Title: PSI 2017/2018 - Scattering Amplitudes in QFT & String Theory - Lecture 3

Date: Apr 11, 2018 11:30 AM

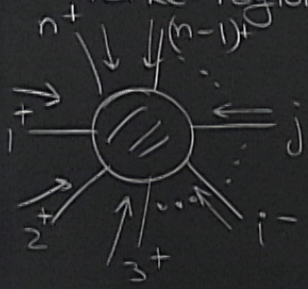
URL: http://pirsa.org/18040037

Abstract:

Lecture III

$$\{k_a^\mu, \epsilon_a^\mu, a_a\} \to \{\lambda_\alpha^{(a)}, \tilde{\lambda}_{\dot{\alpha}}^{(a)}, h_a, a_a\}$$

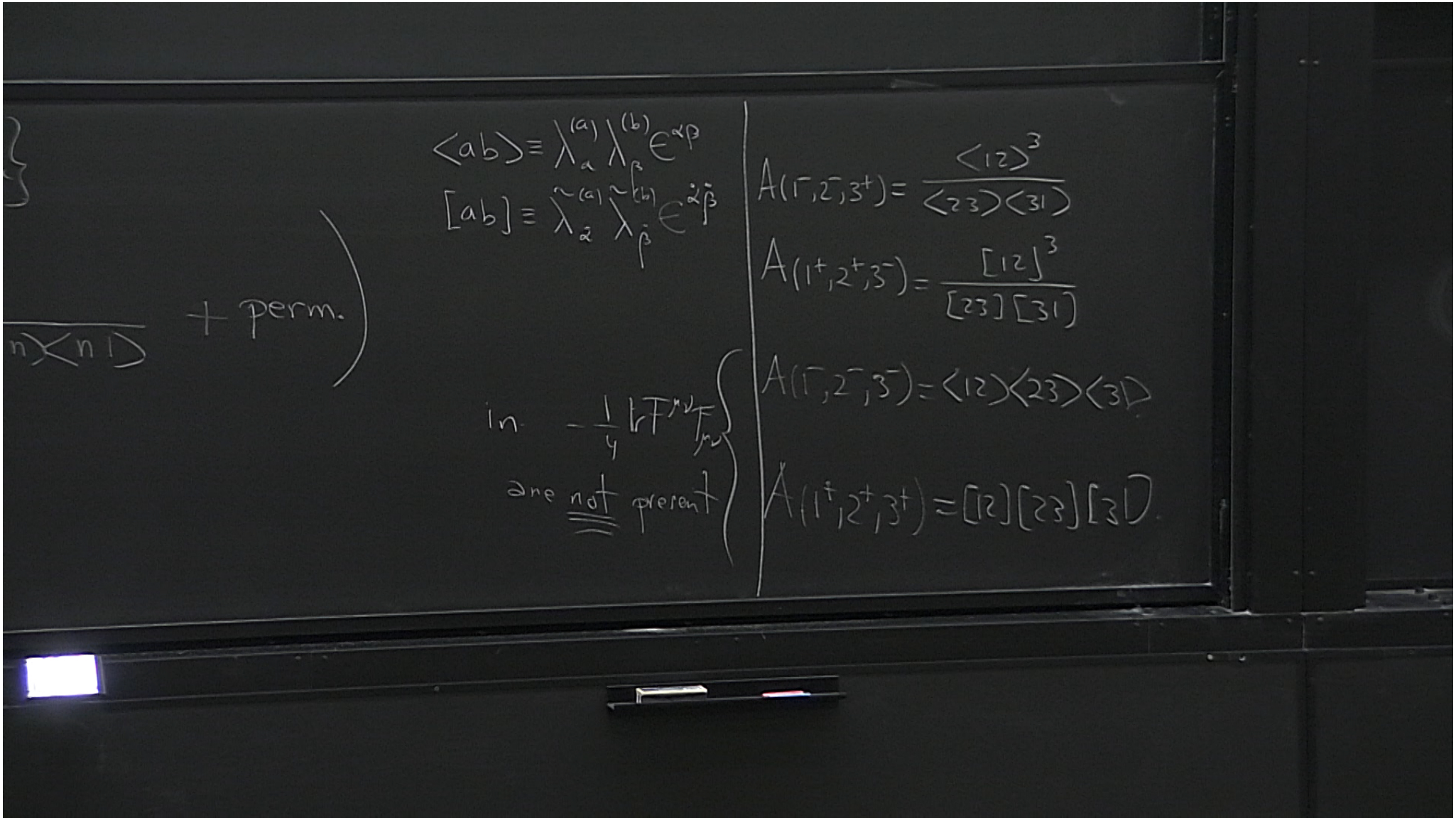
Parke-Taylor Formula

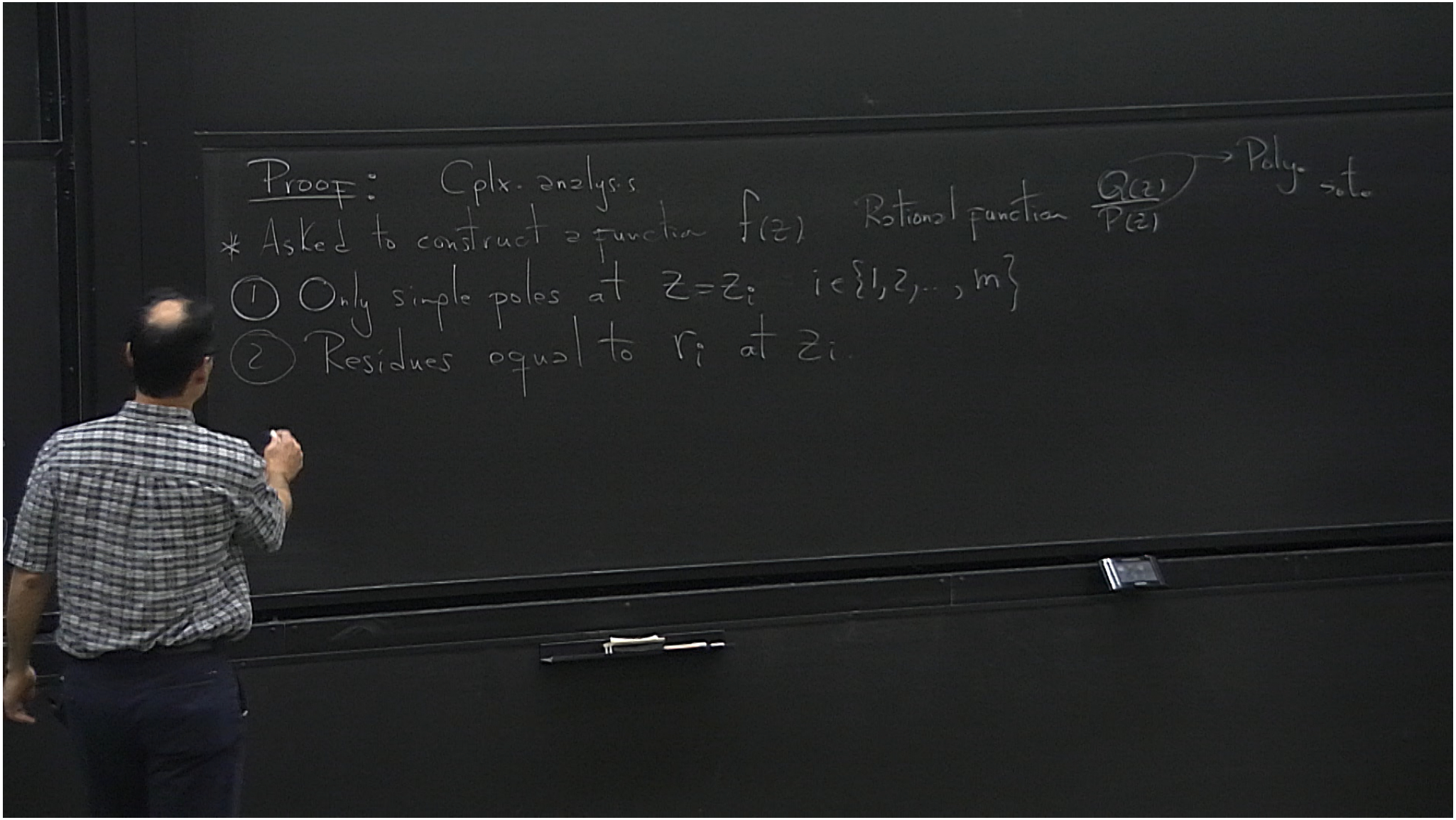


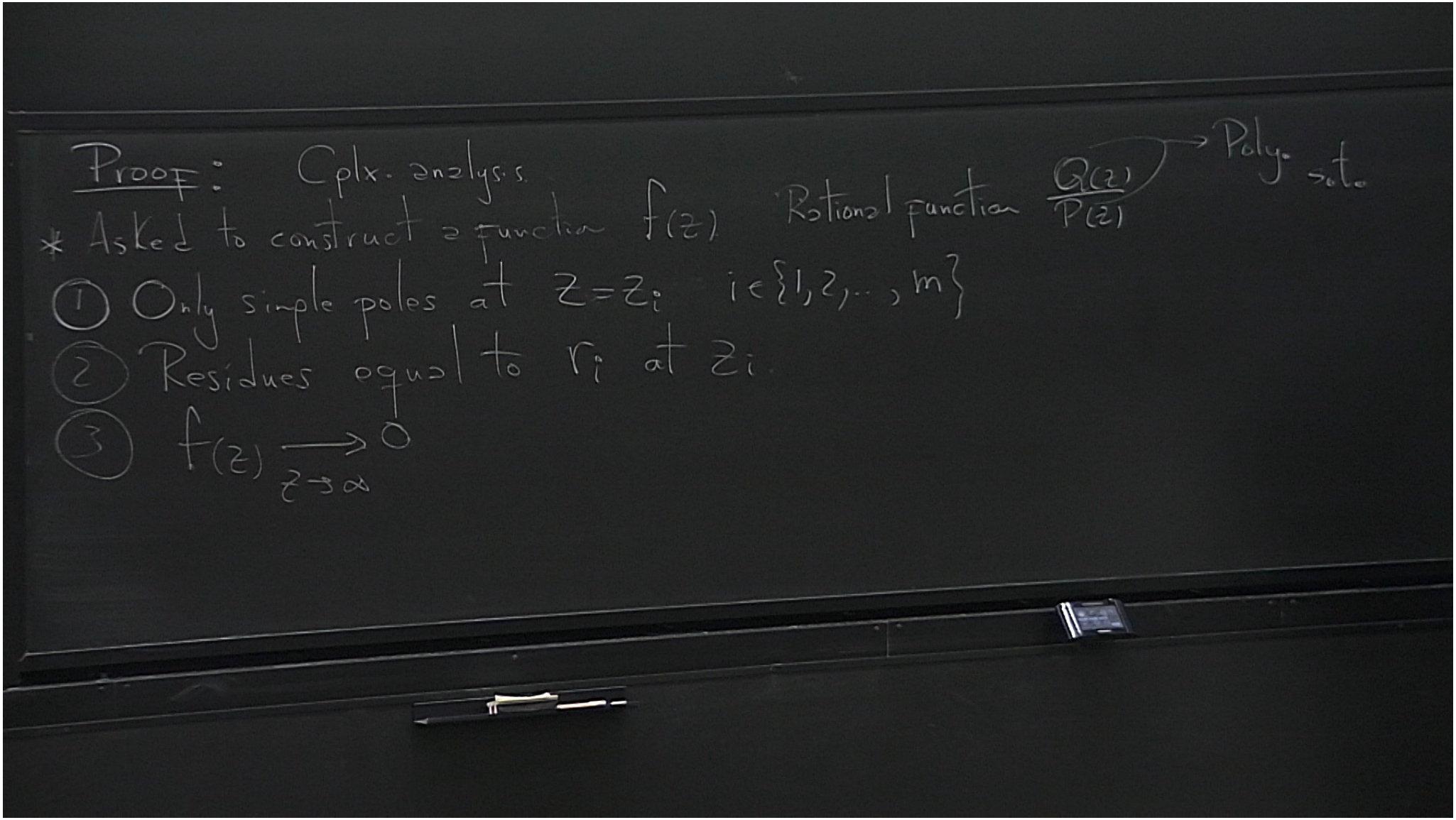
$$= j^- = \delta^4\left(\sum_{a=1}^{n} k_a^\mu\right)\left(\mathrm{tr}(T^{a_1}T^{a_2}\cdots T^{a_n})\frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n-1\ n\rangle\langle n1\rangle} + \text{perm.}\right)$$

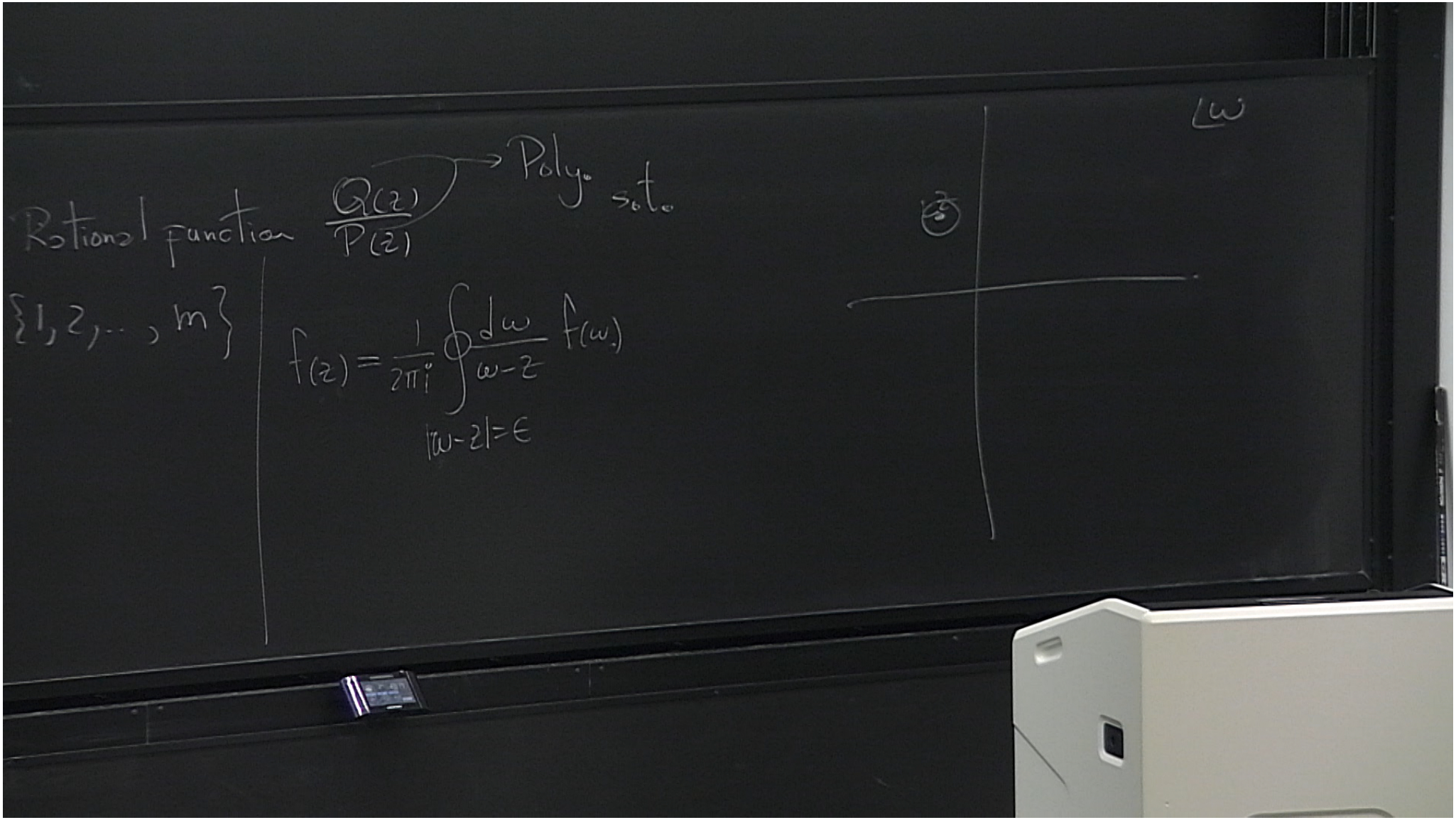
All incoming.

