Title: PSI 2017/2018 - Quantum Integrable Models - Lecture 13

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Abstract:

$$M^a_b = \delta^a_b + z^{-1} \rho^a_b$$

a, b from $1 \ldots 2$

If e_a basis of $\mathbb{C}^2$

$$\rho^a_b \, e_a = e_b$$

$e_a \otimes e_b$ a basis of $\mathbb{C}^2 \otimes \mathbb{C}^2$

$$\rho^a_{b1} \, e_r \otimes e_s = \delta^a_r \, e_b \otimes e_s$$

$$\rho^a_{b2} \, e_r \otimes e_s = \delta^a_s \, e_r \otimes e_b$$

$$m^a_b = \delta^a_b + z^{-1} \rho^a_b$$

a,b from 1..2

If e_a basis of $\mathbb{C}^2$

$$\rho^a_b e_r = e_b \delta^a_r$$

$e_a \otimes e_b$ a basis of $\mathbb{C}^2 \otimes \mathbb{C}^2$

$$\rho^a_{b1}(e_r \otimes e_s) = \delta^a_r \, e_b \otimes e_s$$

$$\rho^a_{b2}(e_r \otimes e_s) = \delta^a_s \, e_r \otimes e_b$$

$$P(e_a \otimes e_b) = e_b \otimes e_a$$

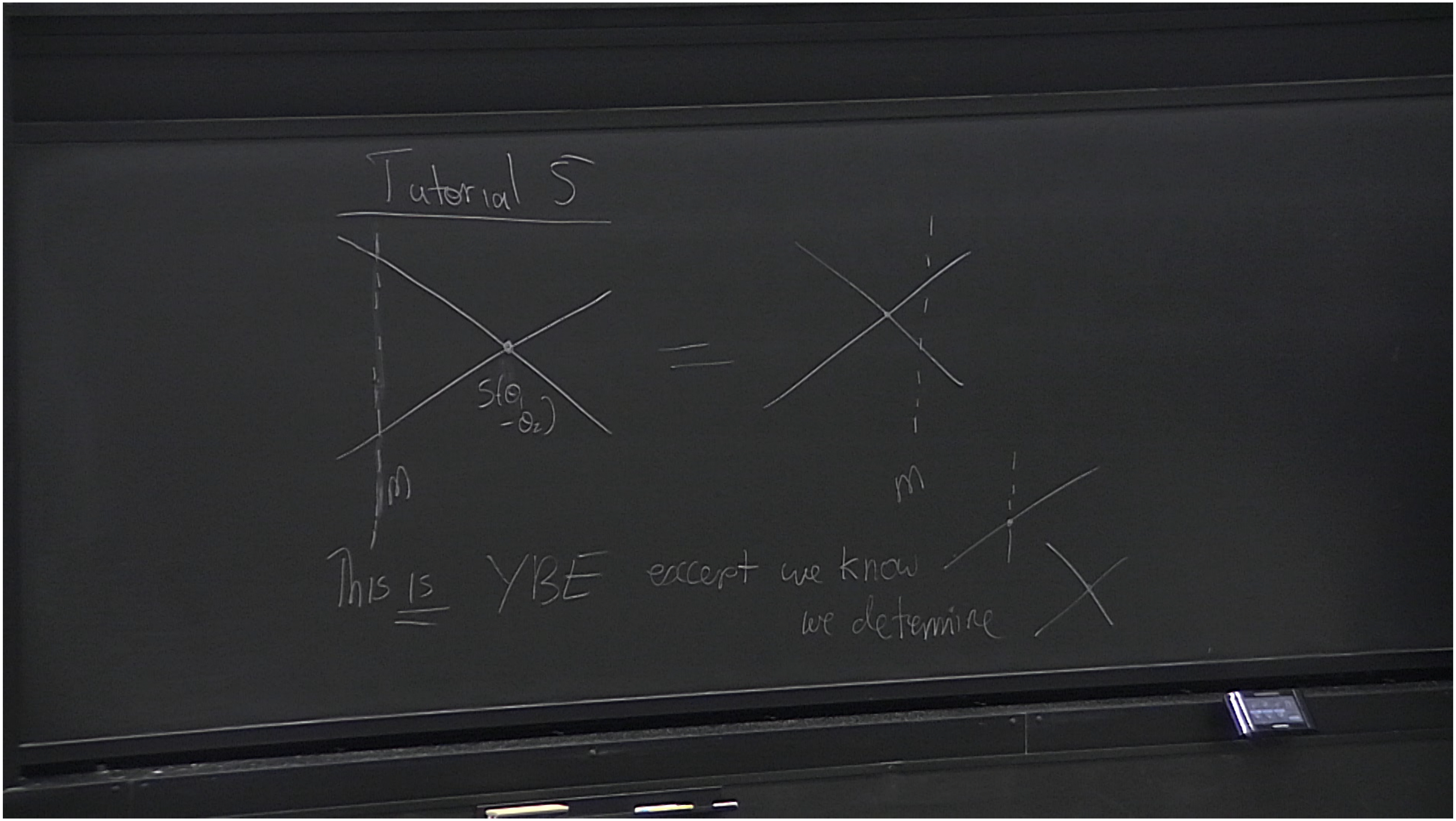
$$P = \sum_{r,s} \rho^r_{s1} \rho^s_{r2}$$

