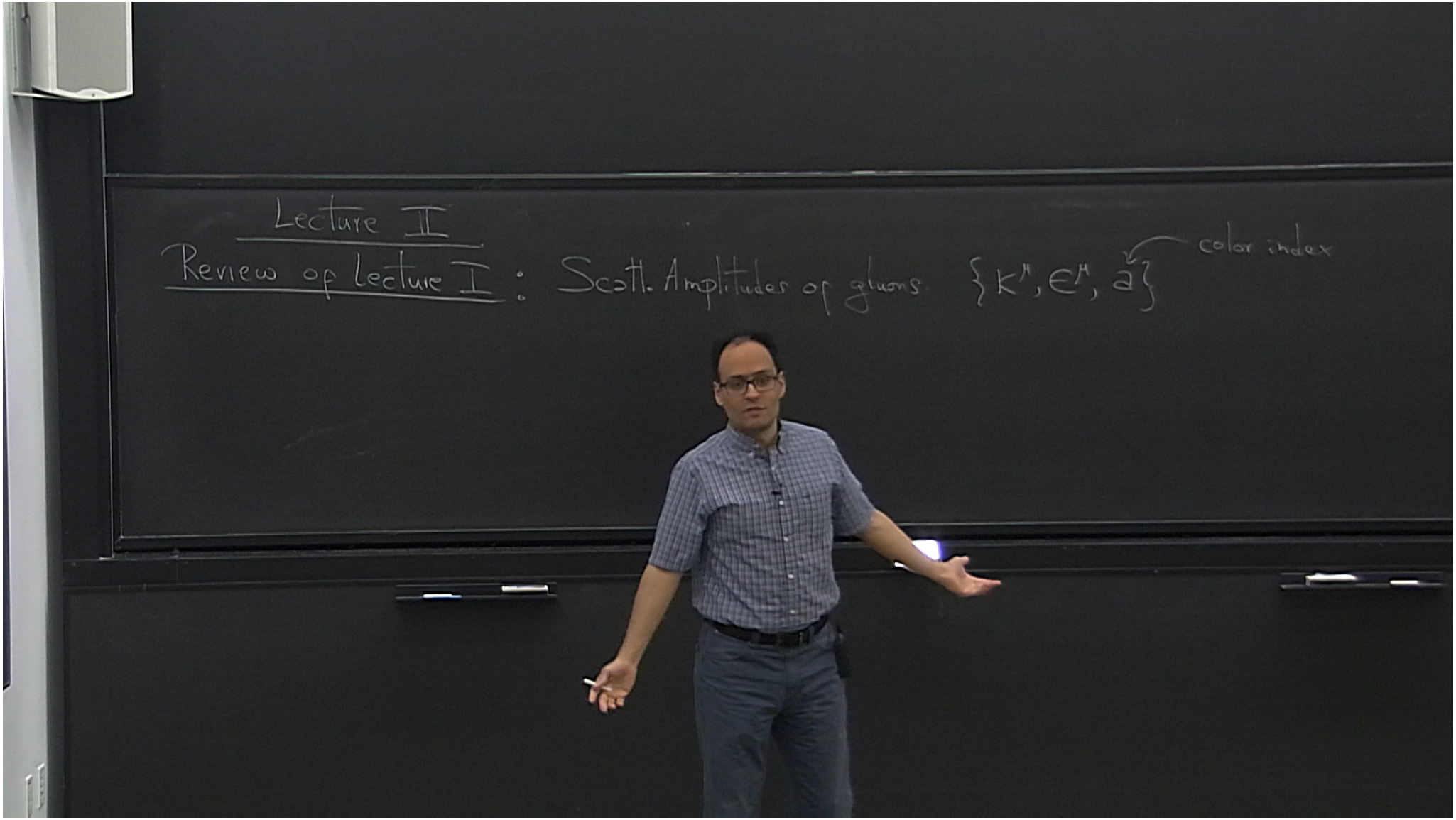
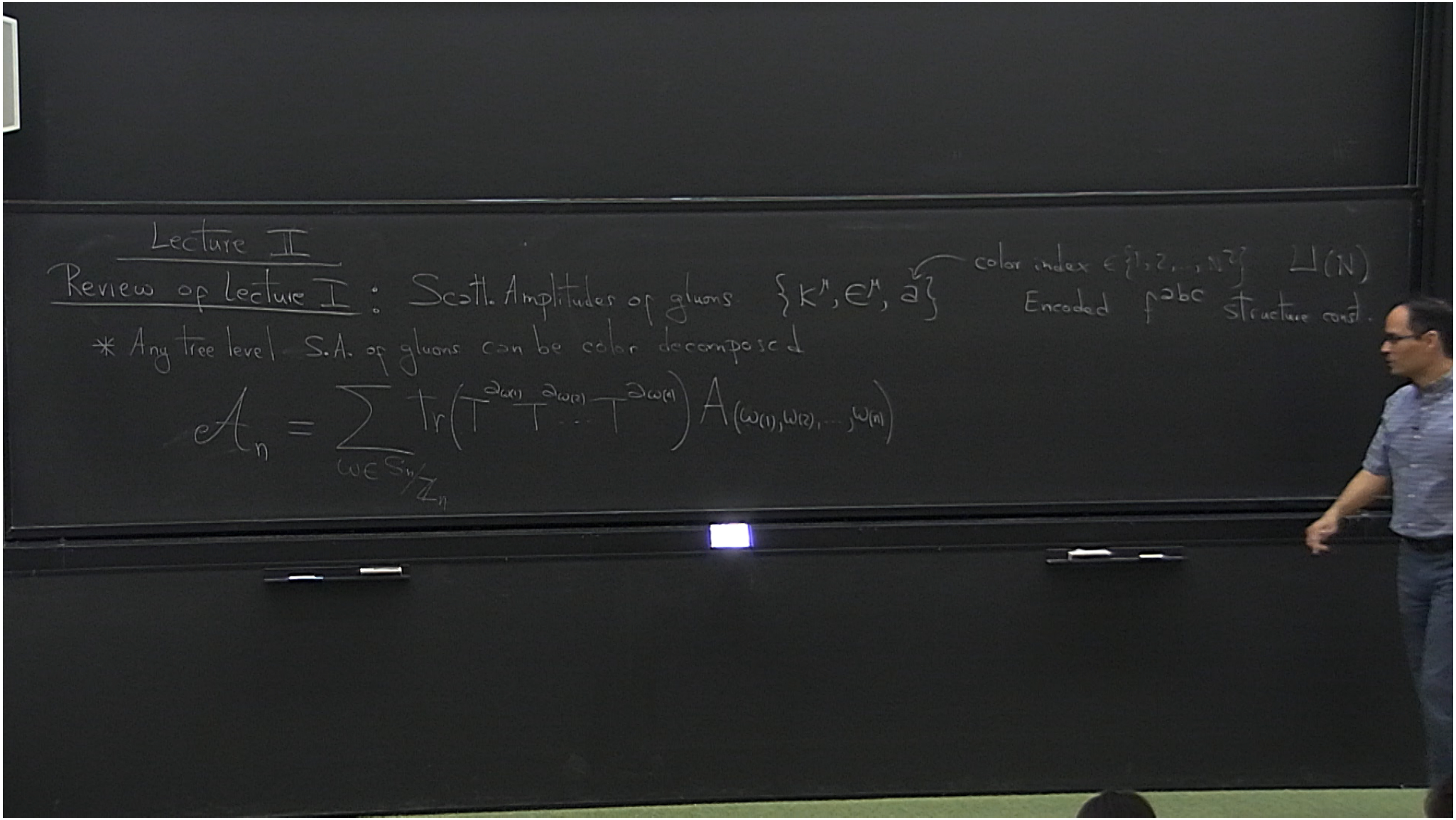
Title: PSI 2017/2018 - Scattering Amplitudes in QFT & String Theory - Lecture 2

Date: Apr 10, 2018 11:30 AM

URL: http://pirsa.org/18040036

Abstract:


Lecture II
Review of Lecture I : Scatt. Amplitudes of gluons
{k^μ, ε^μ, a}
color index


Lecture II
Review of Lecture I : Scatt. Amplitudes of gluons $\{k^\mu, \epsilon^\mu, a\}$
color index $\in \{1, 2, \ldots, N^2\}$ $\sqcup(N)$
Encoded f^{abc} structure const.
* Any tree level S.A. of gluons can be color decomposed
$$\mathcal{A}_n = \sum_{\omega \in S_n/Z_n} \mathrm{Tr}\left(T^{a_{\omega(1)}} T^{a_{\omega(2)}} \cdots T^{a_{\omega(n)}}\right) A_{(\omega(1),\omega(2),\ldots,\omega(n))}$$

* Any tree level S.A. of gluons can be color decomposed

$$\mathcal{A}_n = \sum_{\omega \in S_n/Z_n} \mathrm{Tr}\left(T^{a_{\omega(1)}} T^{a_{\omega(2)}} \cdots T^{a_{\omega(n)}}\right) A_{(\omega(1),\omega(2),\ldots,\omega(n))}$$