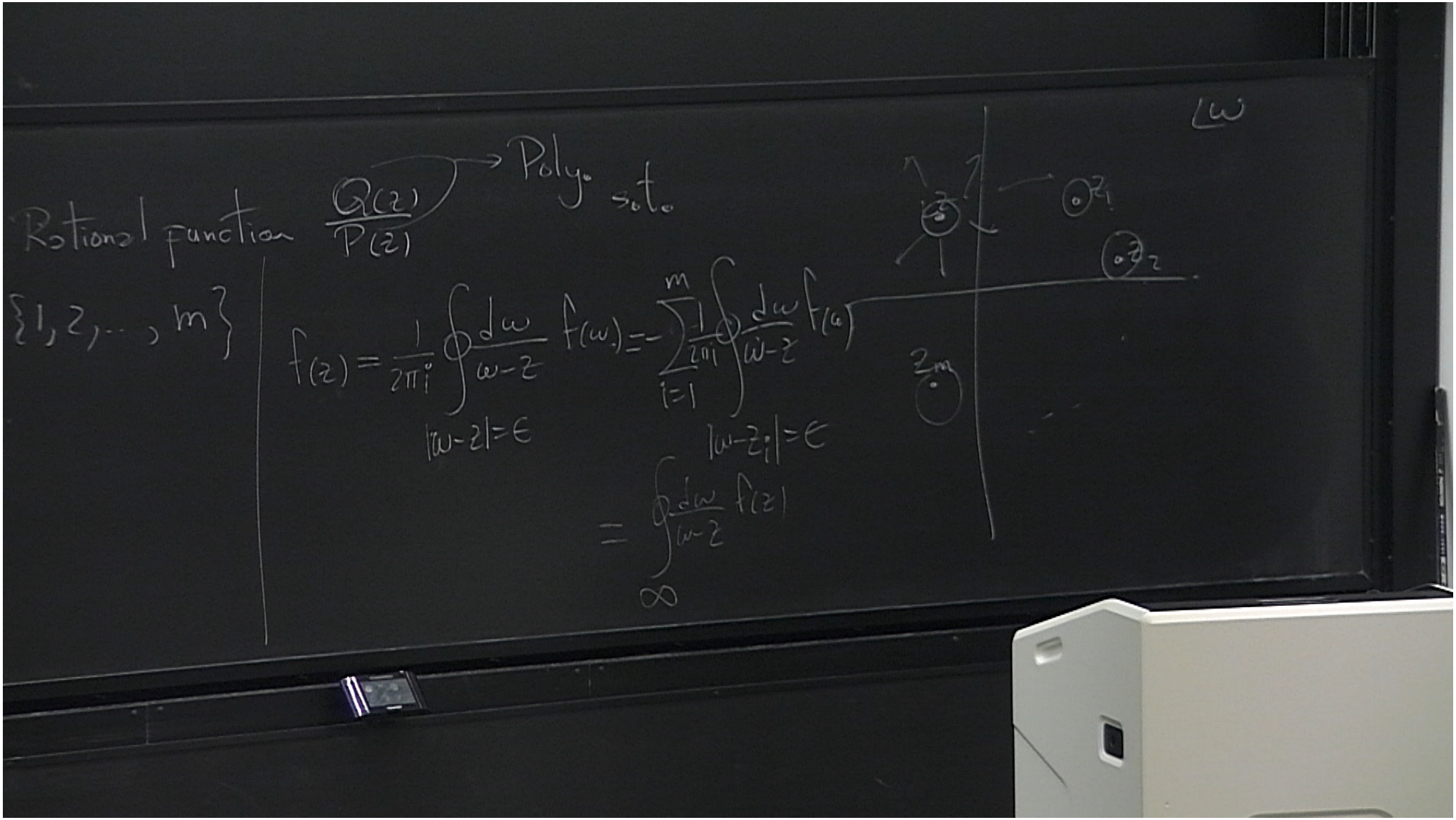
$$k_a^\mu \sigma_\mu = k_a = \lambda^{(a)}\tilde{\lambda}^{(a)}$$


$\langle ab\rangle \equiv \lambda_\alpha^{(a)} \lambda_\beta^{(b)} \epsilon^{\alpha\beta}$
$[ab] \equiv \tilde{\lambda}_{\dot\alpha}^{(a)} \tilde{\lambda}_{\dot\beta}^{(b)} \epsilon^{\dot\alpha\dot\beta}$
$A(1^-,2^-,3^+) = \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}$
$A(1^+,2^+,3^-) = \frac{[12]^3}{[23][31]}$
+ perm.
in $-\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$
are not present
$A(1^-,2^-,3^-) = \langle 12\rangle\langle 23\rangle\langle 31\rangle$
$A(1^+,2^+,3^+) = [12][23][31]$


Proof: Cplx. analysis
* Asked to construct a function f(z) Rational function Q(z)/P(z) → Poly. s.t.
① Only simple poles at z=z_i i∈{1,2,..,m}
② Residues equal to r_i at z_i.


Proof: Cplx. analysis
* Asked to construct a function f(z) Rational function Q(z)/P(z) → Poly. s.t.
① Only simple poles at z=z_i i∈{1,2,..,m}
② Residues equal to r_i at z_i.
③ f(z) → 0 as z→∞


Rational function $\frac{Q(z)}{P(z)}$ → Poly s.t.
$\{1, 2, \ldots, m\}$
$f(z) = \frac{1}{2\pi i} \oint_{|w-z|=\epsilon} \frac{dw}{w-z} f(w)$
$\mathbb{C}_w$


Rational function $\frac{Q(z)}{P(z)}$ → Poly. + t.
$\{1, 2, \ldots, m\}$
$f(z) = \frac{1}{2\pi i}\oint_{|w-z|=\epsilon} \frac{dw}{w-z} f(w) = -\sum_{i=1}^{m} \frac{1}{2\pi i}\oint_{|w-z_i|=\epsilon} \frac{dw}{w-z} f(w)$
$= \oint_{\infty} \frac{dw}{w-z} f(z)$
$\mathbb{C}_w$
z_1
z_2
z_m

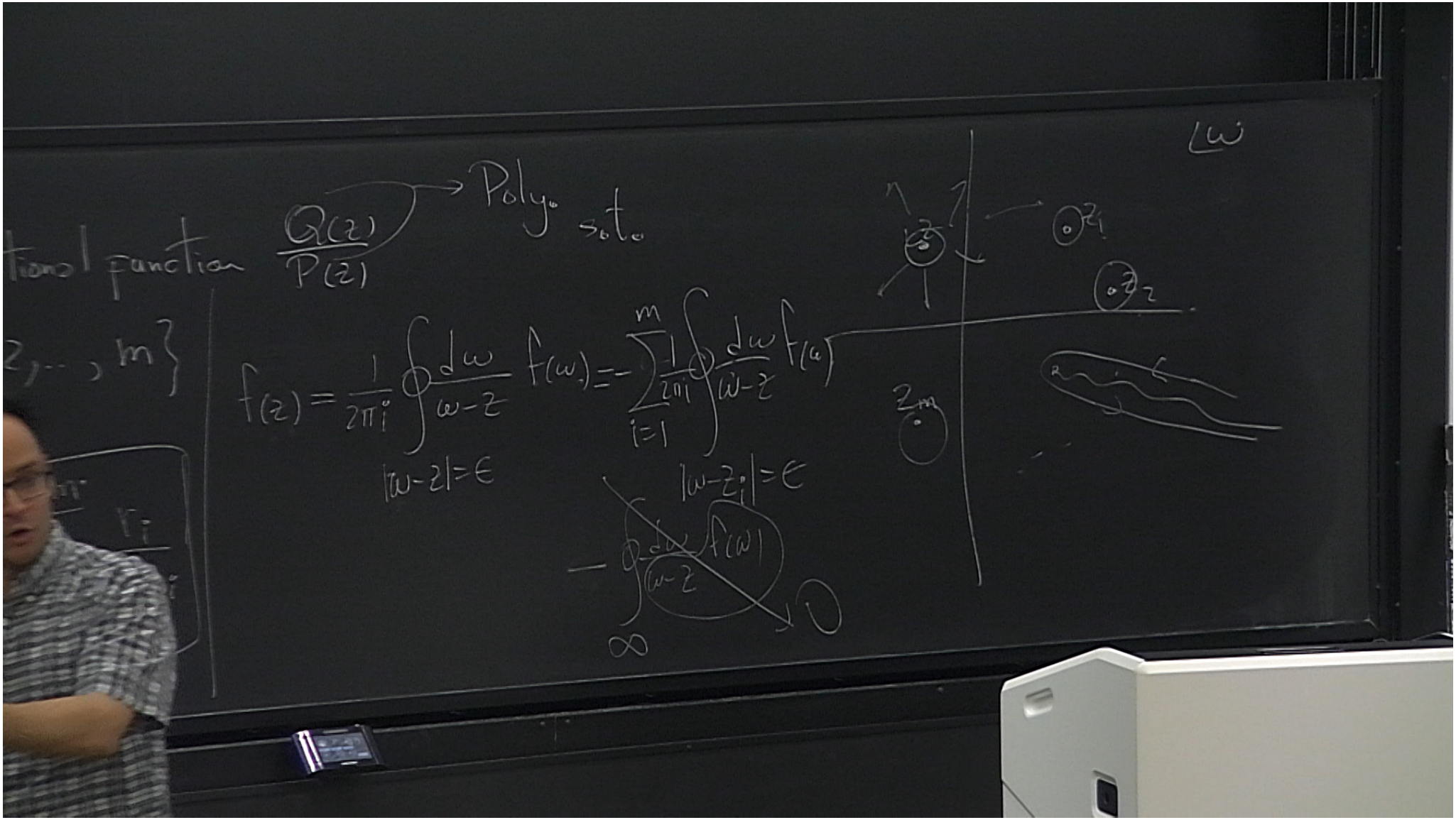
Proof: Cplx. analysis.

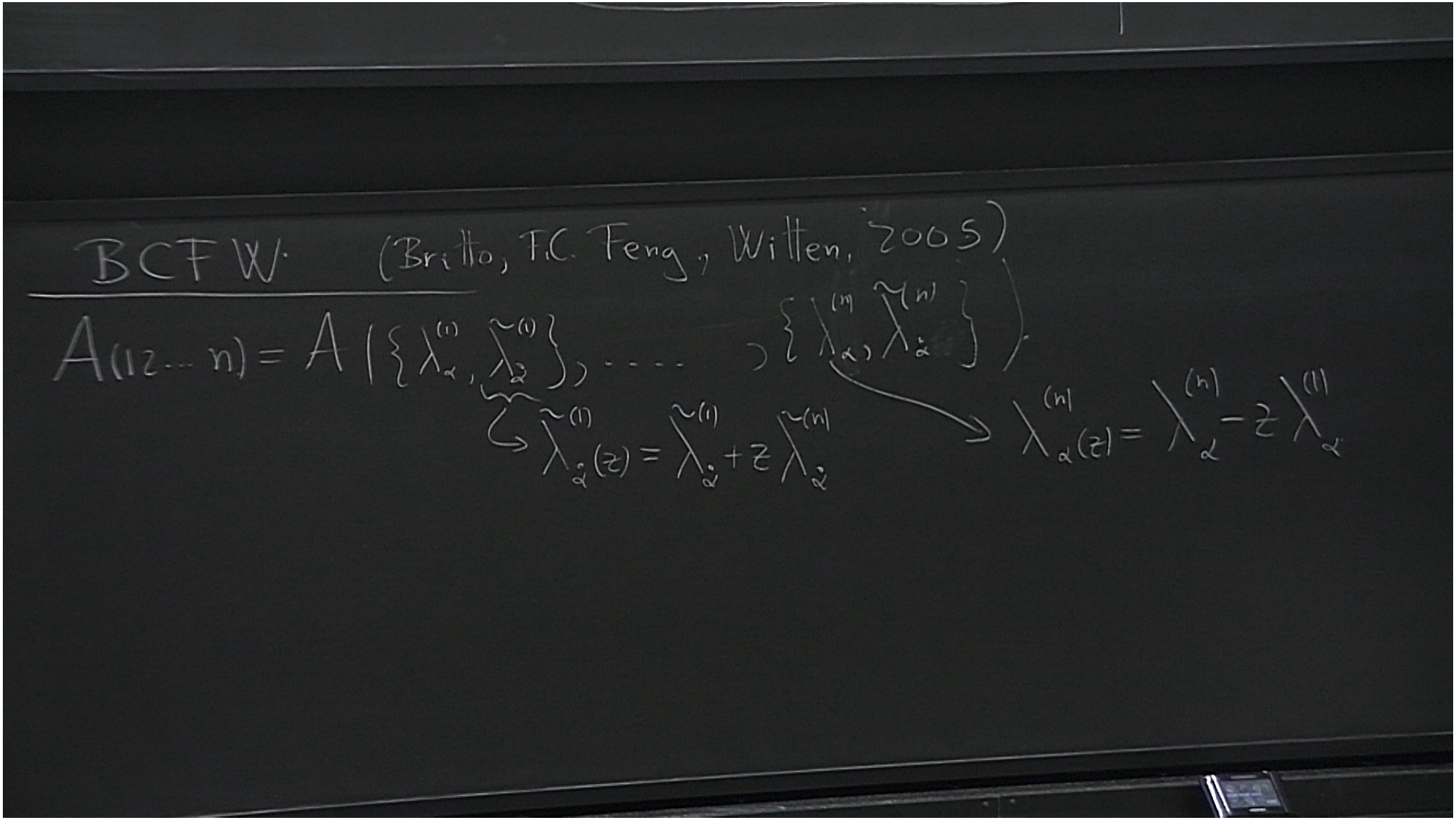
* Asked to construct a function $f(z)$ Rational function $\frac{Q(z)}{P(z)}$

1. Only simple poles at $z = z_i$ $i \in \{1, 2, \ldots, m\}$
2. Residues equal to r_i at z_i.
3. $f(z) \xrightarrow[z \to \infty]{} 0$

$$f(z) = -\sum_{i=1}^{m} \frac{r_i}{z_i - z} = \sum_{i=1}^{m} \frac{r_i}{z - z_i}$$

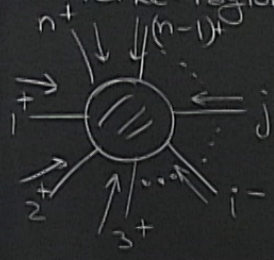
$f(z) =$


tional function $\frac{Q(z)}{P(z)}$ → Poly. s.t.
$z_1, \dots, z_m\}$
$f(z) = \frac{1}{2\pi i}\oint \frac{dw}{w-z} f(w) = -\sum_{i=1}^{m} \frac{1}{2\pi i}\oint \frac{dw}{w-z} f(w)$
$|w-z| = \epsilon$
$|w-z_i| = \epsilon$
$-\oint_{\infty} \frac{dw}{w-z} f(w) \to 0$
γ_i
z_m
z_1
z_2
$\mathbb{C}_w$


BCFW (Britto, F.C. Feng, Witten, 2005)
$A(12\cdots n) = A\left(\{\lambda_\alpha^{(1)}, \tilde{\lambda}_{\dot\alpha}^{(1)}\}, \cdots\cdots, \{\lambda_\alpha^{(n)}, \tilde{\lambda}_{\dot\alpha}^{(n)}\}\right)$
$\tilde{\lambda}_{\dot\alpha}^{(1)}(z) = \tilde{\lambda}_{\dot\alpha}^{(1)} + z\tilde{\lambda}_{\dot\alpha}^{(n)}$
$\lambda_\alpha^{(n)}(z) = \lambda_\alpha^{(n)} - z\lambda_\alpha^{(1)}$

Lecture III

Parke-Taylor Formula



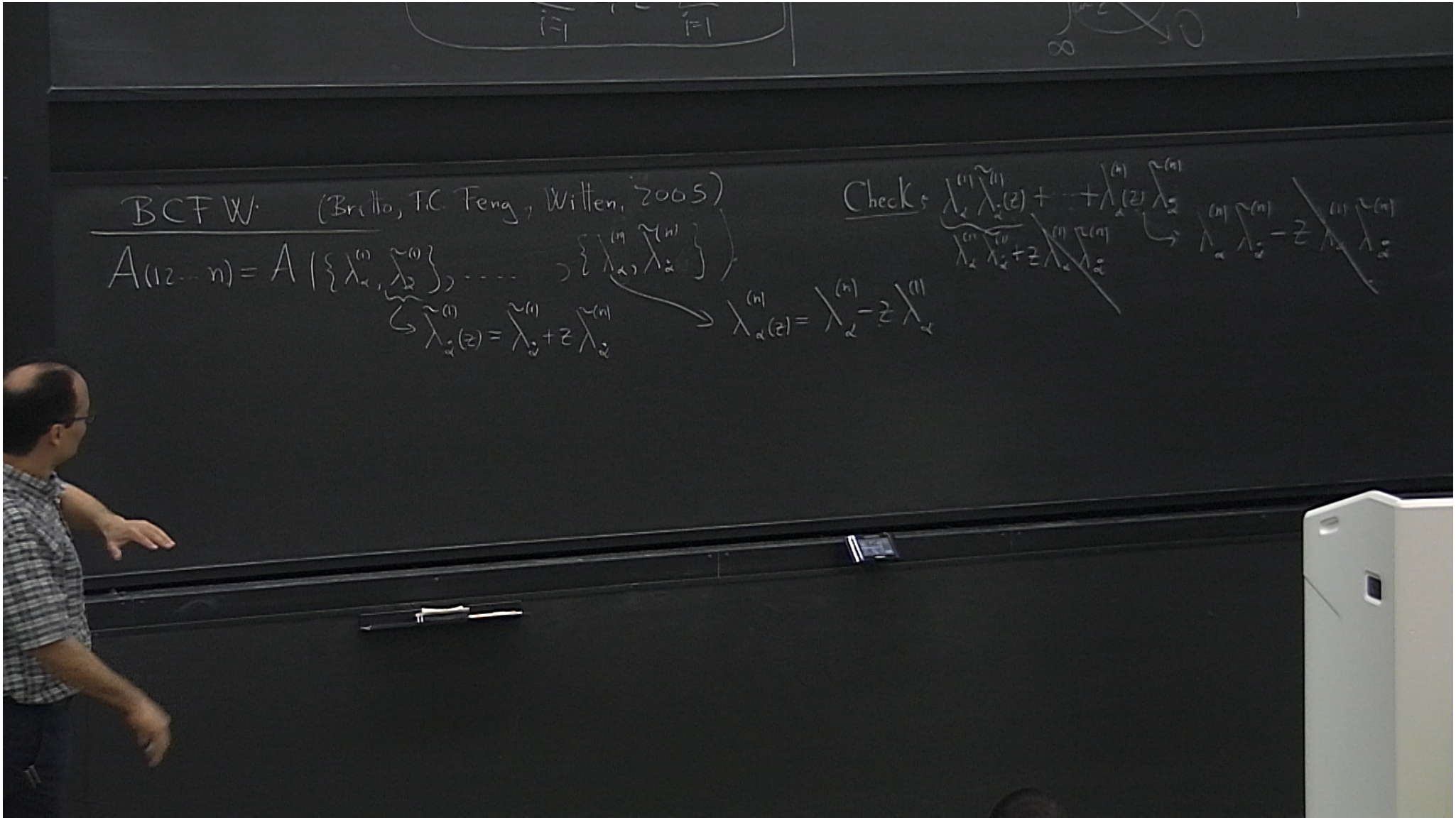
$$j^- = \underbrace{\delta^4\left(\sum_{a=1}^{n} k_a^\mu\right)}_{2\times 2}\int\left(\sum_{a=1}^{n}\lambda_\alpha^{(a)}\tilde{\lambda}_{\dot\alpha}^{(a)}\right)\left(\mathrm{tr}\left(T^{a_1}T^{a_2}\cdots T^{a_n}\right)\frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\cdots\langle n-1\ n\rangle\langle n1\rangle} + \text{perm.}\right)$$

$$k_a^\mu \sigma_\mu = k_a = \lambda^{(a)}\tilde{\lambda}^{(a)}$$

$$\{k_a^\mu, \epsilon_a^\mu, a_a\} \to \{\lambda_\alpha^{(a)}, \tilde{\lambda}_{\dot\alpha}^{(a)}, h_a, a_a\}$$

$\langle ab$

$[ab$


BCFW (Britto, F.C. Feng, Witten, 2005)
Check:

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$A(12\cdots n) = A\left(\{\lambda_\alpha^{(1)}, \tilde{\lambda}_\alpha^{(1)}\}, \cdots, \{\lambda_\alpha^{(n)}, \tilde{\lambda}_\alpha^{(n)}\}\right)$

