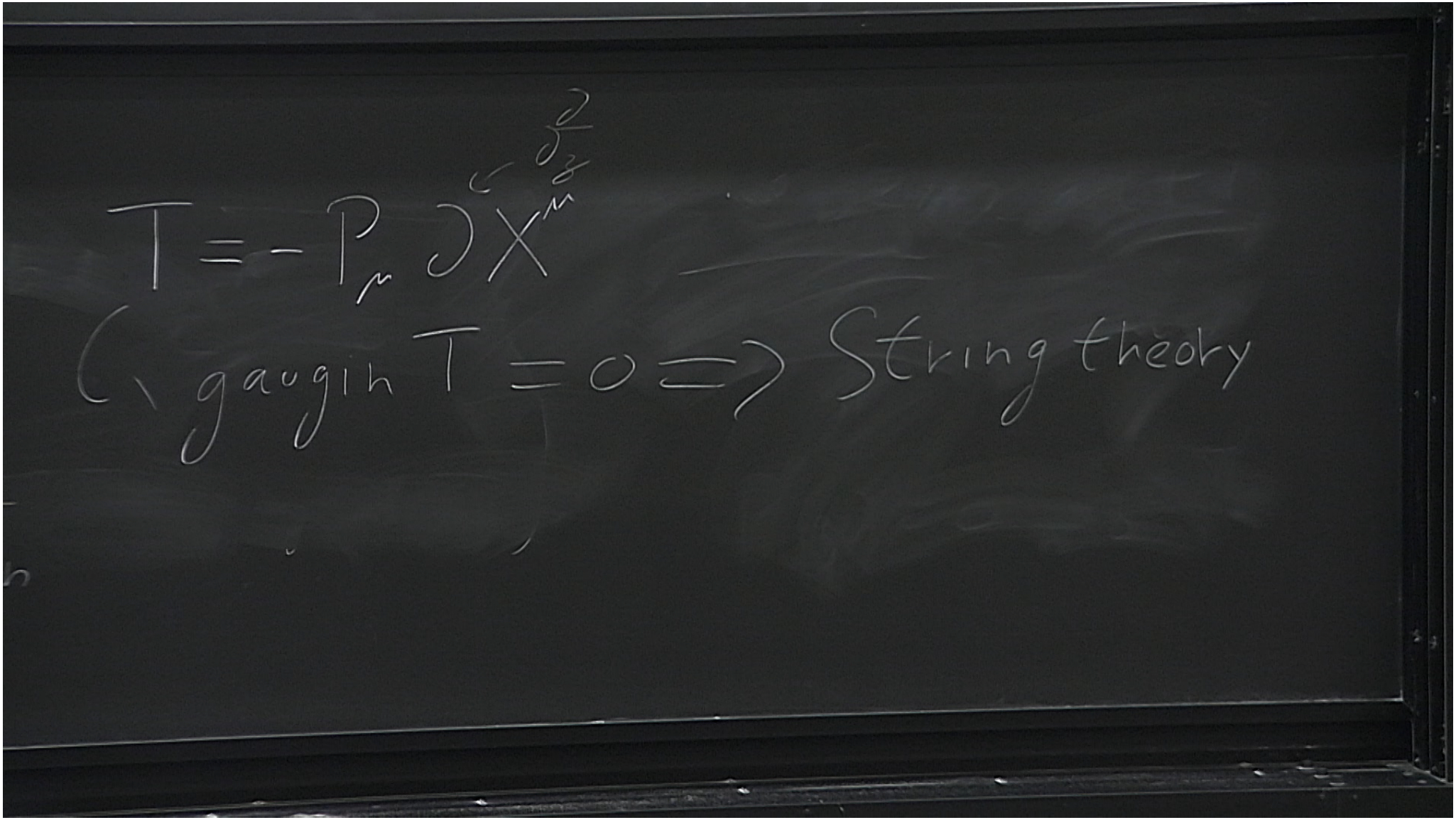

e transformations
slation along null Directions
$(P,X)/\!/\{P^2=0\} \rightsquigarrow$ Flat space
$T^*M/\!/\{P^2=0\} = \mathbb{A} \sim$ Ambitwistor space
↳ Symplectic quotient

another gauge Sym.

$$\delta X^\mu = \nu \partial X^\mu \,, \quad \delta P_\mu = \partial(\nu P_\mu)$$