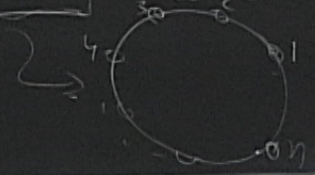
$A_{(\omega(1),\omega(2),\ldots,\omega(n))}$ → Partial amplitudes

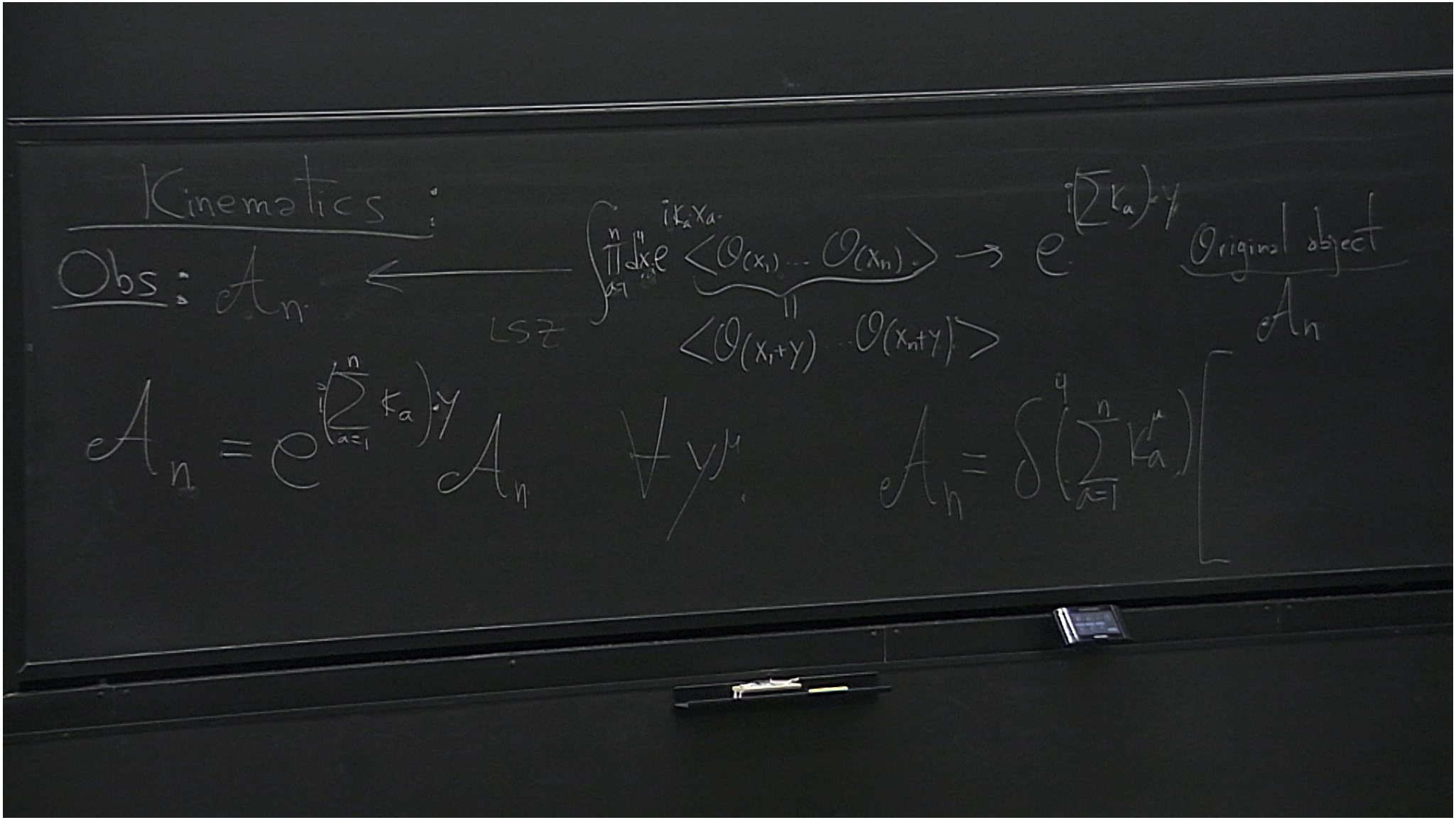
Is it useful?

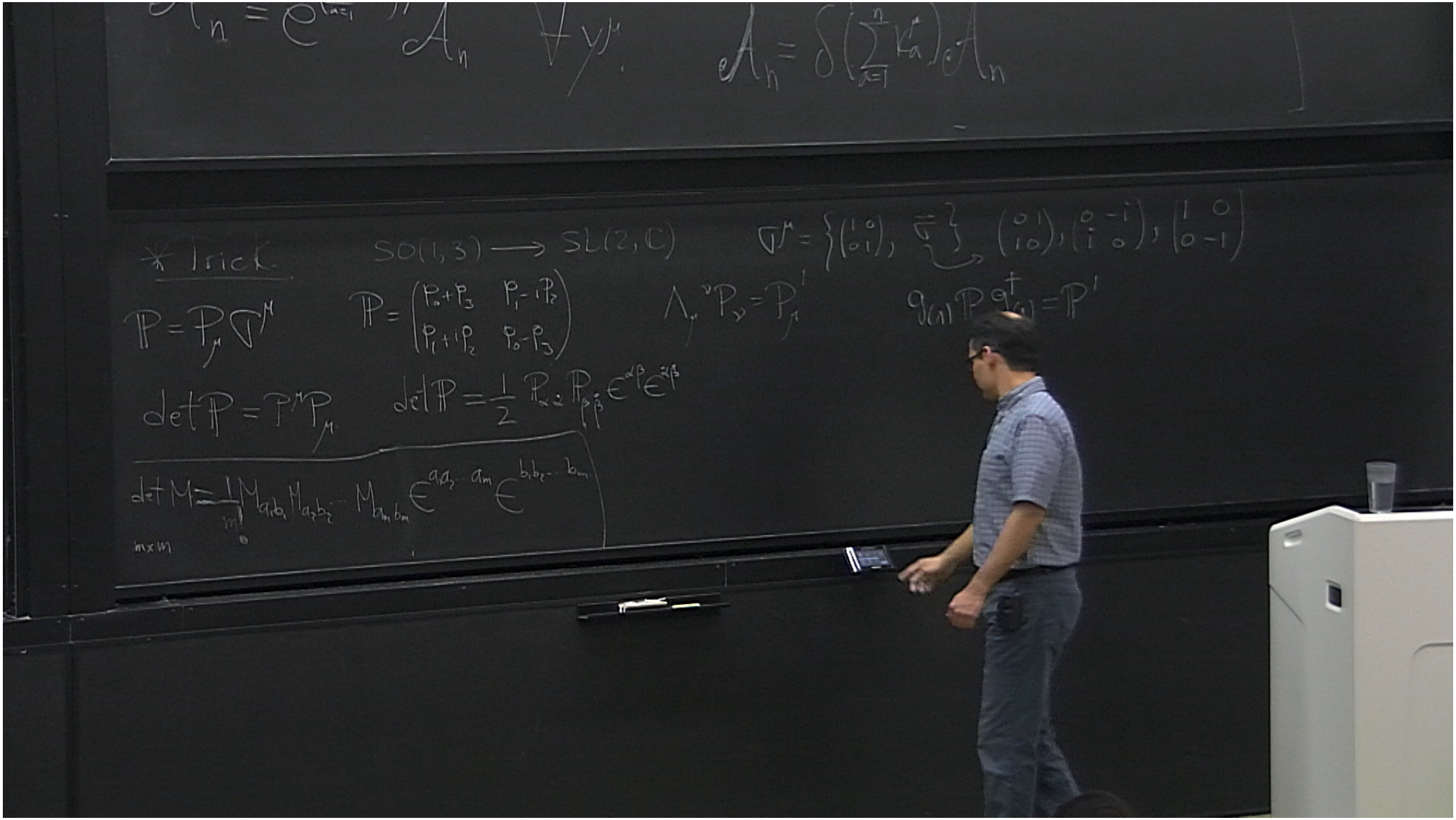
* What kind of T.D. can contribute to $A(1234)$

