

Title: PSI 2017/2018 - Scattering Amplitudes in QFT & String Theory - Lecture 5

Date: Apr 13, 2018 11:30 AM

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Abstract:

Lecture V

Generating Function of Parke-Taylor Amplitudes $\longrightarrow$ Witten-RSV formula

Lecture V

Generating Function of Parke-Taylor Amplitudes

A_n Contains all Amplitudes with $k=2$ negative helicity gluons
 $n-k$ had + helicity.

→ Witten-RSV formula

realizes PT to $k \in \{2, 3\}$

Ke-Taylor Amplitudes



Witten-RSV formula

es with $K=2$ negative
helicity gluons
 $n-K$ had + helicity.

Generalizes PT to any
 $K \in \{2, 3, \dots, n-2\}$

$K = -$ helicity
 $n-K = (+)$ helicity.

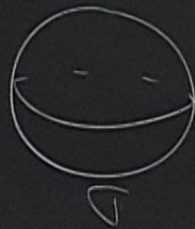
Lecture V

Generating Function of Parke-Taylor Amplitudes

→ Witten-RSV form

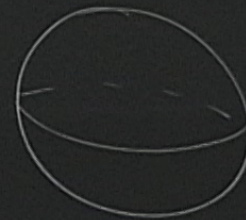
A_n Contains all Amplitudes with $k=2$ negative helicity gluons
 $n-k$ had + helicity.

Generalizes PT
 $k \in \{2, 3, \dots\}$



degree d .

$$d = k - 1$$



$$\lambda_\alpha(\sigma) = p_0 + p_1 \sigma + \dots + p_d \sigma^d$$

$$A(- - +) = \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 31 \rangle}$$

$$A(+ + -) = \frac{[12]^2}{[23][31]}$$

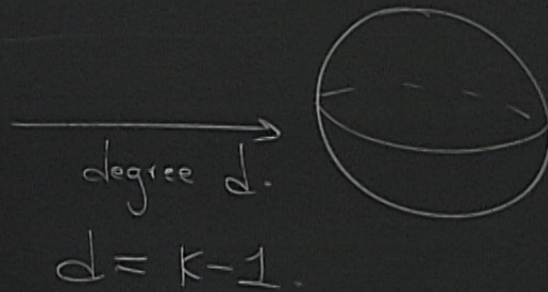
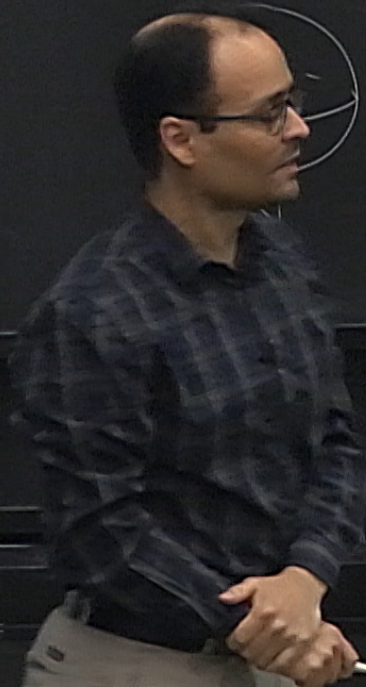
$$\lambda \leftrightarrow \tilde{\lambda}$$

Lecture V

Generating Function of Parke-Taylor Amplitudes

A_n Contains all Amplitudes with $k=2$ negative helicity gluons
 $\tilde{k} = n - k$ had + helicity.

Witten-RSV form
 Generalizes PT
 $k \in \{2, 3, \dots\}$



$$\lambda_\alpha(\sigma) = \rho_0 + \rho_1 \sigma + \dots + \rho_d \sigma^d$$

- Taylor Amplitudes $\longrightarrow$ Witten-RSV formula
 Generalizes PT to any $K \in \{2, 3, \dots, n-2\}$
 $K = n - k$ with $k = 2$ negative helicity gluons
 $K = n - k$ had $+$ helicity. $K = -$ helicity
 $n - K = (+)$ helicity.

$\lambda \longleftrightarrow \bar{\lambda}$

$\lambda_\alpha(\sigma) = \int_0^1 + \int_d \sigma^d$

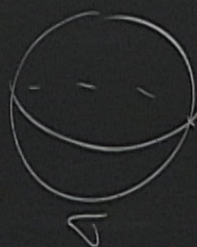
degree

Witten-RSV formula

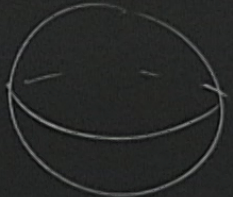
Generalizes PT to any
 $\kappa \in \{2, 3, \dots, n-2\}$

$\tilde{\lambda} \leftrightarrow \lambda$

$\kappa = -$ helicity
 $n - \kappa = +$ helicity



degree d
 $\sum_{i=1}^d$



$$\tilde{\lambda}_d(\sigma) = \tilde{p}_0 + \tilde{p}_1 \sigma + \dots + \tilde{p}_d \sigma^d$$

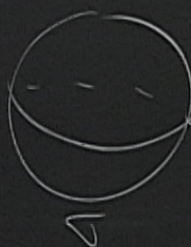
Witten-RSV formula

Generalizes PT to any
 $k \in \{2, 3, \dots, n-2\}$

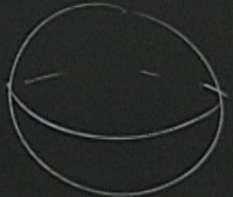
$\tilde{\lambda} \leftrightarrow \lambda$

$+ \int_d \sigma^d$

$k = -$ helicity
 $n-k = (+)$ helicity

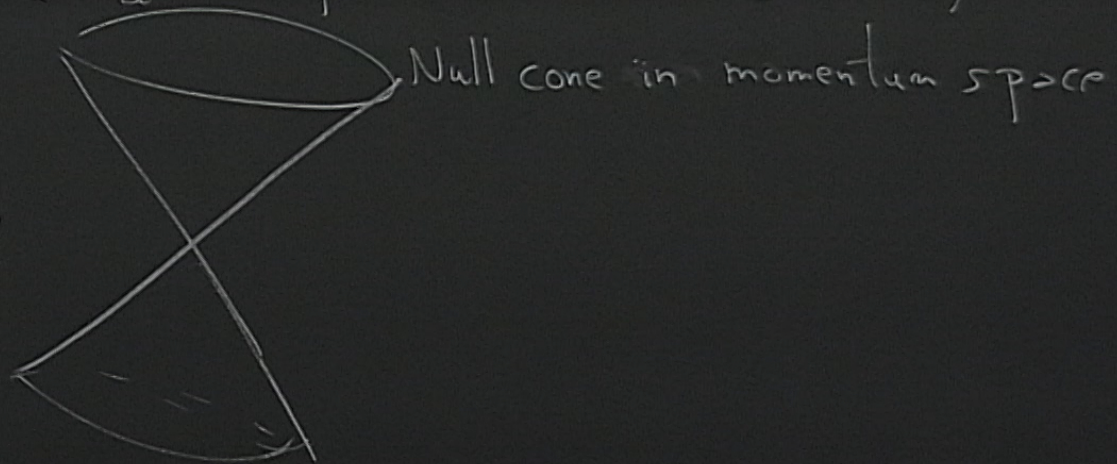


degree d
 $d = k-1$

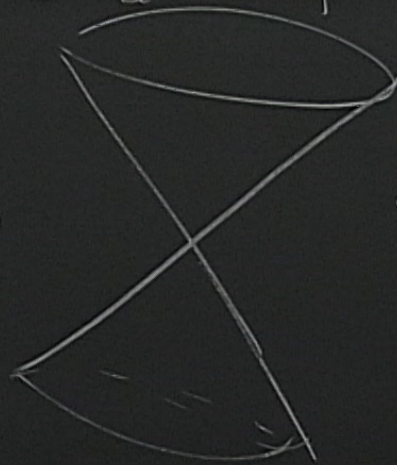
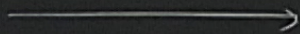
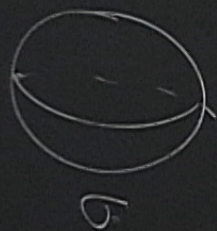


$$\tilde{\lambda}_d(\sigma) = \tilde{p}_0 + \tilde{p}_1 \sigma + \dots + \tilde{p}_d \sigma^d$$

Combine $\lambda_a \tilde{\lambda}_i$ to produce a Vector $\vec{P} \rightarrow P_\mu$.



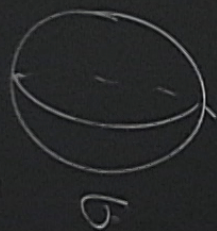
Combine $\lambda_\alpha \tilde{\lambda}_i$ to produce a Vector $\mathcal{P} \rightarrow P_\mu$.



Null cone in momentum space

$$P_{\alpha i}(\sigma) = \lambda_\alpha(\sigma) \tilde{\lambda}_i(\sigma)$$

Combine $\lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$ to produce a Vector $\mathcal{P} \rightarrow P_\mu$.



Null cone in momentum space

$$P_{\alpha\dot{\alpha}}(\sigma) = \underbrace{\lambda_\alpha(\sigma)}_{d} \underbrace{\tilde{\lambda}_{\dot{\alpha}}(\sigma)}_{\bar{d}}$$

$$d = k - 2$$

duce a Vector $\vec{p} \rightarrow P_\mu$.

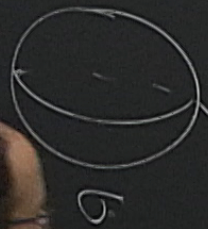
Null cone in momentum space

$$P_{\alpha\tilde{\alpha}}(\sigma) = \underbrace{\lambda_\alpha(\sigma)}_d \underbrace{\tilde{\lambda}_{\tilde{\alpha}}(\sigma)}_{\tilde{d}}$$

$$d + \tilde{d} = k + \tilde{k} - 2 = k + (n - k) - 2 = n - 2$$

k independent $\vec{p}$

Combine $\lambda_\alpha \tilde{\lambda}_i$ to produce a Vector $\mathcal{P} \rightarrow$



Degree $n-2$.



Null cone in momentum

$$P_{\alpha i}(\sigma) = \underbrace{\lambda_\alpha(\sigma)}_{d} \underbrace{\tilde{\lambda}_i(\sigma)}_{d} =$$

Connect to external data:

$$\lambda_{\alpha}^{(2)} = t_{\alpha} \underbrace{\lambda(\sigma)}_{\alpha}$$

$$\tilde{\lambda}_{\alpha}^{(a)} = \tilde{t}_{\alpha} \tilde{\lambda}_{\alpha}(\sigma)$$

Connect to external data:

$$\lambda_{\alpha}^{(2)} = t_{\alpha} \lambda_{\alpha}(\sigma_a)$$

$$\tilde{\lambda}_{\alpha}^{(a)} = \tilde{t}_{\alpha} \tilde{\lambda}_{\alpha}(\sigma_a)$$

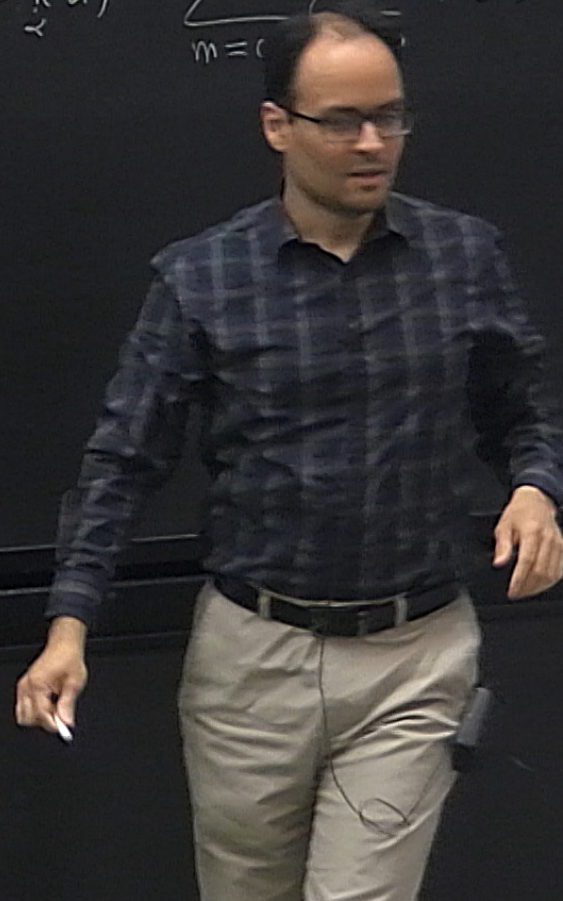
$$\lambda_{\alpha}^{(a)} \tilde{\lambda}_{\alpha}^{(a)} = K_{\alpha\alpha}^{(a)} = t_{\alpha} \tilde{t}_{\alpha} \lambda_{\alpha}(\sigma_a) \tilde{\lambda}_{\alpha}(\sigma_a)$$

Key: Momentum Conservation.

$$\sum_{\alpha=1}^n K_{\alpha\alpha}^{(a)} = 0 = \sum_{\alpha=1}^n t_{\alpha} \hat{\lambda}_{\alpha}(\Phi) \hat{\lambda}_{\alpha}(\Phi)$$

Key: Momentum Conservation.

$$\sum_{a=1}^n K_{\alpha\tilde{\alpha}}^{(a)} = 0 = \sum_{a=1}^n t_a \tilde{t}_a \lambda_{\alpha}(\Gamma_a) \hat{\lambda}_{\tilde{\alpha}}(\Gamma_a) = \sum_{m=1}^d \sum_{\tilde{m}=1}^{\tilde{d}} p_{m,\alpha} \tilde{p}_{\tilde{m},\tilde{\alpha}} \sum_{a=1}^n t_a \tilde{t}_a \nabla_a^{m+\tilde{m}}$$



Key: Momentum Conservation.

$$\sum_{a=1}^n K_{\alpha\tilde{\alpha}}^{(a)} = 0 = \sum_{a=1}^n t_a \tilde{t}_a \lambda_{\alpha}(\tilde{a}) \hat{\lambda}_{\tilde{\alpha}}(a) = \sum_{m=0}^d \sum_{\tilde{m}=0}^{\tilde{d}} p_{m,\alpha} \tilde{p}_{\tilde{m},\tilde{\alpha}} \left(\sum_{a=1}^n t_a \tilde{t}_a \right) \lambda_{\alpha}(\tilde{a}) \hat{\lambda}_{\tilde{\alpha}}(a)$$

For any $m+\tilde{m} \in \{0, 1, 2, \dots, n-2\}$

Connect to external data:

$$\lambda_{\alpha}^{(a)} = t_a \lambda_{\alpha}(\sigma_a) \quad \tilde{\lambda}_{\alpha}^{(a)} = \tilde{t}_a \tilde{\lambda}_{\alpha}(\sigma_a)$$

$$\tilde{\lambda}_{\alpha}^{(a)} K_{\alpha\alpha'}^{(a)} = t_a \tilde{t}_a \lambda_{\alpha}(\sigma_a) \tilde{\lambda}_{\alpha'}(\sigma_a)$$

Key: Momentum Conservation.

$$\sum_{a=1}^n K_{\alpha\alpha'}^{(a)} = 0 = \sum_{a=1}^n t_a \tilde{t}_a \lambda_{\alpha}(\sigma_a) \tilde{\lambda}_{\alpha'}(\sigma_a) = \sum_{m=0}^d \sum_{\tilde{m}=0}^{\tilde{d}} p_{m,\tilde{m}} \tilde{\lambda}_{\alpha'}(\sigma_a)$$

$\Rightarrow n-1$ equations to solve for $T_a \equiv t_a \tilde{t}_a$ in terms of σ_a .

