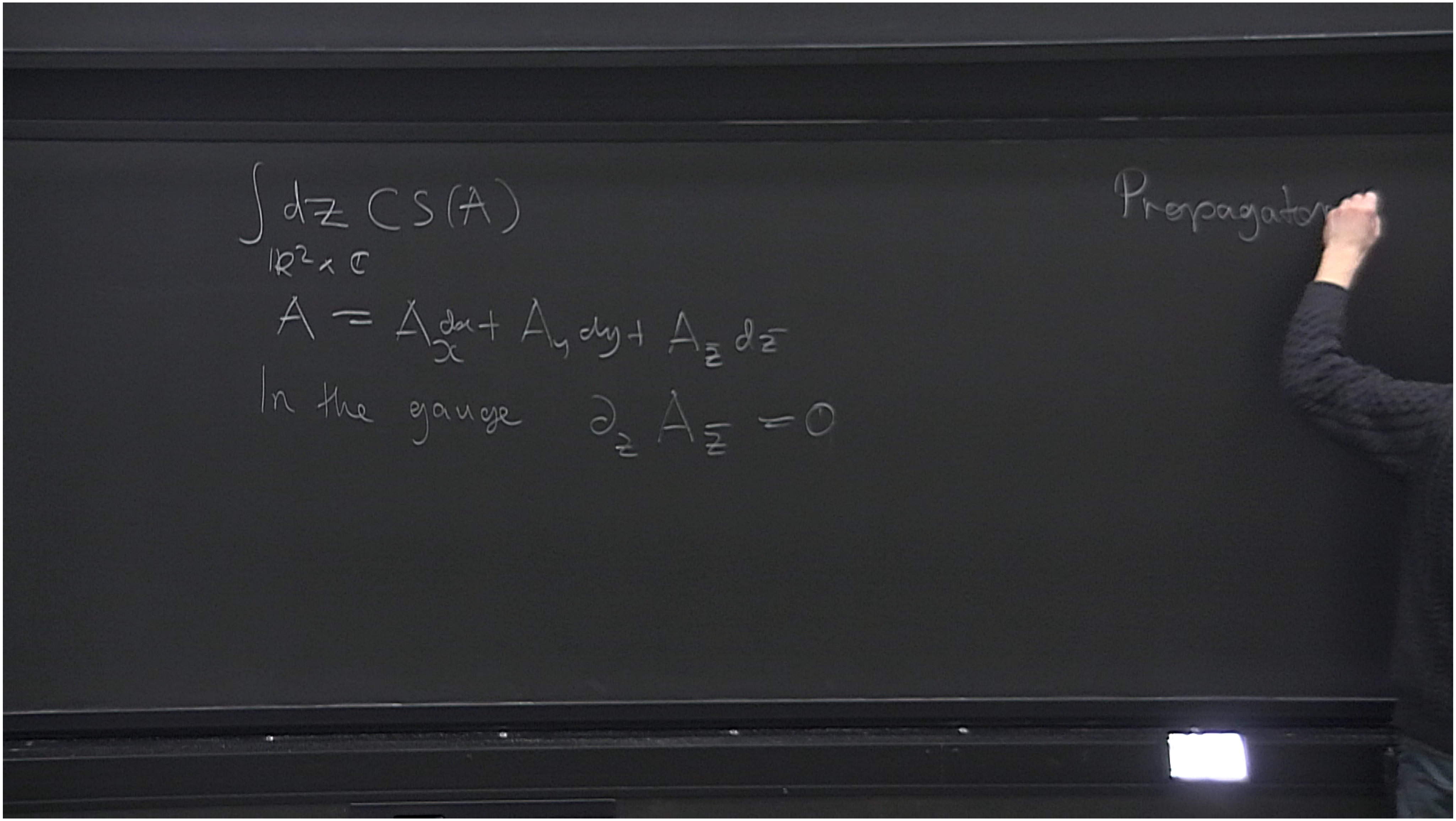
Title: PSI 2017/2018 - Quantum Integrable Models - Lecture 14

Date: Apr 06, 2018 11:30 AM

URL: http://pirsa.org/18040034

Abstract:


$\int_{\mathbb{R}^2 \times \mathbb{C}} dz \, CS(A)$
$A = A_x dx + A_y dy + A_{\bar{z}} d\bar{z}$
In the gauge $\partial_z A_{\bar{z}} = 0$
Propagato

Propagator is such that

$$\langle A^a_x(0), A^b_y(u,v,z)\rangle = \frac{1}{2\pi i}\frac{1}{z}\delta_{u=0}\delta_{v=0}\delta_{ab}$$

To find propagator:

$$\int dz\, A\, dA + A^a_x(0)$$

Solve EOM δA_x we find $\partial_z A^a_y + \partial_y A^a_z = \delta_0$

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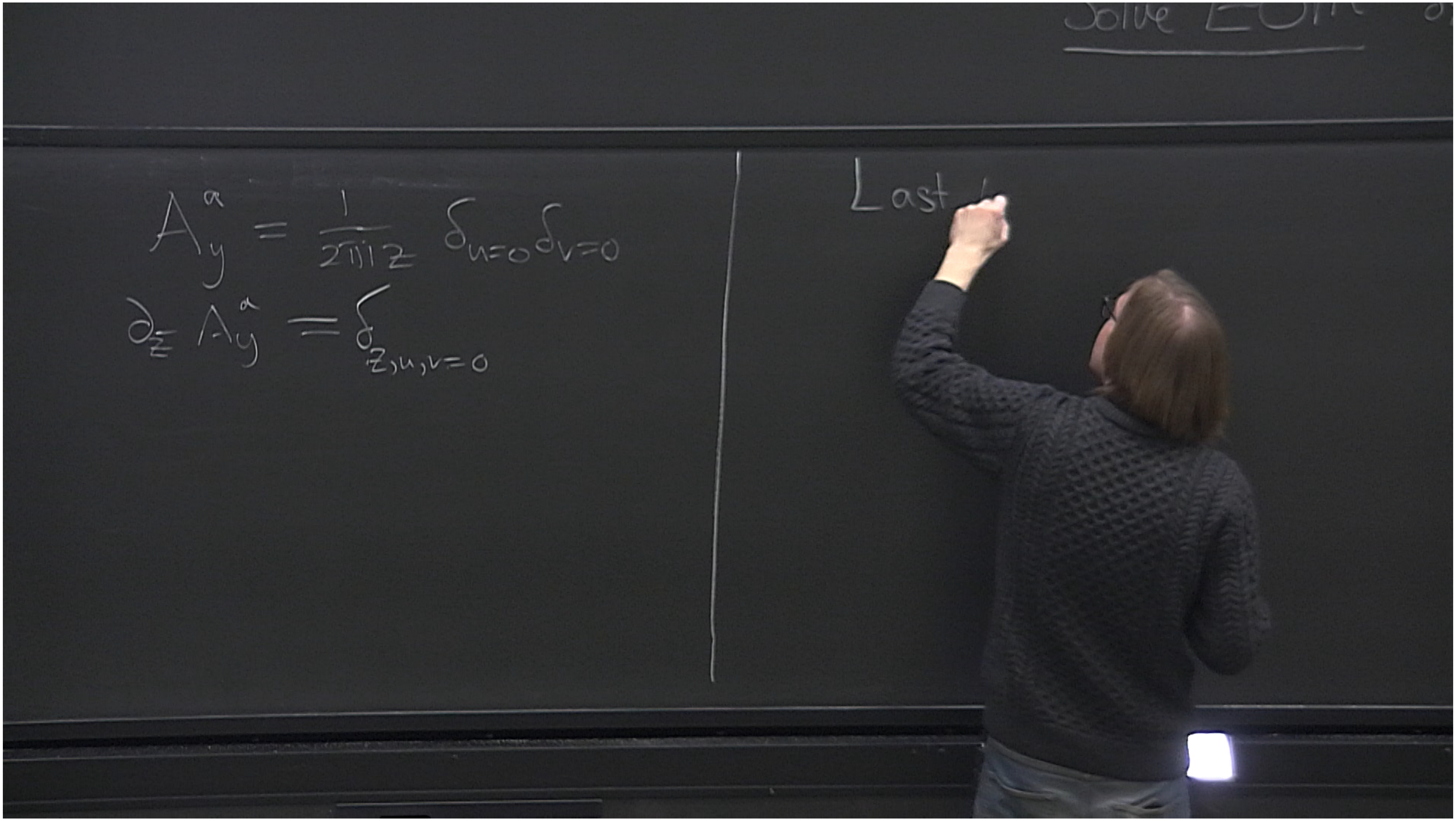
To find propagator:

$$\int dz\, A\, dA + A^a_x(0)$$

Solve EOM δA_x we find $\partial_z A^a_y + \partial_y A^a_{\bar z} = \delta_0$

$$\partial_{\bar z} A^a_{\bar z} = 0$$

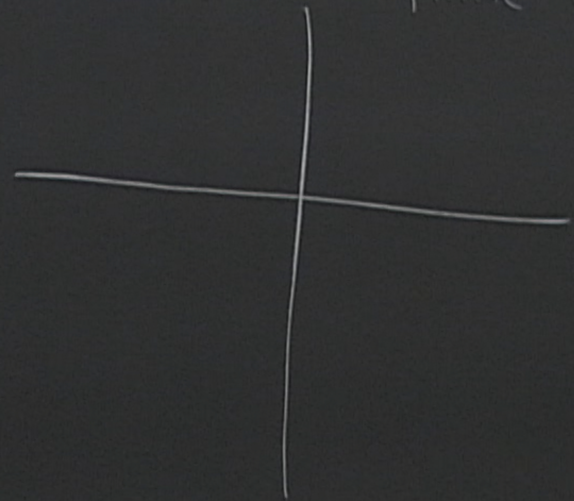

Solve EOM
$A^a_y = \frac{1}{2\pi i z} \delta_{u=0}\delta_{v=0}$
$\partial_{\bar z} A^a_y = \delta_{z,u,v=0}$
Last