$f(z)$

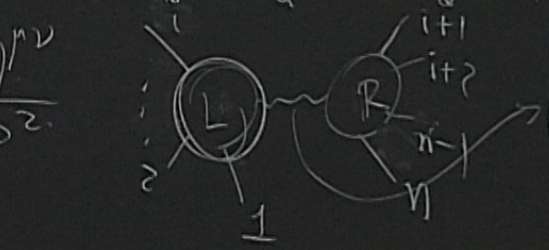
$\tilde{\lambda}_\alpha^{(1)}(z) = \tilde{\lambda}_\alpha^{(1)} + z\tilde{\lambda}_\alpha^{(n)}$

$\lambda_\alpha^{(n)}(z) = \lambda_\alpha^{(n)} - z\lambda_\alpha^{(1)}$

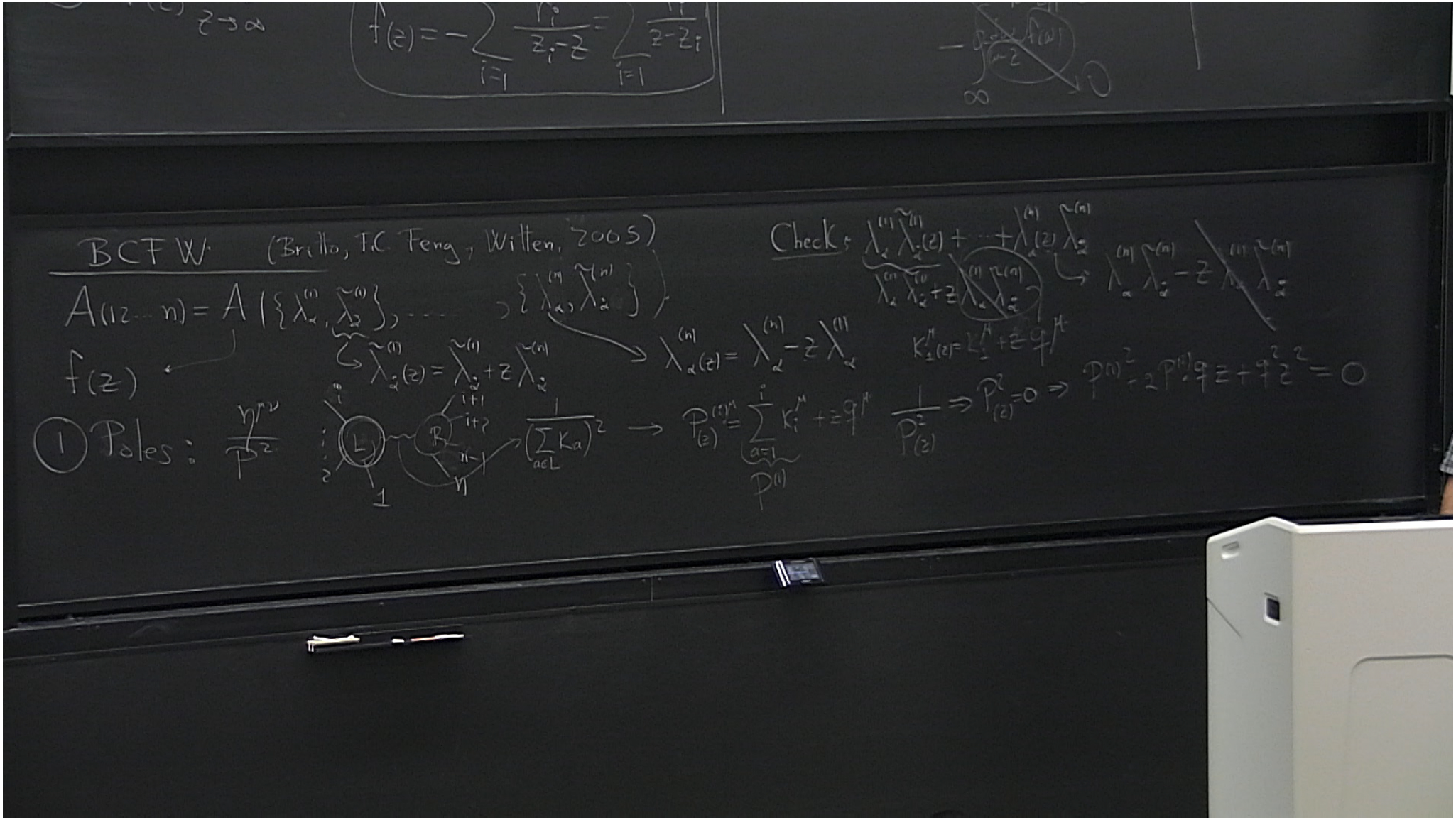
Checks: $\lambda_\alpha^{(1)}\tilde{\lambda}_\alpha^{(1)}(z) + \cdots$

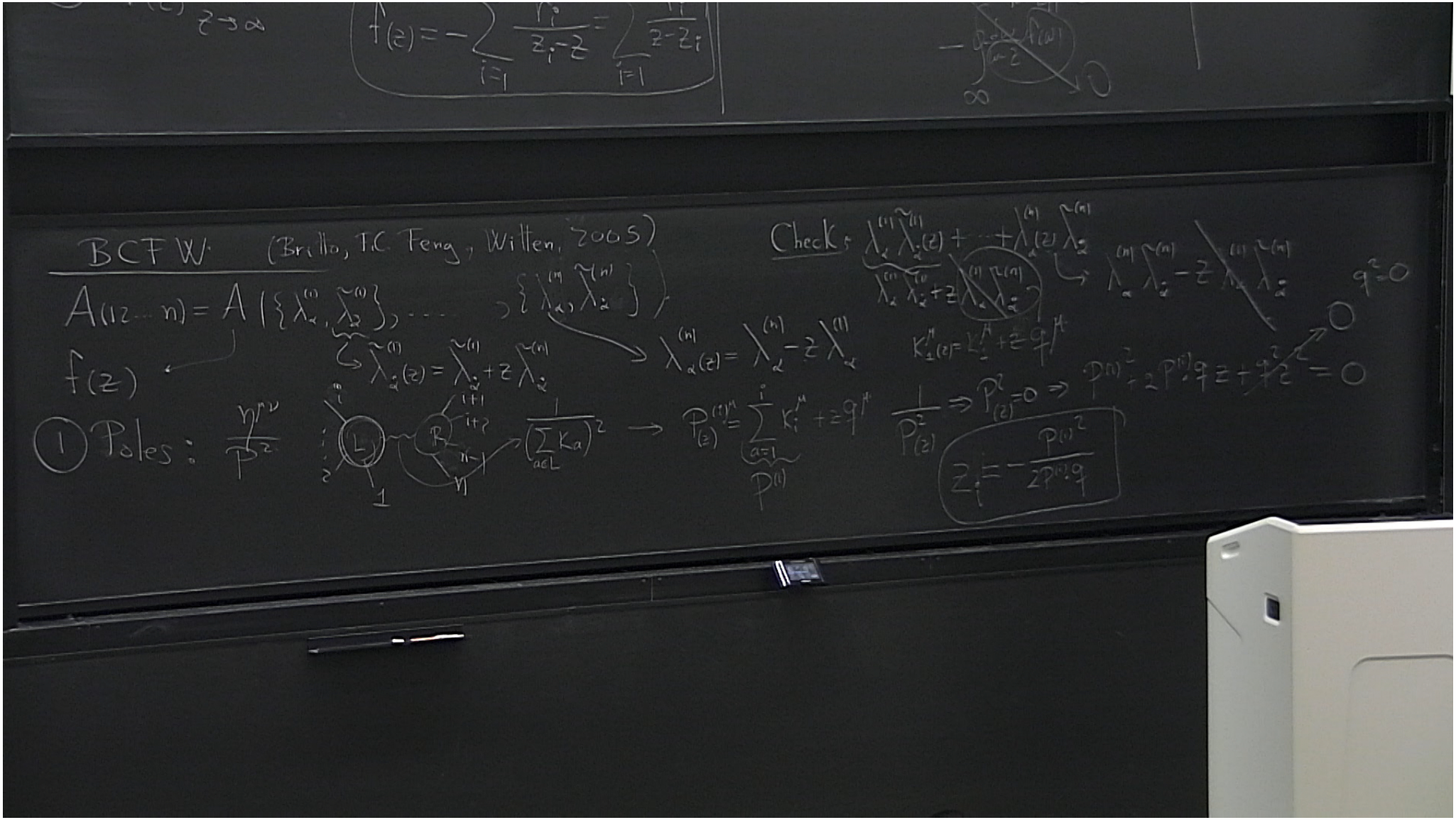
$\lambda_\alpha^{(1)}\tilde{\lambda}_\alpha^{(1)} \quad \lambda_\alpha^{(1)}\tilde{\lambda}_\alpha^{(1)} + z\lambda_\alpha^{(1)}$

(1) Poles: $\frac{\eta^{\mu\nu}}{P^2}$



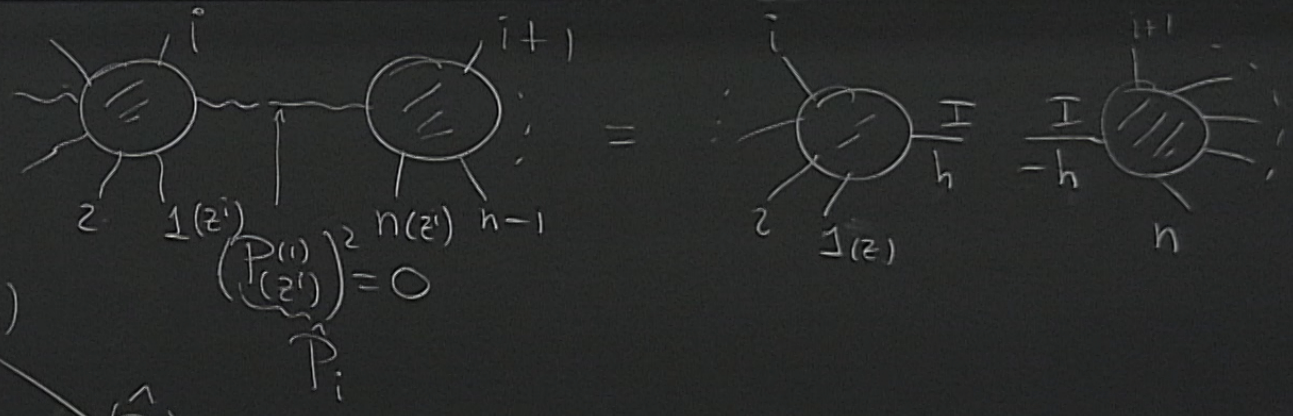
$\frac{1}{\left(\sum_{a\in L} K_a\right)^2} \to P(z)^\mu = \sum_{a=1}^{i} K_i^\mu$


BCFW (Britto, F.C. Feng, Witten, 2005)
Check:
Poles:


BCFW (Britto, F.C. Feng, Witten, 2005)
Check:
Poles:
f(z)

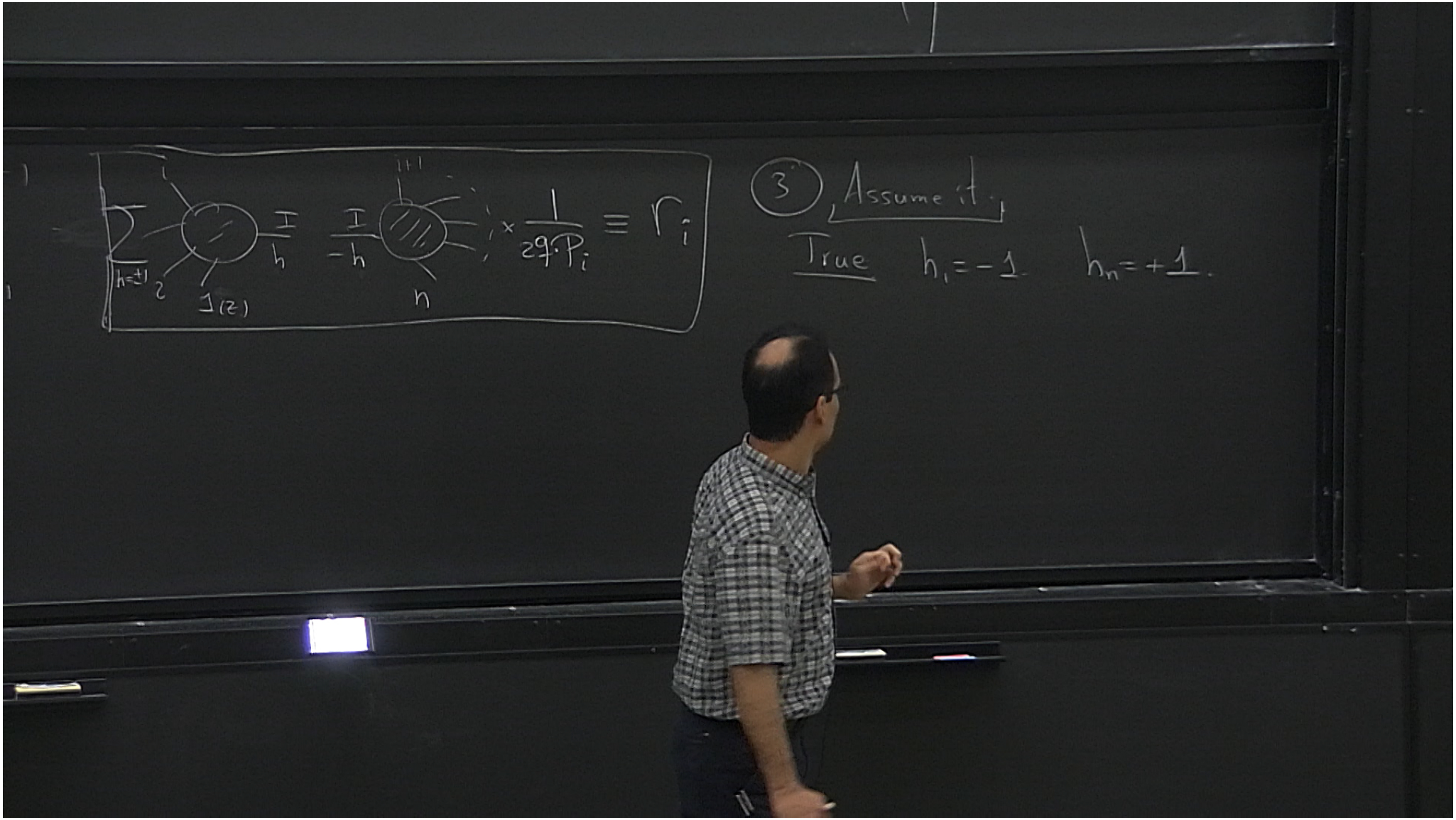
$K_a V_\mu = K_a = \lambda^{(a)} \lambda^{(a)}$

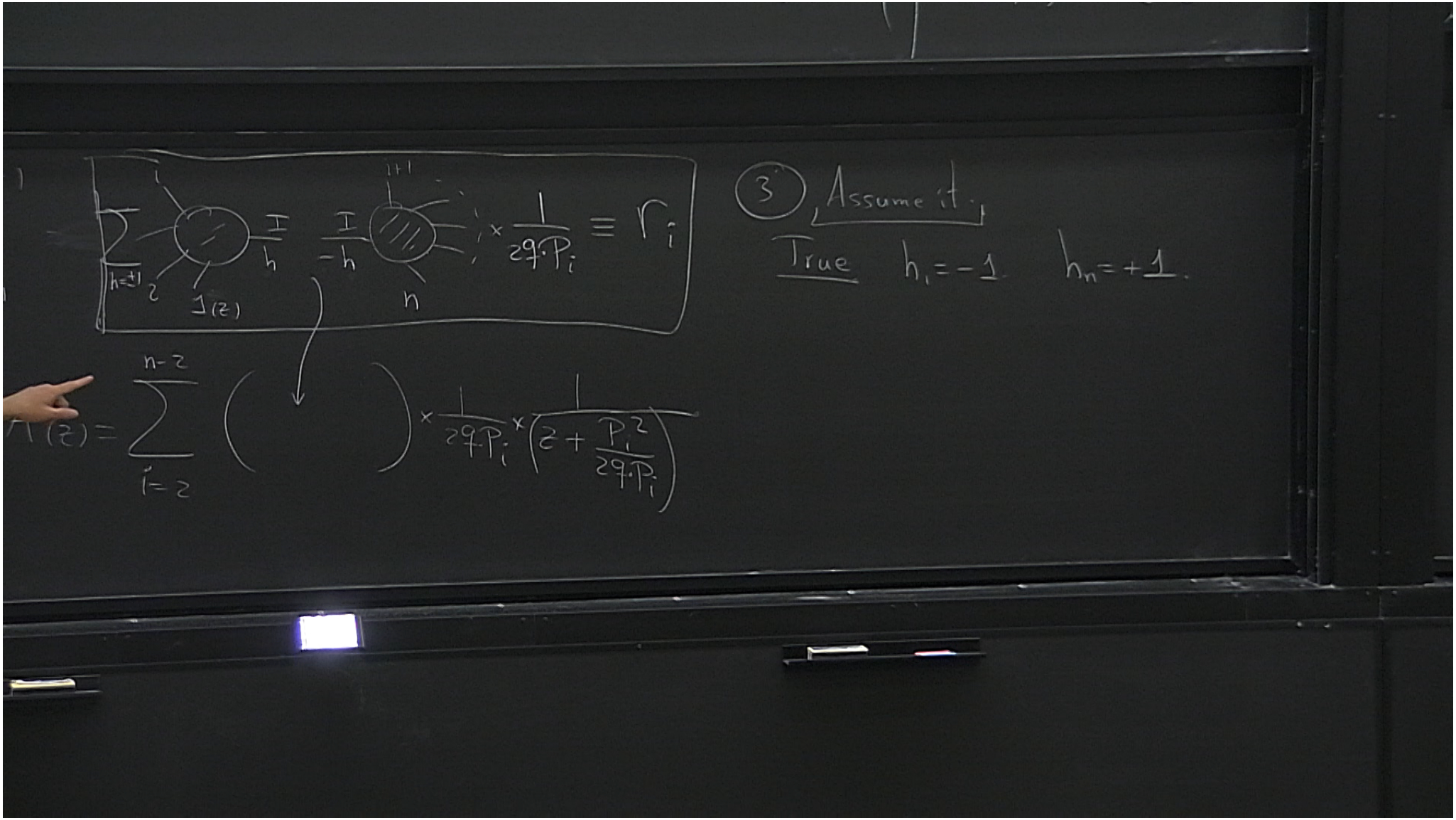
② Residues :