Connect to external data:

$$\lambda_{\alpha}^{(a)} = t_a \lambda_{\alpha}(\sigma_a) \quad \tilde{\lambda}_{\alpha}^{(a)} = \tilde{t}_a \tilde{\lambda}_{\alpha}(\sigma_a)$$

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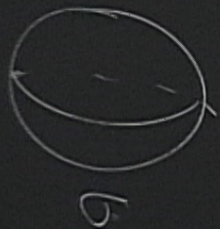
Key: Momentum Conservation.

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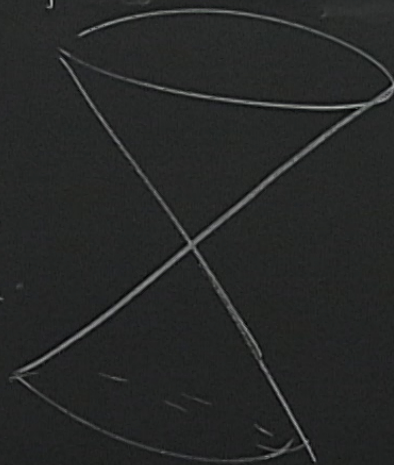
$\Rightarrow n-1$ equations to solve for $T_a = t_a \tilde{t}_a$ in terms of σ 's.

$$T_a = \frac{1}{\prod_{b \neq a} (\sigma_a - \sigma_b)}$$

New Maps



→
Degree $n-2$



Null cone in momentum space

$$P^M(\sigma) = p_0^M + p_1^M \sigma + \dots + p_{n-2}^M \sigma^{n-2}$$

Connect to data using.

momentum space

$$p_1^M \sigma + \dots + p_{n-2}^M \sigma^{n-2}$$

$$K^{(a)M} = \frac{P_1^M(\sigma_a)}{\prod_{\substack{b=1 \\ b \neq a}}^n (\sigma_a - \sigma_b)}$$

momentum space

$$p_1^M \sigma + \dots + p_{n-2}^M \sigma^{n-2}$$

Connect to data using.

$$K^{(a)M} = \frac{P_1^M(\sigma_a)}{\prod_{\substack{b=1 \\ b \neq a}}^n (\sigma_a - \sigma_b)}$$

Valid
for all
Dimensions D

Momentum Conservation (again)

$$\sum_{a=1}^n k_a^{\mu} = 0 = \sum_{a=1}^n \frac{P_a^{\mu}}{\Pi(\cdot)} = \sum_{m=0}^{n-2} p_m^{\mu} \left(\sum_{a=1}^n \frac{\sigma_a^m}{\prod_{b \neq a} (\sigma_a - \sigma_b)} \right) \stackrel{!}{=} 0 \quad \forall m \in \{0, \dots, n-2\}$$

Need:

Momentum Conservation (again)

$$\sum_{a=1}^n k_a^{\mu} = 0 = \sum_{a=1}^n \frac{P_a^{\mu}}{\Pi(\cdot)} = \sum_{m=0}^{n-2} p_m^{\mu} \left(\sum_{a=1}^n \frac{\sigma_a^m}{\prod_{b \neq a} (\sigma_a - \sigma_b)} \right) \quad \forall m \in \{0, \dots, n-2\}$$

$\stackrel{!}{=} 0$ → Not a condition
it is an identity

Proof:

Momentum Conservation (again)

$$\sum_{a=1}^n k_a^{\mu} = 0 = \sum_{a=1}^n \frac{P_a^{\mu}}{\prod_{b \neq a} (z - \zeta_b)} = \sum_{m=0}^{n-2} p_m^{\mu} \left(\sum_{a=1}^n \frac{\zeta_a^m}{\prod_{b \neq a} (\zeta_a - \zeta_b)} \right) \quad \forall m \in \{0, \dots, n-2\}$$

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Proof:

$$\frac{1}{2\pi i} \oint_{|z-\zeta_a|=\epsilon} dz \frac{z^m}{\prod_{b=1}^n (z - \zeta_b)} = \frac{\zeta_a^m}{\prod_{b \neq a} (\zeta_a - \zeta_b)}$$

Momentum Conservation (again)

$$\sum_{a=1}^n k_a^\mu = 0 = \sum_{a=1}^n \frac{P_a^\mu}{\prod_{b \neq a} (z - z_b)} = \sum_{m=0}^{n-2} p_m^\mu \left(\sum_{a=1}^n \frac{z_a^m}{\prod_{b \neq a} (z_a - z_b)} \right) \quad \forall m \in \{0, \dots, n-2\}$$

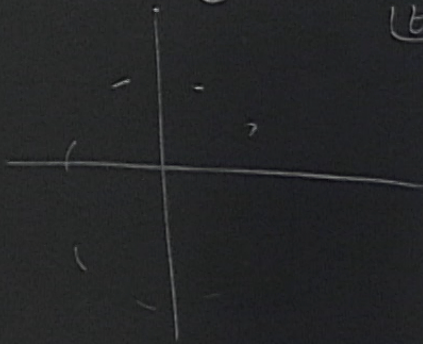
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$$\sum_{a=1}^n \frac{1}{2\pi i} \oint_{|z-z_a|=\epsilon} dz \frac{z^m}{\prod_{b=1}^n (z - z_b)} = \sum_{a=1}^n \frac{z_a^m}{\prod_{b \neq a} (z_a - z_b)}$$

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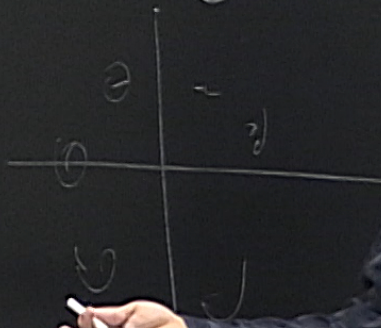
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$$\sum_{a=1}^n \frac{1}{2\pi i} \oint_{|z-\sigma_a|=\epsilon} dz \frac{z^m}{\prod_{b=1}^n (z - \sigma_b)} = \sum_{a=1}^n \frac{\sigma_a^m}{\prod_{b \neq a} (\sigma_a - \sigma_b)}$$


Momentum Conservation (again)

$$\sum_{a=1}^n k_a^{\mu} = 0 = \sum_{a=1}^n \frac{P_a^{\mu}}{\pi(\cdot)} = \sum_{m=0}^{n-2} p_m^{\mu} \left(\sum_{a=1}^n \frac{\sigma_a^m}{\prod_{b \neq a} (\sigma_a - \sigma_b)} \right) \quad \forall m \in \{0, \dots, n-2\}$$

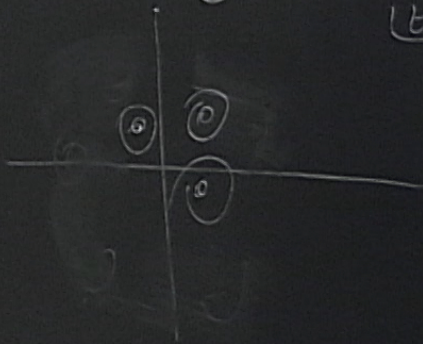
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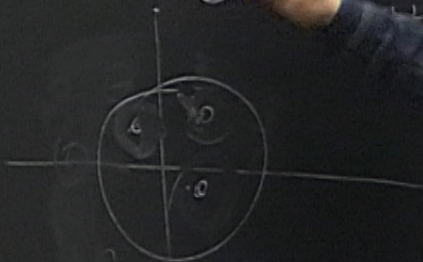
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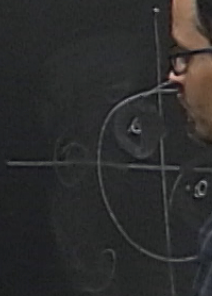
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Momentum Conservation (again)

$$\sum_{a=1}^n k_a^{\mu} = 0 = \sum_{a=1}^n \frac{P_a^{\mu}}{\pi(\cdot)} = \sum_{m=0}^{n-2} p_m^{\mu} \left(\sum_{a=1}^n \frac{\sigma_a^m}{\prod_{b \neq a} (\sigma_a - \sigma_b)} \right) \quad \forall m \in \{0, \dots, n-2\}$$

Not a condition
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L

$$\sum_{a=1}^n \frac{1}{2\pi i} \oint_{|z-\sigma_a|=\epsilon} dz \frac{z^m}{\prod_{b=1}^n (z-\sigma_b)} = \sum_{a=1}^n \frac{\sigma_a^m}{\prod_{b \neq a} (\sigma_a - \sigma_b)}$$

$$\oint_{\infty} dz \frac{z^m}{\pi(\cdot)} = \oint dz \frac{z^m}{z^n}$$


Momentum Conservation (again)

$$\sum_{a=1}^n k_a^{\mu} = 0 = \sum_{a=1}^n \frac{P_a^{\mu}}{\pi(\cdot)} = \sum_{m=0}^{n-2} p_m^{\mu} \left(\sum_{a=1}^n \frac{\zeta_a^m}{\prod_{b \neq a} (\zeta_a - \zeta_b)} \right) \quad \forall m \in \{0, \dots, n-2\}$$

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$$\sum_{a=1}^n \frac{1}{2\pi i} \oint_{|z-\zeta_a|=\epsilon} dz \frac{z^m}{\prod_{b=1}^n (z-\zeta_b)} = \sum_{a=1}^n \frac{\zeta_a^m}{\prod_{b \neq a} (\zeta_a - \zeta_b)}$$

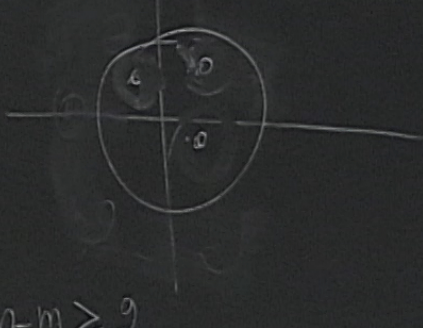
$$\oint_{\infty} dz \frac{z^m}{\pi(\cdot)} \rightarrow \oint_{\infty} dz \frac{z^m}{z^n} = 0 \quad \frac{1}{z^{n-m}} \quad \text{iff } n-m \geq 2$$


Momentum Conservation (again)

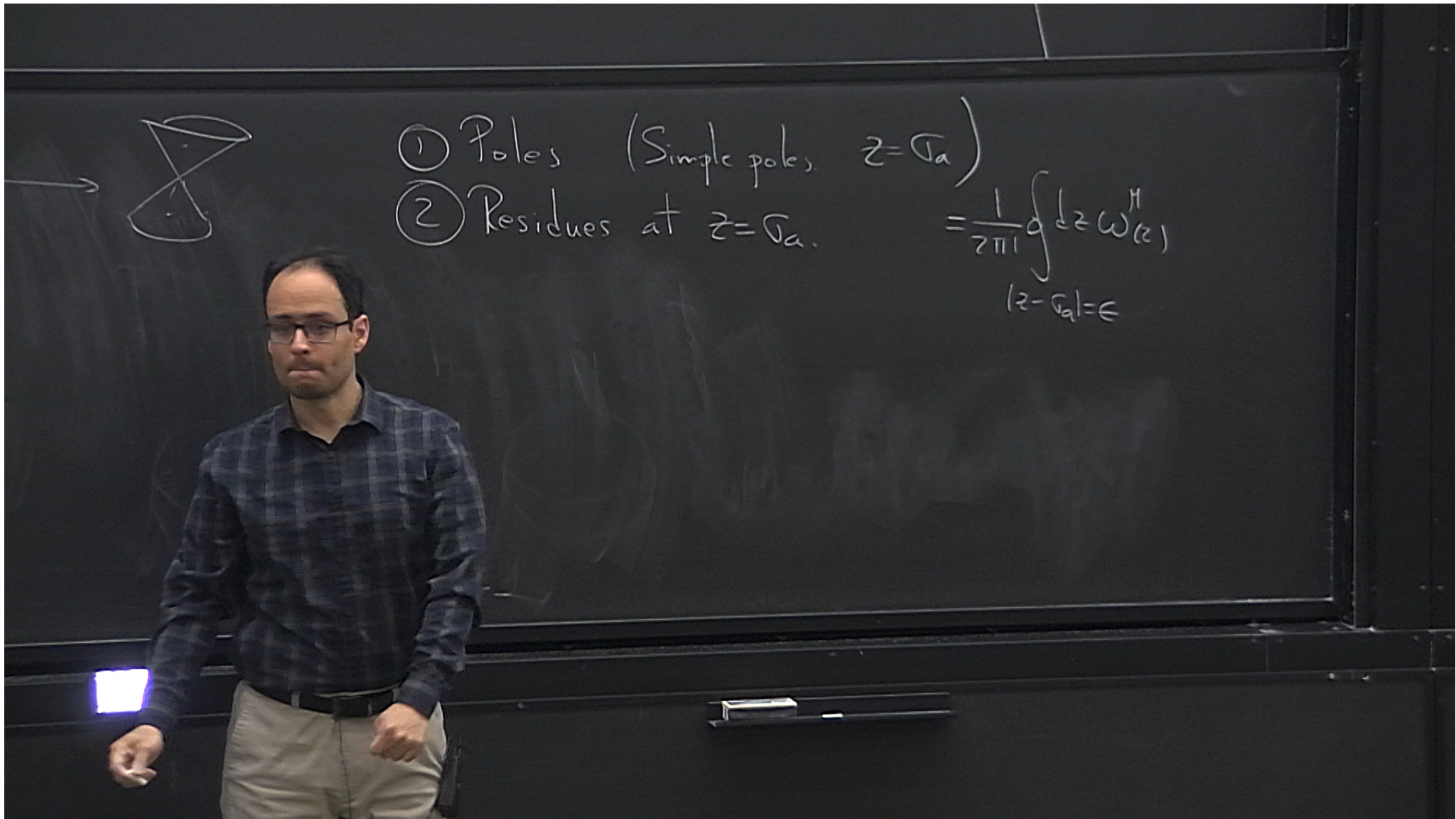
$$\sum_{a=1}^n k_a^m = 0 = \sum_{a=1}^n \frac{P_a^m}{\prod_{b \neq a} (z_a - z_b)} = \sum_{m=0}^{n-2} p_m^m \left(\sum_{a=1}^n \frac{z_a^m}{\prod_{b \neq a} (z_a - z_b)} \right) \quad \forall m \in \{0, \dots, n-2\}$$