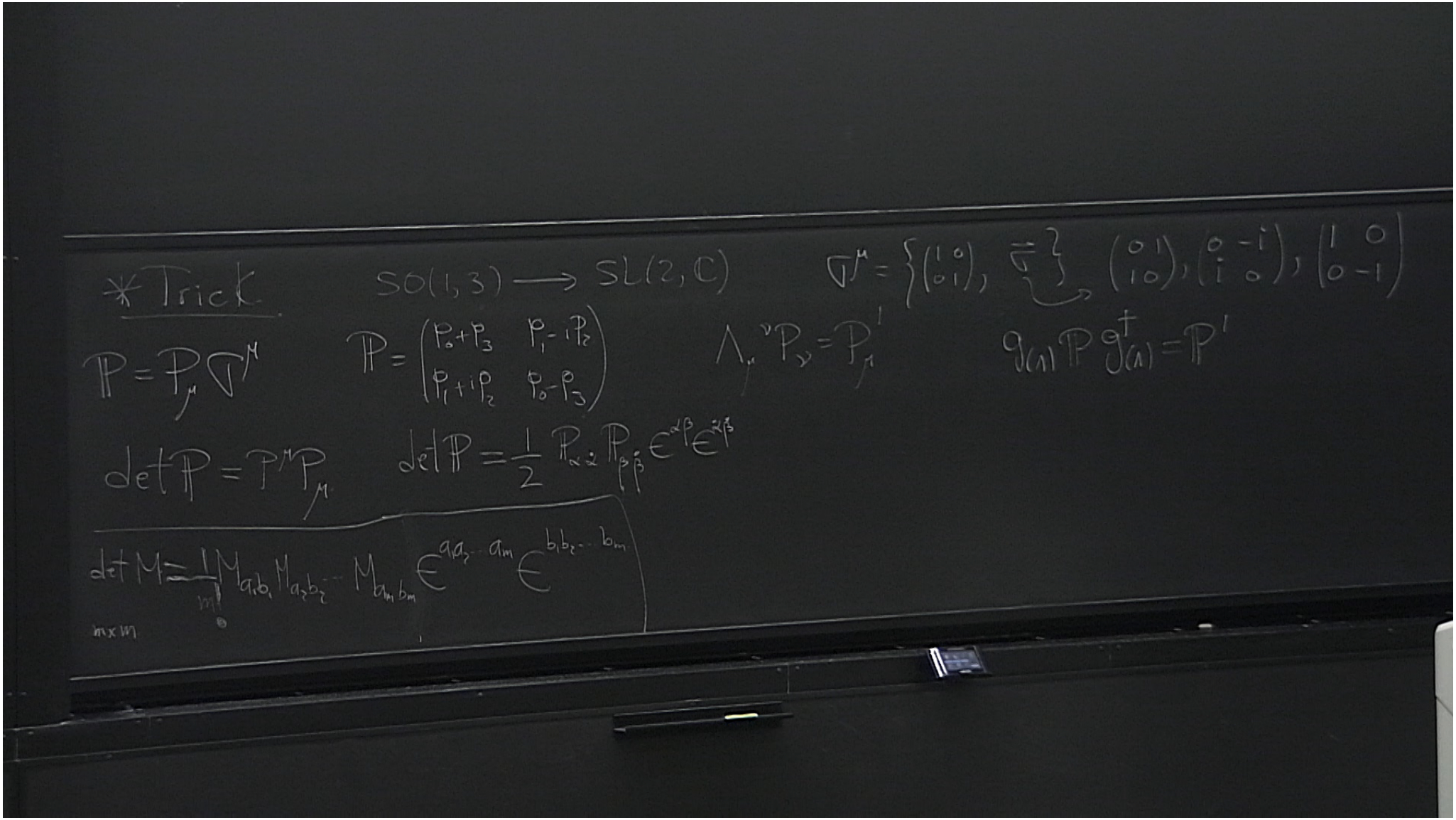


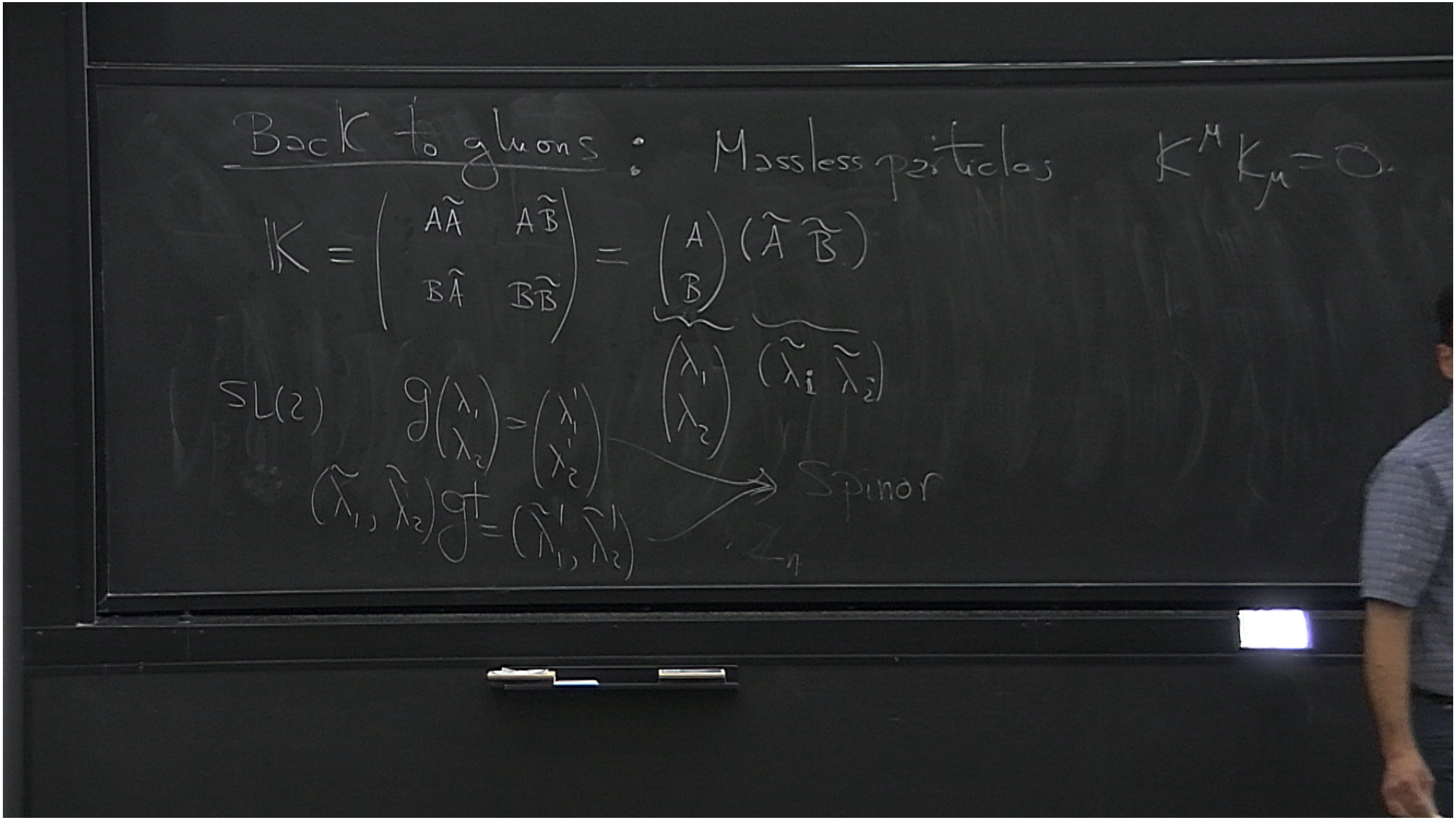
$A(12\ldots n)$ only Planar trees contribute.




Kinematics:
Obs: $\mathcal{A}_n$
LSZ
$\int \prod_{a=1}^{n} d^4x_a e^{ik_a x_a} \langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle \to e^{i(\sum k_a)\cdot y}$
$\langle \mathcal{O}(x_1+y) \cdots \mathcal{O}(x_n+y) \rangle$
original object
$\mathcal{A}_n$
$\mathcal{A}_n = e^{i(\sum_{a=1}^{n} k_a)\cdot y} \mathcal{A}_n \quad \forall y^\mu$
$\mathcal{A}_n = \delta^4(\sum_{a=1}^{n} k_a^\mu)$


* Trick
SO(1,3) ⟶ SL(2,C)
det P = P^μ P_μ
det P = 1/2 P_αα' P_ββ' ε^αβ ε^α'β'
det M = 1/m! M_a1b1 M_a2b2 ... M_ambm ε^a1a2...am ε^b1b2...bm
m×m


* Trick
$SO(1,3) \longrightarrow SL(2,\mathbb{C})$
$\sigma^\mu = \{\begin{pmatrix}1&0\\0&1\end{pmatrix}, \vec{\sigma}\}, \begin{pmatrix}0&1\\1&0\end{pmatrix}, \begin{pmatrix}0&-i\\i&0\end{pmatrix}, \begin{pmatrix}1&0\\0&-1\end{pmatrix}$
$P = P_\mu \sigma^\mu$
$P = \begin{pmatrix}p_0+p_3 & p_1-ip_2\\ p_1+ip_2 & p_0-p_3\end{pmatrix}$
$\Lambda_\mu{}^\nu P_\nu = P'_\mu$
$g_{(\Lambda)} P g^\dagger_{(\Lambda)} = P'$
$\det P = P^\mu P_\mu$
$\det P = \frac{1}{2} P_{\alpha\dot\alpha} P_{\beta\dot\beta} \epsilon^{\alpha\beta} \epsilon^{\dot\alpha\dot\beta}$
$\det M = \frac{1}{m!} M_{a_1 b_1} M_{a_2 b_2} \cdots M_{a_m b_m} \epsilon^{a_1 a_2 \cdots a_m} \epsilon^{b_1 b_2 \cdots b_m}$
$m \times m$