Tutorial!

$$R(z) = F(z)\, Id + G(z) P$$

Ask that R solves YBE

$$\Longrightarrow \frac{F(z)}{G(z)} = \left(z \text{ or } \frac{1}{z}\right)$$

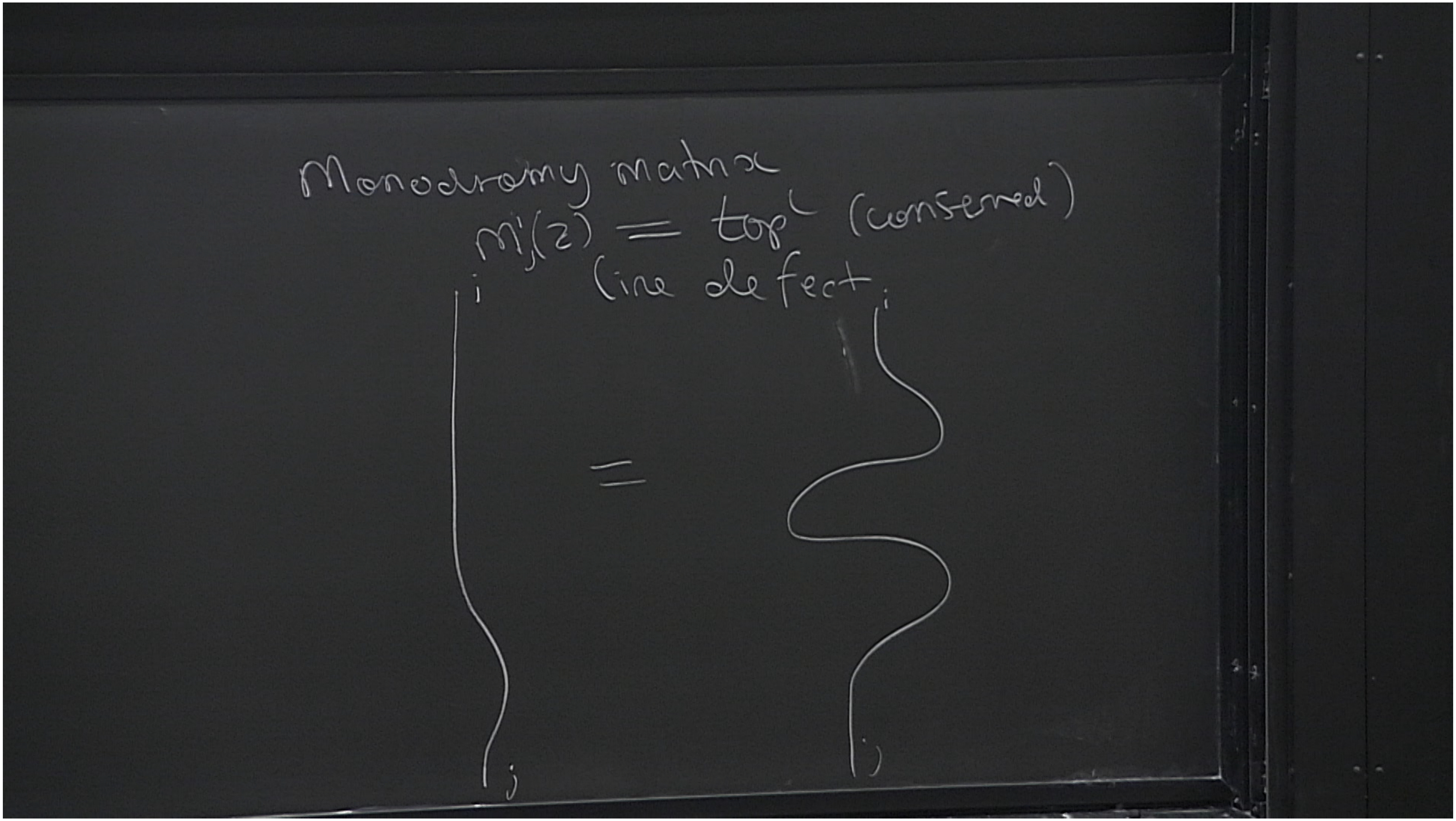

Tutorial 5
S(θ1 - θ2)
m
=
m
This is YBE except we know
we determine

Result

$$S^{ab}_{cd}(\theta_1 - \theta_2)$$

$$= \underbrace{H(\theta_1 - \theta_2)}_{\text{not determined by YBE "Form factor"}} R^{ab}_{cd}(\theta_1 - \theta_2)$$

R-matrix for 6 vertex model


Monodromy matrix
$M^i_j(z) = \mathrm{top}^L$ (conserved)
line defect
i
=
i
j
j

4d gauge theory + integrability

On $\underset{w,\bar{w}}{\mathbb{R}^2} \times \underset{z}{\mathbb{C}}$

Field is a gauge field $A \in \Omega^1(\mathbb{R}^2 \times \mathbb{C}) \otimes gl_n$

Lagrangian is

$$\int dz\, CS(A) = \frac{1}{2}\int dz \operatorname{Tr} A\, dA + \frac{1}{3}\int dz \operatorname{Tr} A^3$$