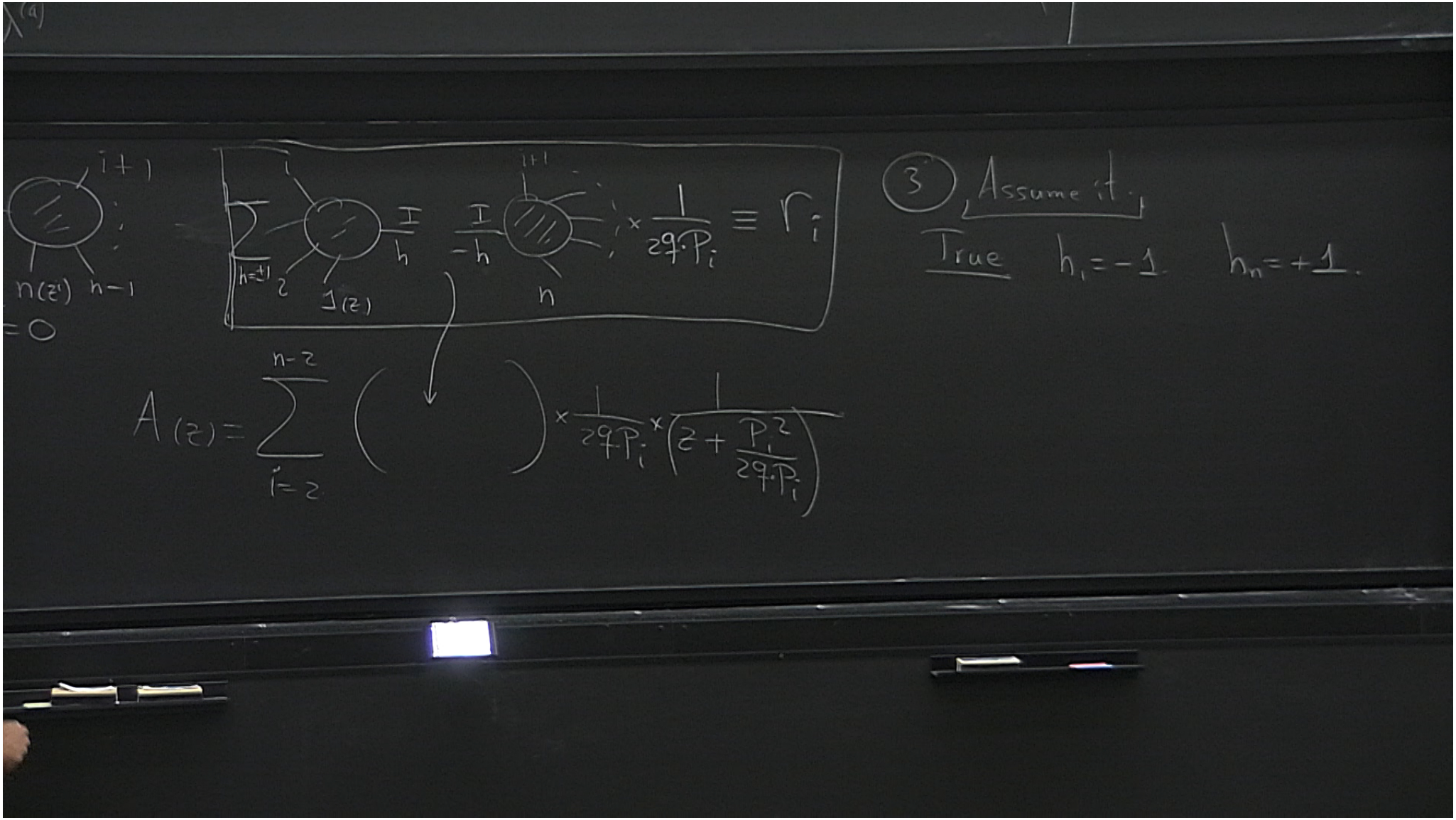


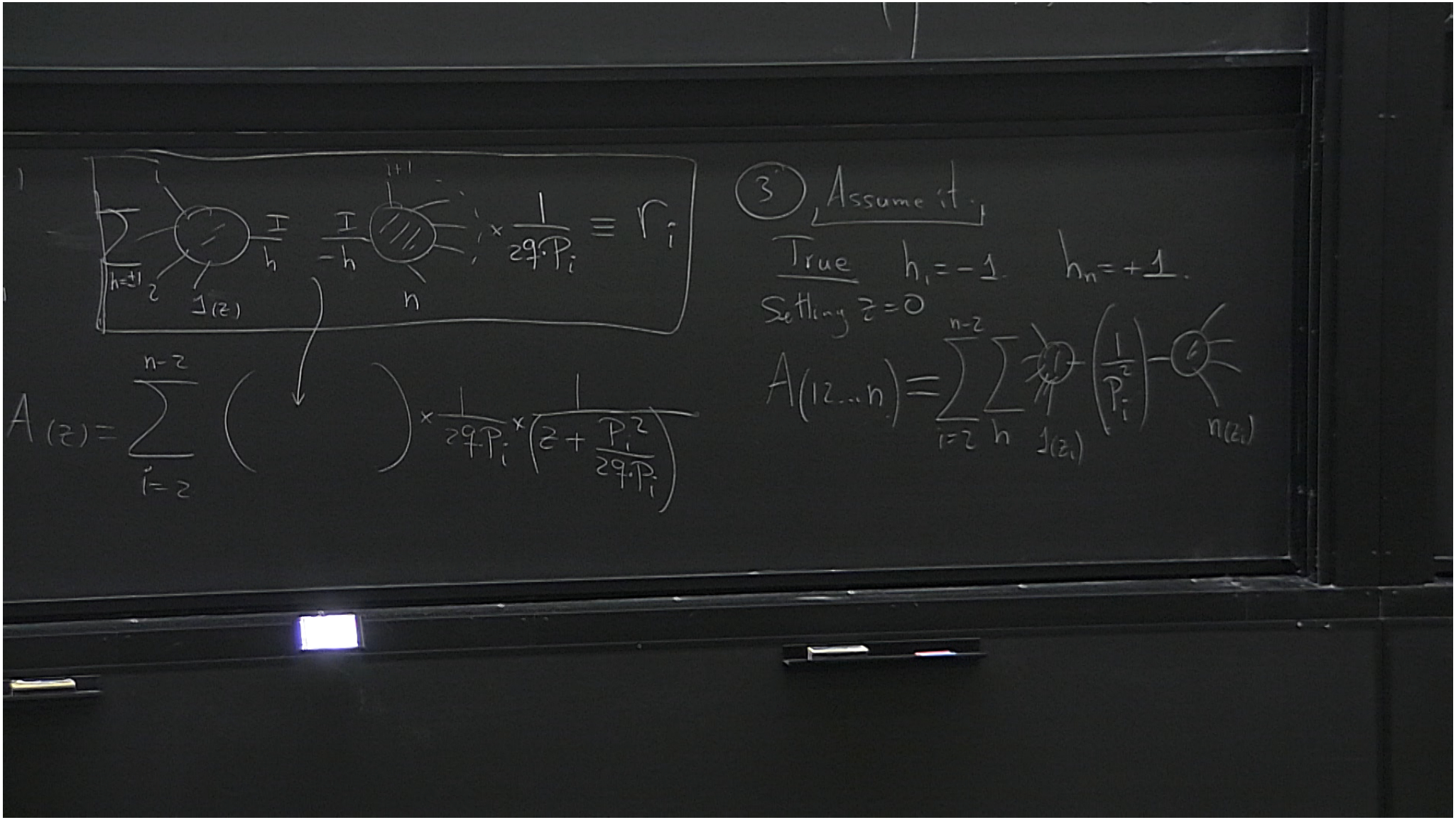
$$\eta_{\mu\nu} = \sum_{h=\pm 1} \epsilon_\mu^h(\hat{P}_i)\,\epsilon_\nu^{-h}(\hat{P}_i)$$

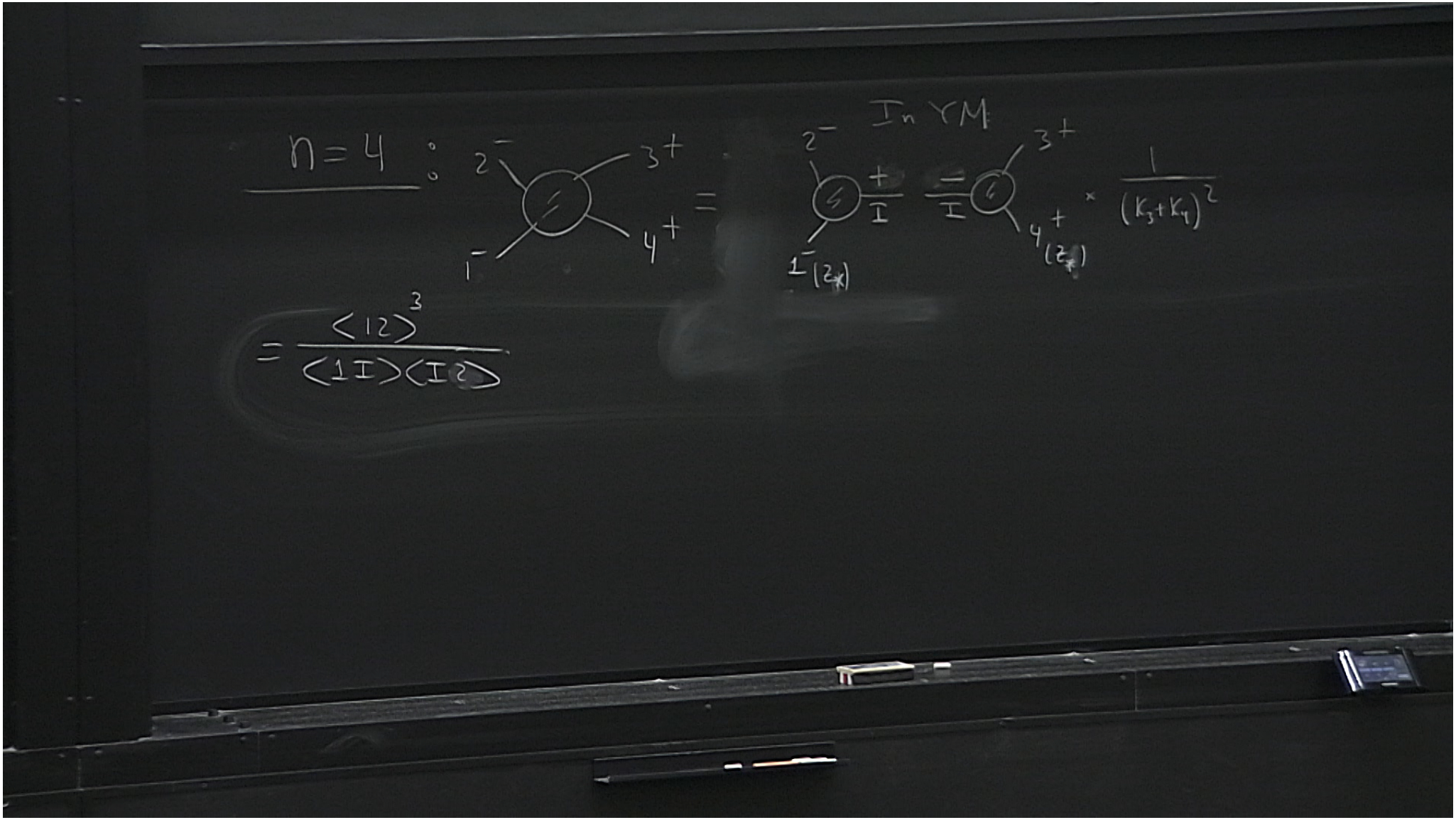
$$+ (\hat{P}_i)_\mu (Q)_\nu + (Q)_\mu (\hat{P}_i)_\nu$$

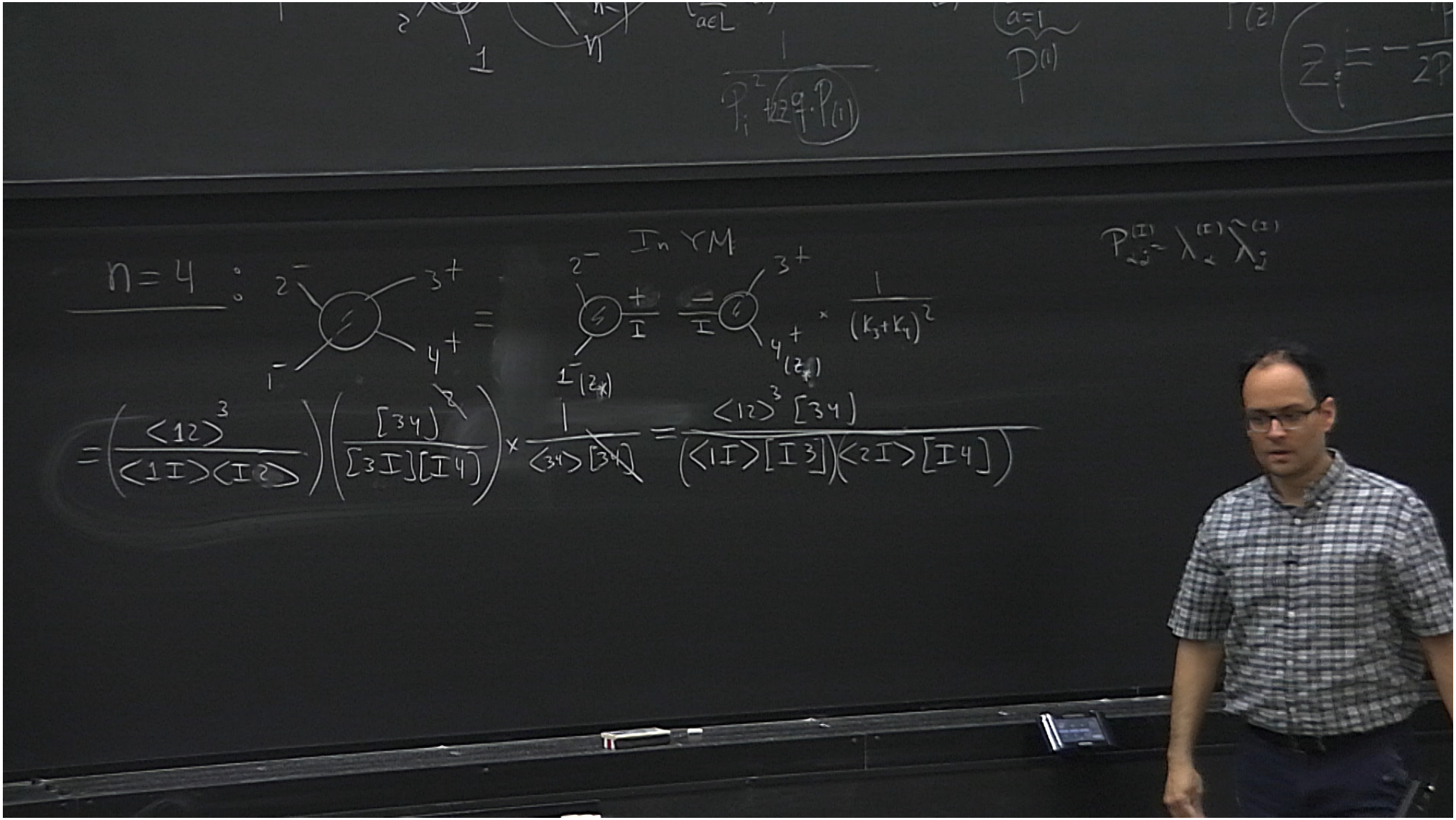

h=+1
2
J(z)
h
-h
n
+1
$\times \frac{1}{2q \cdot P_i} \equiv r_i$
3. Assume it.
True $h_1 = -1$ $h_n = +1$.

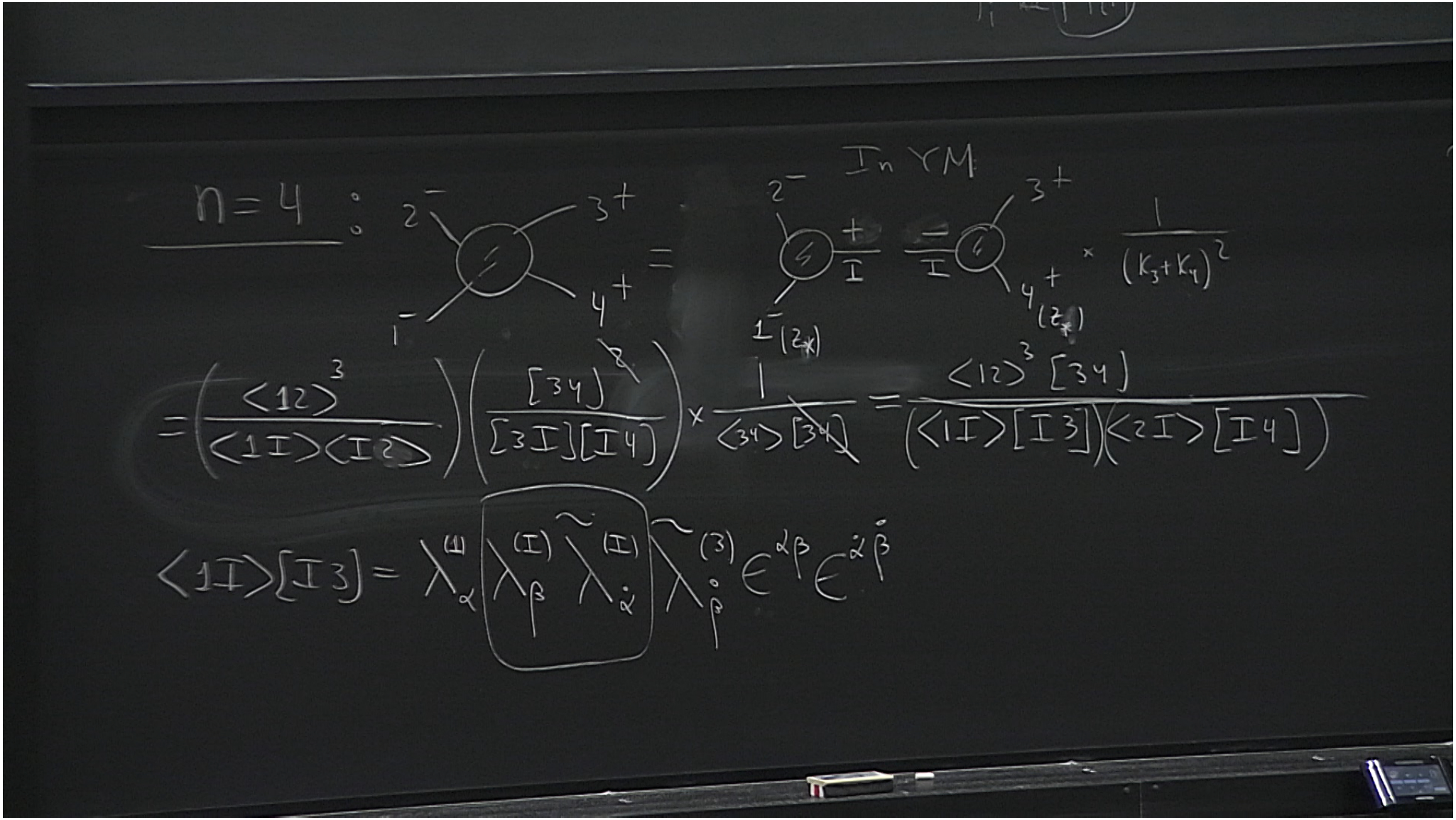

i+1
I
h
I
-h
$\times \frac{1}{2q \cdot P_i} \equiv r_i$
h=±1
2
1(z)
n
3 Assume it.
True $h_1 = -1$ $h_n = +1$.
$1(z) = \sum_{i=2}^{n-2} (\quad) \times \frac{1}{2q \cdot P_i} \times \frac{1}{z + \frac{P_i^2}{2q \cdot P_i}}$