$$\delta e = \nu \partial e - e \partial \nu$$

ν → Vector on Σ_h

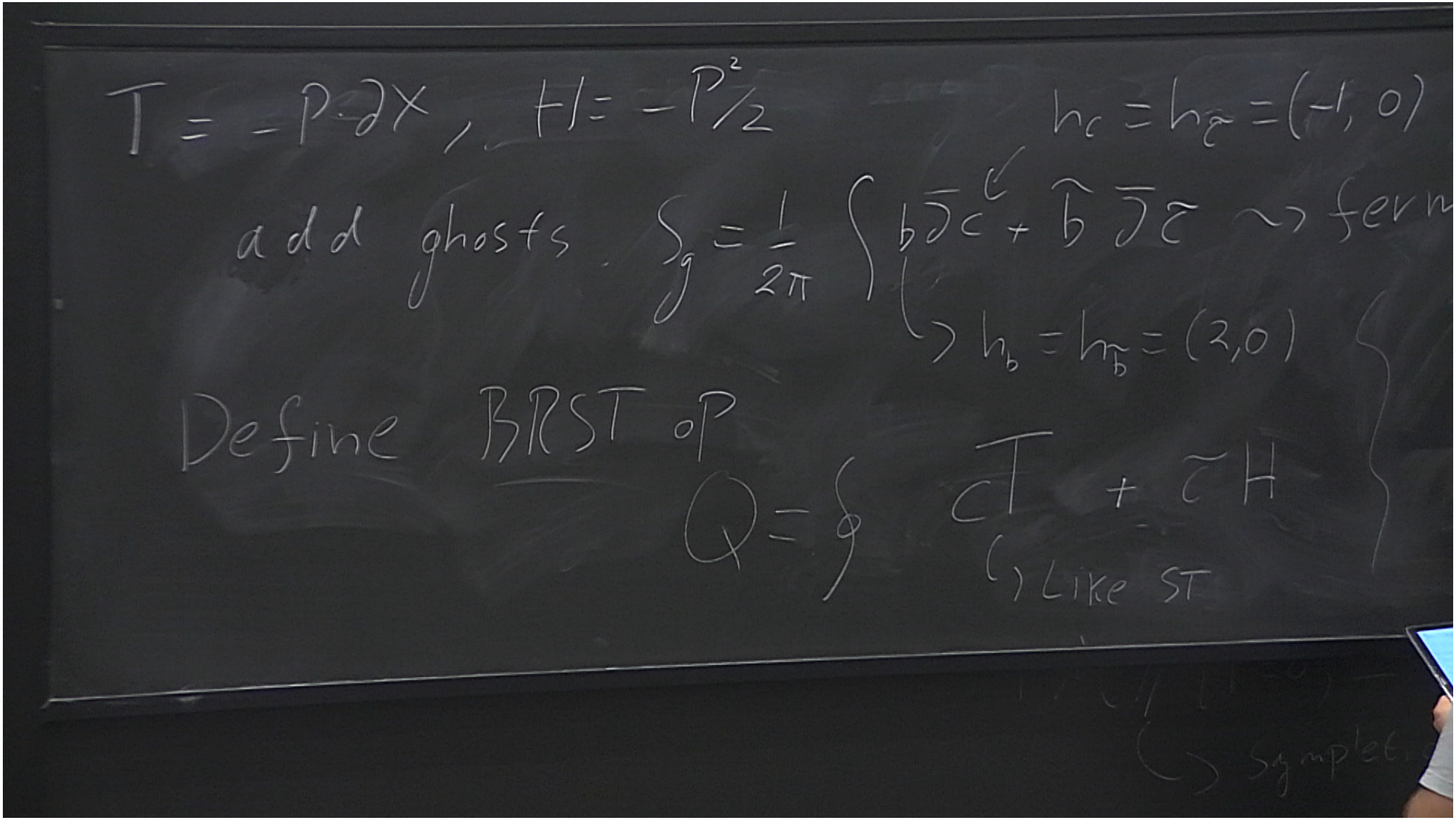

$T = -P_\mu \partial_\sigma X^\mu$
gaugin $T = 0 \Rightarrow$ String theory

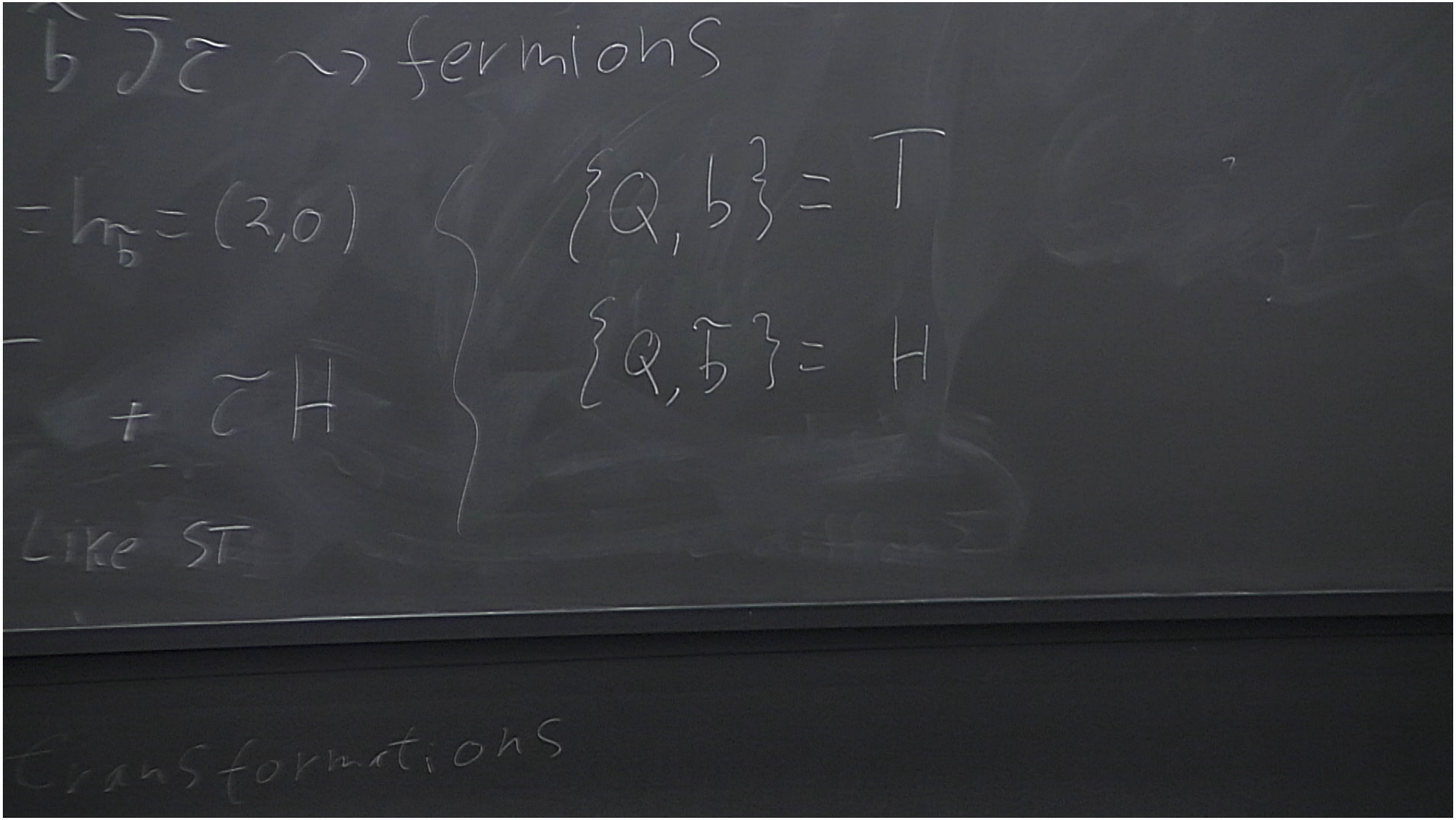
$T = -P_\mu \partial X^\mu$

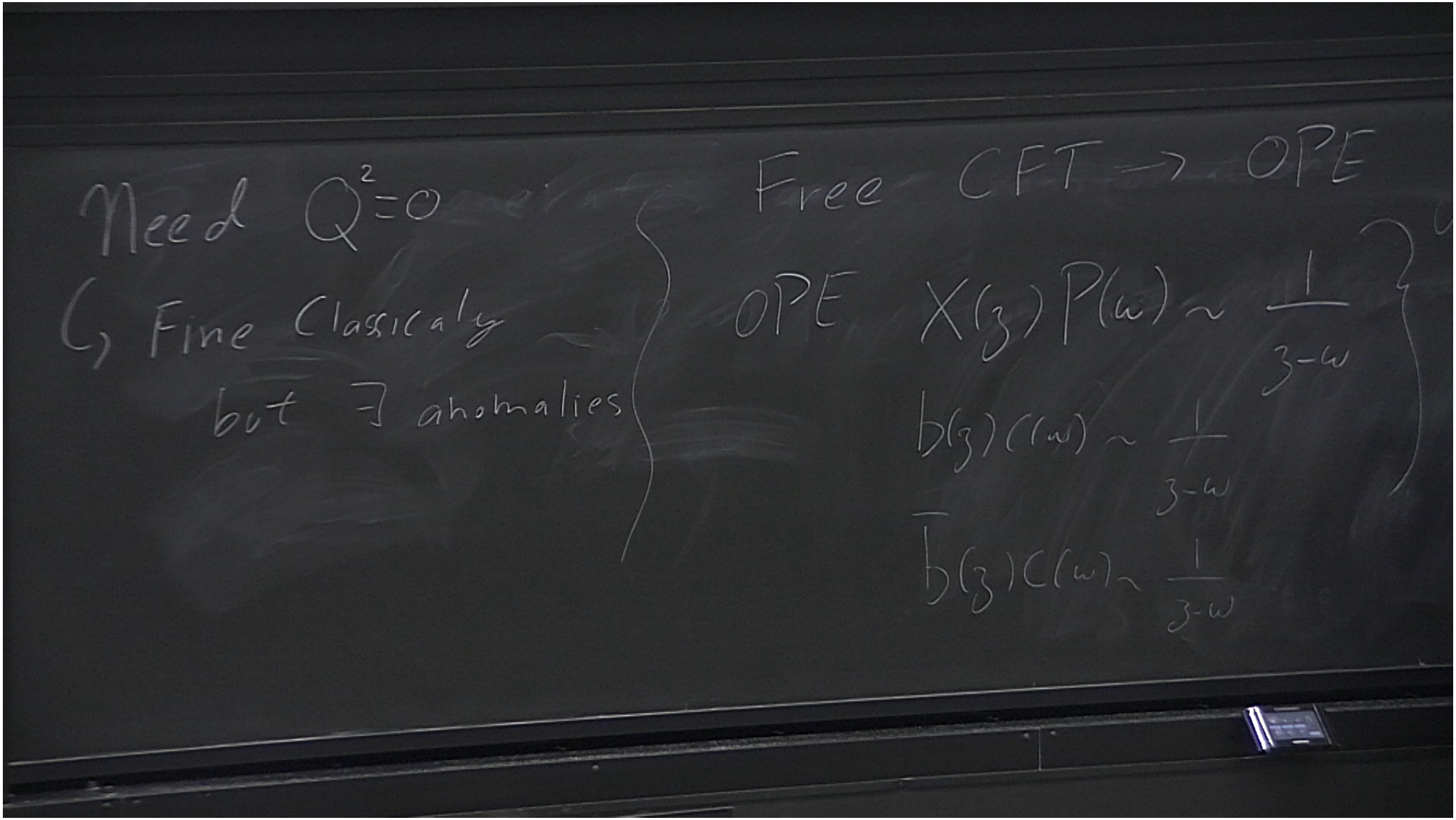
gaugin $T = 0 \Rightarrow$ String theory