Back to gluons : Massless particles $K^\mu K_\mu = 0$.
$K = \begin{pmatrix} A\tilde{A} & A\tilde{B} \\ B\tilde{A} & B\tilde{B} \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} (\tilde{A}\ \tilde{B})$
$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$ $(\tilde{\lambda}_1\ \tilde{\lambda}_2)$
$SL(2)$ $g\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} \lambda_1' \\ \lambda_2' \end{pmatrix}$
$(\tilde{\lambda}_1, \tilde{\lambda}_2) g^\dagger = (\tilde{\lambda}_1', \tilde{\lambda}_2')$
Spinor
Z_n

ssless particles $K^\mu K_\mu = 0$. $\det \mathbb{K} = 0$

$(\mathbb{K}_a)_{\alpha\dot\alpha} = \lambda_\alpha^{(a)} \tilde\lambda_{\dot\alpha}^{(a)}$

$(\mathbb{K}_b)_{\beta\dot\beta} = \lambda_\beta^{(b)} \tilde\lambda_{\dot\beta}^{(b)}$

$\tilde B)$

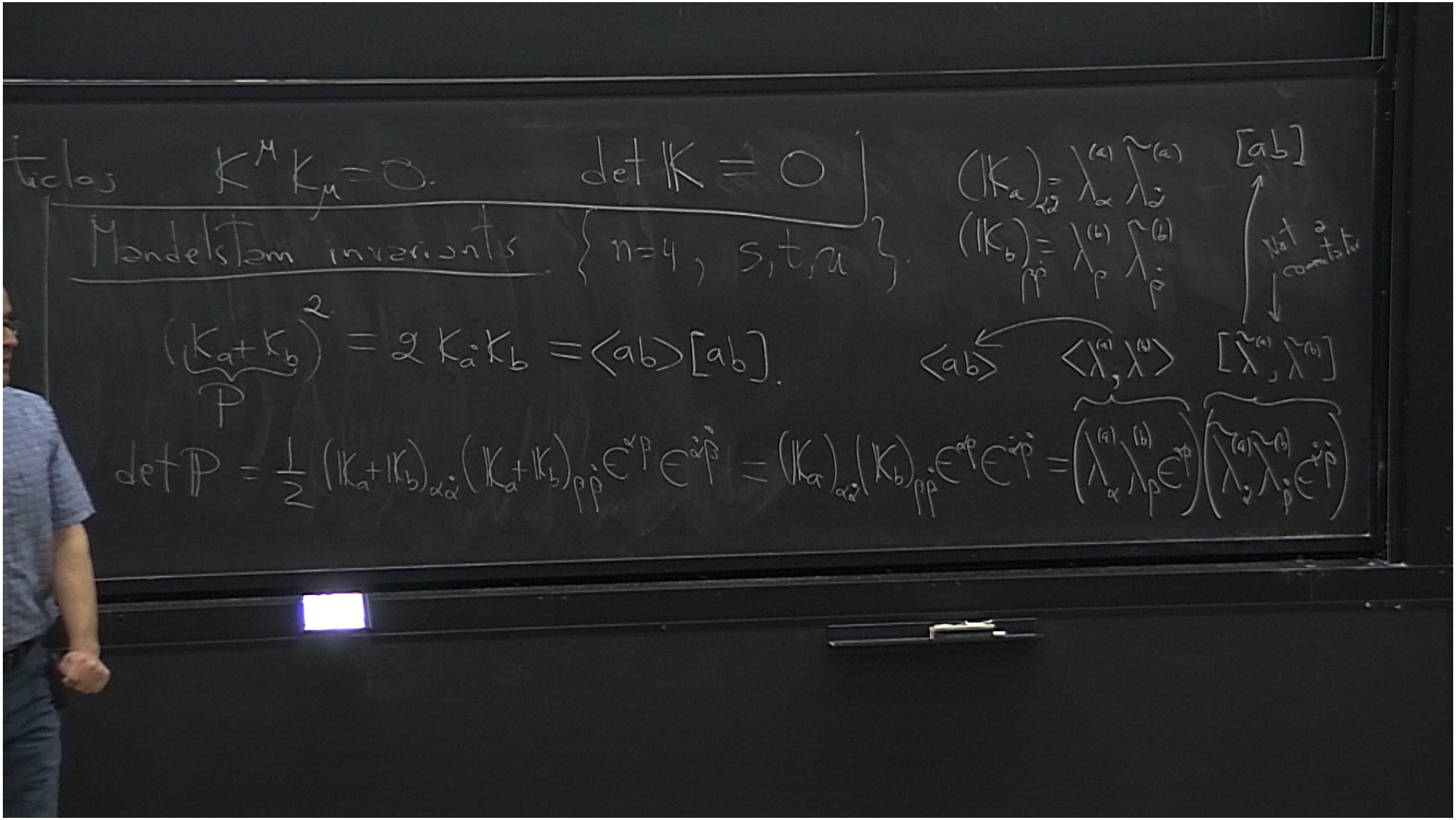
$\widetilde{\lambda_i \lambda_j})$

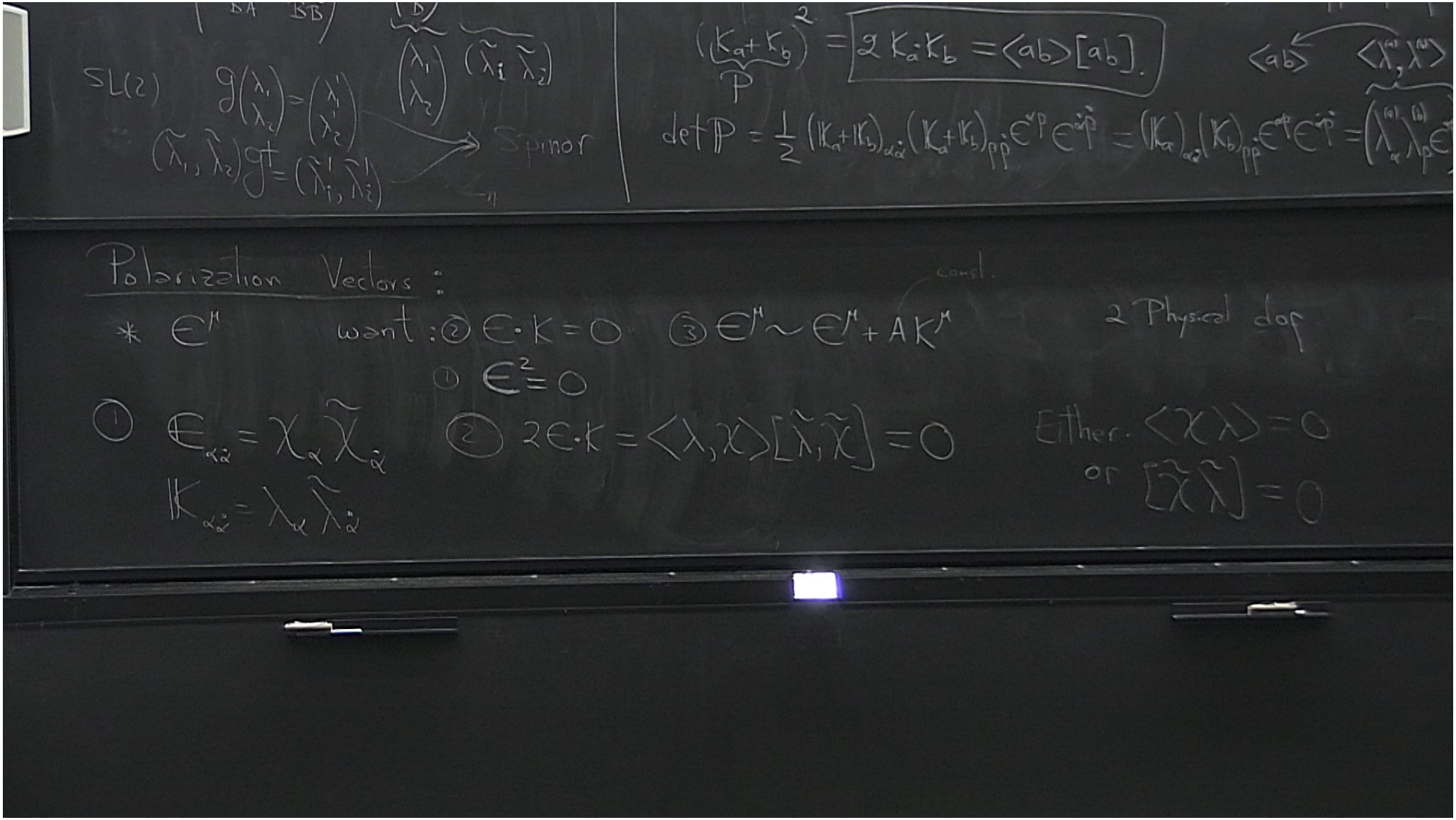
Spinor

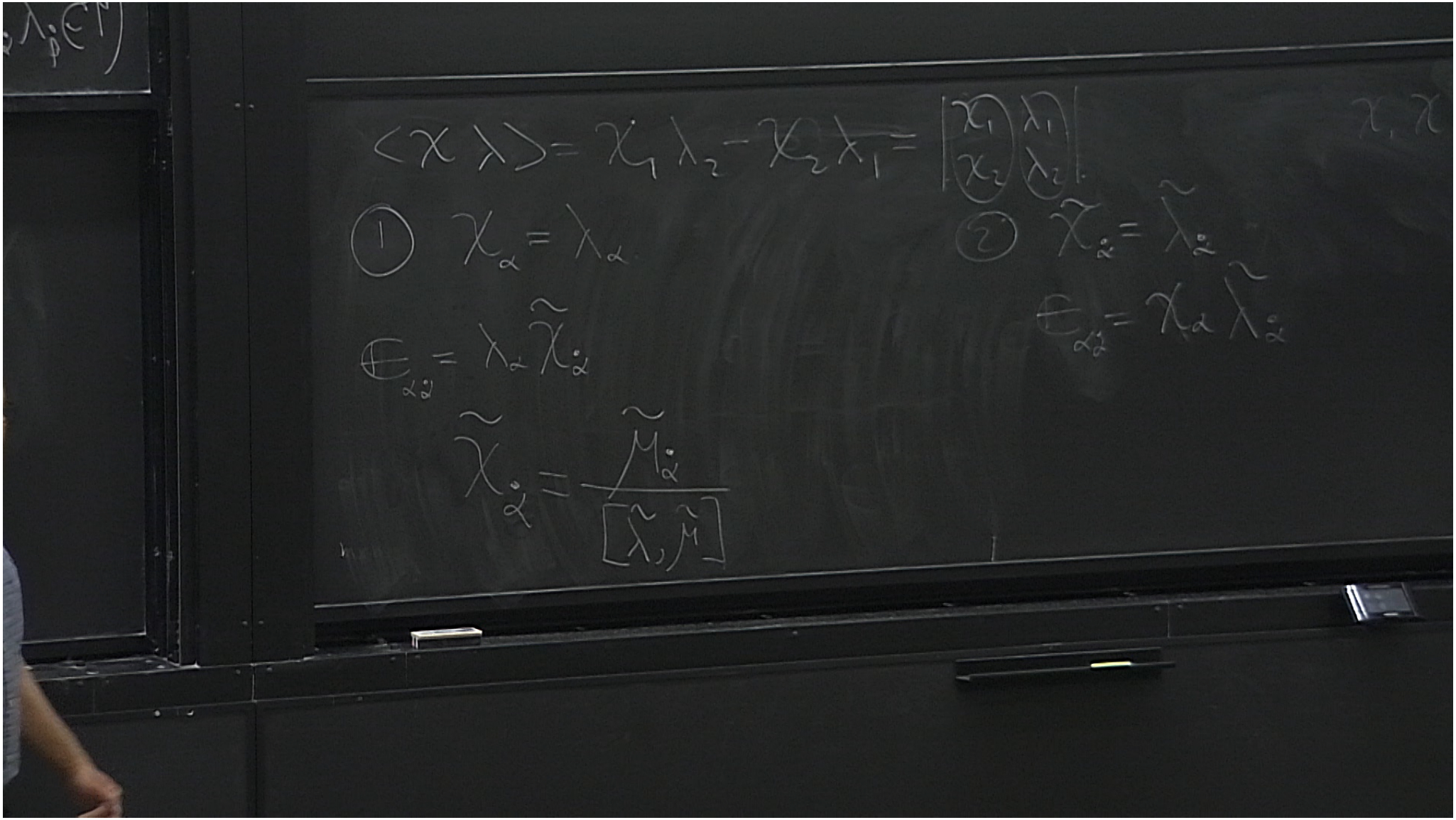
Mandelstam invariants $\{ n=4 ; s, t, u \}$

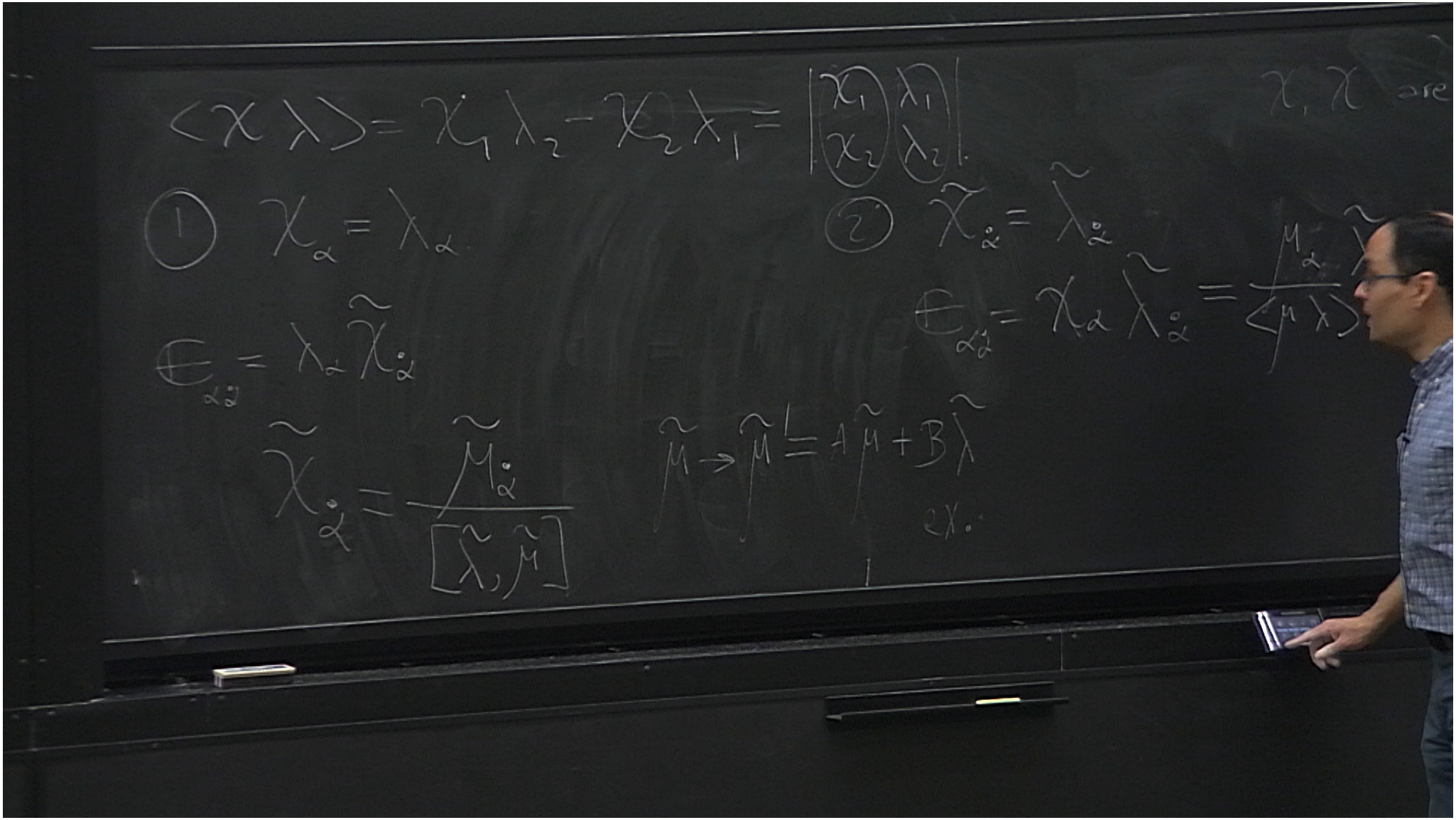
$$(\underbrace{K_a + K_b}_{P})^2 = 2 K_a \cdot K_b$$