Solve EOM $\partial A_{\bar{z}}$ we find

$$\partial_z A^a_{\bar{z}} = 0$$

Last time: we claimed that $w = x + iy$

if we have 2 infinite Wilson lines



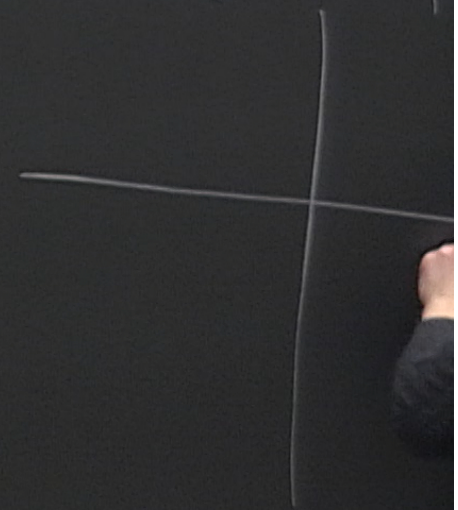
Solve E

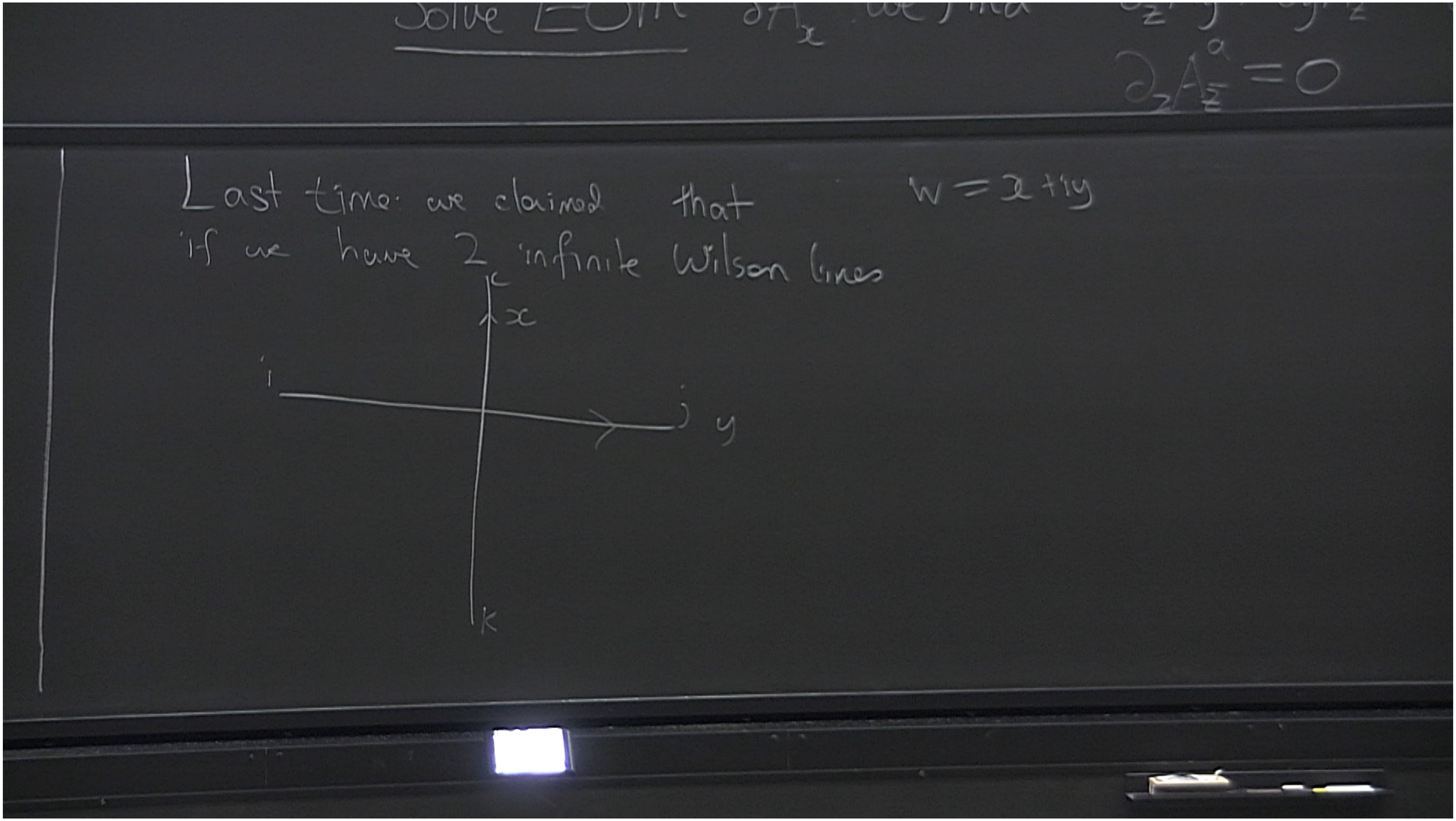
$$A^a_y = \frac{1}{2\pi i z}\,\delta_{u=0}\,\delta_{v=0}$$

$$\partial_{\bar z} A^a_y = \delta_{z,u,v=0}$$

$u = x$ } sorry
$v = y$ } for confusion

Last time: we claim
if we have 2 inf




Solve EOM
$\partial_z A^a_{\bar z} = 0$
Last time: we claimed that
$w = x + iy$
if we have 2 infinite Wilson lines
l
x
i
j
y
k

Solve EOM

$$\partial_z A^a_{\bar z} = 0$$

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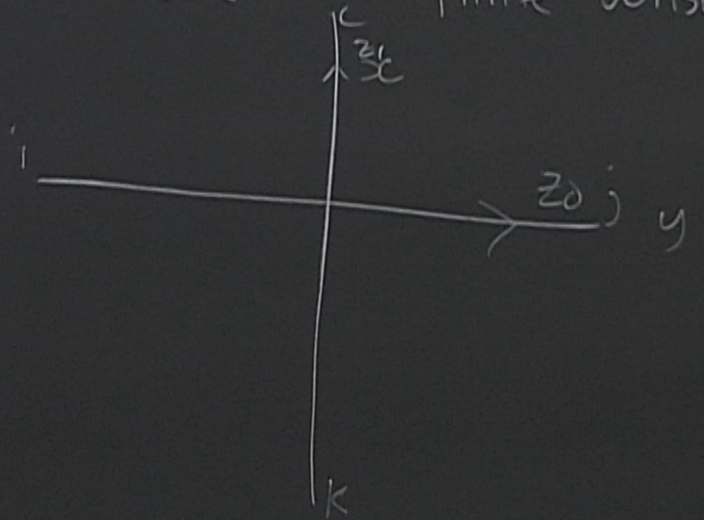

Then, the expectation value is

$$R^{ik}_{jl}(z_0 - z_1) = \delta^i_j \delta^k_l + \frac{\hbar}{z} \delta^i_l \delta^k_j + \cdots$$

Solve EOM δA_x we find $\partial_z A^a_y + \partial_y A^a_{\bar z} = \delta_0$

$$\partial_z A^a_{\bar z} = 0$$

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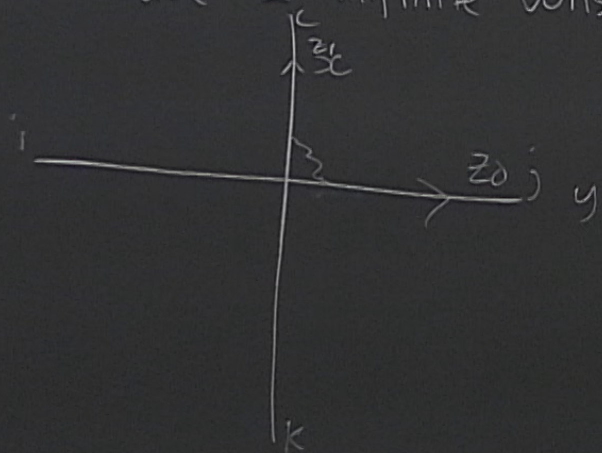
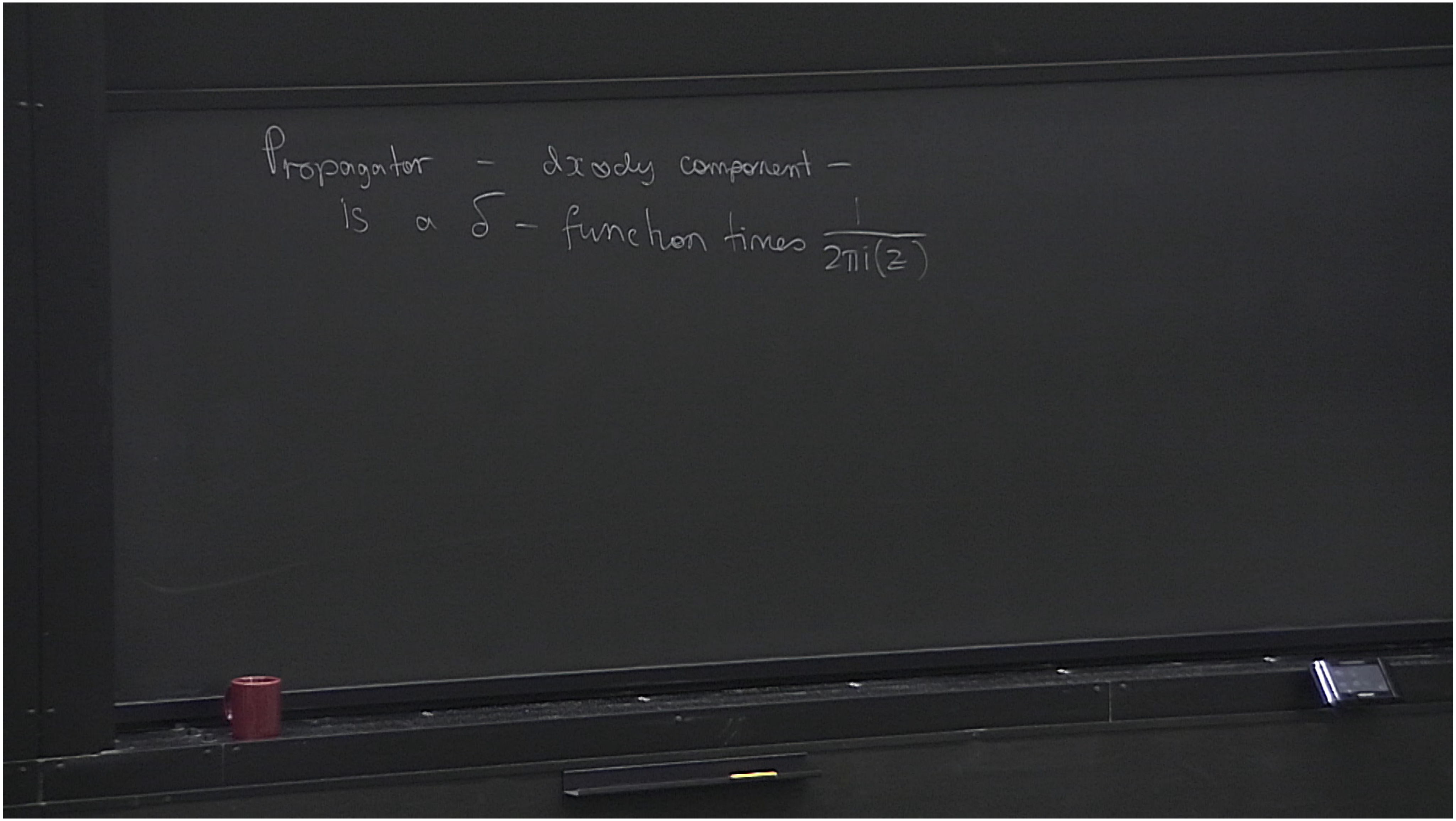