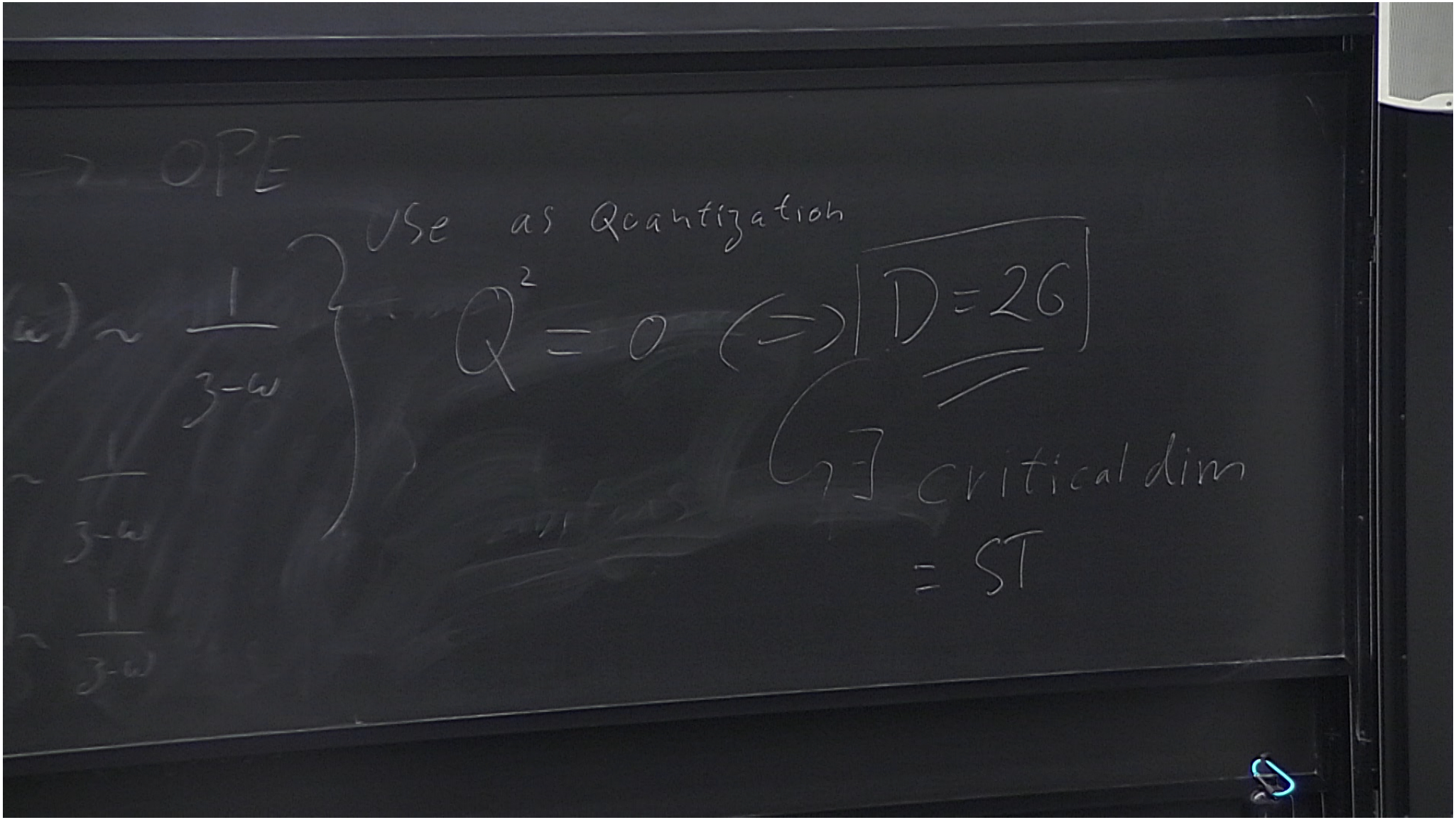
S is partially g.f. to conformal gauge

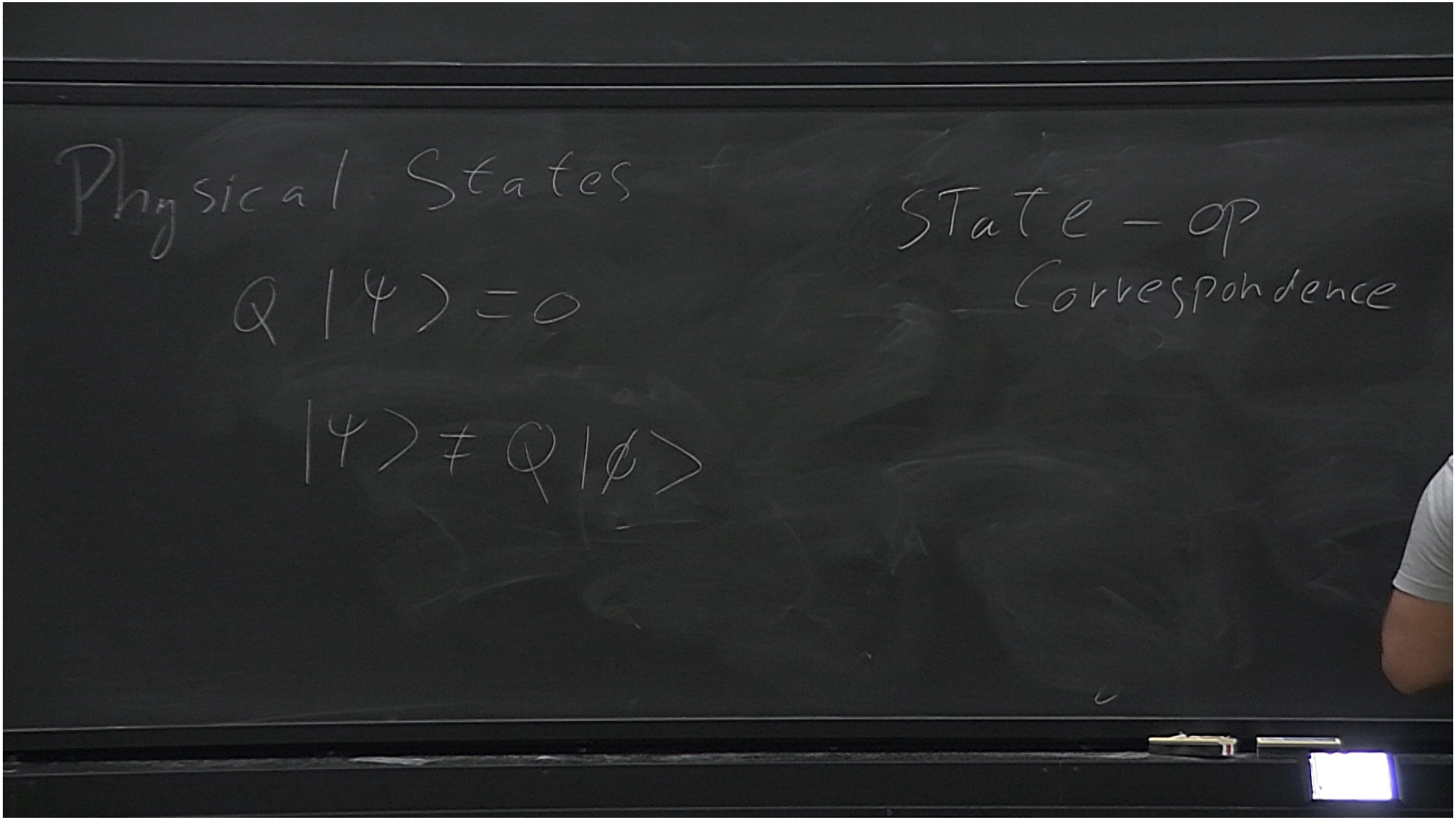
(in $ST \sim \int \partial X \cdot \bar{\partial} X$ + $T = 0$, $\bar{T} = 0$)

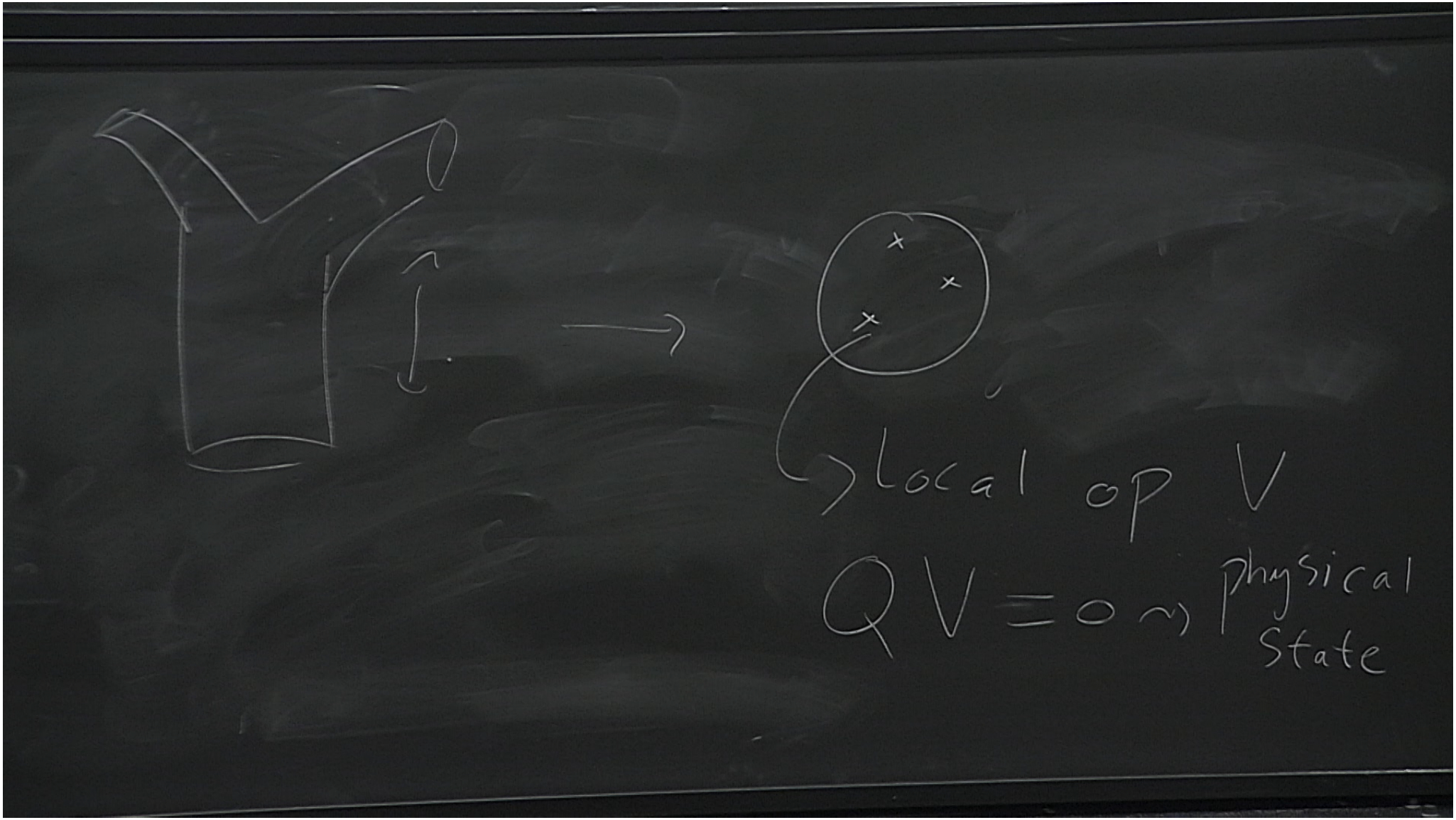

$T = -P\cdot\partial X$, $H = -P^2/2$
$h_c = h_{\tilde{c}} = (-1, 0)$
add ghosts $S_g = \frac{1}{2\pi}\int b\bar{\partial}c + \tilde{b}\bar{\partial}\tilde{c}$ ⇝ ferm
$h_b = h_{\tilde{b}} = (2,0)$
Define BRST oP
$Q = \oint cT + \tilde{c}H$
Like ST
symplect