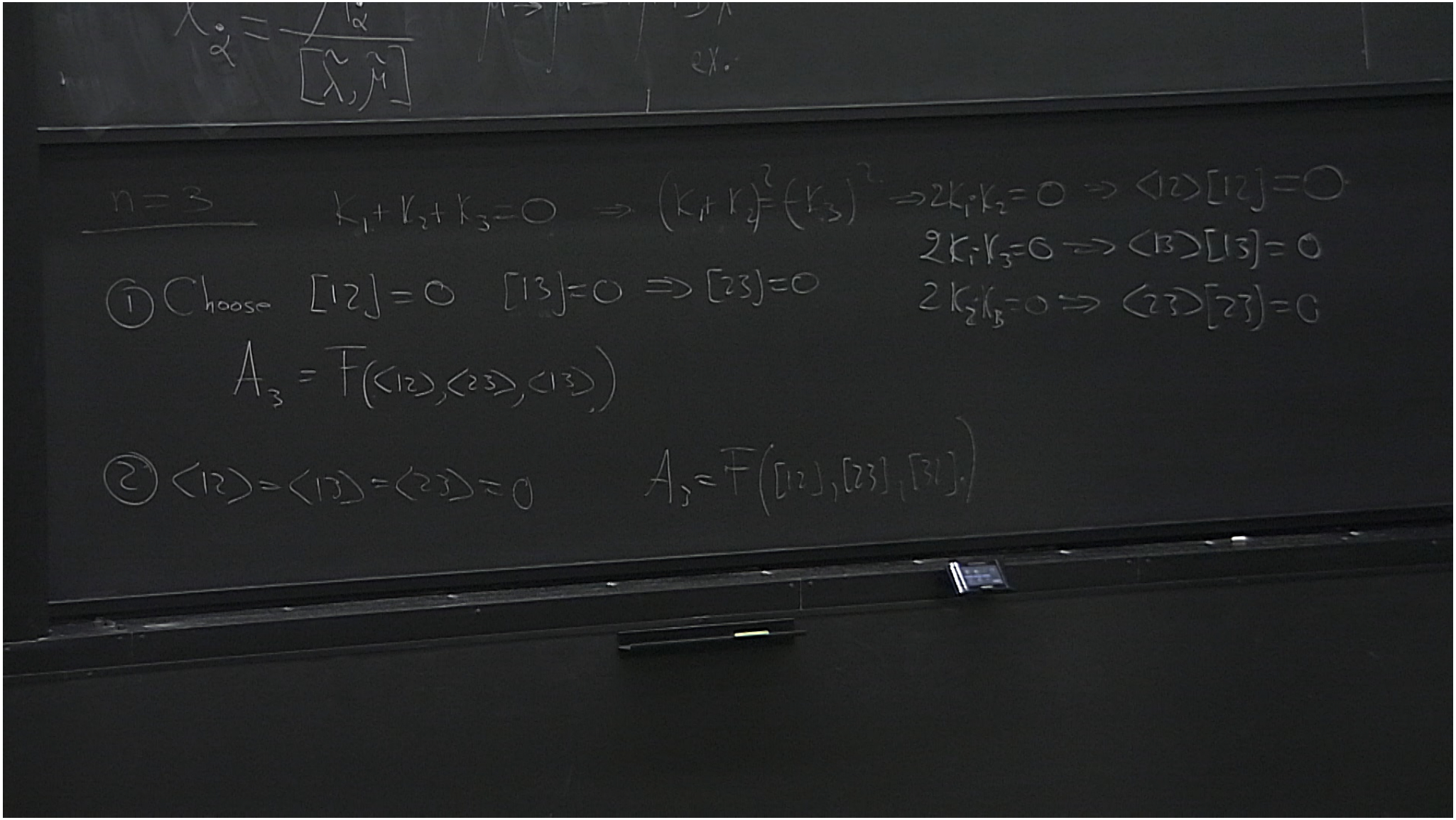
$$\det \mathbb{P} = \frac{1}{2} (\mathbb{K}_a + \mathbb{K}_b)_{\alpha\dot\alpha} (\mathbb{K}_a + \mathbb{K}_b)_{\beta\dot\beta} \epsilon^{\alpha\beta} \epsilon^{\dot\alpha\dot\beta}$$

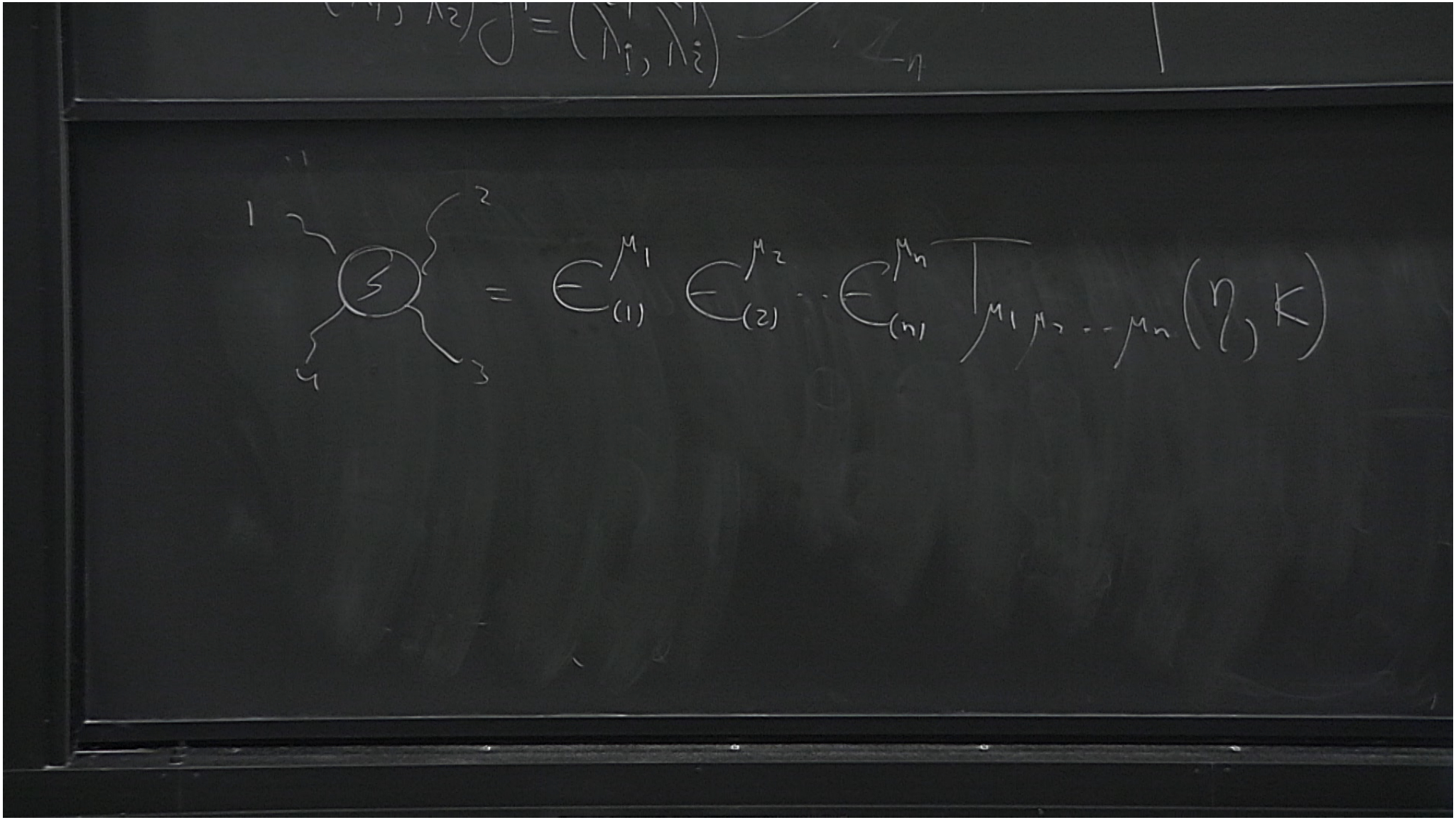

ticles $K^\mu K_\mu = 0$. $\det \mathbb{K} = 0$
Mandelstam invariants $\{n=4, s,t,u\}$.
$(\mathbb{K}_a)_{\alpha\dot\alpha} = \lambda_\alpha^{(a)} \tilde\lambda_{\dot\alpha}^{(a)}$
$(\mathbb{K}_b)_{\beta\dot\beta} = \lambda_\beta^{(b)} \tilde\lambda_{\dot\beta}^{(b)}$
$[ab]$
Not a commutator
$(\underbrace{K_a + K_b}_{P})^2 = 2 K_a \cdot K_b = \langle ab\rangle [ab]$.
$\langle ab\rangle$ $\langle \lambda^{(a)}, \lambda^{(b)}\rangle$ $[\tilde\lambda^{(a)}, \tilde\lambda^{(b)}]$
$\det \mathbb{P} = \frac{1}{2} (\mathbb{K}_a + \mathbb{K}_b)_{\alpha\dot\alpha} (\mathbb{K}_a + \mathbb{K}_b)_{\beta\dot\beta} \epsilon^{\alpha\beta} \epsilon^{\dot\alpha\dot\beta} = (\mathbb{K}_a)_{\alpha\dot\alpha} (\mathbb{K}_b)_{\beta\dot\beta} \epsilon^{\alpha\beta} \epsilon^{\dot\alpha\dot\beta} = \left(\lambda_\alpha^{(a)} \lambda_\beta^{(b)} \epsilon^{\alpha\beta}\right)\left(\tilde\lambda_{\dot\alpha}^{(a)} \tilde\lambda_{\dot\beta}^{(b)} \epsilon^{\dot\alpha\dot\beta}\right)$