Then, the expectation value is

$$R^{ik}_{jl}(z_0 - z_1) = \delta^i_j \delta^k_l + \frac{\hbar}{z} \delta^i_l \delta^k_j + \dots$$

Leading term: exchange of 1 gluon


Propagator – $dx \otimes dy$ component –
is a δ – function times $\frac{1}{2\pi i (z)}$

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is a δ – function times $\frac{1}{2\pi i (z\ )}$

So we get

$$\frac{1}{2\pi i (z_0 - z_1)} \times (\text{Lie algebra factor})$$

For general $\mathfrak{g}$, Lie alg. factor

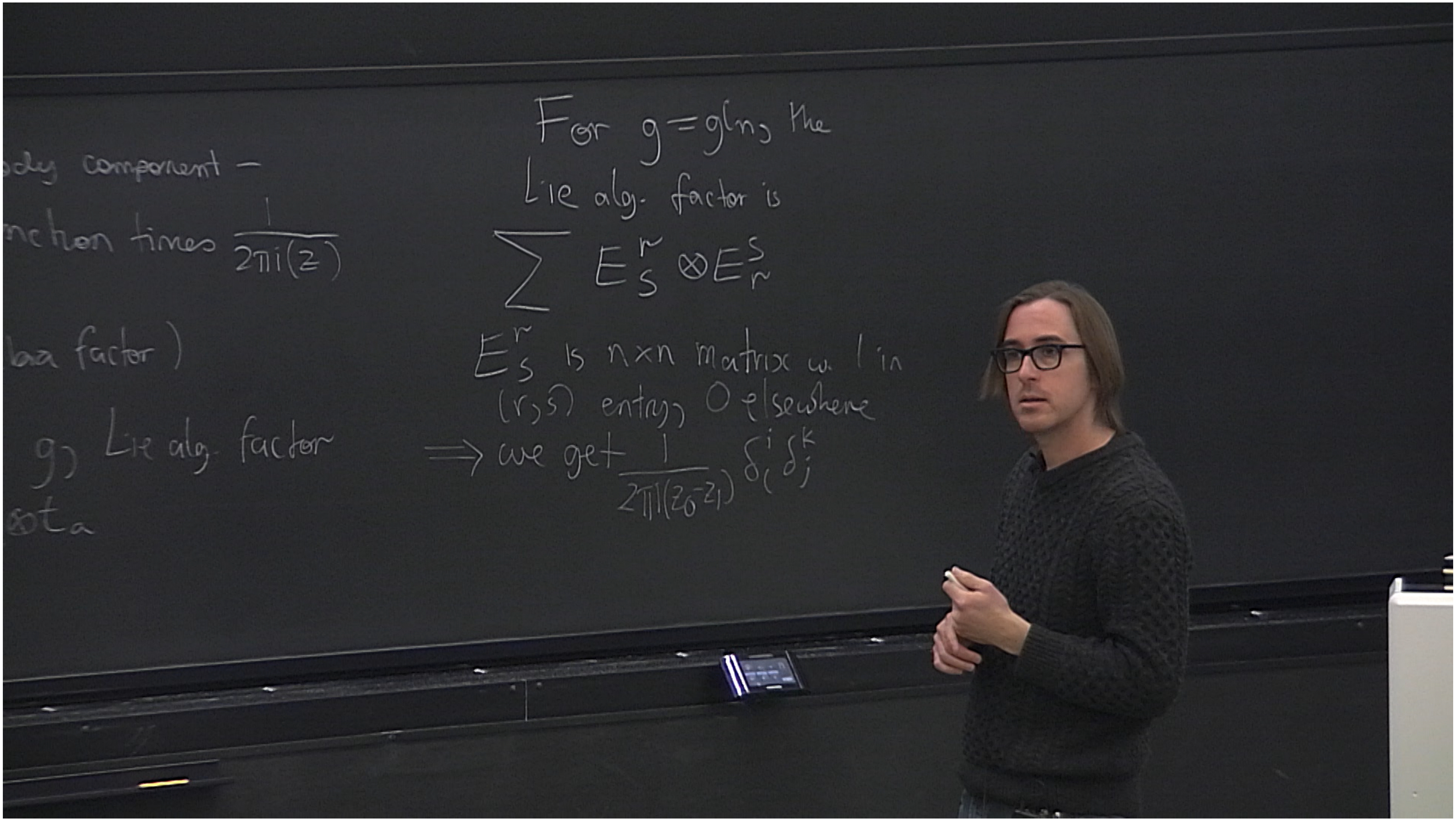
is $\sum t_a \otimes t_a$

For $\mathfrak{g} = \mathfrak{gl}_n$, the

Lie alg. factor is

$$\sum E^r_s \otimes E^s_r$$

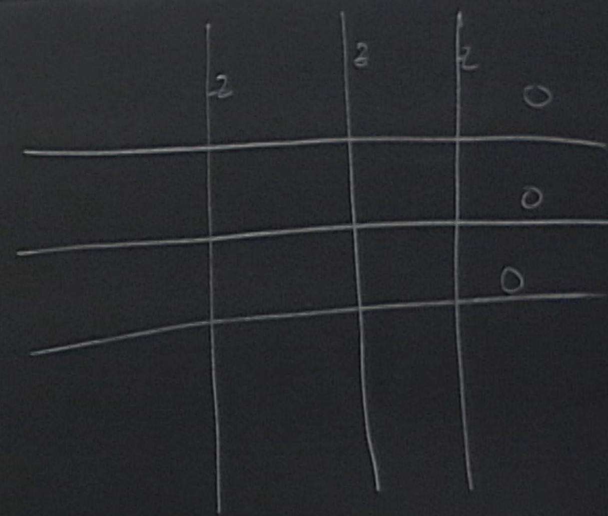
E^r_s is $n \times n$ matrix w. 1 in (r,s) entry, 0 elsewhere


ody component –
nction times $\frac{1}{2\pi i(z\ )}$
bra factor)
g, Lie alg. factor
$\otimes t_a$
For $g = gl_n$, the
Lie alg. factor is
$\sum E^r_s \otimes E^s_r$
E^r_s is $n \times n$ matrix w. 1 in
(r,s) entry, 0 elsewhere
$\Longrightarrow$ we get $\frac{1}{2\pi i(z_0 - z_1)} \delta^i_l \delta^k_j$

Manifestations of 'integrability'

1) Vertex models:

= Lattice of Wilson lines, where vertical ones are at z horizontal at 0



Expectation value of Wilson lines

= Vertex model partition function.

If x, y variables are periodic

$\Longrightarrow$ doubly periodic vertex model.

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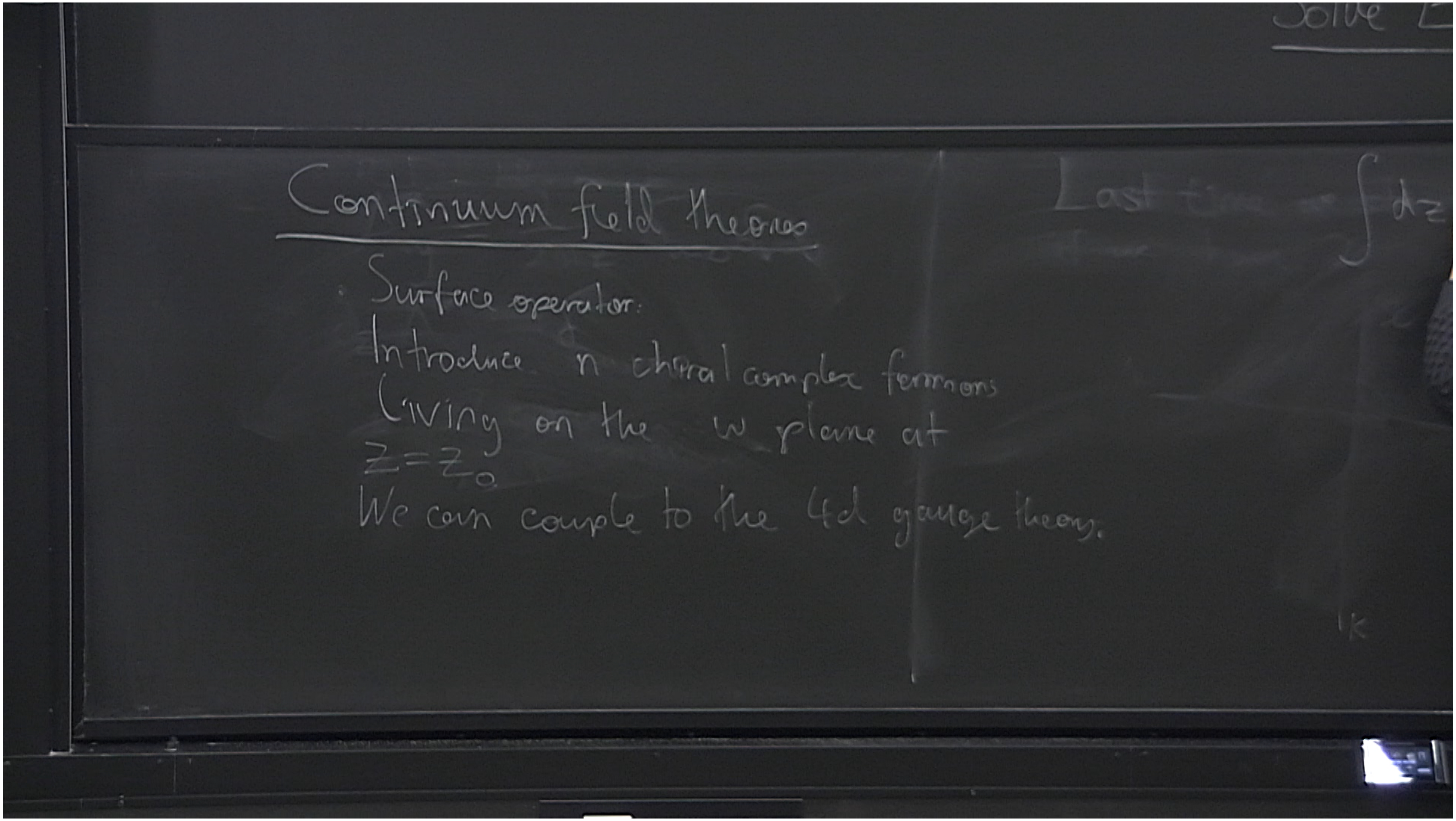
$\Longrightarrow$ doubly periodic vertex model.