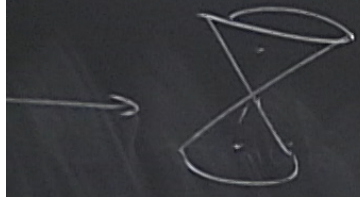
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$$\sum_{a=1}^n \frac{1}{2\pi i} \oint_{|z-z_a|=\epsilon} dz \frac{z^m}{\prod_{b=1}^n (z-z_b)} = \sum_{a=1}^n \frac{z_a^m}{\prod_{b \neq a} (z_a - z_b)}$$

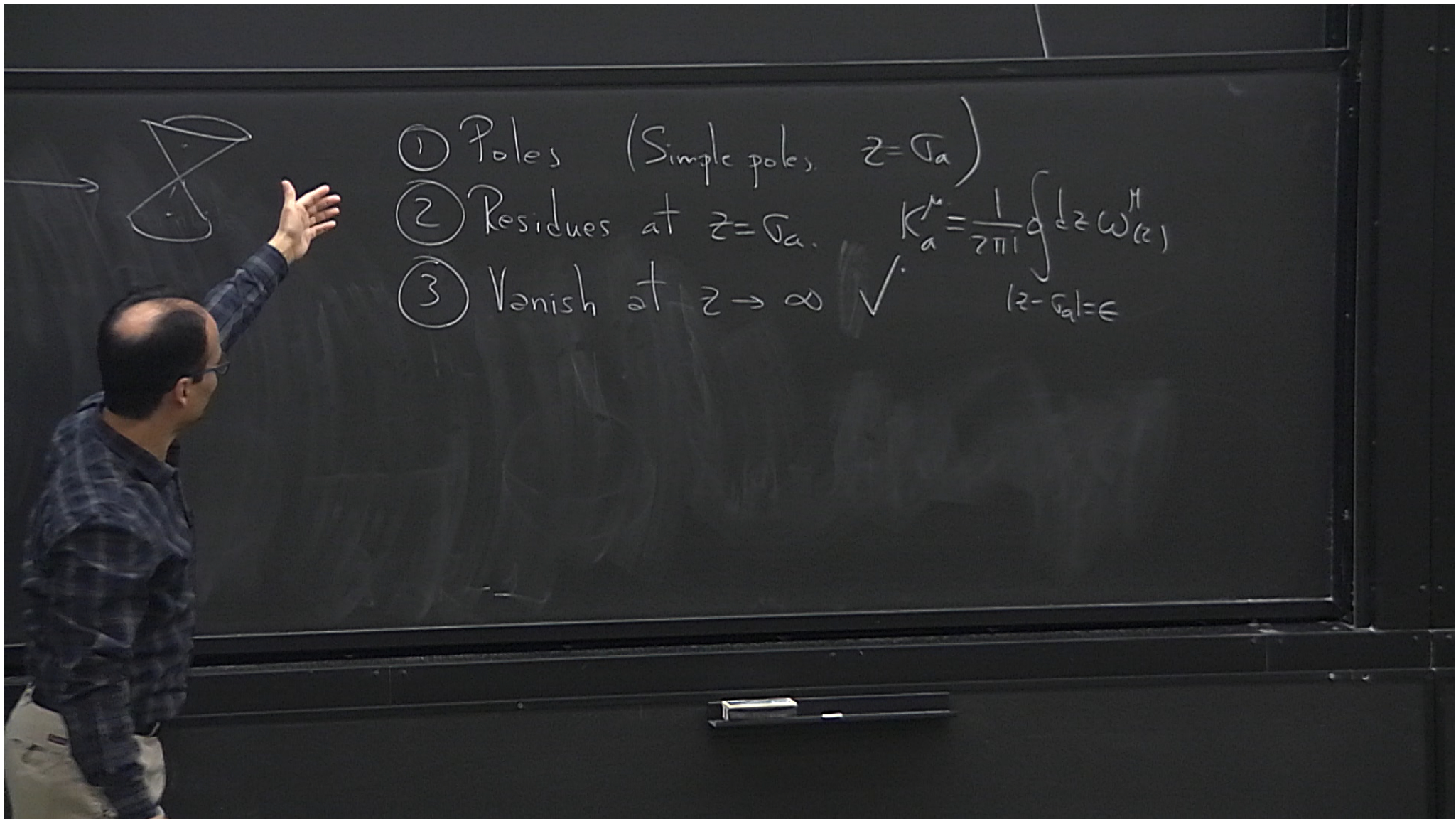
$$\oint_{\infty} dz \frac{z^m}{\prod_{b=1}^n (z-z_b)} = \oint_{\infty} dz \frac{z^m}{z^n} = 0 \quad \text{if } n-m \geq 2$$


$$\sum_{a=1}^n \oint_{|z-\zeta_a|=\epsilon} dz \left(\frac{P^M(z)}{\prod_{b=1}^n (z-\zeta_b)} \right) = 0 \quad \rightarrow \quad \omega^M(z) : z \rightarrow \text{figure-eight}$$

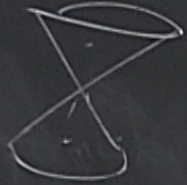




- ① Poles (Simple poles, $z = \sigma_a$)
- ② Residues at $z = \sigma_a$. $K_a^\mu = \frac{1}{2\pi i} \oint_{|z - \sigma_a| = \epsilon} dz \omega^\mu(z)$
- ③ Vanish at $z \rightarrow \infty$?



- ① Poles (Simple poles, $z = \zeta_a$)
- ② Residues at $z = \zeta_a$. $K_a^\mu = \frac{1}{2\pi i} \oint_{|z - \zeta_a| = \epsilon} dz \omega^\mu(z)$
- ③ Vanish at $z \rightarrow \infty$ ✓



① Poles (Simple poles, $z = \sigma_a$)

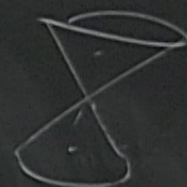
② Residues at $z = \sigma_a$. $K_a^\mu = \frac{1}{2\pi i} \oint_{|z - \sigma_a| = \epsilon} dz \omega^\mu(z)$

③ Vanish at $z \rightarrow \infty$ ✓ $|z - \sigma_a| = \epsilon$

$$\omega^\mu(z) = \sum_{a=1}^n \frac{K_a^\mu}{z - \sigma_a}$$

$$\sum_{a=1}^n \oint_{|z-\zeta_a|=\epsilon} dz \left(\frac{P^M(z)}{\prod_{b=1}^n (z-\zeta_b)} \right) = 0$$

$$\omega^M_{(z)} : z \longrightarrow$$

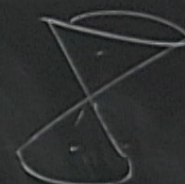


④ $M \rightarrow p$ to the null cone.

$$\omega^M_{(z)} \omega_{\mu(z)} = 0 \quad \forall z.$$

$$\sum_{a=1}^n \oint_{|z-\sigma_a|=\epsilon} dz \left(\frac{P^M(z)}{\prod_{b=1}^n (z-\sigma_b)} \right) = 0$$

$$\omega^M_{(z)} : z \longrightarrow$$



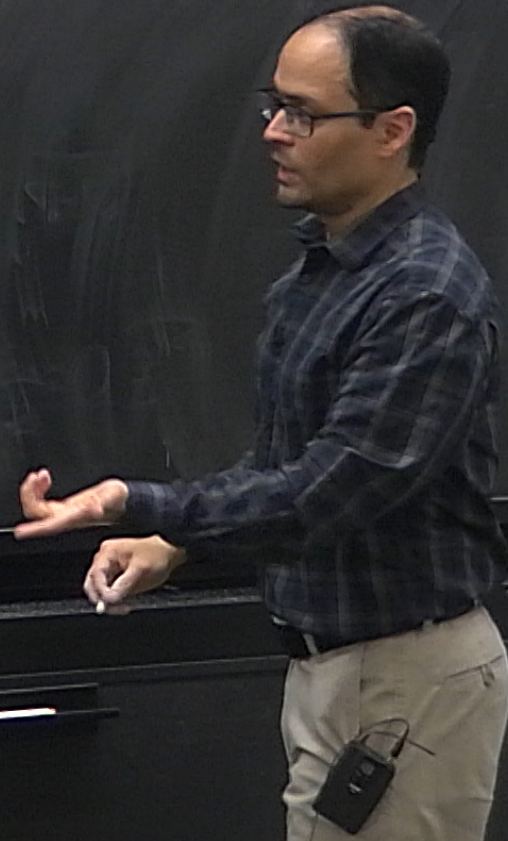
④ Map to the null cone.

$$\omega^M_{(z)} \omega_{\mu(z)} = 0 \quad \forall z.$$

Mandelstam Variable
 $S_{ab} = (k_a + k_b)^2$

$$\sum_{1 \leq a < b \leq n} \frac{2 k_a \cdot k_b}{(z-\sigma_a)(z-\sigma_b)} = 0$$

Given S_{ab} 's Find G 's s.t.
Residue at $z = G_a$ vanishes.



Given S_{ab} 's Find σ_a s.t.
Residue at $z=\sigma_a$ vanishes.

$$\sum_{\substack{b=1 \\ b \neq a}}^n \frac{S_{ab}}{(\sigma_a - \sigma_b)} = 0 \quad \forall a \in \{1, 2, \dots, n\}$$

Given S_{ab} 's Find G_a s.t.
 Residue at $z=G_a$ vanishes.

$$\sum_{\substack{b=1 \\ b \neq a}}^n \frac{S_{ab}}{(G_a - G_b)} = 0 \quad \forall a \in \{1, 2, \dots, n\}$$

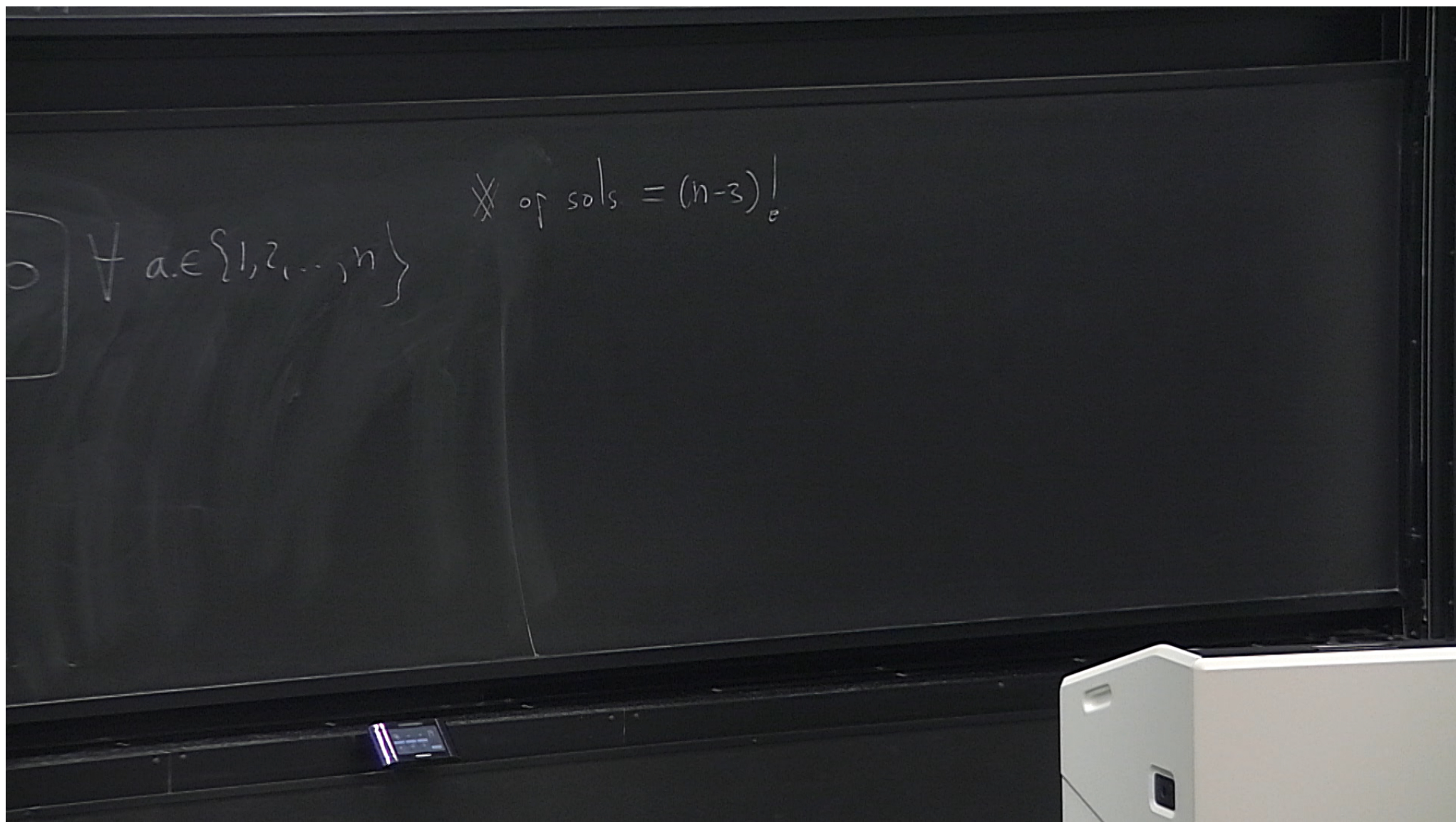
Scattering Equations.