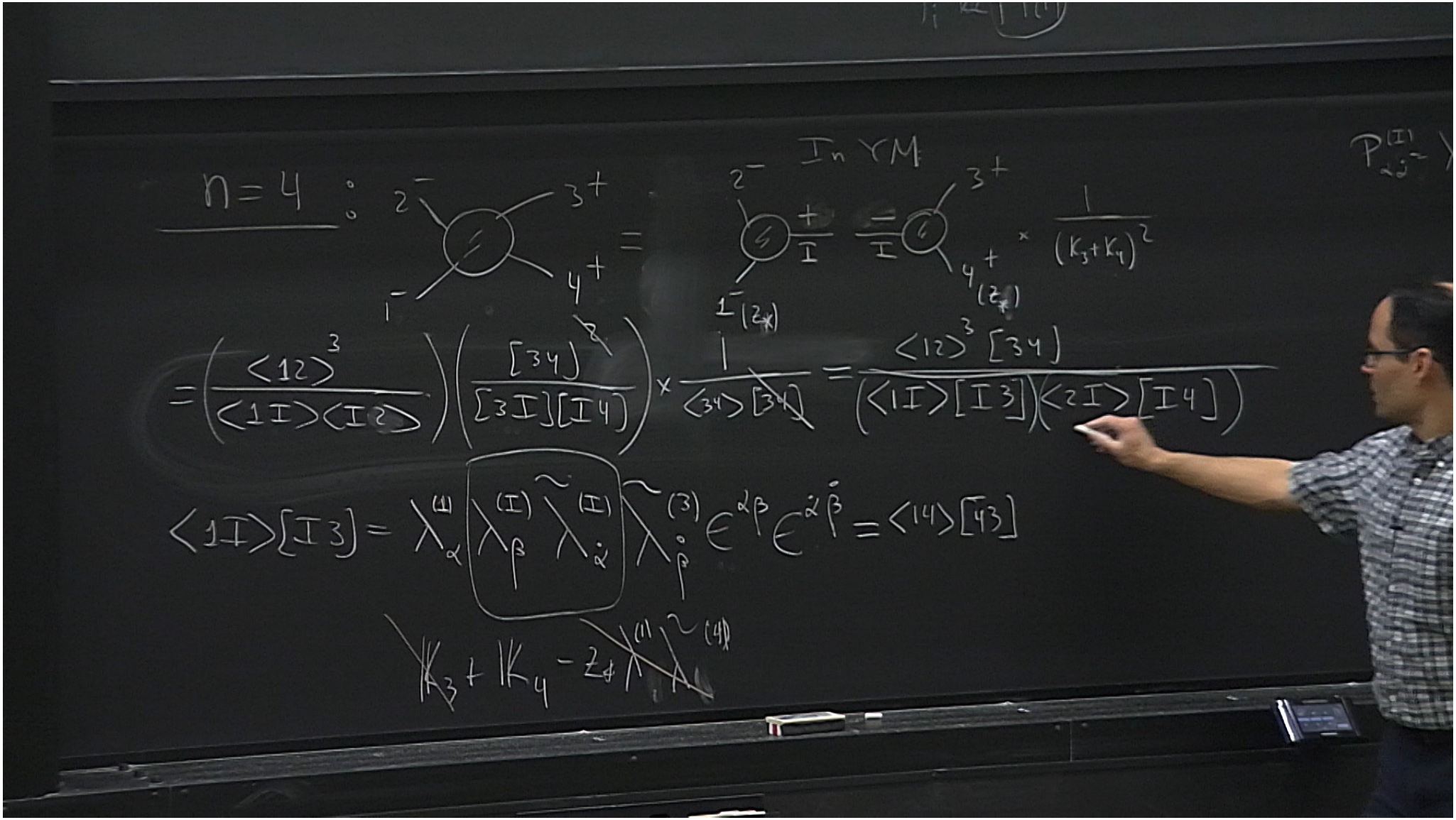

i+1
n(z') n-1
=0
h=±1
2
1(z)
I
h
I
-h
i+1
n
× 1/(2q·P_i) ≡ r_i
3 Assume it.
True h_1 = -1 h_n = +1.
A(z) = Σ_{i=2}^{n-2} ( ) × 1/(2q·P_i) × 1/(z + P_i^2/(2q·P_i))

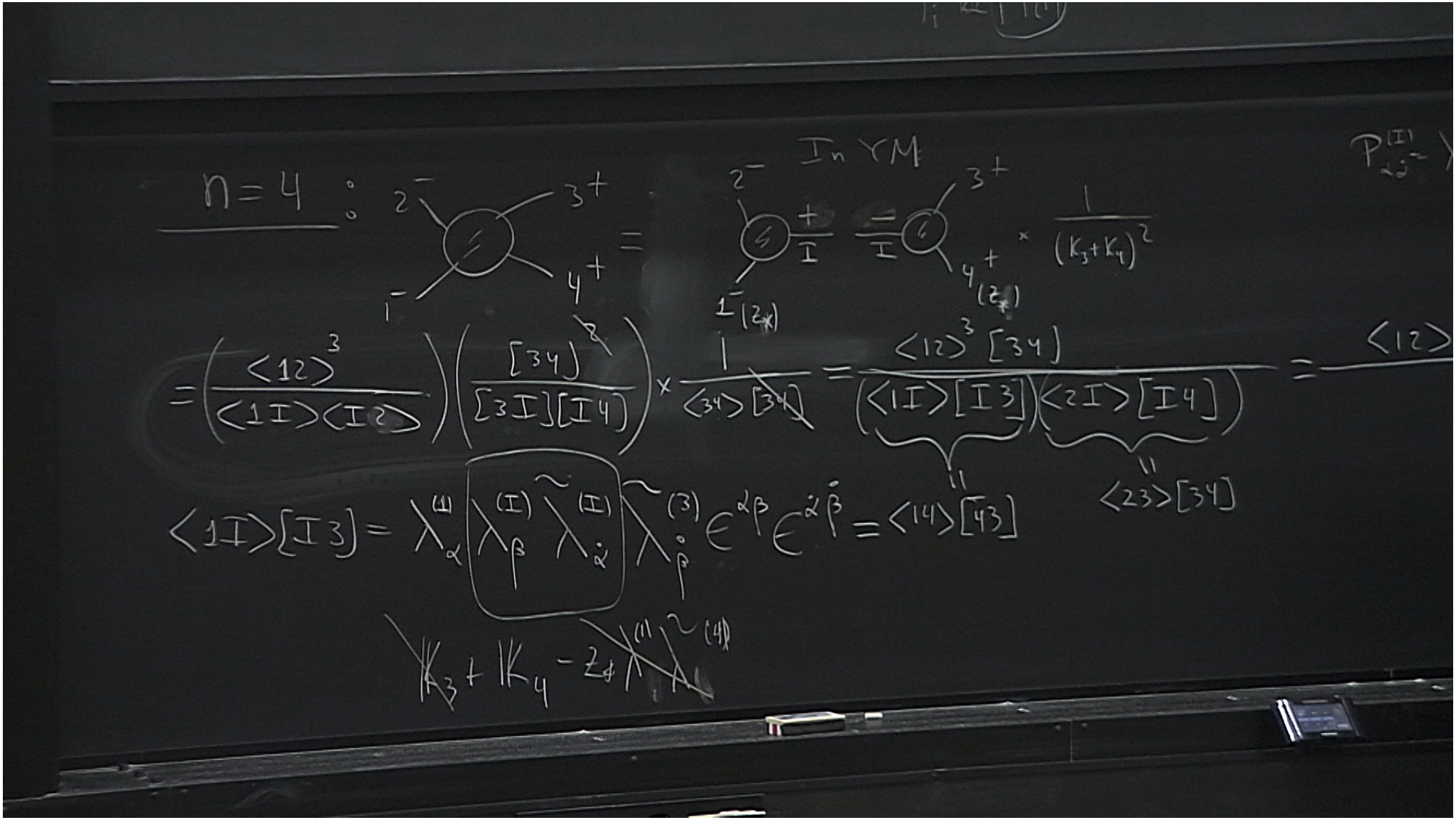

h=+1
2
1(z)
h
-h
i+1
n
$\times \frac{1}{2q \cdot P_i} \equiv r_i$
3 Assume it.
True $h_1 = -1$ $h_n = +1$.
Setting $z = 0$
$A_{(z)} = \sum_{i=2}^{n-2} (\quad) \times \frac{1}{2q \cdot P_i} \times \frac{1}{\left(z + \frac{P_i^2}{2q \cdot P_i}\right)}$
$A_{(12\ldots n)} = \sum_{i=2}^{n-2} \sum_{h} \left(\frac{1}{P_i^2}\right)$
1(z)
n(z)

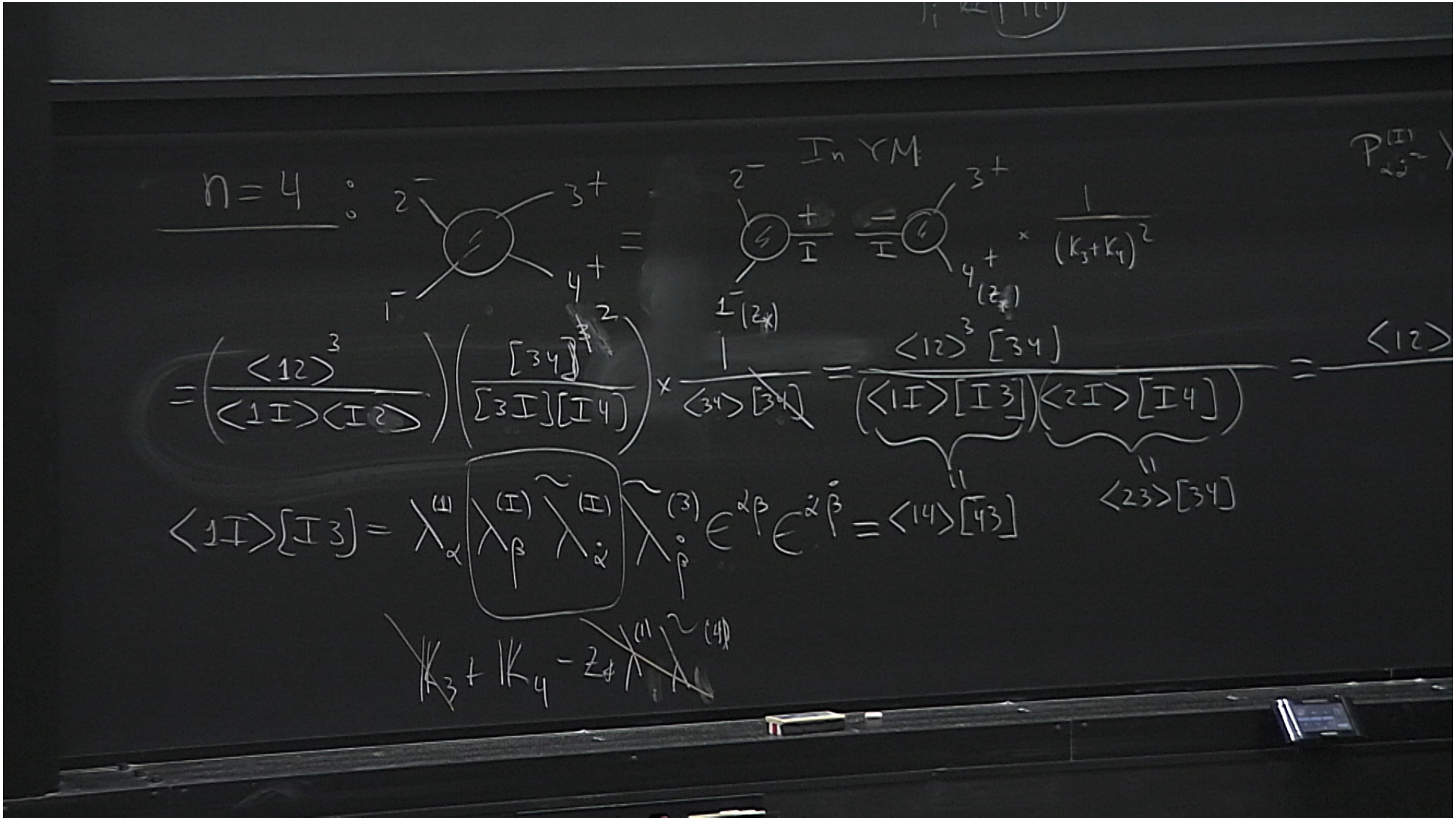

n=4 :
2⁻
3⁺
1⁻
4⁺
=
In YM
2⁻
3⁺
+
I
=
I
4⁺
1⁻(z*)
(z*)
×
1/(K₃+K₄)²
= ⟨12⟩³ / ⟨1I⟩⟨I2⟩

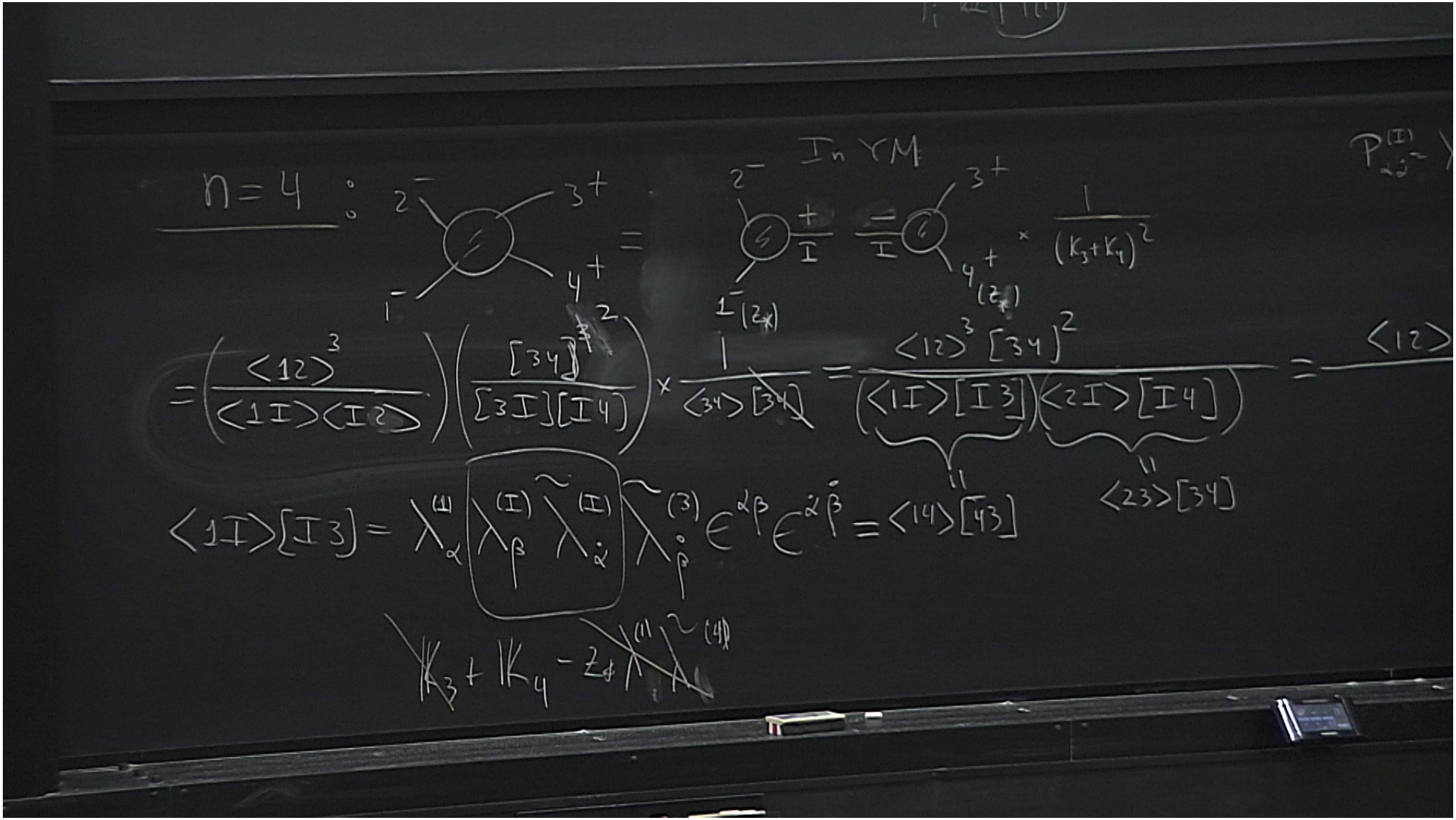

In YM
n=4 :
2⁻
3⁺
1⁻
4⁺
=
I
(K3+K4)²
=
<12>³
<1I><I2>
[34]⁸
[3I][I4]
<34>[34]
<12>³[34]
<1I>[I3]<2I>[I4]