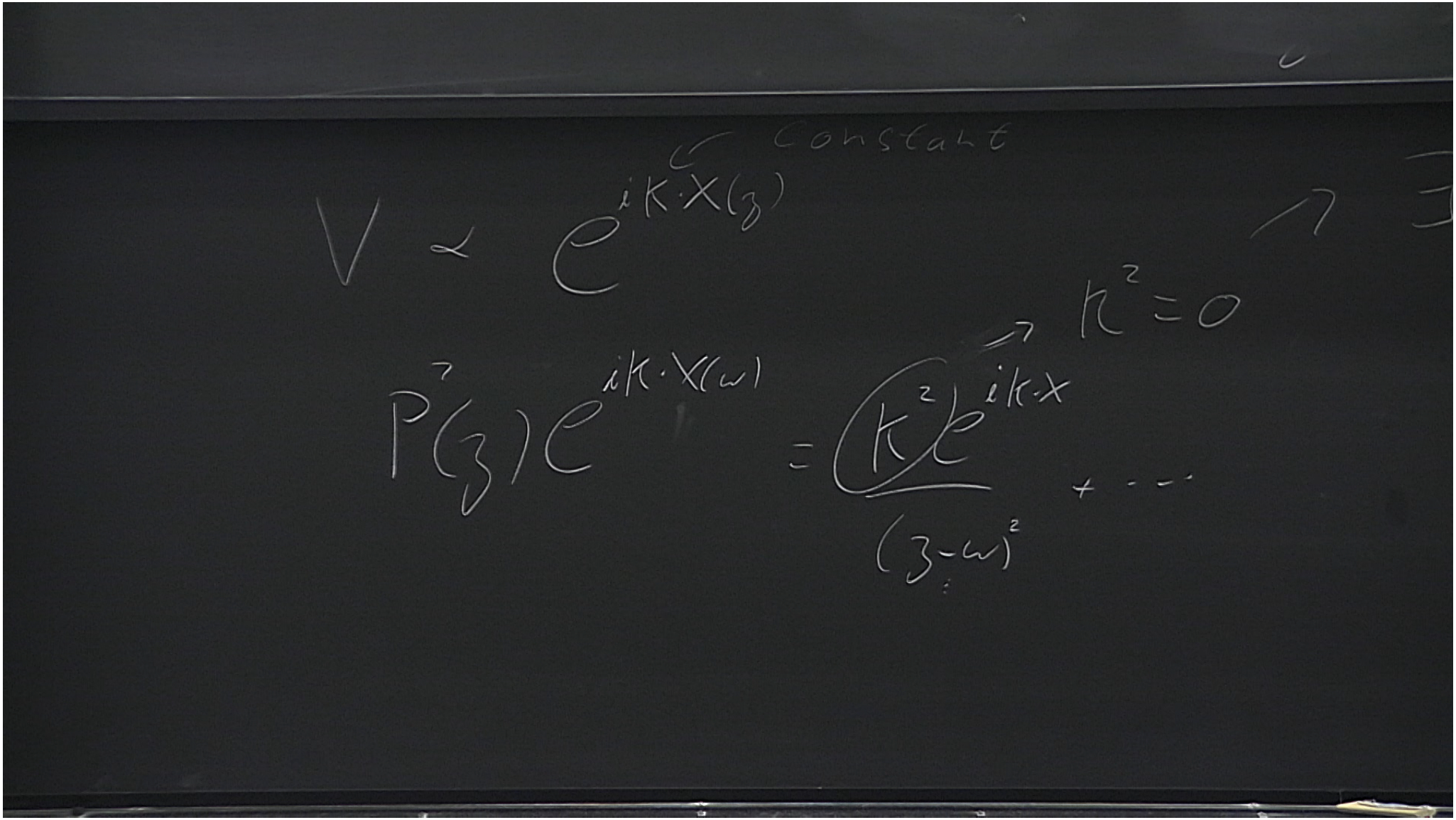

$b\,\tilde{J}\,\tilde{c} \rightsquigarrow$ fermions
$= h_{\tilde{b}} = (2,0)$
$+ \tilde{c}H$
$\{Q, b\} = T$
$\{Q, \tilde{b}\} = H$
Like ST
transformations

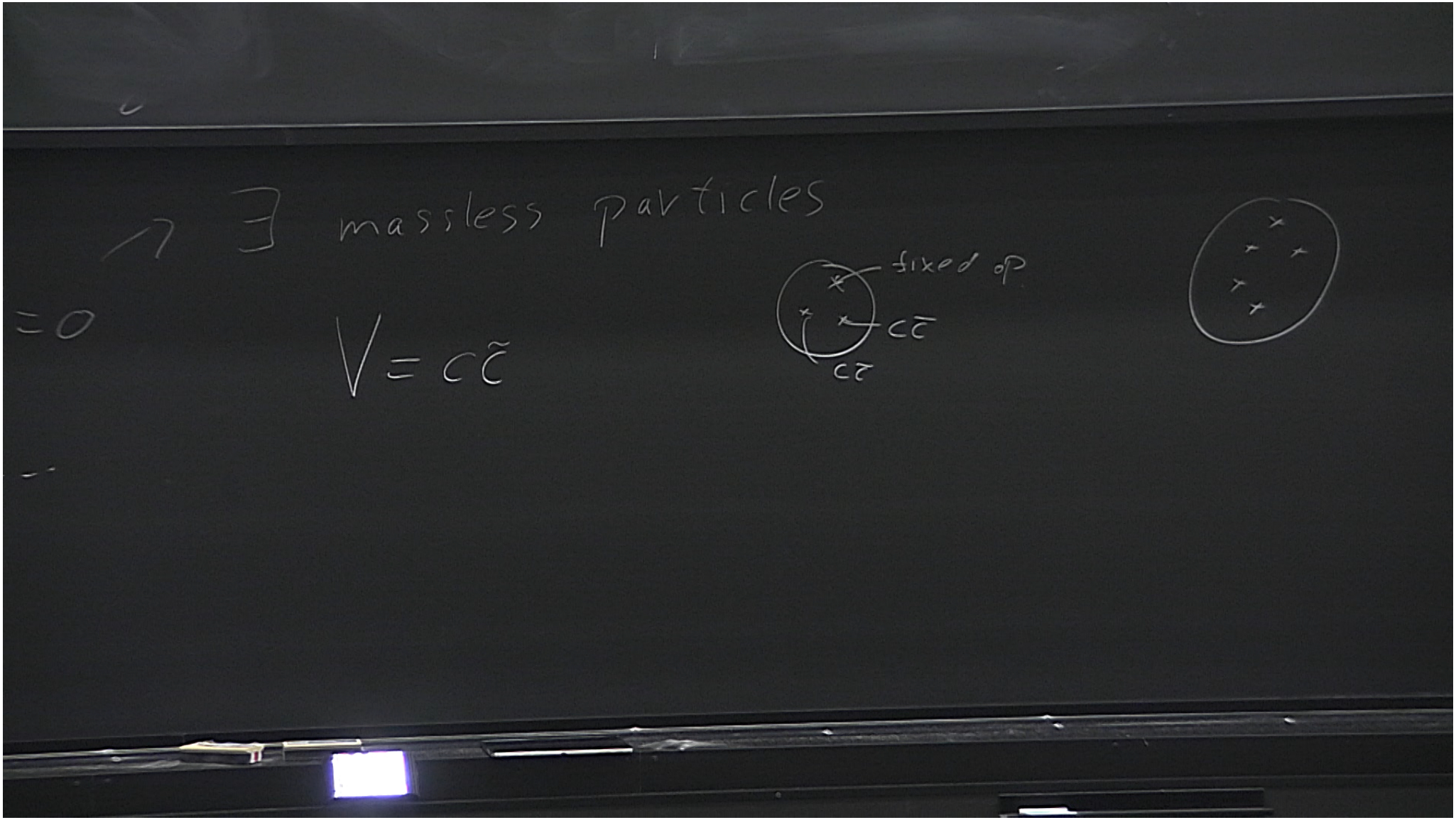

Need $Q^2=0$
Fine Classicaly
but ∃ anomalies
Free CFT → OPE
OPE
$X(z)P(w) \sim \frac{1}{z-w}$
$b(z)c(w) \sim \frac{1}{z-w}$
$\bar{b}(z)\bar{c}(w) \sim \frac{1}{z-w}$


OPE
use as Qcantization
$Q^2 = 0 \Leftrightarrow D = 26$
critical dim
$= ST$


Physical States
$Q|\psi\rangle = 0$
$|\psi\rangle \neq Q|\phi\rangle$
State – op
Correspondence


local op V
QV = 0 ~ physical state

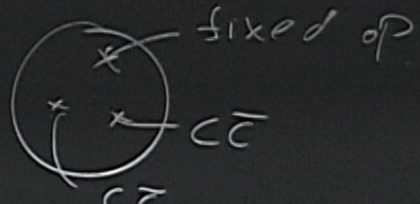

constant
$V \propto e^{ik\cdot X(z)}$
$\Rightarrow k^2 = 0$
$P(z)\, e^{ik\cdot X(w)} = \frac{k^2 e^{ik\cdot X}}{(z-w)^2} + \cdots$