(F.C., Song He, Ellis Yuan 2013)

PI.

PSI

m



Mandelstam
Variable

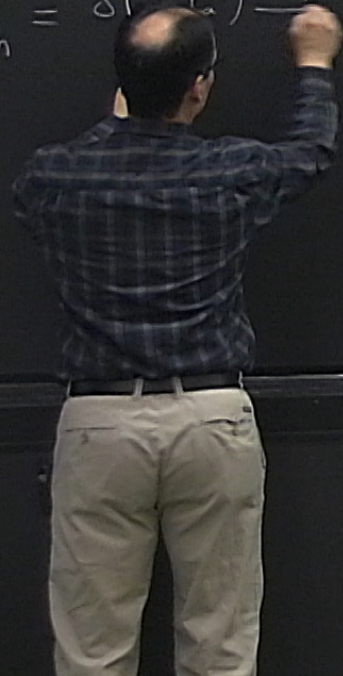
$$\omega(z)\omega_{\mu}(z)=0 \quad \forall z.$$

$$\sum_{1 \leq a < b \leq n} \frac{2k_a \cdot k_b}{(z-\sigma_a)(z-\sigma_b)} = 0$$

$$\omega(z) = \sum_{a=1}^n \frac{k_a}{z-\sigma_a}$$

Hope: These eqs could serve as the engine not only of Witten-RSV but also for any theory of massless particles.

$$t_n = \int^D \delta(\sum k^{\mu})$$



$$\prod_{1 \leq a < b \leq n} (z - \sigma_a)(z - \sigma_b) = 0$$

Hope: These eqs. could serve as the engine not only of Witten-RSV but also for

$$A_n = \delta(\sum k_a) \frac{1}{\text{Vol } SL(2, \mathbb{C})} \int \prod d\sigma_a \delta\left(\sum \frac{s_{ab}}{\sigma_a - \sigma_b}\right) \cdot \underbrace{\mathcal{I}(k, \epsilon, \sigma)}_{\text{Theory}}$$

→ Only a Poly. dep. on k^i, ϵ^i .

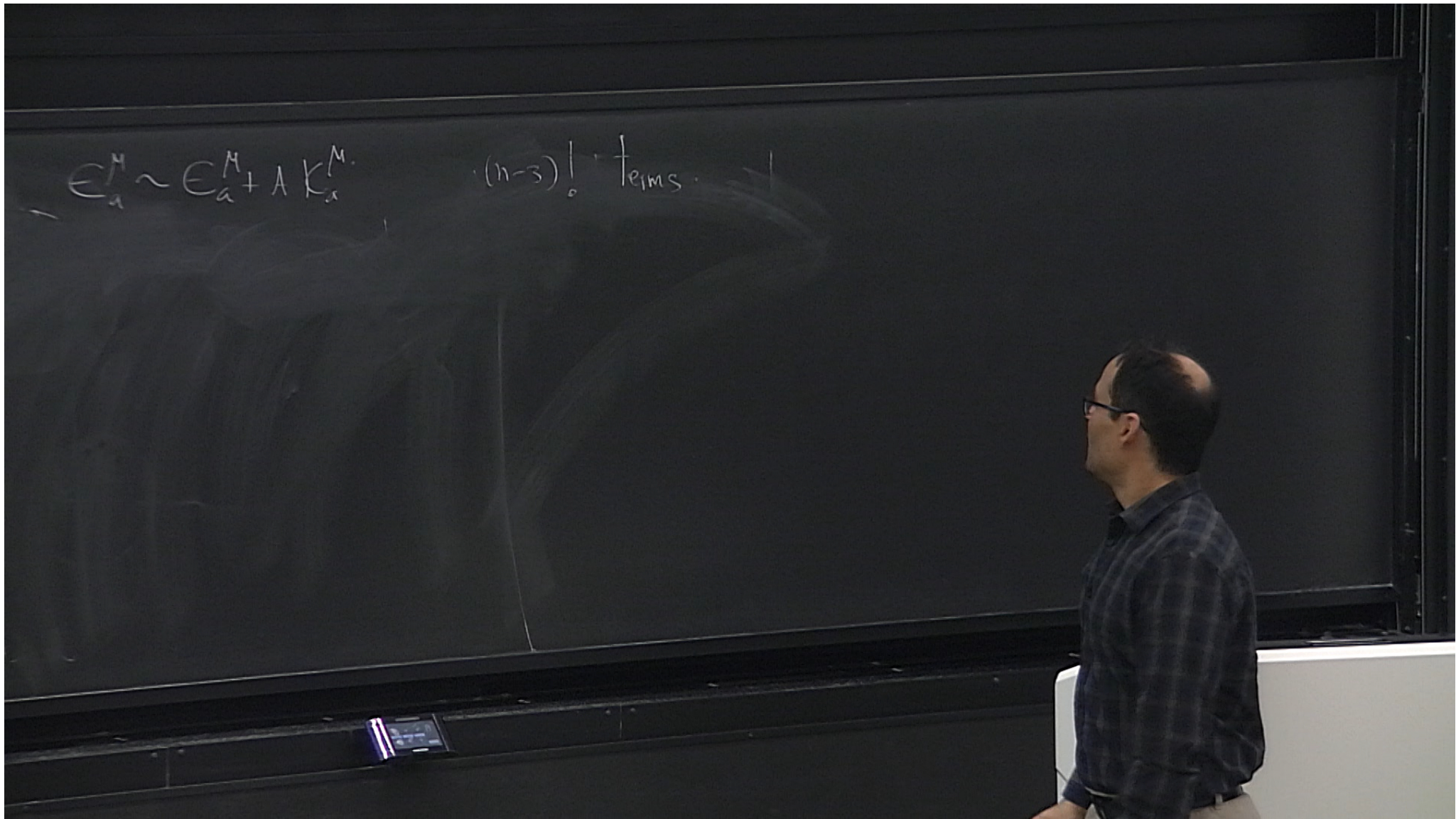
Hope: These eqs. could serve as the engine not only of Witten-RSV Theory.

$$A_n = \int^D (\sum k_a^\mu) \frac{1}{\text{Vol } SL(2, \mathbb{C})} \int \prod d\sigma_a \delta\left(\sum \frac{S_{ab}}{\sigma_a - \sigma_b}\right) \times \underbrace{\mathcal{I}(k, \epsilon, \sigma)}_{\rightarrow \text{Only a Poly. dep.}}$$

$$\underbrace{\mathcal{I}_{\text{Tot}}^{\text{Yang-Mills}}}_{\text{Total}} = \left(\frac{\text{tr}(T^{\sigma_1} T^{\sigma_2} \dots T^{\sigma_n})}{\underbrace{(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3) \dots (\sigma_n - \sigma_1)}_{\mathcal{I}_L}} + \text{perm} \right) \times \left(\mathcal{I}_R \right)$$

Want: $\tilde{\mathcal{L}}_R(k, \epsilon, \sigma)$ Gauge invariant $\rightarrow$

Went: $\widetilde{L}_R(k, \epsilon, \sigma)$ Gauge invariant $\rightarrow E_a^M \sim E_a^M + \Lambda K_a^M$



$\text{nt: } \tilde{\mathcal{L}}_R(k, \epsilon, \sigma)$ Gauge invariant $E_a^M \sim E_a^M + \Lambda K_a^M$ $(n-3)!$ terms.
 For each sol's to the S.E.'s.

nt: $\tilde{L}_R(k, E, \vec{\sigma})$ Gauge invariant $E_a^M \sim E_a^M + A K_a^M$ $(n-s)!$ terms.
 For each sol's to the S.E.'s.

$$\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \\ E \\ \vec{\sigma}_1 \\ \vdots \\ \vec{\sigma}_s \end{bmatrix} \quad 4 \times 2n$$

$$\tilde{L}_R(k, E, \vec{\sigma})$$

Gauge invariant

$$E_a^M \sim E_a^M + A K_a^M$$

For each sol to the S.E.'s

$$\begin{array}{c} k_1 \dots k_n \quad G_i \dots \quad E_b \\ \left[\begin{array}{c} 0 \\ \frac{k_2 k_1}{g_2 - g_1} \\ \vdots \\ \frac{k_n k_1}{g_n - g_1} \\ \hline E_1 \\ E_2 \\ \vdots \\ E_n \end{array} \right] \end{array}$$

42x42

$$\tilde{L}_R(k, E, G)$$

Gauge invariant

$$E_a^M \sim E_a^M + A K_a^M$$

For each sol's to the S.E.'s

$$\begin{array}{c} k_1 \dots k_n \\ \begin{array}{c} k_1 \\ k_2 \\ \vdots \\ k_n \end{array} \left[\begin{array}{c} 0 \\ \frac{k_2 k_1}{g_2 - g_1} 0 \\ \vdots \\ \frac{k_n k_1}{g_n - g_1} 0 \end{array} \right] \\ \begin{array}{c} E_1 \\ E_2 \\ \vdots \\ E_n \end{array} \left[\begin{array}{c} * \\ \frac{E_2 k_1}{g_2 - g_1} \\ \vdots \\ \frac{E_n k_1}{g_n - g_1} \end{array} \right] \end{array}$$

$$\tilde{L}_R(k, \epsilon, \sigma)$$

Gauge invariant

$$\epsilon_a^M \sim \epsilon_a^M + A K_a^M$$

For each sol's to the S.E.'s

k_1	$k_1 \dots k_n$	ϵ_1
k_1	$\begin{bmatrix} 0 \\ \frac{k_2 \cdot k_1}{\sigma_2 - \sigma_1} \\ \vdots \\ \frac{k_n \cdot k_1}{\sigma_n - \sigma_1} \end{bmatrix}$	$\begin{bmatrix} * \\ \frac{k_2 \cdot \epsilon_1}{\sigma_2 - \sigma_1} \\ \vdots \\ \frac{k_n \cdot \epsilon_1}{\sigma_n - \sigma_1} \end{bmatrix}$
ϵ_1	$\begin{bmatrix} * \\ \frac{\epsilon_2 \cdot \epsilon_1}{\sigma_2 - \sigma_1} \\ \vdots \\ \frac{\epsilon_n \cdot \epsilon_1}{\sigma_n - \sigma_1} \end{bmatrix}$	$\begin{bmatrix} 0 \\ \frac{\epsilon_2 \cdot \epsilon_1}{\sigma_2 - \sigma_1} \\ \vdots \\ \frac{\epsilon_n \cdot \epsilon_1}{\sigma_n - \sigma_1} \end{bmatrix}$