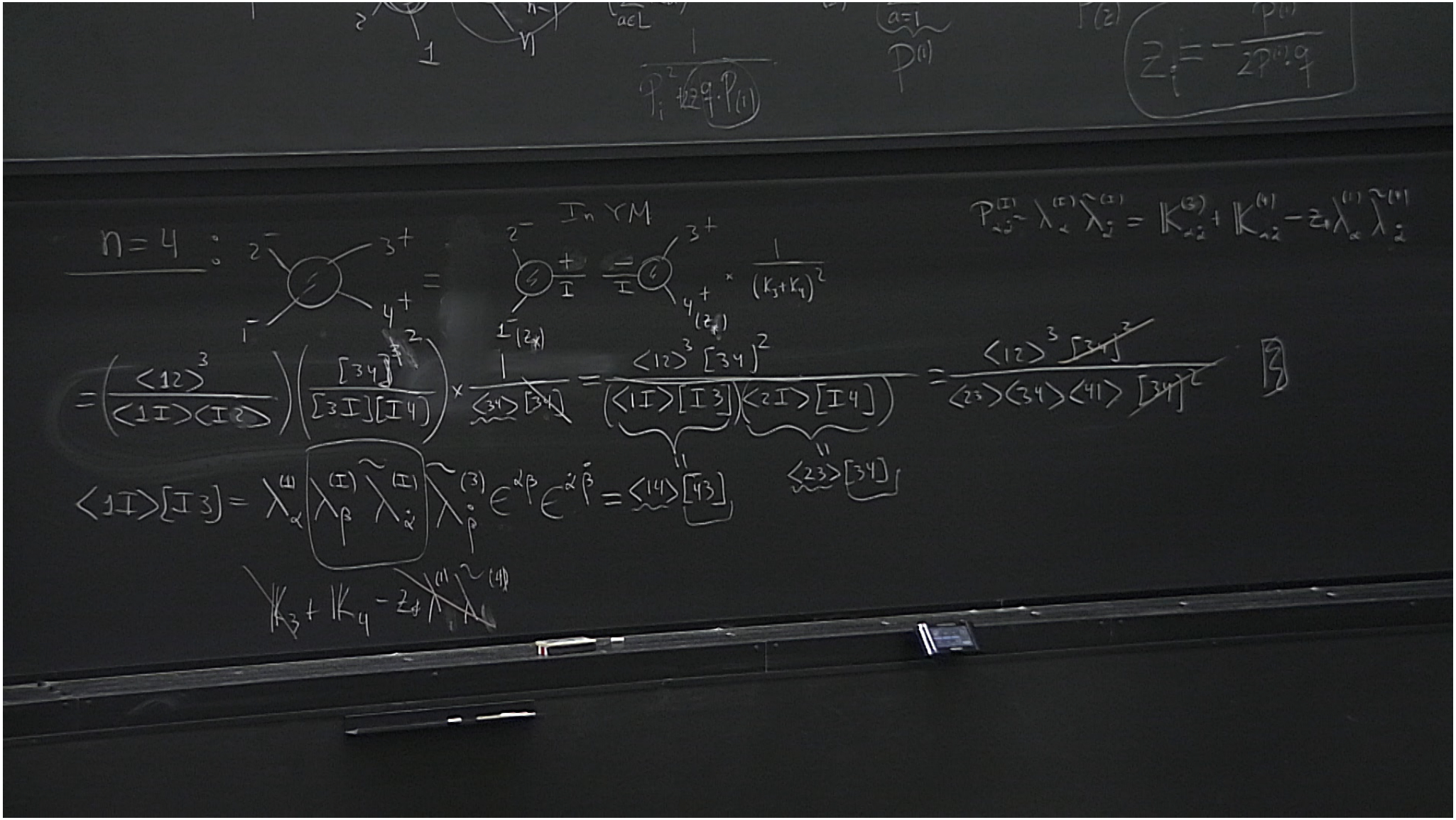

n=4 :
2⁻
3⁺
1⁻
4⁺
=
In YM
2⁻
3⁺
+
I
1⁻(z*)
4⁺(z*)
×
1/(K₃+K₄)²
= (⟨12⟩³/(⟨1I⟩⟨I2⟩)) (([34]³/([3I][I4])) × 1/⟨34⟩[34]) = ⟨12⟩³[34] / (⟨1I⟩[I3]⟨2I⟩[I4])
⟨1I⟩[I3] = λ̃(4)α λ(I)β λ̃(I)β̇ λ̃(3)β̇ ε^αβ ε^α̇β̇

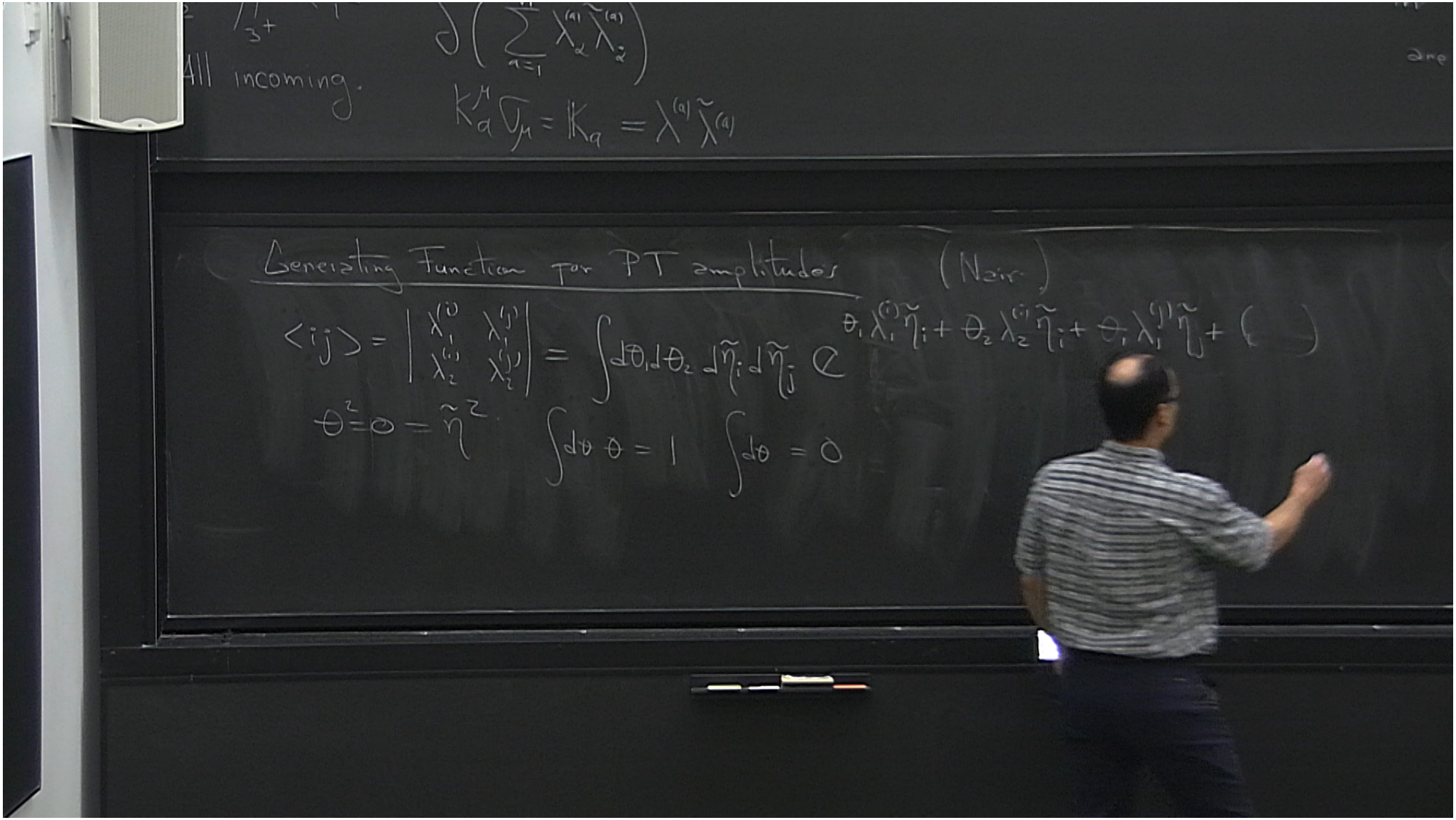

n=4 :
2⁻
3⁺
1⁻
4⁺
=
In YM
2⁻
3⁺
1⁻(z_k)
4⁺(z_k)
+
I
×
1/(K_3+K_4)²
= ⟨12⟩³/(⟨1I⟩⟨I2⟩) ([34]³/([3I][I4])) × 1/⟨34⟩[34] = ⟨12⟩³[34]/(⟨1I⟩[I3]⟨2I⟩[I4])
⟨1I⟩[I3] = λ̃^(1)_α λ^(I)_β λ̃^(I)_β̇ λ̃^(3)_β̇ ε^αβ ε^α̇β̇ = ⟨14⟩[43]
K_3 + K_4 − z_4


n = 4 :
In YM
$= \left(\frac{\langle 12\rangle^3}{\langle 1I\rangle\langle I2\rangle}\right)\left(\frac{[34]^3}{[3I][I4]}\right) \times \frac{1}{\langle 34\rangle[34]} = \frac{\langle 12\rangle^3[34]}{\langle 1I\rangle[I3]\langle 2I\rangle[I4]} = \frac{\langle 12\rangle}{\langle 23\rangle[34]}$
$\langle 1I\rangle[I3] = \lambda_\alpha^{(1)} \lambda_\beta^{(I)} \tilde{\lambda}_{\dot\alpha}^{(I)} \tilde{\lambda}_{\dot\beta}^{(3)} \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta} = \langle 14\rangle[43]$
$K_3 + K_4 - Z_I$

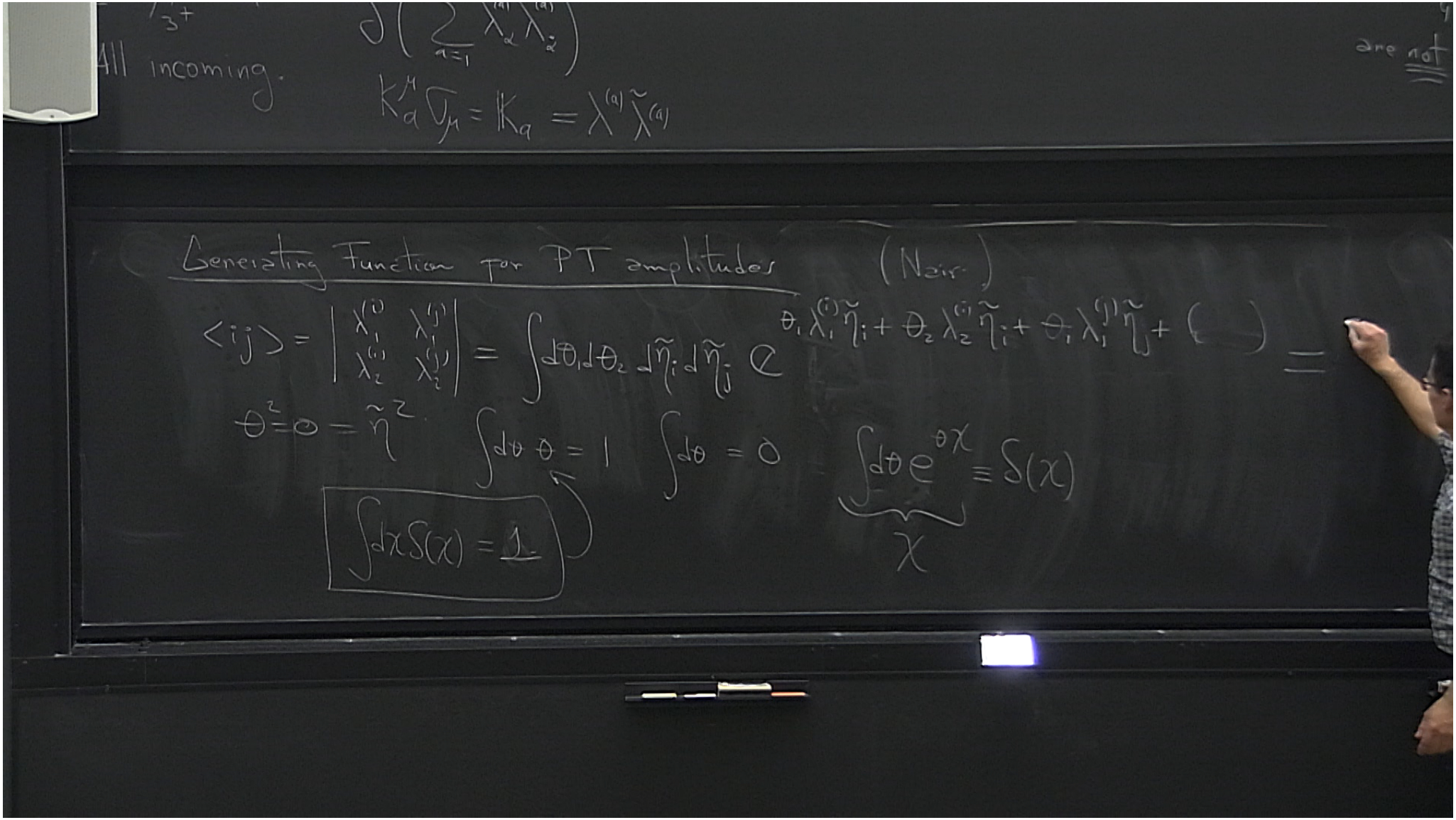

n = 4 :
In YM
$= \left(\frac{\langle 12\rangle^3}{\langle 1I\rangle\langle I2\rangle}\right)\left(\frac{[34]^3}{[3I][I4]}\right) \times \frac{1}{\langle 34\rangle[34]} = \frac{\langle 12\rangle^3[34]}{\langle 1I\rangle[I3]\langle 2I\rangle[I4]} = \frac{\langle 12\rangle}{\langle 1I\rangle[I3]\langle 2I\rangle[I4]}$
$\langle 1I\rangle[I3] = \lambda_\alpha^{(1)} \lambda_\beta^{(I)} \tilde{\lambda}_{\dot\alpha}^{(I)} \tilde{\lambda}_{\dot\beta}^{(3)} \epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta} = \langle 14\rangle[43]$
$\langle 23\rangle[34]$
$K_3 + K_4 - Z_\phi$

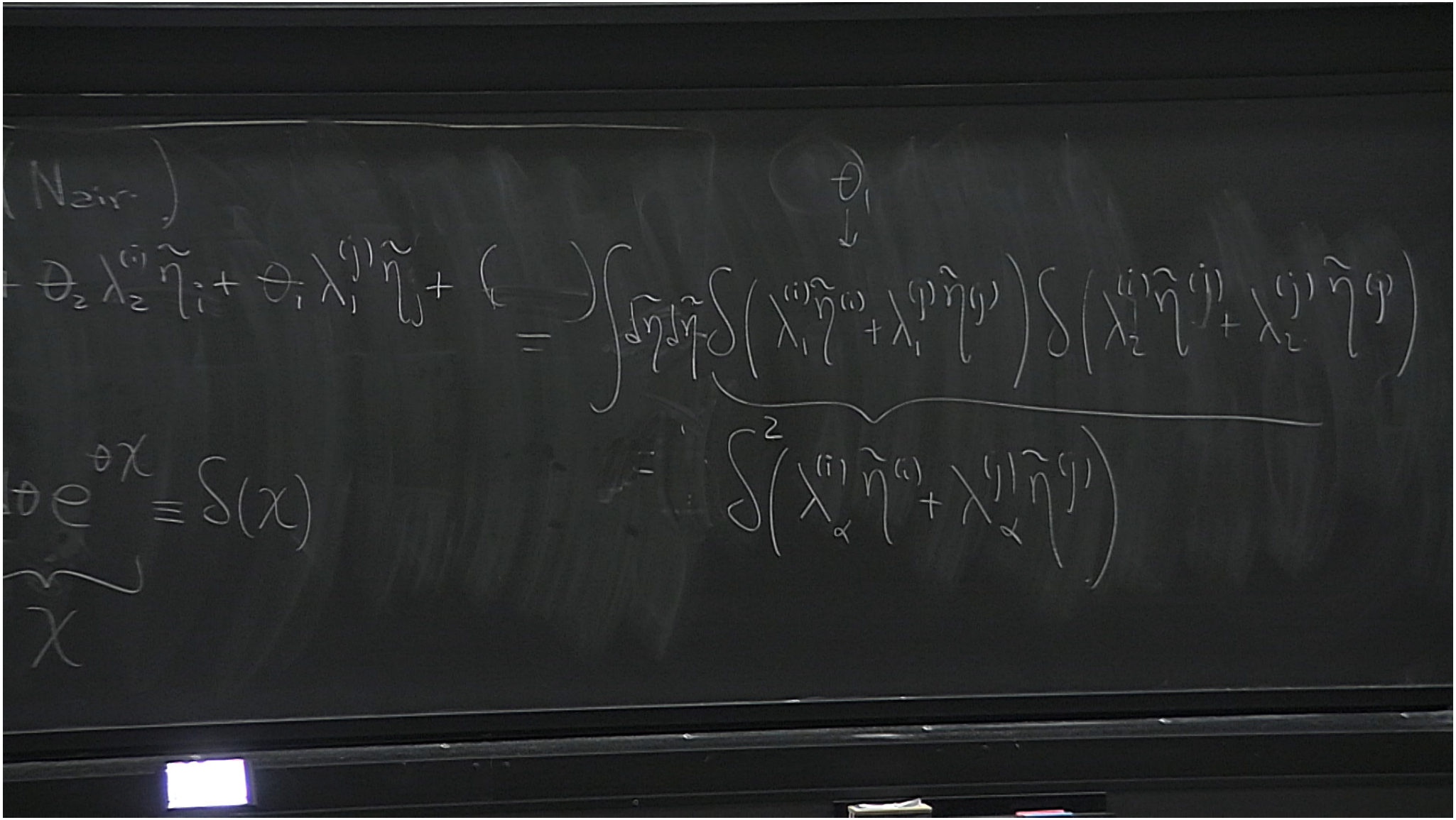

n = 4 :
In YM
$= \left(\frac{\langle 12\rangle^3}{\langle 1I\rangle\langle I2\rangle}\right)\left(\frac{[34]^3}{[3I][I4]}\right) \times \frac{1}{\langle 34\rangle[34]} = \frac{\langle 12\rangle^3[34]^2}{\langle 1I\rangle[I3]\langle 2I\rangle[I4]} = \frac{\langle 12\rangle}{\langle 1I\rangle[I3]\langle 2I\rangle[I4]}$
$\langle 1I\rangle[I3] = \lambda_\alpha^{(1)}\lambda_\beta^{(I)}\tilde{\lambda}_{\dot\alpha}^{(I)}\tilde{\lambda}_{\dot\beta}^{(3)}\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta} = \langle 14\rangle[43]$
$\langle 23\rangle[34]$
$K_3 + K_4 - Z_\phi$