massless particles
fixed op
$c\bar{c}$
$c\bar{c}$
$V = c\bar{c}$

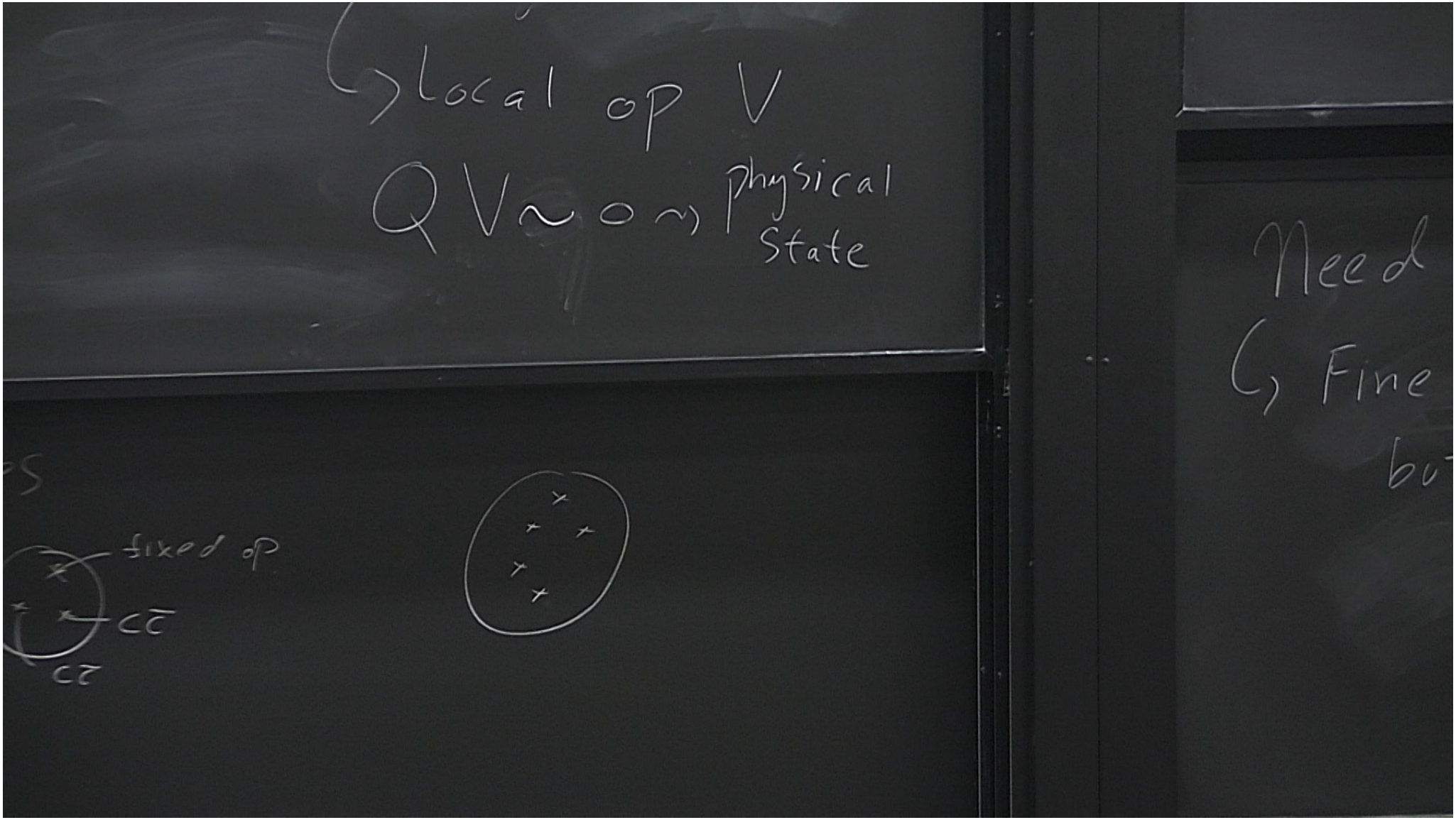
$\exists$ massless particles

$k^2 = 0$

$V = c\tilde{c} f(P, \partial P) e^{ik\cdot X}$



↳ fixed Vertex op.

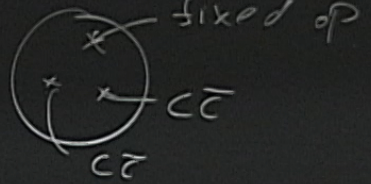

↳ local op V
QV ~ 0 ~ physical state
Need
↳ Fine
fixed op
cc̄
cc̄

∃ massless particles

$h=-2$

$V = \tilde{c}\tilde{c} f(P,\partial P) e^{ik\cdot X}$

$\tilde{c}\tilde{c}$: $h=0$



↳ fixed Vertex op.

$\Rightarrow$ $cT(z)V(w) \sim 0$

$\Downarrow$

$QV \sim 0$

$f(P,\partial P)$ must have $h=2$

s 0 conformal

another sector

new constraints $G = \psi_1 \cdot P$

$\bar{G} = \psi_2 \cdot P$

→ Generate SUSY

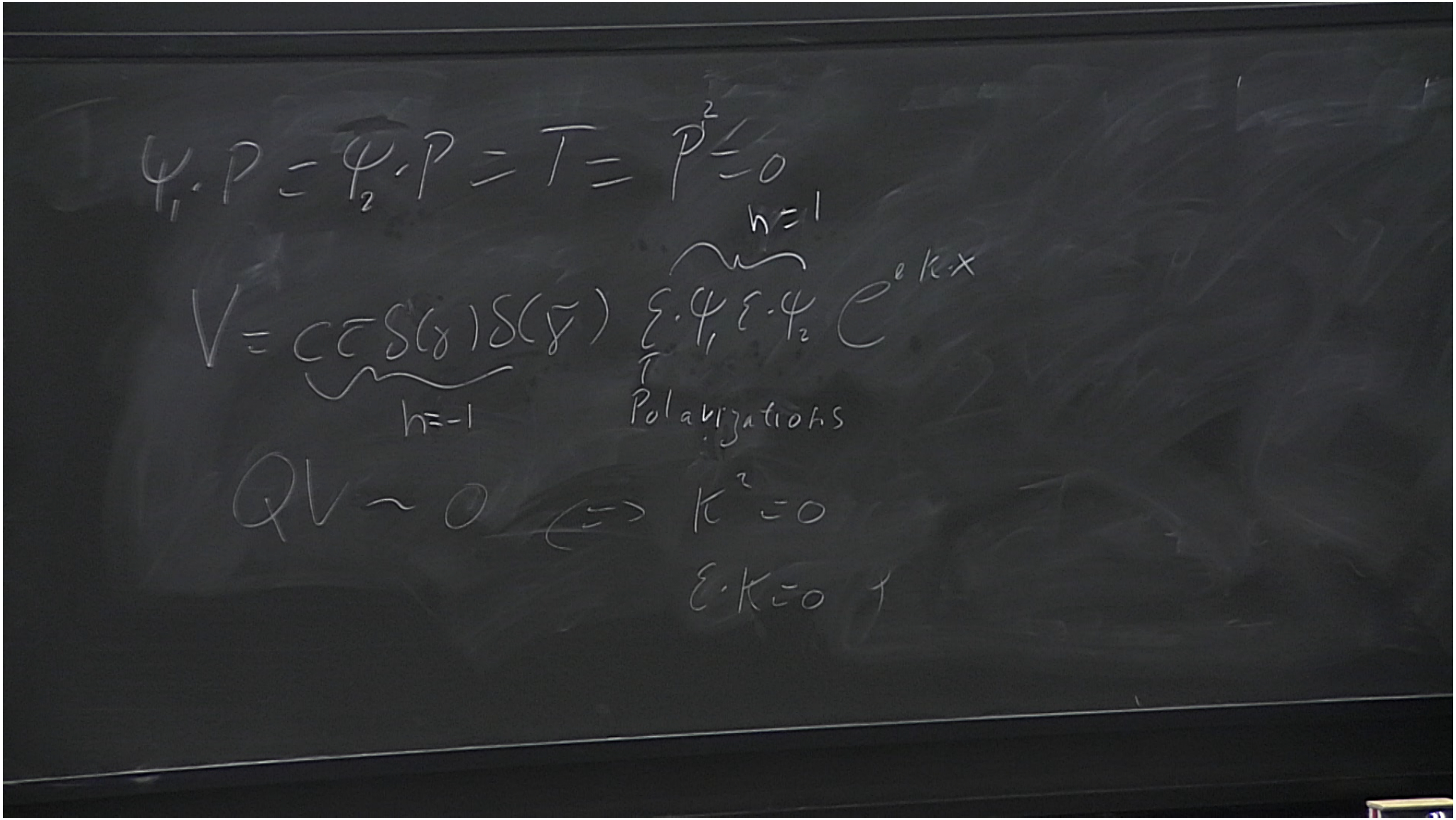
$G(z)G(w) \sim \bar{G}(z)\bar{G}(w) \sim \frac{P^2}{(z-w}$