[Key point: gauge theory is massive.

e.g. $\partial_z A_{\bar{z}} = 0$

only sol^n is $A_{\bar{z}} = 0$]

$(A_{\bar{z}} \to 0$ at $\infty)$


Continuum field theories
Surface operator:
Introduce n chiral complex fermions
Living on the w plane at
z=z0
We can couple to the 4d gauge theory.
Last time
Solve

Solve EOM

$$\partial_z A^a_{\bar z} = 0$$

$$\int_{\mathbb{R}^2 \times \mathbb{C}} dz\, CS(A) + \int_{\mathbb{R}^2} \psi_i \partial_{\bar w} \psi^i + \int_{\mathbb{R}^2} \psi_i A_{\bar w}(w, z_0) \psi^i$$

$$\partial_z A^a_{\bar z} = 0$$

$$\int_{\mathbb{R}^2\times\mathbb{C}} dz\, CS(A) + \int_{\mathbb{R}^2} \psi_i \partial_{\bar w} \psi^i + \int_{\mathbb{R}^2} \psi_i A_{\bar w}(w, z_0)\psi^i$$

If we also put anti-chiral surface defects at $z = z_1$,

$\int \bar\psi_i \partial_w + A_w(w, z_1)\bar\psi^i$, we can then

$$\partial_z A^a_{\bar{z}} = 0$$

$$\int_{\mathbb{R}^2\times\mathbb{C}} dz\, CS(A) + \int_{\mathbb{R}^2} \psi_i \partial_{\bar{w}} \psi^i + \int_{\mathbb{R}^2} \psi_i A_{\bar{w}}(w, z_0)\psi^i$$

If we also put anti-chiral surface defects at $z = z_1$, $\int \bar{\psi}_i \partial_w + A_w(w, z_1)\bar{\psi}^i$, we can then $\int$ out gauge field to get a 2d theory.

Solve EOM δA_x : we find

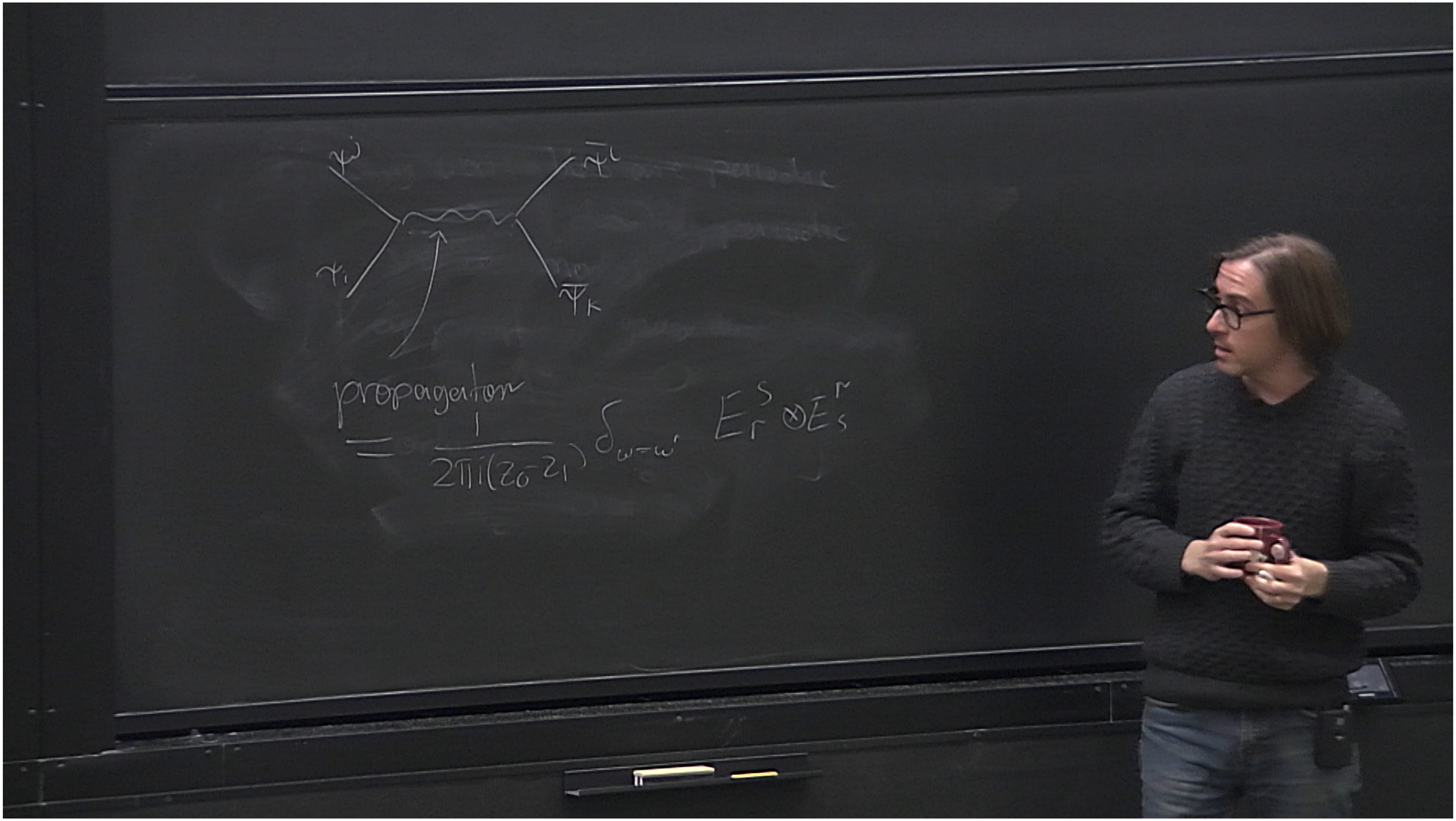
$$\partial_z A^a_{\bar z} = 0$$

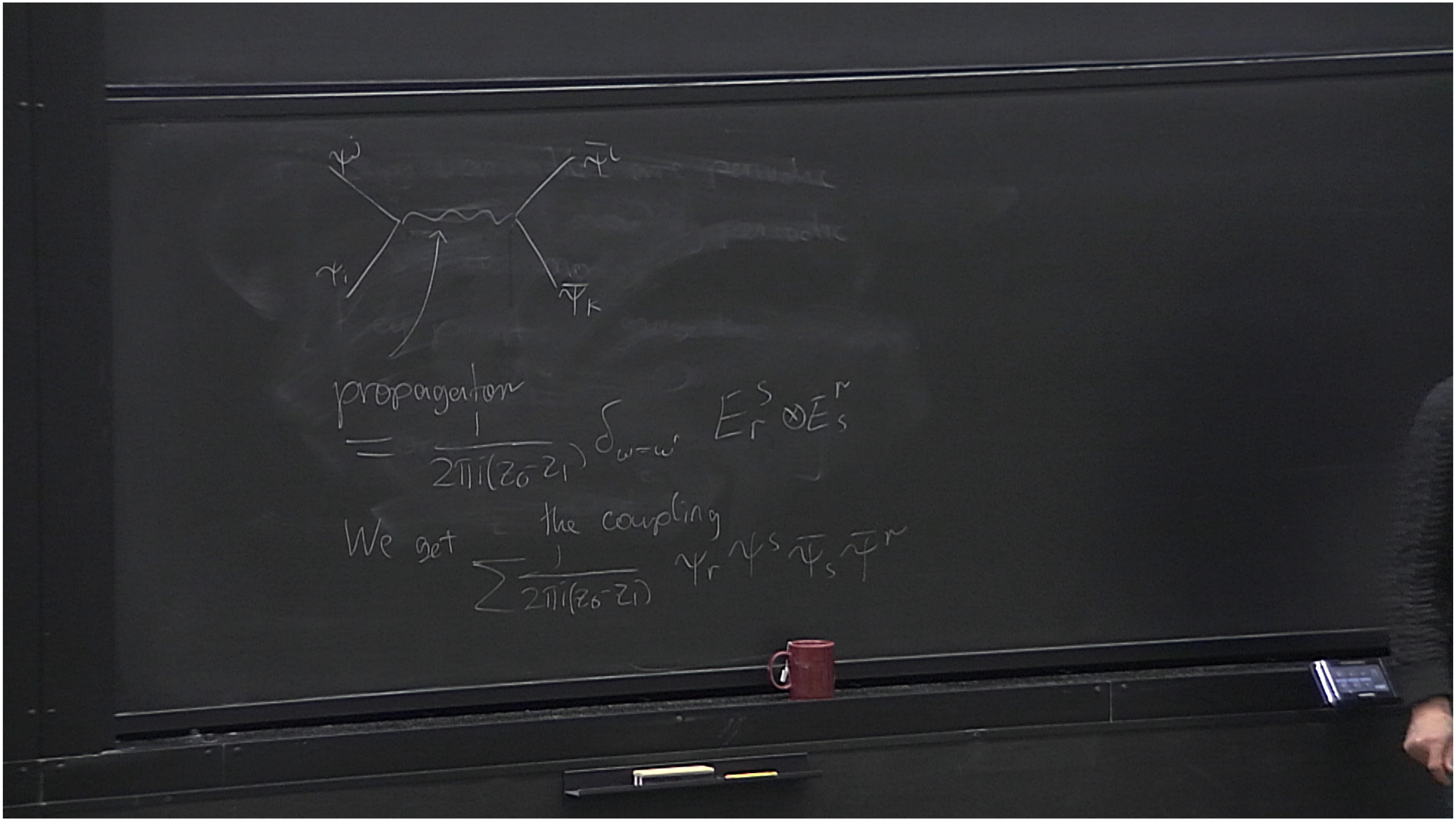
$$\int_{\mathbb{R}^2\times\mathbb{C}} dz\, CS(A) + \int_{\mathbb{R}^2} \psi_i \partial_{\bar w} \psi^i + \int_{\mathbb{R}^2} \psi_i A_{\bar w}(w, z_0) \psi^i$$