Extra gauge symmetry as well as

$$A \longrightarrow A + \varepsilon d\chi + \varepsilon [A, \chi]$$

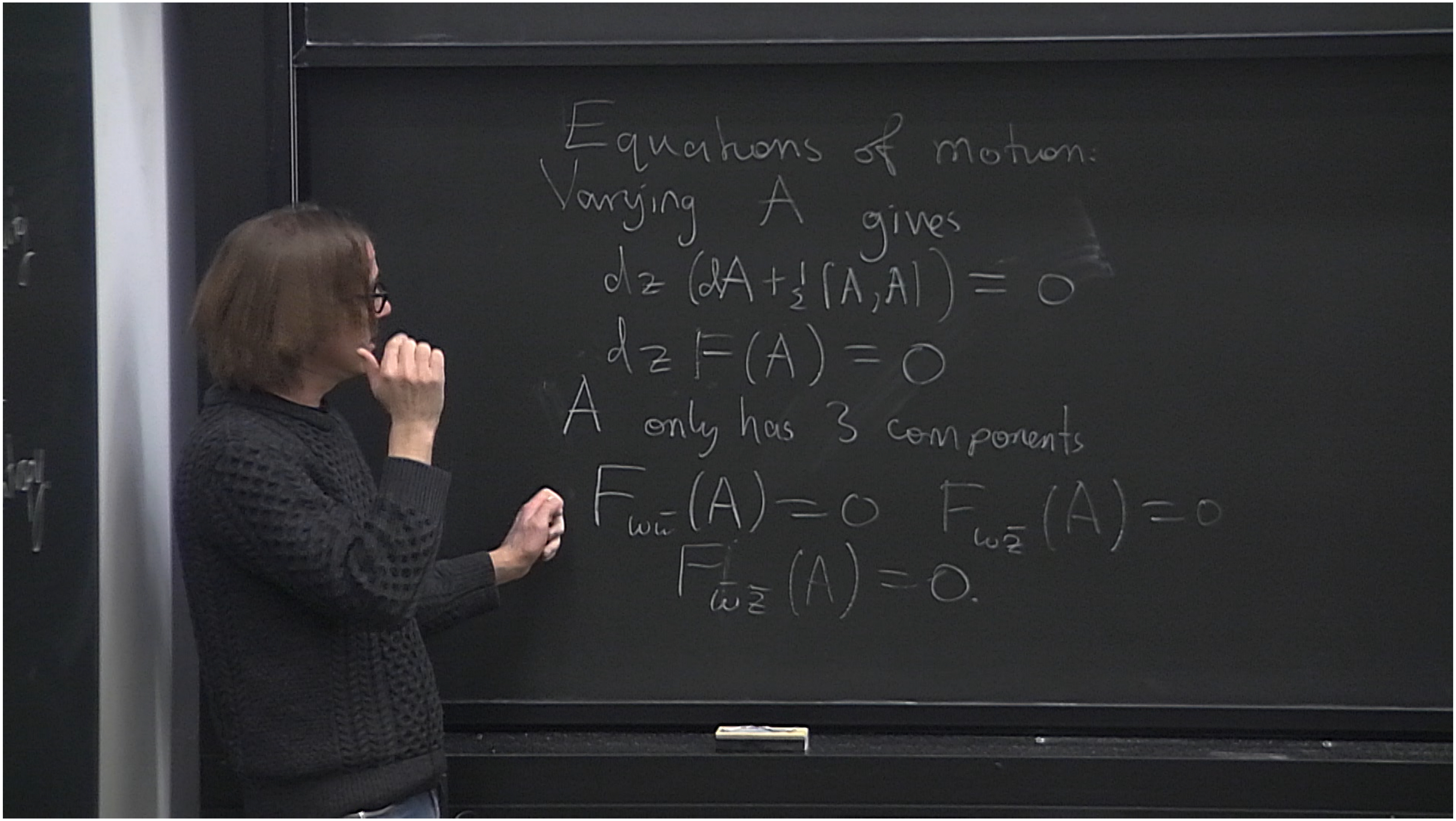
the transformation

$$A \longrightarrow A + dz\, \eta$$

leaves action invariant (η is adj. valued scalar)

Gauge fix: $A_z = 0$

So, $A = A_w dw + A_{\bar{w}} d\bar{w} + A_{\bar{z}} d\bar{z}$


Equations of motion:
Varying A gives
$d_z(dA + \frac{1}{2}[A,A]) = 0$
$d_z F(A) = 0$
A only has 3 components
$F_{w\bar{u}}(A) = 0$ $F_{w\bar{z}}(A) = 0$
$F_{\bar{w}\bar{z}}(A) = 0.$

Wilson lines in this theory are as usual given by $P\mathrm{Exp}\, A$

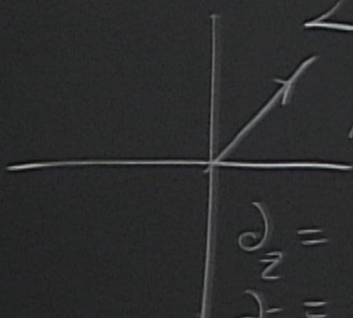
__But__ these are only gauge invariant for a path in w-$\bar{w}$ plane where $z, \bar{z}$ are constant.

If the line moves in z-plane

$P\,\mathrm{Exp}\,A$ involves A_z

Cannot be gauge invariant under

2^{nd} kind of gauge transformation.



$A\partial_x + B\partial_y$

$\partial_z = \partial_x - i\partial_y$

$\partial_{\bar{z}} = \partial_x + i\partial_y$

t coordinate on the line

$(f_1(t), f_2(t))$ are x, y coordinates

$z = x + iy$

1-form we integrate on the line is

$$A_x(f_1(t), f_2(t)) \cdot f_1'(t)\,dt$$
$$+ A_y(f_1(t), f_2(t))\, f_2'(t)\,dt$$