4x2n

Gauge invariant

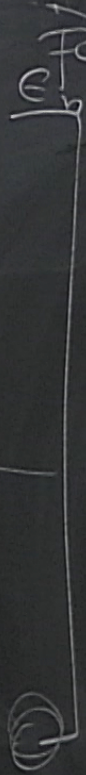
$$E_a^M \sim E_a^M + A K_a^M$$

$(n-3)!$ terms

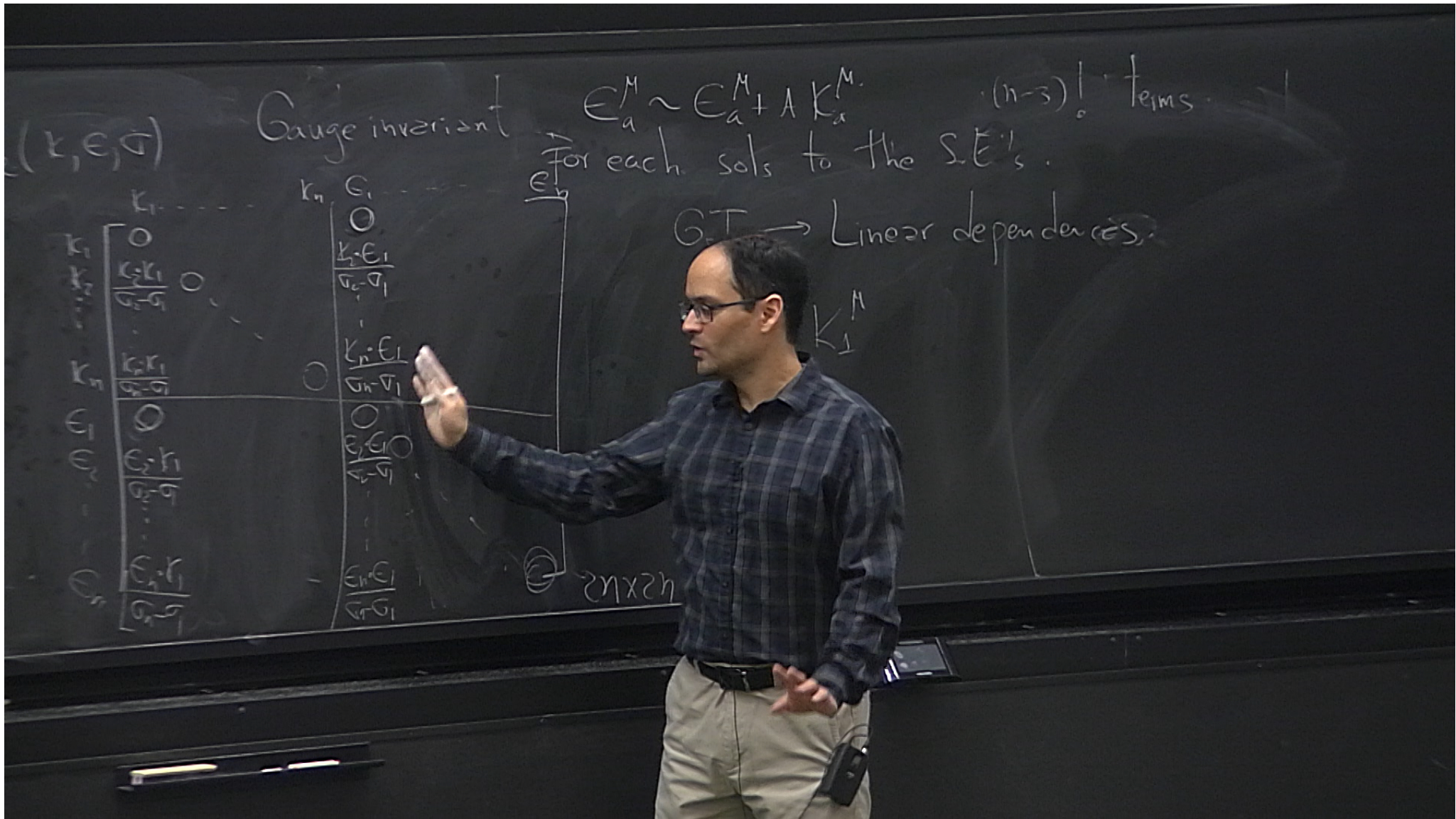
For each sol's to the S.E.'s.

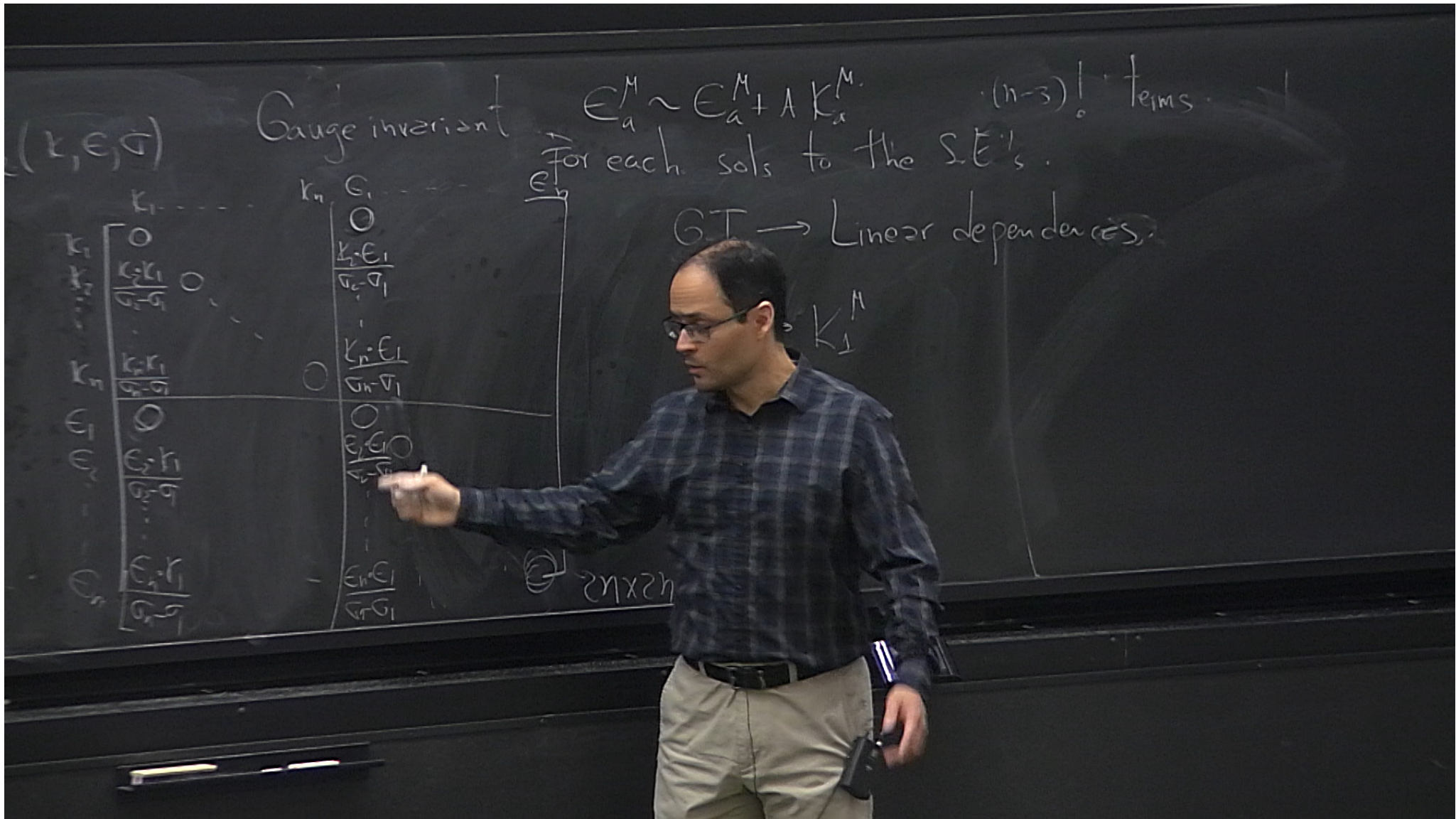
G.I. $\rightarrow$ Linear dependencies.

$$\begin{array}{c} k_n \cdot E_1 \\ * \\ \frac{k_2 \cdot E_1}{\nabla_2 \cdot \nabla_1} \\ \vdots \\ \frac{k_n \cdot E_1}{\nabla_n \cdot \nabla_1} \\ \circ \\ \frac{E_2 \cdot E_1}{\nabla_2 \cdot \nabla_1} \\ \vdots \\ \frac{E_n \cdot E_1}{\nabla_n \cdot \nabla_1} \end{array}$$



2n x 2n





Went: $\tilde{L}_R(k, \epsilon, \sigma)$ Gauge invariant $E_a^M \sim E_a^M + A K_a^M$ (n-s)

For each sol's to the S.E.'s.

$G \cdot B \rightarrow$ Linear dependence

$E_1^M \rightarrow K_1^M$

$\Psi =$

k_1	$\begin{bmatrix} 0 \\ \frac{k_2 k_1}{g_2 - g_1} \\ \vdots \\ \frac{k_n k_1}{g_n - g_1} \end{bmatrix}$	$\begin{bmatrix} 0 \\ \frac{k_2 \cdot \epsilon_1}{g_2 - g_1} \\ \vdots \\ \frac{k_n \cdot \epsilon_1}{g_n - g_1} \end{bmatrix}$
k_2	$\begin{bmatrix} \frac{k_2 k_1}{g_2 - g_1} \\ 0 \\ \vdots \\ \frac{k_n k_2}{g_n - g_2} \end{bmatrix}$	$\begin{bmatrix} \frac{k_2 \cdot \epsilon_1}{g_2 - g_1} \\ \frac{k_2 \cdot \epsilon_2}{g_2 - g_2} \\ \vdots \\ \frac{k_n \cdot \epsilon_2}{g_n - g_2} \end{bmatrix}$
$\vdots$	$\vdots$	$\vdots$
k_n	$\begin{bmatrix} \frac{k_n k_1}{g_n - g_1} \\ \vdots \\ \frac{k_n k_{n-1}}{g_n - g_{n-1}} \end{bmatrix}$	$\begin{bmatrix} \frac{k_n \cdot \epsilon_1}{g_n - g_1} \\ \vdots \\ \frac{k_n \cdot \epsilon_{n-1}}{g_n - g_{n-1}} \end{bmatrix}$
ϵ_1	$\begin{bmatrix} 0 \\ \frac{\epsilon_2 k_1}{g_2 - g_1} \\ \vdots \\ \frac{\epsilon_n k_1}{g_n - g_1} \end{bmatrix}$	$\begin{bmatrix} 0 \\ \frac{\epsilon_2 \cdot \epsilon_1}{g_2 - g_1} \\ \vdots \\ \frac{\epsilon_n \cdot \epsilon_1}{g_n - g_1} \end{bmatrix}$
ϵ_2	$\begin{bmatrix} \frac{\epsilon_2 k_1}{g_2 - g_1} \\ 0 \\ \vdots \\ \frac{\epsilon_n k_2}{g_n - g_2} \end{bmatrix}$	$\begin{bmatrix} \frac{\epsilon_2 \cdot \epsilon_1}{g_2 - g_1} \\ \frac{\epsilon_2 \cdot \epsilon_2}{g_2 - g_2} \\ \vdots \\ \frac{\epsilon_n \cdot \epsilon_2}{g_n - g_2} \end{bmatrix}$
$\vdots$	$\vdots$	$\vdots$
ϵ_n	$\begin{bmatrix} \frac{\epsilon_n k_1}{g_n - g_1} \\ \vdots \\ \frac{\epsilon_n k_{n-1}}{g_n - g_{n-1}} \end{bmatrix}$	$\begin{bmatrix} \frac{\epsilon_n \cdot \epsilon_1}{g_n - g_1} \\ \vdots \\ \frac{\epsilon_n \cdot \epsilon_{n-1}}{g_n - g_{n-1}} \end{bmatrix}$

$2n \times 2n$

$\tilde{L}_R(k, \epsilon, \sigma)$ Gauge invariant $\epsilon_a^M \sim \epsilon_a^M + A K_a^M$ $(n-3)!$ terms
 For each sol's to the S.E.'s.

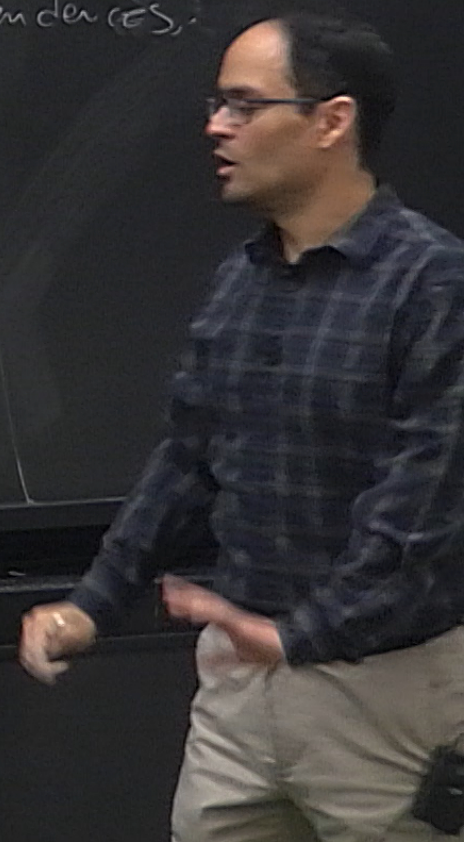
$G \cdot B_i \rightarrow$ Linear dependences.