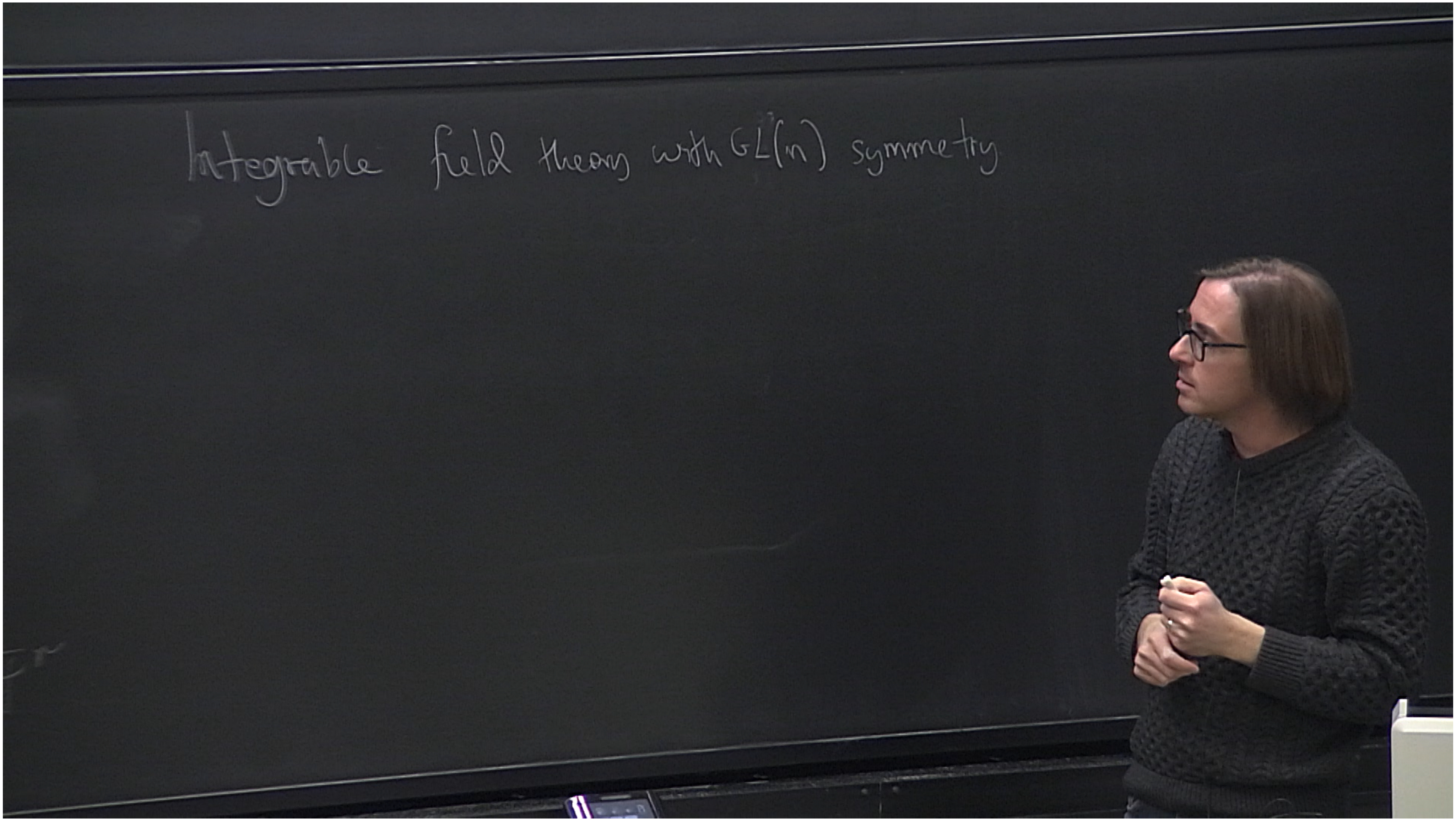
If we also put anti-chiral surface defects at $z = z_1$,

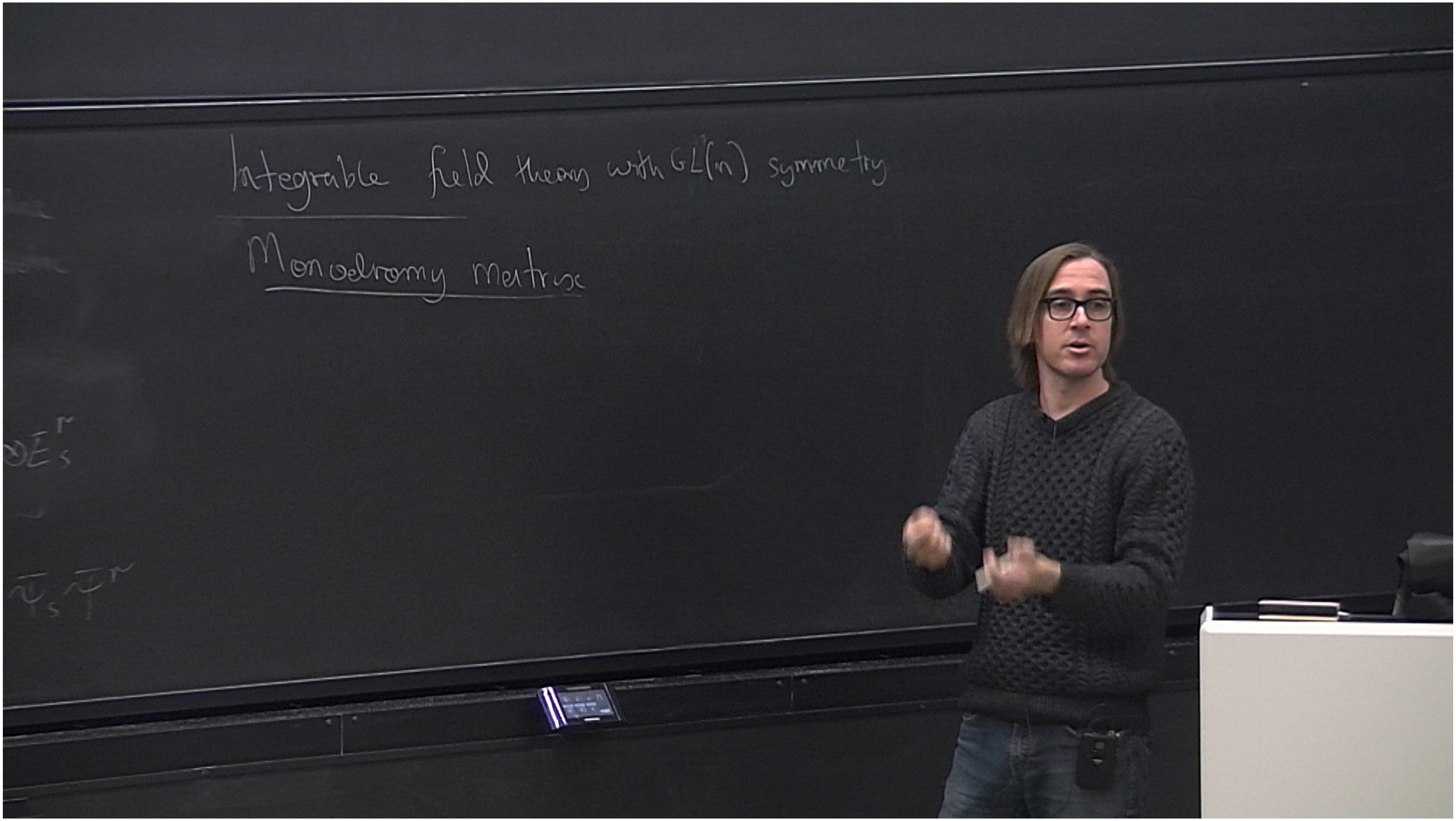
$\int \tilde\psi_i \partial_w + A_w(w, z_1)\tilde\psi^i$, we can then $\int$ out gauge field to get a 2d theory.

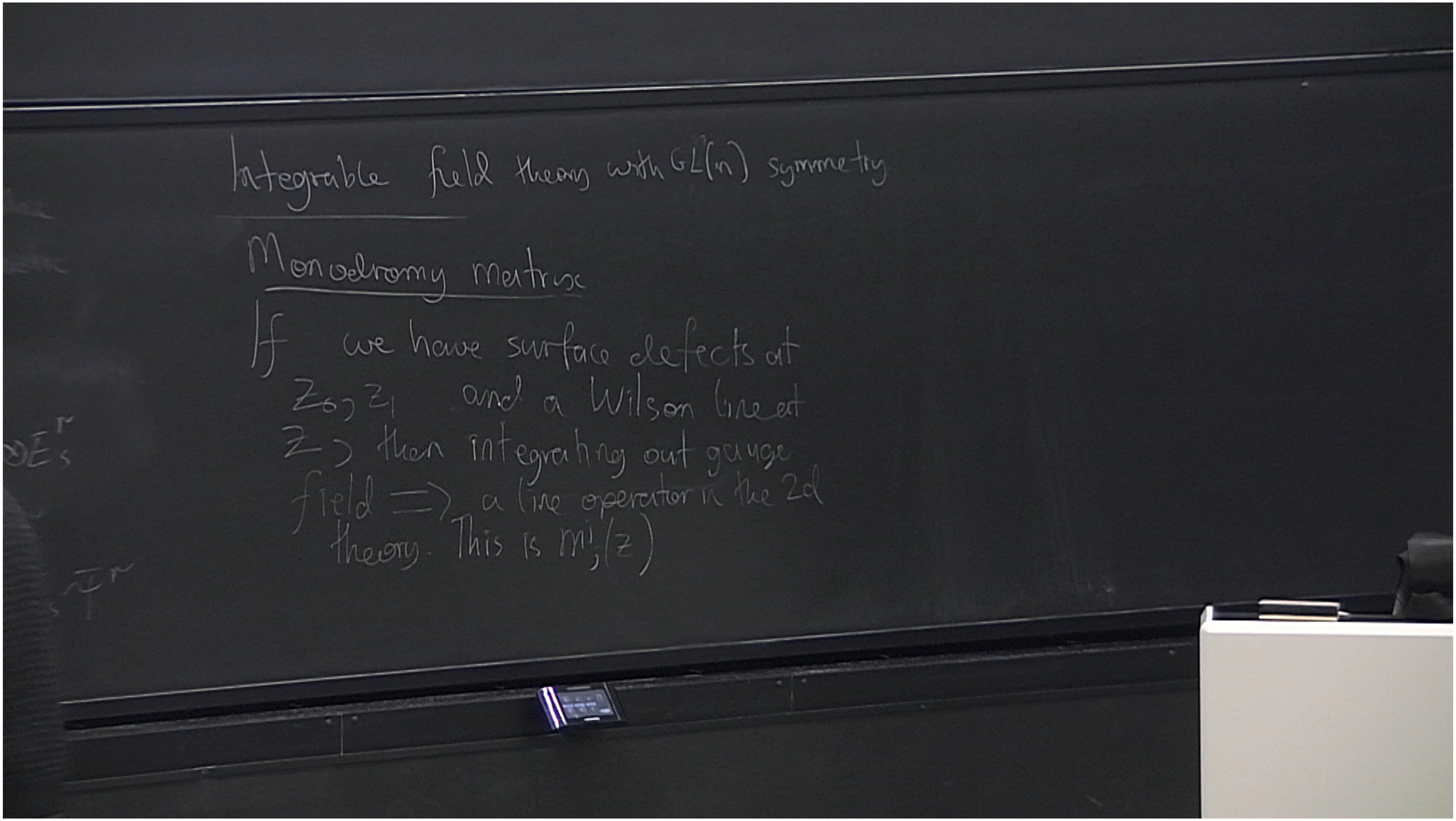
To leading order, ψ, $\tilde\psi$ are coupled by exchange of 1 gluon:

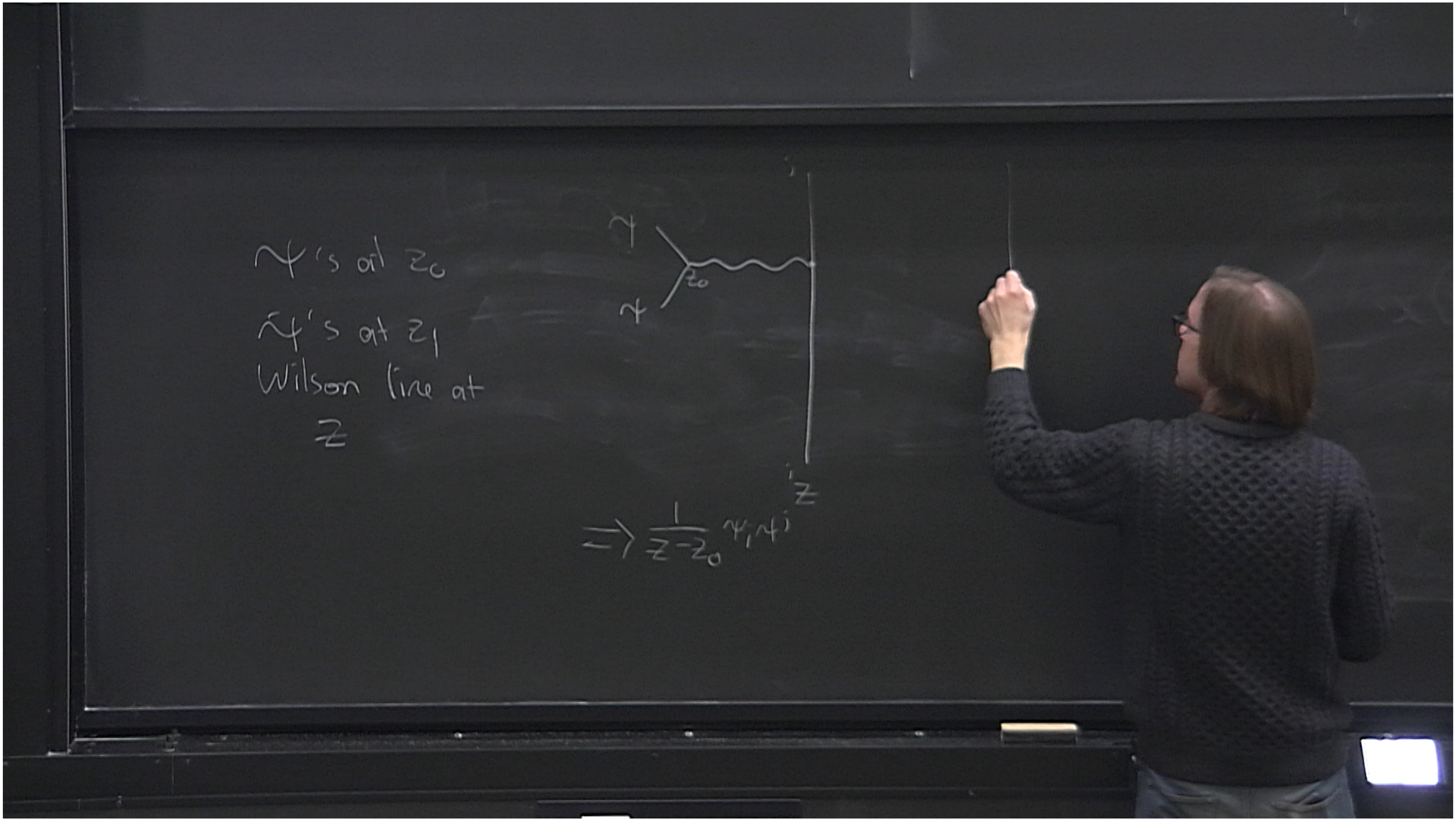

propagator

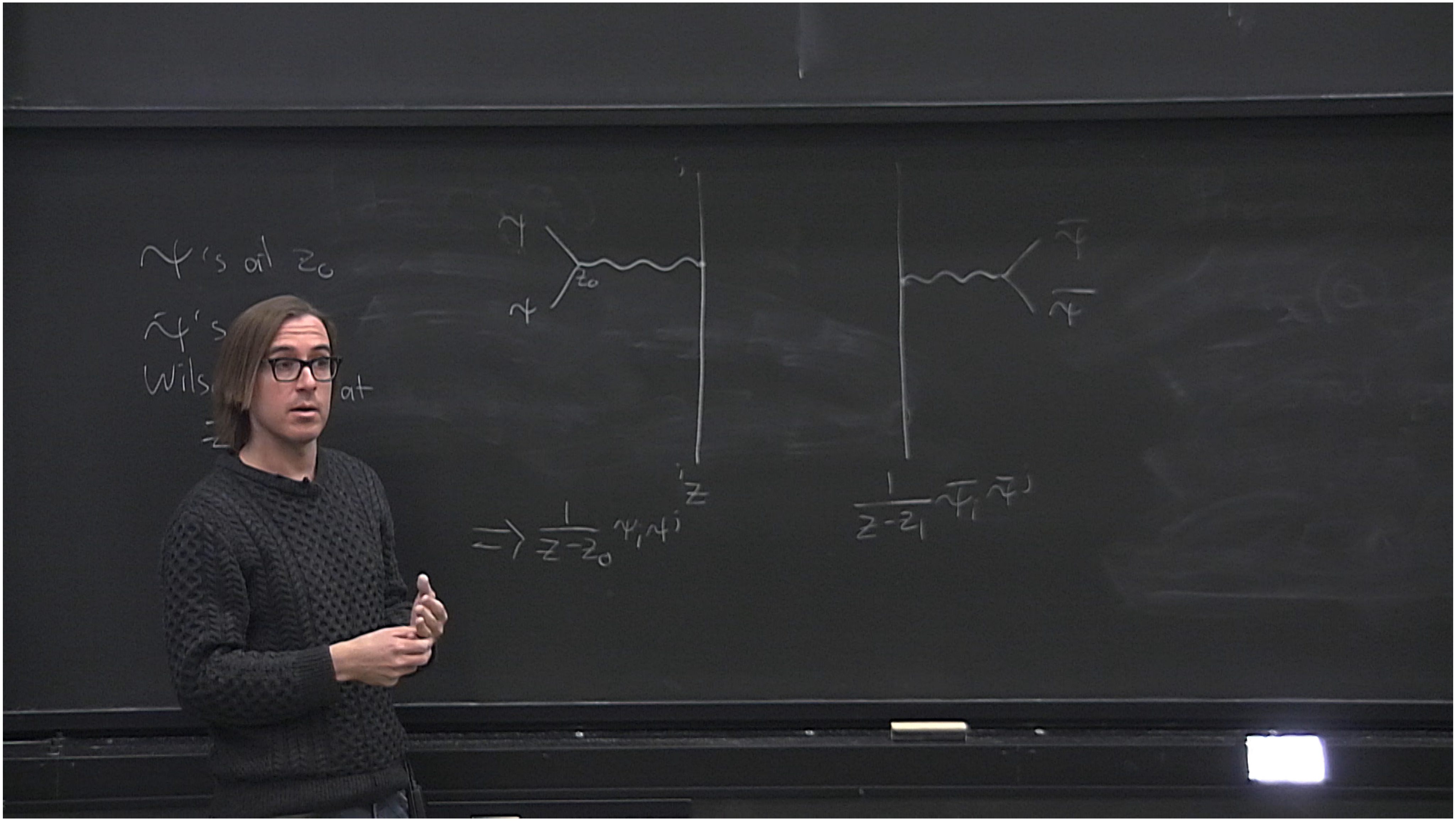

ψ^j
$\bar{\psi}^l$
ψ_i
$\bar{\psi}_k$
propagator
$= \frac{1}{2\pi i(z_0 - z_1)} \delta_{\omega=\omega'} E_r^s \otimes E_s^r$
We get the coupling
$\sum \frac{1}{2\pi i(z_0 - z_1)} \psi_r \psi^s \bar{\psi}_s \bar{\psi}^r$

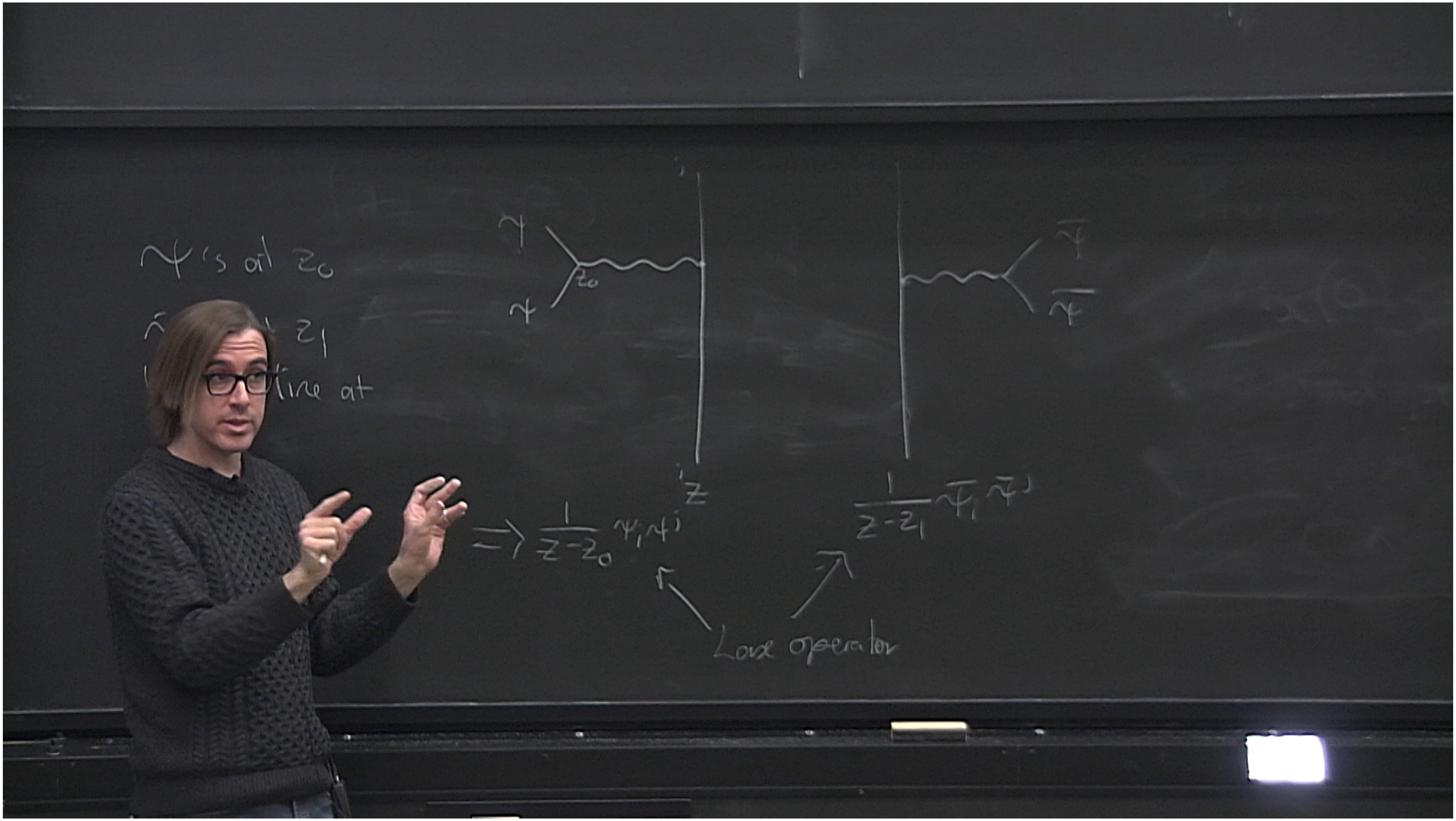

Integrable field theory with GL(n) symmetry


Integrable field theory with GL(n) symmetry
Monodromy matrix


Integrable field theory with GL(n) symmetry
Monodromy matrix
If we have surface defects at z_0, z_1 and a Wilson line at z, then integrating out gauge field $\Rightarrow$ a line operator in the 2d theory. This is $m^i_j(z)$