$$A = A_x\,dx + A_y\,dy$$

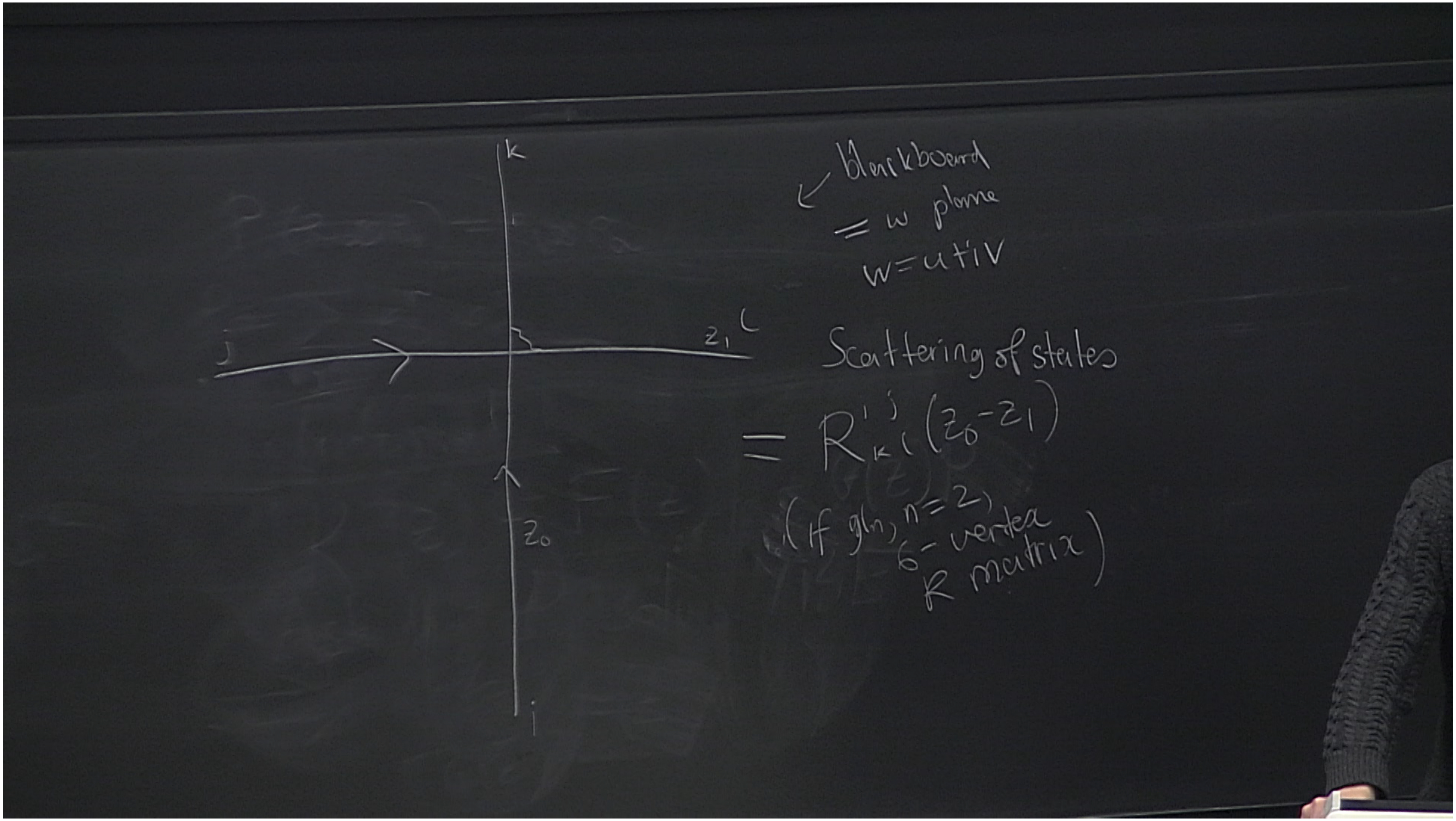
This always involves A_z
because

$$A_x = \frac{1}{2}(A_z + A_{\bar{z}})$$

$$A_y = \frac{1}{2i}(A_z - A_{\bar{z}})$$

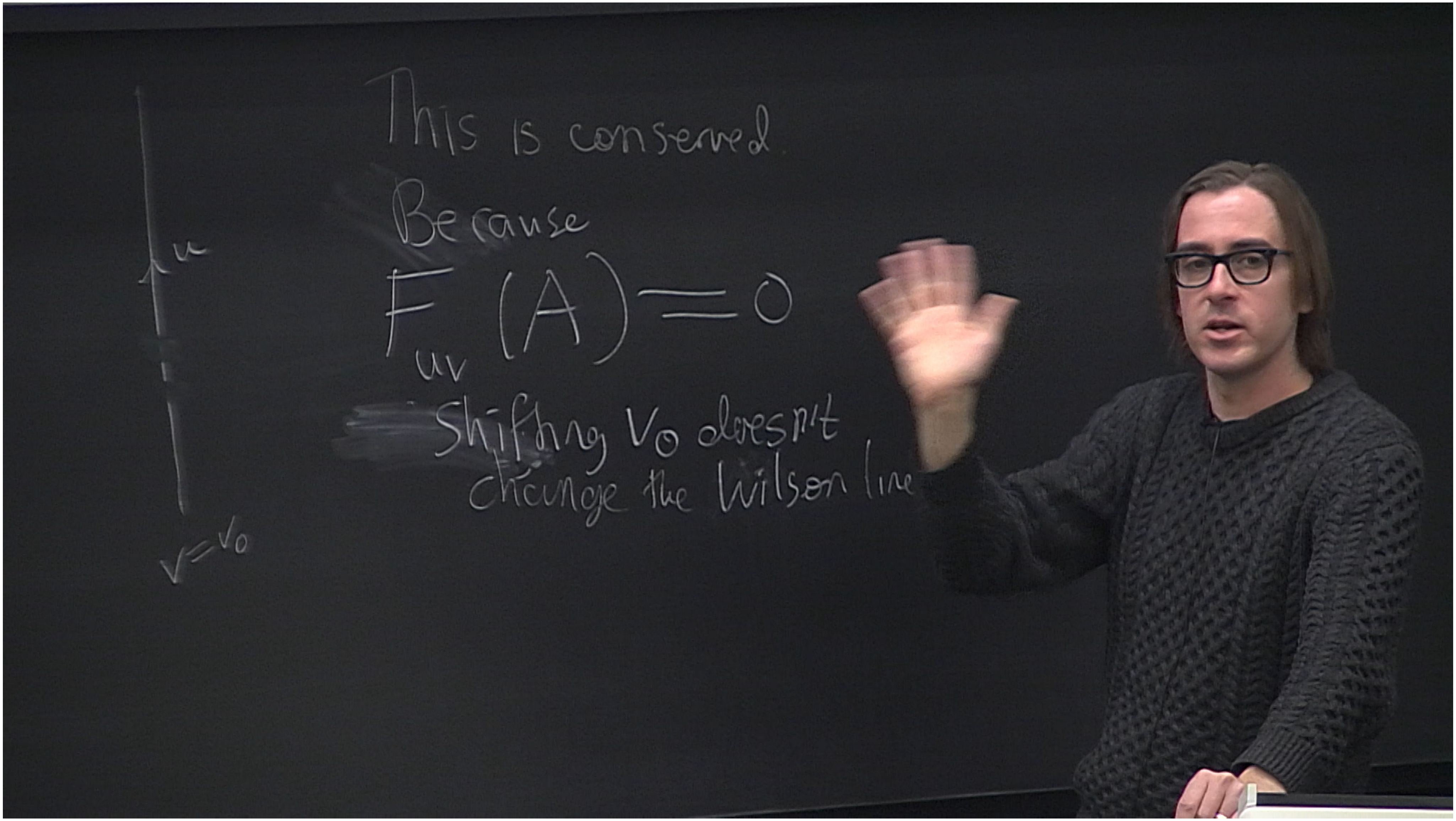
$f_1(t), f_2(t)$ are real

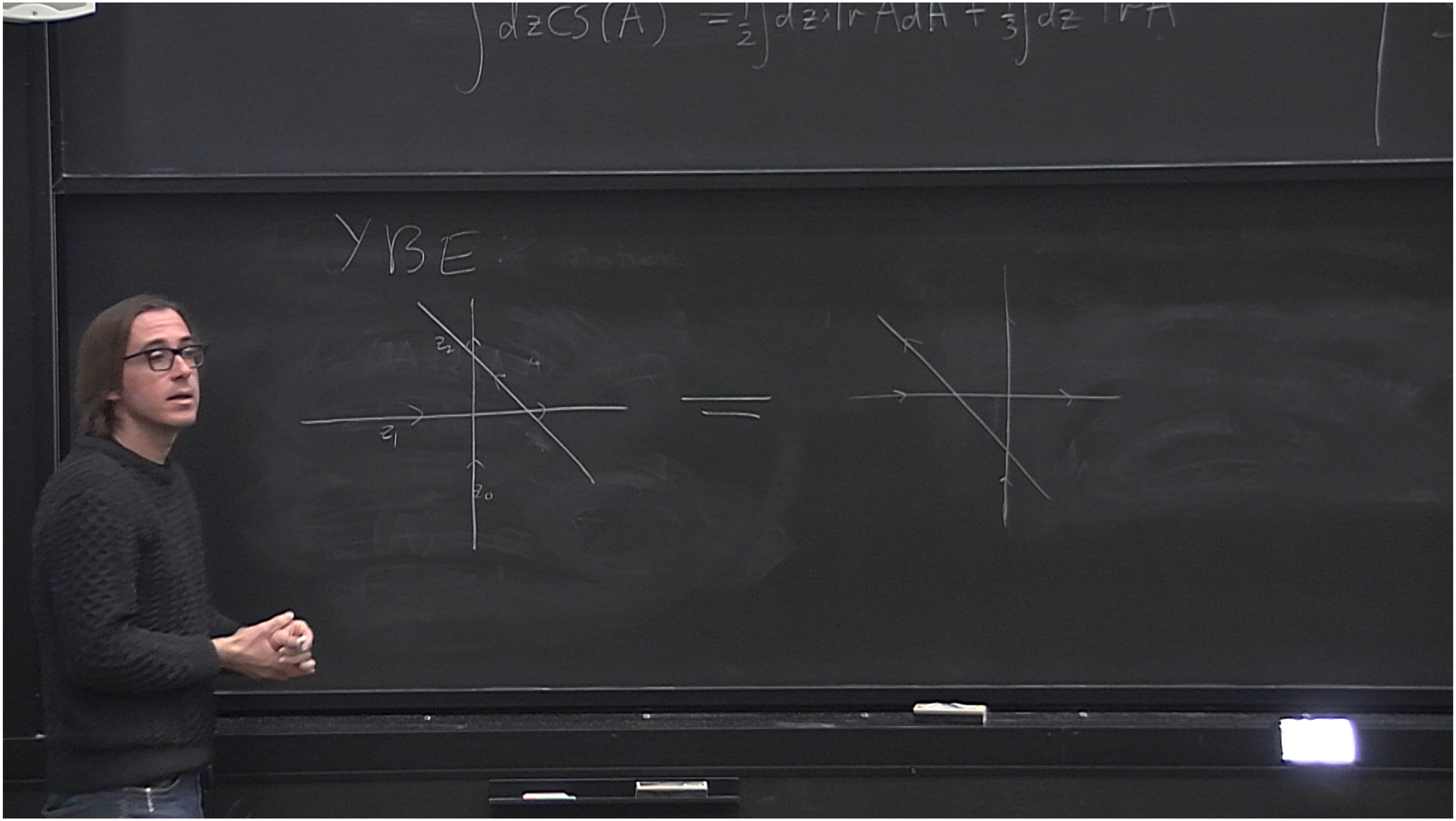
then we can't cancel the A_z.


k
j
z_1, l
z_0
i
blackboard
= w plane
$w = u + iv$
Scattering of states
$= R^{ij}_{kl}(z_0 - z_1)$
(if gl_n, $n=2$, 6-vertex R matrix)