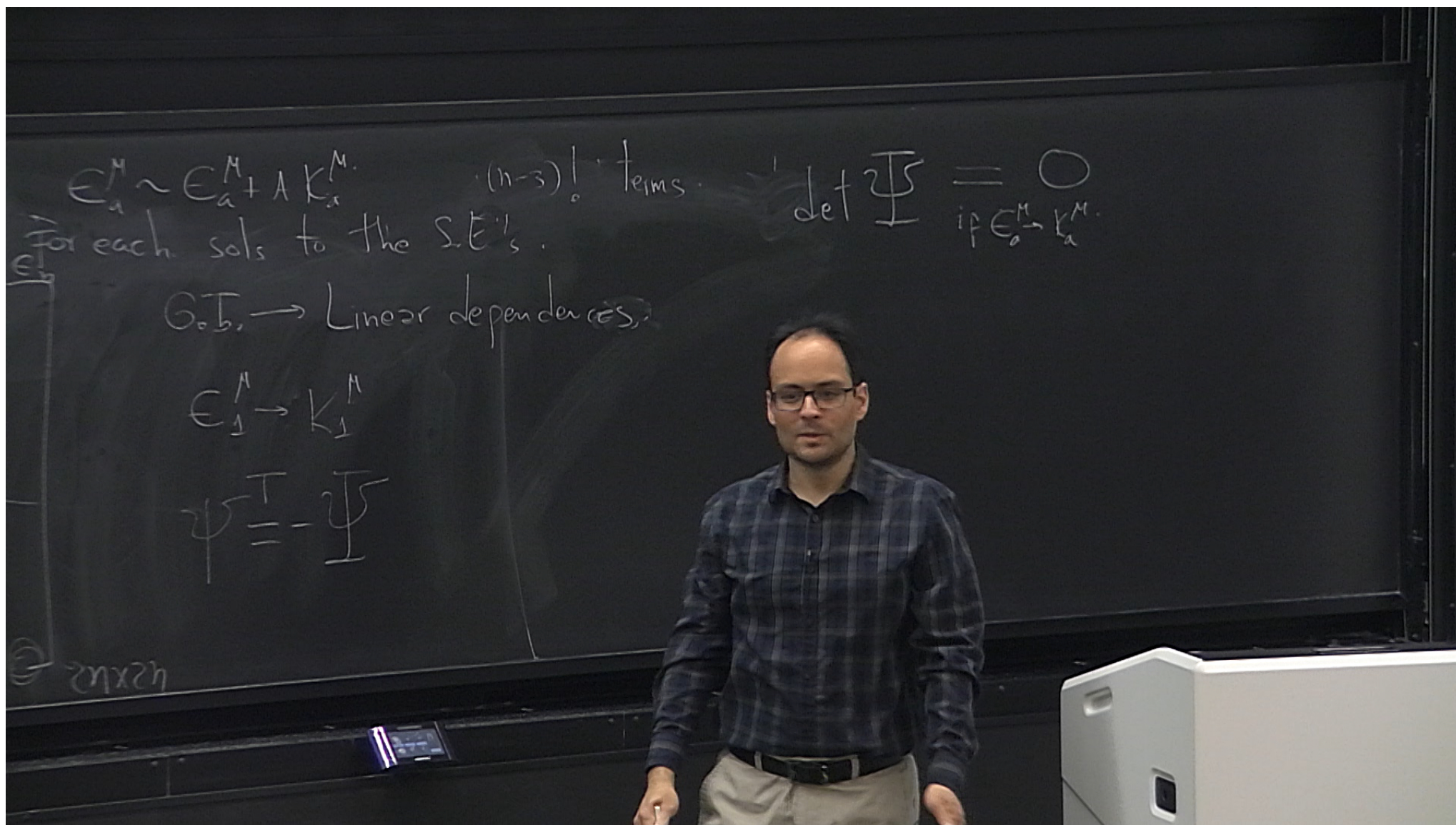
$\epsilon_1^M \rightarrow K_1^M$

$\psi^T = -\psi$

$2n \times 2n$

k_1	0	0
k_2	$\frac{k_2 \cdot k_1}{g_2 - g_1}$	$\frac{k_2 \cdot \epsilon_1}{g_2 - g_1}$
$\vdots$	0	$\vdots$
k_n	$\frac{k_n \cdot k_1}{g_n - g_1}$	$\frac{k_n \cdot \epsilon_1}{g_n - g_1}$
ϵ_1	0	0
ϵ_2	$\frac{\epsilon_2 \cdot k_1}{g_2 - g_1}$	$\frac{\epsilon_2 \cdot \epsilon_1}{g_2 - g_1}$
$\vdots$	$\vdots$	$\vdots$
ϵ_n	$\frac{\epsilon_n \cdot k_1}{g_n - g_1}$	$\frac{\epsilon_n \cdot \epsilon_1}{g_n - g_1}$





Hope: These eqs. could serve as the engine not only of Witten
Theory.

$$A_n = \int^D (\sum k_a^\mu) \frac{1}{\text{Vol } SL(2, \mathbb{C})} \int \prod d\sigma_a \delta\left(\sum \frac{S_{ab}}{\sigma_a - \sigma_b}\right) \times \underbrace{\mathcal{I}(k, \epsilon, \sigma)}_{\rightarrow \text{only poly.}}$$

$$\underbrace{\mathcal{I}}_{\text{Total}}^{\text{Yang-Mills}} = \left(\frac{\text{tr}(T^{\alpha_1} T^{\alpha_2} \dots T^{\alpha_n})}{\underbrace{(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3) \dots (\sigma_n - \sigma_1)}_{\mathcal{I}_L}} + \text{perm} \right) \times \left(\det \Psi \right)$$

Hope: These eqs could serve as the engine not only of Witten
Theory.

$$A_n = \int^D (\sum k_a^\mu) \frac{1}{\text{Vol } SL(2, \mathbb{C})} \int \prod d\sigma_a \delta\left(\sum \frac{S_{ab}}{\sigma_a - \sigma_b}\right) \times \underbrace{\mathcal{I}(k, \epsilon, \sigma)}_{\rightarrow \text{Only a Poly.}}$$

$$\underbrace{\mathcal{I}}_{\text{Total}}^{\text{Yang-Mills}} = \left(\frac{\text{tr}(T^{\alpha_1} T^{\alpha_2} \dots T^{\alpha_n})}{\underbrace{(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3) \dots (\sigma_n - \sigma_1)}_{\mathcal{I}_L}} + \text{perm} \right) \times \left(\det \Psi \right)$$

These eqs. could serve as the engine not only of Witten-RSV but also for

$$\left(\prod_a k_a^\mu \right) \frac{1}{\text{Vol} SL(2, \mathbb{C})} \int \prod_a d\sigma_a \delta \left(\sum \frac{s_{ab}}{\sigma_a - \sigma_b} \right) \times \underbrace{\mathcal{L}(k, \epsilon, \sigma)}_{\text{Theory}}$$

→ Only a Poly. dep. on k^μ, ϵ^μ .

$$= \left(\underbrace{\frac{\text{tr}(T^{\sigma_1} T^{\sigma_2} \dots T^{\sigma_n})}{(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3) \dots (\sigma_n - \sigma_1)} + \text{perm}}_{\mathcal{L}_L} \right) \times \left(\det \Psi \right) = \epsilon_1^\mu \epsilon_1^\nu \dots$$

not only of Witten-RSV but also for any theory of massless particles.

(k, ϵ, σ)

→ Only a Poly. dep. on k^μ, ϵ^μ .

$$\left(\begin{array}{c} \Psi \\ \vdots \end{array} \right) = \epsilon_i^\mu \epsilon_i^\nu \dots$$

$$M^T = -M$$

$$\det M_{4 \times 4} = (ab - cd + fg)^2$$

$$M_{6 \times 6} = \left(\begin{array}{ccccc} - & - & - & - & - \end{array} \right)^2$$

not only of Witten-RSV but also for any theory of massless particles.

(k, ϵ, σ)

→ Only a Poly. dep. on k^μ, ϵ^μ

$$\Psi = \epsilon^\mu_i \epsilon^\nu_j$$

$$M^T = -M$$

$$\det M_{4 \times 4} = (ab - cd + fg)^2$$

$$M_{6 \times 6} = \left(\underbrace{\quad}_{11'} \right)^2$$

Pfaffian M

Hope: These eqs. could serve as the engine not only of Witten's Theory

$$A_n = \delta(\sum k_a) \frac{1}{\text{Vol } SL(2, \mathbb{C})} \int \prod d\sigma_a \delta\left(\sum \frac{S_{ab}}{\sigma_a - \sigma_b}\right) \cdot \underbrace{\mathcal{I}(k, \epsilon, \sigma)}_{\text{Only a}}$$

$$\underbrace{\mathcal{I}_{\text{Total}}}_{\text{Yang-Mills}} = \left(\frac{\text{tr}(T^2 T^2 \dots T^2)}{(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3) \dots (\sigma_n - \sigma_1)} + \text{perm} \right) \times \left(\mathcal{P}_f \mathcal{I} \right)$$

$\underbrace{\hspace{10em}}_{\mathcal{L}_L}$

not only of Witten-RSV but also for any theory of massless particles.

(k, ϵ, σ)

→ Only a Poly. dep. on k^μ, ϵ^μ .

$$\Psi = \epsilon_1^\mu \epsilon_2^\nu \epsilon_3 \dots$$

$$M^T = -M$$

$$\det M_{4 \times 4} = (ab - cd + fg)^2$$

$$M_{6 \times 6} = \left(\underbrace{\quad\quad\quad}_{11'} \right)^2$$

Pfaffian M

These eqs. could serve as the engine not only of Witten-RSV but also for $\mathcal{N}=2$ Supergravity Theory.

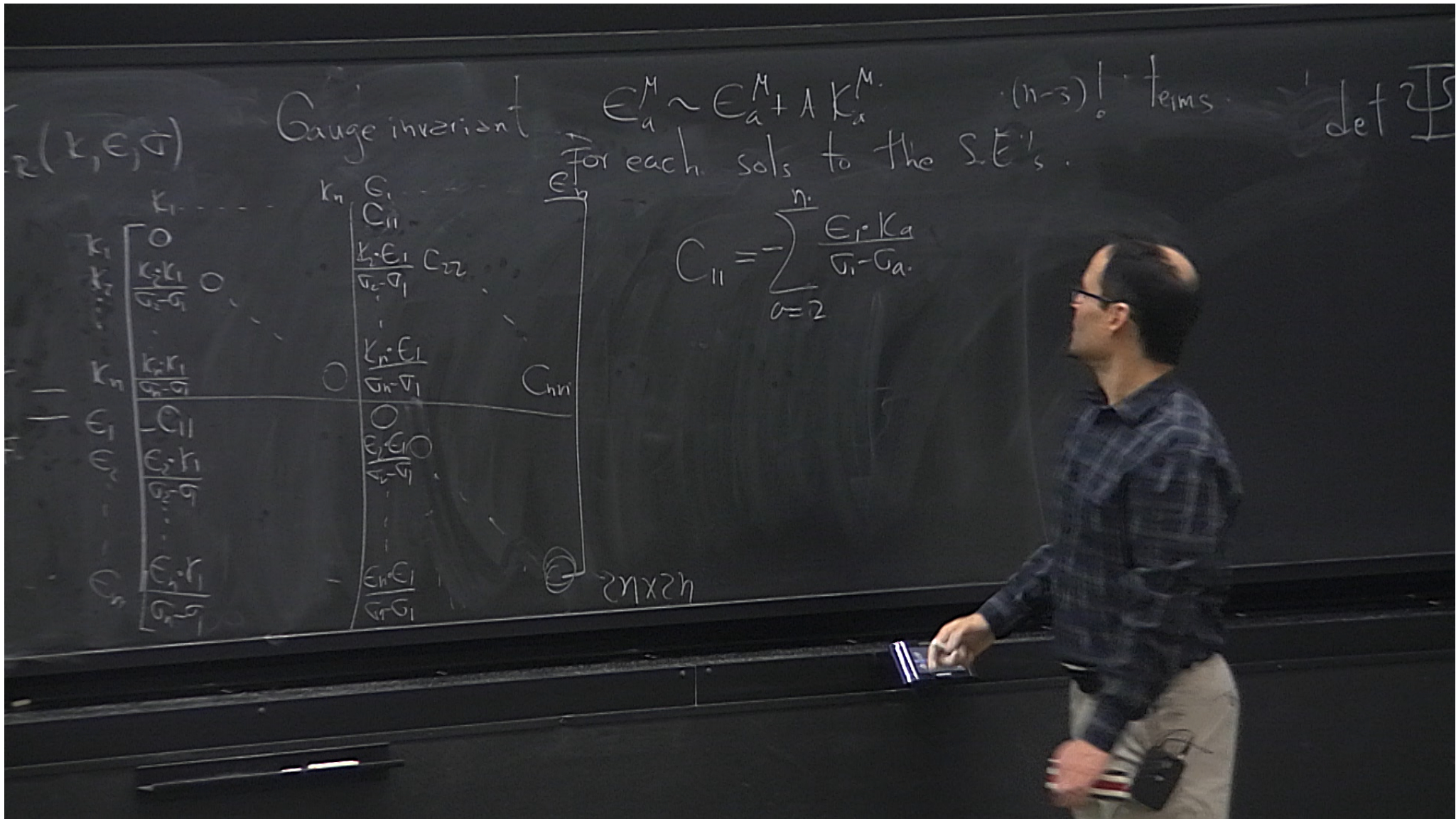
$$\sum k_a^m) \frac{1}{\text{Vol } SL(2, \mathbb{C})} \int \prod d\sigma_a \delta\left(\sum \frac{s_{ab}}{\sigma_a - \sigma_b}\right) \times \underbrace{\mathcal{L}(k, \epsilon, \sigma)}_{\rightarrow \text{Only a Poly. dep. on } k^m, \epsilon^m}$$

$$15. = \left(\frac{\text{tr}(T^{\alpha_1} T^{\alpha_2} \dots T^{\alpha_n})}{\underbrace{(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3) \dots (\sigma_n - \sigma_1)}_{\mathcal{L}_L}} + \text{perm} \right) \times \begin{pmatrix} P & \Psi \\ f & I \end{pmatrix} = \epsilon_1^{\mu_1} \epsilon_2^{\mu_2} \epsilon_3^{\mu_3} \dots$$

Want: $\tilde{L}_R(k, \epsilon, \sigma)$ Gauge invariant $E_a^M \sim E_a^M + A K_a^M$ (n-3)
 For each sol's to the S.E.'s.

$$\tilde{\Psi} = \begin{array}{c} k_1 \dots k_n \\ \begin{array}{c|c} \begin{array}{c} 0 \\ \frac{k_2 k_1}{g_2 - g_1} \circ \\ \vdots \\ \frac{k_n k_1}{g_n - g_1} \end{array} & \begin{array}{c} \epsilon_1 \\ C_{11} \\ \frac{k_1 \cdot \epsilon_1}{g_1 - g_1} \\ \vdots \\ \frac{k_n \cdot \epsilon_1}{g_n - g_1} \end{array} \\ \hline \begin{array}{c} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{array} & \begin{array}{c} -C_{11} \\ \frac{\epsilon_2 \cdot \epsilon_1}{g_2 - g_1} \\ \vdots \\ \frac{\epsilon_n \cdot \epsilon_1}{g_n - g_1} \end{array} \end{array} \quad \begin{array}{c} \epsilon_1 \\ C_{11} \\ \frac{k_1 \cdot \epsilon_1}{g_1 - g_1} \\ \vdots \\ \frac{k_n \cdot \epsilon_1}{g_n - g_1} \\ \hline \begin{array}{c} 0 \\ \frac{\epsilon_1 \cdot \epsilon_1}{g_1 - g_1} \\ \vdots \\ \frac{\epsilon_n \cdot \epsilon_1}{g_n - g_1} \end{array} \end{array}$$

(n x n)



$\Psi(k, E, \sigma)$

k_1	0	$\frac{k_1 \cdot E_1}{\sigma_1 - \sigma_1}$	0
k_2	$\frac{k_2 \cdot E_1}{\sigma_2 - \sigma_1}$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$
k_n	$\frac{k_n \cdot E_1}{\sigma_n - \sigma_1}$	0	0
E_1	$-C_{11}$	0	0
E_2	$\frac{E_2 \cdot E_1}{\sigma_2 - \sigma_1}$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$
E_n	$\frac{E_n \cdot E_1}{\sigma_n - \sigma_1}$	0	0

Gauge invariant

$E_a^M \sim E_a^M + A K_a^M$

For each sol's to the S.E.'s.