ψ's at z₀
ψ̃'s at z₁
Wilson line at
z


ψ's at z₀
Wils
at
z₀
z
1/(z−z₀) ψ₁ψʲ
1/(z−z₁) ψ̃₁ψ̃ʲ


ψ's at z₀
z₁
line at
Local operator

Other terms in monodromy matrix

$\longleftrightarrow$ other terms in PExp expression for the Wilson line

Other terms in monodromy matrix

$\longleftrightarrow$ other terms in PExp expression for the Wilson line

Lax operator

$m^i_j(z)$ are conserved

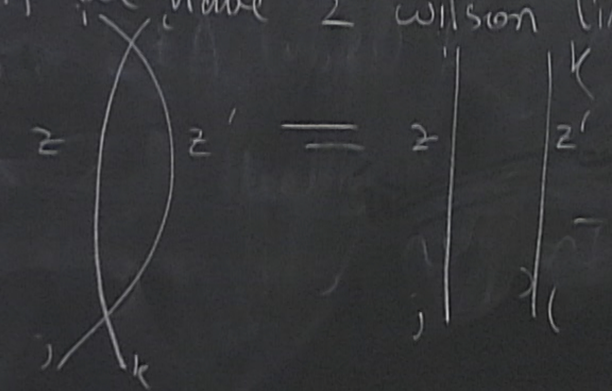
From 4d point of view this is obvious

As, moving the position of Wilson line in w-plane does nothing.

Yangian symmetry

In both vertex model and the integrable field theories, the Yangian symmetry has a simple explanation:

If we have 2 wilson lines



by top^{l} symmetry in w plane

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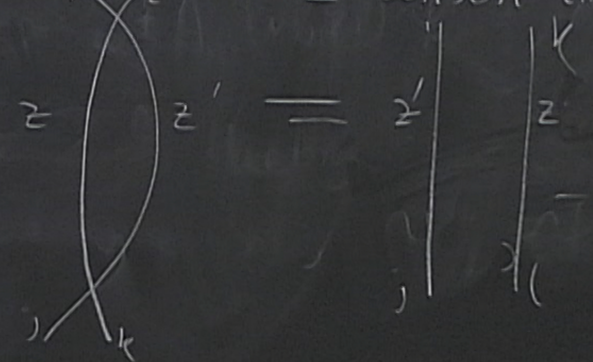
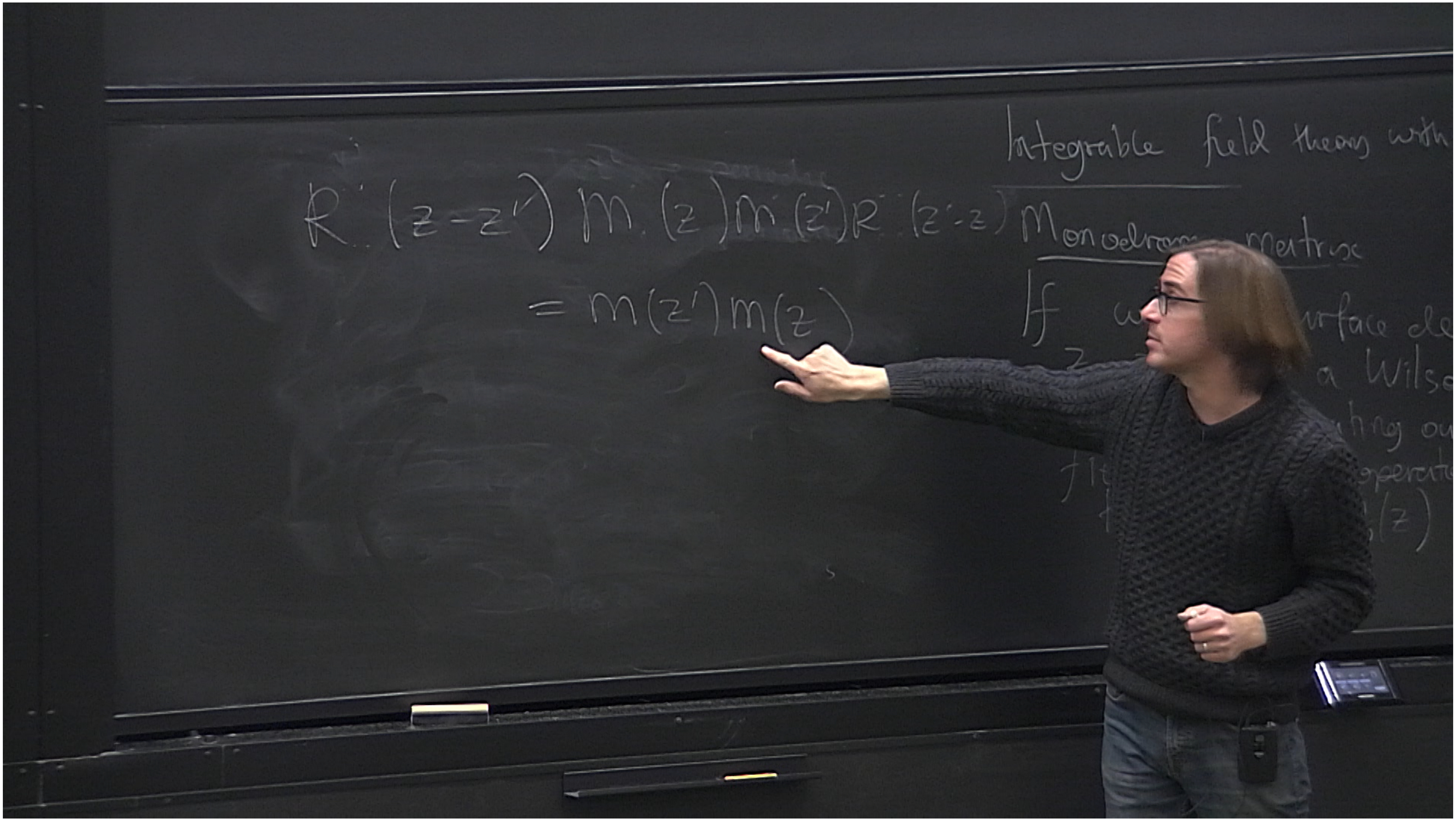
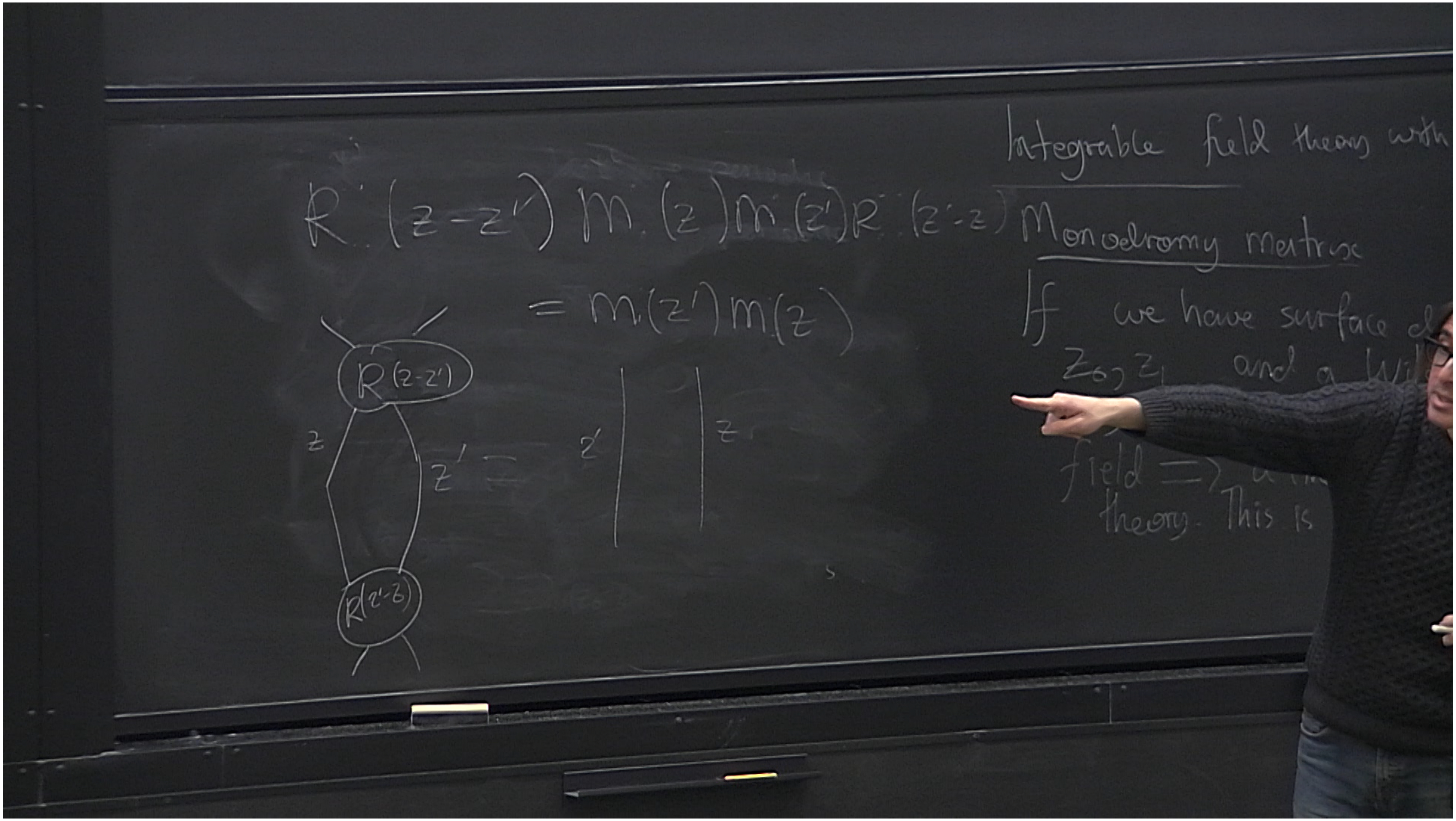


by top'l symmetry in w plane



Integrable field theory with
$R(z-z')\, m(z)\, m(z')\, R(z'-z) = m(z')\, m(z)$
Monodromy matrix