First: why does the YBE hold for these scattering processes?

Answer Consider a Wilson line at $z = z_0$, in u-direction, at a fixed value of v, $v = v_0$


This is conserved.
Because
$F_{uv}(A) = 0$
Shifting v_0 doesn't
change the Wilson line
u
$v = v_0$

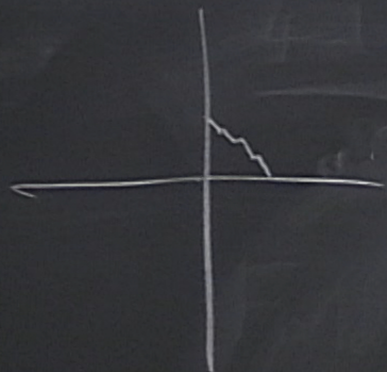

∫dz CS(A) = 1/2 ∫dz∧Tr AdA + 1/3 ∫dz Tr A
YBE:
z2
z1
z0

Leading order computation.

We can introduce a loop expansion param. $\hbar$ so that

$$R(z) = \mathrm{Id} + \frac{\hbar}{z}(t_a \otimes t_a) + O(\hbar^2)$$

t_a = orthonormal basis for the Lie algebra of gauge group.



$$A = A_u\,du + A_v\,dv + A_{\bar z}\,d\bar z$$

Choose the gauge where

$$\partial_z A_{\bar z} = 0$$

[Lorentz gauge

$$\partial_u A_u + \partial_v A_v + \partial_z A_{\bar z} = 0$$

If we rescale u-v plane, find]

In this gauge the propagator is

$$P = \frac{1}{2\pi i (z_1 - z_2)} (t_a \otimes t_a) \left(d(u_1-u_2)\, d(v_1-v_2) \right) \delta_{u_1=u_2}\, \delta_{v_1=v_2}$$