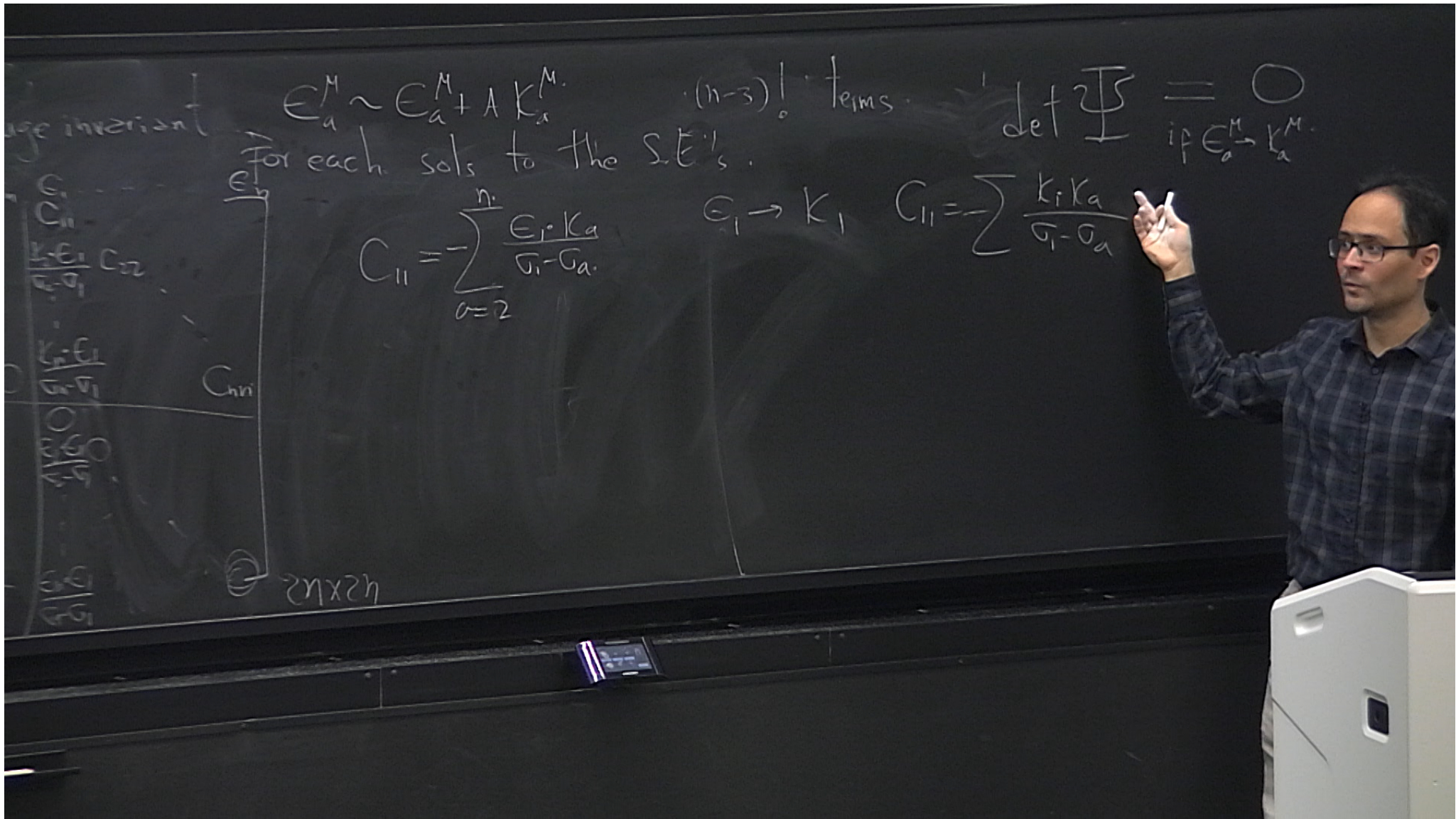
$C_{11} = - \sum_{a=2}^n \frac{E_1 \cdot K_a}{\sigma_1 - \sigma_a}$

$E_1 \rightarrow K_1$

$(n-3)!$ terms

$\det \Psi$

$2n \times 2n$



pe: These eqs. could serve as the engine not only of Witten-RSV but also

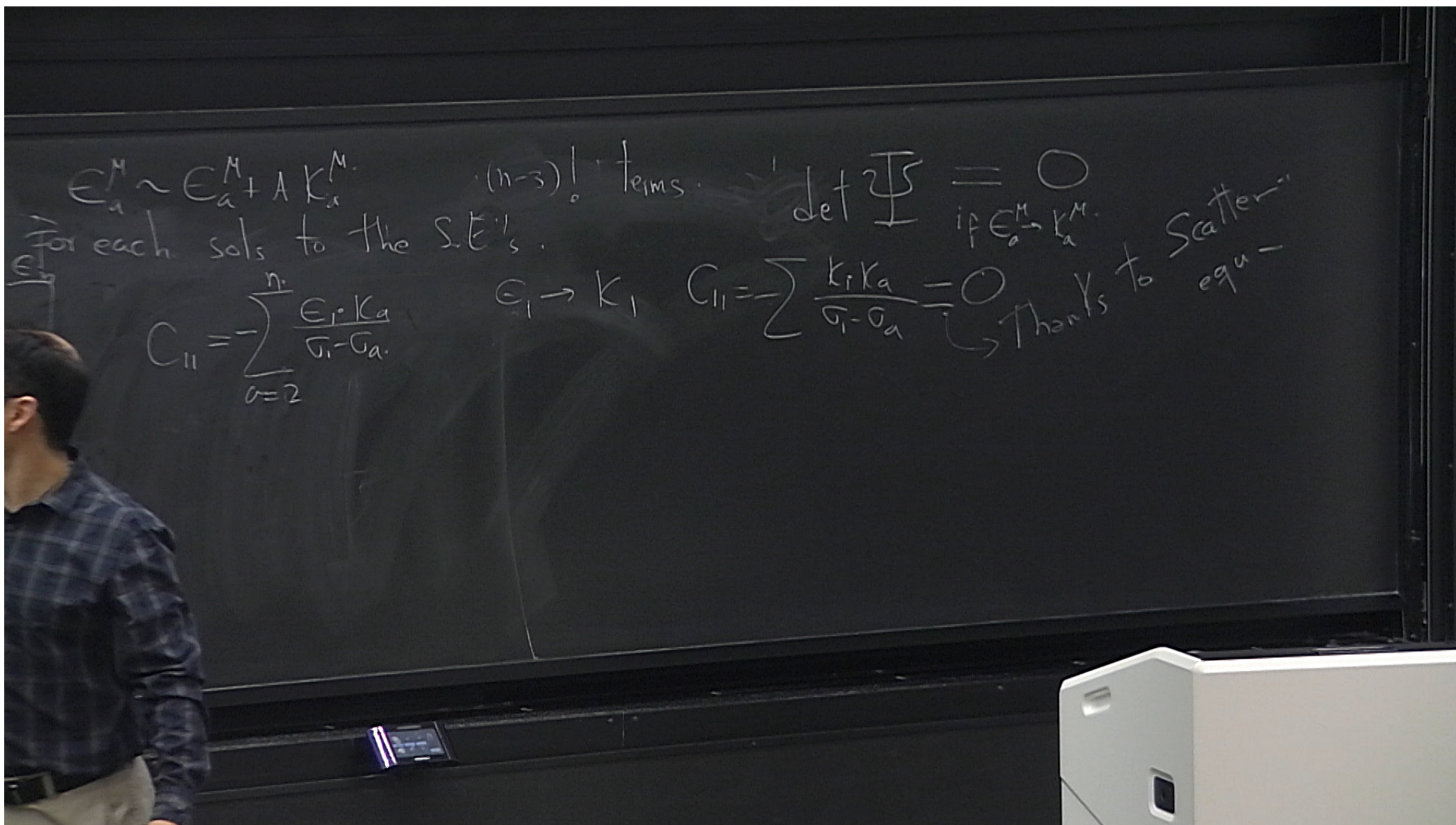
$$\int^D (\sum k_a) \frac{1}{\text{Vol } SL(2, \mathbb{C})} \int \prod d\sigma_a \delta\left(\sum \frac{s_{ab}}{\sigma_a - \sigma_b}\right) \times \underbrace{\mathcal{Z}(k, \epsilon, \sigma)}_{\text{Theory}}$$

→ Only a Poly. dep. on k^a, ϵ

Eng-Mills.

$$= \left(\frac{\text{tr}(T^2 T^2 \dots T^2)}{(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3) \dots (\sigma_n - \sigma_1)} + \text{perm} \right) \times \begin{pmatrix} P \\ f \\ \mathcal{I} \end{pmatrix} = \epsilon_1^1 \epsilon_2^2 \epsilon_3^3 \dots$$

$\underbrace{\hspace{10em}}_{\mathcal{L}_L}$



Went: $\tilde{L}_R(k, \epsilon, \sigma)$ Gauge invariant $E_a^M \sim E_a^M + A K_a^M$ (n-3)

For each sol's to the S.E.'s.

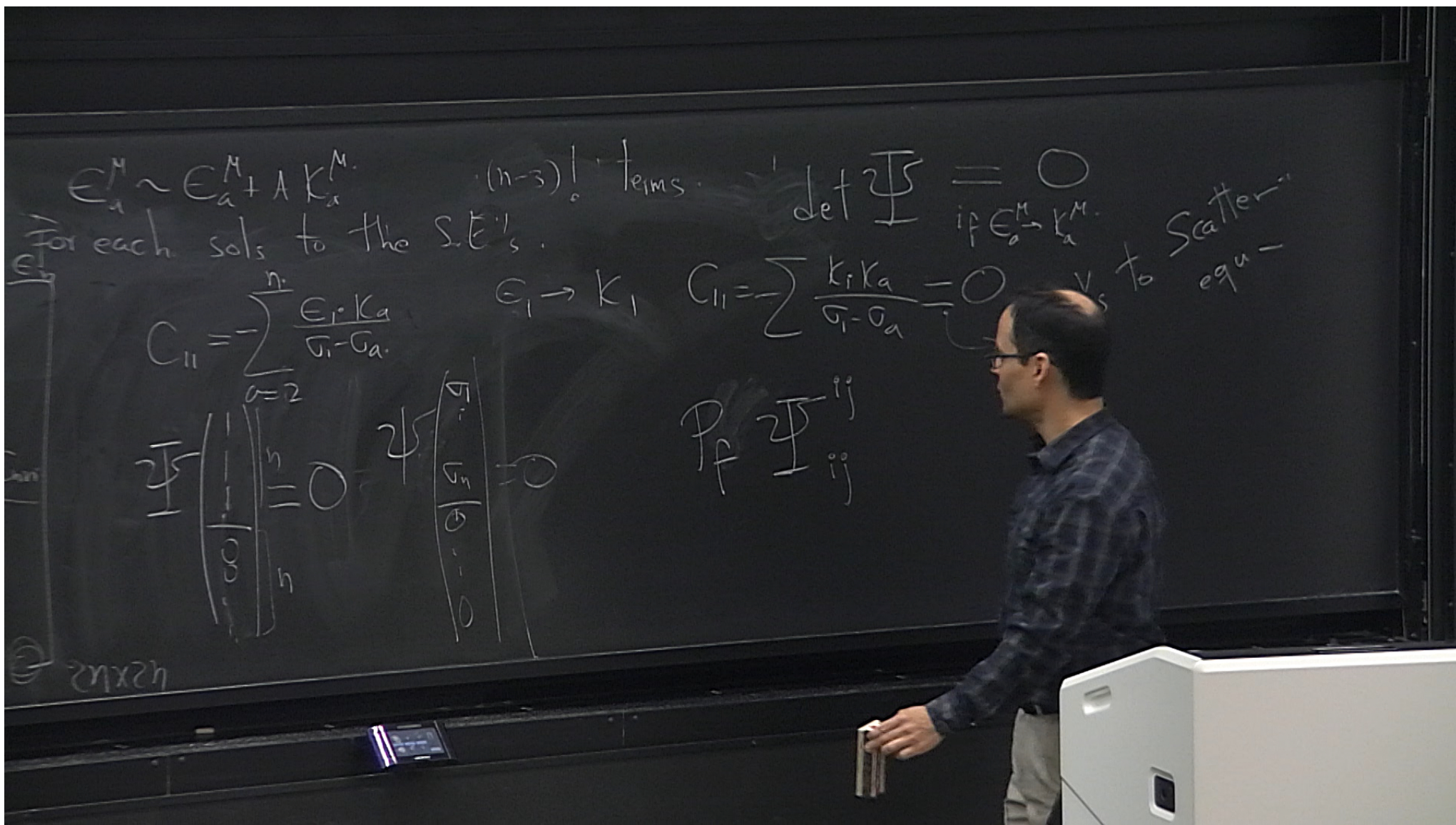
$$C_{11} = - \sum_{a=2}^n \frac{E_1 \cdot K_a}{\sigma_1 - \sigma_a}$$