Integrable field theory with
R(z-z')m(z)m(z')R(z'-z)
= m(z')m(z)
Monodromy matrix
If we have surface
and a
field =>
theory. This is
R(z-z')
R(z'-z)
z
z'

Integrable field theory with

$$R(z-z')\, m(z)\, m(z')\, R(z'-z)$$

Monodromy matrix

If we have surface de
z_0, z_1 and a Wils
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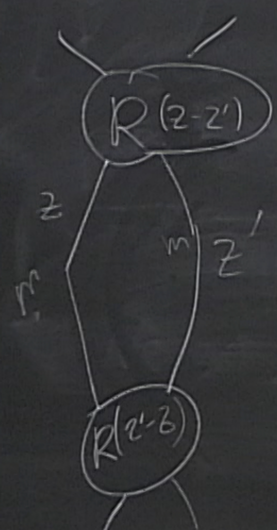
theory. This is $m^i_j(z$

$$= m(z')\, m(z)$$



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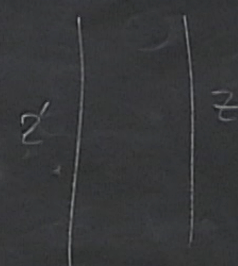
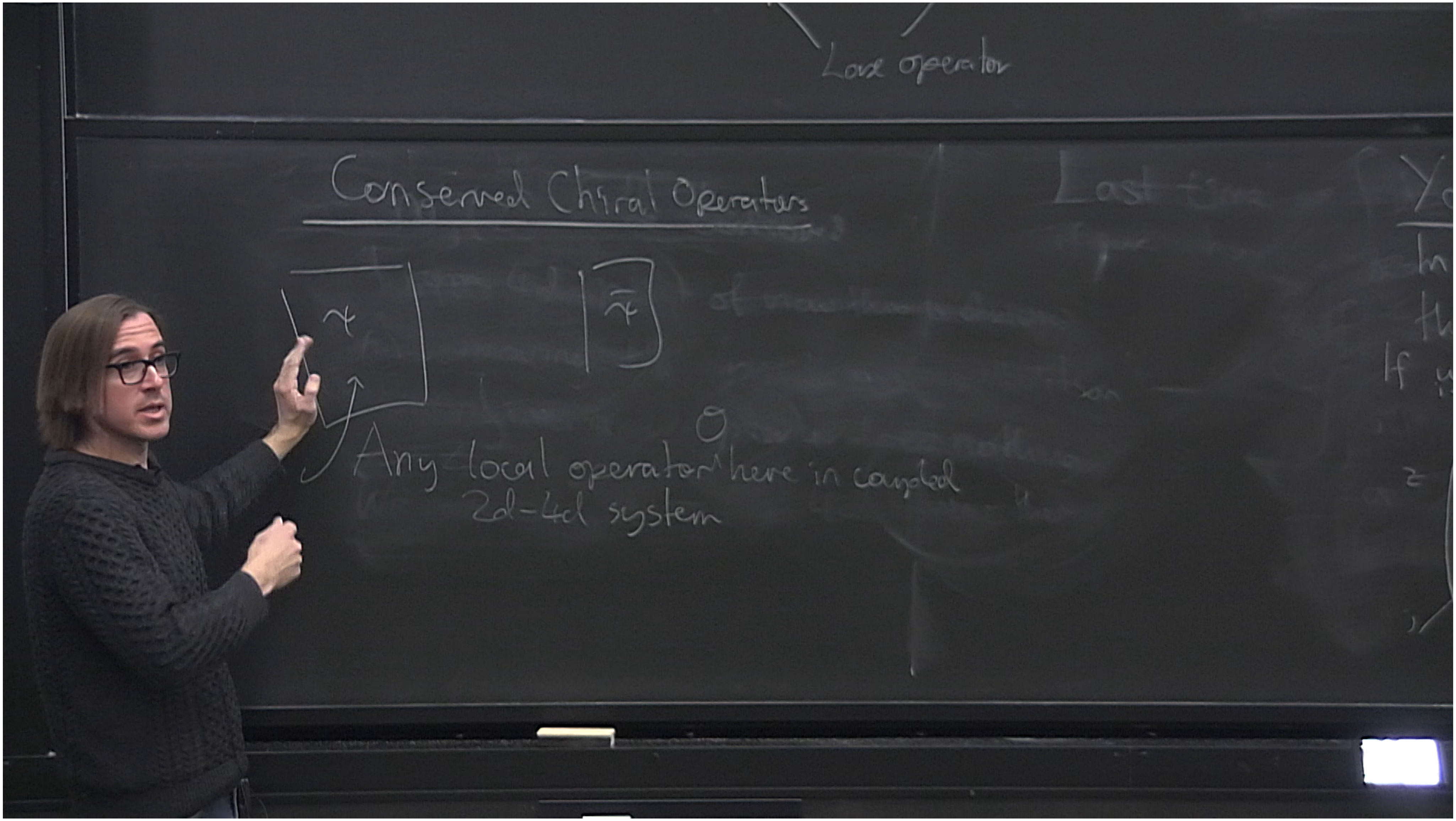
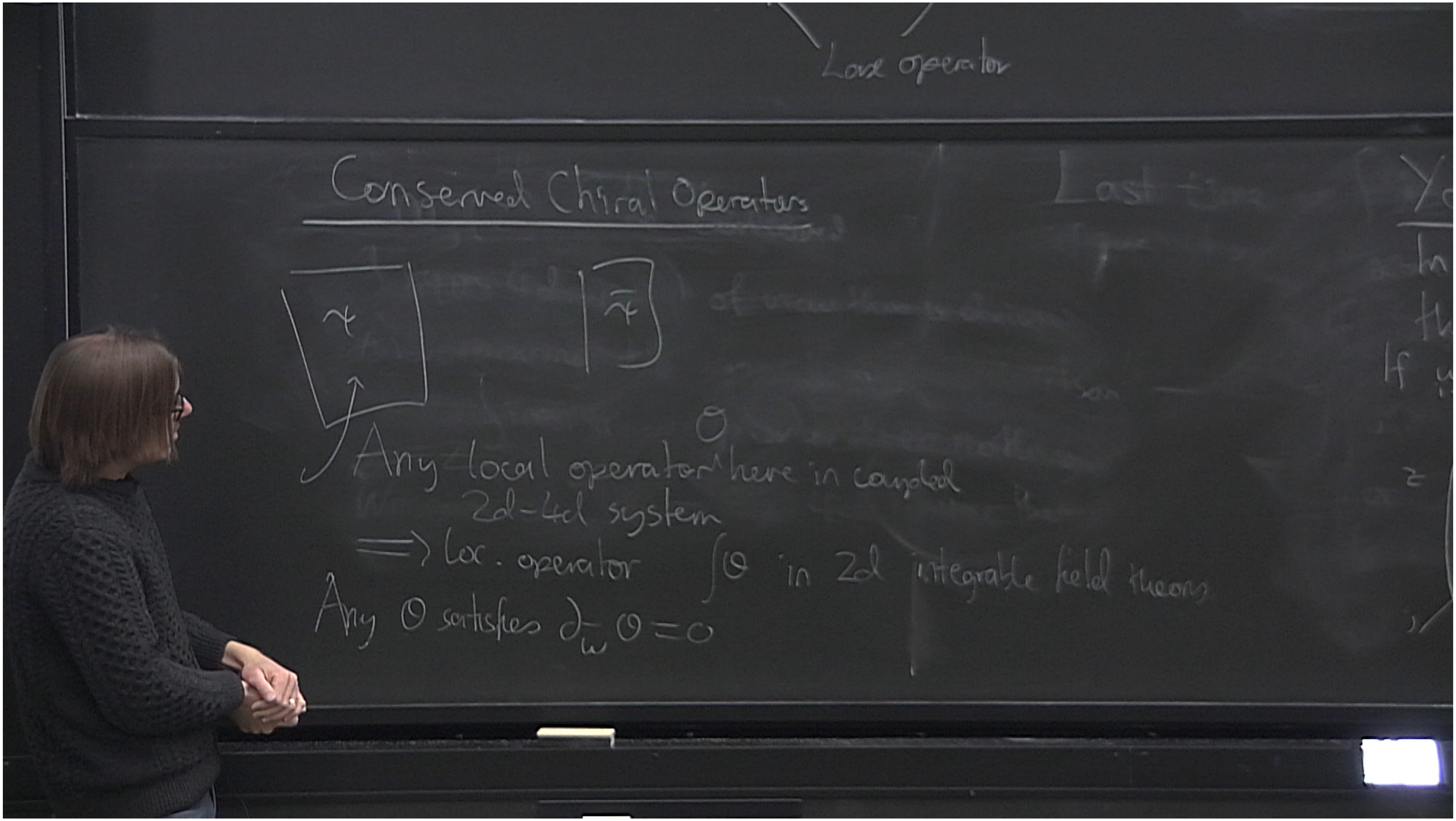



Conserved Chiral Operators
Any local operator here in coupled
2d-4d system
Last time


Loop operator
Conserved Chiral Operators
Any local operator here in coupled 2d-4d system
⟹ Loc. operator ∫O in 2d integrable field theory
Any O satisfies ∂_w̄ O = 0
Last time

At the classical level, Gl_n-invariant operators of Ψ can be made gauge inv. operators.

i.e. those $\mathcal{O}$ with $\oint J \cdot \mathcal{O} = 0$

At the classical level, G_N-invariant operators of $\tilde{\psi}$ can be made gauge inv. operators

i.e. those $\mathcal{O}$ with $\oint J \cdot \mathcal{O} = 0$

At the quantum level, we need

$$\sum_{n>0} \frac{1}{n} J_n J_{-n} \mathcal{O} = 0$$

At the classical level, Gl_n-invariant operators of $\mathcal{Y}$ can be made gauge inv. operators

i.e. those $\mathcal{O}$ with $\oint J \cdot \mathcal{O} = 0$

At the quantum level, we need

$$\sum_{n>0} \frac{1}{n} J_n J_{-n} \mathcal{O} = 0$$

(includes anything in the coset – non-singular OPE w. J)