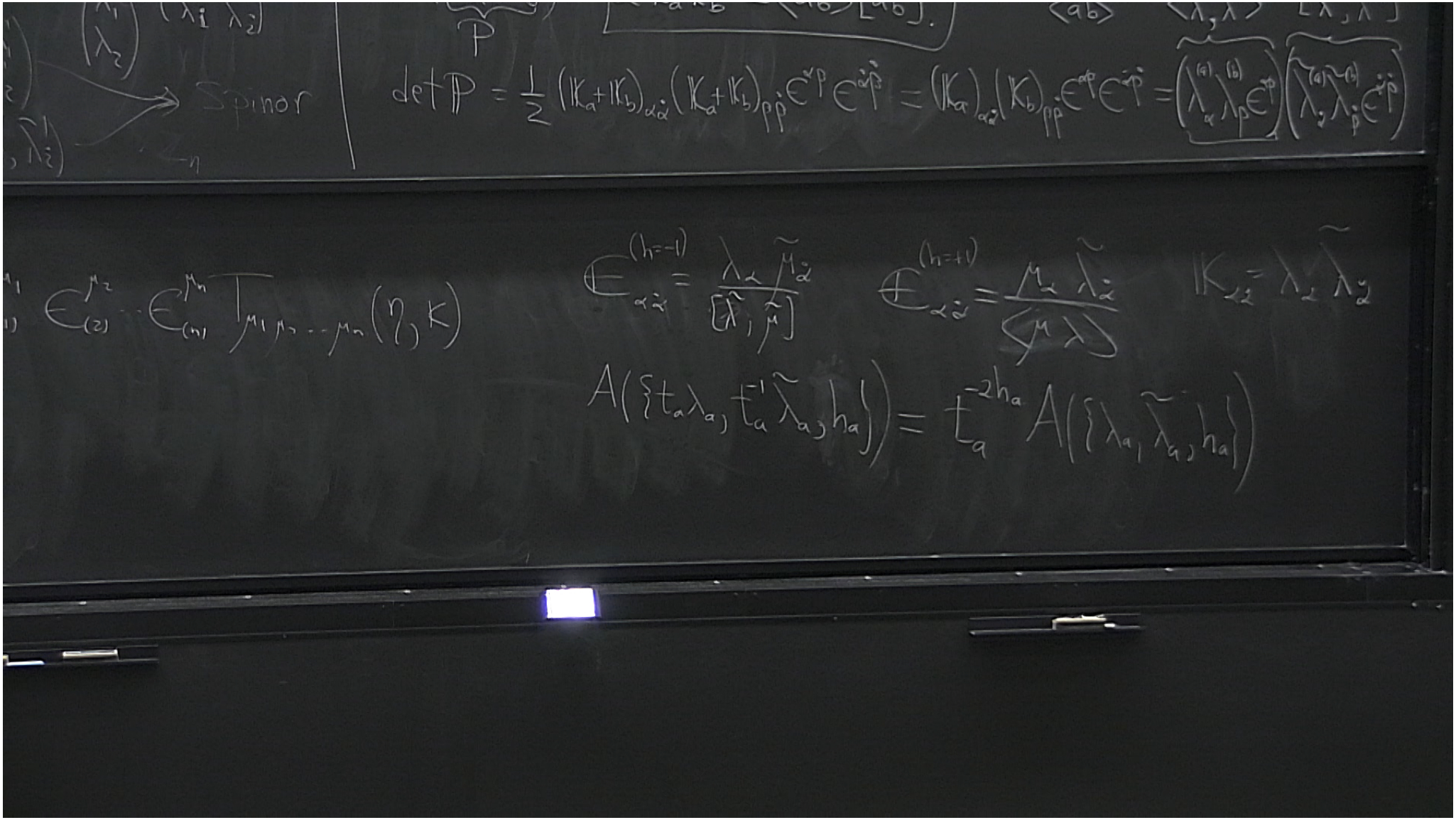

SL(2)
Spinor
$(K_a+K_b)^2 = 2K_a\cdot K_b = \langle ab\rangle[ab]$.
$\det \mathbb{P} = \frac{1}{2}(\mathbb{K}_a+\mathbb{K}_b)_{d\dot{d}}(\mathbb{K}_a+\mathbb{K}_b)_{p\dot{p}}\epsilon^{dp}\epsilon^{\dot{d}\dot{p}} = (\mathbb{K}_a)_{\alpha\dot{\alpha}}(\mathbb{K}_b)_{\beta\dot{\beta}}\epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}} = \langle\lambda_a^{(a)}\lambda_b^{(b)}\epsilon$
Polarization Vectors:
* ϵ^μ want: ② $\epsilon\cdot k = 0$ ③ $\epsilon^\mu \sim \epsilon^\mu + Ak^\mu$ 2 Physical dof
const.
① $\epsilon^2 = 0$
① $\epsilon_{\alpha\dot{\alpha}} = \chi_\alpha\tilde{\chi}_{\dot{\alpha}}$ ② $2\epsilon\cdot k = \langle\lambda,\chi\rangle[\tilde{\lambda},\tilde{\chi}] = 0$ Either $\langle\chi\lambda\rangle = 0$
or $[\tilde{\chi}\tilde{\lambda}] = 0$
$\mathbb{K}_{\alpha\dot{\alpha}} = \lambda_\alpha\tilde{\lambda}_{\dot{\alpha}}$


$\langle \chi \lambda \rangle = \chi_1 \lambda_2 - \chi_2 \lambda_1 = \begin{vmatrix} \chi_1 & \lambda_1 \\ \chi_2 & \lambda_2 \end{vmatrix}$
① $\chi_\alpha = \lambda_\alpha$
② $\tilde{\chi}_{\dot\alpha} = \tilde{\lambda}_{\dot\alpha}$
$\epsilon_{\alpha\beta} = \lambda_\alpha \tilde{\lambda}_{\dot\beta}$
$\epsilon_{\dot\alpha\dot\beta} = \chi_\alpha \lambda_{\dot\alpha}$
$\tilde{\chi}_{\dot\alpha} = \frac{\tilde{\mu}_{\dot\alpha}}{[\tilde{\lambda}, \tilde{\mu}]}$


$\langle \chi \lambda \rangle = \chi_1 \lambda_2 - \chi_2 \lambda_1 = \begin{vmatrix} \chi_1 & \lambda_1 \\ \chi_2 & \lambda_2 \end{vmatrix}$
χ, λ are
① $\chi_\alpha = \lambda_\alpha$
② $\tilde{\chi}_{\dot\alpha} = \tilde{\lambda}_{\dot\alpha}$
$\epsilon_{\alpha\dot\beta} = \lambda_\alpha \tilde{\lambda}_{\dot\beta}$
$\epsilon_{\alpha\dot\alpha} = \chi_\alpha \tilde{\lambda}_{\dot\alpha} = \frac{\mu_\alpha}{\langle \mu \lambda \rangle}$
$\tilde{\chi}_{\dot\alpha} = \frac{\tilde{\mu}_{\dot\alpha}}{[\tilde{\lambda}, \tilde{\mu}]}$
$\tilde{\mu} \to \tilde{\mu}' = A\tilde{\mu} + B\tilde{\lambda}$
ex.


$n=3$
$k_1+k_2+k_3=0 \Rightarrow (k_1+k_2)^2=(-k_3)^2 \Rightarrow 2k_1\cdot k_2=0 \Rightarrow \langle 12\rangle[12]=0$
$2k_1\cdot k_3=0 \Rightarrow \langle 13\rangle[13]=0$
$2k_2\cdot k_3=0 \Rightarrow \langle 23\rangle[23]=0$
① Choose $[12]=0$ $[13]=0 \Rightarrow [23]=0$
$A_3=F(\langle 12\rangle,\langle 23\rangle,\langle 13\rangle)$
② $\langle 12\rangle=\langle 13\rangle=\langle 23\rangle=0$
$A_3=F([12],[23],[31])$


1
2
3
4
S
$= \epsilon^{\mu_1}_{(1)} \epsilon^{\mu_2}_{(2)} \cdots \epsilon^{\mu_n}_{(n)} T_{\mu_1 \mu_2 \cdots \mu_n}(p_i, k)$


Spinor
$\det P = \frac{1}{2}(K_a+K_b)_{\alpha\dot\alpha}(K_a+K_b)_{\beta\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta} = (K_a)_{\alpha\dot\alpha}(K_b)_{\beta\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta} = \langle\lambda_a^{(a)}\lambda_b^{(b)}\rangle[\tilde\lambda_a^{(a)}\tilde\lambda_b^{(b)}]$
$\epsilon_{(2)}^{\mu_2}\cdots\epsilon_{(n)}^{\mu_n}T_{\mu_1\mu_2\cdots\mu_n}(\eta,K)$
$\epsilon^{(h=-1)}_{\alpha\dot\alpha} = \frac{\lambda_\alpha\tilde\mu_{\dot\alpha}}{[\tilde\lambda,\tilde\mu]}$
$\epsilon^{(h=+1)}_{\alpha\dot\alpha} = \frac{\mu_\alpha\tilde\lambda_{\dot\alpha}}{\langle\mu\lambda\rangle}$
$K_{\alpha\dot\alpha} = \lambda_\alpha\tilde\lambda_{\dot\alpha}$
$A(\{t_a\lambda_a, t_a^{-1}\tilde\lambda_a, h_a\}) = t_a^{-2h_a}A(\{\lambda_a,\tilde\lambda_a,h_a\})$

$$\tilde{\chi}_\alpha = \frac{\tilde{M}_\alpha}{[\tilde{\lambda}, \tilde{M}]} \qquad \tilde{M} \to \tilde{M}' = A\tilde{M} + B\tilde{\lambda}$$

ex.