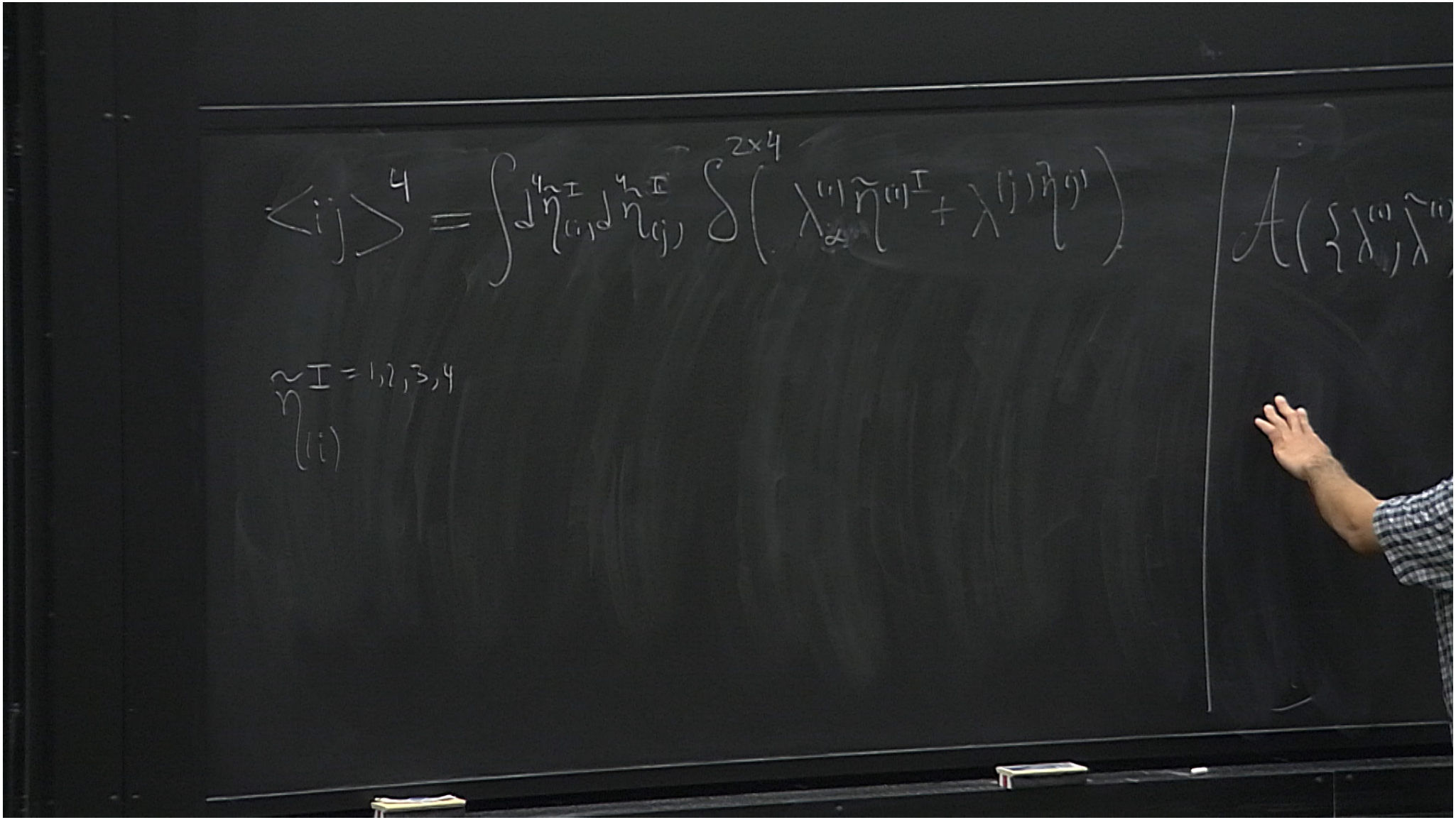

n=4 :
In YM

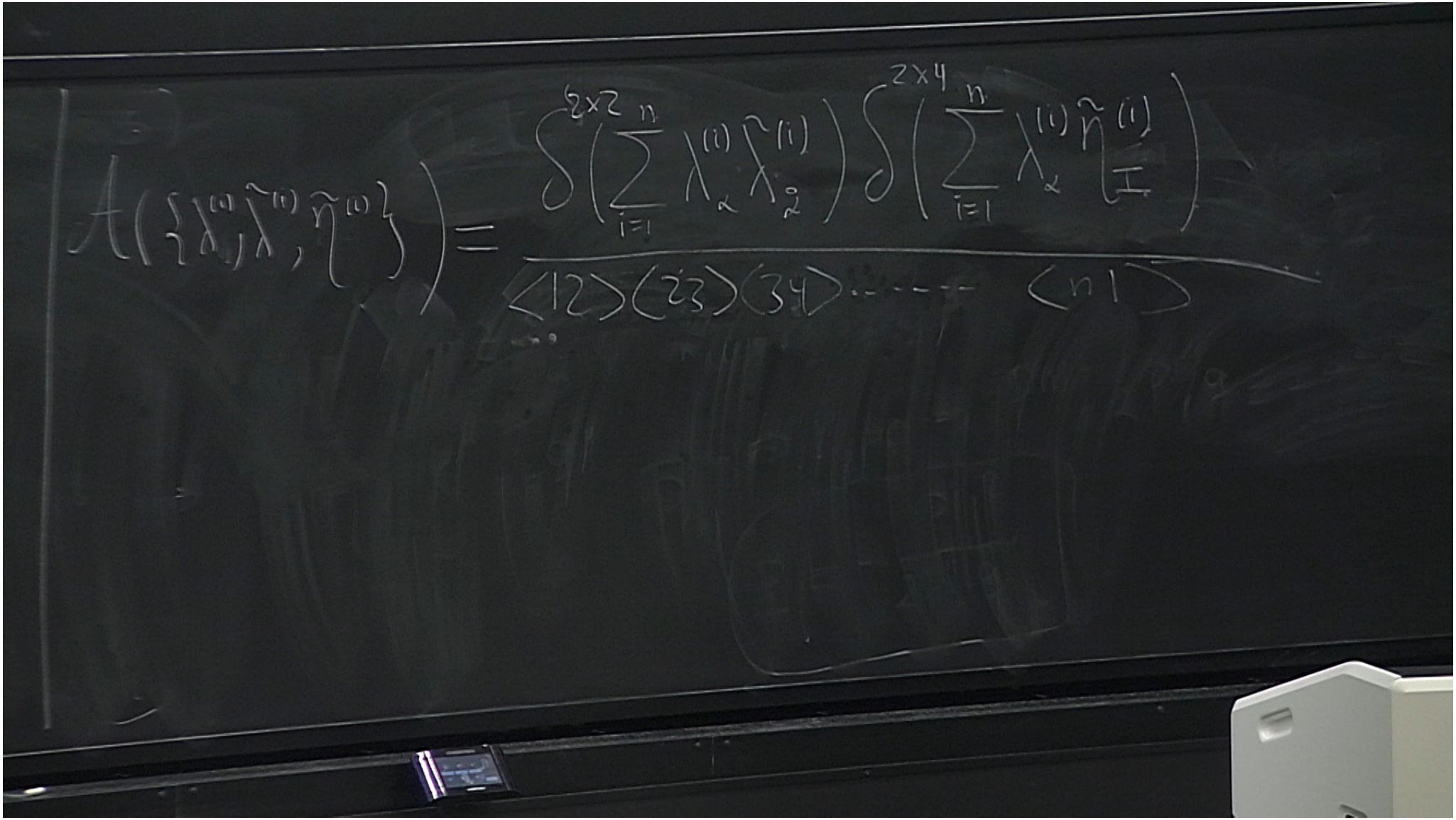

All incoming.
Generating Function for PT amplitudes (Nair)

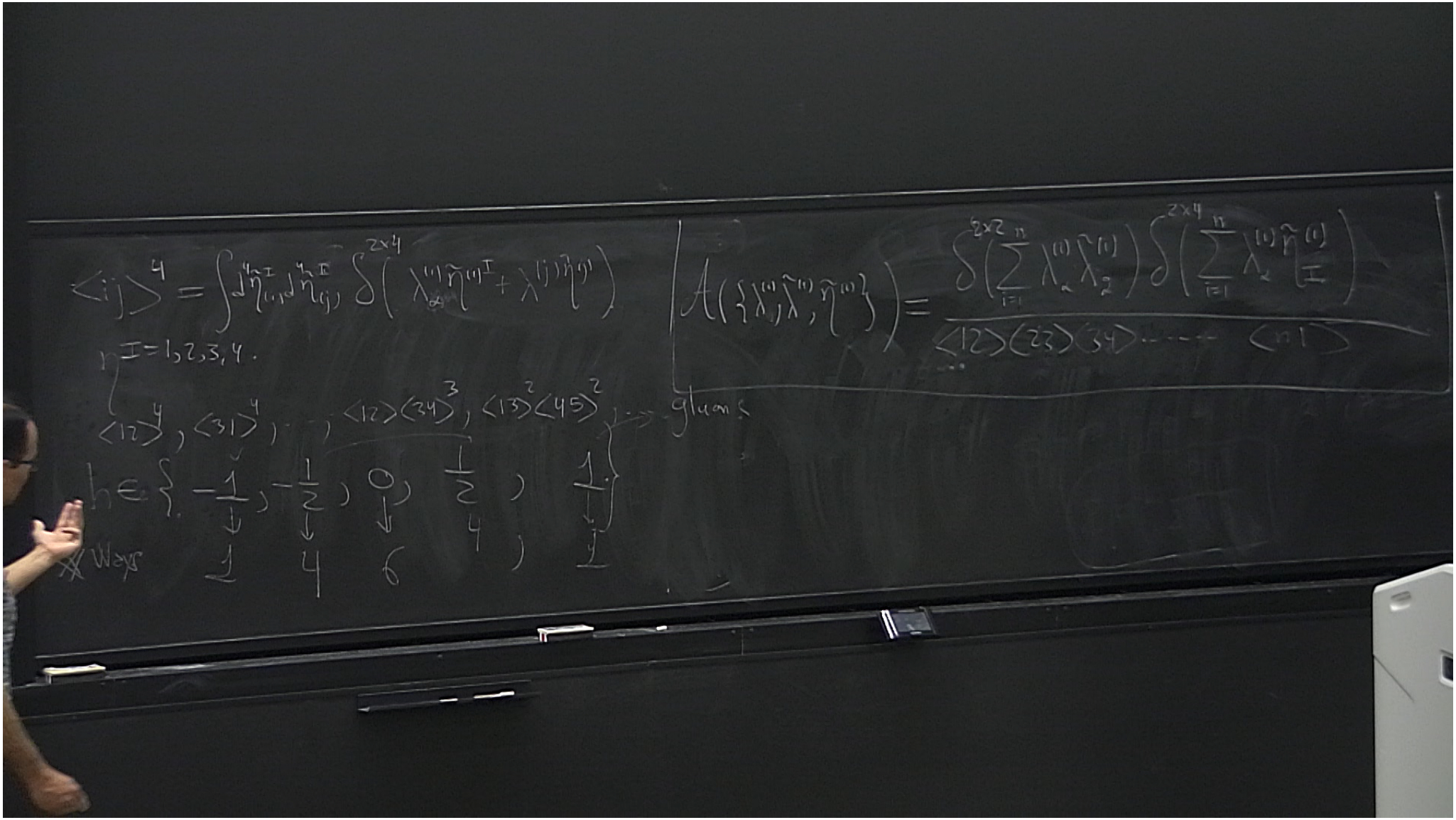


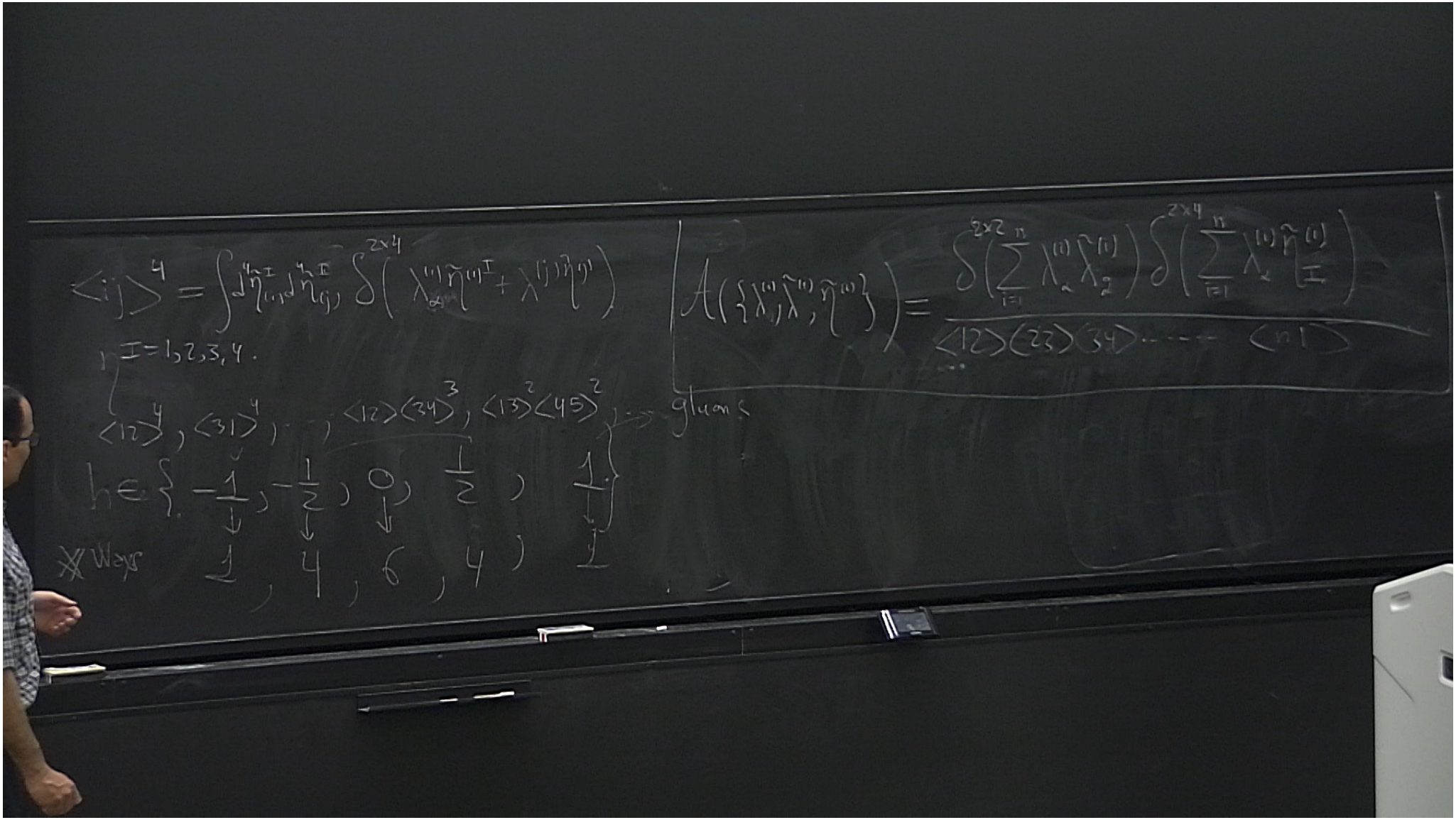

All incoming.
are not
Generating Function for PT amplitudes (Nair.)
$\theta^2 = 0 = \tilde{\eta}^2$
$\int d\theta\, \theta = 1 \qquad \int d\theta = 0$
$\int d\chi\, \delta(\chi) = 1$
$\int d\theta\, e^{\theta\chi} \equiv \delta(\chi)$
χ

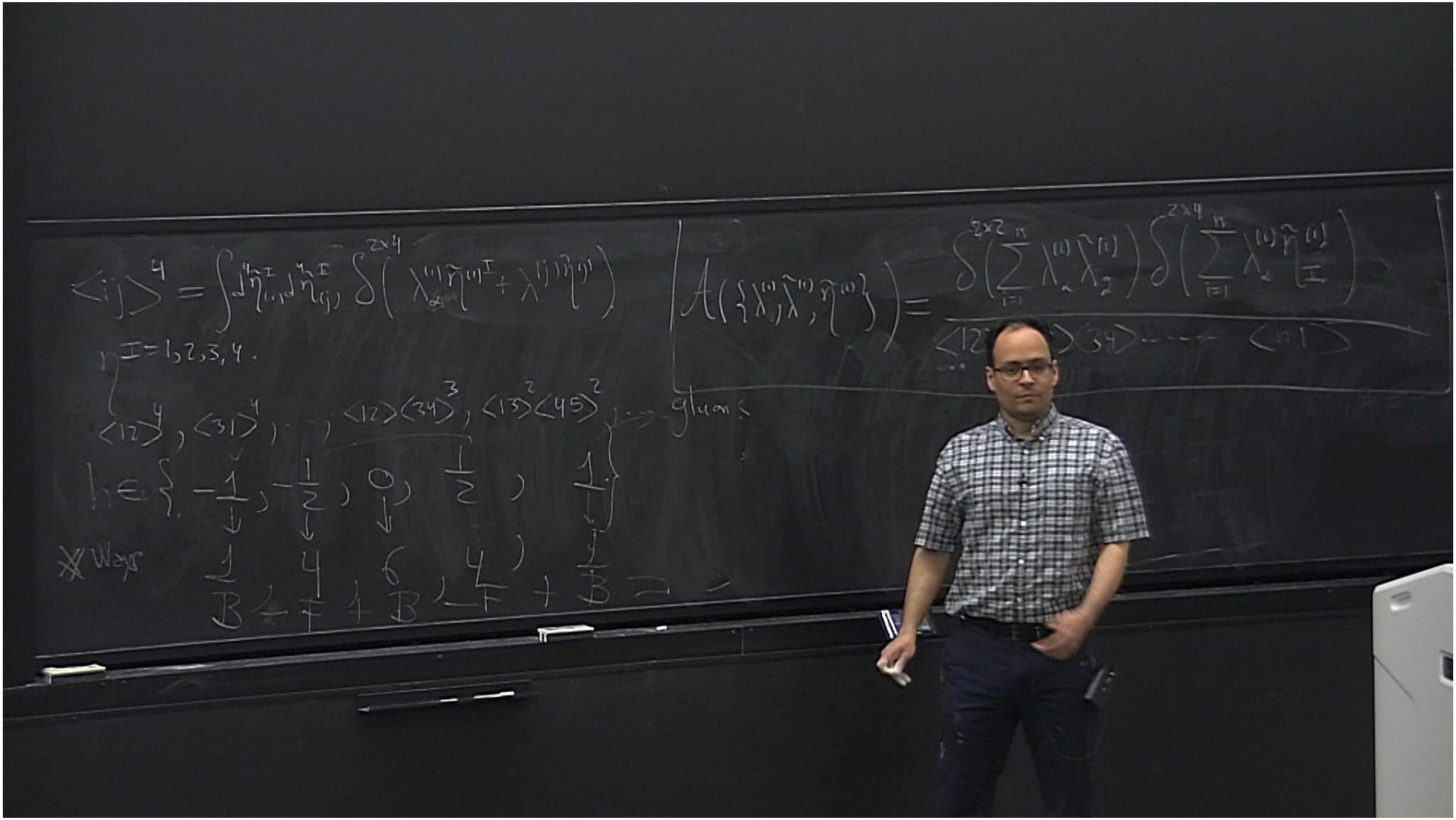

(N air-)
$+\theta_2 \lambda_2^{(i)} \tilde{\eta}_i + \theta_1 \lambda_1^{(j)} \tilde{\eta}_j + ($
$= \int d\tilde{\eta} d\tilde{\eta} \, \delta(\lambda_1^{(i)} \tilde{\eta}^{(i)} + \lambda_1^{(j)} \tilde{\eta}^{(j)}) \, \delta(\lambda_2^{(i)} \tilde{\eta}^{(i)} + \lambda_2^{(j)} \tilde{\eta}^{(j)})$
θ_1
$= \int^2 (\lambda_\alpha^{(i)} \tilde{\eta}^{(i)} + \lambda_\alpha^{(j)} \tilde{\eta}^{(j)})$
$\theta e^{\theta x} \equiv \delta(x)$
$\tilde{\chi}$