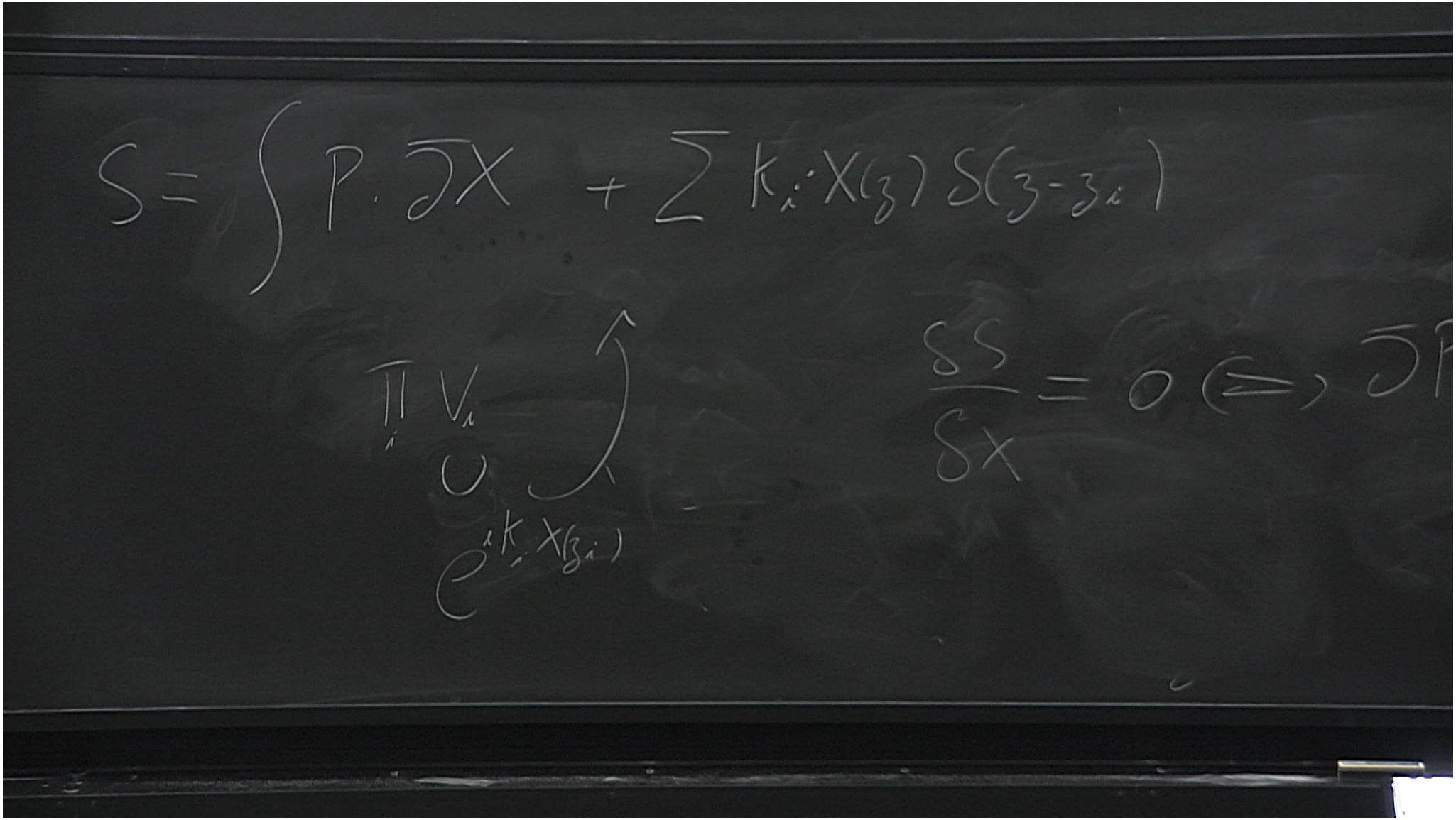
get gravity need to add another sector

new constr

$$S_F = \frac{1}{2\pi}\int_{\Sigma_n} \psi_{1\mu}\bar{\partial}\psi_1^{\mu} + \psi_{2\mu}\bar{\partial}\psi_2^{\mu} \supset S$$

$\hookrightarrow h_\psi(1/2, 0)$


$\psi_1 \cdot P = \psi_2 \cdot P = T = P^2 = 0$
$h=1$
$V = c\tilde{c}\,\delta(\gamma)\delta(\tilde{\gamma})\;\xi\cdot\psi_1\,\xi\cdot\psi_2\,e^{ik\cdot x}$
$h=-1$
Polarizations
$QV \sim 0 \iff k^2 = 0$
$\xi \cdot k = 0$

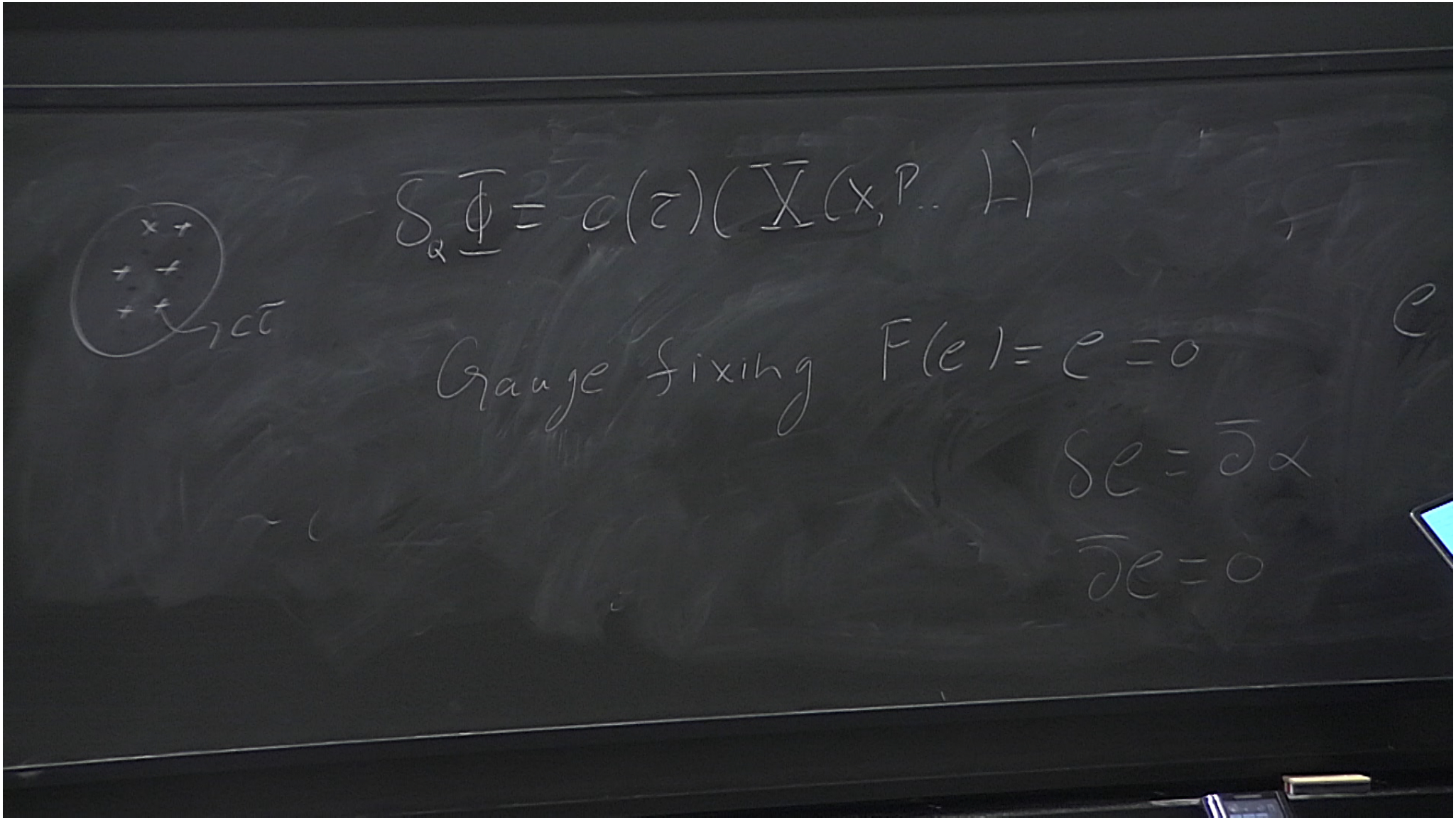

$S = \int P \cdot \partial X + \sum k_i \cdot X(z) \, \delta(z - z_i)$
$\prod_i V_{k_i}$
$\int$
$e^{i k_i \cdot X(z_i)}$
$\frac{\delta S}{\delta X} = 0 \Leftrightarrow \partial$

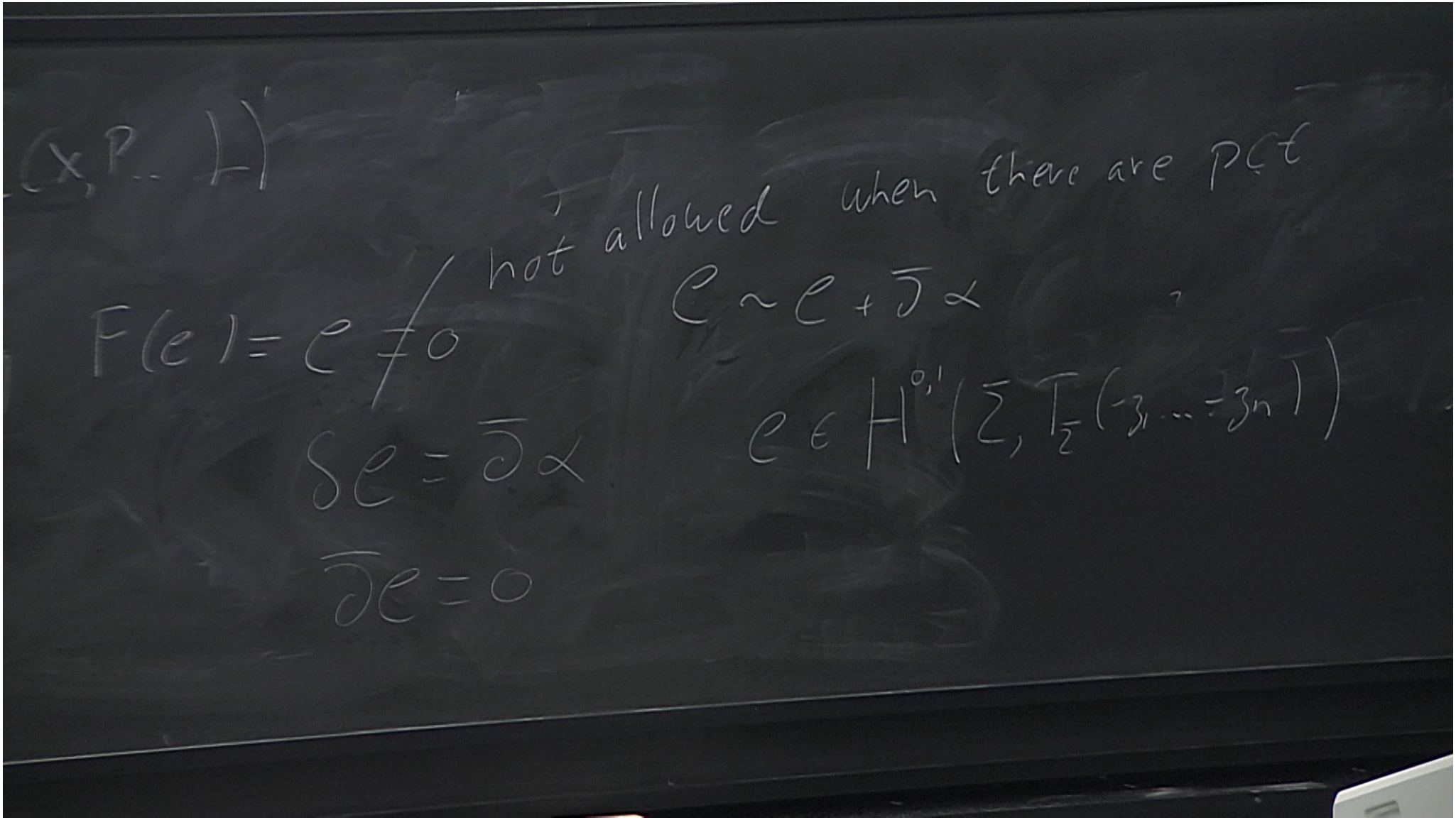
$$\bar{\partial} P = \sum K_i \, \delta(z - z_i) \longrightarrow P(z) = \sum \frac{K_i}{z - z_i}$$