$n=3$ $\quad K_1 + K_2 + K_3 = 0 \Rightarrow (K_1 + K_2)^2 = (-K_3)^2 \Rightarrow 2K_1 K_2 = 0 \Rightarrow \langle 12\rangle[12] = 0$

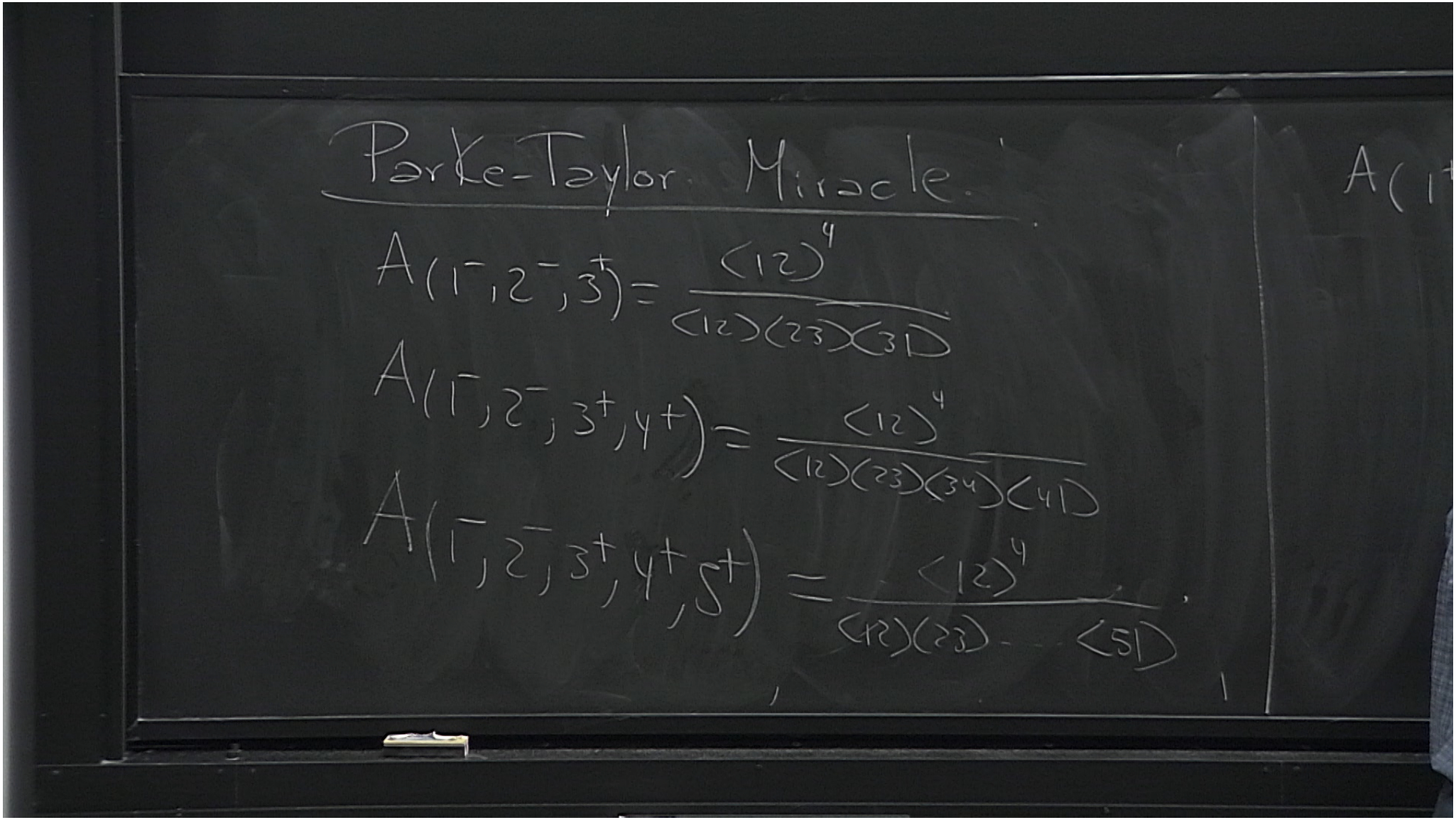
$2K_1 K_3 = 0 \Rightarrow \langle 13\rangle[13] = 0$

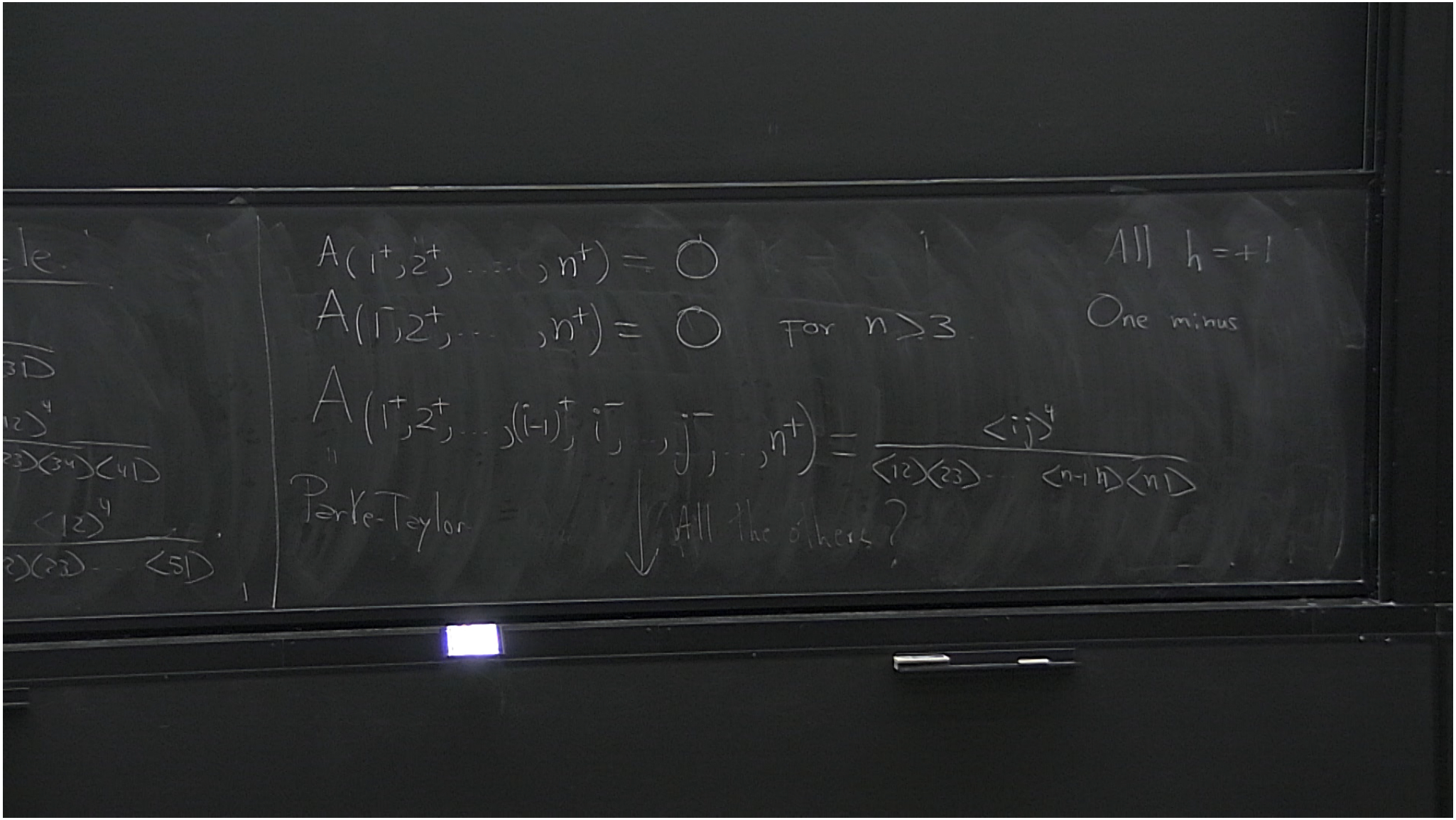
$2K_2 K_3 = 0 \Rightarrow \langle 23\rangle[23] = 0$

① Choose $[12] = 0 \quad [13] = 0 \Rightarrow [23] = 0$

$$A_3 = F(\langle 12\rangle, \langle 23\rangle, \langle 13\rangle) = A(1^{h=-1}, 2^{h=-1}, 3^{h=+1}) = \langle 12\rangle^* \langle 23\rangle^* \langle 31\rangle^* = \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle}$$

② $\langle 12\rangle = \langle 13\rangle = \langle 23\rangle = 0 \qquad A_3 = F([12], [23], [31])$


Parke-Taylor Miracle
$A(1^-,2^-,3^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}$
$A(1^-,2^-,3^+,4^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$
$A(1^-,2^-,3^+,4^+,5^+) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle \cdots \langle 51\rangle}$


$A(1^+, 2^+, \ldots, n^+) = 0$
$A(1^-, 2^+, \ldots, n^+) = 0$ For $n \geq 3$
All $h = +1$
One minus
$A(1^+, 2^+, \ldots, (i-1)^+, i^-, \ldots, j^-, \ldots, n^+) = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n-1\, n \rangle \langle n1 \rangle}$
Parke-Taylor
All the others?