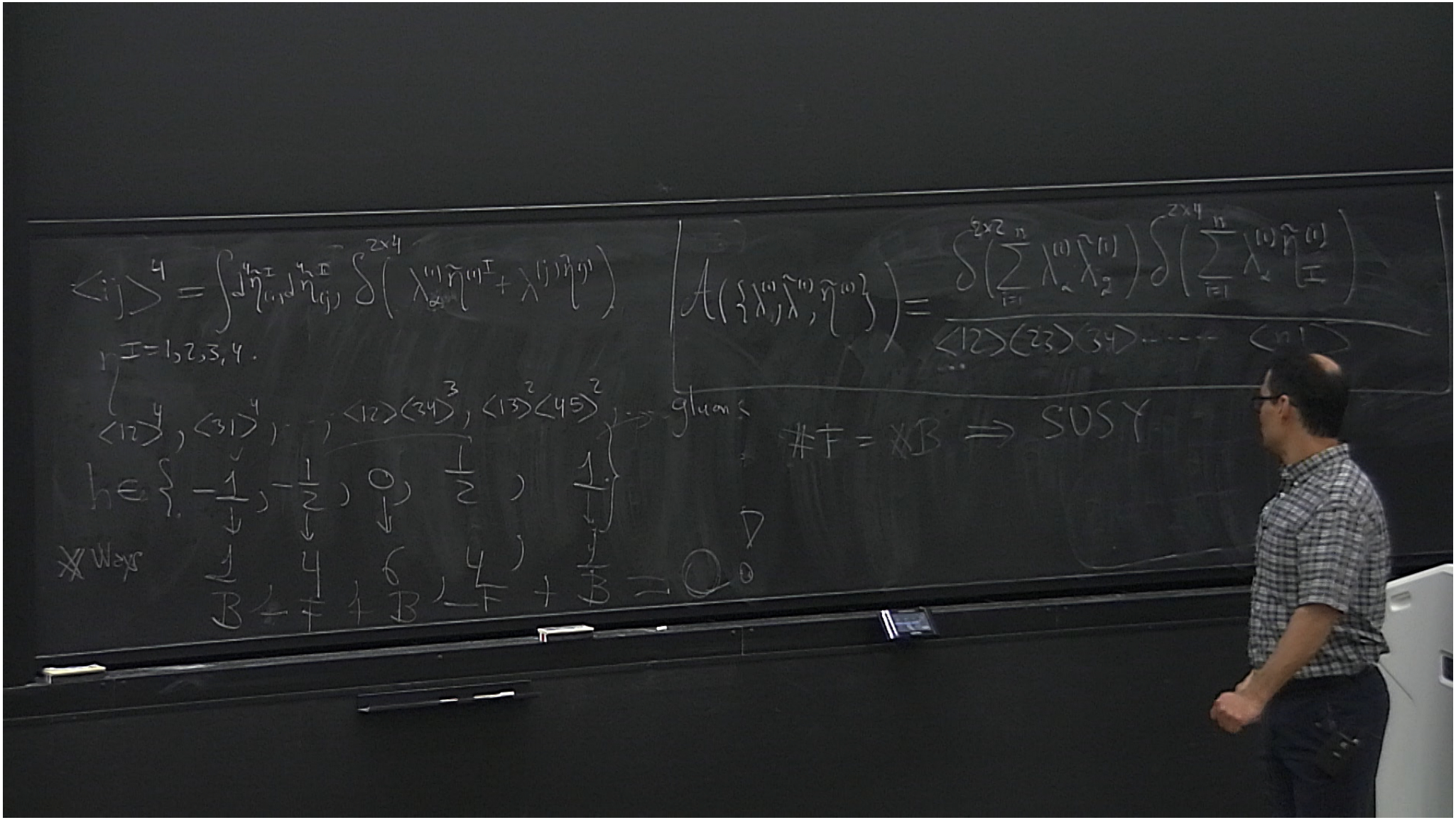

$\langle ij\rangle^4 = \int d^4\tilde{\eta}^I_{(i)} d^4\tilde{\eta}^I_{(j)}\, \delta^{2\times 4}\left(\lambda^{(i)}_\alpha \tilde{\eta}^{(i)I} + \lambda^{(j)}_\alpha \tilde{\eta}^{(j)}\right)$
$\tilde{\eta}^I_{(i)}$ $I = 1,2,3,4$
$A(\{\lambda^{(i)}, \tilde{\lambda}^{(i)}$

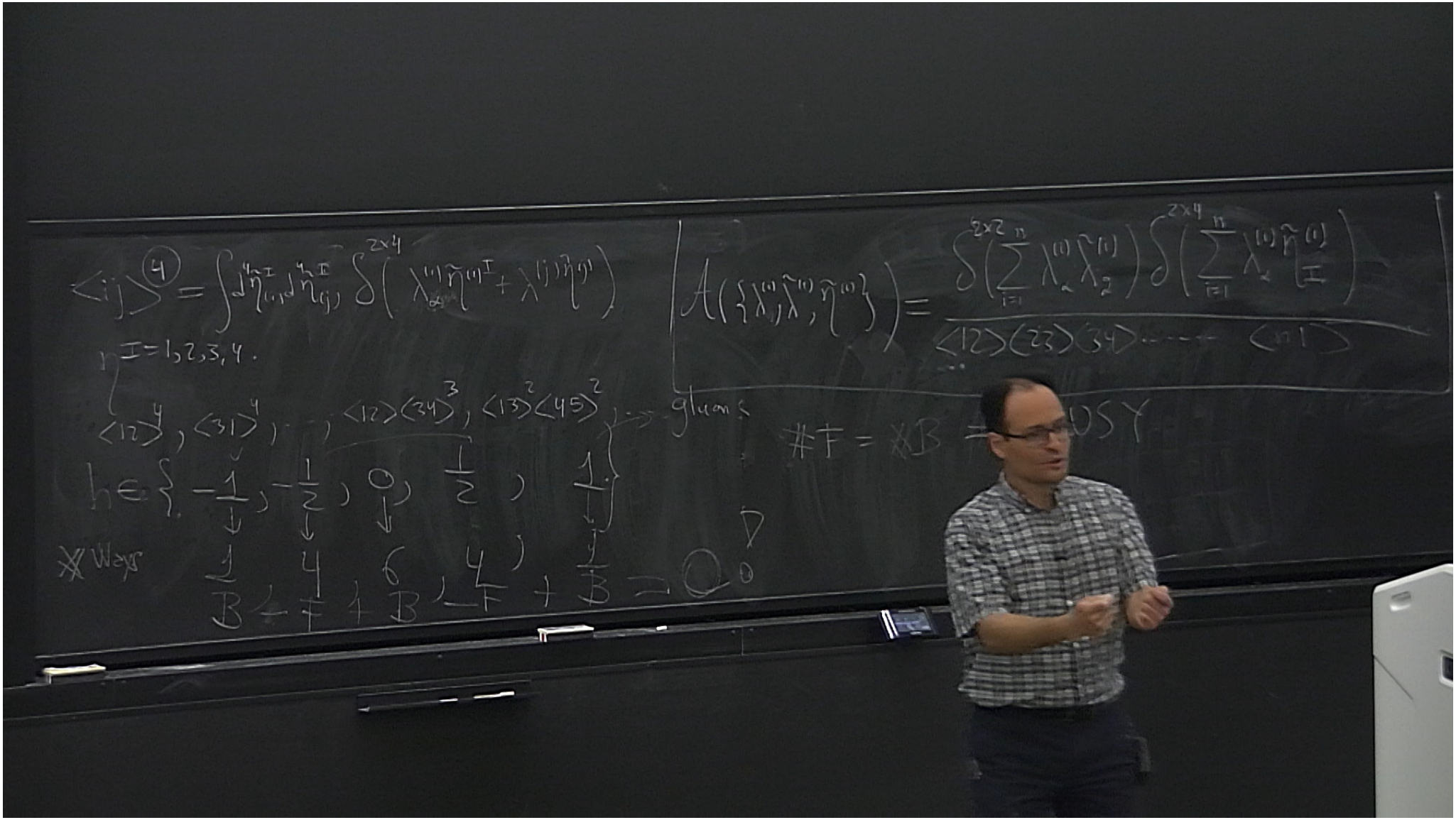

$A(\{\lambda^{(i)}, \tilde{\lambda}^{(i)}, \tilde{\eta}^{(i)}\}) = \frac{\delta^{2\times 2}\left(\sum_{i=1}^{n} \lambda_\alpha^{(i)} \tilde{\lambda}_\beta^{(i)}\right) \delta^{2\times 4}\left(\sum_{i=1}^{n} \lambda_\alpha^{(i)} \tilde{\eta}_I^{(i)}\right)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle \cdots \langle n1\rangle}$

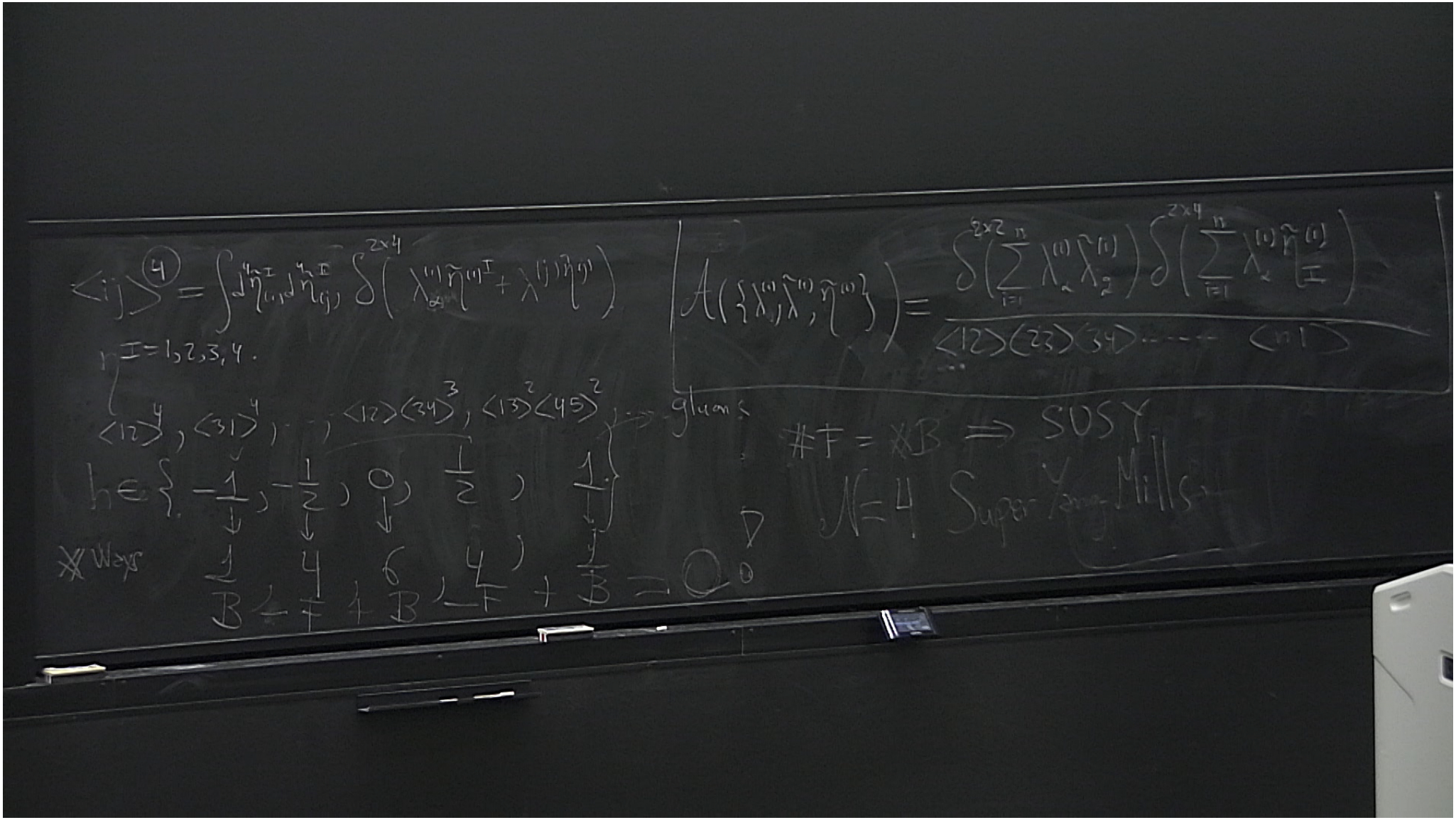

$\langle ij\rangle^4 = \int d^{4|4}\tilde{\eta}_{(i)} d^{4|4}\tilde{\eta}_{(j)}\, \delta^{2\times 4}\left(\lambda^{(i)}_\alpha \tilde{\eta}^{(i)I} + \lambda^{(j)}_\alpha \tilde{\eta}^{(j)I}\right)$
$I = 1,2,3,4.$
$\mathcal{A}(\{\lambda^{(i)}, \tilde{\lambda}^{(i)}, \tilde{\eta}^{(i)}\}) = \frac{\delta^{2\times 2}\left(\sum_{i=1}^{n} \lambda_\alpha^{(i)} \tilde{\lambda}_\beta^{(i)}\right) \delta^{2\times 4}\left(\sum_{i=1}^{n} \lambda_\alpha^{(i)} \tilde{\eta}_I^{(i)}\right)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle \cdots \langle n1\rangle}$
$\langle 12\rangle^4, \langle 31\rangle^4, \ldots, \langle 12\rangle\langle 34\rangle^3, \langle 13\rangle^2\langle 45\rangle^2, \ldots$ gluons
$h \in \{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\}$
Ways 1 4 6 4 1


$\langle ij\rangle^4 = \int d^{4n_i^I} d^{4n_j^I} \delta^{2\times 4}\left(\lambda_\alpha^{(i)} \tilde{\eta}^{(i)I} + \lambda^{(j)} \tilde{\eta}^{(j)}\right)$
$I = 1,2,3,4.$
$\langle 12\rangle^4, \langle 31\rangle^4, \ldots, \langle 12\rangle\langle 34\rangle^3, \langle 13\rangle^2\langle 45\rangle^2, \ldots$ gluons
$h \in \{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\}$
$\mathcal{A}(\{\lambda^{(i)}, \tilde{\lambda}^{(i)}, \tilde{\eta}^{(i)}\}) = \frac{\delta^{2\times 2}\left(\sum_{i=1}^n \lambda_\alpha^{(i)} \tilde{\lambda}_\beta^{(i)}\right) \delta^{2\times 4}\left(\sum_{i=1}^n \lambda_\alpha^{(i)} \tilde{\eta}_I^{(i)}\right)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle \cdots \langle n1\rangle}$


$\langle ij\rangle^4 = \int d^4\eta_{(i),I}\, d^4\eta_{(j),I}\, \delta^{2\times 4}\left(\lambda^{(i)}_{\alpha}\tilde{\eta}^{(i)I} + \lambda^{(j)}\eta^{(j)}\right)$
$I = 1,2,3,4.$
$\langle 12\rangle^4, \langle 31\rangle^4, \ldots, \langle 12\rangle\langle 34\rangle^3, \langle 13\rangle^2\langle 45\rangle^2, \ldots$ gluons
$h \in \{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\}$
MHV
$\mathcal{A}(\{\lambda^{(i)}, \tilde{\lambda}^{(i)}, \eta^{(i)}\}) = \frac{\delta^{2\times 2}\left(\sum_{i=1}^{n} \lambda_1^{(i)}\tilde{\lambda}_2^{(i)}\right)\delta^{2\times 4}\left(\sum_{i=1}^{n} \lambda_\alpha^{(i)}\tilde{\eta}_I^{(i)}\right)}{\langle 12\rangle \ldots \langle 34\rangle \ldots \langle n1\rangle}$


$\langle ij\rangle^4 = \int d^{4n_i^I} \eta_{(i)}^I d^{4n_j^I} \eta_{(j)}^I \, \delta^{2\times 4}\left(\lambda_\alpha^{(i)} \tilde{\eta}^{(i)I} + \lambda^{(j)} \tilde{\eta}^{(j)}\right)$
$I = 1,2,3,4.$
$\mathcal{A}(\{\lambda^{(i)}, \tilde{\lambda}^{(i)}, \tilde{\eta}^{(i)}\}) = \frac{\delta^{2\times 2}\left(\sum_{i=1}^{n} \lambda_\alpha^{(i)} \tilde{\lambda}_2^{(i)}\right) \delta^{2\times 4}\left(\sum_{i=1}^{n} \lambda_\alpha^{(i)} \tilde{\eta}_I^{(i)}\right)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle \cdots \langle n1\rangle}$
$\langle 12\rangle^4, \langle 31\rangle^4, \ldots, \langle 12\rangle\langle 34\rangle^3, \langle 13\rangle^2\langle 45\rangle^2$ ⟶ gluons
$h \in \{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\}$
$\#F = \#B \Rightarrow$ SUSY
Weyl
$B + \frac{1}{2}F + B + \frac{1}{2}F + B = 0$?


I=1,2,3,4.
gluons
#F = #B = SUSY


I=1,2,3,4.
gluons
#F = #B ⇒ SUSY
N=4 Super Yang-Mills