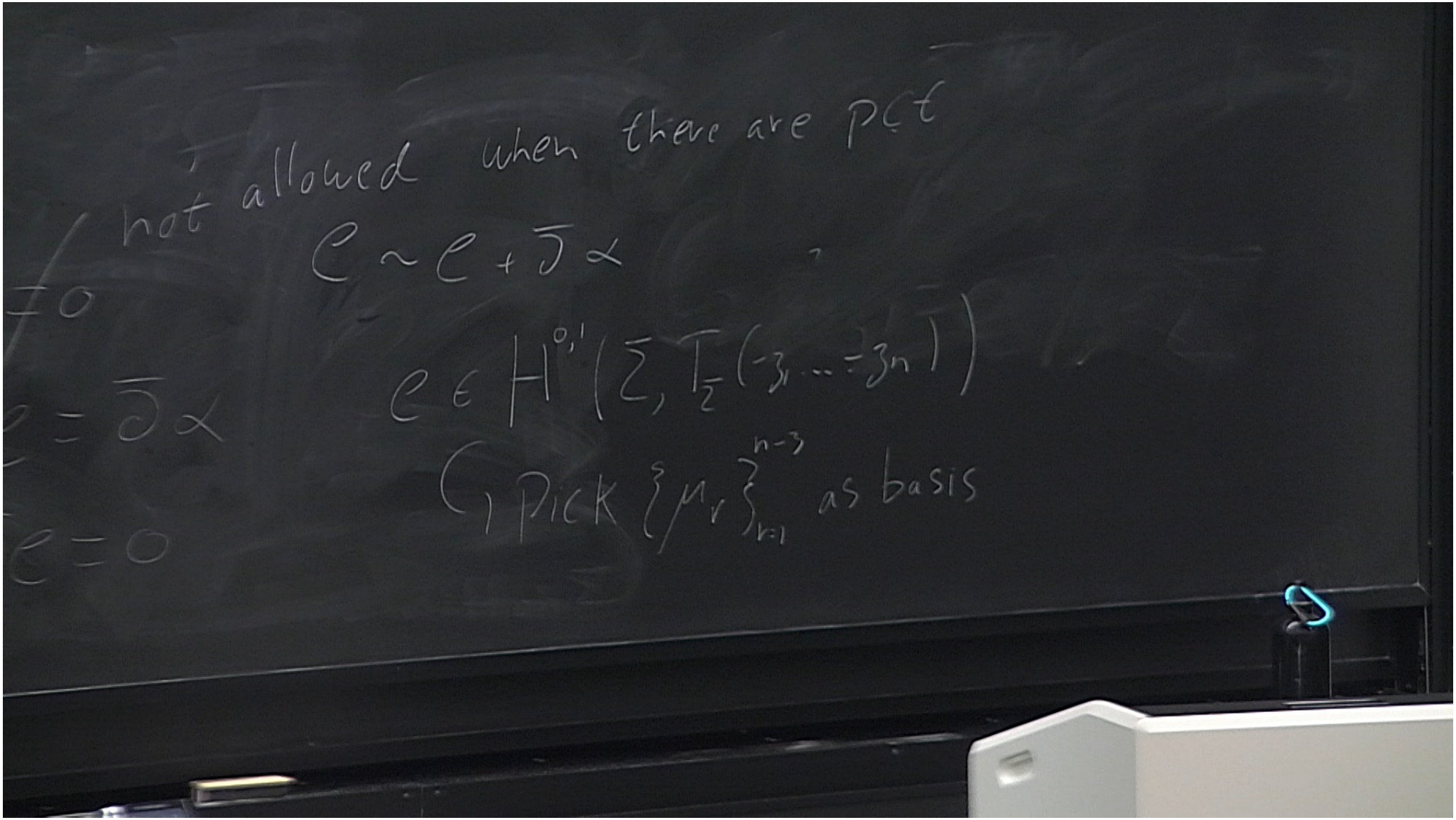
Same P as CHY

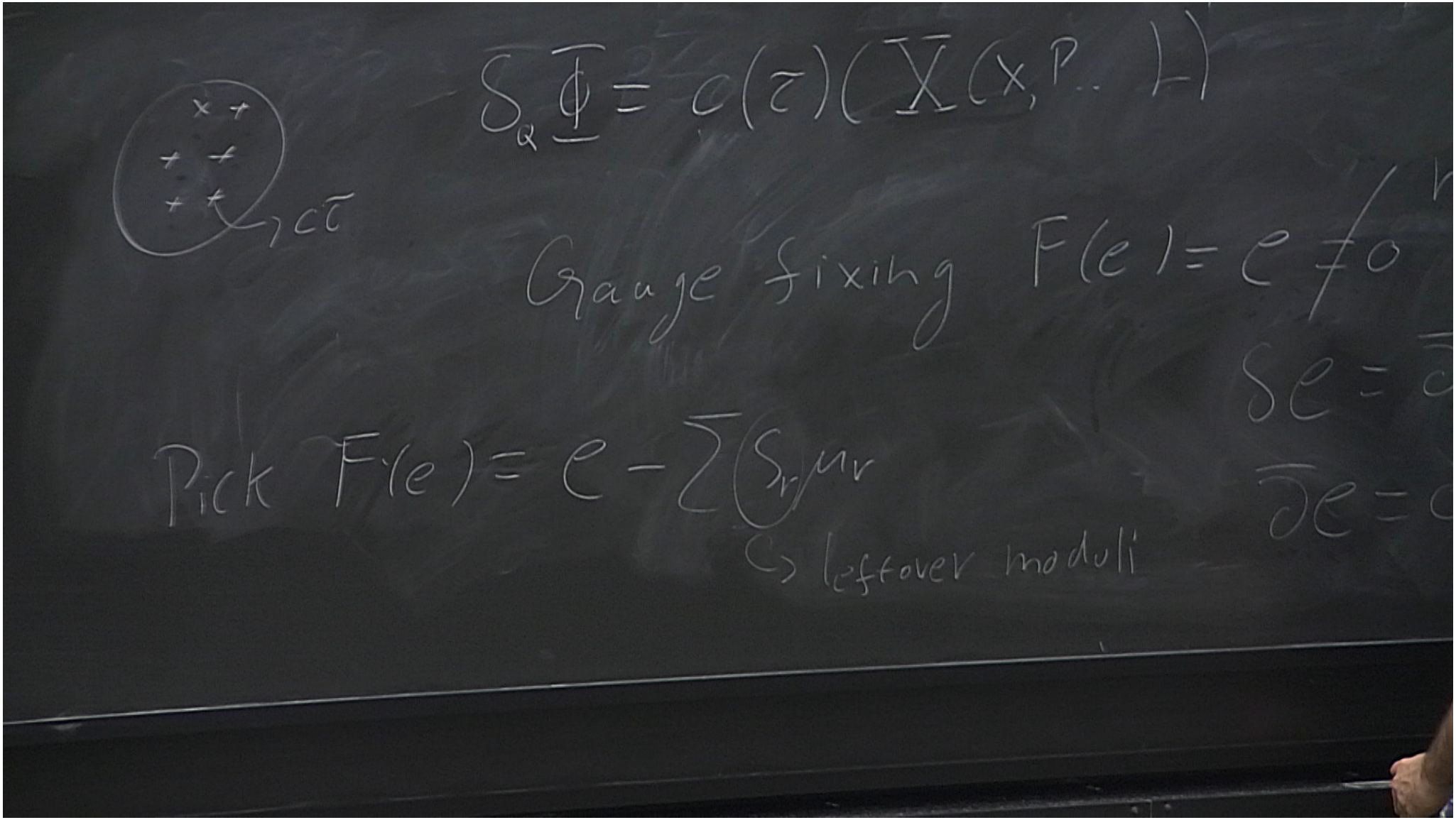
massless particles




$\delta_Q \underline{\Phi} = c(\tau)(\underline{X}(x, p, \ldots))'$
Gauge fixing $F(c) = c = 0$
$\delta c = \partial \alpha$
$\partial c = 0$


(x, P, ...)
F(e) = e ≠ 0
not allowed when there are pcf
e ~ e + ∂̄α
δe = ∂̄α
e ∈ H^{0,1}(Σ, T_Σ(-z_1 - ... - z_n))
∂̄e = 0