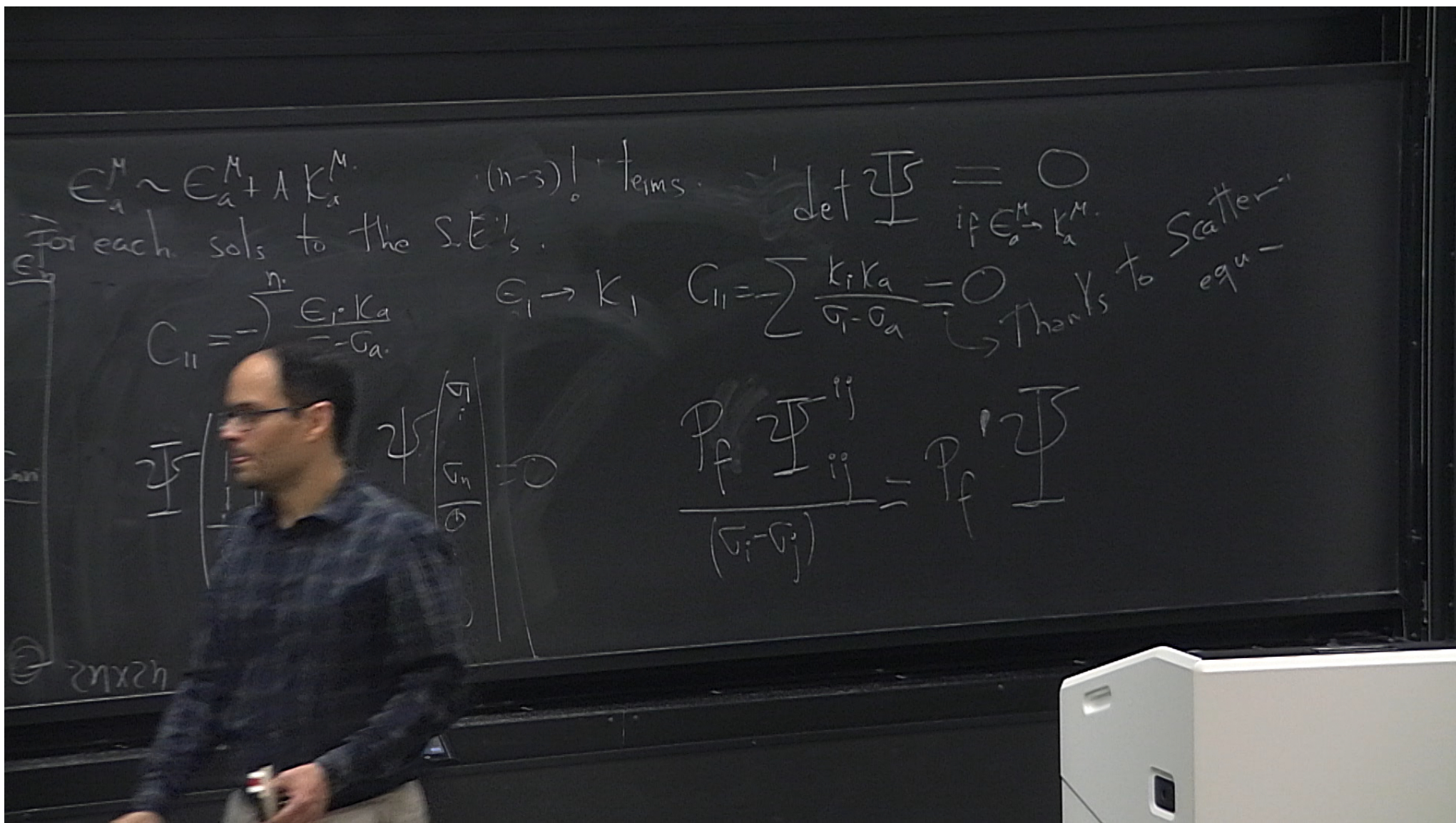
$$\psi = \begin{pmatrix} k_1 & \dots & k_n \\ \frac{k_1 k_1}{\sigma_1 - \sigma_1} & \dots & \frac{k_n k_1}{\sigma_n - \sigma_1} \\ \vdots & \ddots & \vdots \\ \frac{k_1 k_n}{\sigma_1 - \sigma_n} & \dots & \frac{k_n k_n}{\sigma_n - \sigma_n} \end{pmatrix} \begin{pmatrix} \epsilon_1 \\ C_{11} \\ \vdots \\ \epsilon_n \\ C_{nn} \end{pmatrix} = 0$$

$$\psi = \begin{pmatrix} \epsilon_1 \\ \vdots \\ \epsilon_n \end{pmatrix} = 0$$

$$\psi = \begin{pmatrix} \epsilon_1 \\ \vdots \\ \epsilon_n \end{pmatrix} = 0$$

2n x 2n





$$1 \leq a < b \leq n$$

These eqs. could serve as the engine not only of Witten-RSV but also for any theory.

$$\sum k_a^* \frac{1}{\text{Vol} SL(2, \mathbb{C})} \int \prod d\sigma_a \delta\left(\sum \frac{s_{ab}}{\sigma_a - \sigma_b}\right) \times \underbrace{\mathcal{L}(k, \epsilon, \sigma)}_{\text{Theory}}$$

→ Only a Poly. dep. on k^M, ϵ^M .

$$15. = \left(\frac{\text{tr}(T^{\sigma_1} T^{\sigma_2} \dots T^{\sigma_n})}{(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3) \dots (\sigma_n - \sigma_1)} + \text{perm} \right) \times \begin{pmatrix} P & \Psi \\ f & \Psi \end{pmatrix}$$

$\underbrace{\hspace{10em}}_{\mathcal{L}_L}$

Yang-Mills in any Dimension.

Hope: These eqs. could serve as the engine not only of
Theory.

$$A_n = \delta(\sum k_a^M) \left(\frac{1}{\text{Vol } SL(2, \mathbb{C})} \int \prod d\sigma_a \delta\left(\sum \frac{S_{ab}}{\sigma_a - \sigma_b}\right) \right) \times \underbrace{\mathcal{I}(k, \sigma)}_{\text{Yang-Mills.}}$$

$$\underbrace{\mathcal{I}_{\text{Total}}}_{\text{Yang-Mills.}} = \left(\frac{\text{tr}(T^{\alpha_1} T^{\alpha_2} \dots T^{\alpha_n})}{\underbrace{(\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3) \dots (\sigma_n - \sigma_1)}_{\mathcal{I}_L}} + \text{perm} \right) \times \left(\mathcal{P}_f \right)$$

It serve as the engine not only of Witten-RSV but also for any theory of n

$$\int \prod d\sigma_a \delta\left(\sum \frac{S_{ab}}{\sigma_a - \sigma_b}\right) \times \underbrace{\mathcal{L}(k, \epsilon, \sigma)}_{\text{Theory}} \Rightarrow \int \prod_{a \in \{1, \dots, k\}} d\sigma_a \prod_{a \in \{1, \dots, k\}} \delta\left(\sum \frac{S_{ab}}{\sigma_a - \sigma_b}\right)$$

→ Poly. dep. on k^M, ϵ^M .

$$\left(\frac{-T^{2n}}{(\sigma_n - \sigma_1)} + \text{perm} \right) \times \left(\mathcal{P}_f \Psi \right)$$

$\mathcal{L}_L$

Yang-Mills in
any Dimension.

$$M^T = -$$

$$\det M_{4 \times 4} =$$

$$M_{6 \times 6} =$$

not only of Witten-RSV but also for any theory of
 Theory.

$$L(k, \epsilon, \sigma) \Rightarrow \int \prod_{a \in \{i,j,k\}} d\sigma_a \prod_{a \in \{p,q,r\}} S(\sum \frac{S_{ab}}{\sigma_a - \sigma_b}) (\sigma_i - \sigma_j)(\sigma_j - \sigma_k)(\sigma_k - \sigma_i) \\ \times (\sigma_p - \sigma_q)(\sigma_q - \sigma_r)(\sigma_r - \sigma_p)$$

only a Poly. dep. on k^M, ϵ^M .

Yang-Mills in
 any Dimension.

$$M^T =$$

$$\det M_{4 \times 4} =$$

$$M_{6 \times 6}$$

not only of Witten-RSV but also for any theory of massless particles.

$$(k, \epsilon, \sigma) \Rightarrow \int \prod_{a \in \{1, \dots, r\}} d\sigma_a \prod_{a \in \{p, q, r\}} \delta\left(\sum \frac{\sigma_{ab}}{\sigma_a \sigma_b}\right) \frac{(\sigma_i - \sigma_j)(\sigma_j - \sigma_k)(\sigma_k - \sigma_i)}{(\sigma_i - \sigma_q)(\sigma_q - \sigma_r)(\sigma_r - \sigma_i)} \times \mathcal{L}_L \times \mathcal{L}_R$$

→ Only a Poly. dep. on k^μ, ϵ^μ .

Ψ Yang-Mills in any Dimension.

only of Witten-RSV but also for any theory of massless particles.

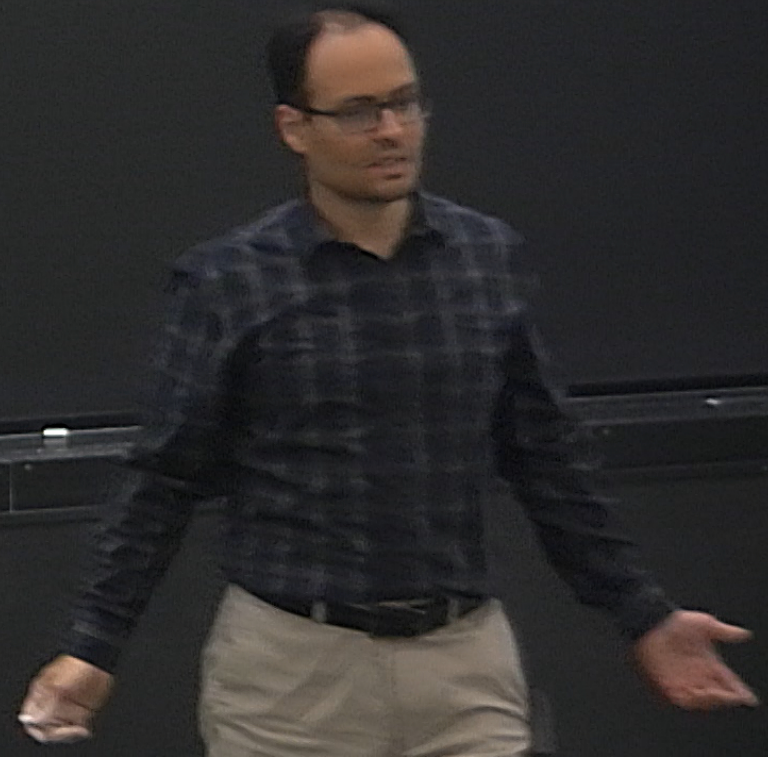
$$k, \epsilon, \sigma \Rightarrow \left(\prod_{a \in \{1, \dots, r\}} d\sigma_a \prod_{a \in \{p, q, r\}} \delta \left(\sum \frac{S_{ab}}{\sigma_a \sigma_b} \right) \frac{(\sigma_i - \sigma_j)(\sigma_i - \sigma_k)(\sigma_j - \sigma_k)}{(\sigma_i - \sigma_q)(\sigma_j - \sigma_q)(\sigma_k - \sigma_q)} \right) \times \mathcal{L}_L \times \mathcal{L}_R$$

→ Only a Poly. dep. on k^M, ϵ^M .

Yang-Mills in any Dimension.

Q: Could $\mathcal{I}_L$ & $\mathcal{I}_R$ be indep. units or building blocks of theories?

$$\mathcal{I}_{\frac{1}{2}} = \left(\frac{\text{tr}(T^2 \dots)}{(\sigma_1 - \sigma_2) \dots (\sigma_n - \sigma_1)} + \text{pen} \right) \left(\frac{\text{tr}(T^2 \dots)}{(\sigma_1 - \sigma_2) \dots} + \text{pen} \right)$$



Q: Could $\mathcal{I}_L$ & $\mathcal{I}_R$ be indep. units or building blocks of theories?

$$\mathcal{I}_{\{L,R\}} = \left(\frac{\text{tr}(T^2 \dots)}{(\sigma_1 - \sigma_2) \dots (\sigma_n - \sigma_1)} + \text{pen.} \right) \left(\frac{\text{tr}(T^2 \dots)}{(\sigma_1 - \sigma_2) \dots} + \text{pen.} \right)$$

Scalar (cubic interaction) $\mathcal{L}_{\text{int}} = f_{abc} f_{abc} \phi^{aa} \phi^{bb} \phi^{cc}$

Biadjoint $U(N) \times U(\tilde{N})$

Q: Could $\mathcal{L}_L$ & $\mathcal{L}_R$ be indep. units or building blocks of the

$$\mathcal{L}_{\text{total}} = \left(\frac{\text{tr}(T^2 \dots)}{(\sigma_1 - \sigma_2) \dots (\sigma_n - \sigma_1)} + \text{pen.} \right) \left(\frac{\text{tr}(T^2 \dots)}{(\sigma_1 - \sigma_2) \dots} + \text{pen.} \right)$$

Scalar
(cubic interaction) $\mathcal{L}_{\text{int}} = f_{abc} f_{abc} \phi^a \phi^b \phi^c$
Biadjoint.

$$\mathcal{L}_{\text{Total}} = P_f^{\psi} \mathcal{L}(k, \epsilon, \sigma) \times P_f^{\psi} \mathcal{L}(k, \epsilon, \sigma)$$

$\hat{T}_R$ be indep. units or building blocks of theories?

$$\left(\frac{\text{tr}(T^2 \dots)}{(G_1 G_2) \dots} + \text{pen.} \right)$$

Scalar
 (cubic interaction)

$$\mathcal{L}_{\text{int}} = f_{abc} f_{abc} \phi^{aa} \phi^{bb} \phi^{cc}$$

Biadjoint

$$U(N) \times U(\hat{N})$$

Tensor

$$E_i^\mu E_i^\nu \rightarrow E_j^{\mu\nu}$$

$(\epsilon, \sigma) \times P_f^{1/25} |(\epsilon, \sigma) \Rightarrow$

$\mathcal{I}_R$ be indep. units or building blocks of theories?
 $\left(\frac{\text{tr}(T^2 \dots)}{(G_1 G_2) \dots} + \text{pen.} \right)$
 Scalar (cubic interaction)
 $\mathcal{L}_{\text{int}} = f_{abc} f_{abc} \phi^{aa} \phi^{bb} \phi^{cc}$
 Bifundamental
 $U(N) \times U(\hat{N})$

$(\epsilon, \sigma) \times P_f \psi_{(k \in \sigma)} \Rightarrow \epsilon^\mu_i \epsilon^\nu_i \rightarrow \epsilon^\mu_\nu \Rightarrow$ Einstein Gravity,
 Tensor