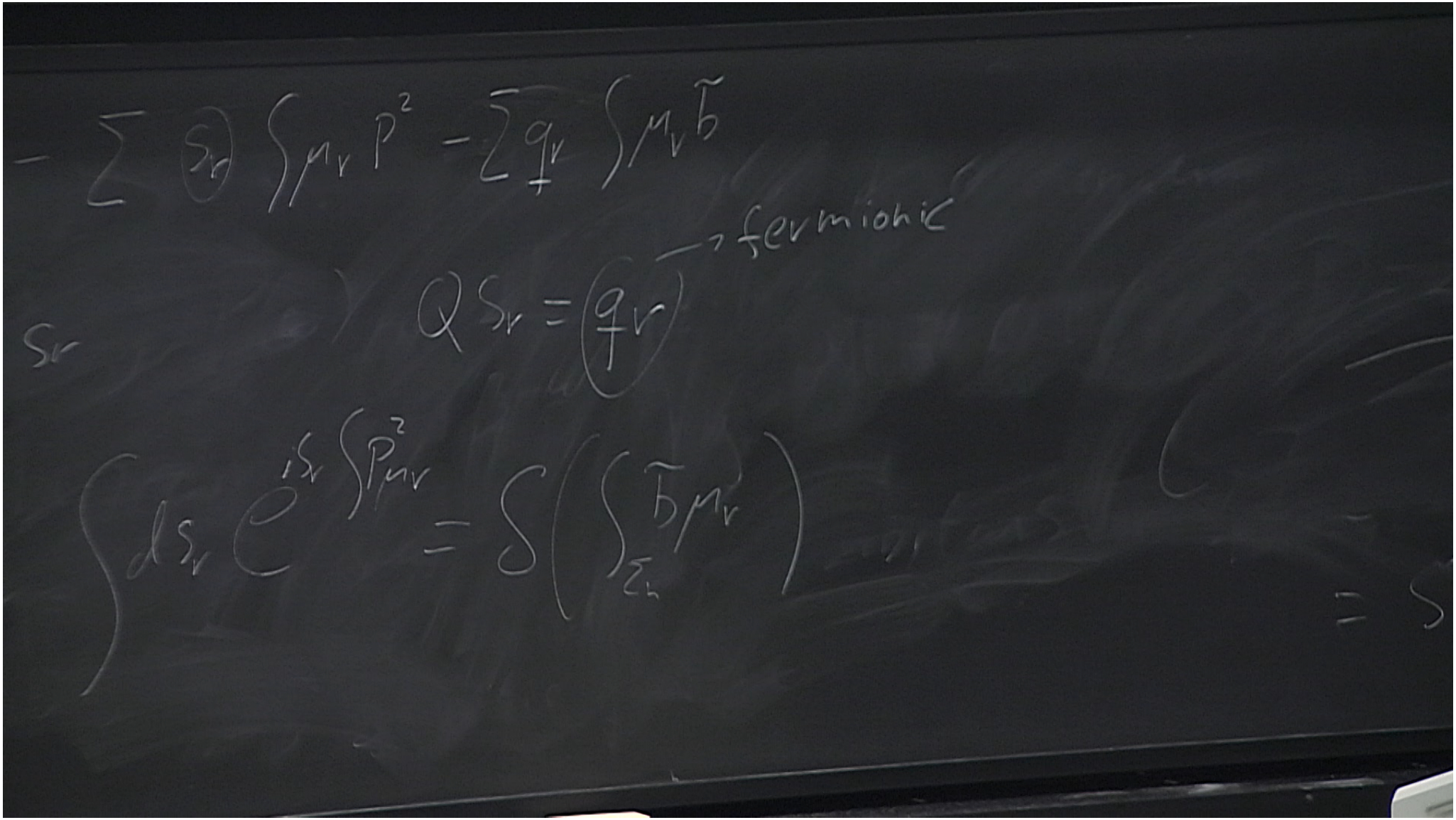

not allowed when there are pCt
$e \sim e + \bar{\partial}\alpha$
$= 0$
$= \bar{\partial}\alpha$
$e = 0$
$e \in H^{0,1}(\Sigma, T_{\frac{1}{2}}(-z_1 \cdots - z_n))$
pick $\{\mu_r\}_{r=1}^{n-3}$ as basis


$\delta_Q \bar{\Phi} = c(\tau)(\bar{X}(x, p, \ldots))$
$\sim 1/c\tau$
Gauge fixing $F(e) = e \neq 0$
$\delta e = \bar{c}$
Pick $F(e) = e - \sum_r s_r \mu_r$
↳ leftover moduli
$\partial e = c$

$$\psi_g = \{Q, F(e)\} = \frac{1}{2\pi} \int \tilde{b}\,\partial\tilde{c} - \sum_r \oint_{\Sigma_r} \int_{\mu_r} P^2 - \mathcal{L}$$

need: to integrate q_r, s_r

$$\int dq_r e^{i q_r \int \tilde{b}_{\mu_r}} = \int_{\Sigma_h} \tilde{b}_{\mu_r}, \quad \int ds_r e^{i s_r \int P^2_{\mu_r}} =$$

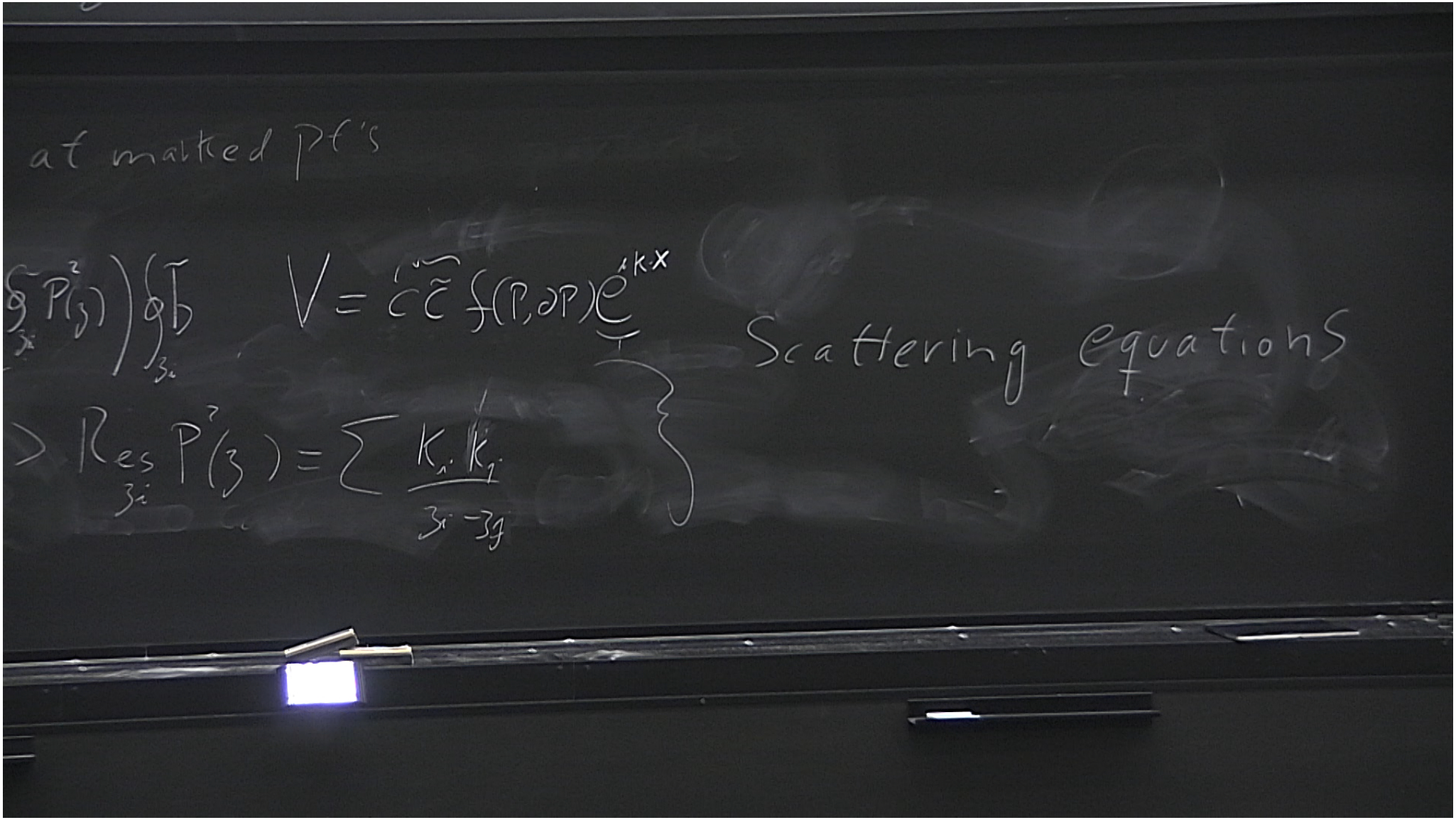

$-\sum s_r \int \mu_r p^2 - \sum \bar{q}_r \int \mu_r \bar{b}$
$Q s_r = q_r$
fermionic
$\int ds_r e^{i s_r \int p^2 \mu_r} = S\left(\int_{\Sigma_h} \bar{b} \mu_r\right)$

Pick μ_r to extract residues at marked P

$$\prod_{r=1}^{n-3} \delta\left(\int_{\Sigma_h} P^2 \mu_r\right) \int_{\Sigma_h} \widehat{b}\mu_r \longrightarrow \prod_{i=1}^{n-3} S\left(\sum_{z_i} P^2(z)\right) \oint_{z_i} b$$

$$V =$$

$$\hookrightarrow \text{Res}_{z_i} P^2(z) =$$


at marked pt's
$V = \tilde{c}\tilde{c}\tilde{c}\tilde{c} f(P, \partial P) e^{ik\cdot x}$
Scattering equations
$\Rightarrow \operatorname{Res}_{3_i} P(3) = \sum \frac{k_i \cdot k_j}{3_i - 3_j}$